

# 1 Theory

FIRST LINE OF THEORY ANOTHER LINE OF THEORY

## 1.1 Notations

In this study, the distribution network is accounted as a tree (connected graph) having  $N$  number of buses (indexed with  $i$ ,  $j$ , and  $k$ ) and the study is conducted for  $T$  time steps (indexed by  $t$ ). The distribution line connecting two buses  $i$  and  $j$  are denoted by  $ij$  and magnitude of the current flowing through the line at time  $t$  is denoted by  $I_{ij}^t$  ( $l_{ij}^t = (I_{ij}^t)^2$ ). The voltage magnitude of bus  $i$  at time  $t$  is given by  $V_i^t$  ( $v_i^t = (V_i^t)^2$ ).

## 1.2 Centralized Multi-Period OPF with Batteries

On similar lines to the branch flow equations in [?], the network is modeled as a function of time, considering the interaction of batteries.

$$p_j^t = \sum_{(j,k) \in \mathcal{L}} P_{jk}^t - \sum_{(i,j) \in \mathcal{L}} \{P_{ij}^t - r_{ij} l_{ij}^t\} - P_{d_j}^t + P_{c_j}^t \quad (1)$$

$$q_j^t = \sum_{(j,k) \in \mathcal{L}} Q_{jk}^t - \sum_{(i,j) \in \mathcal{L}} \{Q_{ij}^t - x_{ij} l_{ij}^t\} - q_{D_j}^t - q_{B_j}^t \quad (2)$$

$$v_j^t = v_i^t + \{r_{ij}^2 + x_{ij}^2\} l_{ij}^t - 2(r_{ij} P_{ij}^t + x_{ij} Q_{ij}^t) \quad (3)$$

$$l_{ij}^t = \frac{(P_{ij}^t)^2 + (Q_{ij}^t)^2}{v_i^t} \quad (4)$$

where  $P_{ij}^t, Q_{ij}^t, l_{ij}^t$  denote the sending-end real power, reactive power and the square of the magnitude of the current flowing in the branch  $(i, j)$  respectively.  $v_j^t$  denotes the square of the magnitude of the voltage at node  $j$ . The superscript  $t$  specifies the time-period for the corresponding variable. Node  $i$  denotes the ‘parent’ node of node  $j$ , which itself may be the parent of a set of  $k$  ‘children’ nodes (the set may contain one, many or even zero nodes, if  $j$  is a leaf node). It may be noted that for a radial distribution system, each node  $j$  can have only one ‘parent’ node  $i$ , and thus the summation for the second term in equations Equations (1) to (4) may be dropped.

**(Integer Constraint Relaxed) Naive Brute Force Full Optimization Model - Full Horizon**

$$\begin{aligned}
\min_{\substack{P_{ij}^t, Q_{ij}^t, v_j^t, l_{ij}^t, \\ q_{D_j}^t, B_j^t, P_{c_j}^t, P_{d_j}^t, q_{B_j}^t}} & \sum_{t=1}^T \sum_{(i,j) \in \mathcal{L}} (r_{ij} l_{ij}^t) \\
& + \alpha \sum_{t=1}^T \sum_{j \in \mathcal{B}} \left\{ (1 - \eta_c) P_{c_j}^t + \left( \frac{1}{\eta_d} - 1 \right) P_{d_j}^t \right\} \\
& + \gamma \sum_{j \in \mathcal{B}} \left\{ (B_j^T - B_{ref_j})^2 \right\} \tag{5}
\end{aligned}$$

s.t.

$$eqs. (1) to (4) \tag{6}$$

$$B_j^t = B_j^{t-1} + \Delta t \eta_c P_{c_j}^t - \Delta t \frac{1}{\eta_d} P_{d_j}^t \tag{7}$$

$$where, \tag{8}$$

$$(i, j) : \text{Branch connecting nodes } i \text{ and } j \tag{9}$$

$$p_j^t = p_{D_j}^t - p_{L_j}^t \tag{10}$$

$$q_j^t = -q_{L_j}^t \tag{11}$$

$$t = \{1, 2, \dots, T\} \tag{12}$$

**1.3 ENApp based Distributed Multi-Period OPF with Batteries**