

A Spatially Distributed Multi-Period Optimal Power Flow Analysis of Radial Active Distribution Networks with Distributed Battery Units

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Abstract—insert abstract here

Index Terms—Batteries, distribution network, distributed energy resources (DERs), equivalent network approximation (ENApp)

I. INTRODUCTION

A. Background and Prior Arts

Presently, optimal power flow (OPF) tools are developed to run the MV/LV distribution grids in the most economical, reliable, and secure manner. The usefulness of OPF studies is gaining more interest due to penetration of distributed energy resources (DERs), especially solar photovoltaic panels. Power generation from these DERs are influenced majorly by the weather conditions, hence highly intermittent nature. Presently, deployment of battery units are becoming more pertinent to mitigate the uncertainty effect and maintain the power balance by controlling the charging and/or discharging operations [1]. However, inclusion of batteries converts the conventional single period time decoupled OPF problem into a multi-period time coupled OPF analysis.

Traditionally, centralized OPF methods were popular where required data are accumulated at a central controller location [2]. The central controller is responsible to process all the accumulated data, solving the OPF algorithm and dispatch control signals to the controlling resources. Yuan et al. [3] proposed a linear OPF model for distribution network depending upon the locational marginal price (LMP). The LMP is calculated by including reactive power components and voltage constraints.

Guo et al. [4] developed a linear OPF model after linearizing the second-order cone constraints with polyhedral approximations. The OPF problem is formulated by considering the variable solar power generation as parameters and hence the overall problem takes form of a parametric distribution OPF.

B. Research Gaps and Contributions

A taxonomy table to compare the existing studies and the present work is provided in I.

The specific contributions are as follows:

- 1) The overall problem is formulated as a non-convex programming and the

TABLE I
TAXONOMY TABLE FOR COMPARISON

References	DERs	Batteries	Single period OPF	Multi-period OPF	Centralized OPF	Distributed OPF	Framework
[3]			✓		✓		Linear
[4]	✓		✓	✓			Linear
[1], [1]	✓	✓				✓	
[1]- [1]	✓			✓			✓
[1], [1]		✓		✓			✓
[1]- [1]	✓			✓			✓
This paper	✓	✓		✓		✓	Non-convex

II. PROBLEM FORMULATION

A. Notations

In this study, the distribution network is accounted as a tree (connected graph) having N number of buses (indexed with i, j , and k) and the study is conducted for T time steps (indexed by t), each of interval length Δt . The distribution line connecting two buses i and j are denoted by ij (having resistance and reactance of r_{ij} ohm and x_{ij} ohm, respectively) and magnitude of the current flowing through the line at time t is denoted by I_{ij}^t ($I_{ij}^t = (I_{ij}^t)^2$). The voltage magnitude of bus i at time t is given by $V_i^t \in [V_{min}, V_{max}]$ ($v_i^t = (V_i^t)^2$). Apparent power demand at a node j at time t is $s_{Lj}^t (= p_{Lj}^t + jq_{Lj}^t)$. The uncontrolled active power generation from the DER present at bus j at time step t is denoted by p_{Dj}^t and controlled reactive power dispatch from the DER inverter is q_{Dj}^t . Static capacitance attached to a node j is denoted by q_{Cj} . The apparent power flow through line ij at time step t is $S_{ij}^t (= P_{ij}^t + jQ_{ij}^t)$. The battery state of charge (soc) or energy level is B_j^t . Charging and discharging active power from battery inverter (of apparent power capacity $S_{R,j}^t$) are denoted by $P_{c_j}^t$ and $P_{d_j}^t$, respectively. The total state of charge capacity of the batteries are denoted by $E_{R,j}$, and the Rated battery powers are

denoted by $P_{B_{R,j}}$. The reactive power support of the battery inverter is $q_{B_j}^t$. Rated apparent powers of DERs and Batteries at node j are denoted by $S_{D_{R,j}}$ and $S_{B_{R,j}}$ respectively.

B. Centralized Multi-Period OPF with Batteries

The OPF problem aims to minimize the total network loss for the entire time period, as specified in (1):

$$\min \sum_{t=1}^T \sum_{(i,j) \in \mathcal{L}} (r_{ij} l_{ij}^t) \quad (1)$$

Subject to the constraints (2) to (14) given below:

$$p_j^t = \sum_{(j,k) \in \mathcal{L}} P_{jk}^t - \{P_{ij}^t - r_{ij} l_{ij}^t\} - P_{d_j}^t + P_{c_j}^t \quad (2)$$

$$q_j^t = \sum_{(j,k) \in \mathcal{L}} Q_{jk}^t - \{Q_{ij}^t - x_{ij} l_{ij}^t\} - q_{D_j}^t - q_{B_j}^t \quad (3)$$

$$p_j^t = p_{D_j}^t - p_{L_j}^t \quad (4)$$

$$q_j^t = q_{C_j}^t - q_{L_j}^t \quad (5)$$

$$v_j^t = v_i^t + \{r_{ij}^2 + x_{ij}^2\} l_{ij}^t - 2(r_{ij} P_{ij}^t + x_{ij} Q_{ij}^t) \quad (6)$$

$$l_{ij}^t = \frac{(P_{ij}^t)^2 + (Q_{ij}^t)^2}{v_i^t} \quad (7)$$

$$v_j^t \in [V_{min}^2, V_{max}^2] \quad (8)$$

$$l_{ij}^t \in [0, I_{R,ij}^2] \quad (9)$$

$$B_j^t = B_j^{t-1} + \Delta t \eta_c P_{c_j}^t - \Delta t \frac{1}{\eta_d} P_{d_j}^t \quad (10)$$

$$B_j^t \in [soc_{min} E_{R,j}, soc_{max} E_{R,j}] \quad (11)$$

$$P_{c_j}^t, P_{d_j}^t \in [0, P_{B_{R,j}}] \quad (12)$$

$$q_{B_j}^t \in \left[-\sqrt{S_{B_{R,j}}^2 - P_{B_{R,j}}^2}, \sqrt{S_{B_{R,j}}^2 - P_{B_{R,j}}^2} \right] \quad (13)$$

$$q_{D_j}^t \in \left[-\sqrt{S_{D_{R,j}}^2 - p_{D_j}^t{}^2}, \sqrt{S_{D_{R,j}}^2 - p_{D_j}^t{}^2} \right] \quad (14)$$

The distribution network is represented with the help of the branch power flow equations (2) to (7). Constraints (2) and (3) signify the active and reactive power balance equations. The net active and reactive power injections at any bus j are represented by (4) and (5) respectively. The KVL equation is represented by (6), while the equation describing the relationship between current magnitude, voltage magnitude and apparent power magnitude is (7). The limits of node voltage and branch current are enforced via (8) and (9). The trajectory of the state of charge of batteries versus time is given by (10) and is the only class of constraints in this paper coupling the optimal power flow problem in time. Battery charging and discharging efficiency values used in this paper are $\eta_c = 95\%$ and $\eta_d = 95\%$ literature?. For a safe and sustainable operation of the batteries based on what literature?, the state of charge B_j^t is constrained to be within some percentage limits of the rated battery soc capacity, as given in (11). In this paper, we're using $soc_{min} = 30\%$ and $soc_{max} = 95\%$. Similarly, battery charging and discharging powers should not exceed

its rated power capacity, as given by (12). (13) and (14) describe the limits for two-quadrant operation of the controlled reactive power support of DERs and Batteries respectively. It may be noted that while both of these limits are non-controllable, only the limits for DERs are time-varying, due to $p_{D_j}^t$ component. For this simulation study, the limits for battery reactive support have been curtailed, i.e. the bounds of the limit have been artificially set smaller than what would be physically permissible. The reason for doing so was to avoid a non-linear inequality coupling decision variables. Should I specify this justification?

(Integer Constraint Relaxed) Naive Brute Force Full Optimization Model - Full Horizon

C. ENApp based Distributed Multi-Period OPF with Batteries

III. CASE STUDY DEMONSTRATION

The energy price data is taken from the ComEd hourly live prices data [5] for 10 November 2023.

A. Simulation Data: IEEE 123 Bus Test System

B. Simulation Results

Case 1: centralized OPF with battery Case 2: ENApp based distributed OPF with battery

IV. RESULTS

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V. CONCLUSIONS

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REFERENCES

- [1] T. Gangwar, N. P. Padhy, and P. Jena, "Storage allocation in active distribution networks considering life cycle and uncertainty," *IEEE Trans. Ind. Inform.*, vol. 19, no. 1, pp. 339–350, Jan. 2023.
- [2] S. Paul and N. P. Padhy, "Real-time advanced energy-efficient management of an active radial distribution network," *IEEE Syst. J.*, vol. 16, no. 3, pp. 3602–3612, Sept. 2022.
- [3] H. Yuan, F. Li, Y. Wei, and J. Zhu, "Novel linearized power flow and linearized opf models for active distribution networks with application in distribution Imp," *IEEE Trans. Smart Grid*, vol. 9, no. 1, pp. 438–448, Jan. 2018.
- [4] Z. Guo, W. Wei, L. Chen, Z. Dong, and S. Mei, "Parametric distribution optimal power flow with variable renewable generation," *IEEE Trans. Power Syst.*, vol. 37, no. 3, pp. 1831–1841, May 2022.

- [5] "Live Prices - ComEd's Hourly Pricing," Jun. 2023, [Online; accessed 16. Nov. 2023]. [Online]. Available: <https://hourlypricing.comed.com/live-prices/?date=20231110>
- [6] M. Farivar and S. H. Low, "Branch flow model: Relaxations and convexification," *2012 IEEE 51st IEEE Conference on Decision and Control (CDC)*, pp. 3672–3679, Dec. 2012.
- [7] N. Nazir and M. Almassalkhi, "Receding-Horizon Optimization of Unbalanced Distribution Systems with Time-Scale Separation for Discrete and Continuous Control Devices," pp. 1–7, Jun. 2018.
- [8] N. Nazir, P. Racherla, and M. Almassalkhi, "Optimal multi-period dispatch of distributed energy resources in unbalanced distribution feeders," Jun. 2019.
- [9] A. Agarwal and L. Pileggi, "Large Scale Multi-Period Optimal Power Flow With Energy Storage Systems Using Differential Dynamic Programming," pp. 1750–1759, Sep. 2021.
- [10] X. Qian and Y. Zhu, "Differential Dynamic Programming for Multistage Uncertain Optimal Control," pp. 88–92, Jul. 2014.