

# Stochastic Dual Dynamic Programming for Operation of DER Aggregators Under Multi-Dimensional Uncertainty

Panagiotis Fatouros<sup>1b</sup>, *Student Member, IEEE*, Ioannis Konstantelos<sup>1b</sup>, *Member, IEEE*,  
Dimitrios Papadaskalopoulos<sup>1b</sup>, *Member, IEEE*, and Goran Strbac, *Member, IEEE*

**Abstract**—The operation of aggregators of distributed energy resources (DER) is highly complex, since it entails the optimal coordination of a diverse portfolio of DER under multiple sources of uncertainty. The large number of possible stochastic realizations that arise can lead to complex operational models that become problematic in real-time market environments. Previous stochastic programming approaches resort to two-stage uncertainty models and scenario reduction techniques to preserve the tractability of the problem. However, two-stage models cannot fully capture the evolution of uncertain processes and the *a priori* scenario selection can lead to suboptimal decisions. In this context, this paper develops a novel stochastic dual dynamic programming approach which does not require discretization of either the state space or the uncertain variables and can be efficiently applied to a multi-stage uncertainty model. Temporal dependencies of the uncertain variables as well as dependencies among different uncertain variables can be captured through the integration of any linear multidimensional stochastic model, and it is showcased for a p-order vector autoregressive model. The proposed approach is compared against a traditional scenario-tree-based approach through a Monte-Carlo validation process, and is demonstrated to achieve a better trade-off between solution efficiency and computational effort.

**Index Terms**—Aggregator, distributed energy resources, multi-dimensional uncertainty, stochastic dual dynamic programming, vector autoregressive modeling.

## NOMENCLATURE

### Indexes

|     |   |
|-----|---|
| $t$ | Index of time periods, running from 1 to $T$ .              |
| $b$ | Index of energy storage (ES) units, running from 1 to $B$ . |
| $f$ | Index of flexible loads (FL), running from 1 to $F$ .       |
| $w$ | Index of wind turbines (WT), running from 1 to $W$ .        |
| $m$ | Index of micro-generators, running from 1 to $M$ .          |

### Parameters

|              |   |
|--------------|---|
| $C_t^{grid}$ | Energy market price at period $t$ .     |
| $C_m^{gen}$  | Operating cost of micro-generator $m$ . |

|                     |  |
|---------------------|--|
| $C^{dem}$           | Cost of demand shedding.   |
| $E_{0,b}$           | Initial energy level of ES unit $b$ .  |
| $\eta_b$            | Round-trip efficiency of ES unit $b$ .                                       |
| $E_b^{max}$         | Maximum energy level of ES unit $b$ .  |
| $E_b^{min}$         | Minimum energy level of ES unit $b$ .  |
| $P_b^s$             | Maximum power rating of ES unit $b$ .  |
| $D_{t,f}^{base}$    | Baseline demand of FL $f$ at period $t$ .                                    |
| $S_f$               | Load shifting limit of FL $f$ at period $t$ .                                |
| $P_t^{g,sell}$      | Maximum power sold to the market at period $t$ .                             |
| $P_t^{g,buy}$       | Maximum power bought from the market at period $t$ .                         |
| $P_m^{gen}$         | Maximum power rating of micro-generator $m$ .                                |
| $P_{t,w}^{wind}$    | Available wind power output of WT $w$ at period $t$ .                        |
| $P_{t-n,w}^{wind}$  | Available wind power output of WT $w$ , $n$ periods before $t$ .             |
| $W_w^{max}$         | Power capacity of WT $w$ .   |
| $\varphi_{n,w}$     | $n^{th}$ autoregressive coefficient associated with power output of WT $w$ . |
| $\varepsilon_{t,w}$ | White noise term associated with power output of WT $w$ at period $t$ .      |
| $d_t^{inf}$         | Inflexible demand at period $t$ .  |
| $\Delta t$          | Temporal resolution of the market.   |

### Variables

|                   |  |
|-------------------|--|
| $p_t^{grid}$      | Power sold to (positive)/bought from (negative) the market at period $t$ . |
| $p_{t,m}^{gen}$   | Power output of micro-generator $m$ at period $t$ .                        |
| $p_t^{dem}$       | Demand shed at period $t$ .  |
| $e_{t,b}$         | Energy level of ES unit $b$ at period $t$ .                                |
| $p_{t,b}^s$       | Power input (positive) / output (negative) of ES unit $b$ at period $t$ .  |
| $d_{t,f}^{sh}$    | Change of demand of FL $f$ at period $t$ due to load shifting.             |
| $d_{t,f}^{total}$ | Demand of FL $f$ at period $t$ after load shifting.                        |
| $p_{t,w}^{wind}$  | Dispatched wind power output of WT $w$ at period $t$ .                     |

## I. INTRODUCTION

### A. Background

A FUNDAMENTAL feature of the emerging *Smart Grid* paradigm involves the integration of a large number of *distributed energy resources* (DER), such as flexible loads, renewable and controllable micro-generators and energy storage units, in order to support the economic operation of the future

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The authors are with the Department of Electrical and Electronic Engineering, Imperial College London, London SW7 2BT, U.K. (e-mail: p.fatouros12@imperial.ac.uk; i.konstantelos@imperial.ac.uk; d.papadaskalopoulos08@imperial.ac.uk; g.strbac@imperial.ac.uk).

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low-carbon power system [1], [2]. However, the large number, small individual size and inherent stochasticity characterizing these DER have complicated system scheduling and market coordination. Furthermore, driven by the wide integration of renewable generation in power systems, there is a general consensus to move energy trading as close as possible to real-time [3], [4], which intensifies the complexity of DER coordination.

These challenges have triggered the introduction of DER aggregators, which group a substantial number of DER (not necessarily located in the same local area or connected through the same distribution network) and represent them in the market, by coordinating their operation according to market opportunities and their individual preferences and requirements [1], [5]. Nevertheless, the operation of a DER aggregator constitutes a challenging high-dimensional stochastic problem, since it entails the optimal real-time coordination of a large number of diverse DER, subject to numerous sources of uncertainty, such as the output of renewable micro-generators and consumers' demand and flexibility patterns.

Existing literature modeling the optimal operation of DER aggregators has mainly focused on *stochastic programming* (SP) approaches, employing two-stage uncertainty models [6]–[14]. A simplified representation of the uncertainty space is usually adopted, utilizing various scenario generation and reduction techniques. In [6], a risk-constrained two-stage stochastic formulation is presented, where stochasticity is captured through a Monte-Carlo scenario based approach and uncertain parameters are assumed to follow normal zero-mean distributions. Ignoring potential dependencies among these uncertainties, 5000 scenarios are initially sampled and are then reduced to 100 for computational tractability purposes. Authors in [7] model uncertainties surrounding load and renewable power output using univariate normal distributions, fit to historical data. In [8] and [9], a fixed number of scenarios is generated via Monte-Carlo simulation to model net wind output and day-ahead (DA) price uncertainty. In [10], a simplistic uniform distribution is assumed to capture the uncertainty associated with the demand of flexible loads. Authors in [11] and [12] develop scenarios describing electric vehicle (EV) uncertain characteristics. In both cases a very limited number of scenarios (100 and 185 respectively) are retained for a 24h scheduling horizon. A wide variety of uncertain variables, including solar irradiance, wind speed, inflexible demand and the availability of distributed generation (DG), energy storage units and the main grid are explored in [13] and [14]. In both cases, even though a large number of scenarios is initially sampled, only a small number is preserved after the scenario reduction procedure, resulting in a coarse coverage of the uncertainty space.

## B. Motivation

These previous SP approaches employ two-stage uncertainty models that exhibit fundamental limitations in capturing the evolution of stochastic processes in a real-time market environment. More specifically, the typical two-stage stochastic problem formulation consists of (i) a first-stage problem to identify

the optimal day-ahead schedule of resources, carried out under uncertainty (ii) multiple second-stage problems, where each problem corresponds to one possible uncertainty realization and is tasked with identifying the optimal schedule re-adjustment given the first-stage commitment and that uncertainty realization; note that the second stage assumes perfect information. It is critical to highlight that this problem structure uses a very simplistic uncertainty description; it is assumed that following the submission of the day-ahead schedule, the uncertainty for the next 24 hours is automatically revealed and the schedule can be fully re-optimized under perfect information. However, in practice, uncertainty is revealed only gradually (i.e., only for the current hour). This necessitates the movement from a two-stage scenario fan to a multi-stage scenario tree description capable of capturing inter-temporal resolution. As such, typical two-stage problem formulations severely underestimate the effect of uncertainty. Despite their shortcomings, existing literature has largely focused on such two-stage stochastic programming approaches due to their simplicity. More sophisticated uncertainty descriptions using multi-stage scenario trees entail a combinatorial explosion of the possible realizations, possibly leading to intractability.

For the same reasons, the existing literature also largely disregards multivariate dependencies among the uncertain variables. However, simplistic descriptions of uncertainty can lead to inefficient solutions since they disregard parts of the uncertainty structure which can be leveraged to make more informed decisions. For example, temporal dependencies may exist between the current state and future evolution of an uncertain variable. In addition, different uncertainty sources may not be independent since relationships exist between different uncertain variables, due to confounding factors, such as weather conditions. In [15] for example, the authors demonstrate that ignoring the stochastic dependence characterizing the multivariate uncertainty around wind farms' output, can lead to suboptimal planning and operation decisions. In a similar vein, authors in [16] present numerous composite modeling approaches for capturing complex non-linear dependency patterns in large power system datasets. Finally, in [17] a multivariate conditional parametric model is proposed for forecasting power output from multiple wind farms, while modeling their spatio-temporal dependence; authors show substantial improvement in forecast quality when the dependence structure is explicitly modeled.

These insights reveal fundamental limitations of the approaches adopted in [6]–[14], in dealing effectively with the high-dimensional stochastic problem faced by the DER aggregator. When such approaches consider multivariate uncertainty in a multi-stage context, a combinatorial explosion of the possible realizations is expected as the planning horizon expands, and the problem soon becomes intractable. Consequently, the deployment of scenario reduction techniques is unavoidable. In general, such techniques determine the final set of considered scenarios 'a priori' (i.e., before the actual problem is solved), according to relevant distance metrics. Therefore, the impact of the stochastic process on the problem is not explicitly evaluated and potentially important scenarios may be ignored. All in all, the requirement for computational tractability imposes limits on

the number of scenarios that can be considered, inadvertently leading to disregarding temporal and multivariate dependencies.

### C. Contributions

In order to overcome the computational tractability problems associated with the incorporation of multi-stage uncertainty and multivariate dependence effects in the DER aggregator operating problem, this paper will be based on *stochastic dual dynamic programming (SDDP)*. This method was firstly introduced in [18] and [19] for the optimal scheduling of hydrothermal generation systems, driven by the need to model the reservoir interconnections for the future inflow sequences. SDDP has been used to model a variety of operational problems [20]. The ability of SDDP to refine solution quality around areas of the state space most likely to occur ('areas of interest') instead of searching the entire state space, facilitates the solution of high dimensional problems.

The fundamental contribution of this paper lies on developing a novel SDDP approach for the optimal operation of a DER aggregator in real-time markets, which can be efficiently applied to a multi-stage uncertainty model and can handle both time-dependent and multivariate uncertainty, through the integration of a *vector autoregressive (VAR)* model of order  $p$ . The traditional SDDP algorithm is extended to account for potential temporal and cross-variable correlations of the stochastic process. This is the first work that employs VAR models for the representation of multivariate uncertainty, driven by their ability to associate current realizations of each of the uncertain variables with the previous  $p$  instances of the entire set of uncertain variables. In this way, both temporal dependencies of uncertain variables and dependencies among different uncertain variables are captured. The stochastic process is integrated in the SDDP algorithm and constitutes an internal part of the solution process, explicitly considering stochastic dependencies.

The proposed SDDP approach is compared against a traditional scenario-tree-based approach with varying tree complexities, in a case study involving an aggregator with diverse generation, demand and storage resources, and facing uncertainty regarding the level of demand to be served and the available wind power output. In order to meaningfully and comprehensively compare the two approaches, Monte-Carlo validation for different demand and wind power starting points is carried out. The proposed approach is demonstrated to achieve a better trade-off between solution efficiency and computational performance, since it yields a similar aggregator's expected cost with the one achieved by the most complex scenario trees, while it exhibits a similar computational performance with the simplest scenario trees. The computational superiority of the proposed approach becomes more significant when longer operating horizons are investigated, further demonstrating the scalability potential of the extended SDDP algorithm to large-scale problems. Finally, the proposed approach is demonstrated to yield better solutions compared to the traditional SDDP framework, which does not consider temporal and cross-variable dependencies of the stochastic process. The proposed framework is bound to increase in importance in the future as the number of controllable elements and sources of uncertainties

increase, rendering operational problems even more computationally challenging.

### D. Paper Structure

The rest of this paper is organized as follows: Section II describes and formulates the DER aggregator's optimal operation problem. The basic principles of the SDDP algorithm are outlined in Section III and its novel extension for incorporating a multivariate stochastic model is detailed in Section IV. The application of the developed model on the examined DER aggregator problem and illustrative results are presented in Section V. Finally, Section VI discusses conclusions of this work.

## II. PROBLEM DEFINITION

In this section, we formulate the multi-stage SP problem of the optimal operation of a DER aggregator participating in a real-time market and facing uncertainty regarding the level of inflexible demand to be served and the available wind power output. We assume that the aggregator portfolio consists of a number of WTs with uncertain power output, controllable micro-generators, uncertain inflexible demand, a group of flexible loads and energy storage units. Therefore, the aggregator must schedule the use of the available wind power output, the amount of energy that will be bought/sold from/to the market, the output of the micro-generators, the flexible loads' consumption levels and the storage units' charging/discharging schedule at each time period  $t$ , in order to minimize the overall cost. Wind output and demand can be curtailed, if required. Fundamentally, this is a multi-stage stochastic problem with recourse, where the stochastic outputs are gradually revealed and decisions are made considering both the already observed and the anticipated outputs.

It is assumed that all the stochastic variables are defined in a common and complete probability space  $(\Omega, \mathcal{F}, \mathcal{P})$ . Then,  $\omega_t$  corresponds to the random variable at period  $t$  and  $\omega_t = (d_t^{inf}, P_{t,w}^{wind}) \in \Omega_t$  represents the realization of the stochastic process  $\omega$  at time  $t$ , where  $d_t^{inf}$  and  $P_{t,w}^{wind}$  correspond to the inflexible demand and wind realizations at  $t$ , respectively.

We first present the mathematical formulation pertaining to a single stage  $t$ . Equation (1) is the objective function at period  $t$  and comprises of the cost of the energy transactions (buying or selling) with the market, the cost of using the micro-generator and the demand shedding cost. In (1),  $x_t$  corresponds to the decision variables vector and is defined as  $x_t \triangleq [p_t^{grid}, p_{t,m}^{gen}, p_t^{dem}, p_{t,b}^s, d_{t,f}^{sh}, p_{t,w}^{wind}]$  where  $x_t \triangleq x_t(\omega_t)$ , but  $\omega_t$  has been dropped for notational convenience. Decisions are taken at the beginning of each period  $t$ , where the uncertainty for the current period has been resolved. This is a fundamental difference compared to scenario tree approaches, since decisions are made sequentially, as the stochastic process has been realized.

$$c_t^T \cdot x_t = -C_t^{grid} \cdot p_t^{grid} + \sum_m C_m^{gen} \cdot p_{t,m}^{gen} + C^{dem} \cdot p_t^{dem} \quad \forall t \quad (1)$$



The operational characteristics of energy storage unit  $b$  are expressed by (2)–(5) [21], [22]. Constraint (2) expresses the energy balance in the storage unit. Constraint (3) corresponds to its maximum depth of discharge and state of charge ratings. Constraint (4) represents its power limits. In order to maintain energy neutrality, the storage energy content at the start and the end of the operating horizon are assumed equal (5).

$$e_{t,b} = \begin{cases} E_{0,b} \cdot \eta_b + p_{t,b}^s \cdot \Delta t, & \text{if } t = 1 \\ e_{t-1,b} \cdot \eta_b + p_{t,b}^s \cdot \Delta t, & \text{if } 1 < t \leq T \end{cases} \quad \forall b \quad (2)$$

$$E_b^{\min} \leq e_{t,b} \leq E_b^{\max} \quad \forall t, b \quad (3)$$

$$-P_b^s \leq p_{t,b}^s \leq P_b^s \quad \forall t, b \quad (4)$$

$$e_{T,b} = E_{0,b} \quad \forall b \quad (5)$$

The flexibility of the demand side is primarily associated with the ability to shift the operation of some loads from periods of higher prices to periods of lower prices [23]. In other words, load reduction during certain periods is accompanied by a *load recovery effect* during preceding or succeeding periods. This implies that the representation of demand flexibility requires the consideration of time-coupling operational characteristics. In this paper, the generic, technology-agnostic model of [24] has been employed for the representation of this time-shifting flexibility, expressed by (6)–(8). The variable  $d_{f,t}^{sh}$  represents the change of demand with respect to the baseline level  $D_{t,f}^{base}$  at period  $t$  due to load shifting, taking negative/positive values when demand is moved away from/towards  $t$ . Constraint (7) expresses the limits of demand change at each period due to load shifting as a ratio  $S_f$  ( $0 \leq S_f \leq 1$ ) of the baseline demand;  $S_f = 0$  implies that load  $f$  does not exhibit any time-shifting flexibility, while  $S_f = 1$  implies that the whole demand can be shifted in time. Finally, constraint (8) ensures that load shifting is energy neutral within the operation horizon i.e., the total size of demand reductions is equal to the total size of demand increases (load recovery), assuming without loss of generality that load shifting does not involve energy losses.

$$d_{t,f}^{total} = D_{t,f}^{base} + d_{t,f}^{sh} \quad \forall t, f \quad (6)$$

$$-S_f \cdot D_{t,f}^{base} \leq d_{t,f}^{sh} \leq S_f \cdot D_{t,f}^{base} \quad \forall t, f \quad (7)$$

$$\sum_t d_{t,f}^{sh} = 0 \quad \forall f \quad (8)$$

Moreover, lower and upper limits of the power associated with the wind turbines, the market and the micro-generators are expressed in (9)–(11) respectively, and the portfolio power balance is ensured by (12).

$$p_{t,w}^{wind} \leq P_{t,w}^{wind} \quad \forall t, w \quad (9)$$

$$-P_t^{g,buy} \leq p_t^{grid} \leq P_t^{g,sell} \quad \forall t \quad (10)$$

$$p_{t,m}^{gen} \leq P_m^{gen} \quad \forall t, m \quad (11)$$

$$\begin{aligned} & \sum_b p_{t,b}^s - \sum_w p_{t,w}^{wind} + p_t^{grid} \\ & + \sum_f d_{t,f}^{total} + d_t^{inf} - p_{t,m}^{gen} = 0 \quad \forall t \end{aligned} \quad (12)$$

Finally, the multi-stage SP is formulated as in (13), where  $\mathcal{C}$  is the total operating cost,  $\omega_t | \omega_{t-1}$  corresponds to the probability of  $\omega_t$  conditioned on  $\omega_{t-1}$  and  $\mathbb{E}_\omega[\cdot]$  denotes the expectation of the terms in the bracket over the stochastic process  $\omega$ . Equation (13) clearly demonstrates the recursive nature of the problem, where decisions are made sequentially as the stochastic process unfolds and more information becomes available to the aggregator. This structure is perfectly aligned with the concept of SDDP that follows.

$$\begin{aligned} \mathcal{C} = \min_{x_1} & \left[ c_1^T \cdot x_1 + \mathbb{E}_{\omega_2} \left[ \min_{x_2} c_2^T \cdot x_2 \right. \right. \\ & + \mathbb{E}_{\omega_3 | \omega_2} \left[ \min_{x_3} c_3^T \cdot x_3 \right. \\ & \left. \left. + \mathbb{E}_{\omega_T | \omega_{T-1}} \left[ \min_{x_T} c_T^T \cdot x_T \right] \right] \right] \quad (13) \\ \text{s.t.} & \quad (1) - (12) \end{aligned}$$

### III. STOCHASTIC DUAL DYNAMIC PROGRAMMING

The key principle of SDDP is that the original multi-stage stochastic problem can be decomposed into a series of single-stage master problems and sub-problems, with appropriate dual variables employed for their coordination. At each stage  $t$ , the corresponding master problem  $M_t$  captures the immediate costs (i.e., for the current stage), along with the approximation of the costs for the remaining stages of the operating horizon, which are incurred as a consequence of the current stage decisions. When  $M_t$  is solved, the decisions pertaining to stage  $t$  are optimized. Then, the respective sub-problem  $S_{t+1}$  optimizes the *future costs* incurred by the aggregator at the remaining stages of the operating horizon (i.e., stages  $t+1$  to  $T$ ), for given stage  $t$  decisions.

Thus, the problem expressed in (13) is now reformulated as in (14), where  $M_1$  represents the first-stage master problem and  $\mathcal{M}_1$  corresponds to the optimal costs for the entire operating horizon, which are incurred by the immediate first stage decisions. Then,  $S_2(x_1, \omega_2)$  represents the respective optimal value for the sub-problem  $S_2$  related to  $M_1$ , where the future cost for horizon  $t' = \{2, \dots, T\}$  is minimized, given the optimal decisions  $x_1$  of  $M_1$  for realization  $\omega_2 \in \Omega_2$ . The realizations  $\omega_t$  are derived either from scenario tree structures, in case there is a finite number of scenarios, or by sampling an appropriate forecasting model (e.g., ARIMA models), when there is a continuous distribution [25].

$$\begin{aligned} M_1 : \mathcal{C} = \mathcal{M}_1 = \min_{x_1} & [c_1^T x_1 + \mathbb{E}_{\omega_2} [S_2(x_1, \omega_2)]] \\ \text{s.t.} & \quad (1) - (12) \end{aligned} \quad (14)$$

The same decomposition process is repeated for every stage of the operating horizon, until  $t$  master problems and  $t-1$  sub-problems (no sub-problem pertains to stage  $t$ ) have emerged. The problem formulation for the subsequent stages is illustrated in (15), which constitutes a generalization of (14) and symbols are used in the same context. It is assumed that the terminal


$$\begin{aligned} \mathcal{M}_t \quad & (x_{t-1}, \omega_t) = \min_{x_1} [c_t^T x_t \\ & M_t : \quad \quad \quad + \mathbb{E}_{\omega_{t+1} | \omega_t} [\mathcal{S}_{t+1}(x_t, \omega_{t+1})]] \quad (15) \\ \text{s.t.} \quad & (1)-(12) \end{aligned}$$
$$\begin{aligned} \alpha_{t+1} &\geq \mathcal{S}_{t+1,k}^i \left( e_{t,b,k,i}, a_{t,f,k,i}^{agg} \right) \\ &\quad + \sum_b \lambda_{t,b,k,i}^E \cdot (e_{t,b} - e_{t,b,k,i}) \quad \forall k, i \\ &\quad + \sum_f \lambda_{t,f,k,i}^D \cdot (a_{t,f}^{agg} - a_{t,f,k,i}^{agg}) \end{aligned} \quad (16)$$

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and significant computational savings are achieved. Each run of the forward pass involves the sampling of different values of the stochastic variables  $\omega_{t,k,i}$ . However, multiple runs of the forward pass  $k = \{1, \dots, NK\}$  pertaining at the same stage  $t$  can be performed simultaneously at each iteration  $i$ . The parallelization of the solution process introduces further computational benefits.

During the backward pass, the approximation of the future cost functions  $\mathfrak{S}_{t+1}$  is gradually improved for  $t = \{1, \dots, T-1\}$  at each iteration with Bender's cuts being constructed for the areas of the state space  $s_{t,k,i}$  identified during the forward pass. Consequently, optimal decisions for  $M_t$ , corresponding to the values of the state variables  $s_{t,k,i}$  obtained during the forward pass run, are applied on  $S_{t+1}$  with their impact on  $S_{t+1}$  captured through the corresponding dual variables. For each point of interest  $k$  identified at the forward pass, the backward pass is solved for all the  $s = \{1, \dots, NS\}$  different samples of the stochastic variables, leading to  $k \cdot s$  problems for each period.

In order to alleviate the computational burden that arises due to the large number of problems, the sub-problems referring to the same period can be solved in parallel. The sub-problems  $S_{t+1}$  consist of all the constraints included in master problems  $M_t$ , augmented by constraints (17)–(18), which are introduced for the dual variable calculation. The auxiliary variables  $\tilde{e}_{t,b,s}$  and  $\tilde{d}_{t,f,s}^{agg}$  are deployed to capture the impact of the  $s_{t,k,i}$ , which have been identified during the forward pass, on the optimal value of the sub-problems  $S_{t+1}$ . After solving all the sub-problems referring to stage  $t$ , a single cut for each point of interest  $k$  is built as per (16) at each iteration  $i$ . The required values for the dual variables  $\lambda_{t,b,k,i}^E$ ,  $\lambda_{t,f,k,i}^D$  and the optimal future cost  $a_{t,k,i}$  per point  $k$  is calculated from (19)–(21), as the expected value over the  $NS$  runs. The derived cuts from  $S_{t+1}$  are appended to the master problem  $M_t$  pertaining to period  $t$ .

$$\tilde{e}_{t,b,s} = e_{t,b,k,i} : \lambda_{t,b,k,s,i}^E \quad \forall t, b, k, s, i \quad (17)$$

$$\tilde{d}_{t,f,s}^{agg} = d_{t,f,k,i}^{agg} : \lambda_{t,f,k,s,i}^D \quad \forall t, f, k, s, i \quad (18)$$

$$\lambda_{t,b,k,i}^E = \sum_s \lambda_{t,b,k,s,i}^E / NS \quad \forall t, b, k, i \quad (19)$$

$$\lambda_{t,f,k,i}^D = \sum_s \lambda_{t,f,k,s,i}^D / NS \quad \forall t, f, k, i \quad (20)$$

$$a_{t,k,i} = \sum_s \alpha_{t,k,s,i} / NS \quad \forall t, k, i \quad (21)$$

Iterations between forward and backward passes continue until  $\mathfrak{S}_{t+1}$  has reached an acceptable accuracy level and an optimal solution of a target quality has been achieved. More specifically, upper and lower bounds of the optimal solution are defined and calculated at each iteration, and the algorithm terminates when the distance between these bounds is within a tolerance value. The upper and lower bounds are given by (22)–(23), and convergence of the two bounds is checked at the end of each forward pass run, according to (25), where  $\bar{z}_{mean}^i$ ,  $\sigma^i$

represent the mean value and standard deviation of the forward pass optimal objective function values at iteration  $i$  [26].

$$\underline{z}^i = \mathcal{M}_1 \quad (22)$$

$$\bar{z}^i = \sum_{t,k} c_{t,k}^T x_{t,k,i} / NK \quad (23)$$

$$\sigma^i = \sqrt{\frac{1}{K-1} \sum_{t,k} (c_{t,k}^T x_{t,k,i} - \bar{z}_{mean}^i)^2} \quad (24)$$

$$\bar{z}_{mean}^i - \frac{1.96 \cdot \sigma^i}{\sqrt{NS}} \leq \underline{z} \leq \bar{z}_{mean}^i + \frac{1.96 \cdot \sigma^i}{\sqrt{NS}} \quad (25)$$

#### IV. SDDP EXTENSION FOR CAPTURING MULTIVARIATE DEPENDENT UNCERTAINTY

The SDDP algorithm presented in the previous section entails the sampling of realizations  $\{\omega_t\}_{t=1}^T$ , which can be obtained either from a continuous or a discrete stochastic process  $\omega$ . Sampling from a continuous distribution exponentially increases the problem complexity, when the operating horizon and the dimensionality of the stochastic process are enhanced, and may lead to intractability issues. This intractability is mainly overcome by assuming that  $\omega$  is stage-wise independent (i.e.,  $\omega$  does not exhibit temporal correlation), a prevailing assumption in literature (e.g., see [18], [19], [25]–[28]). The temporal independence assumption implies that the future cost functions at each  $t$  do not depend on the evolution of  $\omega$ , which allows cut-sharing among all  $M_t/S_t$  corresponding to the same time-period at the cost of ignoring time-dependence of  $\omega$ .

An alternative approach lies in deriving a discretized representation (e.g., scenario tree), where a trade-off between representation complexity and accuracy exists [29]. When multidimensional uncertainty is considered, in order to avoid a combinatorial explosion, this approach ends up with very simple tree structures, unable to accurately capture  $\omega$ .

Both approaches presented above deal with the issue of computational tractability by resorting to simplification. However, the proposed algorithm can accommodate the incorporation of stochastic models which capture complex relationships in  $\omega$  (e.g temporal and among the stochastic variables), as long as the linearity of the stochastic problem is preserved, so that the computation of the required dual variables is not inhibited. VAR models constitute suitable candidates, since they can articulate complex stochastic relationships, while retaining the linearity of the problem. A VAR model depicts the evolution of a multivariate stochastic process  $\omega$ , as a linear function of the previous  $p$  instances of the process. Consequently, a  $p^{th}$  order VAR model has been adopted to represent the  $y$ -dimensional stochastic process  $\omega$ , as expressed in (26), where  $\omega_t$  corresponds to the random variables at  $t$ ,  $A_p$  correspond to  $p$  time-invariant  $y \times y$  matrices,  $\omega_{t-p}$  represents the values of  $\omega_t$   $p$  time-periods before  $t$  and  $\varepsilon_t$  is a  $y \times 1$  error term matrix. The error matrix  $\varepsilon_t$  should contain zero-mean terms (i.e.,  $\mathbb{E}(\varepsilon_t) = 0$ ), while  $\mathbb{E}(\varepsilon_t \varepsilon_t') = K$  and  $\mathbb{E}(\varepsilon_t \varepsilon_{t-p}') = 0$ ,  $\forall p$  should stand for their

covariances, denoting that there is no serial correlation in  $\varepsilon_t$ .

$$\omega_t = \sum_p A_p \cdot \omega_{t-p} + \varepsilon_t \quad \forall t \quad (26)$$

The consideration of temporal and cross-variable correlations of  $\omega$  and the explicit integration of the stochastic model in the solution process of the algorithm, suggest that additional state variables, expressing the evolution of  $\omega$  until the current stage, should be introduced. Therefore, the new state variable vector is expanded to  $s_t \triangleq \{e_{t,b}, d_{t,f}^{agg}, \omega_{t-1}, \dots, \omega_{t-p}\}$  depending on the order of the VAR model and the modified cut is expressed in (27), where  $S_{t+1,k}$  has been enhanced with the expanded  $s_t$  and a new term representing the impact of the previous  $p$  instances of  $\omega_t$  on  $S_{t+1}$  has been added. The dual variables associated with  $\omega_{t-p,k,i}$  are obtained according to (28)–(29) when problem  $S_t$  is solved, where  $\tilde{\omega}_{t-p,s}$  are slack variables introduced for the calculation of  $\lambda_{t,k,s,i}^\omega$ . We should stress out that a complex multidimensional stochastic model, collapses into naïve white error sampling, when integrated in the SDDP formulation, while only increasing the state space dimensionality. However, it is crucial that  $\omega_{t-p,k,i}$  refer to deterministic and already-resolved quantities, when  $M_t/S_t$  are solved, so they comprise inputs of the model. Consequently, the state space expansion does not entail increased computational burden and complexity.

$$a_{t+1} \geq S_{t+1,k}^{i-1} \left( e_{t,b,k,i}, d_{t,f,k,i}^{agg}, \omega_{t-1,k,i}, \dots, \omega_{t-p,k,i} \right) + \sum_b \lambda_{t,b,k,i}^E \cdot (e_{t,b} - e_{t,b,k,i}) \quad \forall k, i \quad (27)$$

$$+ \sum_f \lambda_{t,f,k,i}^D \cdot (d_{t,f}^{agg} - d_{t,f,k,i}^{agg}) + \sum_p \lambda_{t,k,i}^\omega \cdot (\omega_{t-p} - \omega_{t-p,k,i})$$

$$\tilde{\omega}_{t-p,s} = \omega_{t-p,k,i} : \lambda_{t,k,s,i}^\omega \quad \forall p, k, s, i \quad (28)$$

$$\lambda_{t,k,i}^\omega = \sum_s \lambda_{t,k,s,i}^\omega / NS \quad \forall p, k, s, i \quad (29)$$

Another important implication is that when sampling  $\omega$  from a VAR model, output cannot be forced to conform to the underlying process bounds (e.g., maximum wind power output). Ex-post adjustment of these offending realizations to within the acceptable domain (e.g., negative values corrected to 0) undermines model convexity [28]. This issue can be mitigated by employing penalty variables for each of the terms of  $\omega_t$  with upper and lower limits. For the DER aggregator model, this refers to both terms of  $\omega_t$ , which physically cannot be negative and cannot exceed their maximum values. Thus, variables  $y_t = [y_{t,w}^1, y_{t,w}^2, y_t^{d1}, y_t^{d2}]$  are introduced and penalized with factor  $c^{pen}$ , which is a vector consisting of  $2(1+w)$  elements; appropriate penalty terms are appended to all  $M_t$  and  $S_t$ , as in (30). Constraints (31)–(36) are included to restrict  $\omega_t$  and (9) has been omitted, while  $P_{t,w}^{wind}$  and  $D_t^{inf}$  are the values sampled from (26) for the available wind power output and demand to

TABLE I  
CASE STUDY PARAMETERS

| Energy storage parameters   |                      |
|-----------------------------|----------------------|
| $E_b^{\max} / E_b^{\min}$   | 700/140 kW·h         |
| $P_b^s$                     | 280 kW               |
| $E_{0,b}$                   | 350 kW·h             |
| $\eta_b$                    | 0.9                  |
| Flexible demand parameters  |                      |
| $D_{t,f}^{base}$            | 200 kW               |
| $S_f$                       | 0%, 10%, 20%, 30%    |
| System parameters           |                      |
| $W_1^{\max} / P_t^{g,sell}$ | 700 kW/0 kW          |
| $P_t^{g,buy} / P_t^{gen}$   | 300 kW/300 kW        |
| $C_t^{grid}, C_t^{gen}$     | 0.15 £/kW·h/1 £/kW·h |

be served.

$$\mathcal{M}_t(x_{t-1}, \omega_t) = \min_{x_1} [c'_t x_t + c^{pen'} y_t + \mathbb{E}_{\omega_{t+1} | \omega_t} [S_{t+1}(x_t, \omega_{t+1})]] \quad (30)$$

$$\text{s.t.} \quad (1) - (8), (10) - (12), (26)$$

$$P_{t,w}^{wind} \leq P_{t,w}^{wind} + y_{t,w}^1 + y_{t,w}^2 \quad (31)$$

$$M_t : y_{t,w}^1 \leq W_w^{\max} - P_{t,w}^{wind} \quad (32)$$

$$y_{t,w}^1 \leq 0 \quad (33)$$

$$d_t^{inf} = D_t^{inf} + y_t^{d1} + y_t^{d2} \quad (34)$$

$$y_t^{d1} \leq D^{\max} - D_t^{inf} \quad (35)$$

$$y_t^{d1} \leq 0 \quad (36)$$

We should emphasize that, when  $S_t$  is solved for an out-of-bounds value of  $\omega_t$ , the specific realization of  $\omega_t$  does not constitute a valid point of the state space and the respective dual variables do not interpret the real impact of  $\omega_t$  on  $S_t$ . As such, the construction of the respective cut is omitted.

Finally, even though a VAR model has been selected for the examined DER aggregator problem, any linear stochastic model could be incorporated in the traditional SDDP algorithm in a straightforward way, following the process outlined in this section.

## V. CASE STUDIES

### A. Description, Data and Implementation

The examined study focuses on a DER aggregator participating in a real-time market, operating a portfolio consisting of one wind turbine, one controllable micro-generator, inflexible and flexible demand and one energy storage unit, and facing uncertainty regarding the level of inflexible demand to be served and the available wind power output. The deterministic parameters of the case study are presented in Table I. In order to capture these uncertainties, historical wind power output and demand data from the Northern UK area and for a period of one month (so that seasonal effects are avoided) have been employed [30]. A market resolution of  $\Delta t = 1$  h is considered and two different operating horizons of 6 and 24 hours are investigated. The



TABLE II  
SCENARIO TREE STRUCTURE FOR THE 6h HORIZON STUDY

| Complexity level | Structure (nodes per stage) | Number of nodes | Number of scenarios |
|------------------|-----------------------------|-----------------|---------------------|
| C1               | 1-1-1-1-1-1                 | 6               | 1                   |
| C2               | 1-2-2-2-2-2                 | 11              | 2                   |
| C3               | 1-2-4-6-8-10                | 31              | 10                  |
| C4               | 1-2-4-8-12-16               | 43              | 16                  |
| C5               | 1-2-4-8-16-32               | 63              | 32                  |
| C6               | 1-3-9-27-81-243             | 364             | 243                 |

required models have been implemented in MATLAB R2015a [31] and FICO Xpress [32] on a 3.33 GHz Intel Xeon computer.

### B. 6 h Operating Horizon Case Study

The 6 h horizon case study aims at comparing the proposed SDDP approach against a traditional stochastic programming approach employing scenario trees. Regarding the latter approach, a multivariate copula model [16] has been fit to the available data, and 5,000 scenarios have been sampled from this model. In order to comprehensively investigate the performance of this approach, six scenario trees of varying complexity have been formed based on the initial set of 5,000 scenarios (see Table II), by employing a standard scenario reduction process based on Kantorovich distance [33].

Regarding the proposed SDDP approach, the dependencies of the multidimensional uncertainty set are captured through a VAR (1) model, which has been fit to the same set of 5,000 scenarios. Autoregressive models require that the fitted data are stationary i.e., statistical properties are time-invariant. In order to achieve that, time-series differentiation [34] has been employed in the case of wind power data, while for inflexible demand data, mean subtraction and division by the standard deviation—to remove the diurnal component—has preceded the differentiation process.

In order to meaningfully compare the solution efficiency of the seven implemented models (six models corresponding to the six different scenario trees and one model corresponding to the extended SDDP approach, which is denoted SDDPe in the remainder), the first-stage decisions taken by each model are used for Monte Carlo validation i.e., given the obtained first-stage decisions, the aggregator's expected cost over the whole set of 5,000 realizations is calculated. In this way, we can capture the impact of sub-optimal first-stage decisions on subsequent operation. In order to ensure that this comparison is not prone to the selection of the starting point, this process is repeated for 10 starting points regarding wind power output (corresponding to 5%–95% of the wind turbine's capacity  $W_w^{\max}$  with a step of 10%) and 24 starting points for inflexible demand (representing typical demand levels at different hours of the day). Thus, a problem of 1.2 million scenarios has been solved.

For benchmarking purposes, the results of two additional models are reported. Firstly, the traditional SDDP algorithm, where temporal and cross-variable independence of the stochastic process is assumed, is simulated. In this context, a lattice approach, where all scenarios are equiprobable and stochas-

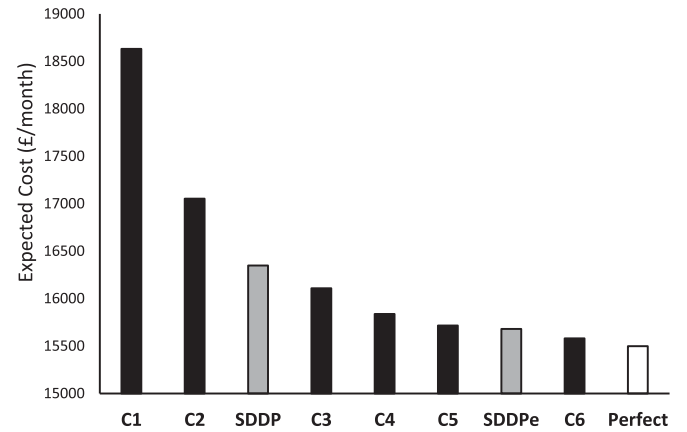


Fig. 2. Expected cost over all starting points for SDDP, SDDPe, different scenario tree types and under perfect information conditions in the 6 h horizon case study.

tic transitions do not consider the previous realizations of the stochastic process, is adopted. Secondly, an idealized deterministic model is also implemented, where the aggregator is assumed to have perfect information regarding the future evolution of uncertain variables. In that case, optimal first stage decisions are obtained for each scenario separately.

Fig. 2 compares the expected monthly aggregator cost obtained from each of the six scenario tree models, the proposed (SDDPe) and traditional SDDP models and the model involving perfect information. These costs are derived by taking the expectation over all the 1.2 million realizations and extrapolating the costs of the examined 6-hour horizon to a monthly horizon. The performance of the scenario tree approach is improved as the complexity of the employed tree is enhanced, given that the representation of the stochastic process becomes more accurate and thus better-informed decisions are made. The proposed SDDPe model massively outperforms the two simpler scenario tree models C1 and C2 by 10–20%, while showcasing a benefit of approximately 0.5–3% with respect to models C3–C5. The most complex tree model (C6) slightly outperforms the SDDPe approach (by 0.6%). As expected, the perfect information model yields the lowest cost, but the savings it yields compared to SDDPe and C6 are relatively small (1–1.5%). Finally, it can be observed that the proposed SDDPe approach provides better results than the traditional one, exhibiting a benefit of 4.3%. Intuitively, disregarding the temporal characteristics of the stochastic process by assuming stage-wise independence leads to poorer first-stage decisions.

In order to compare the computational performance of the different models, Table III presents the number of decision variables, total constraints and time-coupling constraints corresponding to each of the seven models, while Fig. 3 showcases the respective computational times. The computational complexity of the scenario tree approach is exponentially aggravated as the complexity of the employed tree is enhanced, as demonstrated in Table III. Regarding the traditional and the proposed SDDP models, the respective numbers correspond to the total number of variables and constraints included in a single-stage problem ( $M_t/S_t$ )—which are solved sequentially—multiplied by the number of such single-stage problems. These



TABLE III  
NUMBER OF DECISION VARIABLES, TOTAL AND COUPLING CONSTRAINTS FOR SDDP, SDDPe AND DIFFERENT SCENARIO TREE TYPES

|              | Decision variables | Total constraints | Time-coupling constraints |
|--------------|--------------------|-------------------|---------------------------|
| <b>SDDP</b>  | 19*11              | 20*11             | 0                         |
| <b>SDDPe</b> | 19*11              | 20*11             | 0                         |
| <b>C1</b>    | 36                 | 68                | 7                         |
| <b>C2</b>    | 66                 | 126               | 14                        |
| <b>C3</b>    | 126                | 258               | 70                        |
| <b>C4</b>    | 186                | 390               | 112                       |
| <b>C5</b>    | 378                | 886               | 224                       |
| <b>C6</b>    | 2184               | 5584              | 1701                      |

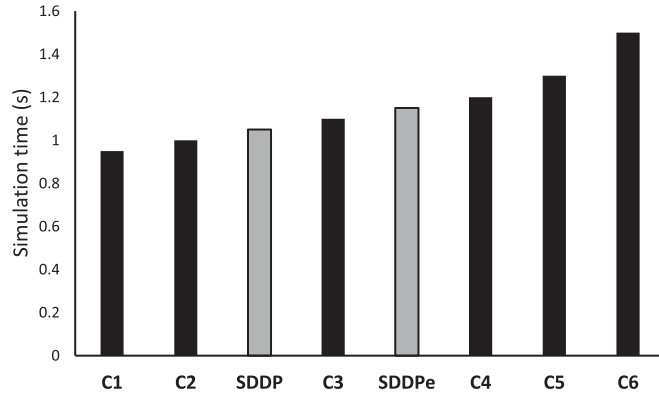


Fig. 3. Simulation time for SDDP, SDDPe and different scenario tree types in the 6 h horizon case study.

numbers are lower than the numbers corresponding to all scenario trees. It should be noted that time-coupling constraints which significantly contribute to problem complexity are inherently avoided by SDDP and SDDPe.

The proposed SDDPe approach also involves less computational time than scenario trees C4-C6, as depicted in Fig. 3. The average computation time of the SDDPe approach was 1.15 s and the algorithm converged after 5–7 iterations. These computational advantages are driven by the fact that SDDP involves stage-wise decomposition of the optimization problem. The computational performance of the traditional SDDP algorithm is slightly better than the proposed one due to the stage-wise independence assumption. However, the computational advantage of SDDP and SDDPe is not particularly pronounced due to the small operating horizon of the study.

All in all, the combination of the insights from Figs. 2–3 and Table III demonstrate that SDDPe achieves a better trade-off between solution efficiency and computational performance with respect to scenario-tree-based approaches, since it yields a similar aggregator's expected cost with the one achieved by the most complex scenario trees, while it exhibits a similar computational performance with the simplest scenario trees.

Fig. 4 presents the expected cost savings achieved by the proposed SDDPe model with respect to the scenario tree models C1, C2, C3 and C5 for the different wind power output starting points, where positive (negative) values signify that SDDPe outperforms (underperforms) the scenario tree solution. First of all, SDDPe outperforms the two simpler scenario tree models

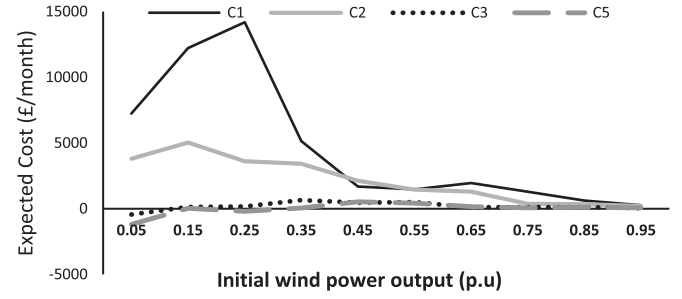


Fig. 4. Difference in expected cost between SDDPe and C1, C2, C3 and C5 respectively for different wind power output starting points in the 6 h horizon case study.

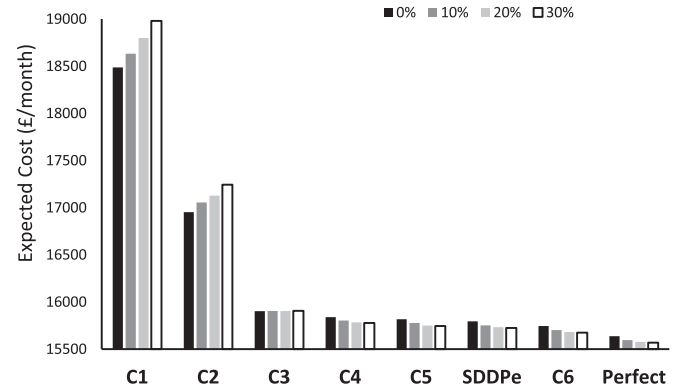


Fig. 5. Expected cost for  $S_f = \{0\%, 10\%, 20\%, 30\%\}$  for SDDPe, different scenario tree types and under perfect information conditions.

C1 and C2 for all starting points. However, this benefit is significantly higher for starting points involving lower wind power and diminishes as we move to starting points involving higher wind power. This effect emerges as low wind power starting points indicate that less wind power is likely available in the operating horizon, increasing the impact of strategic decisions regarding the use of flexible DER in the aggregator's portfolio. On the other hand, high wind power starting points indicate that abundance of wind power is likely available in the operating horizon, and strategic decisions have minor impacts on the aggregator's expected cost.

Compared to the scenario tree models C3 and C5, SDDPe provides consistently better results for middle and high wind power starting points, and slightly worse results for low wind power starting points. This effect emerges since low wind power output starting points are more likely to yield scenarios with negative wind power outputs, in which case the construction of meaningful cuts within the SDDPe algorithm is inhibited and thus the solution quality deteriorates. These results justify the authors' decision to employ multiple starting points in the comparison of the different models.

Fig. 5 compares the expected aggregator cost obtained from the different models in a similar logic with Fig. 2, but this comparison is now made for different cases regarding the extent of demand flexibility in the aggregator's portfolio, as expressed by the load shifting limit  $S_f$ . The particularly interesting result lies in the fact that the simpler scenario tree models C1-C3 exhibit an increase in expected cost with increasing demand flexibil-



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**Panagiotis Fatouros** (S'15) received the Dipl.-Eng. degree in electrical and electronic engineering from NTUA, Athens, Greece, in 2009. He is currently working toward the Ph.D. degree at Imperial College London, London, U.K. His current research focuses on the coordination and integration of distributed energy resources in energy systems under multidimensional uncertainty, employing several decomposition approaches.

**Ioannis Konstantelos** (M'13) is a Research Fellow at Imperial College London, London, U.K. His research interests include mathematical programming and data-driven modeling techniques applied to the planning and operation of energy systems.

**Dimitrios Papadaskalopoulos** (M'13) is a Research Fellow at Imperial College London, London, U.K. His current research focuses on the development and application of distributed and market-based approaches for the coordination of operation and planning decisions in power systems, employing optimization and game theoretic principles.

**Goran Strbac** (M'95) is a Professor of electrical energy systems at Imperial College London, London, U.K. His research interests include electricity system operation, investment and pricing, and integration of renewable generation and distributed energy resources.