A Spatially Distributed Multi-Period Optimal Power Flow Analysis of Active Distribution Networks with Distributed Battery Units

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***Abstract*—**

***Index Terms*—Batteries, distribution network, distributed energy resources (DERs), equivalent network approximation (ENApp)**

I. Introduction

A. Background and Prior Arts

Presently, optimal power flow (OPF) tools are developed to run the MV/LV distribution grids in the most economical, reliable, and secure manner. The usefulness of OPF studies is gaining more interest due to penetration of distributed energy resources (DERs), especially solar photovoltaic panels. Power generation from these DERs are influenced majorly by the weather conditions, hence highly intermittent nature. Presently, deployment of battery units are becoming more pertinent to mitigate the uncertainty effect and maintain the power balance by controlling the charging and/or discharging operations [1]. However, inclusion of batteries converts the conventional single period time decoupled OPF problem into a multi-period time coupled OPF analysis.

Traditionally, centralized OPF methods were popular where required data are accumulated at a central controller location [2]. The central controller is responsible to process all the accumulated data, solving the OPF algorithm and dispatch control signals to the controlling resources. Yuan et al. [3] proposed a linear OPF model for distribution network depending upon the locational marginal price (LMP). The LMP is calculated by including reactive power components and voltage constraints.

Guo et al. [4] developed a linear OPF model after linearizing the second-order cone constraints with polyhedral approximations. The OPF problem is formulated by considering the variable solar power generation as parameters and hence the overall problem takes form of a parametric distribution OPF.

B. Research Gaps and Contributions

A taxonomy table to compare the existing studies and the present work is provided in I.

The specific contributions are as follows:

1) The overall problem is

II. Theory

FIRST LINE OF THEORY

TABLE I
Taxonomy table for comparison

References	DERs	Batteries	Single period OPF	Multi-period OPF	Centralized OPF	Distributed OPF	Framework
[3]			✓		✓		Linear
[4]	✓		✓	✓			Linear
[], []	✓	✓				✓	
[]- []	✓			✓			✓
[], []		✓		✓			✓
[]- []	✓			✓			✓
This paper	✓	✓		✓		✓	Non-convex

A. Notations

In this study, the distribution network is accounted as a tree (connected graph) having N number of buses (indexed with i, j, and k) and the study is conducted for T time steps (indexed by t). The distribution line connecting two buses i and j are denoted by ij and magnitude of the current flowing through the line at time t is denoted by I_{ij}^t ($l_{ij}^t = \left(I_{ij}^t\right)^2$). The voltage magnitude of bus i at time t is given by V_i^t ($v_i^t = (V_i^t)^2$).

B. Centralized Multi-Period OPF with Batteries

On similar lines to the branch flow equations in [5], the network is modeled as a function of time, considering the interaction of batteries.

$$p_j^t = \sum_{(j,k)\in\mathcal{L}} P_{jk}^t - \sum_{(i,j)\in\mathcal{L}} \left\{P_{ij}^t - r_{ij} l_{ij}^t\right\} - P_{d_j}^t + P_{c_j}^t \tag{1}$$

$$q_j^t = \sum_{(j,k)\in\mathcal{L}} Q_{jk}^t - \sum_{(i,j)\in\mathcal{L}} \left\{Q_{ij}^t - x_{ij} l_{ij}^t\right\} - q_{D_j}^t - q_{B_j}^t \tag{2}$$

$$v_j^t = v_i^t + \left\{r_{ij}^2 + x_{ij}^2\right\} l_{ij}^t - 2(r_{ij} P_{ij}^t + x_{ij} Q_{ij}^t) \tag{3}$$

$$l_{ij}^t = \frac{(P_{ij}^t)^2 + (Q_{ij}^t)^2}{v_i^t} \tag{4}$$

where $P_{ij}^t, Q_{ij}^t, l_{ij}^t$ denote the sending-end real power, reactive power and the square of the magnitude of the current flowing in the branch (i, j) respectively. v_j^t denotes the square of the magnitude of the voltage at node j. The superscript t specifies the time-period for the corresponding variable. Node i denotes the 'parent' node of node j, which itself may be the parent of a set of k 'children' nodes (the set may contain one, many or even zero nodes, if j is a leaf node). It may be noted that for a radial distribution system, each node j can have only one 'parent' node i, and thus the summation for the second term in equations Equations (1) to (4) may be dropped.

(Integer Constraint Relaxed) Naive Brute Force Full Optimization Model - Full Horizon

$$\begin{aligned}
\min_{\substack{P_{ij}^t, Q_{ij}^t, v_j^t, l_{ij}^t,\\ q_{D_j}^t, B_j^t, P_{c_j}^t, {P_{d_j}}^t, q_{B_j}^t}} \quad & \sum_{t=1}^{T} \sum_{(i,j)\in\mathcal{L}} (r_{ij} l_{ij}^t) \\
& + \alpha \sum_{t=1}^{T} \sum_{j\in\mathcal{B}} \left\{ (1-\eta_c) P_{c_j}^t + \left(\frac{1}{\eta_d} - 1\right) P_{d_j}^t \right\} \\
& + \gamma \sum_{j\in\mathcal{B}} \left\{ \left(B_j^T - B_{ref_j}]\right)^2 \right\} \qquad (5)
\end{aligned}$$

s.t.

$$eqs.\ (1) to\ (4) \tag{6}$$

$$B_j^t = B_j^{t-1} + \Delta t \eta_c P_{c_j}^t - \Delta t \frac{1}{\eta_d} P_{d_j}^t \tag{7}$$

$$B_j^0 = 0.5(soc_{max} + soc_{min}) E_{Rated} = 0.625 E_{Rated} \tag{8}$$

$$where, \tag{9}$$

$$(i, j) : \text{Branch connecting nodes } i \text{ and } j \tag{10}$$

$$p_j^t = {p_{D_j}}^t - {p_{L_j}}^t \tag{11}$$

$$q_j^t = -{q_{L_j}}^t \tag{12}$$

$$t = \{1, 2, \ldots T\} \tag{13}$$

C. ENApp based Distributed Multi-Period OPF with Batteries

III. Case Study Demonstration

A. Simulation Data: IEEE 123 Bus Test System

B. Simulation Results

Case 1: centralized OPF with battery Case 2: ENApp based distributed OPF with battery

IV. Conclusions

[5]–[9]

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