

$$E_{g_{max}} = mgh_{max}$$

$$E_{g_{curr}} = mgh$$

$$\Delta E_g = mg(h_{max} - h)$$

Assuming all gravitational potential energy transforms into kinetic energy, then

$$E_k = \frac{1}{2}mv^2 = \Delta E_g = mg(h_{max} - h)$$

$$v^2 = g(h_{max} - h)$$

$$v = \sqrt{2g(h_{max} - h)}$$

Let the distance traveled be x, then

$$x = \Delta t v = \Delta t \sqrt{2g(h_{max} - h)}$$

Since $\frac{||dp||}{||du||}$ represents the amount that p changes with respect a change in u, the traveled

distance in u would be $\frac{x}{\frac{||dp||}{||du||}} = \Delta t \frac{\sqrt{2g(h_{max} - h)}}{\frac{||dp||}{||du||}}$,

And thus we have $u_{new} = u_{curr} + \Delta t \frac{\sqrt{2g(h_{max} - h)}}{\frac{||dp||}{||du||}}$. Proved.