I. PROBLEM FORMULATION

A. Notations

In this study, the distribution system is modeled as a tree (connected graph) with N number of buses (indexed with i, j, and k); the study is conducted for T time steps (indexed by t), each of interval length Δt. The sets of buses with DERs and batteries are D and B respectively, such that $D, B \subseteq N$. A directed edge from bus i to j in the tree is represented by ij and the set for edges is given by $\mathcal{L}$. Line resistance and reactance are r_{ij} and x_{ij}, respectively. Magnitude of the current flowing through the line at time t is denoted by I_{ij}^t and $l_{ij}^t = \left(I_{ij}^t\right)^2$. The voltage magnitude of bus j at time t is given by V_j^t and $v_j^t = \left(V_j^t\right)^2$. Apparent power demand at a node j at time t is $s_{L_j}^t (= p_{L_j}^t + jq_{L_j}^t)$. The active power generation from the DER present at bus j at time t is denoted by $p_{D_j}^t$ and controlled reactive power dispatch from the DER inverter is $q_{D_j}^t$. DER inverter capacity is $S_{D_{R_j}}$. The apparent power flow through line ij at time t is $S_{ij}^t (= P_{ij}^t + jQ_{ij}^t)$. The real power flowing from the substation into the network is denoted by P_{Subs}^t and the associated cost involved per kWh is C^t. The battery energy level is B_j^t. Charging and discharging active power from battery inverter (of apparent power capacity $S_{B_{R_j}}$) are denoted by $P_{c_j}^t$ and $P_{d_j}^t$, respectively and their associated efficiencies are η_c and η_d, respectively. The energy capacity of the batteries is denoted by B_{R_j}, and the rated battery power is $P_{B_{R_j}}$. soc_{min} and soc_{max} are fractional values for denoting safe soc limits of a battery about its rated state-of-charge (soc) capacity. The reactive power support of the battery inverter is indicated by $q_{B_j}^t$.

B. MPCOPF with Batteries

The OPF problem aims to minimize two objectives as shown in (1). The first term in (1) aims to minimize the total energy cost for the entire horizon. Including the 'Battery Loss' cost as the second term ($\alpha > 0$) helps eliminate the need for binary (integer) variables typically used to prevent simultaneous charging and discharging. The resulting OPF problem is a non-convex optimization problem [?].

$$\min \sum_{t=1}^{T} \left[C^t P_{Subs}^t \Delta t + \alpha \sum_{j \in \mathcal{B}} \left\{ (1 - \eta_C) P_{C_j}^t \right. \right.$$
$$\left. \left. + \left(\frac{1}{\eta_D} - 1 \right) P_{D_j}^t \right\} \right] \tag{1}$$

Subject to the constraints (2) to (12) as given below:

$$\sum_{(j,k) \in \mathcal{L}} \left\{ P_{jk}^t \right\} - \left(P_{ij}^t - r_{ij} l_{ij}^t \right) = \left(P_{d_j}^t - P_{c_j}^t \right)$$
$$+ p_{D_j}^t - p_{L_j}^t \tag{2}$$

$$\sum_{(j,k) \in \mathcal{L}} \left\{ Q_{jk}^t \right\} - \left(Q_{ij}^t - x_{ij} l_{ij}^t \right) = q_{D_j}^t + q_{B_j}^t - q_{L_j}^t \tag{3}$$

$$v_j^t = v_i^t - 2(r_{ij} P_{ij}^t + x_{ij} Q_{ij}^t) + \left\{ r_{ij}^2 + x_{ij}^2 \right\} l_{ij}^t \tag{4}$$

$$(P_{ij}^t)^2 + (Q_{ij}^t)^2 = l_{ij}^t v_i^t \tag{5}$$

$$P_{Subs}^t \geq 0 \tag{6}$$

$$v_j^t \in \left[V_{min}^2, V_{max}^2 \right] \tag{7}$$

$$q_{D_j}^t \in \left[-\sqrt{S_{D_{R,j}}^2 - p_{D_j}^t{}^2}, \sqrt{S_{D_{R,j}}^2 - p_{D_j}^t{}^2} \right] \tag{8}$$

$$B_j^t = B_j^{t-1} + \Delta t \eta_c P_{c_j}^t - \Delta t \frac{1}{\eta_d} P_{d_j}^t \tag{9}$$

$$P_{c_j}^t, P_{d_j}^t \in \left[0, P_{B_{R_j}} \right], \quad B_j^0 = B_j^T \tag{10}$$

$$q_{B_j}^t \in \left[-\sqrt{0.44} P_{B_{R,j}}, \sqrt{0.44} P_{B_{R,j}} \right] \tag{11}$$

$$B_j^t \in [soc_{min} B_{R,j}, soc_{max} B_{R,j}] \tag{12}$$

A branch power flow model, given by (2) to (5), is used to represent power flow in distribution system. Constraints (2) and (3) model the active and reactive power balance at node j, respectively. The KVL equation for branch (ij) is represented by (4), while the equation describing the relationship between current magnitude, voltage magnitude and apparent power magnitude for branch (ij) is given by (5). Backflow of real power into the substation from the distribution system is avoided using the constraint (6). The allowable limits for bus voltages are modeled via (7). (8) describes the reactive power limits of DER inverters. The trajectory of the battery energy versus time is given by (9) (this is a time-coupled constraint). Battery charging and discharging powers are limited by the battery's rated power capacity, as given by (10). (10) also says that the initial and final energy levels for battery must be the same at the end of the optimization time horizon. Every battery's reactive power is also constrained by the corresponding inverter's rated capacity, modeled in (11). For the safe and sustainable operation of the batteries, the energy B_j^t is constrained to be within some percentage limits of the rated battery SOC capacity, modeled using (12)

C. ENApp based MPDOPF with Batteries

Assuming the presence of a feasible solution for the MP-COPF problem, the MPDOPF is formulated by decomposing the distribution system into multiple small areas. In this paper, the interaction between the two areas will follow the principle of ENApp algorithm. The ENApp algorithm leverages the radial topology of the distribution network to solve the network-level optimization problem in a distributed manner. Each area has a local controller (LC) that solves its specific local optimization problem. The local optimization problem will be the same MPOPF as described in Section II-B but defined using only the variables and parameters specific for the corresponding area. The boundary buses between two areas are accounted as voltage sources for downstream areas and load buses for upstream areas. By knowing the upstream voltage and downstream load data, the LCs will solve their area-specific MPOPF in parallel. Once LC problem converges, the downstream and upstream LCs inform power flows and voltage data to their respective upstream and downstream areas. The

LCs will again solve their local MPOPF and exchange the updated boundary variables with their neighbors. This iterative process terminates after a convergence is achieved at all boundary buses. Comprehensive details of the ENApp algorithm are available in [**?**]. Note that for the case of multi-period optimization with T time steps, each area exchanges a data vector containing $2T$ boundary variables with its neighboring areas. This includes voltage and power flow data for each T time steps.