# 1 Theory

## 1.1 Notations

In this study, the distribution network is accounted as a tree (connected graph) having $N$ number of buses (indexed with $i$, $j$, and $k$) and the study is conducted for $T$ time steps (indexed by $t$). The distribution line connecting two buses $i$ and $j$ are denoted by $ij$ and magnitude of the current flowing through the line at time $t$ is denoted by $I_{ij}^t$ ($l_{ij}^t = \left(I_{ij}^t\right)^2$). The voltage magnitude of bus $i$ at time $t$ is given by $V_i^t$ ($v_i^t = (V_i^t)^2$).

## 1.2 Centralized Multi-Period OPF with Batteries

On similar lines to the branch flow equations in [?], the network is modeled as a function of time, considering the interaction of batteries.

$$p_j^t = \sum_{(j,k)\in\mathcal{L}} P_{jk}^t - \sum_{(i,j)\in\mathcal{L}} \left\{P_{ij}^t - r_{ij}l_{ij}^t\right\} - P_{d_j}^t + P_{c_j}^t \tag{1}$$

$$q_j^t = \sum_{(j,k)\in\mathcal{L}} Q_{jk}^t - \sum_{(i,j)\in\mathcal{L}} \left\{Q_{ij}^t - x_{ij}l_{ij}^t\right\} - q_{D_j}^t - q_{B_j}^t \tag{2}$$

$$v_j^t = v_i^t + \left\{r_{ij}^2 + x_{ij}^2\right\} l_{ij}^t - 2(r_{ij}P_{ij}^t + x_{ij}Q_{ij}^t) \tag{3}$$

$$l_{ij}^t = \frac{(P_{ij}^t)^2 + (Q_{ij}^t)^2}{v_i^t} \tag{4}$$

where $P_{ij}^t, Q_{ij}^t, l_{ij}^t$ denote the sending-end real power, reactive power and the square of the magnitude of the current flowing in the branch $(i,j)$ respectively. $v_j^t$ denotes the square of the magnitude of the voltage at node $j$. The superscript $t$ specifies the time-period for the corresponding variable. Node $i$ denotes the 'parent' node of node $j$, which itself may be the parent of a set of $k$ 'children' nodes (the set may contain one, many or even zero nodes, if $j$ is a leaf node). It may be noted that for a radial distribution system, each node $j$ can have only one 'parent' node $i$, and thus the summation for the second term in equations Equations (1) and (2) may be dropped.

## (Integer Constraint Relaxed) Naive Brute Force Full Optimization Model - Full Horizon

$$\min_{\substack{P_{ij}^t, Q_{ij}^t, v_j^t, l_{ij}^t, \\ q_{D_j}^t, B_j^t, P_{c_j}^t, {P_{d_j}}^t, q_{B_j}^t}} \quad \sum_{t=1}^{T} \sum_{(i,j) \in \mathcal{L}} (r_{ij} l_{ij}^t)$$

$$+ \alpha \sum_{t=1}^{T} \sum_{j \in \mathcal{B}} \left\{ (1 - \eta_c) P_{c_j}^t + \left( \frac{1}{\eta_d} - 1 \right) P_{d_j}^t \right\}$$

$$+ \gamma \sum_{j \in \mathcal{B}} \left\{ \left( B_j^T - B_{ref_j} ] \right)^2 \right\} \tag{5}$$

s.t.

$$p_j^t = \sum_{(j,k) \in \mathcal{L}} P_{jk}^t - \sum_{(i,j) \in \mathcal{L}} \left\{ P_{ij}^t - r_{ij} l_{ij}^t \right\} - P_{d_j}^t + P_{c_j}^t \tag{6}$$

$$q_j^t = \sum_{(j,k) \in \mathcal{L}} Q_{jk}^t - \sum_{(i,j) \in \mathcal{L}} \left\{ Q_{ij}^t - x_{ij} l_{ij}^t \right\} - q_{D_j}^t - q_{B_j}^t \tag{7}$$

$$v_j^t = v_i^t + \left\{ r_{ij}^2 + x_{ij}^2 \right\} l_{ij}^t - 2(r_{ij} P_{ij}^t + x_{ij} Q_{ij}^t) \tag{8}$$

$$l_{ij}^t = \frac{(P_{ij}^t)^2 + (Q_{ij}^t)^2}{v_i^t} \tag{9}$$

$$B_j^t = B_j^{t-1} + \Delta t \eta_c P_{c_j}^t - \Delta t \frac{1}{\eta_d} P_{d_j}^t \tag{10}$$

$$B_j^0 = 0.5(soc_{max} + soc_{min}) E_{Rated} = 0.625 E_{Rated} \tag{11}$$

$$where, \tag{12}$$

$$(i,j) : \text{Branch connecting nodes } i \text{ and } j \tag{13}$$

$$p_j^t = {p_{Dj}}^t - {p_{Lj}}^t \tag{14}$$

$$q_j^t = -{q_{Lj}}^t \tag{15}$$

$$t = \{1, 2, \dots T\} \tag{16}$$

## 1.3   ENApp based Distributed Multi-Period OPF with Batteries