Sufficient Conditions for Exact Relaxation of Complementarity Constraints for Storage-Concerned Economic Dispatch

Zhengshuo Li, Qinglai Guo, Hongbin Sun, and Jianhui Wang

***Abstract*—Storage-concerned economic dispatch (ED) problems with complementarity constraints are strongly non-convex and hard to solve. In this letter, an exact relaxation method is proposed to relax the non-convex ED to a convex form under two sufficient conditions, which can be further generalized for other problems of similar structures. A mathematical proof is provided, and the numerical tests verify the effect of this method.**

***Index Terms*—Complementarity constraint, economic dispatch, relaxation, storage.**

I. Introduction

THE use of energy storage systems is being widely considered in power generation economic dispatch (ED) [1]. However, complementarity constraints, which prevent simultaneous charging and discharging of storage, should be included in a storage-concerned ED model, making the model strongly non-convex and hard to solve efficiently. Mixed-integer programming (MIP) methods are usually used, with exact penalty methods, smoothing methods, and regularization relaxation methods also tested. However, these methods may result in long solution time due to additional integer variables or excessive iterations. Exact relaxation methods were recently proposed in [2] and [3], based on an additional assumption that charging storages does not affect or decrease the total operational cost of the grid. Although the methods neither introduce additional variables nor lead to iterations, they may be challenged under conditions where storage owners should pay the grid for charging energy, or the energy constraints of storage are active. It is thus necessary to investigate if the complementarity constraint can be "exactly" relaxed for general cases. In this letter, an exact relaxation method is proposed to relax the non-convex ED to a convex form under two sufficient conditions, which can be also applied to general cases.

II. Storage-Concerned ED

Consider a power system that has N buses and L lines, and the dispatch horizon lasts from $t=1$ to $t=T$, with a time interval Δt. A direct current (DC) model based storage-concerned ED model [4] can be formulated as follows:

$$\min \sum_{i\in N}\sum_{t\in T}\left(g_i\left(P_i^{dc}(t)\right)-f_i\left(P_i^{ch}(t)\right)\right) + \sum_{i\in N}\sum_{t\in T} h_i\left(P_i^G(t)\right) \quad (1)$$

subject to the following constraints for any $t\in T$, $i\in N$, and $j\in L$:

$$0\le P_i^{ch}(t)\le \overline{P}_i^{ch}(t), \qquad \alpha_{i,1}(t),\ \alpha_{i,2}(t) \quad (2)$$

$$0\le P_i^{dc}(t)\le \overline{P}_i^{dc}(t), \qquad \alpha_{i,3}(t),\ \alpha_{i,4}(t) \quad (3)$$

$$E_i(t)=(1-\varepsilon_i)^t E_i^0+\sum\nolimits_{\tau=1}^{t}(1-\varepsilon_i)^{t-\tau} \times\left(\eta_i^{ch}P_i^{ch}(\tau)-P_i^{dc}(\tau)/\eta_i^{dc}\right)\Delta t \quad (4)$$

$$E_i^{\min}(t)\le E_i(t)\le E_i^{\max}(t), \quad \beta_{i,1}(t),\ \beta_{i,2}(t) \quad (5)$$

$$\sum\nolimits_{t=1}^{T}(1-\varepsilon_i)^{T-t}\left(\eta_i^{ch}P_i^{ch}(t)-P_i^{dc}(t)/\eta_i^{dc}\right)\Delta t \ge E_i^r,\ \varphi_i \quad (6)$$

$$P_i^{ch}(t)P_i^{dc}(t)=0 \quad (7)$$

$$\underline{P}_i^G\le P_i^G(t)\le \overline{P}_i^G \quad (8)$$

$$R_i^{dn}\Delta t\le P_i^G(t+1)-P_i^G(t)\le R_i^{up}\Delta t \quad (9)$$

$$\sum_{i\in N}P_i^G(t)+\sum_{i\in N}\left(P_i^{dc}(t)-P_i^{ch}(t)\right)=\sum_{i\in N}D_i(t),\ \lambda(t) \quad (10)$$

$$\underline{P}_j^{Ln}\le \sum_{i\in N}GSF_{j-i}\left(P_i^G(t)+P_i^{dc}(t)-P_i^{ch}(t)-D_i(t)\right) \le \overline{P}_j^{Ln},\ \mu_{j,1}(t),\ \mu_{j,2}(t) \quad (11)$$

where g_i is the discharging cost of the storage at bus i; f_i is the storage charging fee. $f_i>0$ means storage pays grid for charging, and vice versa if $f_i<0$; h_i is the operational costs of the generator at bus i; P_i^{ch} and $P_i^{dc}(t)$ are the grid-side charging and discharging power of the storage at time t; $\overline{P}_i^{ch}(t)$ and $\overline{P}_i^{dc}(t)$ are the rated limits of the charging and discharging power; $\eta_i^{ch}(t)$ and $\eta_i^{dc}(t)$ are charging and discharging efficiencies; $E_i(t)$ is the stored energy at time t; $E_i^{\min}(t)$ and $E_i^{\max}(t)$ are the lower and upper limits of the stored energy; E_i^0 is the initial energy; ε_i is the self-discharge rate; E_i^r is the total charging demand; $P_i^G(t)$ is the output of the generator at time t; $\underline{P}_i^G$ and $\overline{P}_i^G$ are the lower and upper limits of the output; R_i^{up} and R_i^{dn} are the ramp limits. $D_i(t)$ is the load at bus i at time t; GSF_{j-i} is the generation shift factor to line j from bus i. $\overline{P}_j^{Ln}$ and $\underline{P}_j^{Ln}$ are the upper and lower limits of the transmission capacity; $\lambda(t)$, $\alpha_{i,1}(t)$ to $\alpha_{i,4}(t)$, $\beta_{i,1}(t)$, $\beta_{i,2}(t)$, $\mu_{j,1}(t)$, $\mu_{j,2}(t)$, and φ_i are the multipliers of the corresponding constraints.

The model can be described as follows. The objective in (1) is the total operational cost of the generators and the storages. Usually, h_i is modeled as a convex quadratic function, g_i is a convex non-decreasing function, and f_i is a linear function, so that the objective is convex. If the storage is owned by the power system, g_i and f_i can be zero. Constraints (2) and (3) are the rated charging and discharging power limits of the storage; constrain (4) is the integral relationship between the stored energy and the prior charging and discharging process from $\tau=1$ to t, with self-discharge and a round-trip efficiency considered [2]; in

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Z. Li, Q. Guo, and H. Sun are with the Department of Electrical Engineering, State Key Laboratory of Power Systems, Tsinghua University, Beijing 100084, China (e-mail: shb@tsinghua.edu.cn).

J. Wang is with Argonne National Laboratory, Argonne, IL 60439 USA.

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(5), the stored energy limit is modeled, which equivalently represents the state-of-charge limit; constraint (6) represents the net charging requirements—especially for an aggregation of electric vehicles (EVs), E_i^r is the total charging demand. Constraint (7) is the complementarity constraint, which makes the problem strongly non-convex. If the storage is an aggregation of EVs, the upper and lower limits of (2), (3), and (5) can be time-varying, so time stamps are used. Constraints (8) and (9) describe the generation and ramp limits of a generator; constraint (10) is the power balance of the power system at time t, and (11) describes the bidirectional transmission capacity limits of the lines.

III. Relaxation Conditions and Proof

If (7) is relaxed, i.e., removed from the model, the model becomes convex so that the global optimal solution can be easily obtained [note that Karush-Kuhn-Tucker (KKT) conditions are both necessary and sufficient]. It can be proven that the relaxation is exact under the following sufficient conditions: for any $i \in N$:

Cond. 1: $\inf g_i'(P_i^{dc}(t)) \geq \sup f_i'(P_i^{ch}(t)),\ \forall t$.

Cond. 2: $f_i'(P_i^{ch}(t)) < \lambda(t) + \sum_j GSF_{j-i}(\mu_{j,1}(t) - \mu_{j,2}(t)),\ \forall t$

Proof: Assume there exists $P_i^{ch}(t) > 0$ and $P_i^{dc}(t) > 0$ for storage i at time t in the optimal solution of the relaxed model (RM). Then $\alpha_{i,1}(t) = 0$, $\alpha_{i,3}(t) = 0$, because of the complementary slackness conditions. Let L denote the Lagrangian function of the RM, ξ_i denote $1 - \varepsilon_i$, and $\Gamma(t) = \sum_{\tau \geq t} \xi_i^{\tau - t}(\beta_{i,1}(\tau) - \beta_{i,2}(\tau)) + \xi_i^{T-t}\varphi_i$, then, using KKT conditions, the following equation

$$\frac{\partial L}{\partial P_i^{ch}(t)} = -f_i'\left(P_i^{ch}(t)\right) - \alpha_{i,1}(t) + \alpha_{i,2}(t) - \eta_i^{ch}\Gamma(t)\Delta t + \lambda(t) + \sum_j GSF_{j-i}\left(\mu_{j,1}(t) - \mu_{j,2}(t)\right) = 0 \tag{12}$$

holds. With $\alpha_{i,1}(t) = 0$, $\alpha_{i,2}(t) \geq 0$, and Cond. 2, it can be seen that $\Gamma(t) > 0$ holds for storage i at time t.

Using KKT conditions again, the following equation

$$\frac{\partial L}{\partial P_i^{dc}(t)} = g_i'\left(P_i^{dc}(t)\right) - \alpha_{i,3}(t) + \alpha_{i,4}(t) + \Gamma(t)\Delta t/\eta_i^{dc} - \lambda(t) - \sum_j GSF_{j-i}\left(\mu_{j,1}(t) - \mu_{j,2}(t)\right) = 0 \tag{13}$$

holds. With $\alpha_{i,3}(t) = 0$, by combining (12) and (13)

$$\left(1/\eta_i^{dc} - \eta_i^{ch}\right)\Gamma(t)\Delta t + g_i' - f_i' + \alpha_{i,2}(t) + \alpha_{i,4}(t) = 0 \tag{14}$$

holds. Because of $\alpha_{i,2}(t), \alpha_{i,4}(t) \geq 0$, $1/\eta_i^{dc} - \eta_i^{ch} > 0$ and Cond. 1, it can be inferred that $\Gamma(t) \leq 0$ also holds for storage i at time t. Therefore, it can be seen that $P_i^{ch}(t) > 0$ and $P_i^{dc}(t) > 0$ cannot both appear in the optimal solution of the RM for any storage at any time slot. Hence, the relaxation is exact under sufficient Conds. 1 and 2.

The above conditions and proof can be also applied to an extended storage-concerned ED model with convex additions, e.g., with the generation of renewable energy considered. The proof also holds when (6) is not considered. Conds. 1 and 2 can also be applied to the case in [2] and [3] with $f_i' \leq 0$.

Obviously, with the charging and discharging prices as inputs, Cond. 1 can be easily checked. Moreover, Cond. 1 can be satisfied in two scenarios. First, if the storage is owned by the power system, its operational cost may be neglected in dispatch, so $g_i = f_i = 0$; therefore, Cond. 1 is satisfied. Second, if the storage is owned by a third party, in order to attract those owners to participate in ED, the marginal compensation paid to them for discharging a unit of energy must cover the costs for the owner to charge that amount of energy; therefore, it must still satisfy $\inf g_i'(P_i^{dc}(t)) \leq sup \sum_i'(P_i^{ch}(t))$.

Regarding Cond. 2, because the location marginal price at bus $i(LMP_i)$ in a DC model based ED can be determined from $\lambda(t) + \sum_j GSF_{j-i}(\mu_{j,1}(t) - \mu_{j,2}(t))$, Cond. 2 can be transformed to $f_i'(P_i^{ch}(t)) < LMP_i$, which means the charging price should be (strictly) less than the LMP at the connected location. This condition can be checked prior to solving the RM, if the LMP_i or its lower bound can be predicted by using the available historical data, e.g., with an artificial neural network (ANN) approach, which is consistent with price forecasting employed in practice. In a scenario where a charging price is determined by the government or a power grid company, since flexible charging may benefit the grid, it is reasonable to expect that storage might be rewarded by charging at a low price, even lower than LMP_i, so as to attract more storage to take part in grid dispatch. Hence, Cond. 2 would be satisfied most likely in reality.

The exactness of the RM can be interpreted intuitively: with Conds. 1 and 2 both satisfied, charging and discharging a storage device simultaneously is an uneconomic dispatch, so even if (7) is relaxed, the optimal solution still implicitly satisfies the complementarity constraint.

TABLE I
Relaxation and Computational Time

(f_i', g_i')	Exact	Lowest LMP ($/MWh)	Time of MIP method (s)	Time of solving RM (s)
(0, 0)	yes	13.32	6.248	1.893
(10, 15)	yes	15.68	5.581	1.986

IV. Numerical Simulations

An IEEE 30-bus system with 5 units whose total generation capacity is 520 MW is tested. The output of a wind farm is assumed in the range of 0–50 MW. Fifty storages with a power capacity of 400 kW and energy capacity of 2 MWh are considered [2], with charging and discharging efficiency of 90%. The non-convex ED is solved by an MIP method and CPLEX as a solver. Results are shown in Table I.

With Conds. 1 and 2 satisfied in Table I, the non-convex ED can be exactly relaxed, for the maximum of $P_i^{ch}(t)P_i^{dc}(t)$ for any i and any t is less than 10^{-8}. The objectives of the RM and the original non-convex ED are exactly the same. Since the solution time of solving RM is less than 35% of that using an MIP method, it verifies that the proposed method makes it more efficient to solve the non-convex ED. Moreover, if $(f_i', g_i') = (15, 14)$, or $g_i' = 25 > f_i' = 24 >$ LMP, the maximum of $P_i^{ch}(t)P_i^{dc}(t)$ is over 0.12, which indicates that sufficient Conds. 1 and 2 mutually guarantee the exactness of the relaxation.

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