Optimal Operation of Radial Distribution Systems Using Extended Dynamic Programming

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***Abstract*—An extended dynamic programming (EDP) approach is developed to optimize the ac steady-state operation of radial electrical distribution systems (EDS). Based on the optimality principle of the recursive Hamilton–Jacobi–Bellman equations, the proposed EDP approach determines the optimal operation of the EDS by setting the values of the controllable variables at each time period. A suitable definition for the stages of the problem makes it possible to represent the optimal ac power flow of radial EDS as a dynamic programming problem, wherein the "curse of dimensionality" is a minor concern, since the number of state and control variables at each stage is low and the time complexity of the algorithm grows linearly with the number of nodes of the EDS. The proposed EDP is applied to solve the economic dispatch of the DG units installed in a radial EDS. The effectiveness and the scalability of the EDP approach is illustrated using real-scale systems and comparisons with commercial programming solvers. Finally, generalizations to consider other EDS operation problems are also discussed.**

***Index Terms*—Distributed generation, economic dispatch problem, extended dynamic programming (EDP), optimization of electrical distribution systems (EDS).**

NOTATION

Functions

$f\left(P_{k,t}^{DG}\right)$	Cost of the DG operation
$e_k\left(\mathbf{x}_k, \mathbf{u}_k\right)$	Value of the objective function at node k

Parameters

c_2	Quadratic coefficient of the DG cost function
c_1	Linear coefficient of the DG cost function
c_0	Constant coefficient of the DG cost function
$P_{k,t}^L$	Active demand at node k and time t
$\underline{P_k}$	Lower active flow limit at node k
$\overline{P_k}$	Upper active flow limit at node k
$\underline{P_k}^{DG}$	Lower DG capacity limit at node k
$\overline{P_k}^{DG}$	Upper DG capacity limit at node k
$Q_{k,t}^L$	Reactive demand at node k and time t
$\underline{Q_k}$	Lower reactive flow limit at node k
$\overline{Q_k}$	Upper reactive flow limit at node k
R_k	Resistance of the branch connecting node k
X_k	Reactance of the branch connecting node k
$\underline{V}$	Lower voltage magnitude limit
$\overline{V}$	Upper voltage magnitude limit
α_t	Cost of energy losses at each time period t
Δ_t	Duration of each time period t
ΔP_k^{DG}	Discretization of the DG injection at node k
Δu_k	Discretization of the control variable u_k
Δx_k	Discretization of the state variable x_k

Sets

N	Set of nodes
A_{k+1}	Set of downstream nodes connected to node k
T	Set of time periods

Variables

$P_{k,t}$	Active power flow entering at node k and time t
$P_{k,t}^{DG}$	DG power injected at node k and time t
$Q_{k,t}$	Reactive power flow entering node k and time t
u_k^1, u_k^2	Control variables at node k
$V_{k,t}$	Voltage magnitude at node k and time t
x_k^1, x_k^2, x_k^3	State variables at node k
$y_{k,t}^{DG}$	Binary variable that represents the status of the DG unit at node k and time t

Vectors and Matrices

$\mathbf{F}\left(\mathbf{x}_k\right)$	Multi-dimensional matrix that contains all the optimal costs at node k
$\mathbf{u}_k$	Vector of control variables at node k
$\mathbf{x}_k$	Vector of state variables at node k
$\mathbf{\Pi}\left(\mathbf{x}_k\right)$	Multi-dimensional matrix that contains all the optimal decisions at node k

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I. INTRODUCTION

ENERGY management policies and the economic operation of electrical distribution systems (EDS) have attracted a lot of attention in the last few decades, due to the advent of dispatchable distributed generation (DG) resources, such as microturbines, fuel cells, and storage systems [1], and the increasing investment in distributed automation technologies and energy management systems. The optimal operation of radial EDS derives from the solution of the generalized optimal power flow (OPF) [2]. The OPF is a nonconvex, nonlinear problem, due to the nature of the equations used to represent the operation of AC radial networks and the quadratic function used to represent the operational costs. Furthermore, a set of operational constraints might be considered throughout the optimization process, and binary variables can be used to represent ON-OFF/YES-NO decisions made throughout the operating horizon. Thus, the optimal AC operation of EDS can lead to mixed-integer nonlinear programming (MINLP) problems [3].

Nowadays, various MINLP commercial solvers are available, e.g., BONMIN [4] or KNITRO [5] which cannot guarantee optimality if the MINLP model is non-convex. Other MINLP solvers, such as BARON [6] and COUENNE [7], use Branch-and-Bound techniques to solve most non-convex MINLP problems, but they require high computational times to converge to an optimal solution. Thus, since the early days of the OPF research, most authors have typically adopted four different ways to deal with OPF in EDS [8]: a) using linearization techniques to approximate the OPF into a mixed-integer linear programming problem; b) using heuristic or metaheuristic approaches that provide quality (although, not optimal) solutions; c) using alternative mathematical representations to convexify the OPF problem; and d) decomposing the OPF into a nonlinear subproblem that deals with the AC power flow variables and an integer linear subproblem that deals with the binary decision variables [9].

Mixed-integer linear programming (MILP) models are flexible and easy to reproduce and represent using commercial programming languages, such as AMPL [10] or GAMS. Moreover, MILP can be solved via classical optimization techniques [11], [12]. Nonetheless, MILP models are approximations of the original nonlinear OPF, and depending on the characteristics of the problem, linearized constraints can result in imprecise solutions. Furthermore, the computational time of MILP solvers is highly influenced by the size of the problem and the number of binary decision variables.

With regard to metaheuristics, many instances of the OPF in EDS have been efficiently solved using these techniques [13]–[16]. Metaheuristics are optimization algorithms that use specialized search strategies, mostly based on randomly generated functions, in order to provide quality solutions to difficult mathematical programming problems. The main advantage of metaheuristics is its scalability factor, which makes it possible to solve MINLP with many control variables, within a reasonable amount of time, and with limited computational resources. However, global optimality is not guaranteed by any of them, and in general, metaheuristics are neither flexible nor easy to reproduce or adapt if the conditions of the problem change.

More recently, many authors have addressed the use of equivalent convex relaxation in order to solve the OPF in radial AC networks [17]. Based on the works presented by authors in [18] and [19], it has been proved that the OPF problem can be expressed as a convex, second-order programming (SOP) problem, and it can be efficiently solved using SOP solvers, such as CPLEX [20]. Although the convexification of OPF guarantees optimality, its scalability factor considering binary variables is very poor, and it is impractical for large-size EDS.

The decomposition of the OPF problem is also a popular technique, since it improves the computational complexity of the problem [21]. However, as with metaheuristics, the decomposition does not guarantee optimality of the final solution. Thus, to the best of our knowledge, no works have yet explored the use of dynamic programming (DP) as an optimization technique to efficiently solve OPF problems in practical AC networks, considering multiple dispatchable DG units, active and reactive power flow assessment, and operational constraints, such as nodal voltages and branch flow limits.

To overcome these issues, this paper presents an extended dynamic programming (EDP) approach to optimize the AC steady-state operation of radial EDS. The EDP approach was originally proposed by authors in [22] to solve the capacitor allocation problem in radial EDS, and first adapted to operate simplified, active, small-scale networks by authors in [23]. In this paper, the EDP approach has been generalized to solve OPF problems in radial EDS. The main advantage of the proposed EDP is that optimality is guaranteed by a generalization of the Hamilton-Jacobi-Bellman equation, regardless of the objective function's nature [24]. A complete AC load flow formulation is used to estimate the steady-state operating point of the network and the optimal value of the decision variables at each time period. In this case, the "curse of dimensionality" is a minor concern since the number of state and control variables at each stage is low and the computational complexity of the algorithm grows linearly with the number of nodes of the EDS. Throughout this paper, we focus on the economic dispatch problem, which is an instance of the OPF, wherein the grid's owner is interested in optimizing the operation of the dispatchable generation resources. However, as discussed in Section IV, the EDP can be extended to other operational problems. The effectiveness and scalability of the proposed EDP approach are compared with the solutions provided by mixed-integer nonlinear commercial solvers.

The main contributions of this paper are as follows:

1) A novel extension of the DP approach is proposed to solve the optimal AC operation of radial EDS, considering active and reactive power flows and the physical constraints of the network.
2) The proposed EDP approach makes it possible to solve the OPF of radial EDS as a DP problem, wherein optimality is guaranteed at each time period and the computational complexity of the algorithm grows linearly with the size of the network.

II. The Economic Dispatch Problem

The mathematical model of the economic dispatch problem is presented in this section as an instance of the optimal AC operation of radial EDS. The objective is to familiarize the reader with the formulation of a practical operating problem and the mathematical notation used to represent it. However, as discussed in Section IV-D, other operational problems can be represented and solved using the same methodology proposed in this paper.

The mathematical optimization model used to represent the economic load dispatch problem is given by the mixed-integer nonlinear optimization model in (1)–(9). The model is based on the *DistFlow* formulation originated in [25], [26], and adapted in [22]. Note that if no DG units are connected at node k, then both DG limits ($\underline{P}_k^{\mathrm{DG}}$ and $\overline{P}_k^{\mathrm{DG}}$) and the binary decision variable are set to zero in (1) and (2).

$$\min \quad \sum_{t \in T} \sum_{k \in N} \left[y_{k,t}^{\mathrm{DG}} f\left(P_{k,t}^{\mathrm{DG}}\right) + \alpha_t \Delta_t R_k \left(\frac{P_{k,t}^2 + Q_{k,t}^2}{V_{k,t}^2} \right) \right] \tag{1}$$

subject to:

$$\sum_{j \in A_{k+1}} \left[P_{k+1,j,t} + R_{k+1,j} \left(\frac{P_{k+1,j,t}^2 + Q_{k+1,j,t}^2}{V_{k+1,j,t}^2} \right) \right] = P_{k,t} - P_{k,t}^{\mathrm{L}} + y_{k,t}^{\mathrm{DG}} P_{k,t}^{\mathrm{DG}}; \quad \forall k \in N, t \in T \tag{2}$$

$$\sum_{j \in A_{k+1}} \left[Q_{k+1,j,t} + X_{k+1,j} \left(\frac{P_{k+1,j,t}^2 + Q_{k+1,j,t}^2}{V_{k+1,j,t}^2} \right) \right] = Q_{k,t} - Q_{k,t}^{\mathrm{L}}; \quad \forall k \in N, t \in T \tag{3}$$

$$V_{k+1,j,t}^2 = V_{k,t}^2 - 2(R_{k+1,j} P_{k+1,j,t} - X_{k+1,j} Q_{k+1,j,t}) - \left(R_{k+1,j}^2 + X_{k+1,j}^2\right) \left[\frac{P_{k+1,j,t}^2 + Q_{k+1,j,t}^2}{V_{k+1,j,t}^2} \right] \quad \forall k \in N, j \in A_{k+1}, t \in T | A_{k+1} \neq \emptyset \tag{4}$$

$$\underline{V} \leq V_{k,t} \leq \overline{V} \qquad \forall k \in N, t \in T \tag{5}$$

$$\underline{P}_k \leq P_{k,t} \leq \overline{P}_k \qquad \forall k \in N, t \in T \tag{6}$$

$$\underline{Q}_k \leq Q_{k,t} \leq \overline{Q}_k \qquad \forall k \in N, t \in T \tag{7}$$

$$\underline{P}_k^{\mathrm{DG}} \leq P_{k,t}^{\mathrm{DG}} \leq \overline{P}_k^{\mathrm{DG}} \qquad \forall k \in N, t \in T \tag{8}$$

$$y_{k,t}^{\mathrm{DG}} \in \{0, 1\} \qquad \forall k \in N, t \in T \tag{9}$$

The mixed-integer non-linear objective function in (1) aims at minimizing the operational cost of the DG units and the total active power losses of the EDS. The function $f(P_{k,t}^{\mathrm{DG}})$ represents the quadratic generation cost of each dispatchable DG unit, given by (10). Other objectives can be minimized, e.g., generation cost at the substation, improvement of voltage profile, number of DG operations. However, the minimization of losses is an interesting choice because it is a non-linear non-convex function and, together with the minimization of $f(P_{k,t}^{\mathrm{DG}})$, they produce a conflicted, multipurpose, objective function.

$$f\left(P_{k,t}^{\mathrm{DG}}\right) = c_2 \left(\Delta_t P_{k,t}^{\mathrm{DG}}\right)^2 + c_1 \Delta_t P_{k,t}^{\mathrm{DG}} + c_0 \quad \forall k \in N, t \in T \tag{10}$$

Constraints (2) and (3) represent the active and reactive power flow balance equations, defined for each node $k \in N$ (including the substation node) and time period $t \in T$. As shown in Fig. 1, the active and reactive power flows leaving node k are identified with the subscript $k+1$, and each individual downstream branch is an element of the set $j \in A_{k+1}$. Note that flows entering node k are equal to the summation of the outflows at each branch $j \in A_{k+1}$, considering losses. If node k does not have downstream nodes connected to it, it is a *leaf node*, and $A_{k+1} = \emptyset$. Note that constraint (2) includes the DG power

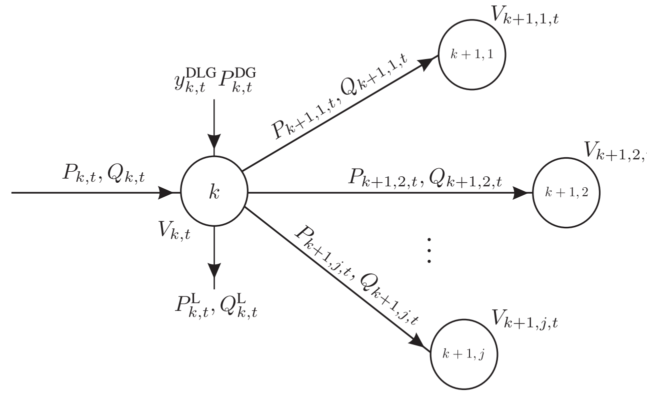

Fig. 1. Active and reactive flow balance at each node.

injection $P_{k,t}^{\mathrm{DG}}$ multiplied by the binary decision variable $y_{k,t}^{\mathrm{DG}}$, and the active power flow $P_{k,t}$ entering at node k. The binary variable $y_{k,t}^{\mathrm{DG}}$ determines whether the DG unit at node k is operating or not. It is important to notice that $P_{k,t}$ can take negative values if the DG injection is higher than the overall demand. Constraint (4) represents the application of Kirchhoff's voltage law for each non-leaf node $k \in N$ and time period $t \in T$. For nodes with voltage regulation, additional constraints must be set in order to specify voltage values. Otherwise, no additional voltage constraints would be needed for leaf nodes, i.e., for $k \in N | A_{k+1} = \emptyset$. Constraints (5)–(8) represent the limits of the voltage magnitudes, active and reactive power flows, and DG units, respectively. Constraint (9) determines the binary nature of the decision variable $y_{k,t}^{\mathrm{DG}}$.

The problem formulated in (1)–(9) can be decoupled into independent subproblems, one for each time-step $t \in T$. After the solution of each independent subproblem is determined, the objective function in (1) can be computed by (11).

$$\sum_{t \in T} \left\{ \min \left[\sum_{k \in N} y_{k,t}^{\mathrm{DG}} f\left(\Delta_t P_{k,t}^{\mathrm{DG}}\right) + \alpha_t \Delta_t R_k \left(\frac{P_{k,t}^2 + Q_{k,t}^2}{V_{k,t}^2} \right) \right] \right\} \tag{11}$$

III. The Extended Dynamic Programming Approach

Since the economic dispatch problem can be decoupled in independent subproblems, one for each time period t, as in (11), the subscript t is dropped throughout this section in order to simplify the notation. In general, dynamic programming is mainly used to solve problems in which the state variables are coupled in time; in this case, since an independent subproblem is solved for each time period, the coupling between the state variables is physical (not temporal) and given by the branches of the EDS. Thus, instead of a time domain DP approach, the proposed EDP uses the radial topology of the EDS to link each stage of the problem with the others.

In the proposed EDP approach, each node of the EDS k is defined as a *stage*. A three-dimensional vector ($\mathbf{x}_k$) configures the *state* variable space at each node (i.e., at each stage). As defined in (12), the first dimension of $\mathbf{x}_k$ represents the active power flow entering at node k, the second represents the reactive power flow entering at node k, and the third represents the voltage magnitude at node k. On the other hand, as shown in (13), the two-dimensional vector $\mathbf{u}_k$ contains the *control* variables of the problem, i.e., the power injection of the DG unit at node k, given by $P_k^{\rm DG}$, and the binary decision variable $y_k^{\rm DG}$. If the reactive injections of the DG units are significant, they can be included in $\mathbf{u}_k$ as a new control variable.

$$\mathbf{x}_k = \begin{bmatrix} x_k^1 \\ x_k^2 \\ x_k^3 \end{bmatrix} = \begin{bmatrix} P_k \\ Q_k \\ V_k \end{bmatrix} \tag{12}$$

$$\mathbf{u}_k = \begin{bmatrix} u_k^1 \\ u_k^2 \end{bmatrix} = \begin{bmatrix} P_k^{\rm DG} \\ y_k^{\rm DG} \end{bmatrix} \tag{13}$$

Since $\mathbf{u}_k$ is defined for each node, new DG units can be connected to the EDS without increasing the dimension of the problem. Thus, the computational burden does not become an obstacle if new DG units are connected into the system, as in other DP approaches [27]. Note that, as shown in (14), the $\mathbf{u}_k$ discretization set can be simplified by combining the binary with the continuous decision variable, where $\Delta P_k^{\rm DG}$ is the discretization used for the DG's injection.

$$P_k^{\rm DG} \in \left\{0, \underline{P}_k^{\rm DG}, \underline{P}_k^{\rm DG} + \Delta P_k^{\rm DG}, \ldots, \overline{P}_k^{\rm DG} - \Delta P_k^{\rm DG}, \overline{P}_k^{\rm DG}\right\} \tag{14}$$

Based on the foregoing definitions, the economic dispatch problem can now be formulated as in (15)–(24), which is a standard DP representation [24].

$$\min \sum_{k \in N} e_k\left(\mathbf{x}_k, \mathbf{u}_k\right) \tag{15}$$

subject to:

$$\begin{aligned} e_k\left(\mathbf{x}_k, \mathbf{u}_k\right) = u_k^2 \left(c_2 \left(\Delta_t u_k^1\right)^2 + c_1 \Delta_t u_k^1 + c_0\right) \\ + \alpha_t \Delta_t R_k \left[\frac{\left(x_k^1\right)^2 + \left(x_k^2\right)^2}{\left(x_k^3\right)^2}\right] \end{aligned} \tag{16}$$

$$\begin{aligned} \sum_{j \in A_{k+1}} \left\{ x_{k+1,j}^1 + R_{k+1,j} \left[\frac{\left(x_{k+1,j}^1\right)^2 + \left(x_{k+1,j}^2\right)^2}{\left(x_{k+1,j}^3\right)^2}\right]\right\} \\ = x_k^1 - P_k^{\rm L} + u_k^1 u_k^2 \end{aligned} \tag{17}$$

$$\begin{aligned} \sum_{j \in A_{k+1}} \left\{ x_{k+1,j}^2 + X_{k+1,j} \left[\frac{\left(x_{k+1,j}^1\right)^2 + \left(x_{k+1,j}^2\right)^2}{\left(x_{k+1,j}^3\right)^2}\right]\right\} \\ = x_k^2 - Q_k^{\rm L} \end{aligned} \tag{18}$$

$$\begin{aligned} &\left(x_{k+1,j}^3\right)^4 - \left[\left(x_k^3\right)^2 - 2\left(R_{k+1} x_{k+1,j}^1 - X_{k+1} x_{k+1,j}^2\right)\right] \\ &\times \left(x_{k+1,j}^3\right)^2 + \left(R_{k+1}^2 + X_{k+1}^2\right)\left[\left(x_{k+1,j}^1\right)^2 + \left(x_{k+1,j}^2\right)^2\right] = 0 \\ &j \in A_{k+1} | A_{k+1} \neq \emptyset \end{aligned} \tag{19}$$

$$\underline{x_k^1} \leq x_k^1 \leq \overline{x_k^1} \tag{20}$$

$$\underline{x_k^2} \leq x_k^2 \leq \overline{x_k^2} \tag{21}$$

$$\underline{x_k^3} \leq x_k^3 \leq \overline{x_k^3} \tag{22}$$

$$\underline{u_k^1} \leq u_k^1 \leq \overline{u_k^1} \tag{23}$$

$$u_k^2 \in \{0, 1\} \tag{24}$$

The term $e_k\left(\mathbf{x}_k, \mathbf{u}_k\right)$ in (15) represents the total cost at each node (stage) of the problem. The state variables $x_{k+1,j}^1$ and $x_{k+1,j}^2$ in (17) and (18) represent the active and reactive power flows at the downstream nodes $k+1, j$. Equation (19) represents the voltage drop at each branch, where x_k^3 and $x_{k+1,j}^3$ are the voltage magnitudes at nodes k and $k+1, j$, respectively.

In order to solve the optimization problem in (15)–(24) using the proposed EDP approach, a preprocessing procedure, a backward process, and a forward process are executed, as explained in the following subsections.

A. Preprocessing Procedure

Initially, the EDS parameters in p.u. are discretized using a proper discretization value for each state variable given by $\Delta \mathbf{x}_k$ and for the control variables $\Delta \mathbf{u}_k$. Then, a renumbering algorithm is executed in order to organize the number of each node, beginning from the substation ($k \rightarrow 0$) and continuing to the farthest node of the system ($k \rightarrow N$) as in [28].

B. Backward Process

Starting at the farthest node of the system, the backward process of the proposed EDP determines the set of optimal values for the discretized state and control variables at each node, via the recursive Hamilton-Jacobi-Bellman equations in (25)–(26) [29]. $\mathbf{F}\left(\mathbf{x}_k\right)$ is a multi-dimensional matrix that contains all the optimal costs for each discretization of $\mathbf{x}_k$. As shown in (25), each element of $\mathbf{F}\left(\mathbf{x}_k\right)$ represents the minimization over the control variables $\mathbf{u}_k$ of the objective function e_k, plus the future cost $\mathbf{F}\left(\mathbf{x}_k\right)$. The optimal policy $\mathbf{\Pi}\left(\mathbf{x}_k\right)$ in (26) saves the optimal decisions made at stage k.

$$\mathbf{F}\left(\mathbf{x}_k\right) = \min_{\mathbf{u}_k}\left[e_k\left(\mathbf{x}_k, \mathbf{u}_k\right) + \mathbf{F}\left(\mathbf{x}_{k+1}\right)\right] \tag{25}$$

$$\mathbf{\Pi}\left(\mathbf{x}_k\right) = \mathbf{u}_k^* = \arg\left\{\min_{\mathbf{u}_k}\left[e_k\left(\mathbf{x}_k, \mathbf{u}_k\right) + \mathbf{F}\left(\mathbf{x}_{k+1}\right)\right]\right\} \tag{26}$$

At each node k, the calculation of (25) and (26) depends on the set A_{k+1} as follows:

1) If $A_{k+1} = \emptyset$: This means that k is a leaf node and no future state exists; thus, the element $\mathbf{F}\left(\mathbf{x}_{k+1}\right) = 0$. In order to fulfill the $\mathbf{F}\left(\mathbf{x}_k\right)$ and $\mathbf{\Pi}\left(\mathbf{x}_k\right)$ matrices, all the discretized values of $\mathbf{x}_k$ and $\mathbf{u}_k$ are compared, and the optimal decision ($\mathbf{u}_k^*$) is found, using an exhaustive search.

2) If $A_{k+1} \neq \emptyset$: In this case, the optimal future cost $\mathbf{F}\left(\mathbf{x}_{k+1}\right)$ in (25) and (26) is calculated using (27). An intuitive derivation of (27) is presented in the Appendix. The optimization problem in (27) guarantees that only the optimal power flows and voltage magnitudes at each child node $k+1, j$ will

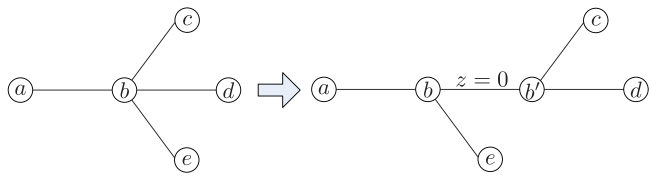

Fig. 2. Downstream branch transformation.

be used when calculating the optimal costs and decisions made at node k.

$$\mathbf{F}\left(\mathbf{x}_{k+1}\right) = \begin{cases} \min\limits_{\forall \mathbf{x}_{k+1,j}} \left[\sum\limits_{j \in A_{k+1}} \mathbf{F}\left(\mathbf{x}_{k+1,j}\right) \right] \\ \text{s.t.:} \\ \text{(17)–(24)} \end{cases} \tag{27}$$

Some comments related to (27) are in order:

1) The expression in (27) is solved using an enumerative search procedure that compares all the discretized values of the future state variables ($\mathbf{x}_{k+1,j}$), and it determines the optimal future costs and state variables that satisfy constraints (17)–(24).
2) Constraint (19) can be expressed as a quadratic function given by $y^4 + ay^2 + b$, and the roots are found using the standard quadratic formula. The solution of y is selected based on the EDS voltage limits.
3) In order to enhance the algorithm's performance, the nodes with three or more downstream branches can be replaced by a series of nodes with only two branches, as shown in Fig. 2. This transformation reduces the number of future state variables and makes (27) easier to compute. The transformation is done during the preprocessing procedure.

C. Forward Process

Once the backward process is completed, the optimal value of the state and control variables at each node can be recovered using the multi-dimensional matrices $\mathbf{F}\left(\mathbf{x}_k\right)$ and $\mathbf{\Pi}\left(\mathbf{x}_k\right)$, according to the operation at the substation node (node $k = 0$). Note that, if $\mathbf{F}\left(\mathbf{x}_0\right)$ is empty, then the problem has no feasible solution. The optimal operation at the substation is given by the set of state variables that produces the minimum present cost, as indicated by the expression in (28).

$$\mathbf{x}_0^* = \arg\left\{\min\left[\mathbf{F}\left(\mathbf{x}_0\right)\right]\right\} \tag{28}$$

The subsequent state variables are calculated using (17)–(19), and the optimal policy function $\mathbf{\Pi}\left(\mathbf{x}_k\right)$. Whenever a bifurcation is found during the forward process, the optimal state variables of nodes $j \in A_{k+1}$ are recovered from the solution obtained after solving the optimization problem (27) at node k.

D. Algorithm

The algorithm shown below summarizes the proposed EDP approach that is used to solve the economic dispatch problem in radial EDS.

Algorithm 1: Extended Dynamic Programming Approach.

```
1:  procedure PRE-PROCESSING PROCEDURE
2:  for each t ∈ T do
3:    for each k ∈ {N ... 0} do     ▷ Backward Process
4:      for each discrete value of x_k do
5:        for each discrete value of u_k do
6:          if A_{k+1} = ∅ then
7:            F(x_{k+1}) = 0
8:          else if A_{k+1} ≠ ∅ then
9:            F(x_{k+1}) ← (27)
10:         F(x_k) ← (25)
11:         Π(x_k) ← (26)
12:   x_0^* ← (28)
13:   for each k ∈ {1 ... N} do     ▷ Forward Process
14:     Use (17)–(19), and Π(x_k) to recover the optimal
         injections at each DG unit
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Note that the algorithm solves an independent DP instance for each time period $t \in T$. Since only three space variables are used for each stage of the problem, the computational complexity and the size of $\mathbf{F}\left(\mathbf{x}_k\right)$ and $\mathbf{\Pi}\left(\mathbf{x}_k\right)$ are constants, and they are given by the number of discrete values used to represent each variable within its limits. Moreover, the $\mathbf{u}_k$ loop in line 5 is only computed if node k has a dispatchable DG unit installed; otherwise, $\mathbf{\Pi}\left(\mathbf{x}_k\right) = 0$. Thus, the computational complexity of the algorithm grows linearly with the number of nodes (N), as defined by the backward and forward process [22].

IV. Simulation Results and Discussions

To shown the effectiveness of the proposed EDP approach, three case studies are presented. First, a deterministic simulation is performed using the radial EDS shown in Fig. 3. Then, uncertainties in the loads are considered using Monte Carlo simulations. Finally, the performance of the EDP algorithm is assessed for large-size networks.

The EDP algorithm was developed in C++ and executed using a discretization of 0.1 p.u. for the active and reactive power flows and injections (i.e., $\Delta x_k^{1,2} = 0.1$ and $\Delta u_k^1 = 0.1$) and 0.01 p.u. for voltage magnitudes (i.e., $\Delta x_k^3 = 0.01$). The maximum values of the active and reactive power flows are 5 and 3 p.u., respectively. The time period is $\Delta_t = 1$ hour. All simulations were driven in a workstation with an Intel Core processor i7-6700 (3.40 GHz), and 8.00 GB of RAM.

A. Deterministic Case

In the EDS shown in Fig. 3, two DG units are available, DG_1 and DG_2, located at nodes 48 and 43, respectively. Electrical data, such as line impedance and average active and reactive power loads are available in [30]. Daily load profiles have been established using the procedures in [31] for commercial and industrial loads. Information related to the DG units and for the EDS are listed in Table I and Table II, respectively.

1) Optimal Dispatch: The active power dispatches obtained for both DG units are shown in Fig. 4. According to these results,

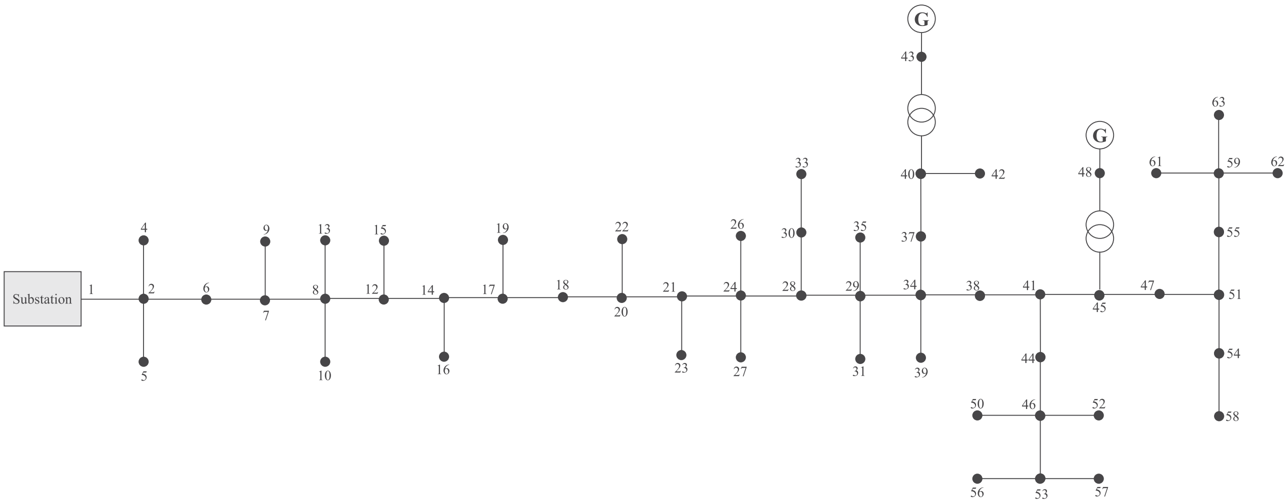

Fig. 3. MV Distribution System.

TABLE I
DG UNITS INFORMATION

$\overline{P}^{\mathrm{DG}}$	5.0	MV
$\underline{P}^{\mathrm{DG}}$	0.0	MV
c_2	4.0	USD$/MWh^2
c_1	10.0	USD$/MWh
c_0	5.0	USD$

TABLE II
EDS INFORMATION

α_t	600	USD$/MWh
S_b	5.0	MVA
V_b	25	kV
$\overline{V}$	1.05	p.u.
$\underline{V}$	0.95	p.u.

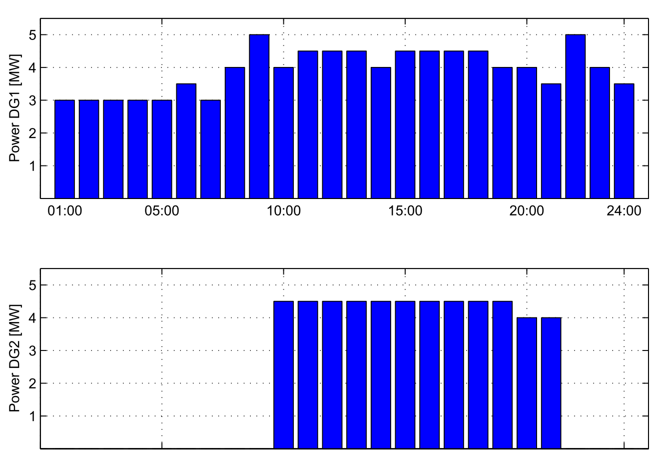

Fig. 4. Optimal dispatches of the DG units.

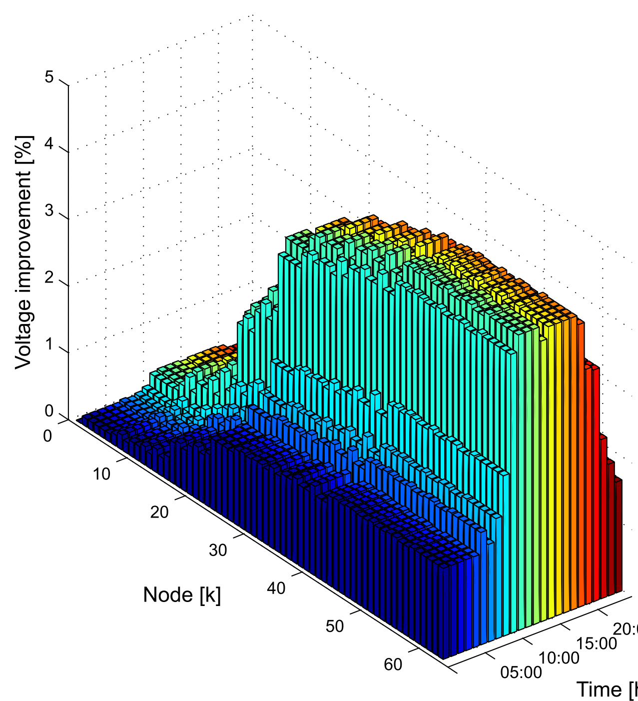

Fig. 5. Voltage improvement profile of the EDS. Deterministic case.

the DG_1 and DG_2 units dispatched approximately 111.4 MWh and 66.3 MWh throughout the day. If no DG units were considered, the substation would supply a total amount of energy equal to 409.69 MWh. Thus, if the optimal dispatch is used, a reduction of 46.34 % of generated energy at the substation can be achieved.

With regard to the voltage magnitudes, Fig. 5 shows the voltage profile improvement for each node and time period. Voltage improvement is assessed based on the operation of the same system without DG operation. According to Fig. 5, a maximum improvement of approximately 4 % can be achieved near to peak periods. This improvement is significant in the farthest nodes of

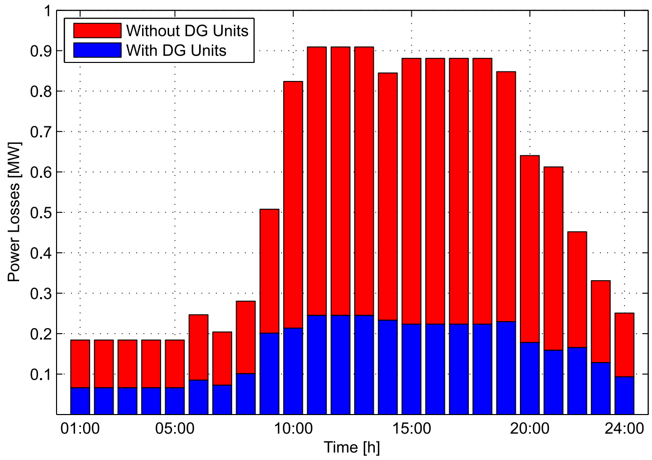

Fig. 6. Total active power losses reduction.

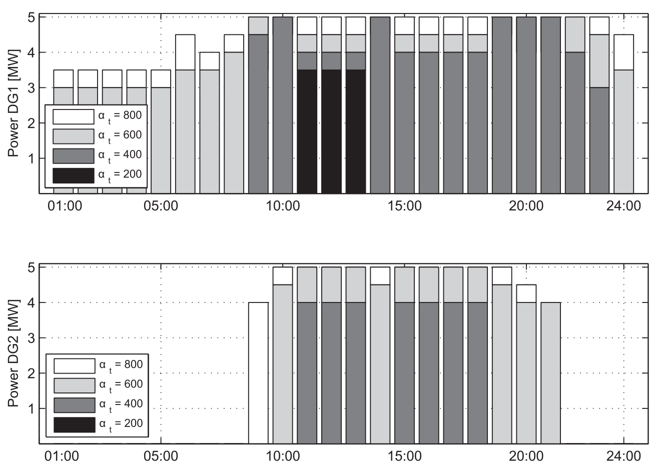

Fig. 7. Sensitivity analysis: $\alpha_t = 200$, 400, 600, and 800 USD$/MWh.

the EDS and in the neighbor nodes of the DG units, due to the influence of the active power supplied by the DG units.

Finally, Fig. 6 compares the hourly power losses of the EDS with and without the DG dispatch. The energy losses without DG units reaches 18.15 MWh. When the DG units are dispatched, a total loss reduction of 66.7 % can be achieved. This reduction is a consequence of the distributed power injections and the improvement of the voltage profile.

2) Sensitivity Analysis: Based on the flexibility of the proposed EDP approach, various sensitivity analyses can be carried out. Constraints, such as voltage magnitude limits or circuit termal and generation capacities can be tightened; or the location, number, and parameters of the dispatchable DG units can be modified. A complementary analysis is the impact of the energy prices in the DG operation. Since the cost of energy can be a highly volatile and uncertain parameter, Fig. 7 shows the resulting dispatch of both DG units of the EDS in Fig. 3 for various energy prices α_t.

The sensitivity analysis in Fig. 7 shows that the DG dispatch is highly sensitive to the energy prices, especially for DG_1, which is farther from the substation than DG_2. Furthermore, as shown

TABLE III
PERFORMANCE COMPARISON FOR THE 63-NODE EDS

Solver	f [USD$]	Time [s]	Optimal Solution
EDP	1043.0	2.7	Global
CPLEX*	1043.0	21.2	Global
BONMIN	1043.0	10.0	Local
COUENNE	1043.0	287.8	Global

*Using a convex second-order conic model.

TABLE IV
TRADE-OFF BETWEEN THE DISCRETIZATION OF THE VARIABLES AND THE QUALITY OF THE SOLUTION

$\Delta x_k^{1,2}$ and Δu_k^1 [p.u.]	Δx_k^3 [p.u.]	error [%]	Time [s]
0.1	0.1	75.0	0.4
0.1	**0.01**	**2.71**	**3.5**
0.01	0.01	2.09	300.1
0.01	0.001	1.40	6870.1

in Fig. 7, maximum distributed generation is dispatched for both units when $\alpha_t > 600$ USD$/MW at peak hours.

3) Performance of the Algorithm: In order to assess the performance of the EDP algorithm and to compare the optimal schedule obtained for the DG units, the economic dispatch problem for the 63-node EDS in Fig. 3 was modeled using AMPL [10], and it was solved using various mixed-integer non-linear commercial solvers. The results are summarized in Table III.

In all cases, demands were adjusted with the same discretization of the state and control variables used in the proposed EDP approach. As shown in Table III, all the methodologies provided a similar value to the objective function. The main difference resides with the execution time and the nature of the optimal solution found by the solver. Note that none of the solvers surpass the execution time of the proposed EDP. Furthermore, global optimality is not guaranteed by BONMIN [4], and COUENNE [7] requires almost 5 minutes to find the optimal solution. Finally, the CPLEX [20] solution is globally optimal, but the mathematical economic dispatch problem was solved using a convex second-order conic programming version of the problem (see [19]), which can lead to intractable models for large-scale EDS.

4) Discretization Error: Since the proposed approach relies on the discretization of the state and control variables, a trade-off between the quality of the solution and the computational performance of the method exists. Table IV compares various discretization steps for the active and reactive power flows and injections (i.e., $\Delta x_k^{1,2}$ and Δu_k^1), whereas Δx_k^3 represents the discretization step of the voltage magnitudes. The absolute error in Table IV is calculated using (29), where $u_{k,t}^{\text{global}}$ is the value of the decision variables found by global MINLP solvers and $u_{k,t}^{\text{EDP}}$ is the solution provided by the proposed EDP approach. Note that a fast and precise solution is found using 0.1 p.u. for power flows and injections, and 0.01 p.u. for voltage magnitudes. Nonetheless, smaller discretization steps will require higher computational efforts with similar results, whereas

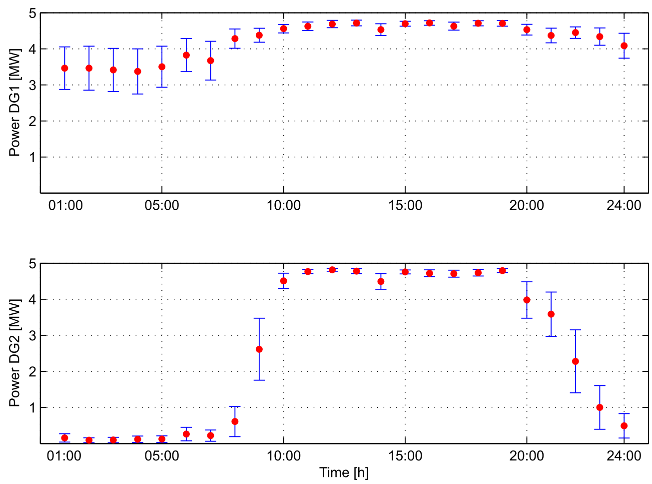

Fig. 8. Means and standard deviations of both DG units' economic dispatch.

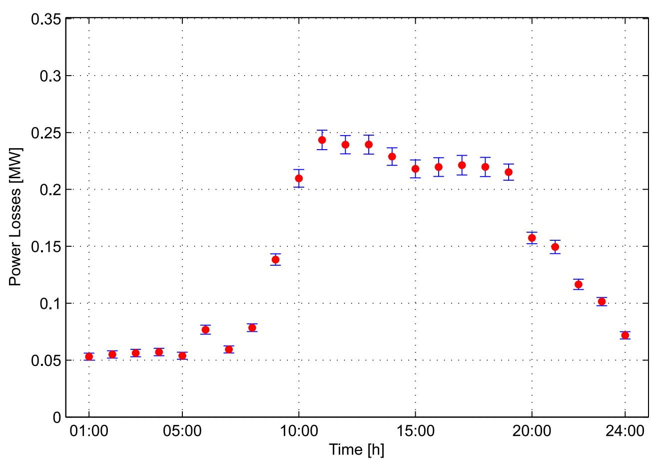

Fig. 9. Means and standard deviations of the total active power losses.

higher discretization values (e.g., $\Delta x_k^{1,2,3} = \Delta u_k^1 = 0.1$ p.u.) would lead to imprecise solutions.

$$error = \max_{k \in N, t \in T} \left\{ \left| \frac{u_{k,t}^{\text{global}} - u_{k,t}^{\text{EDP}}}{u_{k,t}^{\text{global}}} \right| \right\} \cdot 100\% \qquad (29)$$

B. Stochastic Case

In this case, we are interested in finding the optimal average economic dispatch of each DG unit, given a probability density curve defined for each daily load connected to the EDS in Fig. 3 [31]. Thus, a set of Monte Carlo (MC) simulations are executed in order to indicate the average and the standard deviation of each DG scheduled generation, using the proposed EDP approach. The results of the MC, after 1000 simulations, are shown in Figs. 8 and 9. Fig. 8 determines the optimal average dispatch of both units, and Fig. 9 shows the average total active power losses and their corresponding standard deviations. Note that, in almost all the probable scenarios, DG1 is full functional throughout the day, whereas DG2 works as a back-up unit, especially during the peak hours, in order to reduce average energy losses and to improve the voltage profile. If the forecasted probability distribution curves of the loads are accurate enough, the average dispatch in Fig. 8 can be used to deal with most of the daily DG dispatches, resulting in an average active power losses as in Fig. 9. The proposed EDP took approximately 1 hour to complete the MC simulations. The same MC approach would take more than 3 hours using other commercial solvers, such as BONMIN.

TABLE V
PERFORMANCE COMPARISON FOR THE 968-NODE EDS AND 100 DG UNITS

Solver	f [USD\$]	Time [s]	Optimal Solution
EDP	327.3	49.5	Global
CPLEX*	327.3	268.3	Global
BONMIN	327.3	431.1	Local
COUENNE	–	–	Global

*Using a convex second-order conic model.

C. Large-Scale Case

In order to test the performance and the scalability of the proposed EDP algorithm, a large radial network of 968 nodes was used to solve the economic dispatch problem. The data of the 968-node system can be obtained at [32]. To increase the impact of the test, 100 dispatchable DG units are being scheduled. As in Table III, the results of the proposed EDP were compared with the solution provided by other mixed-integer nonlinear commercial solvers. Table V summarizes the results and the nature of the solutions of each method. In all cases, demands were adjusted with the same discretization of the state and control variables from the EDP approach. In terms of computational efficiency, the proposed EDP approach has the lowest execution time. Note that both CPLEX and BONMIN obtained similar results with longer times, and the global MINLP solver COUENNE did not converge to an optimal solution after one hour. Moreover, Branch-and-Bound/Cut solvers, such as CPLEX, BONMIN or COUENNE have an uncertain computational complexity which depends on the intrinsic characteristics of each problem. Thus, in contrast to the proposed EDP method, the scalability of the MINLP solvers cannot be secured.

D. Other EDS Operation Problems

For the sake of simplicity, the EDP approach in this paper was focused on solving the economic dispatch problem in radial and balanced EDS. However, the performance and characteristics of the proposed approach can be adapted to solve other operational problems.

1) Weakly Meshed Networks: The proposed EDP can be implemented in weakly meshed distribution networks by following a similar compensation-based load flow method in [28]. As shown in Fig. 10, in the case of loops within the network, breakpoints are selected and opened in order to execute the EDP's algorithm. When all voltage mismatches between the nodes that comprise the breakpoints are small enough, then the solution has been found. Otherwise, the nodal injections at each breakpoint

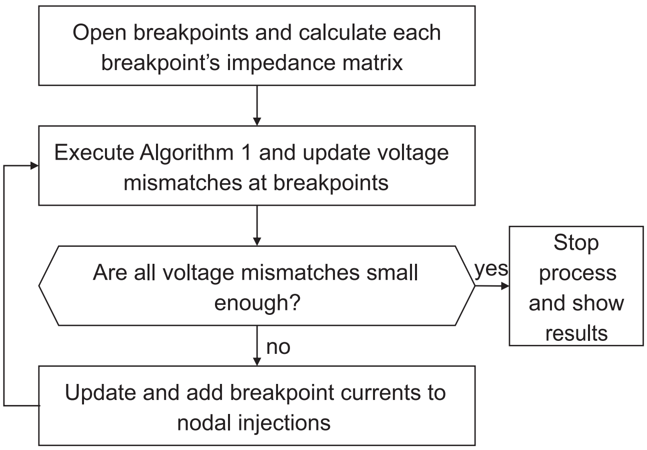

Fig. 10. Compensation-based EDP for weakly meshed networks.

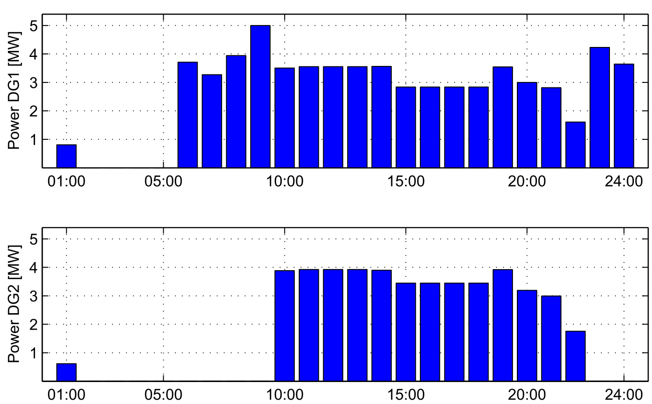

Fig. 11. Optimal dispatches of the DG units. Weakly meshed network.

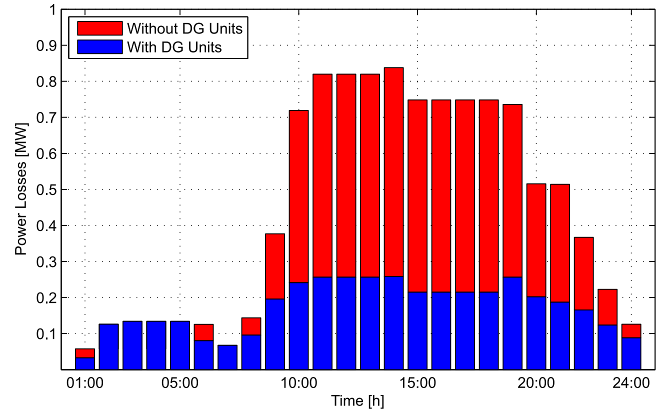

Fig. 12. Total active power losses reduction. Weakly meshed network.

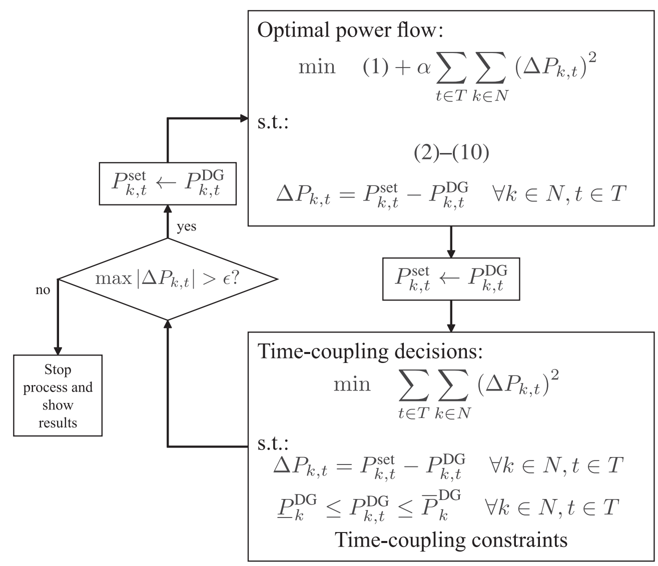

Fig. 13. Decomposition approach to deal with time-coupling constraints.

are updated by adding a breakpoint current, which is calculated based on the voltage mismatches and each breakpoint's impedance matrix [28].

In order to test the method in Fig. 10, a new solid connection between nodes 57 and 58 of the MV distribution system in Fig. 3 has been simulated. This connection produces a loop. Thus, using the compensation-based method in Fig. 10, the new DG dispatches and power losses reduction are shown in Figs. 11 and 12, respectively. Note that both dispatches are different from those in Fig. 4 and, since the system's total impedance is reduced due to the new loop, the active power losses are reduced as well. The total execution time was 14.7 s.

2) Time-Coupling Constraints: The operation of DG and storage devices within the EDS takes into account time-coupled constraints such as, DG ramp-down and ramp-up operation, minimum on and off-line times, start-up and shut-down procedures, or energy charging and discharging. Due to the nature of the proposed EDP algorithm, time-coupling constraints cannot be integrated as such without compromising optimality. Thus, Fig. 13 presents a decomposition strategy to deal with time-coupling constraints [21]. Initially, the optimal power flow is solved using the proposed EDP algorithm, disregarding the existing time-coupling constraints of all dispatchable units. An additional term, $\Delta P_{k,t}$, is calculated and penalized in the objective function. $\Delta P_{k,t}$ is the difference between the dispatched active power generation of each unit and the parameter $P_{k,t}^{\text{set}}$. The weighting factor α is set to zero during the first iteration and then it is set to a big positive number. Once the optimal power flow has been solved, the time-coupling decisions are adjusted using the optimization model at the bottom of Fig. 13. Note that the objective function of the time-coupling decision model minimizes the difference between the generation set by the OPF and the adjusted generation that considers the time-coupling constraints of all DG units. The OPF model in Fig. 13 can be solved using the proposed EDP approach and the time-coupling decision model can be solved using standard dynamic programming. As shown in Fig. 13, the process stops when $|\Delta P_{k,t}|$ is lower than a given threshold.

Similar to the previous adaptation, the method in Fig. 13 has been tested using the MV distribution system in Fig. 3. In this case, both DG units have ramp-up and ramp-down constraints,

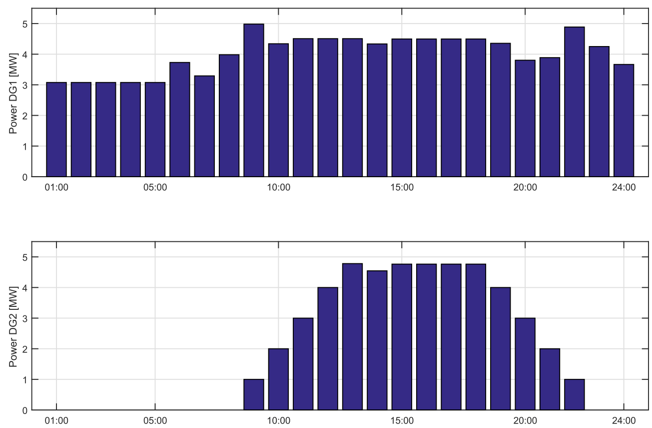

Fig. 14. Optimal dispatches of the DG units. Time-coupling constraints.

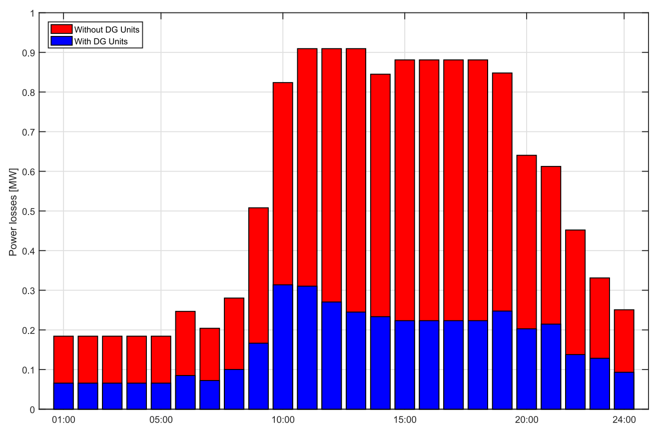

Fig. 15. Total active power losses reduction. Time-coupling constraints.

given by the set of equations in (30), with $\epsilon = 1e^{-3}$.

$$-0.2 \text{ p.u.} \leq P_{k,t}^{\text{DG}} - P_{k,t-1}^{\text{DG}} \leq 0.2 \text{ p.u.} \qquad \forall k \in N, t \in T \tag{30}$$

Thus, after executing the procedure in Fig. 13, the DG dispatches and power losses reduction are shown in Figs. 14 and 15, respectively. As shown in Fig. 14 both dispatches have been reshaped to guarantee the ramp-down and ramp-up constraints of the DG units. The losses profile in Fig. 15 is higher to the one in Fig. 6 due to the search space reduction. Solution in Fig. 14 is not the optimal solution of the time-coupled problem; however, it is a feasible and computationally efficient dispatch. The total execution time was 26.0 s.

3) Others:

1) Since convexity is not a concern for the proposed EDP, binary decision variables can be easily added in order to solve different operation planning problems in a reasonable amount of time. Using binary variables, problems, such as the optimal daily setting of capacitor banks, voltage regulators, or on-line tap changers (OLTC), can be efficiently solved.
2) The proposed EDP can be adapted to an unbalanced three-phase EDS considering the four-dimensional vector in (31) for representing the state variable at each node and phase.

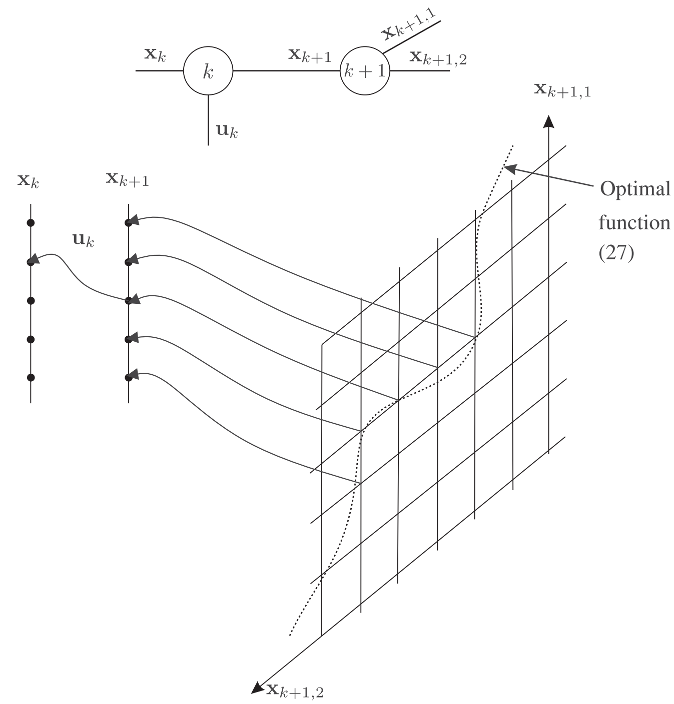

Fig. 16. Derivation of (27) when $A_{k+1} \neq \emptyset$ [22].

$$\mathbf{x}_k^f = \begin{bmatrix} x_k^{1,f} \\ x_k^{2,f} \\ x_k^{3,f} \\ x_k^{4,f} \end{bmatrix} = \begin{bmatrix} \text{Re}\left\{I_k^f\right\} \\ \text{Im}\left\{I_k^f\right\} \\ \text{Re}\left\{V_k^f\right\} \\ \text{Im}\left\{V_k^f\right\} \end{bmatrix} \quad f \in \{a, b, c\} \tag{31}$$

3) Without compromising optimality, the operation of the DG units can be generalized by considering constructive or operational characteristics of the generation units, such as capability curves, power factor limits, or nonlinear mechanical constraints.
4) The recursive EDP approach can use expected values of the objective function in order to explicitly consider random inputs via an extended stochastic dynamic programming approach [24].

V. Conclusion

An extended version of DP is used to optimize the AC steady-state operation of radial EDS. Based on forecasted daily demands at each node, the proposed EDP was adapted to efficiently solve the economic DG dispatch problem. The solution of the economic dispatch problem defines the daily outputs of the dispatchable DG units that minimize the generation costs, while maintaining the voltage magnitudes within their limits, as well as the circuit thermal and DG constraints.

Two networks, with 63 and 968 nodes, were used to demonstrate the applicability and sensibility of the proposed EDP approach. Among the tests that were executed, the paper analysed a deterministic case, including sensitivity and computational performance, and a stochastic case, based on MC simulations. The results illustrate that the EDP is suitable to solve multiple mixed-integer nonlinear, AC operation problems in a reasonable amount of time and with limited computational resources. Finally, generalizations to consider other EDS operation problems such as, weakly-meshed networks and time-coupling constraints, have also been proposed.

Appendix

If $A_{k+1} \neq \emptyset$, it means that the node $k+1$ has one or more downstream branches attached to it. Thus, since the recursive Hamilton-Jacobi-Bellman equations in (25)–(26) only admit one set of optimal future costs $\mathbf{F}\left(\mathbf{x}_{k+1}\right)$ per node, an additional optimization process has to be performed to identify and aggregate the optimal state variables emanating from node $k+1$. As illustrated by Fig. 16, in the case of two downstream branches, the state variable $\mathbf{x}_{k+1}$ embodies the multidimensional information coming from the optimal future costs of $\mathbf{x}_{k+1,1}$ and $\mathbf{x}_{k+1,2}$, considering the physical and operational constraints given by (17)–(24), i.e., the solution for the optimization function in (27) [22]. Eventually, during the forward movement, when the optimal path is being constructed, the optimal values of the state variables $\mathbf{x}_{k+1,1}$ and $\mathbf{x}_{k+1,2}$ will be selected based on the values of $\mathbf{x}_k$.

References

[1] R. A. Walling, R. Saint, R. C. Dugan, J. Burke, and L. A. Kojovic, "Summary of distributed resources impact on power delivery systems," *IEEE Trans. Power Del.*, vol. 23, no. 3, pp. 1636–1644, Jul. 2008.

[2] V. A. Evangelopoulos, P. S. Georgilakis, and N. D. Hatziargyriou, "Optimal operation of smart distribution networks: A review of models, methods and future research," *Elect. Power Syst. Res.*, vol. 140, pp. 95–106, Nov. 2016.

[3] F. Capitanescu, "Critical review of recent advances and further developments needed in AC optimal power flow," *Elect. Power Syst. Res.*, vol. 136, pp. 57–68, Jul. 2016.

[4] P. Bonami *et al.*, "An algorithmic framework for convex mixed integer nonlinear programs," *Discrete Optim.*, vol. 5, no. 2, pp. 186–204, May 2008.

[5] *KNITRO Documentation, Ziena Optimization LLC*, Evanston, IL, USA, Dec. 2012.

[6] N. V. Sahinidis, "Baron: A general purpose global optimization software package," *J. Glob. Optim.*, vol. 8, pp. 201–205, Mar. 1996.

[7] P. Belotti, J. Lee, L. Liberti, F. Margot, and A. Wächter, "Branching and bounds tightening techniques for non-convex MINLP," *Optim. Methods Softw.*, vol. 24, no. 4–5, pp. 597–634, Aug. 2009.

[8] L. Gan, N. Li, S. Low, and U. Topcu, "Exact convex relaxation for optimal power flow in distribution networks," in *Proc. ACM SIGMETRICS*, Pittsburgh, PA, USA, 2013, pp. 351–352.

[9] C. Gamarra and J. M. Guerrero, "Computational optimization techniques applied to microgrids planning: A review," *Renew. Sustain. Energy Rev.*, vol. 48, pp. 413–424, Aug. 2015.

[10] R. Fourer, D. M. Gay, and B. W. Kernighan, *AMPL: A Modeling Language for Mathematical Programming*, 2nd ed. Pacific Grove, CA, USA: Brooks/Cole-Thomson Learning, 2003.

[11] R. R. Gonçalves, R. P. Alves, J. F. Franco, and M. J. Rider, "Operation planning of electrical distribution systems using a mixed integer linear model," *J. Control, Autom. Elect. Syst.*, vol. 24, no. 5, pp. 668–679, Oct. 2013.

[12] R. Palma-Behnke *et al.*, "A microgrid energy management system based on the rolling horizon strategy," *IEEE Trans. Smart Grid*, vol. 4, no. 2, pp. 996–1006, Jun. 2013.

[13] Y. Cao, Y. Tan, C. Li, and C. Rehtanz, "Chance-constrained optimization-based unbalanced optimal power flow for radial distribution networks," *IEEE Trans. Power Del.*, vol. 28, no. 3, pp. 1855–1864, Jul. 2013.

[14] Y.-F. Li, N. Pedroni, and E. Zio, "A memetic evolutionary multi-objective optimization method for environmental power unit commitment," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 2660–2669, Aug. 2013.

[15] A. Trivedi, D. Srinivasan, K. Pal, C. Saha, and T. Reindl, "Enhanced multiobjective evolutionary algorithm based on decomposition for solving the unit commitment problem," *IEEE Trans. Ind. Informat.*, vol. 11, no. 6, pp. 1346–1357, Dec. 2015.

[16] R. Arul, S. Velusami, and G. Ravi, "A new algorithm for combined dynamic economic emission dispatch with security constraints," *Energy*, vol. 79, pp. 496–511, 2015.

[17] S. Huang, Q. Wu, J. Wang, and H. Zhao, "A sufficient condition on convex relaxation of ac optimal power flow in distribution networks," *IEEE Trans. Power Syst.*, vol. 32, no. 2, pp. 1359–1368, Mar. 2017.

[18] R. A. Jabr, "Radial distribution load flow using conic programming," *IEEE Trans. Power Syst.*, vol. 21, no. 3, pp. 1458–1459, Aug. 2006.

[19] M. Farivar and S. H. Low, "Branch flow model: Relaxations and convexification—Part I," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 2554–2564, Aug. 2013.

[20] *CPLEX Optimization Subroutine Library Guide and Reference. ILOG Inc.*, Incline Village, NV, USA, 2008.

[21] D. Olivares, C. Canizares, and M. Kazerani, "A centralized energy management system for isolated microgrids," *IEEE Trans. Smart Grid*, vol. 5, no. 4, pp. 1864–1875, Jul. 2014.

[22] J. F. V. Gonzalez, C. Lyra, and F. L. Usberti, "A pseudo-polynomial algorithm for optimal capacitor placement on electric power distribution networks," *Eur. J. Oper. Res.*, vol. 222, no. 1, pp. 149–156, Oct. 2012.

[23] P. P. Vergara, J. C. López, C. Lyra, and L. C. P. da Silva, "Optimal schedule of dispatchable DG in electrical distribution systems with extended dynamic programming," in *Proc. 17th Int. Conf. Harmonics Qual. Power*, Oct. 2016, pp. 204–208.

[24] W. Powell, *Approximate Dynamic Programming: Solving the Curses of Dimensionality, 2nd ed. (ser. Wiley Series in Probability and Statistics)*. Hoboken, NJ, USA: Wiley, 2011.

[25] M. Baran and F. Wu, "Optimal capacitor placement on radial distribution systems," *IEEE Trans. Power Del.*, vol. 4, no. 1, pp. 725–734, Jan. 1989.

[26] R. Cespedes, "New method for the analysis of distribution networks," *IEEE Trans. Power Deliv.*, vol. 5, no. 1, pp. 391–396, Jan. 1990.

[27] Y. J. Kim, S. J. Ahn, P. I. Hwang, G. C. Pyo, and S. I. Moon, "Coordinated control of a DG and voltage control devices using a dynamic programming algorithm," *IEEE Trans. Power Syst.*, vol. 28, no. 1, pp. 42–51, Feb. 2013.

[28] D. Shirmohammadi, H. W. Hong, A. Semlyen, and G. X. Luo, "A compensation-based power flow method for weakly meshed distribution and transmission networks," *IEEE Trans. Power Syst.*, vol. 3, no. 2, pp. 753–762, May 1988.

[29] R. Bellman, *Dynamic Programming, ser. Dover Books on Computer Science*. New York, NY, USA: Dover, 2013.

[30] F. C. L. Trindade, K. V. do Nascimento, and J. C. M. Vieira, "Investigation on voltage sags caused by DG anti-islanding protection," *IEEE Trans. Power Del.*, vol. 28, no. 2, pp. 972–980, Apr. 2013.

[31] J. Jardini, C. M. Tahan, M. Gouvea, S. U. Ahn, and F. Figueiredo, "Daily load profiles for residential, commercial and industrial low voltage consumers," *IEEE Trans. Power Del.*, vol. 15, no. 1, pp. 375–380, Jan. 2000.

[32] *948 Buses Test System, Sistemas Testes. Laboratorio de Planejamento de Sistemas de Energia Elétrica (LaPSEE)*, 2015. [Online]. Available: http://www.feis.unesp.br/#!/departamentos/engenharia-eletrica/pesquisa s-e-projetos/lapsee/downloads/

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