I. Problem Formulation

A. Notations

In this study, the distribution system is modeled as a tree (connected graph) with N number of buses (indexed with i, j, and k); the study is conducted for T time steps (indexed by t), each of interval length Δt. The sets of buses with DERs and batteries are D and B respectively, such that $D, B \subseteq N$. A directed edge from bus i to j in the tree is represented by ij and the set for edges is given by $\mathcal{L}$. Line resistance and reactance are r_{ij} and x_{ij}, respectively. Magnitude of the current flowing through the line at time t is denoted by I^t_{ij} and $l^t_{ij} = \left(I^t_{ij}\right)^2$. The voltage magnitude of bus j at time t is given by V^t_j and $v^t_j = \left(V^t_j\right)^2$. Apparent power demand at a node j at time t is $s^t_{L_j}$ $(= p^t_{L_j} + jq^t_{L_j})$. The active power generation from the DER present at bus j at time t is denoted by $p^t_{D_j}$ and controlled reactive power dispatch from the DER inverter is $q^t_{D_j}$. DER inverter capacity is $S_{D_{R_j}}$. The apparent power flow through line ij at time t is S^t_{ij} $(= P^t_{ij} + jQ^t_{ij})$. The real power flowing from the substation into the network is denoted by P^t_{Subs} and the associated cost involved per kWh is C^t. The battery energy level is B^t_j. Charging and discharging active power from battery inverter (of apparent power capacity $S_{B_{R_j}}$) are denoted by $P^t_{c_j}$ and $P^t_{d_j}$, respectively and their associated efficiencies are η_c and η_d, respectively. The energy capacity of the batteries is denoted by B_{R_j}, and the rated battery power is $P_{B_{R_j}}$. soc_{min} and soc_{max} are fractional values for denoting safe soc limits of a battery about its rated state-of-charge (soc) capacity. The reactive power support of the battery inverter is indicated by $q^t_{B_j}$.

B. MPCOPF with Batteries

The OPF problem aims to minimize two objectives as shown in (1). The first term in (1) aims to minimize the total energy cost for the entire horizon. Including the 'Battery Loss' cost as the second term ($\alpha > 0$) helps eliminate the need for binary (integer) variables typically used to prevent simultaneous charging and discharging. The resulting OPF problem is a non-convex optimization problem [**?**].

$$\min \sum_{t=1}^{T} \left\{ f^t_0 + f^t_{SCD} \right\} \tag{1}$$

where

$$f^t_0 = C^t P^t_{Subs} \Delta t$$

$$f^t_{SCD} = \alpha \sum_{j \in \mathcal{B}} \left\{ (1 - \eta_C) P^t_{C_j} + \left(\frac{1}{\eta_D} - 1 \right) P^t_{D_j} \right\}$$

Subject to the constraints (2L) to (16) as given below:

$$\sum_{(j,k) \in \mathcal{L}} \left\{ P^t_{jk} \right\} - \left(P^t_{ij} - r_{ij} l^t_{ij} \right) = p^t_j \tag{2NL}$$

$$\sum_{(j,k) \in \mathcal{L}} \left\{ P^t_{jk} \right\} - \left(P^t_{ij} \right) = p^t_j \tag{2L}$$

$$p^t_j = \left(P^t_{d_j} - P^t_{c_j} \right) + p^t_{D_j} - p^t_{L_j} \tag{3}$$

$$\sum_{(j,k) \in \mathcal{L}} \left\{ Q^t_{jk} \right\} - \left(Q^t_{ij} - x_{ij} l^t_{ij} \right) = q^t_j \tag{4NL}$$

$$\sum_{(j,k) \in \mathcal{L}} \left\{ Q^t_{jk} \right\} - \left(Q^t_{ij} \right) = q^t_j \tag{4L}$$

$$q^t_j = q^t_{D_j} + q^t_{B_j} - q^t_{L_j} \tag{5}$$

$$v^t_j = v^t_i - 2(r_{ij} P^t_{ij} + x_{ij} Q^t_{ij}) + \left\{ r^2_{ij} + x^2_{ij} \right\} l^t_{ij} \tag{6NL}$$

$$v^t_j = v^t_i - 2(r_{ij} P^t_{ij} + x_{ij} Q^t_{ij}) \tag{6L}$$

$$(P^t_{ij})^2 + (Q^t_{ij})^2 = l^t_{ij} v^t_i \tag{7NL}$$

$$P^t_{Subs} \geq 0 \tag{8}$$

$$v^t_j \in \left[V^2_{min}, V^2_{max} \right] \tag{9}$$

$$q^t_{D_j} \in \left[-q^t_{D_{Max,j}}, q^t_{D_{Max,j}} \right] \tag{10}$$

$$q^t_{D_{Max,j}} = \sqrt{{S_{D_{R,j}}}^2 - {p^t_{D_j}}^2} \tag{11}$$

$$B^t_j = B^{t-1}_j + \Delta t \left(\eta_c P^t_{c_j} - \frac{1}{\eta_d} P^t_{d_j} \right), \quad B^0_j = B^T_j \tag{12}$$

$$P^t_{c_j}, P^t_{d_j} \in \left[0, P_{B_{R,j}} \right] \tag{13}$$

$$(P^t_{B_j})^2 + (q^t_{B_j})^2 \leq S^2_{B_{R,j}} \tag{14NL}$$

$$q^t_{B_j} \in \left[-\sqrt{3}(P^t_{B_j} + S_{B_{R,j}}), -\sqrt{3}(P^t_{B_j} - S_{B_{R,j}}) \right] \tag{14L-a}$$

$$q^t_{B_j} \in \left[-\frac{\sqrt{3}}{2} S_{B_{R,j}}, \frac{\sqrt{3}}{2} S_{B_{R,j}} \right] \tag{14L-b}$$

$$q^t_{B_j} \in \left[\sqrt{3}(P^t_{B_j} - S_{B_{R,j}}), \sqrt{3}(P^t_{B_j} + S_{B_{R,j}}) \right] \tag{14L-c}$$

$$P^t_{B_j} = P^t_{d_j} - P^t_{c_j} \tag{15}$$

$$B^t_j \in [soc_{min} B_{R,j}, soc_{max} B_{R,j}] \tag{16}$$

A branch power flow model, given by (2L) to (7NL), is used to represent power flow in distribution system. Constraints (2L) and (4L) model the active and reactive power balance at node j, respectively.

The KVL equation for branch (ij) is represented by (6L), while the equation describing the relationship between current magnitude, voltage magnitude and apparent power magnitude for branch (ij) is given by (7NL). Backflow of real power into the substation from the distribution system is avoided using the constraint (8). The allowable limits for bus voltages are modeled via (9). (10) and (11) describe the reactive power limits of DER inverters. The trajectory of the battery energy versus time is given by (12) (this is a time-coupled constraint). Battery charging and discharging powers are limited by the battery's rated power capacity, as given by (13). (13) also says that the initial and final energy levels for battery must be the same at the end of the optimization time horizon. Reactive

Power Output from Battery Inverters are is constrained by the quadratic inequality (14NL). A linearized set of equations approximating the same are given by (14)L, which utilize a hexagonal approximation of the inequality [**?**]. For the safe and sustainable operation of the batteries, the energy B_j^t is constrained to be within some percentage limits of the rated battery SOC capacity, modeled using (16)