I. Problem Formulation

A. Notations

In this study, the distribution network is accounted as a tree (connected graph) having N number of buses (indexed with i, j, and k) and the study is conducted for T time steps (indexed by t), each of interval length Δt. The sets of buses with DERs and batteries are D and B respectively, such that $D, B \subseteq N$. A directed edge from bus i to j in the tree is represented by ij and the set for edges is $\mathcal{L}$. Line resistance and reactance are r_{ij} ohm and x_{ij} ohm, respectively. Magnitude of the current flowing through the line at time t is denoted by I^t_{ij} and $l^t_{ij} = \left(I^t_{ij}\right)^2$. The voltage magnitude of bus j at time t is given by V^t_j and $v^t_j = \left(V^t_j\right)^2$. Apparent power demand at a node j at time t is $s^t_{L_j}$ $(= p^t_{L_j} + jq^t_{L_j})$. The active power generation from the DER present at bus j at time t is denoted by $p^t_{D_j}$ and controlled reactive power dispatch from the DER inverter is $q^t_{D_j}$. DER inverter capacity is $S_{D_{R_j}}$. The apparent power flow through line ij at time t is S^t_{ij} $(= P^t_{ij} + jQ^t_{ij})$. The real power flowing from the substation into the network is denoted by P^t_{Subs} and the associated cost involved per kWh is C^t. The battery energy level is B^t_j. Charging and discharging active power from battery inverter (of apparent power capacity $S^t_{R_j}$) are denoted by $P^t_{c_j}$ and $P^t_{d_j}$, respectively and their associated efficiencies are η_c and η_d respectively. The energy capacity of the batteries is denoted by B_{R_j}, and the rated battery power is $P_{B_{R_j}}$. soc_{min} and soc_{max} are fractional values for denoting safe soc limits of a battery about its rated soc capacity. The reactive power support of the battery inverter is $q^t_{B_j}$.

B. MPCOPF with Batteries

The OPF problem aims to minimize two objectives as shown in (1). The first term in (1) aims to minimize the total energy cost for the entire horizon. The incorporation of 'Battery Loss' cost as the second term ($\alpha > 0$) helps to avoid the usage of binary (integer) variables generally used to refrain from simultaneous charging/discharging. This is done to make the OPF problem a simple non-convex problem but not a MINCP. The term still ensures the complementarity of charging and discharging operations during a particular time [?].

$$\min \sum_{t=1}^{T} \left[C^t P^t_{Subs} \Delta t + \alpha \sum_{j \in \mathcal{B}} \left\{ (1-\eta_C) P^t_{C_j} + \left(\frac{1}{\eta_D} - 1\right) P^t_{D_j} \right\} \right] \quad (1)$$

Subject to the constraints (2) to (13) as given below:

$$0 = \sum_{(j,k)\in\mathcal{L}} \left\{P^t_{jk}\right\} - \left(P^t_{ij} - r_{ij} l^t_{ij}\right) - \left(P^t_{d_j} - P^t_{c_j}\right) - p^t_{D_j} + p^t_{L_j} \quad (2)$$

$$0 = \sum_{(j,k)\in\mathcal{L}} \left\{Q^t_{jk}\right\} - \left(Q^t_{ij} - x_{ij} l^t_{ij}\right) - q^t_{D_j} - q^t_{B_j} + q^t_{L_j} \quad (3)$$

$$0 = v^t_i - v^t_j - 2(r_{ij} P^t_{ij} + x_{ij} Q^t_{ij}) + \left\{r^2_{ij} + x^2_{ij}\right\} l^t_{ij} \quad (4)$$

$$0 = (P^t_{ij})^2 + (Q^t_{ij})^2 - l^t_{ij} v^t_i \quad (5)$$

$$P^t_{Subs} \geq 0 \quad (6)$$

$$l^t_{ij} \in \left[0, I^2_{R,ij}\right] \quad (7)$$

$$v^t_j \in \left[V^2_{min}, V^2_{max}\right] \quad (8)$$

$$q^t_{D_j} \in \left[-\sqrt{{S_{D_{R,j}}}^2 - {p^t_{D_j}}^2}, \sqrt{{S_{D_{R,j}}}^2 - {p^t_{D_j}}^2}\right] \quad (9)$$

$$0 = B^t_j - \left\{B^{t-1}_j + \Delta t \eta_c P^t_{c_j} - \Delta t \frac{1}{\eta_d} P^t_{d_j}\right\} \quad (10)$$

$$P^t_{c_j}, P^t_{d_j} \in \left[0, P_{B_{R_j}}\right] \quad (11)$$

$$q^t_{B_j} \in \left[-0.44 P_{B_{R,j}}, 0.44 P_{B_{R,j}}\right] \quad (12)$$

$$B^t_j \in \left[soc_{min} B_{R,j}, soc_{max} B_{R,j}\right] \quad (13)$$

The distribution network is represented with the help of the branch power flow equations (2) to (5). Constraints (2) and (3) signify the active and reactive power balance at node j, including contributions from any attached DERs ($p^t_{D_j}, q^t_{D_j}$) and/or batteries ($P^t_{c_j}, P^t_{d_j}, q^t_{B_j}$). The KVL equation for branch (i,j) is represented by (4), while the equation describing the relationship between current magnitude, voltage magnitude and apparent power magnitude for nodes i and j is (5). Back-flow of real power into the substation from the distribution system is avoided using the constraint (6).The box limits for squared branch current and squared node voltage are enforced via (7) and (8). (9) describes the reactive power limits of DER inverters. The trajectory of the state of charge of batteries versus time is given by (10) and is the only class of constraints in this paper coupling the optimal power flow problem in time. Battery charging and discharging powers are non-negative valued variables which should not exceed the battery's rated power capacity, as given by (11). Every battery's reactive power is also constrained based on the associated inverter's rated capacity, as described by (12). For safe and sustainable operation of the batteries, the state of charge B^t_j is constrained to be within some percentage limits of the rated battery soc capacity, as given in (13)

C. ENApp based Distributed Multi-Period OPF with Batteries

Solving a nonlinear nonconvex OPF problem is computationally intensive. In the ENApp paradigm, the original OPF problem is divided into several subproblems, based on physically connected 'parent-child' area relationships in the radial distribution network. ENApp primiarly requires three classes of operations. The first is of course solving for the OPF of each individual subproblem, the second is checking for 'boundary convergence' between related areas to check if further improvement iterations are needed and the third and last operation is exchange of variables between related areas for subsequent iterations. Out of these three operations, only solving the OPF is computationally intensive.

The key advantage of using ENApp in preference to MPCOPF is its efficient division of the OPF problem into several smaller problems via efficient exploitation of power flow physics in a radial network [**?**]. Typically, Nonlinear Programming problems grow superlinearly in complexity with increase in problem size, and thus division of the OPF problem into smaller, more manageable chunks greatly speeds up the solution process.

The solution process for each subproblem can then be parallelized, saving computation time even further.

Additionally, on paper it might appear that ENApp requires solving of the subproblems across multiple iterations, potentially nullifying the benefit from reduction in problem size. However in practice, once the first OPF iteration is completed for each area, the results can be used to 'warm-start' subsequent iterations with significant speedups in solution times.

Thus, in essence, the 'computational bottleneck' for ENApp is computing the OPF solution of its biggest subproblem, which typically translates to the single biggest area as part of the spatial decomposition of the radial distribution grid (assuming the number of DERs and Batteries is not significantly higher in another area).

The advantage of this smaller computational bottleneck will be readily seen in **????** when compared against MPCOPF on the Test System.

For the rest of this paper, we'll be addressing ENAPpp based Distributed Multi-Period OPF simply as MPDOPF for brevity.