A Spatially Distributed Multi-Period Optimal Power Flow Analysis of Radial Active Distribution Networks with Distributed Battery Units

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Abstract—

*Index Terms—***Batteries, distribution network, distributed energy resources (DERs), equivalent network approximation (ENApp)**

I. INTRODUCTION

A. Background and Prior Arts

Presently, optimal power flow (OPF) tools are developed to run the MV/LV distribution grids in the most economical, reliable, and secure manner. The usefulness of OPF studies is gaining more interest due to penetration of distributed energy resources (DERs), especially solar photovoltaic panels. Power generation from these DERs are influenced majorly by the weather conditions, hence highly intermittent nature. Presently, deployment of battery units are becoming more pertinent to mitigate the uncertainty effect and maintain the power balance by controlling the charging and/or discharging operations [1]. However, inclusion of batteries converts the conventional single period time decoupled OPF problem into a multi-period time coupled OPF analysis.

Traditionally, centralized OPF methods were popular where required data are accumulated at a central controller location [2]. The central controller is responsible to process all the accumulated data, solving the OPF algorithm and dispatch control signals to the controlling resources. Yuan et al. [3] proposed a linear OPF model for distribution network depending upon the locational marginal price (LMP). The LMP is calculated by including reactive power components and voltage constraints.

Guo et al. [4] developed a linear OPF model after linearizing the second-order cone constraints with polyhedral approximations. The OPF problem is formulated by considering the variable solar power generation as parameters and hence the overall problem takes form of a parametric distribution OPF.

B. Research Gaps and Contributions

A taxonomy table to compare the existing studies and the present work is provided in I.

The specific contributions are as follows:

1) The overall problem is formulated as a non-convex programming and the

TABLE I
TAXONOMY TABLE FOR COMPARISON

References	DERs	Batteries	Single period OPF	Multi-period OPF	Centralized OPF	Distributed OPF	Framework
[3]			✓		✓		Linear
[4]	✓		✓	✓			Linear
[], []	✓	✓				✓	
[]- []	✓			✓			✓
[], []		✓		✓			✓
[]- []	✓			✓			✓
This paper	✓	✓		✓		✓	Non-convex

II. PROBLEM FORMULATION

A. Notations

In this study, the distribution network is accounted as a tree (connected graph) having N number of buses (indexed with i, j, and k) and the study is conducted for T time steps (indexed by t). The distribution line connecting two buses i and j are denoted by ij (having resistance and reactance of r_{ij} ohm and x_{ij} ohm, respectively) and magnitude of the current flowing through the line at time t is denoted by I_{ij}^t ($l_{ij}^t = \left(I_{ij}^t\right)^2$). The voltage magnitude of bus i at time t is given by $V_i^t \in [V_{min}, V_{max}]$ ($v_i^t = (V_i^t)^2$). Apparent power demand at a node j at time t is $s_{Lj}{}^t$ ($= p_{Lj}{}^t + jq_{Lj}{}^t$). The uncontrolled active power generation from the DER present at bus j at time step t is denoted by $p_{Dj}{}^t$ and controlled reactive power dispatch from the DER inverter is $q_{Dj}{}^t$. Static capacitance attached to a node j is denoted by q_{C_j}. The apparent power flow through line ij at time step t is S_{ij}^t ($= P_{ij}^t + jQ_{ij}^t$). The battery energy level is B_j^t. Charging and discharging active power from battery inverter (of capacity S_j^t) are denoted by $P_{c_j}^t$ and $P_{d_j}^t$, respectively. The reactive power support of the battery inverter is $q_{B_j}^t$.

B. Centralized Multi-Period OPF with Batteries

The OPF problem aims to minimize the total network loss for the entire time period, as written below,

$$\min \sum_{t=1}^{T} \sum_{(i,j)\in\mathcal{L}} (r_{ij}l_{ij}^t) \tag{1}$$

Subject to the following constraints,

$$p_j^t = \sum_{(j,k)\in\mathcal{L}} P_{jk}^t - \left\{P_{ij}^t - r_{ij}l_{ij}^t\right\} - P_{d_j}^t + P_{c_j}^t \tag{2}$$

$$q_j^t = \sum_{(j,k)\in\mathcal{L}} Q_{jk}^t - \left\{Q_{ij}^t - x_{ij}l_{ij}^t\right\} - q_{D_j}^t - q_{B_j}^t \tag{3}$$

$$p_j^t = {p_{Dj}}^t - {p_{Lj}}^t \tag{4}$$

$$q_j^t = q_{Cj} - {q_{Lj}}^t \tag{5}$$

$$v_j^t = v_i^t + \left\{r_{ij}^2 + x_{ij}^2\right\} l_{ij}^t - 2(r_{ij}P_{ij}^t + x_{ij}Q_{ij}^t) \tag{6}$$

$$l_{ij}^t = \frac{(P_{ij}^t)^2 + (Q_{ij}^t)^2}{v_i^t} \tag{7}$$

$$q_{D_j}^t \in \left[-\sqrt{{S_{D_j}}^2 - {p_{D_j}^t}^2}, \sqrt{{S_{D_j}}^2 - {p_{D_j}^t}^2}\right] \tag{8}$$

$$B_j^t = B_j^{t-1} + \Delta t \eta_c P_{c_j}^t - \Delta t \frac{1}{\eta_d} P_{d_j}^t \tag{9}$$

$$v_j^t \in \left[V_{Min}^2, V_{Max}^2\right] \tag{10}$$

$$l_{ij}^t \in \left[0, I_{rated}^2\right] \tag{11}$$

$$B_j^t \in \left[0.30E_{Max_j}, 0.95E_{Max_j}\right] \tag{12}$$

$$P_{c_j}^t, P_{d_j}^t \in \left[0, P_{B_{Max_j}}\right] \tag{13}$$

$$q_{B_j}^t \in \left[-\sqrt{{S_{B_j}}^2 - {P_{B_{Max_j}}}^2}, \sqrt{{S_{B_j}}^2 - {P_{B_{Max_j}}}^2}\right] \tag{14}$$

The distribution network is represented with the help of the branch power flow equations (2) to (7). Constraints (2) and (3) signify the active and reactive power balance equations. The net active and reactive power injections at any bus j are represented by (4) and (5) respectively. The KVL equation is represented by (6), while the equation describing the relationship between current magnitude and apparent power magnitude is (7).

Node i denotes the 'parent' node of node j, which itself may be the parent of a set of k 'children' nodes (the set may contain one, many or even zero nodes, if j is a leaf node). It may be noted that for a radial distribution system, each node j can have only one 'parent' node i.

(Integer Constraint Relaxed) Naive Brute Force Full Optimization Model - Full Horizon

C. ENApp based Distributed Multi-Period OPF with Batteries

III. CASE STUDY DEMONSTRATION

A. Simulation Data: IEEE 123 Bus Test System

B. Simulation Results

Case 1: centralized OPF with battery Case 2: ENApp based distributed OPF with battery

IV. RESULTS

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V. CONCLUSIONS

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REFERENCES

[1] T. Gangwar, N. P. Padhy, and P. Jena, "Storage allocation in active distribution networks considering life cycle and uncertainty," *IEEE Trans. Ind. Inform.*, vol. 19, no. 1, pp. 339–350, Jan. 2023.

[2] S. Paul and N. P. Padhy, "Real-time advanced energy-efficient management of an active radial distribution network," *IEEE Syst. J.*, vol. 16, no. 3, pp. 3602–3612, Sept. 2022.

[3] H. Yuan, F. Li, Y. Wei, and J. Zhu, "Novel linearized power flow and linearized opf models for active distribution networks with application in distribution lmp," *IEEE Trans. Smart Grid*, vol. 9, no. 1, pp. 438–448, Jan. 2018.

[4] Z. Guo, W. Wei, L. Chen, Z. Dong, and S. Mei, "Parametric distribution optimal power flow with variable renewable generation," *IEEE Trans. Power Syst.*, vol. 37, no. 3, pp. 1831–1841, May 2022.

[5] M. Farivar and S. H. Low, "Branch flow model: Relaxations and convexification," *2012 IEEE 51st IEEE Conference on Decision and Control (CDC)*, pp. 3672–3679, Dec. 2012.

[6] N. Nazir and M. Almassalkhi, "Receding-Horizon Optimization of Unbalanced Distribution Systems with Time-Scale Separation for Discrete and Continuous Control Devices," pp. 1–7, Jun. 2018.

[7] N. Nazir, P. Racherla, and M. Almassalkhi, "Optimal multi-period dispatch of distributed energy resources in unbalanced distribution feeders," Jun. 2019.

[8] A. Agarwal and L. Pileggi, "Large Scale Multi-Period Optimal Power Flow With Energy Storage Systems Using Differential Dynamic Programming," pp. 1750–1759, Sep. 2021.

[9] X. Qian and Y. Zhu, "Differential Dynamic Programming for Multistage Uncertain Optimal Control," pp. 88–92, Jul. 2014.