1 Theory

1.1 Notations

1.2 Centralized Multi-Period OPF with Batteries

On similar lines to the branch flow equations in [**?**], the network is modeled as a function of time, considering the interaction of batteries.

$$p_j^t = \sum_{(j,k)\in\mathcal{L}} P_{jk}^t - \sum_{(i,j)\in\mathcal{L}} \left\{P_{ij}^t - r_{ij} l_{ij}^t\right\} - P_{d_j}^t + P_{c_j}^t \tag{1}$$

$$q_j^t = \sum_{(j,k)\in\mathcal{L}} Q_{jk}^t - \sum_{(i,j)\in\mathcal{L}} \left\{Q_{ij}^t - x_{ij} l_{ij}^t\right\} - q_{D_j}^t - q_{B_j}^t \tag{2}$$

$$v_j^t = v_i^t + \left\{r_{ij}^2 + x_{ij}^2\right\} l_{ij}^t - 2(r_{ij} P_{ij}^t + x_{ij} Q_{ij}^t) \tag{3}$$

$$l_{ij}^t = \frac{(P_{ij}^t)^2 + (Q_{ij}^t)^2}{v_i^t} \tag{4}$$

where $P_{ij}^t, Q_{ij}^t, l_{ij}^t$ denote the sending-end real power, reactive power and the square of the magnitude of the current flowing in the branch (i, j) respectively. v_j^t denotes the square of the magnitude of the voltage at node j. The superscript t specifies the time-period for the corresponding variable. It may be noted that for a radial distribution system, each node j can have only one 'parent' node i, and thus the summation for the second term in equations Equation (1) and **??** may be dropped.

(Integer Constraint Relaxed) Naive Brute Force Full Optimization Model - Full Horizon

$$\min_{\substack{P_{ij}^t, Q_{ij}^t, v_j^t, l_{ij}^t,\\ q_{D_j}^t, B_j^t, P_{c_j}^t, {P_{d_j}}^t, q_{B_j}^t}} \quad \sum_{t=1}^{T} \sum_{(i,j)\in\mathcal{L}} (r_{ij} l_{ij}^t) \tag{5}$$

$$+ \alpha \sum_{t=1}^{T} \sum_{j\in\mathcal{B}} \left\{ (1-\eta_c) P_{c_j}^t + \left(\frac{1}{\eta_d} - 1 \right) P_{d_j}^t \right\} \tag{6}$$

$$+ \gamma \sum_{j\in\mathcal{B}} \left\{ \left(B_j^T - B_{ref_j}] \right)^2 \right\} \tag{7}$$

s.t.

$$p_j^t = \sum_{(j,k)\in\mathcal{L}} P_{jk}^t - \sum_{(i,j)\in\mathcal{L}} \left\{ P_{ij}^t - r_{ij} l_{ij}^t \right\} - P_{d_j}^t + P_{c_j}^t \tag{8}$$

$$q_j^t = \sum_{(j,k)\in\mathcal{L}} Q_{jk}^t - \sum_{(i,j)\in\mathcal{L}} \left\{ Q_{ij}^t - x_{ij} l_{ij}^t \right\} - q_{D_j}^t - q_{B_j}^t \tag{9}$$

$$v_j^t = v_i^t + \left\{ r_{ij}^2 + x_{ij}^2 \right\} l_{ij}^t - 2(r_{ij} P_{ij}^t + x_{ij} Q_{ij}^t) \tag{10}$$

$$l_{ij}^t = \frac{(P_{ij}^t)^2 + (Q_{ij}^t)^2}{v_i^t} \tag{11}$$

$$B_j^t = B_j^{t-1} + \Delta t \eta_c P_{c_j}^t - \Delta t \frac{1}{\eta_d} P_{d_j}^t \tag{12}$$

$$B_j^0 = 0.5(soc_{max} + soc_{min}) E_{Rated} = 0.625 E_{Rated} \tag{13}$$

$$where, \tag{14}$$

$$(i,j) : \text{Branch connecting nodes } i \text{ and } j \tag{15}$$

$$p_j^t = {p_D}_j^t - {p_L}_j^t \tag{16}$$

$$q_j^t = -{q_L}_j^t \tag{17}$$

$$t = \{1, 2, \ldots T\} \tag{18}$$

1.3 ENApp based Distributed Multi-Period OPF with Batteries