Investigating Optimality Gap between Linear and Nonlinear Multi-Period OPF Models for Active Distribution Networks

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***Abstract*—**

***Index Terms*—Battery energy storage systems, distribution system, optimal power flow, distributed energy resources.**

I. Introduction

Optimal power flow (OPF) techniques are utilized to efficiently manage controllable grid-edge resources to achieve various system-wide goals, including cost-effectiveness, reliability, and resilience [1]. OPF analysis is becoming increasingly important at the distribution level due to the rising integration of distributed energy resources (DERs), particularly photovoltaic (PV) systems and battery energy storage systems (BESS). BESS plays a crucial role in mitigating DER variability through controlled charging and discharging, ensuring a stable supply-demand balance [2]. However, incorporating BESS into OPF significantly increases the complexity of distribution network (DN) optimization, shifting the problem from a single-period, time-independent framework to a multi-period, time-coupled approach [3].

Conventional OPF methods rely on a central controller that collects grid-edge data, runs the OPF algorithm, and dispatches control signals to manage system resources. Therefore, those are named as centralized OPF (COPF), which are typically formulated as mixed-integer non-convex programming (MINCP) problems. A non-convex active-reactive OPF is formulated in [4] for scheduling the operation of BESS in DN. Safdarian et al. [5] investigated the impact of demand response programs on residential customers by directly solving the the MINCP based OPF problem. Mohapatra et al. [6] combined the gradient method and metaheuristic optimization for solving the MINCP framework. Padilha-Feltrin et al. [7] and Liu et al. [8] employed nondominated sorting genetic algorithm (NSGA-II) and improved grey wolf equilibrium optimizer for solving the MINCP based OPF problems, respectively. Previously, authors also solved the MINCP based OPF for determining the operation of batteries in DN [9].

For making the MINCP OPF models convex, Li et al. [10] proposed a linear power flow model by merging support vector regression (SVR) and ridge regression (RR) algorithms. Lei et al. [11] proposed a privacy-preserving linear OPF model for multi-agent DN having privately owned grid resources. Linear approximation of the non-convex power flow model with Taylor series expansion for reactive power optimization is suggested in [12] and [13]. Vaishya et al. [14] designed a linear ACOPF model for DN using active and reactive power sensitivity factors.

Following research gaps are identified from the above literature survey:

1) Direct solution of MINCP framework, [**?**], [5], [9] may provide the global optimal solution but the solution time is more and mostly efficient for small and medium sized DNs. They are improper for bulk DNs.
2) Metaheuristic optimizations, [6]–[8], may stuck at the local optimum solution and suffer from slow convergence for multi-variate problems.
3) Linear programming frameworks, [10]–[14] are fast converging. However, they possess an optimality gap in the derived solutions. Further, the impact of the network size on the optimality gap is

II. Problem Formulation

A. Notations

In this study, the distribution system is modeled as a tree (connected graph) with N number of buses (indexed with i, j, and k); the study is conducted for T time steps (indexed by t), each of interval length Δt. The sets of buses with DERs and batteries are D and B respectively, such that $D, B \subseteq N$. A directed edge from bus i to j in the tree is represented by ij and the set for edges is given by $\mathcal{L}$. Line resistance and reactance are r_{ij} and x_{ij}, respectively. Magnitude of the current flowing through the line at time t is denoted by I_{ij}^t and $l_{ij}^t = \left(I_{ij}^t\right)^2$. The voltage magnitude of bus j at time t is given by V_j^t and $v_j^t = \left(V_j^t\right)^2$. Apparent power demand at a node j at time t is $s_{L_j}^t$ $(= p_{L_j}^t + jq_{L_j}^t)$. The active power generation from the DER present at bus j at time t is denoted by $p_{D_j}^t$ and controlled reactive power dispatch from the DER inverter is $q_{D_j}^t$. DER inverter capacity is $S_{D_{R_j}}$. The apparent power flow through line ij at time t is S_{ij}^t $(= P_{ij}^t + jQ_{ij}^t)$. The real power flowing from the substation into the network is denoted by P_{Subs}^t and the associated cost involved per kWh is C^t. The battery energy level is B_j^t. Charging and discharging active power from battery inverter (of apparent power capacity $S_{B_{R_j}}$) are denoted by $P_{c_j}^t$ and $P_{d_j}^t$, respectively and their associated efficiencies are η_c and η_d, respectively. The energy capacity of the batteries is denoted by B_{R_j}, and the rated battery power is $P_{B_{R_j}}$. soc_{min} and soc_{max} are fractional values for denoting safe soc limits of a battery about its rated state-of-charge (soc) capacity. The reactive power support of the battery inverter is indicated by $q^t_{B_j}$.

B. Non-linear MPCOPF with Batteries

The OPF problem aims to minimize two objectives as shown in (1). The first term in (1) aims to minimize the total energy cost for the entire horizon. Including the 'Battery Loss' cost as the second term ($\alpha > 0$) helps eliminate the need for binary (integer) variables typically used to prevent simultaneous charging and discharging. The resulting OPF problem is a non-convex optimization problem [15].

$$\min \sum_{t=1}^{T} \left\{ f_0^t + f_{SCD}^t \right\} \tag{1}$$

where

$$f_0^t = C^t P^t_{Subs} \Delta t$$

$$f^t_{SCD} = \alpha \sum_{j \in \mathcal{B}} \left\{ (1-\eta_C) P^t_{C_j} + \left(\frac{1}{\eta_D} - 1 \right) P^t_{D_j} \right\}$$

Subject to the constraints (2L) to (16) as given below:

$$\sum_{(j,k)\in\mathcal{L}} \left\{ P^t_{jk} \right\} - \left(P^t_{ij} - r_{ij} l^t_{ij} \right) = p^t_j \tag{2NL}$$

$$\sum_{(j,k)\in\mathcal{L}} \left\{ P^t_{jk} \right\} - \left(P^t_{ij} \right) = p^t_j \tag{2L}$$

$$p^t_j = \left(P^t_{d_j} - P^t_{c_j} \right) + p^t_{D_j} - p^t_{L_j} \tag{3}$$

$$\sum_{(j,k)\in\mathcal{L}} \left\{ Q^t_{jk} \right\} - \left(Q^t_{ij} - x_{ij} l^t_{ij} \right) = q^t_j \tag{4NL}$$

$$\sum_{(j,k)\in\mathcal{L}} \left\{ Q^t_{jk} \right\} - \left(Q^t_{ij} \right) = q^t_j \tag{4L}$$

$$q^t_j = q^t_{D_j} + q^t_{B_j} - q^t_{L_j} \tag{5}$$

$$v^t_j = v^t_i - 2(r_{ij} P^t_{ij} + x_{ij} Q^t_{ij}) + \left\{ r^2_{ij} + x^2_{ij} \right\} l^t_{ij} \tag{6NL}$$

$$v^t_j = v^t_i - 2(r_{ij} P^t_{ij} + x_{ij} Q^t_{ij}) \tag{6L}$$

$$(P^t_{ij})^2 + (Q^t_{ij})^2 = l^t_{ij} v^t_i \tag{7NL}$$

$$P^t_{Subs} \geq 0 \tag{8}$$

$$v^t_j \in \left[V^2_{min}, V^2_{max} \right] \tag{9}$$

$$q^t_{D_j} \in \left[-q^t_{D_{Max,j}}, q^t_{D_{Max,j}} \right] \tag{10}$$

$$q^t_{D_{Max,j}} = \sqrt{{S_{D_{R,j}}}^2 - {p^t_{D_j}}^2} \tag{11}$$

$$B^t_j = B^{t-1}_j + \Delta t \left(\eta_c P^t_{c_j} - \frac{1}{\eta_d} P^t_{d_j} \right), \quad B^0_j = B^T_j \tag{12}$$

$$P^t_{c_j}, P^t_{d_j} \in \left[0, P_{B_{R,j}} \right] \tag{13}$$

TABLE I: Parameter values

Parameter	Value
V_{min}, V_{max}	0.95 pu, 1.05 pu
$p_{D_{R,j}}$	$0.33 p_{L_{R,j}}$
$S_{D_{R,j}}$	$1.2 p_{D_{R,j}}$
$P_{B_{R,j}}$	$0.33 p_{L_{R,j}}$
$S_{B_{R,j}}$	$1.2 P_{B_{R,j}}$
$B_{R,j}$	$T_{fullCharge} \times P_{B_{R,j}}$
$T_{fullCharge}$	4 h
Δt	1 h
η_c, η_d	0.95, 0.95
soc_{min}, soc_{max}	0.30, 0.95
α	0.001

$$(P^t_{B_j})^2 + (q^t_{B_j})^2 \leq S^2_{B_{R,j}} \tag{14NL}$$

$$q^t_{B_j} \in \left[-\sqrt{3}(P^t_{B_j} + S_{B_{R,j}}), -\sqrt{3}(P^t_{B_j} - S_{B_{R,j}}) \right] \tag{14L-a}$$

$$q^t_{B_j} \in \left[-\frac{\sqrt{3}}{2} S_{B_{R,j}}, \frac{\sqrt{3}}{2} S_{B_{R,j}} \right] \tag{14L-b}$$

$$q^t_{B_j} \in \left[\sqrt{3}(P^t_{B_j} - S_{B_{R,j}}), \sqrt{3}(P^t_{B_j} + S_{B_{R,j}}) \right] \tag{14L-c}$$

$$P^t_{B_j} = P^t_{d_j} - P^t_{c_j} \tag{15}$$

$$B^t_j \in \left[soc_{min} B_{R,j}, soc_{max} B_{R,j} \right] \tag{16}$$

A branch power flow model, given by (2L) to (7NL), is used to represent power flow in distribution system. Constraints (2L) and (4L) model the active and reactive power balance at node j, respectively.

The KVL equation for branch (ij) is represented by (6L), while the equation describing the relationship between current magnitude, voltage magnitude and apparent power magnitude for branch (ij) is given by (7NL). Backflow of real power into the substation from the distribution system is avoided using the constraint (8). The allowable limits for bus voltages are modeled via (9). (10) and (11) describe the reactive power limits of DER inverters. The trajectory of the battery energy versus time is given by (12) (this is a time-coupled constraint). Battery charging and discharging powers are limited by the battery's rated power capacity, as given by (13). (13) also says that the initial and final energy levels for battery must be the same at the end of the optimization time horizon. Reactive Power Output from Battery Inverters are is constrained by the quadratic inequality (14NL). A linearized set of equations approximating the same are given by (14)L, which utilize a hexagonal approximation of the inequality [16]. For the safe and sustainable operation of the batteries, the energy B^t_j is constrained to be within some percentage limits of the rated battery SOC capacity, modeled using (16)

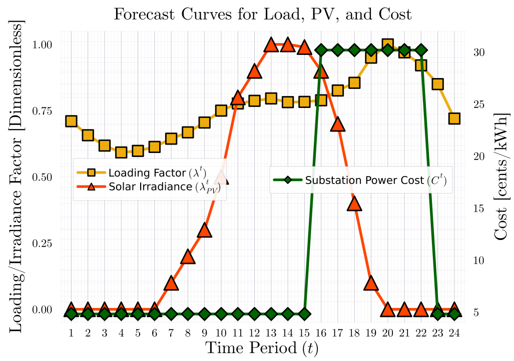

Fig. 1: Forecasts for demand power, irradiance and cost of substation power over a 24 hour horizon

TABLE II: MPOPF performance comparison - ADS10 test system for 24h

Metric	BFM-NL	LinDistFlow
Full horizon		
Substation power cost ($)	204.27	204.28
Substation real power (kW)	1528.35	1528.4
Line loss (kW)	0.28	0.33
Substation reactive power (kVAR)	428.9	795.56
PV reactive power (kVAR)	174.41	-0.69
Battery reactive power (kVAR)	192.8	-0.37
Computation		
Number of Iterations	1	1
Total Simulation Time (s)	2.64	0.77

C. Linear MPCOPF with Batteries

III. Case Study Demonstration

IV. Conclusions

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TABLE III: MPOPF feasibility comparison - ADS10 test system for 24h

Metric	BFM-NL	LinDistFlow
Max. all-time discrepancy		
Voltage (pu)	0.00001	0.00001
Line loss (kW)	0.000009	0.000006
Substation power (kW)	0.000014	0.02410
Substation reactive power (kVAR)	0.070706	0.05618

TABLE IV: MPOPF performance comparison - IEEE123-A test system for 24h

Metric	BFM-NL	LinDistFlow
Largest subproblem		
Decision variables	15144	12096
Linear constraints	18456	22200
Nonlinear constraints	3672	0
Simulation results		
Substation power cost ($)	2787.44	2798.4
Substation real power (kW)	20984.89	21065.89
Line loss (kW)	380.09	461.38
Substation reactive power (kVAR)	6835.82	12259.29
PV reactive power (kVAR)	1972.27	195.12
Battery reactive power (kVAR)	3709.71	204.63
Computation		
Total Simulation Time (s)	17.44	0.85

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TABLE V: MPOPF feasibility comparison - IEEE123-A for 24h

Metric	BFM-NL	LinDistFlow
Max. all-time discrepancy		
Voltage (pu)	0.00007	0.00206
Line loss (kW)	0.01818	1.8074
Substation power (kW)	0.43164	32.362
Substation reactive power (kVAR)	1.0102	64.403