WASHINGTON STATE
UNIVERSITY

# Scalable Multi-Period Optimal Power Flow
## for Active Power Distribution Systems

*or simply, Scalable MP-OPF in ADS*


*Aryan Ritwajeet Jha*

*Graduate Research Assistant*

*Washington State University*

# Dual Variables Comparison

$ADS10_{1ph}$, $T = 24$

$DDP$

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Dual Variables (mu) for SOC Constraints:
mu[5, 1, 3] = 45.17473125550735
mu[6, 1, 3] = 45.185519186176556
mu[5, 2, 3] = -45.174801744427846
mu[6, 2, 3] = -45.18566013739137
mu[5, 3, 3] = -48.239942587228796
mu[6, 3, 3] = -48.25099019620321
mu[5, 4, 3] = -48.24107478564388
mu[6, 4, 3] = -48.2516640313043
mu[5, 5, 3] = -48.241196259159416
mu[6, 5, 3] = -48.25189015114355
mu[5, 6, 3] = -48.241498954100564
mu[6, 6, 3] = -48.25245499800463
mu[5, 7, 3] = -48.241996600606505
mu[6, 7, 3] = -48.253124259218936
mu[5, 8, 3] = -48.242352832568955
mu[6, 8, 3] = -48.25352967582148
mu[5, 9, 3] = -48.242971180373054
mu[6, 9, 3] = -48.25442486213601
mu[5, 10, 3] = -48.243623170596926
mu[6, 10, 3] = -48.25512400205344
mu[5, 11, 3] = -48.243784096347376
mu[6, 11, 3] = -48.25464693444468
mu[5, 12, 3] = -48.243857925739185
mu[6, 12, 3] = -48.254525571498355
mu[5, 13, 3] = -48.243911780436605
mu[6, 13, 3] = -48.25436727134667
mu[5, 14, 3] = -48.243611647078
mu[6, 14, 3] = -48.25382187859363
mu[5, 15, 3] = -48.27767043280479
mu[6, 15, 3] = -48.287965604466486
mu[5, 16, 3] = -301.66935431134704
mu[6, 16, 3] = -301.7358816667674
mu[5, 17, 3] = -301.83189287498675
mu[6, 17, 3] = -301.9069334415277
mu[5, 18, 3] = -301.83806304872076
mu[6, 18, 3] = -301.9232220706423
mu[5, 19, 3] = -301.8524462450138
mu[6, 19, 3] = -301.9549648463871
mu[5, 20, 3] = -301.85952662140807
mu[6, 20, 3] = -301.9698513713682
mu[5, 21, 3] = -301.85603642385746
mu[6, 21, 3] = -301.9630767020347
mu[5, 22, 3] = -302.0049526046895
mu[6, 22, 3] = -302.10666900436934
mu[5, 23, 3] = -48.29845680868607
mu[6, 23, 3] = -48.31416276384528
mu[5, 24, 3] = -177604.01937392345
mu[6, 24, 3] = -88822.93735871241
```

BruteForced

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Dual Variables (mu) for SOC Constraints:
mu[5, 1] = -51.41801881638614
mu[6, 1] = -51.428827793029235
mu[5, 2] = -51.41801979342437
mu[6, 2] = -51.42883047558444
mu[5, 3] = -51.418023465518246
mu[6, 3] = -51.42883904249744
mu[5, 4] = -51.41803524911316
mu[6, 4] = -51.42886076359697
mu[5, 5] = -51.41807640762771
mu[6, 5] = -51.428910924641634
mu[5, 6] = -51.418148149916966
mu[6, 6] = -51.428989919622026
mu[5, 7] = -51.418247324543884
mu[6, 7] = -51.42909387136665
mu[5, 8] = -51.41836979730944
mu[6, 8] = -51.429219943018055
mu[5, 9] = -51.41851776114211
mu[6, 9] = -51.429369368599055
mu[5, 10] = -51.41869015145029
mu[6, 10] = -51.42953880457677
mu[5, 11] = -51.4188863429248
mu[6, 11] = -51.42972686728392
mu[5, 12] = -51.419113183857114
mu[6, 12] = -51.4299440750174
mu[5, 13] = -51.41938428087563
mu[6, 13] = -51.43020441803826
mu[5, 14] = -51.41972842806792
mu[6, 14] = -51.43054027925132
mu[5, 15] = -51.42030280344431
mu[6, 15] = -51.43115597947386
mu[5, 16] = -286.1783903314517
mu[6, 16] = -286.25232257011146
mu[5, 17] = -286.18413492690695
mu[6, 17] = -286.2697067055625
mu[5, 18] = -286.1869461326777
mu[6, 18] = -286.2791023564704
mu[5, 19] = -286.18833870443524
mu[6, 19] = -286.28481722740173
mu[5, 20] = -286.1884103837522
mu[6, 20] = -286.285043875479
mu[5, 21] = -286.1883973978632
mu[6, 21] = -286.2850199931506
mu[5, 22] = -286.1882908057603
mu[6, 22] = -286.28460871257334
mu[5, 23] = -51.424572090777126
mu[6, 23] = -51.44038325614457
mu[5, 24] = -51.42444528454106 5
mu[6, 24] = -51.44010590105099
```

$ADS10_{1ph}$, $T = 3$

$DDP$

```
Dual Variables (mu) for SOC Constraints:
mu[5, 1, 3] = 45.16940459928702
mu[6, 1, 3] = 45.18525275674813
mu[5, 2, 3] = -45.16947508820853
mu[6, 2, 3] = -45.185393707968935
mu[5, 3, 3] = -22553.064658338142
mu[6, 3, 3] = -11279.18311886909
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BruteForced

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Dual Variables (mu) for SOC Constraints:
mu[5, 1] = -51.43296947278457
mu[6, 1] = -51.4444417850802154
mu[5, 2] = -51.433014630013794
mu[6, 2] = -51.444449826072886
mu[5, 3] = -51.43322659704835
mu[6, 3] = -51.44492057842625
```

# Dual Variables Comparison

$IEEE123_{1ph}$, $T = 24$

$DDP$

```
Dual Variables (mu) for SOC Constraints:
mu[3, 1, 3] = 45.60864760521776
mu[116, 1, 3] = 47.08876539391993
mu[3, 2, 3] = -45.60871809413895
mu[116, 2, 3] = -47.088906345152424
mu[3, 3, 3] = -48.501845777200465
mu[116, 3, 3] = -50.02028563871999
mu[3, 4, 3] = -48.49125943942271
mu[116, 4, 3] = -49.939168077339765
mu[3, 5, 3] = -48.49400750612041
mu[116, 5, 3] = -49.958082254667374
mu[3, 6, 3] = -48.50090569669647
mu[116, 6, 3] = -50.005663401497095
mu[3, 7, 3] = -48.512206604238095
mu[116, 7, 3] = -50.08286763445897
mu[3, 8, 3] = -48.520316096659364
mu[116, 8, 3] = -50.138314593731565
mu[3, 9, 3] = -48.53452079210512
mu[116, 9, 3] = -50.2366599987618
mu[3, 10, 3] = -48.54957735426274
mu[116, 10, 3] = -50.341196614655466
mu[3, 11, 3] = -48.55335367549444
mu[116, 11, 3] = -50.36721810258768
mu[3, 12, 3] = -48.5551370821374
mu[116, 12, 3] = -50.380045203808876
mu[3, 13, 3] = -48.55649813697718
mu[116, 13, 3] = -50.39026006222793
mu[3, 14, 3] = -48.54988031444939
mu[116, 14, 3] = -50.343517862988904
mu[3, 15, 3] = -48.582414475949115
mu[116, 15, 3] = -50.382407582723566
mu[3, 16, 3] = -303.64338615003595
mu[116, 16, 3] = -315.08503717457677
mu[3, 17, 3] = -303.94432070460806
mu[116, 17, 3] = -316.2826585511957
mu[3, 18, 3] = -304.0919188015224
mu[116, 18, 3] = -317.31466316868733
mu[3, 19, 3] = -304.43789314137445
mu[116, 19, 3] = -319.8278050520248
mu[3, 20, 3] = -304.6086406827508
mu[116, 20, 3] = -321.1052878385193
mu[3, 21, 3] = -304.5149433341445
mu[116, 21, 3] = -320.42375598320746
mu[3, 22, 3] = -304.4804277976777
mu[116, 22, 3] = -319.4473164224873
mu[3, 23, 3] = -48.6613104154854
mu[116, 23, 3] = -50.825429407582014
mu[3, 24, 3] = -183395.68396762005
mu[116, 24, 3] = -91719.43856902262
```

BruteForced

```
Dual Variables (mu) for SOC Constraints:
mu[3, 1] = -51.51257833256933
mu[116, 1] = -53.173875250036204
mu[3, 2] = -51.512578405332654
mu[116, 2] = -53.173875309107316
mu[3, 3] = -51.51257813731455
mu[116, 3] = -53.17387539100572
mu[3, 4] = -51.512593339424946
mu[116, 4] = -53.17390469427558
mu[3, 5] = -51.51264583900783
mu[116, 5] = -53.17402363849502
mu[3, 6] = -51.51278078349113
mu[116, 6] = -53.17440092574151
mu[3, 7] = -51.51321528994621
mu[116, 7] = -53.1770914568914
mu[3, 8] = -51.51484563582417
mu[116, 8] = -53.188725859055204
mu[3, 9] = -51.517325758602176
mu[116, 9] = -53.20647159896489
mu[3, 10] = -51.52023804567281
mu[116, 10] = -53.227241502546654
mu[3, 11] = -51.523471880889936
mu[116, 11] = -53.25022356877818
mu[3, 12] = -51.52709555057574
mu[116, 12] = -53.27587912950017
mu[3, 13] = -51.53124886730688
mu[116, 13] = -53.30515202141375
mu[3, 14] = -51.53619815448398
mu[116, 14] = -53.3398261016505
mu[3, 15] = -51.54379936789264
mu[116, 15] = -53.39270701318736
mu[3, 16] = -288.35952695269947
mu[116, 16] = -300.2213698875003
mu[3, 17] = -288.5178563452341
mu[116, 17] = -301.38878058997847
mu[3, 18] = -288.6015637627998
mu[116, 18] = -302.01885852965825
mu[3, 19] = -288.65164393053993
mu[116, 19] = -302.40411917860666
mu[3, 20] = -288.65175671309373
mu[116, 20] = -302.4043103325418
mu[3, 21] = -288.6517442533275
mu[116, 21] = -302.40427745233603
mu[3, 22] = -288.6516450221879
mu[116, 22] = -302.40392181133296
mu[3, 23] = -51.6366898379404
mu[116, 23] = -54.050243374223136
mu[3, 24] = -51.636548882408114
mu[116, 24] = -54.04995997455812
```

$IEEE123_{1ph}$, $T = 3$

$DDP$

```
Dual Variables (mu) for SOC Constraints:
mu[3, 1, 3] = 45.63893318054958
mu[116, 1, 3] = 47.301492040235075
mu[3, 2, 3] = -45.639004920833266
mu[116, 2, 3] = -47.30164573295515
mu[3, 3, 3] = -23288.521097996185
mu[116, 3, 3] = -11646.998034908327
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BruteForced

```
Dual Variables (mu) for SOC Const
mu[3, 1] = -51.59309339195896
mu[116, 1] = -53.683406950678155
mu[3, 2] = -51.593131240451726
mu[116, 2] = -53.68349448499136
mu[3, 3] = -51.593343229505564
mu[116, 3] = -53.68391677038888
```

$IEEE123_{1ph}$, $T = 3$ all batteries: $\mu$ values for all batteries for any particular time step $t_0$ are pretty much the same

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mu[101, 1] = -53.47295830616711
mu[106, 1] = -53.58659837946916
mu[111, 1] = -53.589633105833144
mu[116, 1] = -53.683406950678155
mu[3, 2] = -51.593131240451726
mu[7, 2] = -51.64597979885023
mu[11, 2] = -52.07329816772151
mu[18, 2] = -52.34772454007912
mu[22, 2] = -52.619436028467476
mu[30, 2] = -52.63338725105715
mu[34, 2] = -52.653757124520894
mu[37, 2] = -52.65776443635618
mu[41, 2] = -52.68943973495119
mu[47, 2] = -52.79425454302701
mu[50, 2] = -52.8359303591509
mu[53, 2] = -52.84394094777267
mu[58, 2] = -52.707719939257814
mu[62, 2] = -53.274431422352755
mu[67, 2] = -53.57642427152813
mu[71, 2] = -53.48089120605158
mu[75, 2] = -53.53486792317647
mu[79, 2] = -53.5620976422982
mu[84, 2] = -53.60264525252994
mu[87, 2] = -53.6646589847673
mu[92, 2] = -53.64642902580883
mu[97, 2] = -53.649216321434224
mu[101, 2] = -53.473003045789824
mu[106, 2] = -53.586645356621005
mu[111, 2] = -53.5896811122778
mu[116, 2] = -53.68349448499136
mu[3, 3] = -51.593343229505564
mu[7, 3] = -51.646402201677525
mu[11, 3] = -52.0735101498098
mu[18, 3] = -52.347936518186465
mu[22, 3] = -52.61964800263351
mu[30, 3] = -52.63359922502093
mu[34, 3] = -52.65417946925903
mu[37, 3] = -52.65797640996631
mu[41, 3] = -52.68986207763251
mu[47, 3] = -52.79467687966754
mu[50, 3] = -52.83597084273534
mu[53, 3] = -52.84436328154971
mu[58, 3] = -52.70814228088628
mu[62, 3] = -53.27485373132199
mu[67, 3] = -53.57648497078599
mu[71, 3] = -53.48110316772271
mu[75, 3] = -53.53507988406481
mu[79, 3] = -53.56230960279157
mu[84, 3] = -53.602857212435175
mu[87, 3] = -53.66487094377281
mu[92, 3] = -53.6466409850791
mu[97, 3] = -53.64963860880763
mu[101, 3] = -53.47321500757538
mu[106, 3] = -53.586857316758156
mu[111, 3] = -53.589893072371034
mu[116, 3] = -53.68391677038888
```