I. Problem Formulation

A. Notations

In this study, the distribution network is accounted as a tree (connected graph) having N number of buses (indexed with i, j, and k) and the study is conducted for T time steps (indexed by t), each of interval length Δt. The sets of buses with DERs and batteries are D and B respectively, such that $D, B \subseteq N$. A directed edge from bus i to j in the tree is represented by ij and the set for edges is $\mathcal{L}$. Line resistance and reactance are r_{ij} ohm and x_{ij} ohm, respectively. Magnitude of the current flowing through the line at time t is denoted by I_{ij}^t and $l_{ij}^t = \left(I_{ij}^t\right)^2$. The voltage magnitude of bus j at time t is given by V_j^t and $v_j^t = \left(V_j^t\right)^2$. Apparent power demand at a node j at time t is $s_{L_j}^t$ $(= p_{L_j}^t + jq_{L_j}^t)$. The active power generation from the DER present at bus j at time t is denoted by $p_{D_j}^t$ and controlled reactive power dispatch from the DER inverter is $q_{D_j}^t$. DER inverter capacity is $S_{D_{R_j}}$. The apparent power flow through line ij at time t is S_{ij}^t $(= P_{ij}^t + jQ_{ij}^t)$. The real power flowing from the substation into the network is denoted by P_{Subs}^t and the associated cost involved per kWh is C^t. The battery energy level is B_j^t. Charging and discharging active power from battery inverter (of apparent power capacity $S_{R_j}^t$) are denoted by $P_{c_j}^t$ and $P_{d_j}^t$, respectively and their associated efficiencies are η_c and η_d respectively. The energy capacity of the batteries is denoted by B_{R_j}, and the rated battery power is $P_{B_{R_j}}$. soc_{min} and soc_{max} are fractional values for denoting safe soc limits of a battery about its rated soc capacity. The reactive power support of the battery inverter is $q_{B_j}^t$.

B. MPCOPF with Batteries

The OPF problem aims to minimize two objectives as shown in (1). The first term in (1) aims to minimize the total energy cost for the entire horizon. The incorporation of 'Battery Loss' cost as the second term ($\alpha > 0$) helps to avoid the usage of binary (integer) variables generally used to refrain from simultaneous charging/discharging. This is done to make the OPF problem a simple non-convex problem but not a MINCP. The term still ensures the complementarity of charging and discharging operations during a particular time [?].

$$\min \sum_{t=1}^{T} \left[C^t P_{Subs}^t \Delta t + \alpha \sum_{j \in \mathcal{B}} \left\{ (1-\eta_C) P_{C_j}^t + \left(\frac{1}{\eta_D} - 1\right) P_{D_j}^t \right\} \right] \tag{1}$$

Subject to the constraints (2) to (12) as given below:

$$0 = \sum_{(j,k)\in\mathcal{L}} \left\{P_{jk}^t\right\} - \left(P_{ij}^t - r_{ij} l_{ij}^t\right) - \left(P_{d_j}^t - P_{c_j}^t\right) - p_{D_j}^t + p_{L_j}^t \tag{2}$$

$$0 = \sum_{(j,k)\in\mathcal{L}} \left\{Q_{jk}^t\right\} - \left(Q_{ij}^t - x_{ij} l_{ij}^t\right) - q_{D_j}^t - q_{B_j}^t + q_{L_j}^t \tag{3}$$

$$0 = v_i^t - v_j^t - 2(r_{ij}P_{ij}^t + x_{ij}Q_{ij}^t) + \left\{r_{ij}^2 + x_{ij}^2\right\} l_{ij}^t \tag{4}$$

$$0 = (P_{ij}^t)^2 + (Q_{ij}^t)^2 - l_{ij}^t v_i^t \tag{5}$$

$$P_{Subs}^t \geq 0 \tag{6}$$

$$v_j^t \in \left[V_{min}^2, V_{max}^2\right] \tag{7}$$

$$q_{D_j}^t \in \left[-\sqrt{{S_{D_{R,j}}}^2 - {p_{D_j}^t}^2}, \sqrt{{S_{D_{R,j}}}^2 - {p_{D_j}^t}^2}\right] \tag{8}$$

$$0 = B_j^t - \left\{B_j^{t-1} + \Delta t \eta_c P_{c_j}^t - \Delta t \frac{1}{\eta_d} P_{d_j}^t\right\} \tag{9}$$

$$P_{c_j}^t, P_{d_j}^t \in \left[0, P_{B_{R_j}}\right], \quad B_j^0 = B_j^T \tag{10}$$

$$q_{B_j}^t \in \left[-0.44 P_{B_{R,j}}, 0.44 P_{B_{R,j}}\right] \tag{11}$$

$$B_j^t \in \left[soc_{min} B_{R,j}, soc_{max} B_{R,j}\right] \tag{12}$$

The distribution network is represented with the help of the branch power flow equations (2) to (5). Constraints (2) and (3) signify the active and reactive power balance at node j, respectively The KVL equation for branch (ij) is represented by (4), while the equation describing the relationship between current magnitude, voltage magnitude and apparent power magnitude for branch (ij) is (5). Backflow of real power into the substation from the distribution system is avoided using the constraint (6). The box limits for squared node voltage are enforced via (7). (8) describes the reactive power limits of DER inverters. The trajectory of the battery energy versus time is given by (9) and is the only time-coupled constraint in this paper. Battery charging and discharging powers are limited by the battery's rated power capacity, as given by (10). (10) also says that the initial and final energy levels must be the same. Every battery's reactive power is also constrained based on the associated inverter's rated capacity, as described by (11). For the safe and sustainable operation of the batteries, the energy B_j^t is constrained to be within some percentage limits of the rated battery soc capacity, as given in (12)

C. ENApp based MPDOPF with Batteries

Assuming the