Optimality Investigation of Linear and Nonlinear Multi-Period OPF Models for Active Distribution Networks

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***Abstract*—**

***Index Terms*—Battery energy storage systems, distribution system, optimal power flow, distributed energy resources.**

I. Introduction

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II. Problem Formulation

A. Notations

In this study, the distribution system is modeled as a tree (connected graph) with N number of buses (indexed with i, j, and k); the study is conducted for T time steps (indexed by t), each of interval length Δt. The sets of buses with DERs and batteries are D and B respectively, such that $D, B \subseteq N$. A directed edge from bus i to j in the tree is represented by ij and the set for edges is given by $\mathcal{L}$. Line resistance and reactance are r_{ij} and x_{ij}, respectively. Magnitude of the current flowing through the line at time t is denoted by I_{ij}^t and $l_{ij}^t = \left(I_{ij}^t\right)^2$. The voltage magnitude of bus j at time t is given by V_j^t and $v_j^t = \left(V_j^t\right)^2$. Apparent power demand at a node j at time t is $s_{L_j}^t$ $(= p_{L_j}^t + jq_{L_j}^t)$. The active power generation from the DER present at bus j at time t is denoted by $p_{D_j}^t$ and controlled reactive power dispatch from the DER inverter is $q_{D_j}^t$. DER inverter capacity is $S_{D_{R_j}}$. The apparent power flow through line ij at time t is S_{ij}^t $(= P_{ij}^t + jQ_{ij}^t)$. The real power flowing from the substation into the network is denoted by P_{Subs}^t and the associated cost involved per kWh is C^t. The battery energy level is B_j^t. Charging and discharging active power from battery inverter (of apparent power capacity $S_{B_{R_j}}$) are denoted by $P_{c_j}^t$ and $P_{d_j}^t$, respectively and their associated efficiencies are η_c and η_d, respectively. The energy capacity of the batteries is denoted by B_{R_j}, and the rated battery power is $P_{B_{R_j}}$. soc_{min} and soc_{max} are fractional values for denoting safe soc limits of a battery about its rated state-of-charge (soc) capacity. The reactive power support of the battery inverter is indicated by $q_{B_j}^t$.

B. Non-linear MPCOPF with Batteries

The OPF problem aims to minimize two objectives as shown in (1). The first term in (1) aims to minimize the total energy cost for the entire horizon. Including the 'Battery Loss' cost as the second term ($\alpha > 0$) helps eliminate the need for binary (integer) variables typically used to prevent simultaneous charging and discharging. The resulting OPF problem is a non-convex optimization problem [1].

$$\min \sum_{t=1}^{T} \left\{ f_0^t + f_{SCD}^t \right\} \tag{1}$$

where

$$f_0^t = C^t P_{Subs}^t \Delta t$$

$$f_{SCD}^t = \alpha \sum_{j \in \mathcal{B}} \left\{ (1-\eta_C) P_{C_j}^t + \left(\frac{1}{\eta_D} - 1 \right) P_{D_j}^t \right\}$$

Subject to the constraints (2L) to (16) as given below:

$$\sum_{(j,k) \in \mathcal{L}} \left\{ P_{jk}^t \right\} - \left(P_{ij}^t - r_{ij} l_{ij}^t \right) = p_j^t \tag{2NL}$$

$$\sum_{(j,k) \in \mathcal{L}} \left\{ P_{jk}^t \right\} - \left(P_{ij}^t \right) = p_j^t \tag{2L}$$

$$p_j^t = \left(P_{d_j}^t - P_{c_j}^t \right) + p_{D_j}^t - p_{L_j}^t \tag{3}$$

$$\sum_{(j,k) \in \mathcal{L}} \left\{ Q_{jk}^t \right\} - \left(Q_{ij}^t - x_{ij} l_{ij}^t \right) = q_j^t \tag{4NL}$$

$$\sum_{(j,k) \in \mathcal{L}} \left\{ Q_{jk}^t \right\} - \left(Q_{ij}^t \right) = q_j^t \tag{4L}$$

$$q_j^t = q_{D_j}^t + q_{B_j}^t - q_{L_j}^t \tag{5}$$

$$v_j^t = v_i^t - 2(r_{ij} P_{ij}^t + x_{ij} Q_{ij}^t) + \left\{ r_{ij}^2 + x_{ij}^2 \right\} l_{ij}^t \tag{6NL}$$

$$v_j^t = v_i^t - 2(r_{ij} P_{ij}^t + x_{ij} Q_{ij}^t) \tag{6L}$$

$$(P_{ij}^t)^2 + (Q_{ij}^t)^2 = l_{ij}^t v_i^t \tag{7NL}$$

$$P_{Subs}^t \geq 0 \tag{8}$$

$$v_j^t \in \left[V_{min}^2, V_{max}^2 \right] \tag{9}$$

$$q_{D_j}^t \in \left[-q_{D_{Max,j}}^t, q_{D_{Max,j}}^t \right] \tag{10}$$

$$q_{D_{Max,j}}^t = \sqrt{ {S_{D_{R,j}}}^2 - {p_{D_j}^t}^2 } \tag{11}$$

$$B_j^t = B_j^{t-1} + \Delta t \left(\eta_c P_{c_j}^t - \frac{1}{\eta_d} P_{d_j}^t \right), \quad B_j^0 = B_j^T \tag{12}$$

$$P_{c_j}^t, P_{d_j}^t \in \left[0, P_{B_{R,j}}\right] \tag{13}$$

$$(P_{B_j}^t)^2 + (q_{B_j}^t)^2 \leq S_{B_{R,j}}^2 \tag{14NL}$$

$$q_{B_j}^t \in \left[-\sqrt{3}(P_{B_j}^t + S_{B_{R,j}}), -\sqrt{3}(P_{B_j}^t - S_{B_{R,j}})\right] \tag{14L-a}$$

$$q_{B_j}^t \in \left[-\frac{\sqrt{3}}{2} S_{B_{R,j}}, \frac{\sqrt{3}}{2} S_{B_{R,j}}\right] \tag{14L-b}$$

$$q_{B_j}^t \in \left[\sqrt{3}(P_{B_j}^t - S_{B_{R,j}}), \sqrt{3}(P_{B_j}^t + S_{B_{R,j}})\right] \tag{14L-c}$$

$$P_{B_j}^t = P_{d_j}^t - P_{c_j}^t \tag{15}$$

$$B_j^t \in [soc_{min} B_{R,j}, soc_{max} B_{R,j}] \tag{16}$$

A branch power flow model, given by (2L) to (7NL), is used to represent power flow in distribution system. Constraints (2L) and (4L) model the active and reactive power balance at node j, respectively.

The KVL equation for branch (ij) is represented by (6L), while the equation describing the relationship between current magnitude, voltage magnitude and apparent power magnitude for branch (ij) is given by (7NL). Backflow of real power into the substation from the distribution system is avoided using the constraint (8). The allowable limits for bus voltages are modeled via (9). (10) and (11) describe the reactive power limits of DER inverters. The trajectory of the battery energy versus time is given by (12) (this is a time-coupled constraint). Battery charging and discharging powers are limited by the battery's rated power capacity, as given by (13). (13) also says that the initial and final energy levels for battery must be the same at the end of the optimization time horizon. Reactive Power Output from Battery Inverters are is constrained by the quadratic inequality (14NL). A linearized set of equations approximating the same are given by (14)L, which utilize a hexagonal approximation of the inequality [2]. For the safe and sustainable operation of the batteries, the energy B_j^t is constrained to be within some percentage limits of the rated battery SOC capacity, modeled using (16)

C. Linear MPCOPF with Batteries

III. Case Study Demonstration

TABLE III: ACOPF feasibility analyses - 5 hour

Metric	**MPDOPF**	**OpenDSS**
Full horizon		
Substation real power (kW)	4308.14	4308.35
Line loss (kW)	76.12	76.09
Substation reactive power (kVAR)	656.24	652.49
Max. all-time discrepancy		
Voltage (pu)	0.0002	
Line loss (kW)	0.0139	
Substation power (kW)	0.3431	

TABLE I: Parameter values

Parameter	**Value**
V_{min}, V_{max}	0.95 pu, 1.05 pu
$p_{D_{R,j}}$	$0.33 p_{L_{R,j}}$
$S_{D_{R,j}}$	$1.2 p_{D_{R,j}}$
$P_{B_{R,j}}$	$0.33 p_{L_{R,j}}$
$S_{B_{R,j}}$	$1.2 P_{B_{R,j}}$
$B_{R,j}$	$T_{fullCharge} \times P_{B_{R,j}}$
$T_{fullCharge}$	4 h
Δt	1 h
η_c, η_d	0.95, 0.95
soc_{min}, soc_{max}	0.30, 0.95
α	0.001

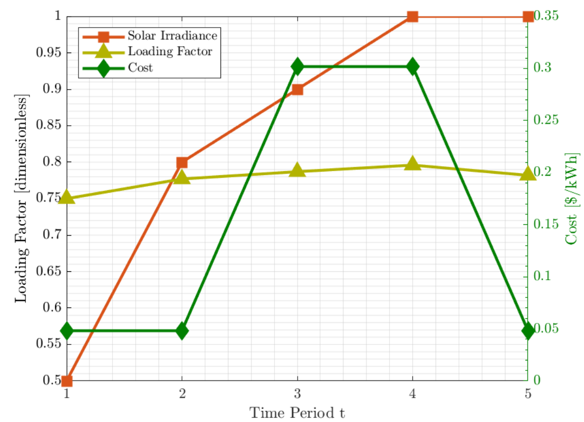

Fig. 1: Forecasts for demand power, irradiance and cost of substation power over a 5 hour horizon

TABLE IV: Comparison between Nonlinear BFM and LinDistFlow - 10 hour

Metric	**Nonlinear BFM**	**LinDistFlow**
Largest subproblem		
Decision variables	6300	2640
Linear constraints	11636	4891
Nonlinear constraints	1270	530
Simulation results		
Substation power cost ($)	1197.87	1197.87
Substation real power (kW)	8544.28	8544.04
Line loss (kW)	148.67	148.94
Substation reactive power (kVAR)	1092.39	1252.03
PV reactive power (kVAR)	222.59	139.81
Battery reactive power (kVAR)	388.52	310.94
Computation		
Number of Iterations	-	5
Total Simulation Time (s)	4620.73	358.69

IV. Conclusions

References

[1] N. Nazir and M. Almassalkhi, "Guaranteeing a Physically Realizable Battery Dispatch Without Charge-Discharge Complementarity Constraints," IEEE Trans. Smart Grid, vol. 14, no. 3, pp. 2473–2476, Sep. 2021.

[2] H. Ahmadi and J. R. Martı', "Linear Current Flow Equations With Application to Distribution Systems Reconfiguration," IEEE Trans. Power Syst., vol. 30, no. 4, pp. 2073–2080, Oct. 2014.

TABLE II: Comparative analyses between BFM-NL and LinDistFlow - IEEE123 System for 24 time-period horizon

Metric	**BFM-NL**	**LinDistFlow**$^{\mathbb{O}}$
Largest subproblem		
Decision variables	15144	12096
Linear constraints	18456	22200
Nonlinear constraints	3672	0
Simulation results		
Substation power cost ($)	2787.44	2798.4
Substation real power (kW)	20984.89	21065.89
Line loss (kW)	380.09	461.38
Substation reactive power (kVAR)	6835.82	12259.29
PV reactive power (kVAR)	1972.27	195.12
Battery reactive power (kVAR)	3709.71	204.63
Computation		
Total Simulation Time (s)	17.44	0.85

TABLE V: ACOPF feasibility analyses - 24 hour

Metric	**BFM-NL**	**LinDistFlow**
Max. all-time discrepancy		
Voltage (pu)	0.00007	0.00206
Line loss (kW)	0.01818	1.8074
Substation power (kW)	0.43164	32.362
Substation reactive power (kVAR)	1.0102	64.403