I. Problem Formulation

A. Notations

In this study, the distribution system is modeled as a tree (connected graph) with N number of buses (indexed with i, j, and k); the study is conducted for T time steps (indexed by t), each of interval length Δt. The sets of buses with DERs and batteries are D and B respectively, such that $D, B \subseteq N$. A directed edge from bus i to j in the tree is represented by ij and the set for edges is given by $\mathcal{L}$. Line resistance and reactance are r_{ij} and x_{ij}, respectively. Magnitude of the current flowing through the line at time t is denoted by I_{ij}^t and $l_{ij}^t = \left(I_{ij}^t\right)^2$. The voltage magnitude of bus j at time t is given by V_j^t and $v_j^t = \left(V_j^t\right)^2$. Apparent power demand at a node j at time t is $s_{L_j}^t$ $(= p_{L_j}^t + jq_{L_j}^t)$. The active power generation from the DER present at bus j at time t is denoted by $p_{D_j}^t$ and controlled reactive power dispatch from the DER inverter is $q_{D_j}^t$. DER inverter capacity is $S_{D_{R_j}}$. The apparent power flow through line ij at time t is S_{ij}^t $(= P_{ij}^t + jQ_{ij}^t)$. The real power flowing from the substation into the network is denoted by P_{Subs}^t and the associated cost involved per kWh is C^t. The battery energy level is B_j^t. Charging and discharging active power from battery inverter (of apparent power capacity $S_{B_{R_j}}$) are denoted by $P_{c_j}^t$ and $P_{d_j}^t$, respectively and their associated efficiencies are η_c and η_d, respectively. The energy capacity of the batteries is denoted by B_{R_j}, and the rated battery power is $P_{B_{R_j}}$. soc_{min} and soc_{max} are fractional values for denoting safe soc limits of a battery about its rated state-of-charge (soc) capacity. The reactive power support of the battery inverter is indicated by $q_{B_j}^t$.

B. MPOPF Model Descriptions

In this section, a head to head comparison of the constraints used for both the nonlinear and linear models is provided for the Multi-Period Optimal Power Flow (MPOPF) problem. For the nonlinear formulation, the branch flow model (BFM-NL) [**?**] is used, while for the linear formulation, the LinDistFlow variation of the Branch Flow Model [**?**] is used. It may be seen that barring a few constraints, most of the constraints are common between the two models as described by (1) to (16). The uncommon constraints are suffixed with an '-NL' for the nonlinear model and an '-L' for the linear model.

(1) describes the objective function of the MPOPF problem. Specifically, for the problem of minimizing the cost of the power borrowed from the substation f_0. The inclusion of a secondary 'Battery Loss' function f_{SCD} penalizes the simultaneous charging and discharging of batteries, while avoiding the need of binary variables [**?**].

$$\min \sum_{t=1}^{T} \left\{ f_0^t + f_{SCD}^t \right\} \tag{1}$$

where

$$f_0^t = C^t P_{Subs}^t \Delta t$$

$$f_{SCD}^t = \alpha \sum_{j \in \mathcal{B}} \left\{ (1-\eta_C) P_{C_j}^t + \left(\frac{1}{\eta_D} - 1 \right) P_{D_j}^t \right\}$$

Subject to the constraints (2NL) to (16) as given below:

$$\sum_{(j,k)\in\mathcal{L}} \left\{ P_{jk}^t \right\} - \left(P_{ij}^t - r_{ij} l_{ij}^t \right) = p_j^t \tag{2NL}$$

$$\sum_{(j,k)\in\mathcal{L}} \left\{ P_{jk}^t \right\} - \left(P_{ij}^t \right) = p_j^t \tag{2L}$$

$$p_j^t = \left(P_{d_j}^t - P_{c_j}^t \right) + p_{D_j}^t - p_{L_j}^t \tag{3}$$

$$\sum_{(j,k)\in\mathcal{L}} \left\{ Q_{jk}^t \right\} - \left(Q_{ij}^t - x_{ij} l_{ij}^t \right) = q_j^t \tag{4NL}$$

$$\sum_{(j,k)\in\mathcal{L}} \left\{ Q_{jk}^t \right\} - \left(Q_{ij}^t \right) = q_j^t \tag{4L}$$

$$q_j^t = q_{D_j}^t + q_{B_j}^t - q_{L_j}^t \tag{5}$$

$$v_j^t = v_i^t - 2(r_{ij} P_{ij}^t + x_{ij} Q_{ij}^t) + \left\{ r_{ij}^2 + x_{ij}^2 \right\} l_{ij}^t \tag{6NL}$$

$$v_j^t = v_i^t - 2(r_{ij} P_{ij}^t + x_{ij} Q_{ij}^t) \tag{6L}$$

$$(P_{ij}^t)^2 + (Q_{ij}^t)^2 = l_{ij}^t v_i^t \tag{7NL}$$

$$P_{Subs}^t \geq 0 \tag{8}$$

$$v_j^t \in \left[V_{min}^2, V_{max}^2 \right] \tag{9}$$

$$q_{D_j}^t \in \left[-q_{D_{Max,j}}^t, q_{D_{Max,j}}^t \right] \tag{10}$$

$$q_{D_{Max,j}}^t = \sqrt{{S_{D_{R,j}}}^2 - {p_{D_j}^t}^2} \tag{11}$$

$$B_j^t = B_j^{t-1} + \Delta t \left(\eta_c P_{c_j}^t - \frac{1}{\eta_d} P_{d_j}^t \right) \tag{12}$$

$$P_{c_j}^t, P_{d_j}^t \in \left[0, P_{B_{R,j}} \right] \tag{13}$$

$$(P_{B_j}^t)^2 + (q_{B_j}^t)^2 \leq S_{B_{R,j}}^2 \tag{14NL}$$

$$q_{B_j}^t \in \left[-\sqrt{3}(P_{B_j}^t + S_{B_{R,j}}), -\sqrt{3}(P_{B_j}^t - S_{B_{R,j}}) \right] \tag{14L-a}$$

$$q_{B_j}^t \in \left[-\frac{\sqrt{3}}{2} S_{B_{R,j}}, \frac{\sqrt{3}}{2} S_{B_{R,j}} \right] \tag{14L-b}$$

$$q_{B_j}^t \in \left[\sqrt{3}(P_{B_j}^t - S_{B_{R,j}}), \sqrt{3}(P_{B_j}^t + S_{B_{R,j}}) \right] \tag{14L-c}$$

$$P_{B_j}^t = P_{d_j}^t - P_{c_j}^t \tag{15}$$

$$B_j^t \in [soc_{min} B_{R,j}, soc_{max} B_{R,j}] \tag{16}$$

The network power flow constraints, notably, the nodal real and reactive power balance equations and the KVL constraint across a branch, a re similar across the two models, with the only difference being whether a term representing losses is included (2NL, 4NL,6NL) or not (2L,4L,6L) not.

Backflow of real power into the substation from the distribution system is avoided using the constraint (8). The allowable limits for bus voltages are modeled via (9). (10) and (11) describe the reactive power limits of DER inverters. The trajectory of the battery energy versus time is given by

(12) (this is a time-coupled constraint). Battery charging and discharging powers are limited by the battery's rated power capacity, as given by (13). Reactive Power Output from Battery Inverters are is constrained by the quadratic inequality (14NL). A linearized set of equations approximating the same are given by (14)L, which utilize a hexagonal approximation of the inequality [**?**]. For the safe and sustainable operation of the batteries, the energy B_j^t is constrained to be within some percentage limits of the rated battery SOC capacity, modeled using (16)

C. Linear MPOPF with Batteries