$\boxed{A3}$ $S_n = 7n^2 + 9n.$

find AP.

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_n = \frac{n}{2}\left(a_1 + a_1 + (n-1)d\right)$$

$$S_n = \frac{n}{2}\left(2a_1 + (n-1)d\right)$$

$$S_n = (n^2 - n)\frac{d}{2} + na_1$$

$$\boxed{S_n = \left(\frac{d}{2}\right)n^2 + \left(a_1 - \frac{d}{2}\right)n}$$

$$\frac{d}{2} = 7 \Rightarrow d = 14$$

$$a_1 - 7 = 9$$

$$a_1 = 16$$

$a_1 = 16$

$a_2 = 23$

$$a_n = 16 + (n-1)\,14$$

$$a_n = 14n + 2$$

$$a_{18} = 254$$

A17

7. In an A.P., the sum of first n terms is $\frac{3n^2}{2} + \frac{5n}{2}$. Find 25^{th} term.

$$S_n = \frac{3n^2}{2} + \frac{5n}{2}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$\text{or } S_n = \frac{n}{2}\left(2a_1 + (n-1)d\right)$$

$$\text{or } S_n = \frac{n}{2}(n-1)d + a_1 n$$

$$\text{or } S_n = \left(\frac{d}{2}\right)n^2 + \left(a_1 - \frac{d}{2}\right)n$$

$$\Rightarrow d = 3 \quad a_1 = 4$$

$$a_n = 4 + (n-1)3$$

$$\text{or } a_n = 1 + 3n$$

$$\boxed{a_{25} = 76} \quad \text{Ans}$$

B/1

1. The 10th term of an arithmetic progression whose sum is $n^2 - 3n$ is

(1) 19 (2) 16

(3) 18 (4) 6

$$S_n = \frac{n}{2}\left(2a_1 + (n-1)d\right)$$

$$\text{or } S_n = \left(\frac{d}{2}\right)n^2 + \left(a_1 - \frac{d}{2}\right)n$$

$$\Rightarrow d = 2 \qquad a_1 = -2$$

$$a_n = -2 + (n-1)2$$

$$\text{or } a_n = -4 + 2n$$

$$\Rightarrow a_{10} = 16$$

$\boxed{(2)\ \text{Ans}}$

22. If $S_n = 3n^2 + 2n$ is the sum of n-terms of an A.P., then the common difference is

(1) 3

(2) 5

(3) 6

(4) 8

$$S_n = \left(\frac{d}{2}\right)n^2 + \left(a_1 - \frac{d}{2}\right)n$$

$$\Rightarrow \boxed{d = 6\ (3)}\quad \underline{\underline{\text{An}}}$$

$\boxed{C/2}$

2. A : Number of terms in the A.P. 8, 13, 18, 23, ..., 163 is 32.

R : In an A.P., if $S_n = (n^2 + 3n)$, then a_{16} equals 34.

$a_1 = 8 \quad a_n = 163 \quad d = 5$

$a_n = a_1 + (n-1)d$

$\frac{163 - 8}{5} + 1 = n$

$n = 32$

$d = 2 \quad a_1 = 4$

$a_n = 4 + (n-1)2$

$\therefore a_n = 2 + 2n$

$\therefore a_{16} = 34$

$\boxed{(2)\ A}$

$\boxed{A|2,(g)}$

(g) ________ term of A.P. 20, 17, 14, ... is the first negative term.

$20, 17, 14, 11, \ldots$

$a_n < 0 \qquad a_1 = 20$

$d = -3$

$\therefore a_n = a_1 + (n-1)d$

$\therefore a_n = 20 + (n-1)(-3)$

$\therefore a_n = -3n + 23$

$a_n < 0$?

$-3n + 23 < 0$

$23 < 3n$

$3n > 23$

$\Rightarrow n > \frac{23}{3}$

$\Rightarrow n > 7\frac{2}{3}$ 7.67

$\boxed{n = 8}$ $\boxed{8^{th}\ \text{term}}$

$\boxed{a_n = a_8 = -1}$ Ans

A/Q (i)

(i) ________ term of the AP – 28, –25, –22, ... is least natural number.

$a_1 = -28$ $d = +3$

$a_n = -28 + (n-1)3$

$\Rightarrow a_n = 3n - 31$

$3n - 31 > 0$

or $3n > 31$

$n > 10.\ldots$

$\boxed{n = 11}$ $\boxed{11^{th}}$ Ans

A/3

3. Prove that the sequence with n^{th} term $a + nb$ is always an A.P., no matter what the real numbers a and b are. What is the common difference?

$2, 5, 8, 11, \ldots$ $a_n = 2 + (n-1)3$

$1, 10, 100, 1000, 10000$

GP $a_n = 1 \times 10^{(n-1)}$

$1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0$

$a_n = \begin{cases} 1 & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even} \end{cases}$

AP: $\boxed{a_{n+1} - a_n = d \quad \forall\, n \geq 1}$

$a_{n+1} = a_n + d$

for $n = 1$

$a_2 = a_1 + d$ (1)

for $n = 2$

$a_3 = a_2 + d$ (2)

But $a_2 = a_1 + d$

$\Rightarrow a_3 = a_1 + 2d$

$\simeq \boxed{a_n = a_1 + (n-1)d}$

Proof:

If $a_n = a + nb$

P.T. a_n is an AP.

$a_n = a + nb$ (1)

$a_{n+1} = a + (n+1)\, b$ (2)

(2) − (1): $\boxed{a_{n+1} - a_n = b} \Rightarrow$ A.P.

(3)

A/4

4. Find the common difference of an A.P. whose first term is 100 and the sum of whose first six terms is five times the sum of next six terms.

$a_1 = 100 \quad d = ?$

(i) $S_6 = \left(\frac{d}{2}\right).36 + \left(100 - \frac{d}{2}\right)6$

$S_{12} - S_6 = \left(\frac{d}{2}\right)108 + \left(100 - \frac{d}{2}\right)6$

(ii) $S_{12} = \left(\frac{d}{2}\right)144 + \left(100 - \frac{d}{2}\right)12$

$\left(\frac{d}{2}\right)36 + \left(100 - \frac{d}{2}\right)6 = \left(\frac{d}{2}\right)540 + \left(100 - \frac{d}{2}\right)30$

Given: $S_6 = 5(S_{12} - S_6)$

$-24 \times \left(100 - \frac{d}{2}\right) = 504 \times \frac{d}{2}$

or $6 S_6 = 5 S_{12}$

or $100 - \frac{d}{2} = -21 \times \frac{d}{2}$

or (iii) $1.2 S_6 = S_{12}$

or $100 = -20 \times \frac{d}{2} \Rightarrow \boxed{d = -10}$

Ans

or $1.2 \times 36 \times \frac{d}{2} + \left(100 - \frac{d}{2}\right) \times 6 \times 1.2$

$= \frac{d}{2} \times 144 + \left(100 - \frac{d}{2}\right) \times 12$

or $(43.2 - 144)\frac{d}{2} = \left(100 - \frac{d}{2}\right) \times 4.8$

$(-99.2)\frac{d}{2} = \left(100 - \frac{d}{2}\right) \times 4.8$

$(-992)d = \left(100 - \frac{d}{2}\right)96$

$-992d = 9600 - 48d$

$-9660 = 944d$

$d = \frac{-9660}{944}$

1	2	3	4	5	6	7	8	9	10	11	12
100	90	80	70	60	50	40	30	20	10	0	-10

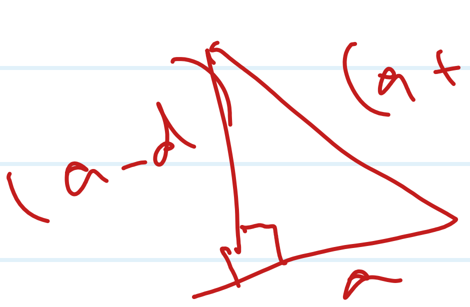
$$\begin{array}{r} 10.23 \\ 944\,)\,9660\,(\\ 2200 \\ 1888 \\ \hline 3120 \end{array}$$

$S_6 = 450$

$S_{7+n} = 90$

A|S

9. If the length of sides of a right angled triangle are in arithmetic progression, then find their ratio.



$a, a+d, a+2d$

$a-d, a, a+d$

$(a-d)^2 + a^2 = (a+d)^2$

$\Rightarrow 2a^2 - 2ad + \cancel{d^2} = a^2 + 2ad + \cancel{d^2}$

$\Rightarrow a^2 = 4ad$

$\Rightarrow \boxed{a = 4d}$ (ignoring $a=0$, nonsense)

$3d, 4d, 5d \Rightarrow \boxed{3:4:5}$ Ans

1. Find the sum of integers between 200 and 300 that are

(i) Divisible by 13

(ii) Not divisible by 13

201 to 299

13 : 208 to 299

$d = 13$ $n = 8$

$S_n^1 = \left(\frac{13}{2}\right) \times 64 + \left(208 - \frac{13}{2}\right) \times 8$

$S_n^1 = 13 \times 32 + 208 \times 8$

$\Rightarrow S_n^1 = 13 \times 28 + 208 \times 8 - 13 \times 4$

$S_n^2 =$ Total sum $- S_n^1$

$S_n^1 = 364 + 1664 = 2028$

$250 \times 99 = 25000$
$- 250$
24750

$$\boxed{S_n^2 = 24750 - 2028 = 22722}$$

Ans

A/M

4. Divide 32 into four parts which are in A.P. such that the ratio of product of extremes is to product of means is 7 : 15.

$32 = a_1 + a_2 + a_3 + a_4$

E

$\underbrace{a-3d,\ a-d,\ a+d,\ a+3d}$

M

$S_4 = 4a = 32 \Rightarrow \boxed{a = 8}$

$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$$

$$15(a^2-9d^2) = 7(a^2-d^2)$$

$$8a^2 = 128d^2$$

$$d^2 = \frac{8a^2}{128}$$

$$d^2 = \frac{1}{16}a^2$$

$$d^2 = 4 \Rightarrow \boxed{d=2}$$

$$\boxed{2, 6, 10, 14}$$ Ans

(21)

$\boxed{A \mid 8}$

8. Sum of the first p, q and r terms of an A.P. are a, b, and c respectively. Prove that

$$\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$$

$$a = S_p = \left(\frac{d}{2}\right)p^2 + \left(a_1 - \frac{d}{2}\right)p$$

$$b = S_q = \left(\frac{d}{2}\right)q^2 + \left(a_1 - \frac{d}{2}\right)q$$

$$c = S_r = \left(\frac{d}{2}\right)r^2 + \left(a_1 - \frac{d}{2}\right)r$$

$$\frac{d}{2} = k_1, \quad a_1 - \frac{d}{2} = k_2$$

$$\frac{a}{p}(q-r) = (k_1 p + k_2)(q-r)$$

$$+ \frac{b}{q}(r-p) = (k_1 q + k_2)(r-p)$$

$$+ \frac{c}{r}(p-q) = (k_1 r + k_2)(p-q)$$

$$k_1\left(\cancel{pq} - \cancel{pr} + \cancel{qr} - \cancel{pq}\right.$$

$$+ p\cancel{r} - \cancel{q}r)$$

$$+ k_2(\cancel{q} - \cancel{r}$$

$$+ \cancel{r} - \cancel{p}$$

$$+ p - \cancel{q}$$

$$= 0$$

Q.E.D.

A/S

5. Ratio of sum of n terms of two A.P.'s is $7n + 1 : 4n + 27$. Find ratio of their m^{th} terms.

$$\frac{S_n^1}{S_n^2} = \frac{7n+1}{4n+27}$$

$$\frac{S_n^1}{S_n^2} = \frac{(7n^2+1n)k}{(4n^2+27n)k}$$

$$\Rightarrow d^1 = 14K \quad a_1^1 = 8K$$

$$d^2 = 8K \quad a_1^2 = 31K$$

$$a_m^1 = 8K + (m-1)14K$$

$$a_m^2 = 31K + (m-1)8K$$

$$\frac{a_m^1}{a_m^2} = \frac{8 + 14(m-1)}{31 + 8(m-1)}$$

or $\boxed{\frac{a_m^1}{a_m^2} = \frac{14m-6}{8m+23}}$ $A_m =$

$$\frac{S_n^1}{S_n^2} = \frac{\left(\frac{d^1}{2}\right)n^2 + \left(a_1^1 - \frac{d^1}{2}\right)n}{\left(\frac{d^2}{2}\right)n^2 + \left(a_1^2 - \frac{d^2}{2}\right)n}$$

a $$\frac{S_n^1}{S_n^2} = \frac{\left(\frac{d^1}{2}\right)n + \left(a_1^1 - \frac{d^1}{2}\right)}{\left(\frac{d^2}{2}\right)n + \left(a_1^2 - \frac{d^2}{2}\right)}$$

$\boxed{C13}$

(1) $a_n = \frac{1}{y} = a_1 + (n-1)d$

(2) $a_y = \frac{1}{n} = a_1 + (y-1)d$

$a_{ny} = ? = a_1 + (ny-1)d$

(1) − (2) = $\frac{1}{y} - \frac{1}{n} = (n-y)d$

$\Rightarrow d = \frac{\frac{1}{y} - \frac{1}{n}}{n-y}$

(1) : $\frac{1}{y} = a_1 + (n-1)\frac{1}{ny}$

$d = \frac{\frac{n-y}{ny}}{n-y} = \frac{1}{ny}$

$a_1 = \frac{1}{y} - \frac{1}{y} + \frac{1}{ny}$

$\boxed{a_1 = \frac{1}{ny}}$ (4) $\boxed{d = \frac{1}{ny}}$ (3)

$a_{ny} = \frac{1}{ny} + (ny-1) \cdot \frac{1}{ny}$

$\boxed{a_{ny} = 1}$ $\Rightarrow$ (1)

C | 4

4. The interior angles of a convex polygon are in arithmetic progression. If the smallest angle is 120° and the common difference is 5°, then the number of sides of the polygon is

(1) 16 (2) 8

(3) 9 (4) 12

$S_n = 180(n-2)$

$\frac{d}{2}n^2 + \left(2a - \frac{d}{2}\right)n$

$= 360(n-2)$

$5n^2 + 235n = 360n - 720$

$n^2 - 25n + 144 = 0$

$\boxed{n = 9 \text{ or } 16}$

120 125 130 ....

120 125 130 ...

120

(3) 9 Ans

11.

$a_1 = a_1$

$a_n = a_1 + (n-1)d$

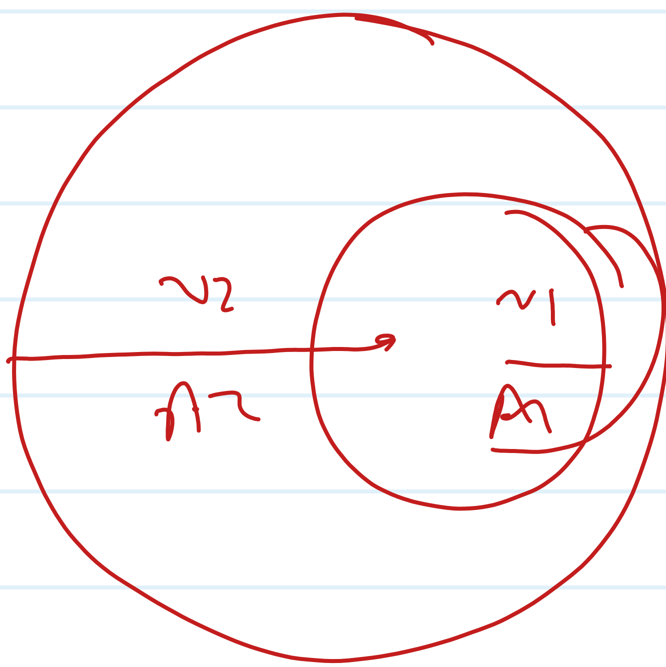
$2a_1 + (n-1)d$

$a_2 = a_1 + 1d$

$a_{n-1} = a_1 + (n-2)d$

$2a_1 + (n-1)d$

C | 21



$A_1, A_2, A_1 + A_2$

$xk \quad (3-x)k$

$9k$

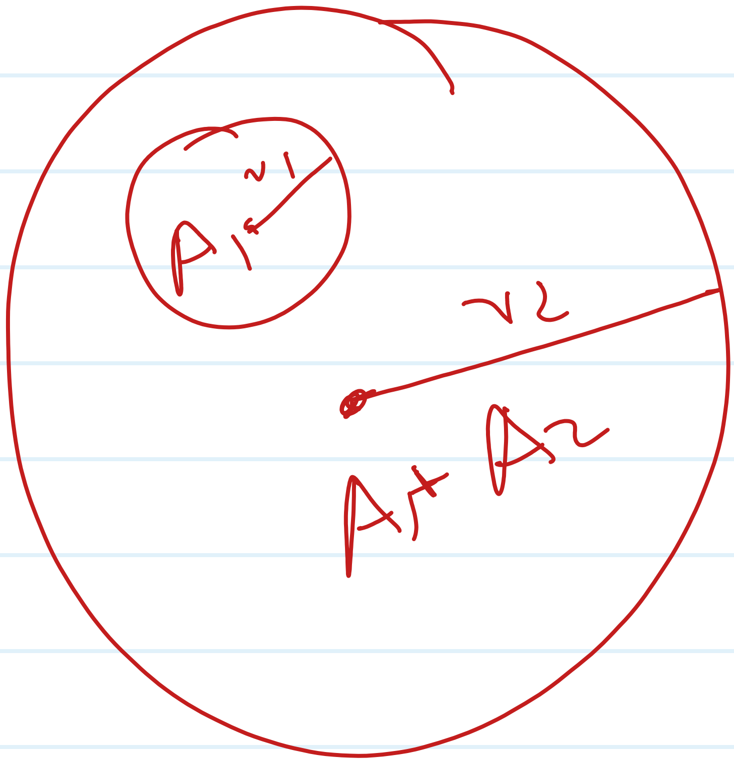
$x \quad 3-x \quad 3$

$18 - 2x = 9 + x$

$9 = 3x$

$x = 3$

$\sqrt{3}$



r_1		r_2
$r_1^2\pi$	$(9-r_1^2)\pi$	9π
A_1	A_2	$= A_1 + A_2$

$a_1 \quad a_2 \quad c_1$

$r_1^2, \quad 9-r_1^2, \quad 9$

$d \qquad d$

$$9 \quad \nu_1$$

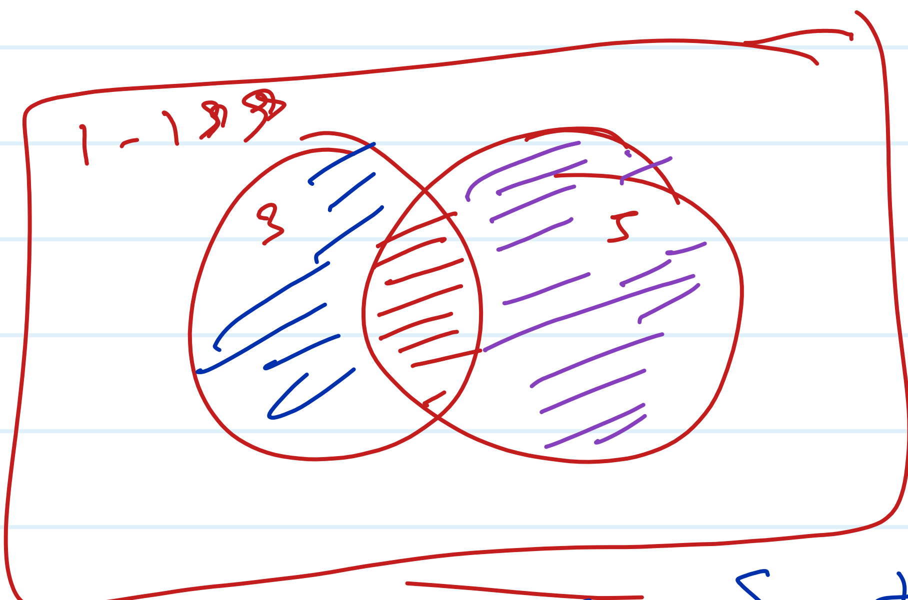
$$(9 - \nu_1^2) - \nu_1^2 = 9 - (9 - \nu_1^2)$$

$$9 - 2\nu_1^2 = \nu_1^2$$

$$\therefore 9 = 3\nu_1^2$$

$$\therefore \nu_1^2 = 3 \qquad \boxed{\therefore \nu_1 = \sqrt{3}} \text{ Ans}$$

20



$$S_{\text{neither 3 nor 5}} = S_{\text{Total}} - S_3 - S_5 + S_{15}$$

C/14

6   13   13

$a-d, a, a+d$

$a = 13$

$a(a^2-d^2)=1560$

$169-d^2=120$

$d^2 = 49$

$d=7$

(2)6

C/17

$a_1, a_2, \ldots a_n$ many and AP.

$S_n = 5$

$5(a_1+20)-20, 5(a_2+20)-20, \ldots, 5(a_n+20)-20$

$5a_1+80, 5a_2+80, \ldots, 5a_n+80$

$55 + 80 \times n$ (2)

$55+80n$