Multi-Period Optimal Power Flow (MP-OPF)

for Power Distribution Systems with Renewables and Battery Storage

Team WSU

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Overall Scope of the problem

System Description and Objective

A radial distribution grid with multiple Grid Edge Resources.

Several EDO Nodes, which aggregate a portion of the Grid Edge Resources, and communicate and control them as per communication received from OpenDSO controllers.

OpenDSO controllers receive state variables from the grid, including those from EDO nodes, and load and DER generation forecast for the entire horizon.

OpenDSO controllers need to communicate amongst themselves (details in the algorithm).

The goal: To minimize some objective function $f(x)$, say line-losses or cost of substation power for the whole horizon of time-period T.

Controllable variables (example): $q_{DER\,j}^{t}, B_j^t, P_{c_j}^t, P_{d_j}^t, q_{B_j}^t$

Overall Scope of the problem

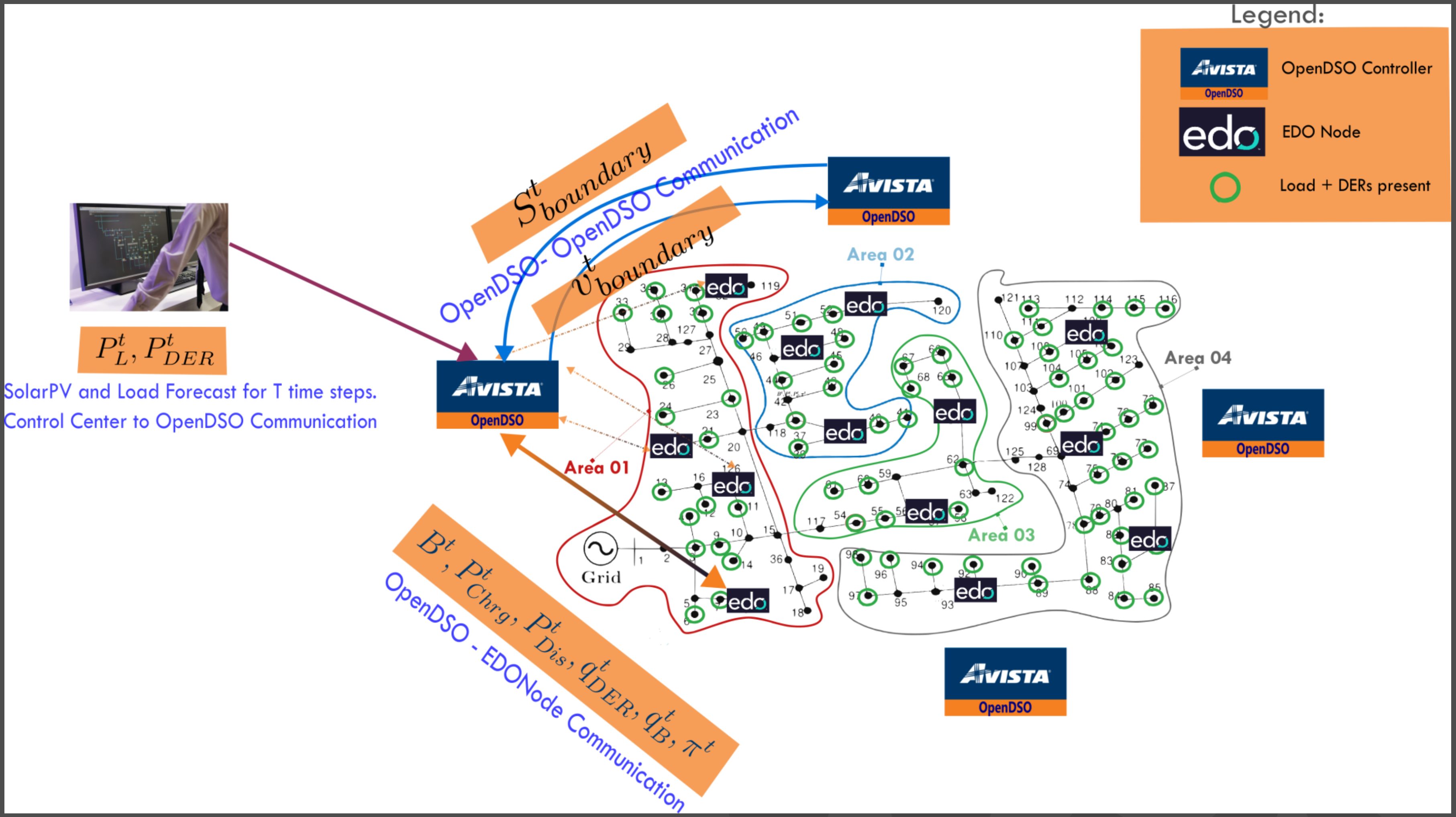
Algorithm and Communication Architecture

OpenDSO-System Operator (Forecast)

OpenDSO-Measurement Devices

OpenDSO-EDO

OpenDSO-OpenDSO



Nature of Aggregated Edo Nodes

Grid Edge Resources over multiple buildings aggregated to form Edo Nodes.

We model them as Batteries, which introduces temporal coupling between OPF solutions at individual time-steps.

This mandates MPOPF

Our model is based on the *Generalized Battery Model* as given in [2].

Exception 1: Lifetime dissipation is not modelled.

Exception 2: Currently limits such as charging/discharging limits are assumed to be constant over time (in one day).

Definition 2.9. A *Generalized Battery Model* $\mathbb{B}$ is a set of signals $u(t)$ that satisfy

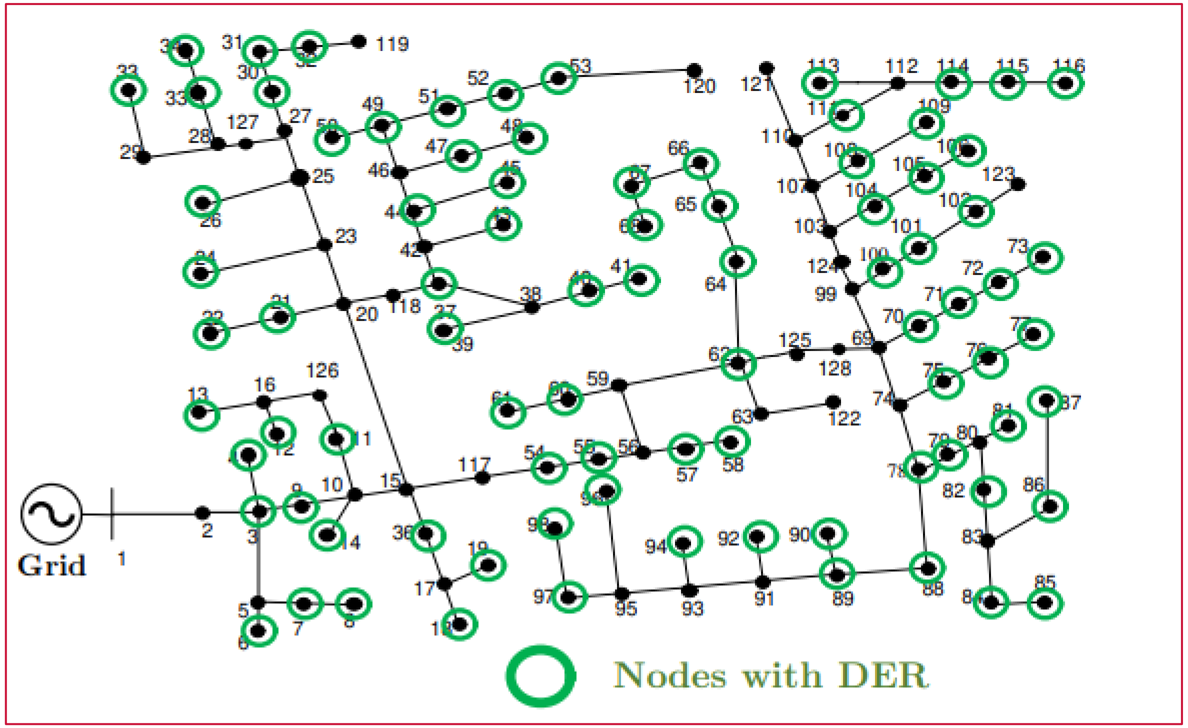
$$-n_- \leq u(t) \leq n_+, \quad \forall\, t > 0,$$

$$\dot{x} = -\alpha x - u,\ x(0) = 0 \ \Rightarrow |x(t)| \leq C, \quad \forall\, t > 0.$$

Network being tested on

Current problem statement

- **Test Case**: IEEE123 Node System with Distributed Energy Resources (DERs) and Batteries.
- Balanced three-phase (with no phase coupling, akin to single-phase) power distribution network.
- **Objective**: Solve for Optimal Power Flow (OPF) for a horizon time interval of T for a given objective function, say loss minimization.



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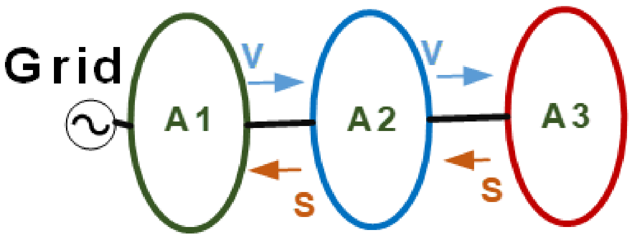
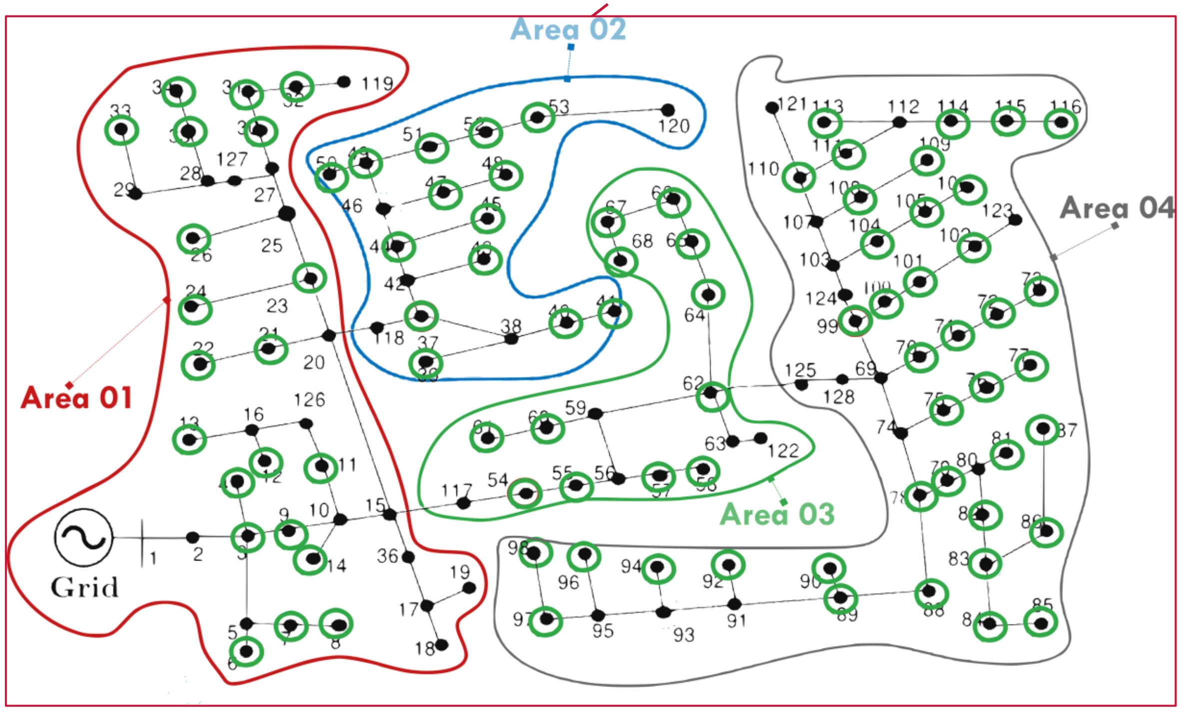


Network being tested on

A previous model of the test system. Currently the system has **DERs and Batteries on 100% of the nodes with non-zero loads**.

ENApp Approach [1]

- Divide the network into several areas to solve for their Optimal Power Flow solutions separately, with exchange of boundary variables after every ‘macro-iteration’.



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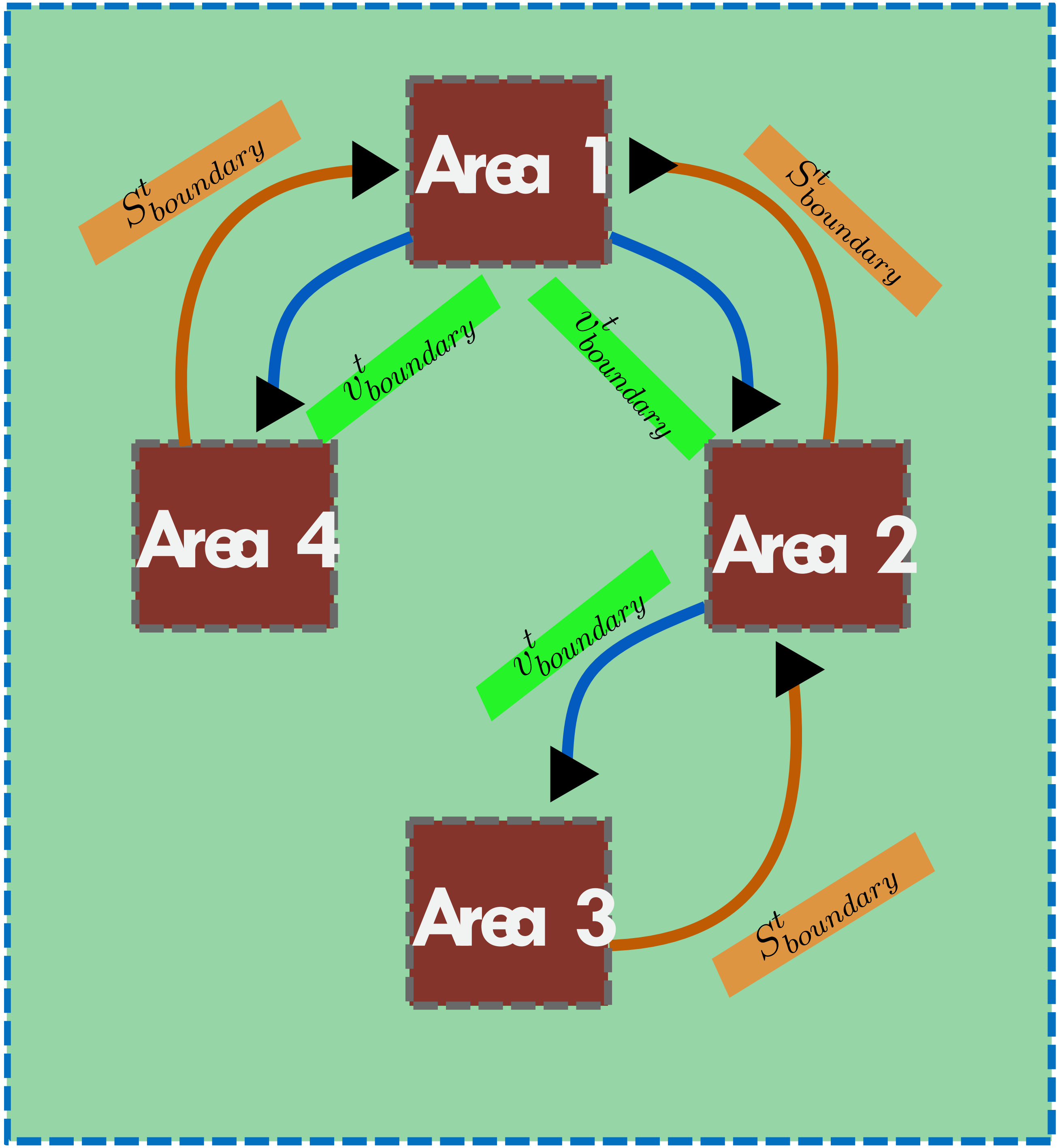
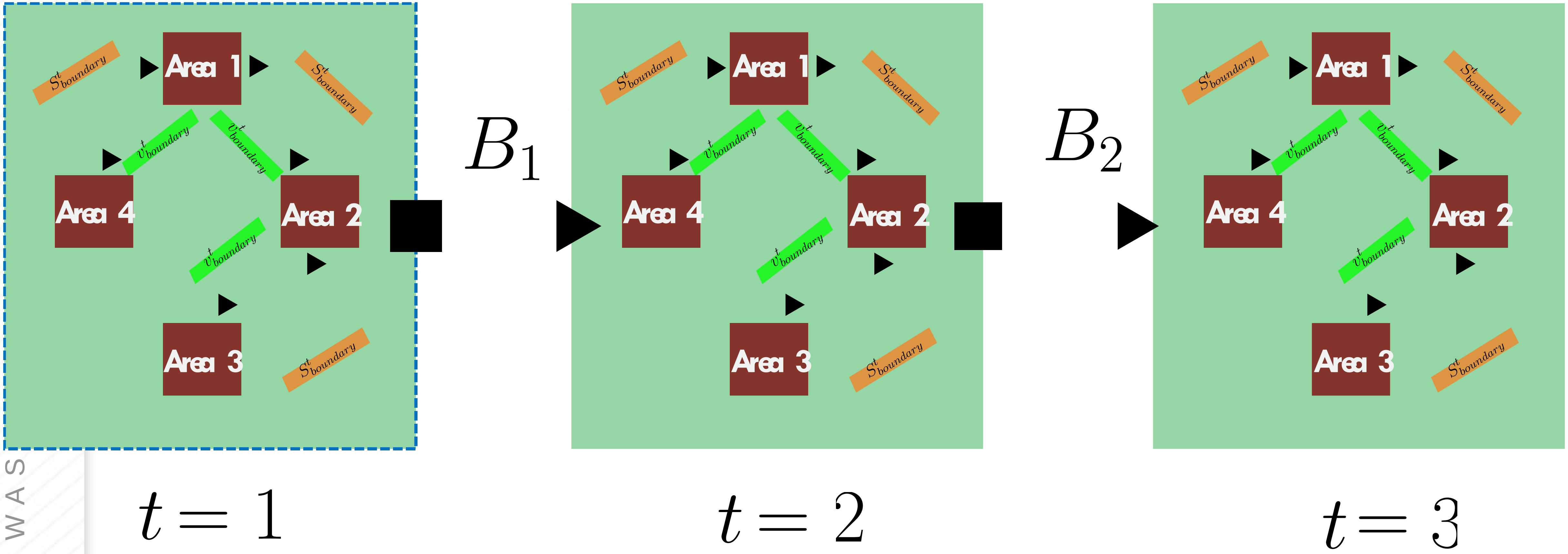
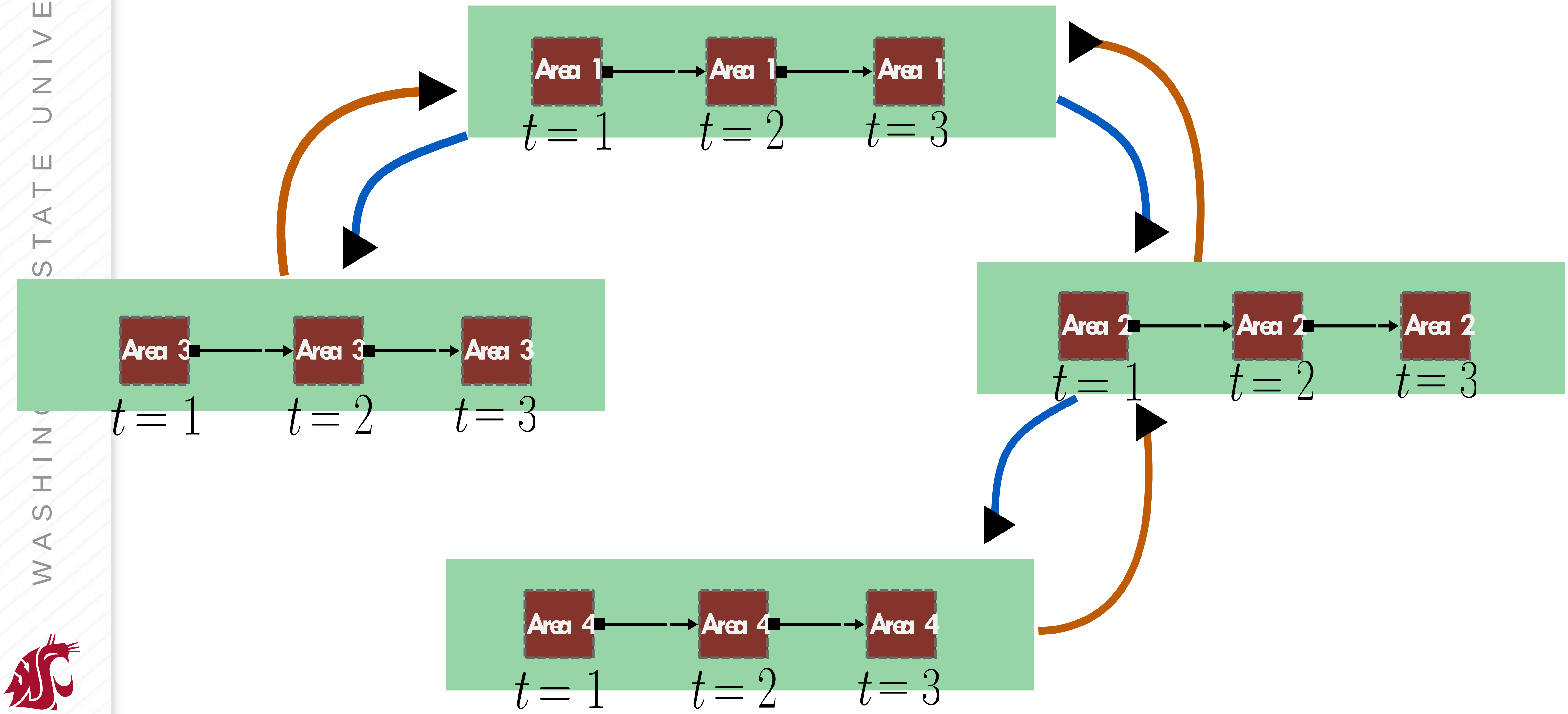
A simpler schematic for D-OPF.



A simpler schematic for MP-D-OPF for three time-steps $(T = 3)$



What is currently being done? MP-DOPF in Brute Force



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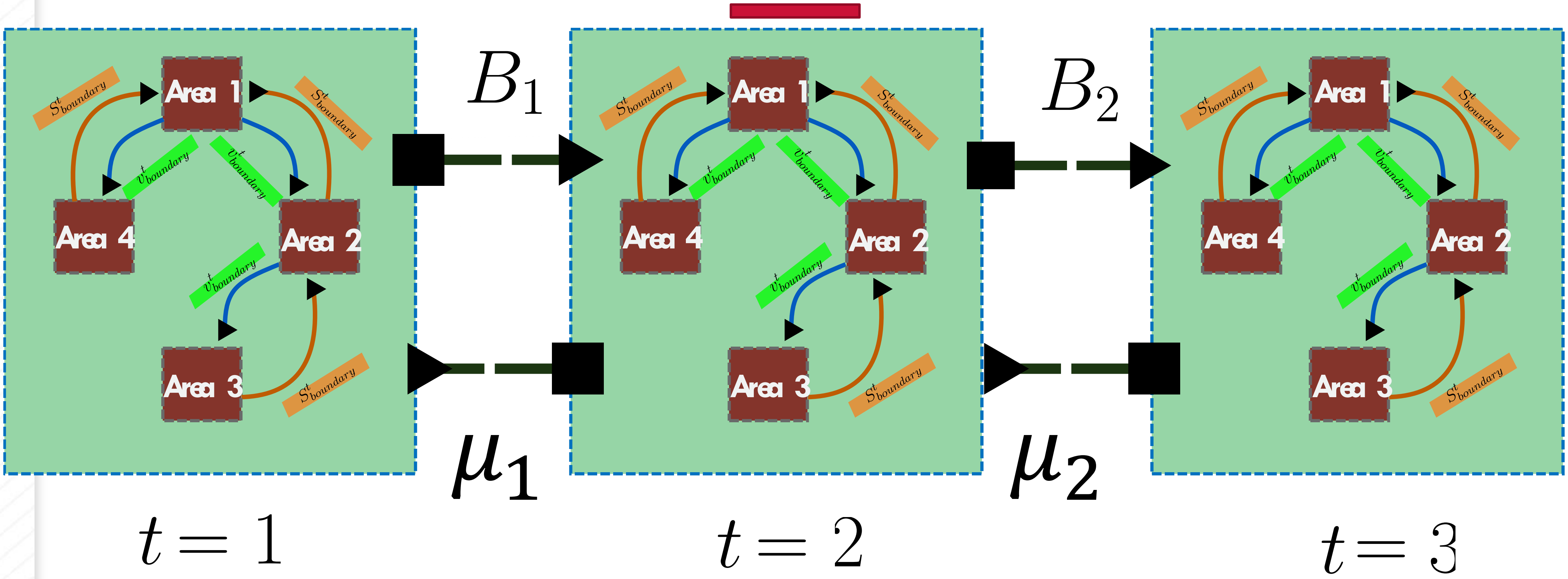
What is will be done? A ‘true’ MP-DOPF using Differential Dynamic Programming (Currently under formulation, variables not explained here) [3]



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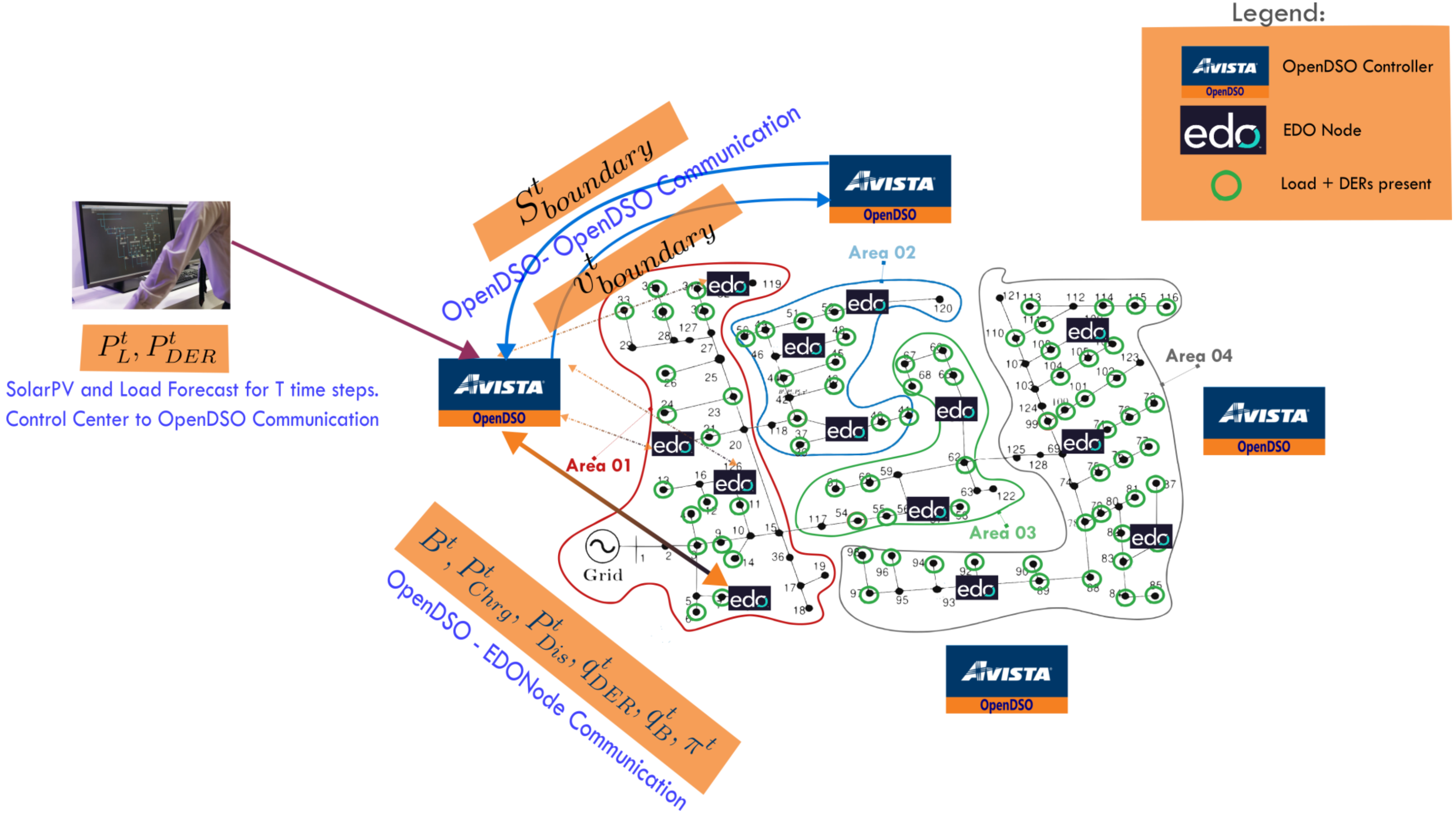


Goal of Avista (problem objective for System Operator)

- Problem objective of System Operator: Currently Modelled Use Case: Loss minimization

- Use case communicated to all OpenDSO controllers

- All OpenDSO controllers procure the models for the respective EDO nodes (the grid edge models: currently we're modelling these as batteries, but generalized battery model can also be used, but we can make any other model work too which captures flexibility over time.)

- Upon getting that, they locally optimize for their area
- Then share the boundary information with the neighbouring OpenDSO nodes
- Previous two steps are repeated until boundary values are converged.

Problem Statement

Given the **State of Charges of all batteries** in a grid **at** the beginning of a horizon interval ($B_j^0, j \in \mathcal{B}, \mathcal{B}$ representing the set of nodes with Batteries), along with predictions for other non-controllable parameter values like $P_{Load}^t, P_{DER}^t\ \forall\ t \in [0, T]$:

Solve for Optimal Power Flow of the system, with the **objective** of **minimizing line losses across** the entire **horizon** of time interval $\boldsymbol{T}$

Problem Statement

Given the **State of Charges of all batteries** in a grid **at** the beginning of a horizon interval ($B_j^0, j \in \mathcal{B}, \mathcal{B}$ representing the set of nodes with Batteries), along with other non-controllable parameter values like P_{Load}, P_{DER}:

Solve for Optimal Power Flow of the system, with the **objective** of **minimizing line losses across** the entire **horizon** of time interval $\boldsymbol{T}$

while **making sure that**:

1. B_t (the **SOC of the battery** at every point of the simulation) **stays in permissible limits**
2. The OPF Solution is physically consistent. For no node containing a battery, is $P_d^t . P_c^t \neq 0$. In other words, make sure that Complementarity of Discharging and Charging is ensured. Alternatively, Simultaneous Charging and Discharging (SCD) is avoided.
3. Other usual OPF physical constraints are also satisfied.
4. Terminal SOC is same as initial.

Simulation Parameters

Objective: Minimize Line Losses for whole horizon

System: IEEE 123 Node System

Phase: Balanced 3∅ (no phase coupling)

Supply: DERs (PVs): Forecasted, varying with t. Batteries at select (about 50% of total) nodes. Substation.

Demand: Forecasted. Varying with t

Control: Inverter Reactive Power Control for DERs and Batteries. Battery Charging/Discharging.

Number of discrete time-steps: $\mathrm{T} = 5$ (can be changed)

Duration per time-step: $\Delta t = 15$ min.

Simulation Process of the current, brute-force algorithm

Start

Known:

$B_j^0, P_{DER_j}^t, P_{Load_j}^t \;\forall\, t \in [0, T], j \in \mathrm{N}$

----

while *"(Spatial) boundary variables are NOT equal in value for all connected parent-child areas"*

i. Run OPF for minimizing the losses across the horizon for each area, in parallel. This is called as a macro-iteration.

ii. Exchange (spatial) boundary variables between all sets of connected parent-child areas.

end

Stop

Optimization Formulation

(Integer-Constraint Relaxed) Non-linear Optimization Problem:
BFM Constraints with Battery and DER decision variables.

$$\min_{\substack{P_{ij}^t, Q_{ij}^t, v_j^t, l_{ij}^t, \\ q_{D_j}^t, B_j^t, P_{c_j}^t, P_{dj}{}^t, q_{B_j}^t}} \sum_{t=1}^{T} \sum_{(i,j)\in\mathcal{L}} (r_{ij} l_{ij}^t)$$

Line Losses

$$p_j^t = \sum_{(j,k)\in\mathcal{L}} P_{jk}^t - \sum_{(i,j)\in\mathcal{L}} \{P_{ij}^t - r_{ij} l_{ij}^t\} - P_{d_j}^t + P_{c_j}^t$$

Real Powerflow from a bus into the network

$$q_j^t = \sum_{(j,k)\in\mathcal{L}} Q_{jk}^t - \sum_{(i,j)\in\mathcal{L}} \{Q_{ij}^t - x_{ij} l_{ij}^t\} - q_{D_j}^t - q_{B_j}^t$$

Reactive Powerflow from a bus into the network

$$v_j^t = v_i^t + \{r_{ij}^2 + x_{ij}^2\} l_{ij}^t - 2(r_{ij} P_{ij}^t + x_{ij} Q_{ij}^t)$$

Kirchhoff's Voltage Law (KVL)

$$l_{ij}^t = \frac{(P_{ij}^t)^2 + (Q_{ij}^t)^2}{v_i^t}$$

Current Magnitude in a branch

Optimization Formulation

(Integer-Constraint Relaxed) Non-linear Optimization Problem:
BFM Constraints with Battery and DER decision variables.

$$\min_{\substack{P_{ij}^t, Q_{ij}^t, v_j^t, l_{ij}^t, \\ q_{D_j}^t, B_j^t, P_{c_j}^t, P_{dj}{}^t, q_{B_j}^t}} \sum_{t=1}^{T} \sum_{(i,j)\in\mathcal{L}} (r_{ij} l_{ij}^t)$$

Line Losses

$$|q_{B_{Max}}| \quad \sim [1, 10]\ \mathrm{kVAr}$$

DER Inverter Reactive Power

$$V_{min} \quad 0.95\, pu$$

$$V_{Max} \quad 1.05\, pu$$

$$v \quad [V_{min}^2, V_{Max}^2]$$

Nodal Voltage Limits

Optimization Formulation

(Integer-Constraint Relaxed) Non-linear Optimization Problem:
BFM Constraints with Battery and DER decision variables.

$$\min_{\substack{P_{ij}^t, Q_{ij}^t, v_j^t, l_{ij}^t, \\ q_{D_j}^t, B_j^t, P_{c_j}^t, P_{dj}{}^t, q_{B_j}^t}} \sum_{t=1}^{T} \sum_{(i,j)\in\mathcal{L}} (r_{ij} l_{ij}^t)$$

Line Losses

$$B_j^t = B_j^{t-1} + \Delta t \eta_c P_{c_j}^t - \Delta t \frac{1}{\eta_d} P_{d_j}^t$$

$$B_j^0 = 0.5(soc_{max} + soc_{min}) E_{Rated} = 0.625 E_{Rated}$$

Battery State of Charge Equation *(this equation is temporally coupled, or has 'memory')*

Optimization Formulation

(Integer-Constraint Relaxed) Non-linear Optimization Problem:
BFM Constraints with Battery and DER decision variables.

$$\min_{\substack{P_{ij}^t, Q_{ij}^t, v_j^t, l_{ij}^t, \\ q_{D_j}^t, B_j^t, P_{c_j}^t, {P_{dj}}^t, q_{B_j}^t}} \sum_{t=1}^{T} \sum_{(i,j)\in\mathcal{L}} (r_{ij} l_{ij}^t)$$

Line Losses

P_{Max}	$\sim [2, 20]$ kW
$\lvert q_{B_{Max}} \rvert$	$\sim [1, 10]$ kVAr
P_d, P_c	$[0, P_{Max}]$
q_B	$[-q_{B_{Max}}, q_{B_{Max}}]$
E_{Rated}	$P_{Max} \times 4$ h
B	$[0.30 E_{Rated}, 0.95 E_{Rated}]$

Battery Charging and Discharging Power Limits

Battery Inverter Reactive Power Limits

Battery SOC Permissible Limits

Optimization Formulation

(Integer-Constraint Relaxed) Non-linear Optimization Problem:
Tweaking the Objective Function to satisfy some more constraints.

$$\min_{\substack{P_{ij}^t, Q_{ij}^t, v_j^t, l_{ij}^t, \\ q_{D_j}^t, B_j^t, P_{c_j}^t, P_{dj}{}^t, q_{B_j}^t}} \sum_{t=1}^{T} \sum_{(i,j)\in\mathcal{L}} (r_{ij} l_{ij}^t)$$

$$+ \alpha \sum_{t=1}^{T} \sum_{j\in\mathcal{B}} \left\{ (1-\eta_c) P_{c_j}^t + \left(\frac{1}{\eta_d} - 1 \right) P_{d_j}^t \right\}$$

$$+ \gamma \sum_{j\in\mathcal{B}} \left\{ \left(B_j^T - B_{ref_j}] \right)^2 \right\}$$

Line Losses

Ensures Complementarity of Charging and Discharging, or alternatively, ensure there's no SCD.

For minimizing the deviation between end-of-day battery SOC and a reference SOC.

Optimization Formulation

(Integer-Constraint Relaxed) The Full Non-linear Optimization Problem

Line Losses

Ensures Complementarity of Charging and Discharging, or alternatively, ensure there's no SCD.

Battery State of Charge Equation *(temporally coupled)* *with complementarity relaxed.*

For minimizing the deviation between end-of-day battery SOC and a reference SOC.

Real Powerflow from a bus into the network

Reactive Powerflow from a bus into the network

Kirchhoff's Voltage Law (KVL)

Current Magnitude in a branch

DER Generation (forecasted, non-controllable)

Loads (forecasted, non-controllable)

$$\min_{\substack{P_{ij}^t, Q_{ij}^t, v_j^t, l_{ij}^t, \\ q_{D_j}^t, B_j^t, P_{c_j}^t, {P_{d_j}}^t, q_{B_j}^t}} \sum_{t=1}^{T} \sum_{(i,j)\in\mathcal{L}} (r_{ij} l_{ij}^t) \tag{E.15}$$

$$+\alpha \sum_{t=1}^{T} \sum_{j\in\mathcal{B}} \left\{(1-\eta_c)P_{c_j}^t + \left(\frac{1}{\eta_d} - 1\right) P_{d_j}^t\right\} \tag{E.16}$$

$$+\gamma \sum_{j\in\mathcal{B}} \left\{\left(B_j^T - B_{ref_j}]\right)^2\right\} \tag{E.17}$$

s.t.

$$p_j^t = \sum_{(j,k)\in\mathcal{L}} P_{jk}^t - \sum_{(i,j)\in\mathcal{L}} \{P_{ij}^t - r_{ij} l_{ij}^t\} - P_{d_j}^t + P_{c_j}^t \tag{E.18}$$

$$q_j^t = \sum_{(j,k)\in\mathcal{L}} Q_{jk}^t - \sum_{(i,j)\in\mathcal{L}} \{Q_{ij}^t - x_{ij} l_{ij}^t\} - q_{D_j}^t - q_{B_j}^t \tag{E.19}$$

$$v_j^t = v_i^t + \{r_{ij}^2 + x_{ij}^2\} l_{ij}^t - 2(r_{ij}P_{ij}^t + x_{ij}Q_{ij}^t) \tag{E.20}$$

$$l_{ij}^t = \frac{(P_{ij}^t)^2 + (Q_{ij}^t)^2}{v_i^t} \tag{E.21}$$

$$B_j^t = B_j^{t-1} + \Delta t \eta_c P_{c_j}^t - \Delta t \frac{1}{\eta_d} P_{d_j}^t \tag{E.22}$$

$$B_j^0 = 0.5(soc_{max} + soc_{min})E_{Rated} = 0.625 E_{Rated} \tag{E.23}$$

where, (E.24)

(i,j) : Branch connecting nodes i and j (E.25)

$$p_j^t = {p_{D_j}}^t - {p_{L_j}}^t \tag{E.26}$$

$$q_j^t = -{q_{L_j}}^t \tag{E.27}$$

$$t = \{1, 2, \ldots T\} \tag{E.28}$$

#Types of variables

$9T$

#Variables

$4mT + n_{DERT}T + 4n_{Batt}T$

#Equality Constraints

$4mT + n_{Batt}T$

(Non-Linear)

Conclusion and Discussion

Major Observations:

Batteries were modelled correctly, i.e. no Simultaneous Charging and Discharging was observed.

Terminal SOC values (SOC values at end of day) mostly kept close to the original SOC values (beginning of day).

Questions:

Currently the EDO Nodes have been modelled as batteries, but different models can be accommodated. What kind of models are Avista expecting?

What kind of objective function is Avista looking to optimize? Currently for line losses, the EDO nodes are operated under direct control, but for price signal inclusion, indirect control can be imposed on the EDO nodes (price signal data will be needed).

Current Status of Simulation and Results

What has been done

A variety of simulation runs have been modelled and run for the sake of benchmarking including:

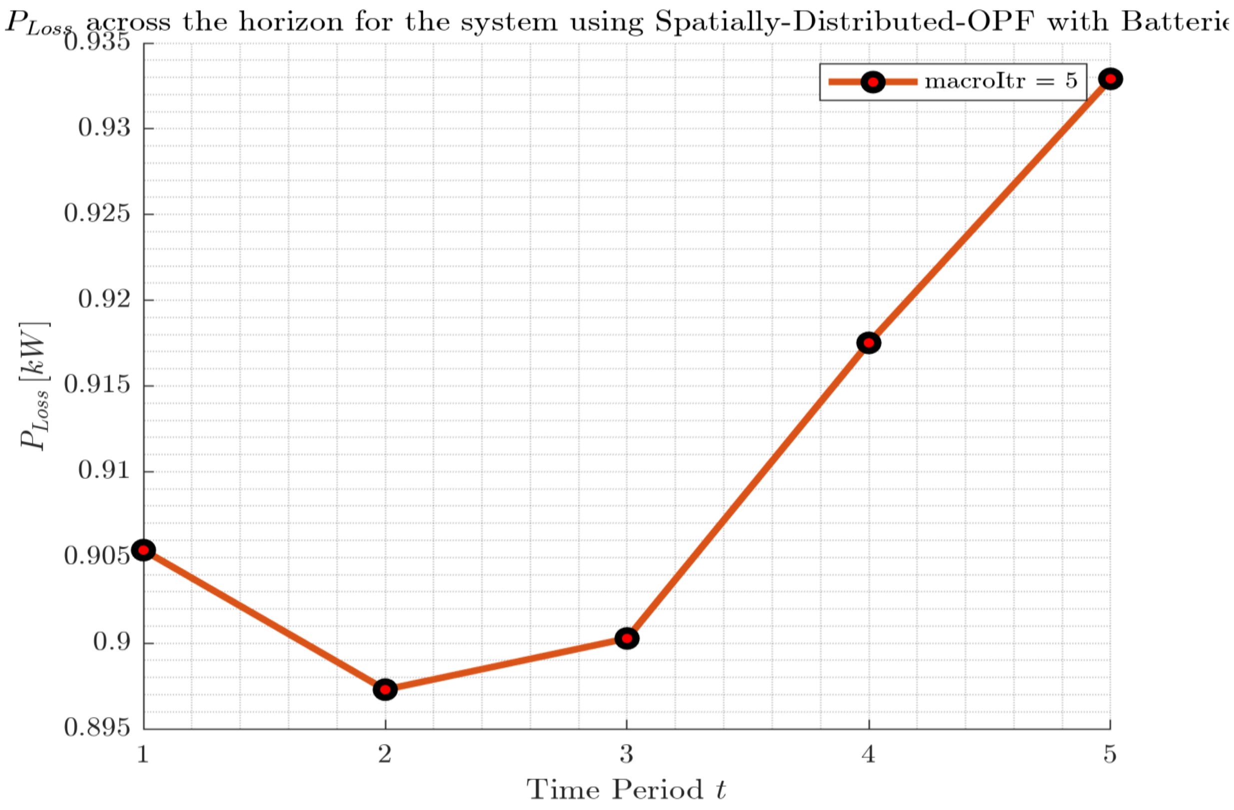
Centralized OPF with and without batteries (T time-steps).
Distributed* OPF with and without batteries. (T time–steps, 4 Areas)

*Spatially Distributed Computation, Brute-forced Temporal Computation.

Previous: Distributed OPF with batteries in a greedy "minimize losses right this time-period, for every time-period" manner. (4 Areas, T time-steps)

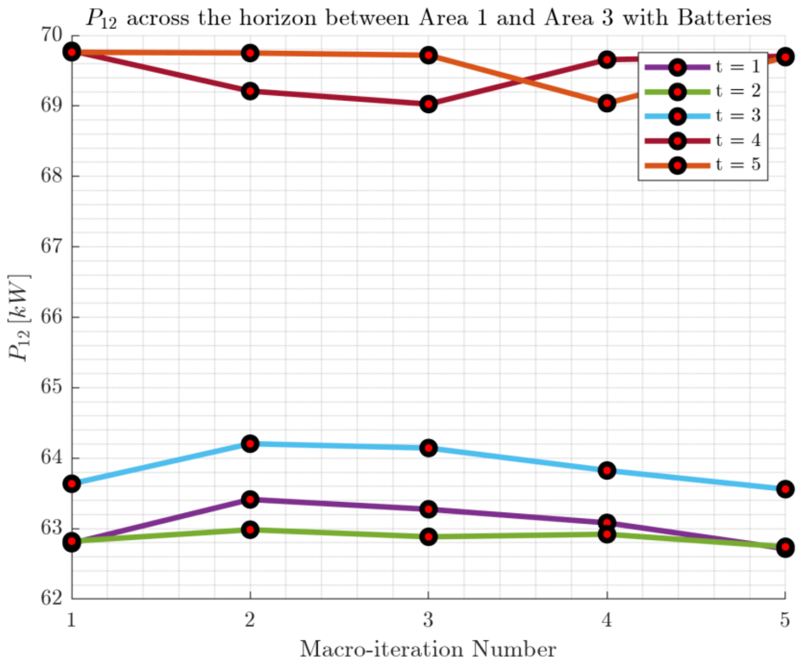
Results: Boundary Variables

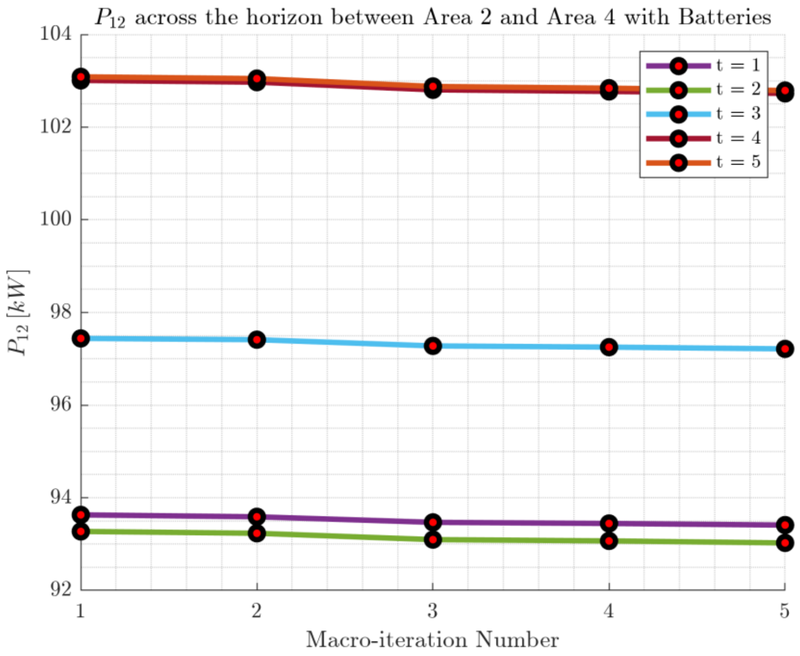
P_{Losses} vs t

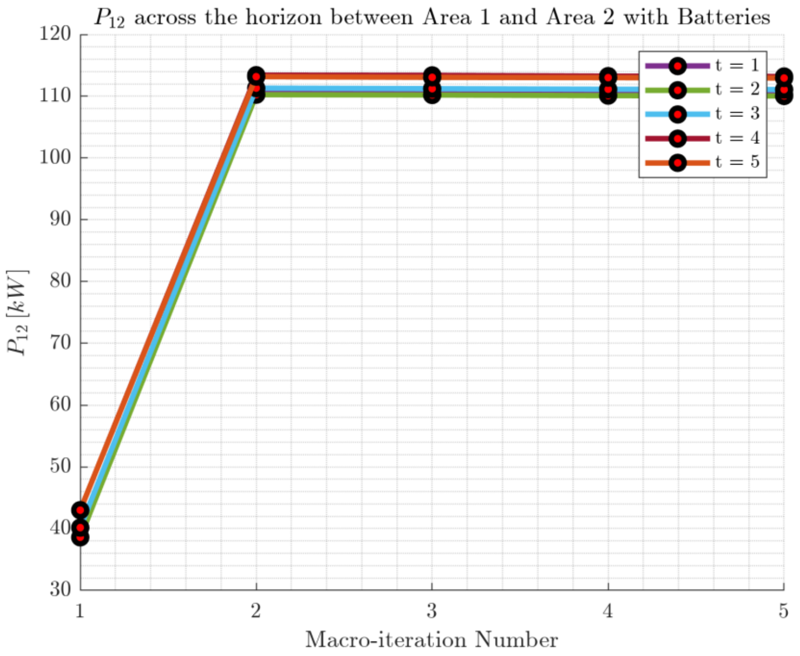


Results: Boundary Variables

P_{12} vs t

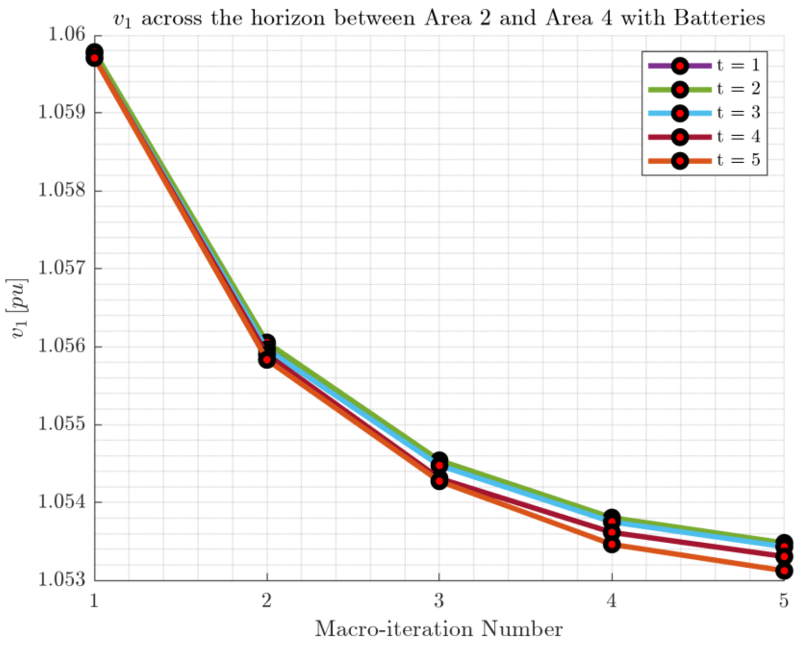


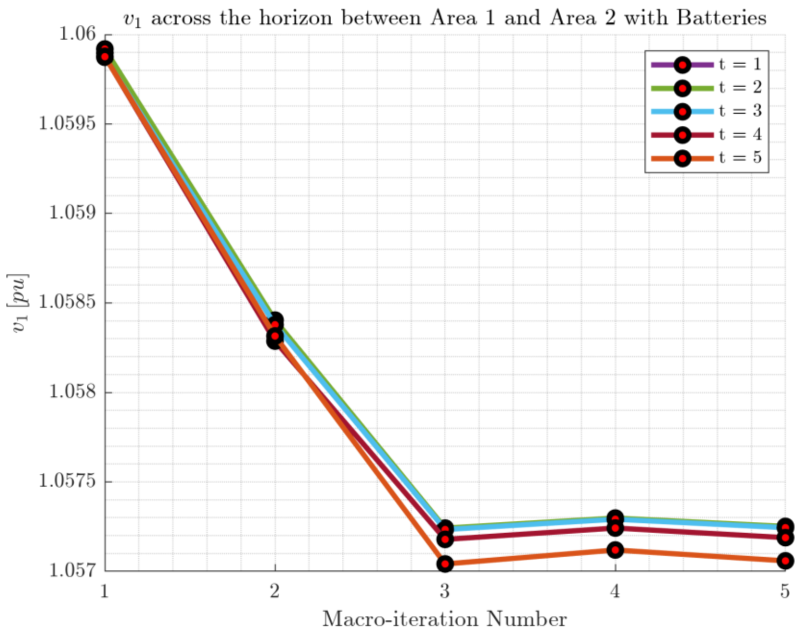


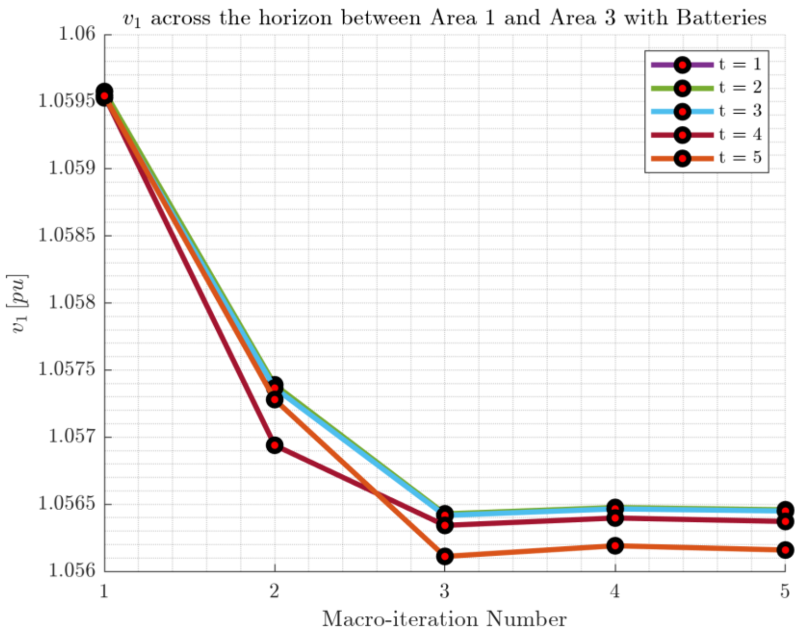


Results: Boundary Variables

V_1 vs t

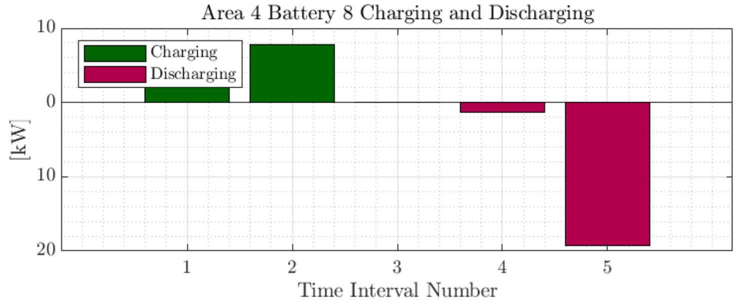


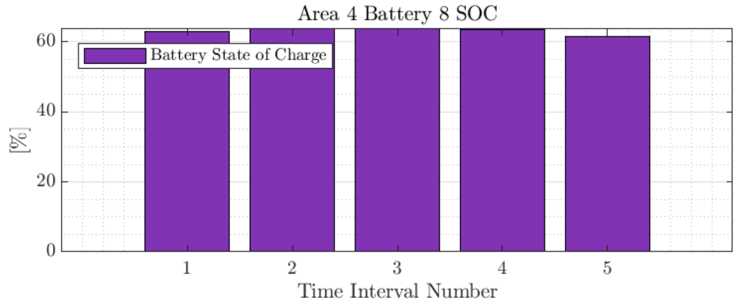




Current Status of Simulation and Results

SCD Plots to check Physical validity of Distributed MPOPF

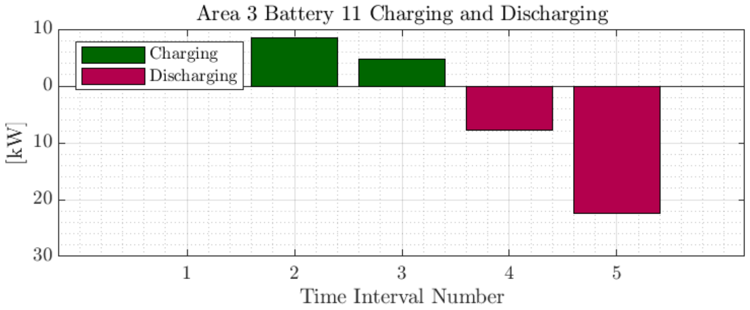


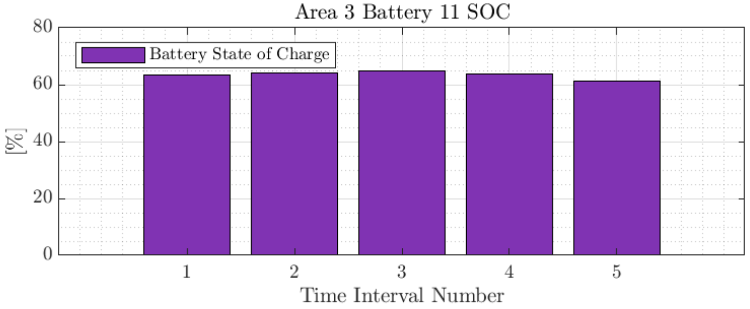


This simulation was done for 5 time-steps (but the algorithm is scalable, and I've done them for 10, 15 time-steps. But the time taken by the *brute force* algorithm grows exponentially with number of time steps)

Charging and Discharging are always Complementary.

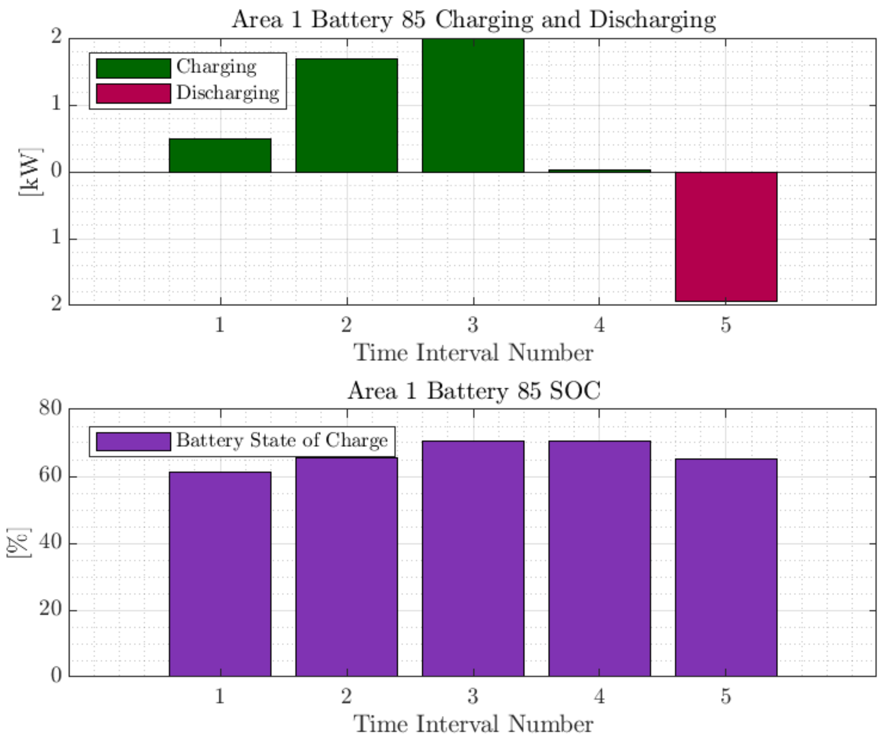
Terminal SOC is almost the same as initial SOC





Current Status of Simulation and Results

Results for 123 Node System from Centralized MPOPF



The plot shown is typical of the charging/discharging cycle for most batteries in the area.

This simulation was done for 5 time-steps (but the algorithm is scalable, and I've done them for 10, 15 time-steps. But the time taken by the *brute force* algorithm grows exponentially with number of time steps)

Charging and Discharging are always Complementary.

Terminal SOC is almost the same as initial SOC

IEEE 123 Node System modelled here has 85 nodes with Nodes and DERs and Batteries.

Current Status of Simulation and Results

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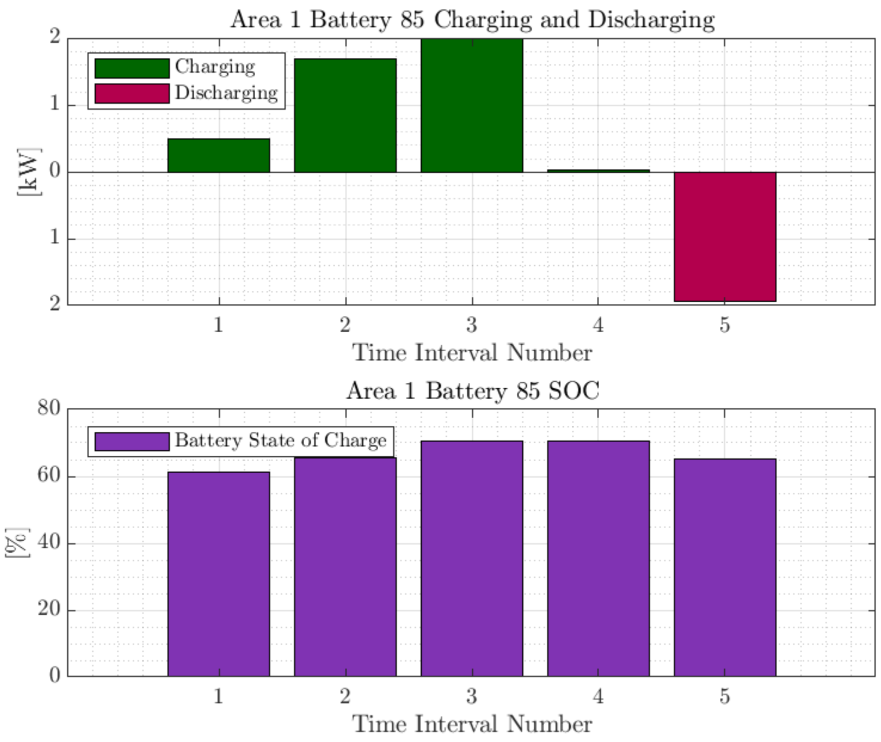


```
Real Power Line Losses for Area 1 for 5 time periods = 3.927813e+01 [kW]
SCD Constraint violation for Area 1 for 5 time periods = 2.659036e+01 [kW]
SOC Level constraint violation for Area 1 for 5 time periods = 2.539628e-01 [kWh]
```

horizonTimes	totalTimes [s]	nLinEqns	nNonLinEqns	nVars	PLoss [kW]
1	8.0925	467	127	934	8.65996
2	71.6502	934	254	1868	15.756
3	99.3195	1401	381	2802	23.5859
4	226.253	1868	508	3736	31.4306
5	740.258	2335	635	4670	39.2745
6	1340.52	2802	762	5604	47.1437
7	1824.28	3269	889	6538	54.9948
8	8109.09	3736	1016	7472	62.8446

Thank You.

Appendix and References follow

Implementation Details

Platform, Input, Output

Platform:

Current Implementation is in **MATLAB**.

Model: Multi-Period Non-Linear OPF

Information needed to be exchanged between EDO Nodes and my area optimization controller. (depends on objective function)

Communication among OpenDSO nodes.

Inputs:

Inputs are the same as Powerflow variables for a distribution system. That means:

Topology:

busData: $(\forall\ j \in N)\ P_{G\,j}, Q_{G_j}, P_{L_j}, Q_{L_j}, |V_j|$ $[kW, kVAr, kV\ or\ pu]$

branchData: $(\forall\ \mathrm{i} \rightarrow j \in L)\ Z_{ij} = R_{ij} + jX_{ij}$ $[\Omega\ or\ pu]$

Power Demand Forecast Profile $[kW]$

PV Forecast Profile $[kW] or\ \lambda$

PV Real Power/Reactive Power Limits $[kW]$

Battery Charging/Discharging Limits $[\boldsymbol{kW}]$

Battery SOC Limits $[\boldsymbol{kWh}]$

What kind of objective function? Cost of Generation? Then give an estimate of Cost of substation power, cost of battery charging and cost of discharging power $[\$/kW]$.

Outputs:

Optimal value of the objective function (say, line losses $\boldsymbol{P_{Loss}}$ in $\boldsymbol{kW}$ or Cost of generation in $).

All state/control/output optimal variables. Such as:

The nine optimization variables for all time instances across the horizon:

$P_{ij}^t, Q_{ij}^t, l_{ij}^t, v_j^t, q_{D_j}^t, B_j^t, P_{c_j}^t, P_{d_j}^t, q_{B_j}^t\ \forall\ t \in [1, T], j \in N, (i,\ j) \in L$

Output variables such as Substation Power P_{Subs}.

Any other relevant variables.

Implementation Details

Optimization Algorithm Needs

Inputs: Data needs for Topology

- Branch
  - To Node, From Node, RX
- Bus
  - Controllable
    - Base PQ
  - Static Load PQ
    - Allocation Factor
- Devices
  - Discover Buses and Branches for Section that it is optimizing
  - Discover List of Upstream Neighbors and Downstream Neighbors
  - Subscribe to their Parameters

Implementation Details

Optimization Algorithm Needs

Inputs: Data needs from In Zone Devices

- Cap Bank
  - Status
  - Simple Expected Behavior
    - Discussion: Need to establish a method to correlate measurements and model with expected controls actions.
- Regulator
  - Tap Position
  - Simple Expected Behavior
  - Discussion: Need to establish a method to correlate measurements and model with expected controls actions.
- Other Measurement Devices Data Needs
  - For Static Load Calibration
  - Solution Feedback

Implementation Details

Optimization Algorithm Needs

Inputs: Data needs from Neighbor Algorithm Agents on OpenDSO

- Power Flow
  - V,I,P,Q
- Algorithm Specific Coordination Parameters
  - To be determined in algorithm system design
- Convergence Indication
- Speed of communication for iterative algorithm convergence
  - Asynchronous vs Synchronous
  - To be determined in algorithm system design

Bounds and Values

Battery State/Control Variables (1/2)

Variable	Value or Limits	Description
P_{Max}	$\sim [2, 20]$ kW	P_{Rated} of corresponding DER.
$\lvert q_{B_{Max}} \rvert$	$\sim [1, 10]$ kVAr	$q_{D_{Rated}}$ of corresponding DER.
P_d, P_c	$[0, P_{Max}]$	
q_B	$[-q_{B_{Max}}, q_{B_{Max}}]$	Currently linearly modeled (is actually quadratic)
E_{Rated}	$P_{Max} \times 4$ h	4 h of one-way Charging/Discharging at Maximum Power
B	$[0.30E_{Rated}, 0.95E_{Rated}]$	2.4 h of one-way Charging/Discharging at Maximum Power
B_0	$0.625E_{Rated}$	Batteries start with an SOC value in the middle of their SOC range.
η_d, η_c	0.95	

Bounds and Values

Battery State/Control Variables (2/2)

Variable	Value or Limits	Description
α	1e−3	Coefficient of auxiliary objective function penalizing SCD. Value depends on the magnitude of the loss term in the objective function
γ	50	Coefficient of auxiliary objective function penalizing deviation of final SOC value from a reference.

[1] A note on α: Too big a value of α would reduce both P_c and P_d terms to zero, whereas too small a value would not penalize SCD, causing physically infeasible solutions.

Bounds and Values

BFM and DER State/Control Variables

Variable	Value or Limits	Description
$P_{DER_{Max}}$	$\sim [2, 20]$ kW	P_{Rated} of corresponding DER.
$\lvert q_{B_{Max}} \rvert$	$\sim [1, 10]$ kVAr	$q_{D_{Rated}}$ of corresponding DER.
V_{min}	$0.95\,pu$	
V_{Max}	$1.05\,pu$	
v	$[V_{min}^2, V_{Max}^2]$	Squared magnitude of nodal voltage

Reference of Notation: Battery Variables

Variable	Description	Dimension	Dimension pu
$B_{n,k}$	State of Charge (SOC) of Battery after time-interval k	$[kWh]$	$[pu\,h]$
$P^c_{n,k}$	Average Charging Power of the Battery during the k-th time interval.	$[kW]$	$[pu]$
$P^d_{n,k}$	Average Discharging Power of the Battery during the k-th time interval.	$[kW]$	$[pu]$
$q_{B_{n,k}}$	Average Reactive Power Output from the Battery Inverter during the k-th time interval.	$[kVAr]$	$[pu]$

Reference of Notation: BFM and DER Variables

Variable	Description
p_j^t	Fixed Real Power Generation minus Fixed Real Power Load. Here, $p_j^t = {p_D}_j^t - {p_L}_j^t$. A known (predicted) value $\forall\, t, j$.
q_j^t	Fixed Reactive Power Generation minus Fixed Reactive Power Load. Here, $q_j^t = -{q_L}_j^t$. A known (predicted) value $\forall\, t, j$.
$p_{D_j}^t$	Real Power Generated by DERs
$p_{L_j}^t$	Real Power Demand
$q_{L_j}^t$	Reactive Power Demand

Helpful for clarifying any variable in the optimization formulations.

Reference of Notation: Grid-level Notation

Variable	Description
$\mathcal{N}$	Set of all nodes. $\mathcal{N} = \{1, 2, \ldots n\}$
$\mathcal{L}$	Set of all branches. $\mathcal{L} = \{1, 2, \ldots l\} = \{(i, k)\} \subset (\mathcal{N} \times \mathcal{N})$.
$\mathcal{D}$	Set of all nodes containing DERs. $\mathcal{D} \subset \mathcal{N}$
$\mathcal{B}$	Set of all nodes containing storage. $\mathcal{B} \subset \mathcal{N}$
Δt	Duration of a single time period. Here $\Delta t = 15$ min. $= 0.25$ h.
T	Prediction Horizon Duration. Total number of time-intervals solved for as part of one instance of MP-OPF.

Helpful for clarifying any variable in the optimization formulations.

Reference of Notation: Branch Flow Model and DER variables

Variable	Description
p_j^t	Fixed Real Power Generation minus Fixed Real Power Load. Here, $p_j^t = {p_D}_j{}^t - {p_L}_j{}^t$. A known (predicted) value $\forall t, j$.
q_j^t	Fixed Reactive Power Generation minus Fixed Reactive Power Load. Here, $q_j^t = -{q_L}_j{}^t$. A known (predicted) value $\forall t, j$.
$p_{D_j}^t$	Real Power Generated by DERs
$p_{L_j}^t$	Real Power Demand
$q_{L_j}^t$	Reactive Power Demand

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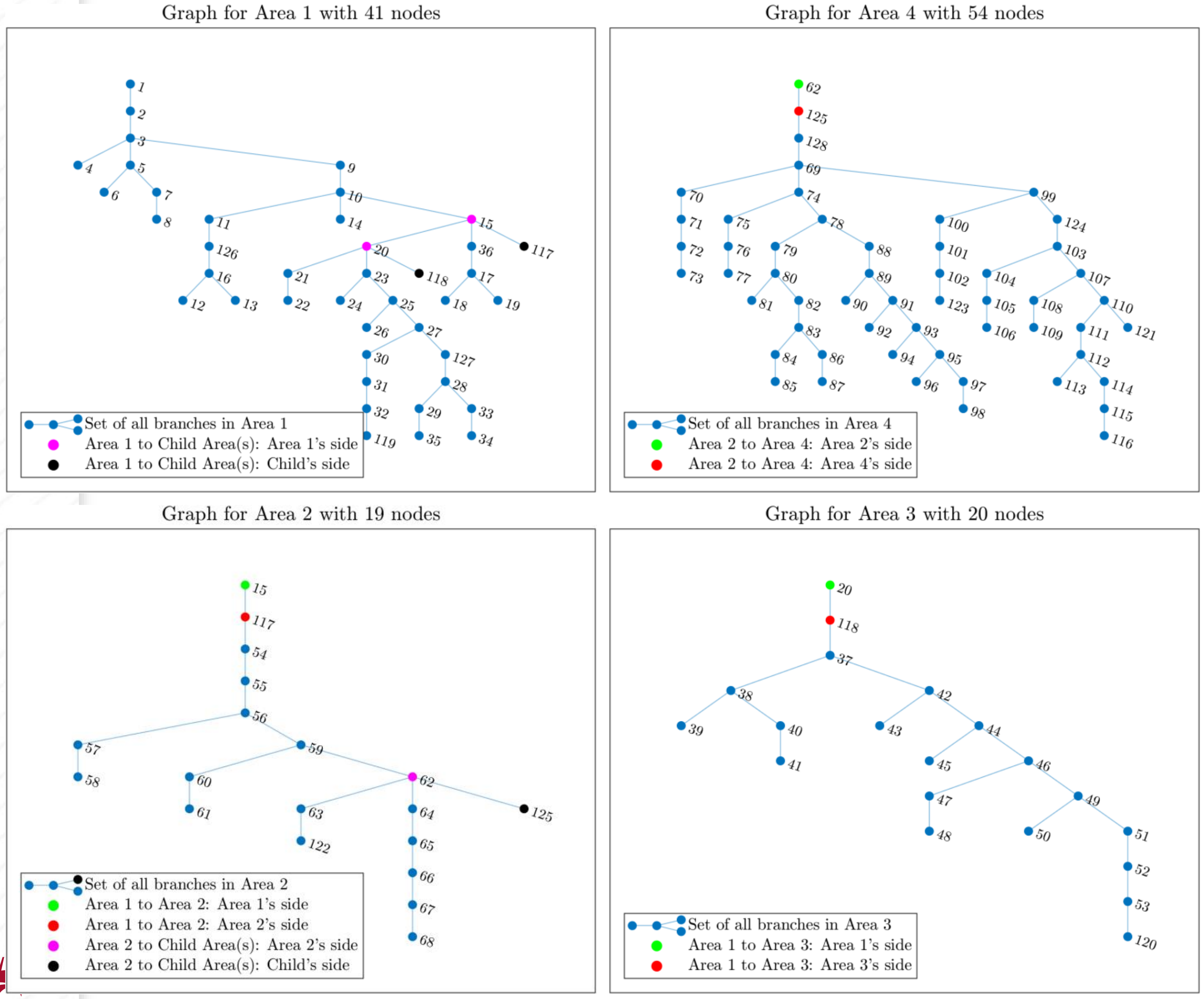

(IEEE 123 System divided into)

4 Area System

[In development] Simulation Process of Spatially and Temporally Decomposed OPF algorithm.

Start

Known:

$B_j^0, P_{DER_j}^t, P_{Load_j}^t \; \forall \, t \in [0, T], j \in \mathrm{N}$

---

while *"temporal boundary variables are NOT equal in value for all consecutive time intervals"*

while *"spatial boundary variables are NOT equal in value for all connected parent-child areas"*

i. Run (D-)OPF for minimizing the losses across the horizon for each area, in parallel. This is called as a macro-iteration.

ii. Exchange spatial boundary variables between all sets of connected parent-child areas.

end

i. Exchange temporal boundary variables between consecutive time intervals

end

Stop

Optimization Formulation

Mixed-Integer Non-linear Optimization Problem (in built-complementarity, but harder to solve)

$$\min_{P_{ij}^t, Q_{ij}^t, v_j^t, l_{ij}^t, q_{D_j}^t, B_j^t, P_{c_j}^t, P_{d_j}{}^t, q_{B_j}^t} \quad \sum_{t=1}^{T} \sum_{(i,j)\in\mathcal{L}} (r_{ij} l_{ij}^t) \tag{E.1}$$

s.t.

$$p_j^t = \sum_{(j,k)\in\mathcal{L}} P_{jk}^t - \sum_{(i,j)\in\mathcal{L}} \{P_{ij}^t - r_{ij} l_{ij}^t\} - P_{d_j}^t + P_{c_j}^t \tag{E.2}$$

$$q_j^t = \sum_{(j,k)\in\mathcal{L}} Q_{jk}^t - \sum_{(i,j)\in\mathcal{L}} \{Q_{ij}^t - x_{ij} l_{ij}^t\} - q_{D_j}^t - q_{B_j}^t \tag{E.3}$$

$$v_j^t = v_i^t + \{r_{ij}^2 + x_{ij}^2\} l_{ij}^t - 2(r_{ij} P_{ij}^t + x_{ij} Q_{ij}^t) \tag{E.4}$$

$$l_{ij}^t = \frac{(P_{ij}^t)^2 + (Q_{ij}^t)^2}{v_i^t} \tag{E.5}$$

$$B_j^t = B_j^{t-1} + z\Delta t \eta_c P_{c_j}^t - (1-z)\Delta t \frac{1}{\eta_d} P_{d_j}^t \tag{E.6}$$

$$B_j^0 = 0.5(soc_{max} + soc_{min}) E_{Rated} = 0.625 E_{Rated} \tag{E.7}$$

$$B_j^T = B_j^0 \tag{E.8}$$

where, (E.9)

(i, j) : Branch connecting nodes i and j (E.10)

$$p_j^t = p_{Dj}{}^t - p_{Lj}{}^t \tag{E.11}$$

$$q_j^t = -q_{Lj}{}^t \tag{E.12}$$

$$t = \{1, 2, \ldots T\} \tag{E.13}$$

$$z = \{0, 1\} \tag{E.14}$$

Line Losses

Real Powerflow from a bus into the network

Reactive Powerflow from a bus into the network

Kirchhoff's Voltage Law (KVL)

Current Magnitude in a branch

DER Generation (forecasted, non-controllable)

Loads (forecasted, non-controllable)

Battery State of Charge Equation *(this equation is temporally coupled, or has 'memory')*

Complementarity of Charging and Discharging during a particular time interval.

References

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