Electric Power Systems Research 182 (2020) 106228

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

Distributed multi-period three-phase optimal power flow using temporal neighbors

Rui Pinto[a,b], Ricardo J. Bessa[a,*], Jean Sumaili[a], Manuel A. Matos[a,b]

[a] INESC TEC, Institute for Systems and Computer Engineering, Technology and Science, Campus da FEUP, Rua Dr. Roberto Frias, 4200-465 Porto, Portugal
[b] Faculty of Engineering, University of Porto, Rua Dr. Roberto Frias, 4200-465 Porto, Portugal

ARTICLE INFO

Keywords:
ADMM
Distributed
Optimal power flow
Energy storage
Multi-period
Low voltage

ABSTRACT

The penetration of distributed generation in medium (MV) and low (LV) voltage distribution grids has been steadily increasing every year in multiple countries, thus creating new technical challenges in grid operation and motivating developments in distributed optimization for flexibility management. The traditional centralized optimal power flow (OPF) algorithm can solve technical constraints violation. However, computational efficiency, new technologies (e.g., edge computing) and control architectures (e.g., web-of-cells) are demanding for distributed approaches. This work formulates a novel distributed multi-period OPF for three-phase unbalanced grids that is essential when integrating energy storage units in operational planning (e.g., day-ahead) of LV or local energy community grids. The decentralized constrained optimization problem is solved with the alternating direction method of multipliers (ADMM) adapted for unbalanced LV grids and multi-period optimization problems. A 33-bus LV distribution grid is used as a case-study in order to define optimal battery storage scheduling along a finite time horizon that minimizes overall grid operational costs, while complying with technical constraints of the grid (e.g., voltage and current limits) and battery state-of-charge constraints.

1. Introduction

Energy policies driven by a growing environmental awareness, together with the cost decrease of photovoltaic (PV) panels and wind turbines technology, are boosting the investment in the Distributed Renewable Energy Sources (DRES). The deployment of DRES started with power plants in the MW-scale connected to medium, high and extra-high voltage grids. Presently, the integration levels of PV panels in rooftops of domestic houses or service buildings has been steadily growing in several countries, alongside with new business models such as local communities [1] and transactive energy [2] for low voltage (LV) prosumers.

As DRES technologies and other types of distributed generation (e.g., combined heat and power units) increase their penetration in LV residential areas, thus possibly affecting the power quality and end-user voltages [3]. In fact, current LV grids were developed under the assumption of unidirectional power flows from the transmission grid to distribution grid and LV consumers. With significant levels of DRES at the LV level, this unidirectional characteristic is not constant, which brings new challenges for current Distribution System Operators (DSO).

The possibility to store electrical energy surplus (e.g., from PV) and to sue it later can help deferring network investments to accommodate high DRES integration levels and improve grid operational performance (e.g. power losses, voltage profiles). Domestic load flexibility is also a crucial component for LV grid management, which can be modeled as a virtual battery storage [4], and can be used to maximize self-consumption and/or supply grid non-frequency ancillary services in order to assist DSO operational procedures both in MV and LV grids [5]. In summary, the path in the energy sector is to head towards empowerment of small-scale grid users and real-world deployment of microgrid technology and business models at the LV (or community) level in order to bring generation that is much closer to demand, thus decreasing grid losses and increasing self-sufficiency of the grid users.

In order to deal with these new challenges and, therefore, to exploit flexibility options, new grid active management strategies have to be developed. In order to maximize the potential of DRES, these new strategies will have to account for demand-side flexibility forecast, while considering the positive impacts of small-scale storage, which involves solving a multi-period optimization problem with high complexity. These algorithms should be designed while taking into account the following requirements (covered by the algorithm proposed):

- Include multi-period constraints from flexible loads (e.g., rebound effect) and storage units (e.g. state-of-charge);

* Corresponding author.
E-mail address: ricardo.j.bessa@inesctec.pt (R.J. Bessa).

https://doi.org/10.1016/j.epsr.2020.106228
Received 2 November 2019; Accepted 13 January 2020
Available online 06 February 2020

- Consider three-phase unbalanced LV grids and handle non-convex optimization problems that may have multiple local optimal points that can take longer (when compared to a convex problem) to identify whether the problem has a global solution or not;
- In contrast to the conventional optimal power flow (OPF) formulation, it should enable a decentralized optimization of the grid operation and address increasing data privacy concerns.

The following section introduces the state-of-the-art regarding distributed OPF formulation. Section 3 describes the proposed multi-period three-phase OPF formulation and in Section 4, the case-study and main results are discussed. Finally, Section 5 presents the main conclusions and future work.

2. Related work and contributions

The use of decentralized techniques for solving the OPF problem is not recent and mathematical decomposition techniques were already applied in the past (see [6] and [7] for more details). With the revisitation of the Alternating Direction Method of Multipliers (ADMM) in [8] for large-scale statistical learning and implementation in cloud computing environments, which was generalized for distributed convex optimization problems, new approaches for decentralized OPF are emerging in the literature [9]. Microgrid operation proneness to the use of decentralized algorithms has been studied with initial works on energy management [10], control of power flows [11], and ADMM-based algorithms of simplified OPF formulations [12–14].

Zhang et al. used linear approximations for the AC power flow equations while using ADMM to develop a distributed control framework that steers DRES output power into a solution of an AC OPF [15]. In [16], Guo et al. showed, by means of extensive simulations and analytical studies that with the proposed partitioning technique, the convergence performance of ADMM-based algorithm can be substantially improved, thus enabling its application for very large-scale systems. Another ADMM-based algorithm for the OPF problem is proposed in [17], with the authors using sequential convex approximations to deal with the non-convex nature of power flow equations.

Kraning et al. considered power system with generators, fixed and flexible loads, and storage devices, each one with dynamic constraints and individual objective and formulated a global problem, having the minimization of the total network operation costs as the objective function [18]. The authors developed a decentralized iterative method for this problem called proximal message passing. The basic idea behind this method is that, at each iteration, each device exchanges messages with its neighboring devices in the network and then solves its own optimization problem, by minimizing its own objective function that was augmented by a term related to the messages received. This message passing method is proved to be converged to a solution when the device's individual objective and constraints are convex. Numerical examples presented in [18] include a 30 million variables problem solved with a serial implementation in five minutes; with decentralized computing, the computation time can be reduced to less than a second. Although the work has merits in what concerns the establishment of a distributed OPF formulation, it remains a very simplified version of the OPF problem using a DC version approximation, which is not suitable for the operational management of distribution grids. The formulation of a multi-period version is said to be possible, however the authors do not present such formulation.

In order to deal with non-convexity, several researchers adopted convex relaxations. One can find several OPF studies that use Semidefinite Program (SDP) [19] and Second Order Cone Program (SOCP) [20] relaxations in the bus injection model, and SCOP relaxation for the branch flow model [21–23]. Peng and Low [24] took advantage of those relaxations in their own research, in which they proposed a distributed OPF algorithm for radial networks. In the proposed method, the authors decomposed a relaxed OPF problem into smaller sub-problems using ADMM. Additionally, they derived closed form solution for the sub-problems and consequently reducing the computation time needed by a factor of 1000. In a later work [25], the authors extended their previous work to formulate an OPF distributed algorithm suited for unbalanced three-phase radial distribution grids. Although being close to fulfill the requirements to be used for microgrid operation management, [25] does not include a multi-period formulation that could be used to efficiently manage energy storage equipment which impose inter-temporal constraints such as state-of-charge.

The work in [26] proposed an algorithm that uses decomposition methods to solve an OPF problem considering energy storage systems. The algorithm is based on moment relaxation and ADMM. Despite the merits of this work, the authors do not propose a multi-phase OPF formulation, which increases the complexity of the formulation but it is essential when studying LV distribution grids. Additionally, in the first sub-problem formulation being proposed, authors decompose the problem among different time steps which fails to separate the OPF problem into node level.

The work in [27] proposed a generalized formulation based on a proximal dual consensus ADMM for multi-agent constrained optimization, which, among other applications, can be used to solve OPF problems. Nevertheless, the authors do not show how inter-temporal constraints such as the state-of-charge (SOC) supervision could be model into this formulation.

Recently, Munsing et al. [28] motivated by security and transparency issues in peer-to-peer energy markets presented an approach that uses blockchain and smart contracts in order to facilitate a decentralized coordination between agents. ADMM is used to decentralize the OPF problem and to schedule batteries and flexible loads within a microgrid while smart contract on blockchain is used as ADMM coordinator. The proposed methodology formulates a single-phase OPF while the decentralized algorithm still remains dependent on a central aggregator. A multi-period formulation is not considered.

When analyzing the state-of-the-art, one can find that decentralized OPF formulation is a popular topic nowadays as the LV distribution grid operation management paradigm is shifting, following the foster of local and distributed generation units and the increase of demand flexibility. Some of the revisited work focused on proposing and validating initial distributed approaches for the OPF problem (like [8,12,14]), which were important as a first iteration, but did not fulfill some requirements that are essential for distribution grid management tools, such as three-phase formulation, the consideration of an unbalanced network, and a multi-period formulation that models flexible equipment like domestic electric batteries or other energy storage equipment, which pose inter-temporal constraint for the OPF problem. On the other hand, there are some papers [24,25] that take a step further and propose OPF formulations and algorithms much closer of being suitable for LV distribution grid operation management. Nevertheless, to the best knowledge of the authors, currently there are no distributed unbalanced three-phase OPF formulations that consider multi-period constraints. The present paper generalizes previously published formulations by proposing a multi-period three-phase OPF formulation that is essential when operating LV distribution grids, such as microgrids with small-scale storage.

3. Distributed multi-period optimal power flow

This section formulates the proposed novel distributed multi-period three-phase OPF. Firstly, the mathematical background of the distributed OPF for radial grids is briefly described, namely the ADMM constrained optimization algorithm and branch flow mathematical model for electrical grids. Secondly, the multi-period formulation is described in detail and the differences to state-of-the-art methodologies are highlighted.

3.1. Background: ADMM and power flow model

3.1.1. ADMM – constrained optimization algorithm

The ADMM-based algorithm, which takes the form of a decomposition-coordination procedure where solutions to small sub-problems are coordinated in order to find a solution to a global problem, is adopted in this work so as to decompose the OPF formulation down to the node level. ADMM combines the decomposability of the dual ascent method with the superior convergence properties from the method of multipliers [8]. This algorithm is used for convex optimization problems of the following form

$$\begin{aligned} &\min\ f(z)\\ &\text{subject to:}\\ &\mathrm{A}z = b \end{aligned} \tag{1}$$

The implementation of ADMM in this type of problems requires the variable z to be split into two parts, x and y, the objective function being separable during this splitting.

$$\begin{aligned} &\min\ f(x) + g(y)\\ &\quad \text{subject to:}\\ &\mathrm{Ax} + \mathrm{By} = c \end{aligned} \tag{2}$$

Similar to the method of multipliers, the augmented Lagrangian comes from

$$L_\rho(x, y, \tau) = f(x) + g(y) + \tau^T(\mathrm{Ax} + \mathrm{By} - c) + \frac{\rho}{2}\ \|\mathrm{Ax} + \mathrm{By} - c\|_2^2 \tag{3}$$

where τ is the Lagrangian multiplier for the constraint in (2), the operator $(*)^T$ refers to the transpose of an array, and ρ is a constant.

The ADMM-based algorithm is an iterative procedure that continuously updates its variables, consisting on the following iterations, with $\rho > 0$

$$x^{k+1} = \underset{x}{\operatorname{argmin}}\, L_\rho(x, y^k, \tau^k) \tag{4}$$

$$y^{k+1} = \underset{y}{\operatorname{argmin}}\, L_\rho(x^{k+1}, y, \tau^k) \tag{5}$$

$$\tau^{k+1} = \tau^k + \rho^*(x^{k+1} - y^{k+1}) \tag{6}$$

The ability to decompose the problem into sub-problems without affecting the global objective function value is explored in order to decompose the OPF problem into local (or nodal) sub-problems, with the innovation of including multi-period constraints from storage operation. The formulation of this multi-period OPF problem with ADMM is presented in Section 3.2.

3.1.2. Mathematical power flow model for radial networks

Before presenting the formulation of the three-phase unbalanced OPF, it is important to briefly describe the power flow mathematical model that is adopted in this work for radial networks. Two main power flow mathematical models can be considered for this work: bus injection model (BIM) and branch flow model (BFM). BIM uses conventional variables in its formulation and the results are easier to directly understand. On the other hand, BFM defines new variables to exclude non-linearity from its equations. These models are described in [21,29] and their equivalence is proved. BFM brings increased numerical stability regarding radial networks and for that reason, it is used in this work and is briefly presented below.

Let one consider a power network, which is modeled by a connected directed graph $G(N_+, E)$ where each node in N_+ represents a bus and each edge in E represents a transmission/distribution power line. An edge is defined by (j, k) if it points from node j to node k. For each edge (j, k) let z_{jk} represent the complex impedance of the power line; let $S_{jk} = P_{jk} + iQ_{jk}$ be the complex power sent from node j to node k; and I_{jk} be the complex current from node j to node k. The complex power injection, which comes from generation minus load at bus j, is defined as s_j. BFM is defined by the following power flow equations:

$$\sum_{k:\mathrm{jk}} S_{\mathrm{jk}} = \sum_{i:\mathrm{ij}} (S_{\mathrm{ij}} - z_{\mathrm{ij}}\ |I_{\mathrm{ij}}|^2) + s_j, \quad \text{for all } j \tag{7}$$

$$I_{\mathrm{jk}} = y_{\mathrm{jk}}(V_j - V_k), \quad \text{for all } j \tag{8}$$

$$S_{\mathrm{jk}} = V_j I_{\mathrm{jk}}^H, \quad \text{for all } j \tag{9}$$

where (7) defines the nodal power balance constraint, (8) is Ohm's law, and (9) defines the branch power flow. In (7) the under-script ij refers to the branches that connect to node j from downward nodes in the radial network. Accordingly, (7) means that the sum of all power flows from node j to nodes k (upward nodes) must be equal to the sum of all power flows coming from nodes i (downward nodes) to node j minus the respective power losses and including the injected power in node j, s_j.

The formulation of the OPF problem for the BFM aims at finding the feasible set regarding variables S, I, V, and s while complying with (7)–(9), and additional technical constraints. Following [22], the notation for BFM can be simplified: assume $l_{\mathrm{ij}} = |I_{\mathrm{ij}}|^2$ and . Considering that G is a tree, the respective network is radial (i.e., for each network node there is a single connection between it and the upstream section of that network). Adopting the direction that makes all edges point towards the root (most upstream node of the network with index = 0), the BFM can be defined without modeling the phase angles of voltage and currents [24]:

$$\sum_{j\in C_i} (S_j - l_j z_j) + s_i - S_i = 0 \tag{10}$$

$$v_{\mathrm{Ai}} - v_i + (z_i - S_i{}^*{+}z_i{}^*S_i) - l_i\ |z_i|^2 = 0 \tag{11}$$

$$|S_i|^2 = v_i l_i \tag{12}$$

where Ai represents the unique node ancestor of node i, C_i represents the set of children nodes of node i, and the operator $(.)^*$ represents the conjugate of a complex scalar. $S_0 = 0$ and $l_0 = 0$ as the root of the tree does not have an ancestor. It is shown in [22,23] that for a feasible solution of (10)–(12), (S, l, v, s), the phase angles of voltages and currents can be uniquely retrieved if the network is a tree (i.e., a radial network).

BFM is chosen for the developed distributed OPF formulation since, when compared with BIM, it shows better numerical stability for radial networks [24]. The focus of this work relates with the three-phase formulation whilst (7)–(9) and (10)–(12) suppose a single-phase formulation. Accordingly, the branch flow model can be defined as [25]:

$$Y(v_{\mathrm{Ai}}) = v_i - z_i S_i^H - z_i{}^H S_i + z_i l_i z_i{}^H \tag{13}$$

$$s_i = -\mathrm{diag}\left(\sum_{j\in C_i} Y(S_{ji} - z_{ji} l_{ji}) - S_i\right) \tag{14}$$

$$\begin{pmatrix} v_i & S_i \\ S_i^H & l_i \end{pmatrix} \in \Psi_+ \tag{15}$$

$$\mathrm{rank}\begin{pmatrix} v_i & S_i \\ S_i^H & l_i \end{pmatrix} = 1 \tag{16}$$

where $Y(v_{Ai})$ represents the projection of v_{Ai} on the set of phases of bus i, $\Upsilon(S_j - z_{ji}l_{ji})$ represents the result of $S_j - z_{ji}l_{ji}$ on the set of phases of bus i and filling some possible phase mismatch regarding its children with 0, and Ψ_+ imposes a positive semidefinite matrix. The superscript H represents the Hermitian transpose.

The following section describes the multi-period formulation of the distributed OPF, which uses the BFM model to include network technical constraints.

3.2. ADMM-based multi-period distributed OPF

This section describes the multi-period OPF formulation, which uses the ADMM optimization algorithm and the BFM power flow model as core mathematical tools, and adopts the notation used in [25]. The formulation will only cover the case of radial networks, which is the standard topology in MV and LV grids.

Accordingly, a radial distribution network is modeled as a directed tree graph ($T := (N, E)$) represented by the set of buses, N, and the set of power lines, E. In this radial network based representation, each bus has a unique ancestor, A_i, and a possible set of children, C_i. The graph orientation adopted in this work makes every line point towards the root. Let a, b, c denote the three phases of a distribution network and Φ_i the set of phases in bus i. For each phase ϕ_i let V_i^{ϕ} represent the bus complex voltage, $s_i^{\phi} = p_i + \mathrm{j}q_i$ denote the complex power injection, and I_i^{ϕ} the complex branch current from bus i to its ancestor. Denote $V_i := (V_i^{\phi}, \phi \in \Phi_i)$, $s_i := (s_i^{\phi}, \phi \in \Phi_i)$, $v_i = V_i{*}V_i^H$, $I_i := (I_i^{\phi}, \phi \in \Phi_i)$, and $l_i = I_i{*}I_i^H$.

Considering the minimization of operating costs (in this case generation costs) as an objective function, the goal becomes the reduction of costs along the considered time horizon (T) along all buses. As in [25], the objective function continues to be separable along all buses and, now, along all time intervals, which is a necessary condition when trying to decompose the problem into local sub-problem using ADMM.

In [25], each bus has its ancestor and all possible children as neighbors, in a radial topology where the direction is defined from the downwards buses to the reference bus. In order to deal with multi-period constraints, the concept of temporal neighbor is introduced in this work. Accordingly, each bus, in which equipment that introduces the inter-temporal constraints is located, needs to consider as its neighbors all buses that represent itself in different time intervals.

In this work, the OPF formulation considers electric batteries as energy storage equipment, introducing inter-temporal constraints regarding the SOC limits. The OPF formulation is presented below. It is important to underline that other multi-period constraints related to load flexibility can be integrated in the same manner and without additional changes in the proposed formulation. On the other hand, the objective function $f_{i,t}^{\phi}((s_{i0,t})^{(x)})$ defined below can be set in order to minimize generation costs, power losses, energy exchange with main grid, or even to minimize the activation of flexibility from local prosumers, depending on what better fits the intended use case.

The power flow model described in (13)–(16) is non-convex due to the rank constraint (16). In [30], the rank constraint is removed, resulting in a SDP relaxation, which, in the same work, is shown to be exact for radial distribution networks. Accordingly, the resulting BFM equations are modeled into the OPF formulation in (17a)–(17c). Within this OPF formulation equations, the superscripts (x) and (y) are used to identify whether the correspondent variables are updated within the x_{update} or y_{update} iterations. Furthermore, N refers to the total number of nodes, T to the total number of periods considered in the multi-temporal formulation.

Technical and operational constraints include maximum bus injected power (17d), nodal voltage limits (17e), and maximum branch power flow (17f). Eqs. (17g) and (17h) introduce the maximum and minimum SOC constraints in the OPF problem. Depending on the objective function and on the operating condition of each time interval, batteries will have different operating points for each time interval. The proposed concept of temporal neighbors is modeled into the OPF formulation by constraints (17g) and (17h), which guarantee that the final battery operation strategy complies with the physical limitations of the equipment regarding the maximum and minimum capacity during the entire considered operation horizon. Considering battery initial SOC, the combination of power injection among temporal neighbors must comply with physical limitation of considered equipment. Battery power constraints are modeled within constraint (17d).

$$\min \sum_{t\in T}\sum_{i\in N}\sum_{\phi\in\Phi_i} f_{i,t}^{\phi}((s_{i0,t})^{(x)}) \tag{17}$$

$$Y(v_{A_i i,t}^{(y)}) = v_{ii,t}^{(y)} - z_i (S_{ii,t}^{(y)})^H - z_i^H S_{ii,t}^{(y)} + z_i l_{ii,t}^{(y)} z_i^H, \quad i \in N, \quad t \in T \tag{17a}$$

$$s_{ii,t}^{(y)} = -\mathrm{diag}\Big(\sum_{j\in C_i} Y(S_{ji,t}^{(y)} - z_{ji} l_{ji,t}^{(y)}) - S_{ii,t}^{(y)}\Big), \quad i \in N, \quad t \in T \tag{17b}$$

$$\begin{pmatrix} v_{i0,t}^{(x)} & S_{i0,t}^{(x)} \\ (S_{i0,t}^{(x)})^H & l_{i0,t}^{(x)} \end{pmatrix} \in \Psi_+, \quad i \in N, \quad t \in T \tag{17c}$$

$$s_i^{\min} \le (s_{i1,t})^{(x)} \le s_i^{\max}, \quad i \in N, \quad t \in T \tag{17d}$$

$$v_i^{\min} \le (v_{i1,t}^{\phi\phi})^{(x)} \le v_i^{\max}, \quad i \in N, \quad t \in T \tag{17e}$$

$$S_i^{\min} \le (S_{i1,t}^{\phi\phi})^{(x)} \le S_i^{\max}, \quad i \in N, \quad t \in T \tag{17f}$$

$$\mathrm{SOC}_i^{\min} \le \mathrm{SOC}_i^{\mathrm{ini}} + (s_{ii,t})^{(y)} + \sum_{j\in T_i} (s_{ji})^{(y)}, \quad i \in N, \quad t \in T \tag{17g}$$

$$\mathrm{SOC}_i^{\mathrm{ini}} + (s_{ii,t})^{(y)} + \sum_{j\in T_i} (s_{ji})^{(y)} \le \mathrm{SOC}_i^{\max}, \quad i \in N, \quad t \in T \tag{17h}$$

$$x_{ir,t} = y_{ii,t}, \quad r = 1, \quad i \in N, \quad t \in T \tag{17i}$$

$$x_{i0,t} = y_{ij,t}, \quad j \in N_i, \quad i \in N, t \in T \tag{17j}$$

Introducing the inter-temporal criteria into the OPF formulation greatly increases its complexity. It is not only about replicating the OPF formulation by the number of periods in study; one must guarantee that variables states in each period are being considered and influencing the correspondent variables in other time period – temporal neighbors. Accordingly, (17g) assures that for nodes in which energy storage equipment is modelled, the minimum SOC is not violated. It does so, despite the sum of the initial SOC, the injected power (which can be positive or negative) by the energy storage system in period t, and also all injected powers by the same equipment at periods that precede t, which represent the temporal neighbors. A similar rational is applied by (17h) to model the maximum SOC constraint. (17g) and (17h) achieve its objective by being represented within the y_{update} iteration, where observation variables are updated following (5) which aims at minimizing the difference between main variables (x_{update}) and the correspondent observations, while complying with the formulated constraints. By modelling inter-temporal criteria, the number of OPF constraints is increased as it is the number of observation variables required – representing the observation of all temporal neighbors. As an example, for an eight-period OPF formulation and for each node in which storage equipment is modelled, there will be an additional seven observation variables per node for the set of variables belonging to $t = 8$, an additional six observation variables per node for the set of variables belonging to $t = 7$, and so on.

Problem (17) is formulated as a general ADMM form:

$$\min \sum_{t\in T}\sum_{i\in N}\sum_{\phi\in\Phi_i} f_{i,t}^{\phi}(x_{i0,t}^{\phi}) \tag{18}$$

$$\text{over to:} \tag{18a}$$

$$x = \{x_{ir} \mid 0 \le r \le R_i, \ \ i \in N\} \tag{18b}$$

$$y = \{y_{ij} \mid j \in N_i \ \ \cup \ \ y_{it} \mid t \in T_i, \ \ i \in N\} \tag{18c}$$

$$\text{subject to:} \tag{18d}$$

$$\sum_{j\in N_i} A_{ij} y_{ji} = 0 \tag{18e}$$

$$\sum_{t\in T_i} B_{it} y_{ti} \leqslant 0 \tag{18f}$$

$$x_i \in K_i, \quad i \in N \tag{18g}$$

$$x_i = y_{ij}, \quad j \in N_i, \quad i \in N \tag{18h}$$

$$x_i = y_{it}, \quad t \in T_i, \quad i \in N \tag{18i}$$

where K_i is a convex set and A_{ij} and B_{ij} are matrices with appropriate dimensions. R_i refers to the different sets of decision variables that are used: set $r = 0$ is used to compute the optimal values of decision variables while set $r = 1$ is used to model range constraints for the decision variables (17d)–(17f). Let τ_{ij} and τ_{it} be the Lagrangian multiplier for the consensus constraints $x_i = y_{ij}$ and $x_i = y_{it}$, respectively, and $\langle\tau_{ij}, x_i - y_{ij}\rangle$ be the inner product between a scalar and a complex number that represents the difference between main variables and observation variables then the augmented Lagrangian emerges

$$\begin{aligned} L_\rho(x, y, \tau) = \ & f_i(x_i) + \sum_{j\in N_i}\left(\langle\tau_{ij}, x_i - y_{ij}\rangle + \frac{\rho}{2}\|x_i - y_{ij}\|_2^2\right) \\ & + \sum_{t\in T_i}\left(\langle\tau_{it}, x_i - y_{it}\rangle + \frac{\rho}{2}\|x_i - y_{it}\|_2^2\right) \end{aligned} \tag{19}$$

In this ADMM-based algorithm, each bus communicates with its neighbors. With the radial topology of the networks in mind, each bus needs to exchange information with its ancestor (a unique upward bus) and with all its possible children (downward buses directly connected to it). Furthermore, in order to incorporate the multi-period nature intended for the developed formulation, buses, in which batteries are allocated, also need to communicate with their temporal neighbors. In this way, each node receives voltage information from its parent Ai, ($v_{Ai\,i}$), branch power flow (S_{ji}) and current (l_{ji}) from its children $j \in C_i$, and injected power (s_i) from its temporal neighbors, if applicable. Thus, the observation variable vector y contains information

$$y_{ij} = \begin{cases} (S_{ii}^{(y)}, l_{ii}^{(y)}, v_{ii}^{(y)}, s_{ii}^{(y)}), & j = i \\ (S_{ij}^{(y)}, l_{ij}^{(y)}), & j = A_i \\ (v_{ij}^{(y)}), & j \in C_i \end{cases} \tag{20}$$

$$y_{it} = (s_{it}^{(y)}), \quad t \in T_i \tag{21}$$

where the underscript ij emans that variable y_{ij} represents the observation regarding the decision variables of bus i, made by bus j.

The penalty term regarding the difference between decision variables x_i and observation variables y_{ij} in (19) comes from

$$x_i - y_{ij} = \begin{cases} (S_i^{(x)} - S_{ii}^{(y)}, l_i^{(x)} - l_{ii}^{(y)}, v_i^{(x)} - v_{ii}^{(y)}, s_i^{(x)} - s_{ii}^{(y)}), & j = i \\ (S_i^{(x)} - S_{iA_i}^{(y)}, l_i^{(x)} - l_{iA_i}^{(y)}), & j = A_i \\ (v_i^{(x)} - v_{ij}^{(y)}), & j \in C_i \end{cases} \tag{22}$$

$$x_i - y_{it} = (s_i^{(x)} - s_{it}^{(y)}), \quad t \in T_i \tag{23}$$

For each of the terms composing $x_i - y_{ij}$ there is a correspondent Lagrangian multiplier τ_{ij}. The two problems are related to the update of x_i (4) and the problem linked to the update of y_{ji} (5) are explicitly formulated below with (24) and (26), and with (30), respectively.

$$\min \ f_i(x_{i0}) \sum_{j\in N_i}\left(\langle\tau_{ij}, x_{i0}\rangle + \frac{\rho}{2}\|x_{i0} - y_{ij}\|_2^2\right) \tag{24}$$

subject to:

$$\begin{pmatrix} v_{i0} & S_{i0} \\ (S_{i0})^H & l_{i0} \end{pmatrix} \in \Psi_+, \quad i \in N \tag{25}$$

$$\min \ \left(\langle\tau_{ir}, x_{ir}\rangle + \frac{\rho}{2}\|x_{ir} - y_{ii}\|_2^2\right) \tag{26}$$

subject to:

$$s_i^{\min} \le s_{ir} \le s_i^{\max}, \quad i \in N \tag{27}$$

$$v_i^{\min} \le (v_{ir}^{\phi\phi})^{(x)} \le v_i^{\max}, \quad i \in N \tag{28}$$

$$S_i^{\min} \le (S_{ir}^{\phi\phi})^{(x)} \le S_i^{\max}, \quad i \in N \tag{29}$$

$$\min \ \sum_{r=1}^{R_i}\left(\langle\tau_{ir}, y_{ii}\rangle + \frac{\rho}{2}\|x_{ir} - y_{ii}\|_2^2\right) + \sum_{j\in N_i}\left(\langle-\tau_{ji}, y_{ji}\rangle + \frac{\rho}{2}\|x_{j0} - y_{ji}\|_2^2\right) \tag{30}$$

subject to:

$$\sum_{j\in N_i} A_{ij} y_{ji} = 0 \tag{31}$$

$$\sum_{t\in T_i} B_{it} y_{ti} \leqslant 0 \tag{32}$$

The sub-problem linked to x-update (24) is solved by modelling the correspondent formulation into CVXOPT [31] solver in order to cope with the semi-definite programming formulation. On the other hand, for (26) and for the y-update sub-problem (30), Gurobi Optimization solver [32] is used. Python programming language was used to implement the OPF optimization problem.

Lagrangian multipliers are updated after the x-update and the y-update by

$$\tau^{k+1} = \tau^k + \rho(x^{k+1} - y^{k+1}) \tag{33}$$

Regarding the stopping criteria of the algorithm, primal and dual residuals need to be within a specified maximum value. Primal residual refers to the Euclidean norm of $x_i - y_{ij}$ at the end of each iteration k; while the dual residual refers to the Euclidean norm between consecutive iteration values of y_{ij}.

$$\text{primal residual} = \|(x^k - y^k)\|_2 \tag{34}$$

$$\text{dual residual} = \rho\ \|(y^k - y^{k-1})\|_2 \tag{35}$$

The main iterations of the proposed algorithm are depicted in Fig. 1, by means of a block diagram. The x_{update} iteration consists of two parallel optimization step. With the set $r = 0$ the OPF variables are optimized following the minimization (24) of the difference between them and their observation values (that come from y_{update}) and considering the respective Lagrangian multipliers, while maintaining the power flow relations between all variables (25). OPF constraints such as nodal voltage limits or maximum line power flow must be modelled by a

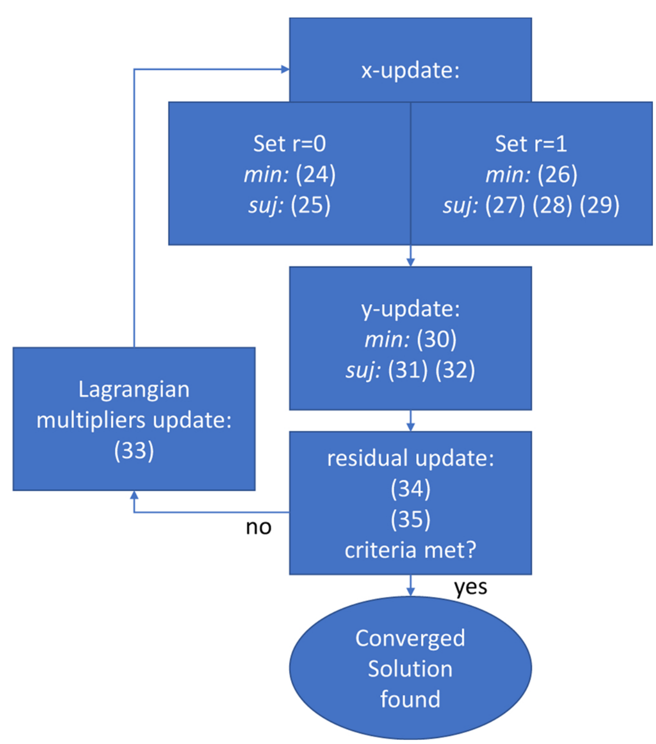

Fig. 1. Algorithm block-diagram.

parallel set of variables, $r > 0$. For each constrained variable, there is a reduction of the difference between them and the respective observation (26) while imposing the pretended constraints (27) and (28). In the y_{update} the observation variables are computed by the minimization (30) between them and the variables values coming from x_{update} while imposing power flow equations (31) and inter-temporal relations (32). By (34) and (35) residuals are computed and convergence is checked. If residuals values are still greater than the defined maximum tolerance, Lagrangian multipliers are updated (33) and used in the following iterations of x_{update} and y_{update}.

In the next section, numerical results from the developed algorithm will be discussed, placing the emphasis on the multi-period constraint compliment and battery final schedule optimized for the reduction of costs regarding LV/MV network interconnection, which represents the main contribution from the formulation and methodology presented in this work.

4. Proof of concept with numerical results

4.1. Test case description

The case study will focus on demonstrating that the SOC limits introduced by the concept of temporal neighbors are being met by the OPF solution. This reflects a correct modeling of inter-temporal technical constraints, which is the main contribution of this work. In order to make this happen, batteries' final schedules are presented, illustrating how the operating strategies react to different electrical energy purchase prices during the operation horizon.

Battery storage is modeled as costless generation units, but battery specific operating costs could be included without any modification to the proposed formulation. In fact, the impact of this type of equipment is accounted in the objective function, by means of its power schedule (charging and discharging cycles). When the battery is charging, demand is increased in the respective bus, and consequently operation costs also increase as reference bus must compensate for the new demand; on the other hand, during discharging cycle, the battery injects power in the grid, thus decreasing the need for costly generation units to operate. Thus, whilst the OPF variable referring to the battery power schedule does not directly introduce changes in the objective function (minimize operation costs), it is indirectly accounted for it imposes changes in the higher-cost generators of the grid (in this case the reference bus). The optimal battery operating strategy should schedule charging cycles during low-price periods and inject power during high-price periods while complying with OPF technical constraints. A more complex operating strategy might arise when a LV grid is, in some periods, injecting power in the upward main grid and the objective function relates to the minimization of energy exchange.

The 33-bus network presented in Fig. 2 is used as case study. The technical parameters of this LV network can be found in [33,34]. The case study considers as unbalanced an operation along the three phases considered with phase *a* accounting for 101.2 kVA contracted power, phase *b* and phase *c* 103.5 kVA; a load simultaneity factor of 0.3 was considered and also a power factor of 0.4. Additionally, total PV installed capacities accounted for 43.124 kW in phase *a* and 39.675 kW in phases *b* and *c*.

Eight time intervals with hourly resolution are considered as operation horizon. Considered load profile accounted for 50%, 50%, 55%, 60%, 60%, 70%, 80%, 65% of the base load, while PV generation profile accounted for 0%, 0%, 20%, 30%, 30%, 40%, 50%, 60% of the installed capacities. LV network has several PV units represented and for certain operation conditions microgeneration-based production can be greater than demand. This is a particularly demanding situation for LV networks, as these were not designed to deal with power flows from downward nodes to upwards nodes of the grid. The importance of the flexible assets of LV grids to mitigate this effect during PV generation peak hours is critical for DSO. This is a major motivation for the work presented in this paper, as flexible equipment such as electric batteries impose multi-period formulation, which is not covered by present state-of-the-art literature regarding decentralized/distributed three-phase OPF algorithm suitable for LV grid operation.

Batteries were connected to network buses #6, #15, and #16, on phases "b", "c", and "a", respectively. Maximum SOC is defined at 1.5 kWh for batteries #6 and #15, and 3.0 kWh for battery #16, while initial SOC is defined at 1.0 kWh for all batteries. Nominal power is defined at 0.80 kW for batteries #6 and #15, and 1.0 kW for battery #16. Different price levels were considered during the eight-hour interval operation horizon. The final OPF solution must schedule the batteries to operate in such way that operational costs are minimized while complying with technical constraints (nodal voltage levels, maximum branch power flow, power limits, and battery SOC). As batteries were modeled as costless equipment, the minimization of operation costs represents the minimization of power exchange in the MV/LV coupling point (bus #1).

4.2. Results

The results presented in this section use the following power and voltage base values for p.u. system: $S_b = 1000VA$ and $V_b = \frac{400}{\text{sqrt}(3)}V$. The power exchange at the MV/LV coupling point (reference bus) for the eight time intervals is depicted in Fig. 3, illustrating the response of energy exchange regarding price fluctuation when batteries are present in the network. Energy exchange at MV/LV coupling point is compared in two different cases: with and without batteries modeled in the grid.

From the analysis of Fig. 3, one can verify that in the first three time intervals, the LV grid is receiving power from the MV grid as its microgeneration based power output is not sufficient to satisfy demand; in the following intervals, the LV grid is injecting power into the MV grid as the total PV generation in all phases is greater than demand. Additionally, and to assist on the assessment of the impact of modelling storage devices, energy price exchange is also depicted. Fig. 3 clearly shows the positive impact of having storage equipment in the grid, as the energy exchange with the MV grid is reduced in almost all periods when compared with the scenario without storage equipment. In addition to this, the impact that the energy price exchange has on the battery schedule is also demonstrated, as the differences between the scenarios tend to increase alongside the price. Battery schedule and SOC variation during the eight-interval operation horizon are presented in Fig. 4. It is expected that flexible equipment schedule is defined by the developed algorithm in such way that its impact reduces the operating costs. Accordingly, it is expected that in periods in which energy exchange price is increased, storage can make a positive impact by either injecting power to meet demand or charging it to accommodate PV surplus.

Both nominal power and SOC constraints are being complied by the computed solution, which can be verified when analyzing Fig. 4. Analyzing the first three time intervals of operation, where the LV grid presents a lack of energy generation regarding its load demand (representing a necessity of energy exchange from the MV grid), all batteries optimally schedule their power injection during interval 3, neglecting time intervals 1 and 2, which reflects the price increase depicted in Fig. 3. Between time intervals 4 and 8, LV grid operation changes, as PV output overcomes demand and power flow is reversed (from LV to MV grid). Time intervals 7 and 8 present the higher power injection values. Accordingly, battery-charging schedule is optimally defined for these two intervals, with particular emphasis on interval eight where the price of energy exchange is higher than in interval seven. These results display the distributed OPF capability of identifying the most demanding periods regarding LV grid operation and optimize the energy storage operation.

Voltage constraints can be either a maximum and minimum voltage levels or a predefined nodal voltage set-points to be maintained during operation. In order to illustrate the algorithm capabilities for modeling such type of criteria, an additional constraint is demonstrated to impose voltage level for phase "c" at the node #23 to be maintained at

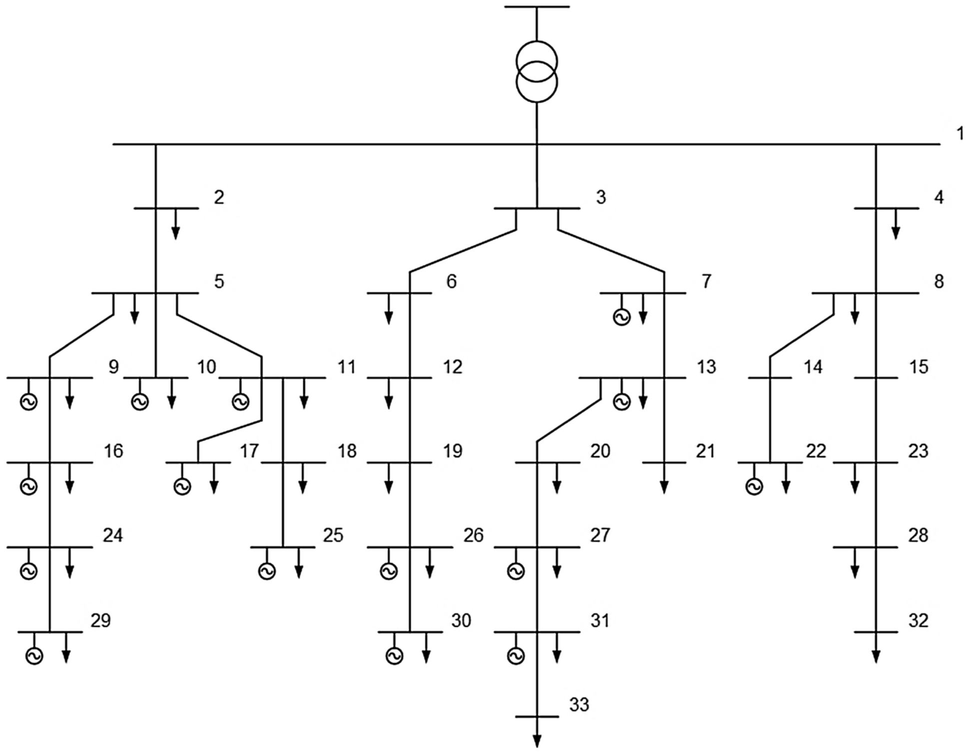

Fig. 2. Low voltage test network topology.

0.99 p.u. from time interval 5 onward since that period was identified as the most critical. Fig. 5 illustrates voltage level comparison at the referred node and phase.

The voltage levels at bus #23 for the two scenarios are compared in Fig. 5, with and without the inclusion of the new voltage constraint for that node. Although it is true that the overall power exchange between microgrid and the MV grid follows the upward direction (microgrid injecting power), this is not verified in this specific branch. The branch downward from node #15 where a battery is connected on phase “c” is composed by nodes #23, #28 and #32, where all of them contain loads and no source of power generation. In addition to that, battery in node #23 is charging in order to minimize power injection from LV grid into upward network. This has the effect of decreasing voltage level along the mentioned branch. Fig. 5 demonstrates that the proposed OPF formulation and our algorithm are capable of respecting not only storage technical limits, but also nodal voltage limits. Although it is not presented here, it is important to stress that the new OPF solution that complies with the additional voltage constraint added on node #23 can only be achieved by changing the battery schedule, which has an impact on the energy exchange with main grid, representing an increase of the operational costs. Similarly, to voltage constraints, branch power flow limits can also impose constraints in grid operation and limit power injections at some nodes, conditioning the charge/discharge schedule of storage. Branch power flow limits are considered in the OPF formulation (see (17f)), nevertheless, this case-study does not present any practical example for the sake of space.

4.3. Optimality and convergence

ADMM-based algorithms are proved to converge to global minimum under sub-problem formulation convexity, x_{update} and y_{update}. As known, AC OPF formulation considers non-convex constraints. The power flow

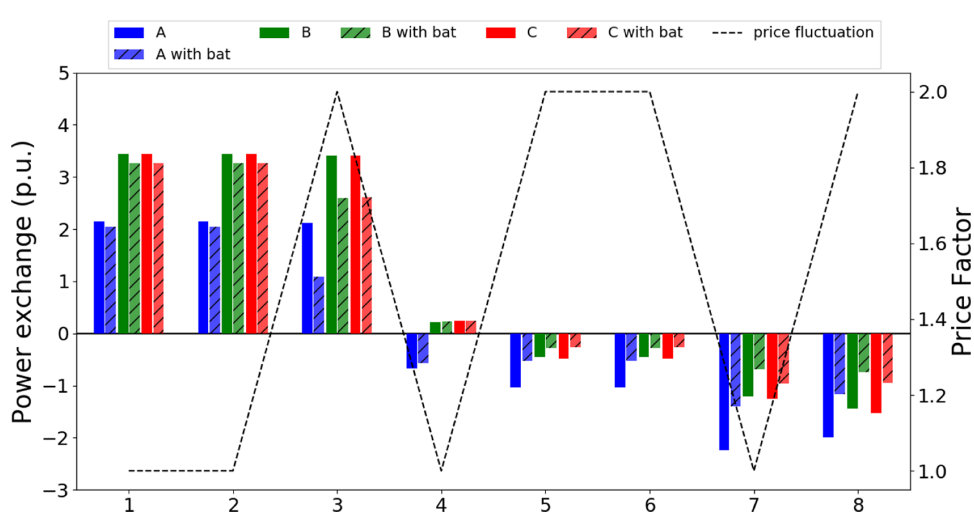

Fig. 3. Energy exchange between LV and MV networks and price variations. (Horizon axis refer to time intervals.)

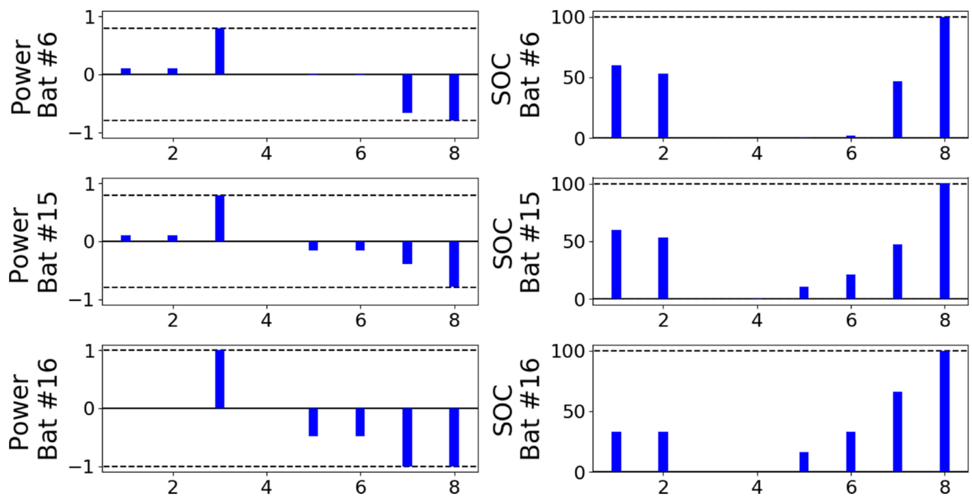

Fig. 4. Batteries power schedule and correspondent SOC evolution. (Horizon axis refer to time intervals.)

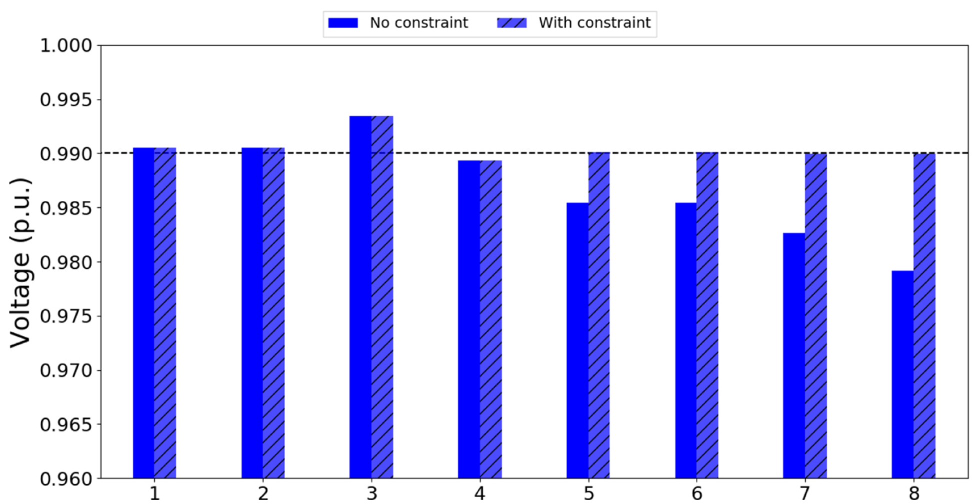

Fig. 5. Voltage level comparison for bus #23 and phase “c”. (Horizon axis refer to time intervals.)

model described in (13)–(16) is non-convex due to the rank constraint (16). In [30], the rank constraint is removed resulting in a SDP relaxation. In the referred work authors show that the SDP relaxation is exact for radial distribution networks. Accordingly, here, the resulting BFM equations are modelled into the OPF formulation in (17a)–(17c), guarantying the convexity of the sub-problems and consequently the convergence to global optimum.

The main focus of this work is the proposal of a multi-temporal formulation for the distributed OPF problem suited for three-phase unbalanced LV distribution grids. Accordingly, the algorithm that was developed had as only objective the validation of the proposed formulation.

The convergence quality of the developed algorithm was not assessed, as optimization software responsible of modelling the ADMM sub-problem mathematical formulation were not selected based on convergence speed criteria nor the algorithm coding was fully-optimized. For those reasons, the computational performance analysis that sometimes appears in related papers sometimes appears, it is not present in this work. It is in the authors’ intentions to perform such analysis in future work. As showed in [35], convergence and computational properties of ADMM-based algorithms require an separated and detailed analysis.

Nevertheless, differently from conventional and centralized OPF formulation, from the authors’ perspective, such decentralized formulation as the one proposed in this work will allow to allocate and share the computational burden among all individual and local equipment, such as HEMS, in each node of the grid.

5. Conclusions

Creating technical conditions in distribution grids to increase the integration levels of DRES is essential to achieve environment goals in what concerns global warming and CO2 emissions reduction. Energy storage at MV and LV distribution grids, as well as small-scale domestic energy storage, introduce the required operational flexibility to maximize DRES integration. As the number of flexible equipment increases in distribution grids, so does the complexity regarding the operation and management of such grids. Conventional (centralized formulation) OPF algorithms are not computationally efficient as the number of control variables exponentially increase, as well as for data-privacy concerns.

This work pushes current literature about distributed OPF one step further by proposing a multi-period formulation for the three-phase unbalanced OPF problem, which is essential to properly integrate distributed energy resources like energy storage and flexible loads. A 33-bus LV distribution grid was used as a test case. Results were focused on the optimal electric battery schedule along the time horizon considered. Energy exchange at interconnection point between LV/MV grids is supervised, thus demonstrating that the final battery schedule computed by the proposed algorithm is price-sensitive, minimizing overall operation costs for the considered distribution grid while state-of-charge constraints are complied. The innovative inclusion of multi-period constraints through the concept of temporal neighbors was demonstrated by the presented results, validating the proposed OPF formulation and adding essential OPF features to the literature, when operating LV distribution grids.

The methodology is in line with recent data-privacy concerns. No sensitive behind-the-meter data is exchanged between agents of the grid, such as levels of DRES generation, load, or flexible equipment set-points. Information exchanged during the optimization process used to assure inter-temporal constraints (SOC) refers to the same physical network bus (agent) but for different time intervals (temporal neighbors), which keeps private data unknown for other agents.

As discussed, ADMM-based algorithms rely on the convexity of its sub-problems. In this work, a SDP relaxation is introduce to cope with the non-convexity of the rank constraint. It is important to stress that this relaxation is only proved to be exact for radial networks. Accordingly, under the formulation proposed in this work, convergence to optimum is only assure for radial networks. Furthermore, related works ([24] and [25]) have discovered that, in similar formulation to the one proposed in this work, the number of iterations tends to increase, not significantly with the number of nodes being modelled, but with number of layers of the radial work in study.

Future work will focus on the time-to-convergence of the developed algorithm, including the development of innovative OPF initialization methods as well as adapting current literature methodologies that speed-up sub-problem optimization processes.

Conflicts of interest

The authors declare no conflicts of interest.

Acknowledgements

This work is funded by the European Regional Development Fund (ERDF) through the Operational Programme for Competitiveness and Internationalisation – COMPETE 2020 Programme, and by National Funds through the Portuguese funding agency, FCT – Fundação para a Ciência e a Tecnologia, within project ESGRIDS – Desenvolvimento Sustentável da Rede Elétrica Inteligente/SAICTPAC/0004/2015-POCI-01-0145-FEDER-016434. Rui Pinto was also supported by Fundação para a Ciência e a Tecnologia (FCT) under Grant SFRH/BD/117428/2016.

References

[1] Y. Parag, B.K. Sovacool, Electricity market design for the prosumer era, Nat. Energy 1 (2016) 1–6.
[2] D. Forfia, M. Knight, R. Melton, The view from the top of the mountain: building a community of practice with the gridwise transactive energy framework, IEEE Power Energy Mag. 14 (2016) 25–33.
[3] P.-C. Chen, R. Salcedo, Q. Zhu, F. de Leon, D. Czarkowski, Z.-P. Jiang, V. Spitsa, Z. Zabar, R.E. Uosef, Analysis of voltage profile problems due to the penetration of distributed generation in low-voltage secondary distribution networks, IEEE Trans. Power Deliv. 27 (2012) 2020–2028.
[4] J.T. Hughes, A.D. Domínguez-García, K. Poolla, Identification of virtual battery models for flexible loads, IEEE Trans. Power Syst. 31 (2016) 4660–4669.
[5] J. Villar, R. Bessa, M. Matos, Flexibility products and markets: literature review, Electr. Power Syst. Res. 154 (2018) 329–340.
[6] A.J. Conejo, J.A. Aguado, Multi-area coordinated decentralized DC optimal power flow, IEEE Trans. Power Syst. 13 (1998) 1272–1278.
[7] B.H. Kim, R. Baldick, A comparison of distributed optimal power flow algorithms, IEEE Trans. Power Syst. 15 (2000) 599–604.
[8] S. Boyd, N. Parikh, E. Chu, B. Peleato, J. Eckstein, Distributed optimization and statistical learning via the alternating direction method of multipliers, Found. Trends® Mach. Learn. 3 (2010) 1–122.
[9] Y. Wang, S. Wang, L. Wu, Distributed optimization approaches for emerging power systems operation: a review, Electr. Power Syst. Res. 144 (2017) 127–135.
[10] W. Shi, X. Xie, C.C. Chu, R. Gadh, Distributed optimal energy management in microgrids, IEEE Trans. Smart Grid 6 (2015) 1137–1146.
[11] H. Dagdougui, R. Sacile, Decentralized control of the power flows in a network of smart microgrids modeled as a team of cooperative agents, IEEE Trans. Control Syst. Technol. 22 (2014) 510–519.
[12] E. Dall'Anese, H. Zhu, G.B. Giannakis, Distributed optimal power flow for smart microgrids, IEEE Trans. Smart Grid 4 (2013) 1464–1475.
[13] P. Scott, S. Thiébaux, Dynamic Optimal Power Flow in Microgrids Using the Alternating Direction Method of Multipliers, (2014) arXiv:1410.7868.
[14] A. Kargarian, J. Mohammadi, J. Guo, S. Chakrabarti, M. Barati, G. Hug, S. Kar, R. Baldick, Toward distributed/decentralized DC optimal power flow implementation in future electric power systems, IEEE Trans. Smart Grid 9 (2018) 2574–2594.
[15] Y. Zhang, M. Hong, E. Dall'Anese, S. Dhople, Z. Xu, Distributed controllers seeking AC optimal power flow solutions using ADMM, IEEE Trans. Smart Grid 9 (2018) 4525–4537.
[16] J. Guo, G. Hug, O.K. Tonguz, A case for nonconvex distributed optimization in large-scale power systems, IEEE Trans. Power Syst. 32 (2017) 3842–3851.
[17] S. Magnússon, P.C. Weeraddana, C. Fischione, A distributed approach for the optimal power-flow problem based on ADMM and sequential convex approximations, IEEE Trans. Control Netw. Syst. 2 (2015) 238–253.
[18] M. Kraning, E. Chu, J. Lavaei, S. Boyd, Dynamic network energy management via proximal message passing, Found. Trends® Optim. 1 (2014) 73–126.
[19] X. Bai, H. Wei, K. Fujisawa, Y. Wang, Semidefinite programming for optimal power flow problems, Int. J. Electr. Power Energy Syst. 30 (2008) 383–392.
[20] R.A. Jabr, Radial distribution load flow using conic programming, IEEE Trans. Power Syst. 21 (2006) 1458–1459.
[21] S.H. Low, Convex relaxation of optimal power flow – part I: Formulations and equivalence, IEEE Trans. Control Netw. Syst. 1 (2014) 15–27.
[22] M. Farivar, S.H. Low, Branch flow model: Relaxations and convexification – Part I, IEEE Trans. Power Syst. 28 (2013) 2554–2564.
[23] M. Farivar, S.H. Low, Branch flow model: relaxations and convexification – Part II, IEEE Trans. Power Syst. 28 (2013) 2565–2572.
[24] Q. Peng, S.H. Low, Distributed optimal power flow algorithm for radial networks, I: Balanced single phase case, IEEE Trans. Smart Grid 9 (2018) 111–121.
[25] Q. Peng, S. Low, Distributed Algorithm for Optimal Power Flow on Unbalanced Multiphase Distribution Networks, (2015) arXiv:1512.06482.
[26] C. Duan, L. Jiang, W. Fang, J. Liu, Multi-period OPF with energy storages and renewable sources: a parallel moment approach, 2016 IEEE Power and Energy Society General Meeting (PESGM) (2016) 1–5.
[27] T. Chang, A proximal dual consensus ADMM method for multi-agent constrained optimization, IEEE Trans. Signal Process. 64 (2016) 3719–3734.
[28] E. Munsing, J. Mather, S. Moura, Blockchains for decentralized optimization of energy resources in microgrid networks, 2017 IEEE Conference on Control Technology and Applications (CCTA), Mauna Lani, HI, USA, 2017.
[29] S.H. Low, Convex relaxation of optimal power flow – Part II: Exactness, IEEE Trans. Control Netw. Syst. 1 (2014) 177–189.
[30] L. Gan, S.H. Low, Convex relaxations and linear approximation for optimal power flow in multiphase radial networks, 2014 Power Systems Computation Conference (2014) 1–9.
[31] CVXOPT – Python Software for Convex Optimization, (2019).
[32] Gurobi Optimization – The State-of-the-Art Mathematical Programming Solver, (2019).
[33] A. Madureira, Coordinated and Optimized Voltage Management of Distribution Networks With Multi-Microgrids, Ph.D. Thesis, Faculty of Engineering, University of Porto (FEUP), 2010 [Online] https://repositorio-aberto.up.pt/handle/10216/58358?locale=en.
[34] P. Olival, A. Madureira, M. Matos, Advanced voltage control for smart microgrids using distributed energy resources, Electr. Power Syst. Res. 146 (2017) 132–140.
[35] F. Moret, T. Baroche, E. Sorin, P. Pinson, Negotiation algorithms for peer-to-peer electricity markets: Computational properties, 2018 Power Systems Computation Conference (PSCC), Dublin, Ireland, 2018.