Multi-Period Optimal Power Flow:
Temporal ADMM (tADMM) Formulation

Decomposition with T Single-Step Blocks

1 Problem Overview

The Temporal ADMM (tADMM) algorithm decomposes the multi-period optimal power flow problem into T single-step subproblems, each corresponding to one time period. This approach enables parallel computation and improved scalability.

1.1 Variable Color Coding

Following the handwritten PDF formulation:

- $\mathbf{B_{t_0}}$: Local SOC variables for subproblem t_0 (blue)
- $\hat{\mathbf{B}}$: Global consensus SOC trajectory (red)
- $\mathbf{u_{t_0}}$: Local scaled dual variables for subproblem t_0 (green)

2 tADMM Algorithm Structure

The algorithm alternates between three update steps:

2.1 Step 1: Primal Update (Blue Variables)

For each subproblem $t_0 \in \{1, 2, \ldots, T\}$:

$$\min_{P_{\text{subs},t_0}, P_{B,t_0}, \mathbf{B_{t_0}}} \quad C_{t_0} \cdot P_{\text{subs},t_0} + \frac{\rho}{2} \left\| \mathbf{B_{t_0}} - \hat{\mathbf{B}} + \mathbf{u_{t_0}} \right\|_2^2 \tag{1}$$

$$\text{s.t.} \quad \mathbf{B_{t_0}}[\mathbf{t_0}] - B_0 + P_{B,t_0} \cdot \Delta t = 0 \quad \text{if } t_0 = 1 \tag{2}$$

$$\mathbf{B_{t_0}}[\mathbf{t_0}] - \hat{\mathbf{B}}[\mathbf{t_0 - 1}] + P_{B,t_0} \cdot \Delta t = 0 \quad \text{if } t_0 > 1 \tag{3}$$

$$P_{\text{subs},t_0} + P_{B,t_0} - P_L[t_0] = 0 \tag{4}$$

$$-P_{B,R} - P_{B,t_0} \leq 0 \tag{5}$$

$$P_{B,t_0} - P_{B,R} \leq 0 \tag{6}$$

$$\text{SOC}_{\min} \cdot E_{\text{Rated}} - \mathbf{B_{t_0}}[\mathbf{t}] \leq 0, \quad \forall t \in \{1, \ldots, T\} \tag{7}$$

$$\mathbf{B_{t_0}}[\mathbf{t}] - \text{SOC}_{\max} \cdot E_{\text{Rated}} \leq 0, \quad \forall t \in \{1, \ldots, T\} \tag{8}$$

2.2 Step 2: Consensus Update (Red Variables)

$$\hat{\mathbf{B}}[\mathbf{t}] = \text{clamp}\left(\frac{1}{T}\sum_{t_0=1}^{T}\left(\mathbf{B}_{\mathbf{t_0}}[\mathbf{t}] + \mathbf{u}_{\mathbf{t_0}}[\mathbf{t}]\right), \text{SOC}_{\min} \cdot E_{\text{Rated}}, \text{SOC}_{\max} \cdot E_{\text{Rated}}\right) \tag{9}$$

$$\forall t \in \{1, 2, \ldots, T-1\} \tag{10}$$

$$\hat{\mathbf{B}}[\mathbf{T}] = B_{T,\text{target}} \quad \text{(if terminal constraint exists)} \tag{11}$$

2.3 Step 3: Dual Update (Green Variables)

$$\mathbf{u}_{\mathbf{t_0}}[\mathbf{t}] := \mathbf{u}_{\mathbf{t_0}}[\mathbf{t}] + \left(\mathbf{B}_{\mathbf{t_0}}[\mathbf{t}] - \hat{\mathbf{B}}[\mathbf{t}]\right) \tag{12}$$

$$\forall t_0 \in \{1, \ldots, T\}, \forall t \in \{1, \ldots, T\} \tag{13}$$

3 Convergence Criteria

The algorithm terminates when both residuals fall below specified thresholds:

3.1 Primal Residual (Consensus Violation)

$$\|r^k\|_2 = \left\|\text{vec}\left(\left\{\mathbf{B}_{\mathbf{t_0}} - \hat{\mathbf{B}}\right\}_{t_0=1}^{T}\right)\right\|_2 \leq \epsilon_{\text{pri}} \tag{14}$$

3.2 Dual Residual (Consensus Change)

$$\|s^k\|_2 = \rho\left\|\hat{\mathbf{B}}^{\mathbf{k}} - \hat{\mathbf{B}}^{\mathbf{k-1}}\right\|_2 \leq \epsilon_{\text{dual}} \tag{15}$$

4 Final Solution Recovery

After convergence, the final power schedules are reconstructed from the consensus trajectory:

$$P_B^*[t] = \begin{cases} \frac{-\left(\hat{\mathbf{B}}[\mathbf{t}] - B_0\right)}{\Delta t} & \text{if } t = 1 \\ \frac{-\left(\hat{\mathbf{B}}[\mathbf{t}] - \hat{\mathbf{B}}[\mathbf{t-1}]\right)}{\Delta t} & \text{if } t > 1 \end{cases} \tag{16}$$

$$P_{\text{Subs}}^*[t] = P_L[t] - P_B^*[t] \tag{17}$$

5 Algorithm Parameters

- **Penalty Parameter**: ρ (typically 1.0 to 10.0)
- **Primal Tolerance**: $\epsilon_{\text{pri}} = 10^{-3}$
- **Dual Tolerance**: $\epsilon_{\text{dual}} = 10^{-3}$
- **Maximum Iterations**: 1000