WASHINGTON STATE UNIVERSITY

Scalable Multi-Period Optimal Power Flow for Active Power Distribution Systems

or simply, Scalable MP-OPF in ADS

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Graph Illustration

Here the *true global* problem has a single consensus variable x which is used (partially/fully) by all the individual B subproblems – as copies $x_1, x_2, \ldots x_B$.

After every subproblem is solved for once -- to get latest values of $x_1, x_2, \ldots x_B$, the value of the consensus variable x is updated.

Next the dual variables $u_1, u_2, \ldots u_B$ are updated

Repeat.

$$x_i^{(k)} = \arg\min_{x_i} f_i(x_i) + \frac{\rho}{2}\|x_i - x^{(k-1)} + w_i^{(k-1)}\|_2^2 \quad i = 1, \ldots, B$$

$$x^{(k)} = \frac{1}{B}\sum_{i=1}^{B}(x_i^{(k)} + w_i^{(k-1)})$$

$$w_i^{(k)} = w_i^{(k-1)} + x_i^{(k-1)} - x^{(k)} \quad i = 1, \ldots, B$$

Consensus ADMM formulation

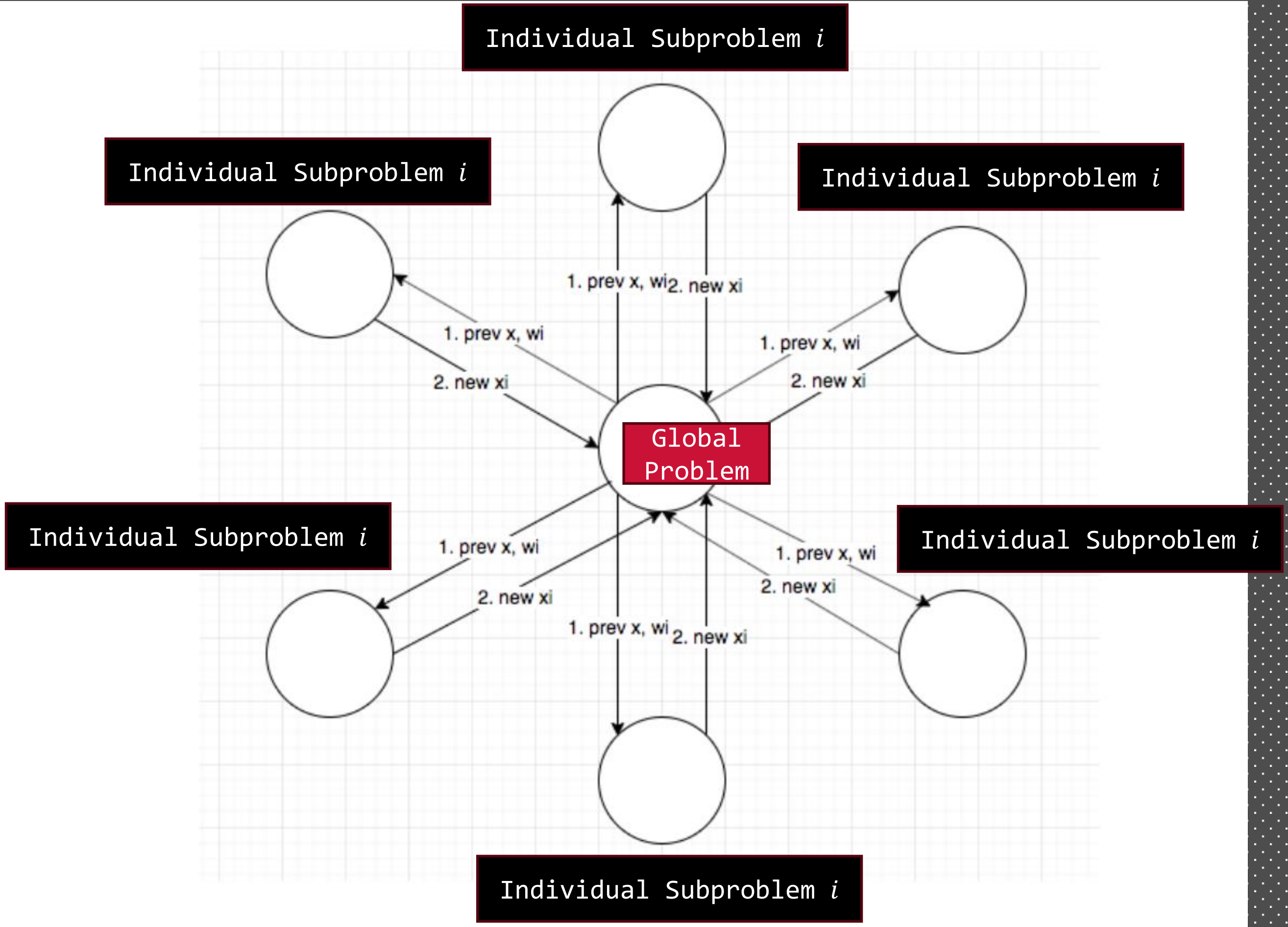

Figure 21.2: A graph illustration of consensus ADMM algorithm

Source: [ADMM notes CMU]

Update 1: Solve for $x_i^k \ \forall i \in 1:B$

Here the *true global* problem has a single consensus variable x which is used (partially/fully) by all the individual B subproblems – as copies $x_1, x_2, \ldots x_B$.

The consensus constraint in this workflow becomes $x_i = x \ \forall i \in 1:B$.

In Update 1 (k^{th} iteration), Latest values of subproblem copies x_i are solved for in parallel using last known copies of x and u_i – namely $x^{\{k-1\}}, u_i^{\{k-1\}}$ respectively. Note that I'm using u instead of w as given in this screenshot.

Consensus ADMM formulation

Note: This is like a *one-sided* ADMM formluation

21.4 Consensus ADMM

Consider a general problem

$$\min_x \sum_{i=1}^{B} f_i(x).$$

The consensus ADMM approach begins by reparametrizing the above problem to the following form:

$$\min_{x_1,\ldots,x_B,x} \sum_{i=1}^{B} f_i(x_i) \quad s.t. \quad x_i = x \quad \forall\, i \in [B].$$

By such transformation, the updates of x_i at each ADMM step are independent and therefore can be run in parallel.

The detailed ADMM steps:

$$\boxed{x_i^{(k)} = \arg\min_{x_i} f_i(x_i) + \frac{\rho}{2}\|x_i - x^{(k-1)} + w_i^{(k-1)}\|_2^2 \quad i = 1, \ldots, B}$$

$$x^{(k)} = \frac{1}{B}\sum_{i=1}^{B}(x_i^{(k)} + w_i^{(k-1)})$$

$$w_i^{(k)} = w_i^{(k-1)} + x_i^{(k-1)} - x^{(k)} \quad i = 1, \ldots, B$$

Source: [ADMM notes CMU]

Update 2: Solve for x^k

In Update 2 (k^{th} iteration), Latest value of global consensus variable x is computed using last known copies of x_i and u_i – namely $x_i^{\{k\}}, u_i^{\{k-1\}}$ respectively.

Consensus ADMM formulation

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$$\boxed{x^{(k)} = \frac{1}{B}\sum_{i=1}^{B}(x_i^{(k)} + w_i^{(k-1)})}$$

$$w_i^{(k)} = w_i^{(k-1)} + x_i^{(k-1)} - x^{(k)} \quad i = 1, \ldots, B$$

Update 3: Solve for u_i^k $\forall\, i \in 1{:}B$

In Update 3 $(k^{th}$ iteration), Latest values of local dual variables u_i are computed using last known copies of x_i, x and u_i – namely $x_i^{\{k\}}, x^{\{k\}}, u_i^{\{k-1\}}$ respectively.

Consensus ADMM formulation

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The consensus ADMM approach begins by reparametrizing the above problem to the following form:

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By such transformation, the updates of x_i at each ADMM step are independent and therefore can be run in parallel.

The detailed ADMM steps:

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Graph Illustration

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Consensus ADMM formulation

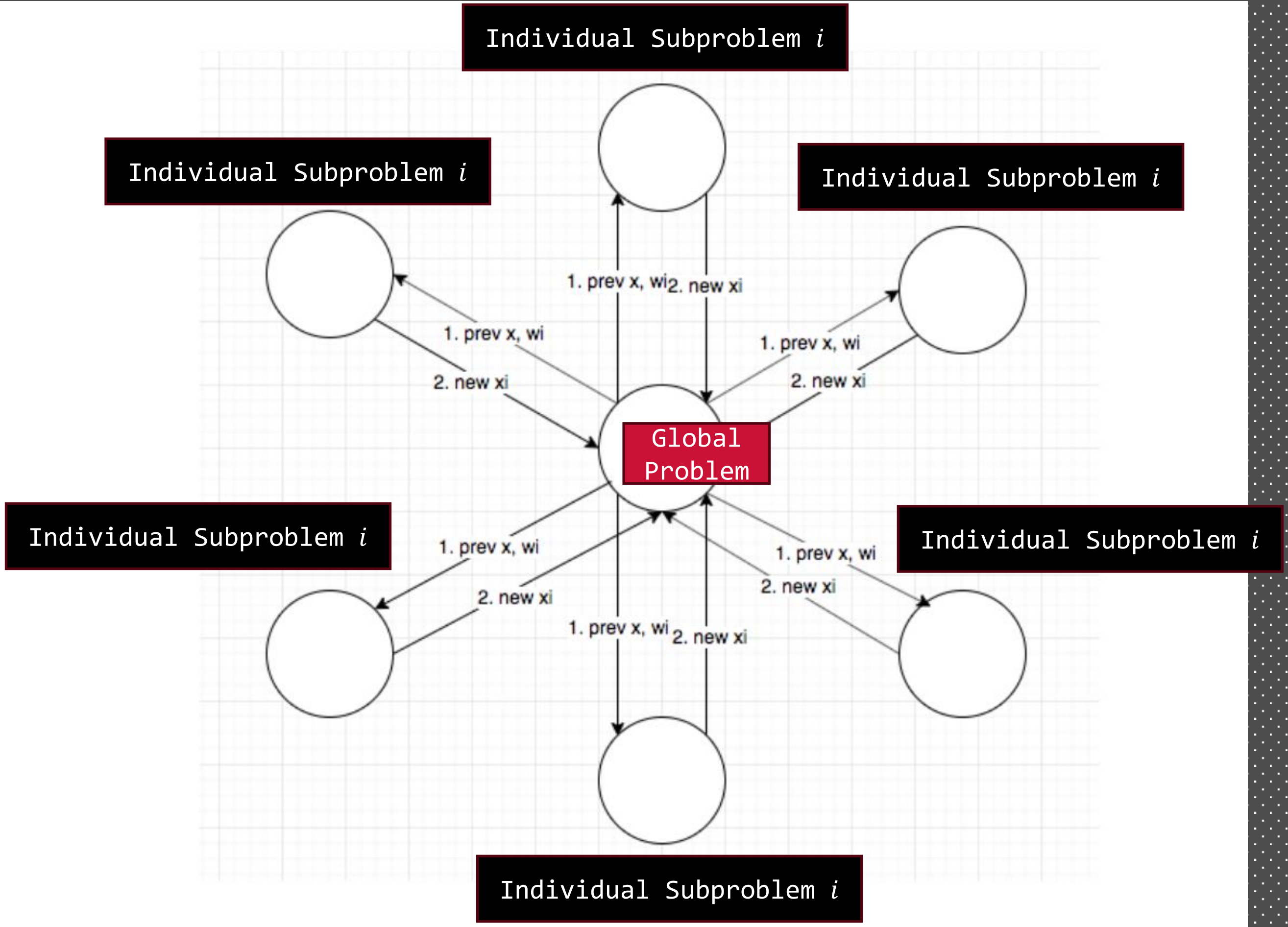
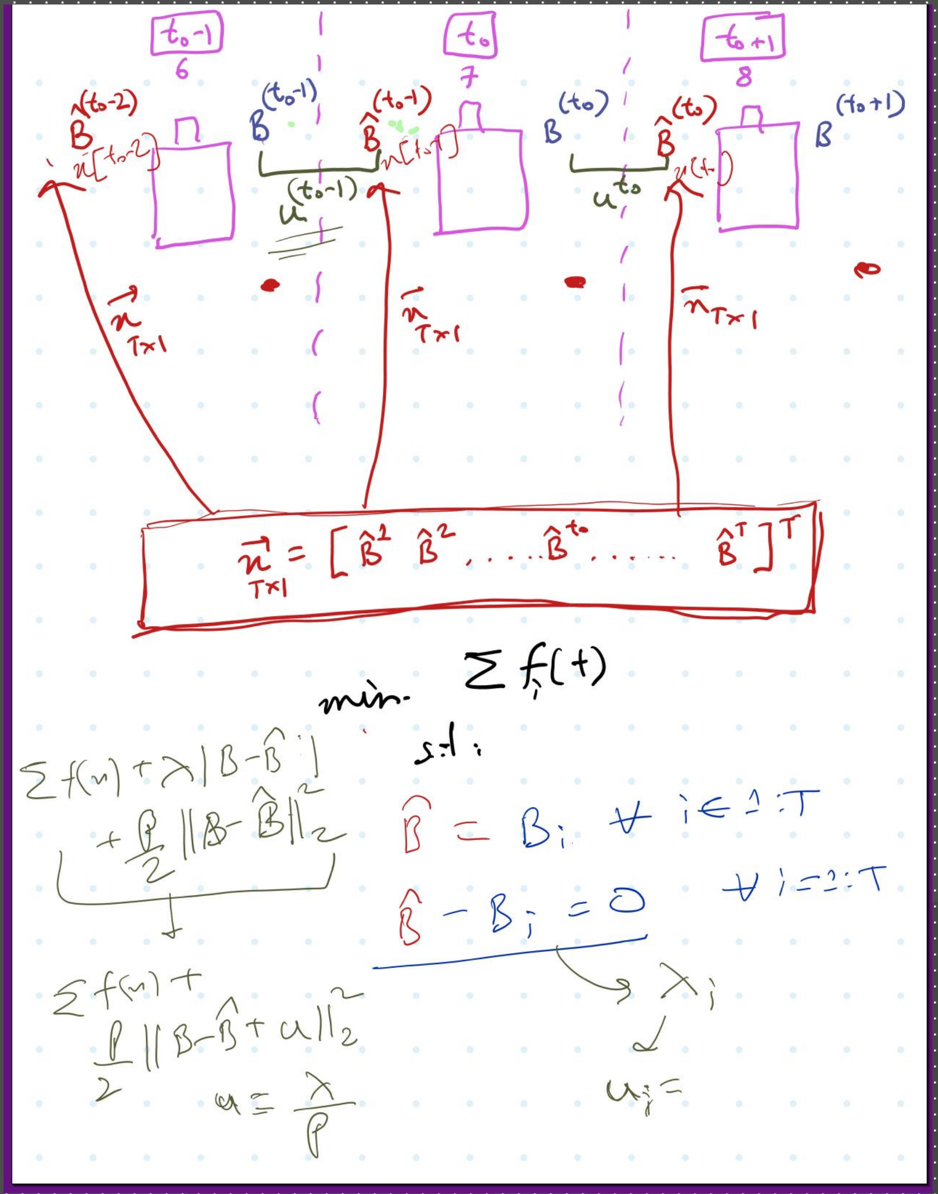


Figure 21.2: A graph illustration of consensus ADMM algorithm

Description

Consensus ADMM formulation for MPOPF



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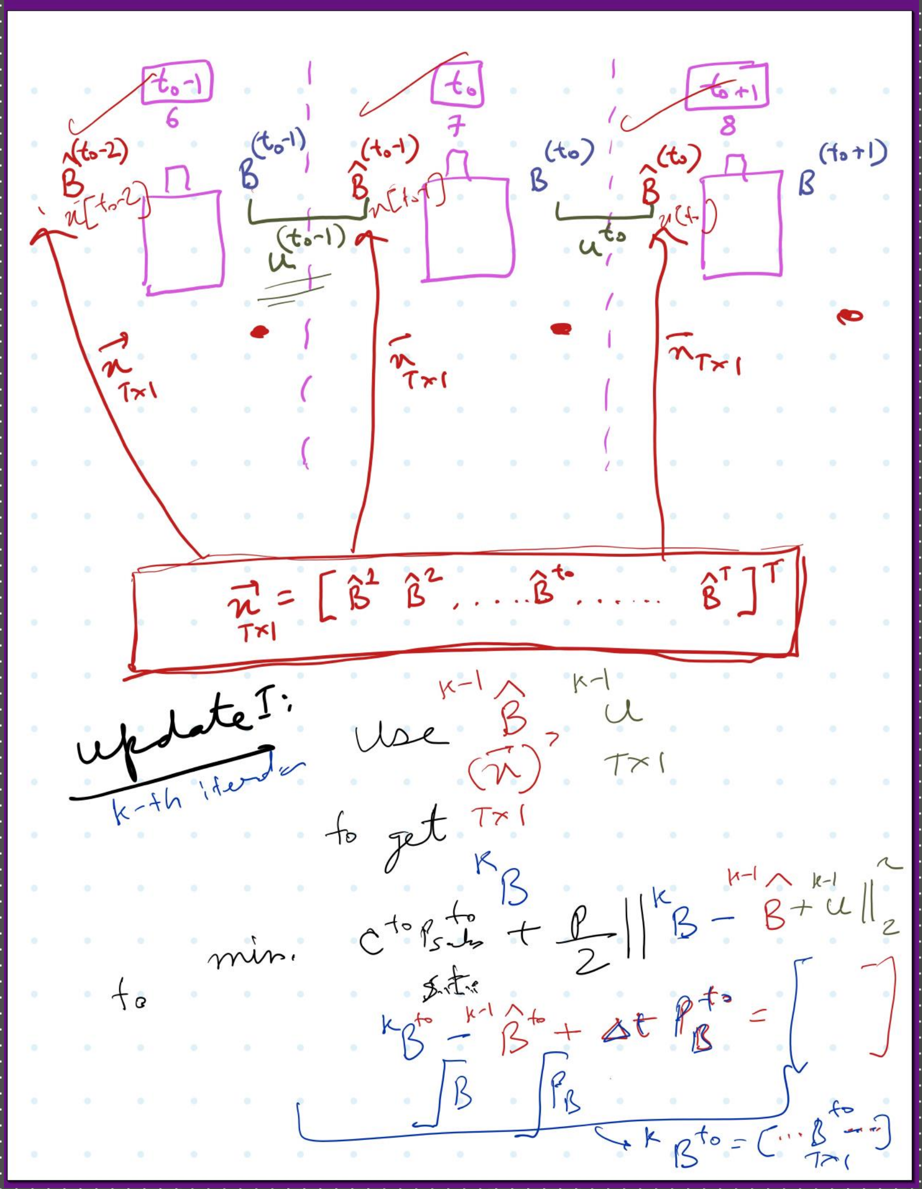
Consensus ADMM formulation for MPOPF



Description

Consensus ADMM formulation for MPOPF

Update II:

k-th iteration — Latest ${}^{k}\hat{B}$ from ${}^{k}B^{t}$, ${}^{k-1}u^{t}$ $\forall\, t = 1:T$

$$\boxed{\underset{T\times 1}{{}^{k}\hat{B}} = \frac{1}{T}\sum_{t=1}^{T}\left({}^{k}B^{t} + {}^{k-1}u^{t}\right)}$$

Update III:

Latest ${}^{k}u^{t}$ $\forall\, t = 1:T$

from ${}^{k}B^{t}$ $\forall\, t = 1:T$

and ${}^{k}\hat{B}$

$$\boxed{{}^{k}u^{t} = {}^{k-1}u^{t} + {}^{k}B^{t} - {}^{k}\hat{B} \quad \forall\, t = 1:T}$$

$${}^{k}u^{t} \rightarrow \frac{{}^{k}\lambda^{t}}{\rho}$$

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Previous Slides Follow.

admm-mpopf-formulation-draft1.pdf

References

- [ADMM notes CMU] Index of /~ryantibs/convexopt/scribes. (2025, September 01). Retrieved from https://www.stat.cmu.edu/~ryantibs/convexopt/scribes