I. Problem Formulation

A. Notations

In this study, the distribution network is accounted as a tree (connected graph) having N number of buses (indexed with i, j, and k) and the study is conducted for T time steps (indexed by t), each of interval length Δt. The distribution line connecting two buses i and j are denoted by ij (having resistance and reactance of r_{ij} ohm and x_{ij} ohm, respectively) and magnitude of the current flowing through the line at time t is denoted by I_{ij}^t ($l_{ij}^t = \left(I_{ij}^t\right)^2$). The voltage magnitude of bus i at time t is given by $V_i^t \in [V_{min}, V_{max}]$ ($v_i^t = (V_i^t)^2$). Apparent power demand at a node j at time t is ${s_L}_j^t$ ($= {p_L}_j^t + j{q_L}_j^t$). The uncontrolled active power generation from the DER present at bus j at time step t is denoted by ${p_D}_j^t$ and controlled reactive power dispatch from the DER inverter is ${q_D}_j^t$. Static capacitance attached to a node j is denoted by q_{C_j}. The apparent power flow through line ij at time step t is S_{ij}^t ($= P_{ij}^t + jQ_{ij}^t$). The battery state of charge (soc) or energy level is B_j^t. Charging and discharging active power from battery inverter (of apparent power capacity $S_{R,j}^t$) are denoted by $P_{c_j}^t$ and $P_{d_j}^t$, respectively. The total state of charge capacity of the batteries are denoted by $E_{R,j}$, and the Rated battery powers are denoted by $P_{B_{R,j}}$. The reactive power support of the battery inverter is $q_{B_j}^t$. Rated apparent powers of DERs and Batteries at node j are denoted by $S_{D_{R,j}}$ and $S_{B_{R,j}}$ respectively.

B. Centralized Multi-Period OPF with Batteries

The OPF problem given in (1) aims to minimize the cost of power borrowed from the substation for the entire horizon. The incorporation of an additional 'Battery Loss' term helps us bypass using binary (integer) constraints for modelling the operation of batteries, which make the optimization problem harder to solve. The term still ensures the complementarity of charging and discharging operations for any battery during a particular time period [?], [?], [?].

$$\min \sum_{t=1}^{T} \left[C^t P_{Subs}^t + \alpha \sum_{j \in \mathcal{B}} \left\{ (1-\eta_C) P_{C_j}^t + \left(\frac{1}{\eta_D} - 1\right) P_{D_j}^t \right\} \right] \tag{1}$$

Subject to the constraints (2) to (14) given below:

$$p_j^t = \sum_{(j,k)\in\mathcal{L}} P_{jk}^t - \left\{P_{ij}^t - r_{ij} l_{ij}^t\right\} - P_{d_j}^t + P_{c_j}^t \tag{2}$$

$$q_j^t = \sum_{(j,k)\in\mathcal{L}} Q_{jk}^t - \left\{Q_{ij}^t - x_{ij} l_{ij}^t\right\} - q_{D_j}^t - q_{B_j}^t \tag{3}$$

$$p_j^t = {p_D}_j^t - {p_L}_j^t \tag{4}$$

$$q_j^t = {q_C}_j - {q_L}_j^t \tag{5}$$

$$v_j^t = v_i^t + \left\{r_{ij}^2 + x_{ij}^2\right\} l_{ij}^t - 2(r_{ij} P_{ij}^t + x_{ij} Q_{ij}^t) \tag{6}$$

$$l_{ij}^t = \frac{(P_{ij}^t)^2 + (Q_{ij}^t)^2}{v_i^t} \tag{7}$$

$$v_j^t \in \left[V_{min}^2, V_{max}^2\right] \tag{8}$$

$$l_{ij}^t \in \left[0, I_{R,ij}^2\right] \tag{9}$$

$$B_j^t = B_j^{t-1} + \Delta t \eta_c P_{c_j}^t - \Delta t \frac{1}{\eta_d} P_{d_j}^t \tag{10}$$

$$B_j^t \in [soc_{min} E_{R,j}, soc_{max} E_{R,j}] \tag{11}$$

$$P_{c_j}^t, P_{d_j}^t \in \left[0, P_{B_{R_j}}\right] \tag{12}$$

$$q_{B_j}^t \in \left[-\sqrt{{S_{B_{R,j}}}^2 - {P_{B_{R,j}}}^2}, \sqrt{{S_{B_{R,j}}}^2 - {P_{B_{R,j}}}^2}\right] \tag{13}$$

$$q_{D_j}^t \in \left[-\sqrt{{S_{D_{R,j}}}^2 - {p_{D_j}^t}^2}, \sqrt{{S_{D_{R,j}}}^2 - {p_{D_j}^t}^2}\right] \tag{14}$$

TABLE I: Parameter Values

Parameter	Value
V_{min}, V_{max}	0.95, 1.05
$P_{DER_{Rated_j}}$	$0.33 P_{L_{Rated_j}}$
$S_{DER_{Rated_j}}$	$1.2 P_{DER_{Rated_j}}$
$P_{B_{Rated_j}}$	$0.33 P_{L_{Rated_j}}$
B_{Rated_j}	$T_{fullCharge} \times P_{B_{Rated_j}}$
$T_{fullCharge}$	4 h
Δt	1 h
η_C, η_D	0.95, 0.95
soc_{min}, soc_{max}	0.30, 0.95
α	0.001

The distribution network is represented with the help of the branch power flow equations (2) to (7). Constraints (2) and (3) signify the active and reactive power balance equations. The net active and reactive power injections at any bus j are represented by (4) and (5) respectively. The KVL equation is represented by (6), while the equation describing the relationship between current magnitude, voltage magnitude and apparent power magnitude is (7). The limits of node voltage and branch current are enforced via (8) and (9). The trajectory of the state of charge of batteries versus time is given by (10) and is the only class of constraints in this paper coupling the optimal power flow problem in time. Battery charging and discharging efficiency values used in this paper are $\eta_c = 95\%$ and $\eta_d = 95\%$ literature?. For a safe and sustainable operation of the batteries based on what literature?, the state of charge B_j^t is constrained to be within some percentage limits of the rated battery soc capacity, as given in (11). In this paper, we're using $soc_{min} = 30\%$ and $soc_{max} = 95\%$. Similarly, battery charging and discharging powers should not exceed its rated power capacity, as given by (12). (13) and (14) describe the limits for two-quadrant operation of the controlled reactive power support of DERs and Batteries respectively. It may be noted that while both of these limits are non-controllable, only the limits for DERs are time-varying, due to $p^t_{D_j}$ component. For this simulation study, the limits for battery reactive support have been curtailed, i.e. the bounds of the limit have been artificially set smaller than what would be physically permissible. The reason for doing so was to avoid a non-linear inequality coupling decision variables. Should I specify this justification?

(Integer Constraint Relaxed) Naive Brute Force Full Optimization Model - Full Horizon

C. ENApp based Distributed Multi-Period OPF with Batteries