Multi-Period Optimal Power Flow: Temporal ADMM (tADMM) Formulation

Decomposition with T Single-Step Blocks

1 Problem Overview

The Temporal ADMM (tADMM) algorithm decomposes the multi-period optimal power flow problem into T single-step subproblems, each corresponding to one time period. The hope is to enable parallel computation and improved scalability while still retaining solution optimality.

1.1 Variable Color Coding

For convenience, we I'm using color coding for the variables to match the handwritten PDF formulation:

- $\mathbf{B_{t_0}}$: Local SOC variables for subproblem t_0 (blue)
- $\hat{\mathbf{B}}$: Global consensus SOC trajectory (red)
- $\mathbf{u_{t_0}}$: Local scaled dual variables for subproblem t_0 (green)

1.2 SOC Bound Definitions

$$\underline{B} = \mathrm{SOC}_{\mathrm{min}} \cdot E_{\mathrm{Rated}} \tag{1}$$
$$\overline{B} = \mathrm{SOC}_{\mathrm{max}} \cdot E_{\mathrm{Rated}} \tag{2}$$

2 tADMM Algorithm Structure

The algorithm alternates between three update steps:

2.1 Step 1: Primal Update (Blue Variables)

For each subproblem $t_0 \in \{1, 2, \ldots, T\}$:

$$\min_{P_{\mathrm{subs},t_0}, P_{B,t_0}, \mathbf{B_{t_0}}} \quad C_{t_0} \cdot P_{\mathrm{subs},t_0} + \frac{\rho}{2} \left\| \mathbf{B_{t_0}} - \hat{\mathbf{B}} + \mathbf{u_{t_0}} \right\|_2^2 \tag{3}$$

$$\text{s.t.} \quad \mathbf{B_{t_0}}[\mathbf{t_0}] - B_0 + P_{B,t_0} \cdot \Delta t = 0 \quad \text{if } t_0 = 1 \tag{4}$$

$$\mathbf{B_{t_0}}[\mathbf{t_0}] - \hat{\mathbf{B}}[\mathbf{t_0 - 1}] + P_{B,t_0} \cdot \Delta t = 0 \quad \text{if } t_0 > 1 \tag{5}$$

$$P_{\text{subs},t_0} + P_{B,t_0} - P_L[t_0] = 0 \tag{6}$$

$$-P_{B,R} \leq P_{B,t_0} \leq P_{B,R} \tag{7}$$

$$\underline{B} \leq \mathbf{B_{t_0}}[\mathbf{t}] \leq \overline{B}, \quad \forall t \in \{1, \ldots, T\} \tag{8}$$

2.2 Step 2: Consensus Update (Red Variables)

$$\hat{\mathbf{B}}[\mathbf{t}] = \text{clamp}\left(\frac{1}{T}\sum_{t_0=1}^{T}\left(\mathbf{B_{t_0}}[\mathbf{t}] + \mathbf{u_{t_0}}[\mathbf{t}]\right), \underline{B}, \overline{B}\right) \tag{9}$$

$$\forall t \in \{1, 2, \ldots, T-1\} \tag{10}$$

$$\hat{\mathbf{B}}[\mathbf{T}] = B_{T,\text{target}} \quad \text{(if terminal constraint exists)} \tag{11}$$

2.3 Step 3: Dual Update (Green Variables)

$$\mathbf{u_{t_0}}[\mathbf{t}] := \mathbf{u_{t_0}}[\mathbf{t}] + \left(\mathbf{B_{t_0}}[\mathbf{t}] - \hat{\mathbf{B}}[\mathbf{t}]\right) \tag{12}$$

$$\forall t_0 \in \{1, \ldots, T\}, \forall t \in \{1, \ldots, T\} \tag{13}$$

3 Convergence Criteria

The algorithm terminates when both residuals fall below specified thresholds:

3.1 Primal Residual (Consensus Violation)

$$\|r^k\|_2 = \left\|\text{vec}\left(\left\{\mathbf{B_{t_0}} - \hat{\mathbf{B}}\right\}_{t_0=1}^{T}\right)\right\|_2 \leq \epsilon_{\text{pri}} \tag{14}$$

3.2 Dual Residual (Consensus Change)

$$\|s^k\|_2 = \rho\left\|\hat{\mathbf{B}}^{\mathbf{k}} - \hat{\mathbf{B}}^{\mathbf{k-1}}\right\|_2 \leq \epsilon_{\text{dual}} \tag{15}$$

4 Final Solution Recovery

After convergence, the final power schedules are reconstructed from the consensus trajectory:

$$P_B^*[t] = \begin{cases} \frac{-(\hat{\mathbf{B}}[\mathbf{t}] - B_0)}{\Delta t} & \text{if } t = 1 \\ \frac{-(\hat{\mathbf{B}}[\mathbf{t}] - \hat{\mathbf{B}}[\mathbf{t-1}])}{\Delta t} & \text{if } t > 1 \end{cases} \tag{16}$$

$$P_{\text{Subs}}^*[t] = P_L[t] - P_B^*[t] \tag{17}$$

5 Algorithm Parameters

- **Penalty Parameter**: ρ (typically 1.0 to 10.0)
- **Primal Tolerance**: $\epsilon_{\text{pri}} = 10^{-3}$
- **Dual Tolerance**: $\epsilon_{\text{dual}} = 10^{-3}$
- **Maximum Iterations**: 1000