

# Analyzing the Optimality Gap between Linear and Nonlinear Multi-Period Optimal Power Flow Models of Active Distribution Networks

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**Abstract**—The development of the multi-period optimal power flow framework is gaining interest nowadays because of the large penetration of battery associated distributed energy resources (DERs) into electricity distribution networks (EDNs). Typically, the OPF problem is modeled as a non-convex programming (NCP) framework by employing the non-convex branch power flow model of EDNs. However, the slow convergence and large computation time of the NCP motivate researchers to explore the possibility of developing the linear programming (LP) based OPF portfolio at the expense of the optimality gap in the derived solutions. To the best of the authors' knowledge, no state-of-the-art research has investigated the impact of EDN size on the optimality gap. In this context, this article aims to develop multi-period OPF models for EDNs and analyze their NCP- and LP-based solution processes across different EDN sizes, specifically 10-bus (small), IEEE-123 bus (medium), and 730-bus (large) test systems. The optimality gaps in the ACOPF solution and decision variables for each network are assessed by simulating the control decisions in the OpenDSS platform. The simulation results indicate that while the optimality gap due to LP increases with network size, it consistently delivers near-optimal solutions with fast convergence.

**Index Terms**—Battery energy storage systems, distribution system, optimal power flow, distributed energy resources.

## I. INTRODUCTION

Optimal power flow (OPF) techniques are utilized to efficiently manage controllable grid-edge resources to achieve various system-wide goals, including cost-effectiveness, reliability, and resilience [1]. OPF analysis is becoming increasingly important at the distribution level due to the rising integration of distributed energy resources (DERs), particularly photovoltaic (PV) systems and battery energy storage systems (BESS). BESS plays a crucial role in mitigating DER variability through controlled charging and discharging, ensuring a stable supply-demand balance [2]. However, incorporating BESS into OPF significantly increases the complexity of distribution network (DN) optimization, shifting the problem from a single-period, time-independent framework to a multi-period, time-coupled approach [3].

Conventional OPF methods rely on a central controller that collects grid-edge data, runs the OPF algorithm, and dispatches control signals to manage system resources. Therefore, those are named as centralized OPF (COPF), which are typically formulated as mixed-integer non-convex programming (MINCP)

problems. A non-convex active-reactive OPF is formulated in [4] for scheduling the operation of BESS in DN. Safdarian et al. [5] investigated the impact of demand response programs on residential customers by directly solving the the MINCP based OPF problem. Mohapatra et al. [6] combined the gradient method and metaheuristic optimization for solving the MINCP framework. Padilha-Feltrin et al. [7] and Liu et al. [8] employed nondominated sorting genetic algorithm (NSGA-II) and improved grey wolf equilibrium optimizer for solving the MINCP based OPF problems, respectively. Previously, authors also solved the MINCP based OPF for determining the operation of batteries in DN [9].

For making the MINCP OPF models convex, Li et al. [10] proposed a linear power flow model by merging support vector regression (SVR) and ridge regression (RR) algorithms. Lei et al. [11] proposed a privacy-preserving linear OPF model for multi-agent DN having privately owned grid resources. Linear approximation of the non-convex power flow model with Taylor series expansion for reactive power optimization is suggested in [12] and [13]. Vaishya et al. [14] designed a linear ACOPF model for DN using active and reactive power sensitivity factors.

Following research gaps are identified from the above literature survey:

- 1) Direct solution of MINCP framework, [4], [5], [9] may provide the global optimal solution but the solution time is more and mostly efficient for small and medium-sized DNs. They possess slow convergence and sometimes fail to converge for bulk DNs.
- 2) Metaheuristic optimizations, [6]–[8], may stuck at the local optimum solution and suffer from slow convergence for multi-variate problems.
- 3) Linear programming frameworks, [10]–[14], are fast converging. However, they possess an optimality gap in the derived solutions and the impact of the size of the network on the optimality gap is not investigated in the existing research works.

To overcome the above mentioned research gaps, this article aims to investigate the impact of the size of the network on the value of the optimality gap between the solutions obtained from non-linear and linear multi-period OPF models for DNs.

The overall study has the following distinguished features:

- 1) At first, the multi-period OPF problem is formulated for DNs consisting of battery-associated solar power generation as both non-linear and linear optimization problems. The non-linear problem is formulated by considering the original non-convex branch flow model of DNs [15]. In contrast, the linear framework is developed by considering the LinDistFlow model of DN [16]. Further, the quadratic inequality constraints are linearized by employing the hexagonal approximation technique.
- 2) The investigation is carried out for distribution networks with different sizes like small (10-bus), medium (IEEE 123-bus), and large (730-bus) with different penetration levels of DERs and batteries.
- 3) The ACOF feasibility of the derived solutions is validated by feeding the determined control outputs in OpenDSS platform.

## II. PROBLEM FORMULATION

### A. Notations

In this study, the distribution system is modeled as a tree (connected graph) with  $N$  number of buses (indexed with  $i$ ,  $j$ , and  $k$ ); the study is conducted for  $T$  time steps (indexed by  $t$ ), each of interval length  $\Delta t$ . The sets of buses with DERs and batteries are  $D$  and  $B$  respectively, such that  $D, B \subseteq N$ . A directed edge from bus  $i$  to  $j$  in the tree is represented by  $ij$  and the set for edges is given by  $\mathcal{L}$ . Line resistance and reactance are  $r_{ij}$  and  $x_{ij}$ , respectively. Magnitude of the current flowing through the line at time  $t$  is denoted by  $I_{ij}^t$  and  $l_{ij}^t = (I_{ij}^t)^2$ . The voltage magnitude of bus  $j$  at time  $t$  is given by  $V_j^t$  and  $v_j^t = (V_j^t)^2$ . Apparent power demand at a node  $j$  at time  $t$  is  $s_{L_j}^t (= p_{L_j}^t + jq_{L_j}^t)$ . The active power generation from the DER present at bus  $j$  at time  $t$  is denoted by  $p_{D_j}^t$  and controlled reactive power dispatch from the DER inverter is  $q_{D_j}^t$ . DER inverter capacity is  $S_{D_{R_j}}$ . The apparent power flow through line  $ij$  at time  $t$  is  $S_{ij}^t (= P_{ij}^t + jQ_{ij}^t)$ . The real power flowing from the substation into the network is denoted by  $P_{Subs}^t$  and the associated cost involved per kWh is  $C^t$ . The battery energy level is  $B_j^t$ . Charging and discharging active power from battery inverter (of apparent power capacity  $S_{B_{R_j}}$ ) are denoted by  $P_{c_j}^t$  and  $P_{d_j}^t$ , respectively and their associated efficiencies are  $\eta_c$  and  $\eta_d$ , respectively. The energy capacity of the batteries is denoted by  $B_{R_j}$ , and the rated battery power is  $P_{B_{R_j}}$ .  $soc_{min}$  and  $soc_{max}$  are fractional values for denoting safe soc limits of a battery about its rated state-of-charge (soc) capacity. The reactive power support of the battery inverter is indicated by  $q_{B_j}^t$ .

### B. Non-linear MPCOPF with Batteries

The OPF problem aims to minimize two objectives as shown in (1). The first term in (1) aims to minimize the total energy cost for the entire horizon. Including the 'Battery Loss' cost as the second term ( $\alpha > 0$ ) helps eliminate the need for binary (integer) variables typically used to prevent

simultaneous charging and discharging. The resulting OPF problem is a non-convex optimization problem [17].

$$\min \sum_{t=1}^T \{f_0^t + f_{SCD}^t\} \quad (1)$$

where

$$f_0^t = C^t P_{Subs}^t \Delta t$$

$$f_{SCD}^t = \alpha \sum_{j \in B} \left\{ (1 - \eta_c) P_{c_j}^t + \left( \frac{1}{\eta_d} - 1 \right) P_{d_j}^t \right\}$$

Subject to the constraints (2L) to (16) as given below:

$$\sum_{(j,k) \in \mathcal{L}} \{P_{jk}^t\} - (P_{ij}^t - r_{ij} l_{ij}^t) = p_j^t \quad (2NL)$$

$$\sum_{(j,k) \in \mathcal{L}} \{P_{jk}^t\} - (P_{ij}^t) = p_j^t \quad (2L)$$

$$p_j^t = (P_{d_j}^t - P_{c_j}^t) + p_{D_j}^t - p_{L_j}^t \quad (3)$$

$$\sum_{(j,k) \in \mathcal{L}} \{Q_{jk}^t\} - (Q_{ij}^t - x_{ij} l_{ij}^t) = q_j^t \quad (4NL)$$

$$\sum_{(j,k) \in \mathcal{L}} \{Q_{jk}^t\} - (Q_{ij}^t) = q_j^t \quad (4L)$$

$$q_j^t = q_{D_j}^t + q_{B_j}^t - q_{L_j}^t \quad (5)$$

$$v_j^t = v_i^t - 2(r_{ij} P_{ij}^t + x_{ij} Q_{ij}^t) + \{r_{ij}^2 + x_{ij}^2\} l_{ij}^t \quad (6NL)$$

$$v_j^t = v_i^t - 2(r_{ij} P_{ij}^t + x_{ij} Q_{ij}^t) \quad (6L)$$

$$(P_{ij}^t)^2 + (Q_{ij}^t)^2 = l_{ij}^t v_i^t \quad (7NL)$$

$$P_{Subs}^t \geq 0 \quad (8)$$

$$v_j^t \in [V_{min}^2, V_{max}^2] \quad (9)$$

$$q_{D_j}^t \in [-q_{D_{Max,j}}^t, q_{D_{Max,j}}^t] \quad (10)$$

$$q_{D_{Max,j}}^t = \sqrt{S_{D_{R,j}}^2 - p_{D_j}^t{}^2} \quad (11)$$

$$B_j^t = B_j^{t-1} + \Delta t \left( \eta_c P_{c_j}^t - \frac{1}{\eta_d} P_{d_j}^t \right), \quad B_j^0 = B_j^T \quad (12)$$

$$P_{c_j}^t, P_{d_j}^t \in [0, P_{B_{R,j}}] \quad (13)$$

$$(P_{B_j}^t)^2 + (q_{B_j}^t)^2 \leq S_{B_{R,j}}^2 \quad (14NL)$$

$$q_{B_j}^t \in [-\sqrt{3}(P_{B_j}^t + S_{B_{R,j}}), -\sqrt{3}(P_{B_j}^t - S_{B_{R,j}})] \quad (14L-a)$$

$$q_{B_j}^t \in \left[ -\frac{\sqrt{3}}{2} S_{B_{R,j}}, \frac{\sqrt{3}}{2} S_{B_{R,j}} \right] \quad (14L-b)$$

$$q_{B_j}^t \in [\sqrt{3}(P_{B_j}^t - S_{B_{R,j}}), \sqrt{3}(P_{B_j}^t + S_{B_{R,j}})] \quad (14L-c)$$

$$P_{B_j}^t = P_{d_j}^t - P_{c_j}^t \quad (15)$$

$$B_j^t \in [soc_{min} B_{R,j}, soc_{max} B_{R,j}] \quad (16)$$

TABLE I: Parameter values

Parameter	Value
$V_{min}, V_{max}$	0.95 pu, 1.05 pu
$p_{DR,j}$	$0.33p_{LR,j}$
$s_{DR,j}$	$1.2p_{DR,j}$
$p_{BR,j}$	$0.33p_{LR,j}$
$s_{BR,j}$	$1.2p_{BR,j}$
$B_{R,j}$	$T_{fullCharge} \times P_{BR,j}$
$T_{fullCharge}$	4 h
$\Delta t$	1 h
$\eta_c, \eta_d$	0.95, 0.95
$soc_{min}, soc_{max}$	0.30, 0.95
$\alpha$	0.001

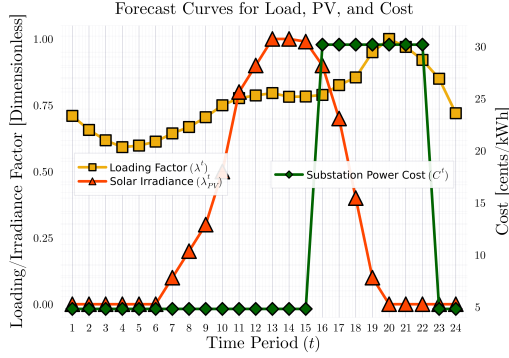


Fig. 1: Forecasts for demand power, irradiance and cost of substation power over a 24 hour horizon

A branch power flow model, given by (2L) to (7NL), is used to represent power flow in distribution system. Constraints (2L) and (4L) model the active and reactive power balance at node  $j$ , respectively.

The KVL equation for branch  $(ij)$  is represented by (6L), while the equation describing the relationship between current magnitude, voltage magnitude and apparent power magnitude for branch  $(ij)$  is given by (7NL). Backflow of real power into the substation from the distribution system is avoided using the constraint (8). The allowable limits for bus voltages are modeled via (9). (10) and (11) describe the reactive power limits of DER inverters. The trajectory of the battery energy versus time is given by (12) (this is a time-coupled constraint). Battery charging and discharging powers are limited by the battery's rated power capacity, as given by (13). (13) also says that the initial and final energy levels for battery must be the same at the end of the optimization time horizon. Reactive Power Output from Battery Inverters are is constrained by the quadratic inequality (14NL). A linearized set of equations approximating the same are given by (14)L, which utilize a hexagonal approximation of the inequality [18]. For the safe and sustainable operation of the batteries, the energy  $B_j^t$  is constrained to be within some percentage limits of the rated battery SOC capacity, modeled using (16)

TABLE II: MPOPF performance comparison - ADS10 test system for 24h

Metric	BFM-NL	LinDistFlow
Full horizon		
Substation power cost (\$)	204.27	204.28
Substation real power (kW)	1528.35	1528.4
Line loss (kW)	0.28	0.33
Substation reactive power (kVAR)	428.9	795.56
PV reactive power (kVAR)	174.41	-0.69
Battery reactive power (kVAR)	192.8	-0.37
Computation		
Number of Iterations	1	1
Total Simulation Time (s)	2.64	0.77

TABLE III: MPOPF feasibility comparison - ADS10 test system for 24h

Metric	BFM-NL	LinDistFlow
Max. all-time discrepancy		
Voltage (pu)	0.00001	0.00001
Line loss (kW)	0.000009	0.000006
Substation power (kW)	0.000014	0.02410
Substation reactive power (kVAR)	0.070706	0.05618

### C. Linear MPCOPF with Batteries

## III. CASE STUDY DEMONSTRATION

## IV. CONCLUSIONS

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TABLE IV: MPOPF performance comparison - IEEE123-A test system for 24h

Metric	BFM-NL	LinDistFlow <sup>⓪</sup>
Largest subproblem		
Decision variables	15144	12096
Linear constraints	18456	22200
Nonlinear constraints	3672	0
Simulation results		
Substation power cost (\$)	2787.44	2798.4
Substation real power (kW)	20984.89	21065.89
Line loss (kW)	380.09	461.38
Substation reactive power (kVAR)	6835.82	12259.29
PV reactive power (kVAR)	1972.27	195.12
Battery reactive power (kVAR)	3709.71	204.63
Computation		
Total Simulation Time (s)	17.44	0.85

TABLE V: MPOPF feasibility comparison - IEEE123-A tests system for 24h

Metric	BFM-NL	LinDistFlow
Max. all-time discrepancy		
Voltage (pu)	0.00007	0.00206
Line loss (kW)	0.01818	1.8074
Substation power (kW)	0.43164	32.362
Substation reactive power (kVAR)	1.0102	64.403

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TABLE VI: MPOPF performance comparison - IEEE123-B test system for 24h

Metric	BFM-NL	LinDistFlow <sup>⓪</sup>
Largest subproblem		
Decision variables	17184	0000
Linear constraints	24168	0000
Nonlinear constraints	4080	0000
Simulation results		
Substation power cost (\$)	1973.83	0000
Substation real power (kW)	16594.11	0000
Line loss (kW)	234.78	0000
Substation reactive power (kVAR)	-404.91	0000
PV reactive power (kVAR)	7255.22	0000
Battery reactive power (kVAR)	5371.97	0000
Computation		
Total Simulation Time (s)	23.75	0000

TABLE VII: MPOPF performance comparison - IEEE730 test system for 24h

Metric	BFM-NL	LinDistFlow <sup>⓪</sup>
Largest subproblem		
Decision variables	00000	67224
Linear constraints	00000	131616
Nonlinear constraints	0000	0
Simulation results		
Substation power cost (\$)	0000	1539.4
Substation real power (kW)	0000	12313.19
Line loss (kW)	0000	176.41
Substation reactive power (kVAR)	0000	4626.23
PV reactive power (kVAR)	0000	-18.69
Battery reactive power (kVAR)	0000	-14.33
Computation		
Total Simulation Time (s)	0000	7.67

TABLE VIII: MPOPF feasibility comparison - IEEE730 test system for 24h

Metric	BFM-NL	LinDistFlow
Max. all-time discrepancy		
Voltage (pu)	0000	0.0227
Line loss (kW)	0000	2.6696
Substation power (kW)	0000	12.1844
Substation reactive power (kVAR)	0000	7.3131