Scalable Multi-Period Optimal Power Flow for Active Power Distribution Systems

or simply, Scalable MP-OPF in ADS

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Steps towards understanding Temporal Decomposition – Which Papers to Read?

- Find work where it has actually been used.
  - Beginning with only those which solve for the MPOPF problem.
    - Right now, Transmission systems are OK too (they have to deal with ramping up/down constraints for generators)
    - But may look into other domains as well afterwards
  - Ascertaining which approach to follow, based on certain desirable features the *future algorithm* should have.

Steps towards understanding Temporal Decomposition – Intended Algorithmic Solution

- Desirable features the *future algorithm* should have:

1. Should be **scalable with** respect to **Time** (horizon duration)
   - At least, something like 96 time-steps should be solvable in one go
2. Should ideally **solve** for **the** 'true' **nonlinear problem**, with as few approximations as possible
3. **Relaxations** however **are fine**, and if they're used, their solutions will obviously be checked for any infeasibility.
4. Changing **optimization formulation** is undesirable for me. Something **close to**, if not exactly the same as **the 'standard' nonlinear Branch Flow Model** [farivar2013] is what I want.
   - So, I'm looking to solve the problem using Branch Flow Variables $P_{ij}, Q_{ij}, v_j, l_{ij}$ only.
   - *Many* papers, bring out their own new set of variables, approximations, customized algorithms to deal with them, in order to solve for the MPOPF problem. This is NOT what I'm looking to do.

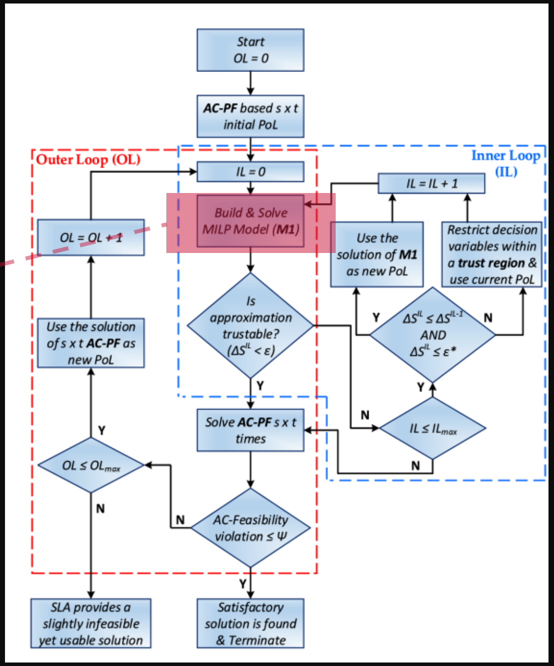
Do others ‘scalably’ solve MPOPF? – Some of them, Yes.

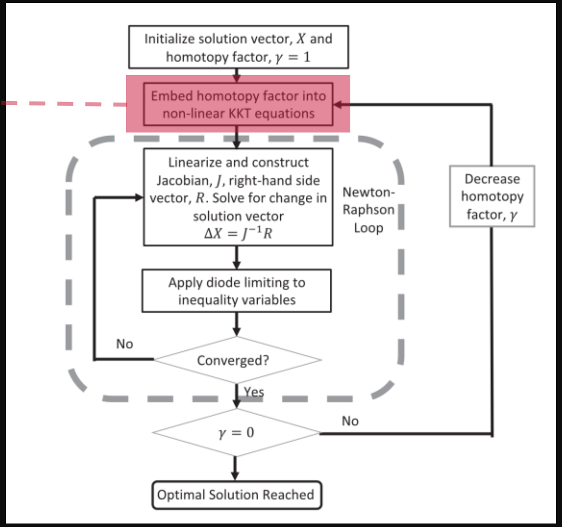
- These days, many papers pose a solution to the MPOPF problem.
  - Many papers don't actually solve the problem in a scalable manner: the test-cases are reasonable (moderate sized, moderate penetration, day-ahead or large sized, low penetration, some hours only), but even they acknowledge that this formulation is not scalable, unless …
  - … Approximations are made

How do others ‘scalably’ solve it? – Linearized Solution, Iteratively Improved

Specifically,

- … Approximations are made at the most *modular form of OPF formulation [usman2022, usman2023]* which itself could be within a loop, where some meta-parameters are adjusted based on successive iterative updates.
  - Some papers model these successive iterative updates on a homotopy based paradigm [agarwal2022]

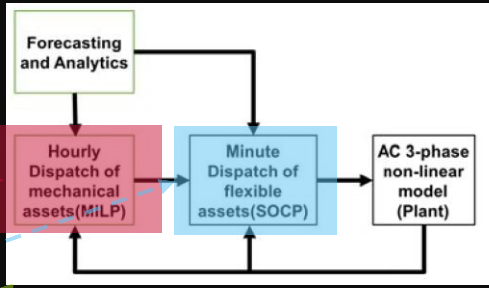


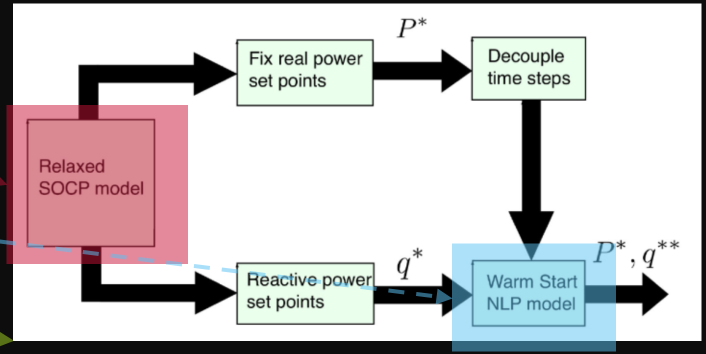


How do others 'scalably' solve it? – Hierarchical Optimization, Approximated (Relaxed) Top Level Step

Specifically,

- The full optimization problem is divided into hierarchies, with the top hierarchy problem solving for an approximate (relaxed) model of the full horizon MINLP (NLP) problem.
- Once the top-level problem is solved and set-points for temporally-coupling decision variables are obtained, the same decision variables are kept as constant for every single time-step's OPF formulation.
- The now temporally-decoupled Single-Period Optimal Power Flow Problems are formulated as SOCP (NLP) in a parallel manner.
- *[nazir2018 (nazir2020)]*





How do others 'scalably' solve it? – Some sort of Algorithms involving Primal Dual Decomposition of the Constraints containing Variables from different time-steps

$$\underline{P_{ramp}} \leq P_G^t - P_G^{t-\Delta t} \leq \overline{P_{ramp}}$$

$$B^t = B^{t-\Delta t} + \left(\eta_c P_c^t - \eta_d P_d^t\right) \Delta t$$

Not fully appreciated (understood) by me yet. But here are two instances:

- In [agarwal2022] Their transmission system has many kinds of constraints, but only two temporally coupling constraints – Generator Ramping (one upper bound inequality and one lower bound inequality for each generator for each 'time-boundary') and Battery SOC (one equality for each battery for each time-boundary). All other equality/inequality constraints are constraints *within* a particular time-step, and thus are also called as *uncoupled constraints*.
- They then formulate a Lagrangian Function for the full OPF problem. After grouping 'like' terms together (as in whether they are temporally coupled or uncoupled), they ended up with this equation, which has (a summation of) temporally coupled terms (which come arise from the three kinds of temporal constraints) and temporally uncoupled terms (all contained in the first term)
- **Finally, from the full horizon problem, a *t*-th time-step problem function** is carved out, which, outside the variables for that time-step, **only contains certain primal and dual variables outside the time-step,** including:
  - $P_G^{t-\Delta t}$, $B^{t-\Delta t}$, $\mu_{P_{ramp}}^{t+\Delta t}$ (both lower and upper bounds), $\lambda_B^{t+\Delta t}$

$$\mathcal{L}_{\text{MPOPF}} = \sum_{t=1}^{t_n} \mathcal{L}_{ACOPF}^t + \sum_{t=2}^{t_n} \overline{\mu_{P_{ramp}}}^t \cdot \left(P_G^t - P_G^{t-\Delta t} - \overline{P_{ramp}}\right) - \underline{\mu_{P_{ramp}}}^t \cdot \left(P_G^t - P_G^{t-\Delta t} - \underline{P_{ramp}}\right) + \sum_{t=2}^{t_n} \lambda_B^t \left(B^t - \left(B^{t-\Delta t} + \left(\eta_c P_c - \eta_d P_d\right) \Delta \mathrm{t}\right)\right) \quad (41)$$

$$\begin{aligned}\mathcal{L}_{MPOPF}^t = {} & \mathcal{L}_{ACOPF}^t + \overline{\mu_{P_{ramp}}}^t \cdot \left(P_G^t - P_G^{t-\Delta t} - \overline{P_{ramp}}\right) \\ & - \underline{\mu_{P_{ramp}}}^t \cdot \left(P_G^t - P_G^{t-\Delta t} - \underline{P_{ramp}}\right) \\ & - P_G^t \left(\overline{\mu_{P_{ramp}}}^{t+\Delta t} - \underline{\mu_{P_{ramp}}}^{t+\Delta t}\right) \\ & + \lambda_B^t \left(B^t - \left(B^{t-\Delta t}\right.\right. \\ & \left.\left. + \left(\eta_c P_c - \eta_d P_d\right) \Delta t\right)\right) - \lambda_B^{t+\Delta t} B^t \end{aligned} \quad (47)$$

How do others 'scalably' solve it? – Some sort of Algorithms involving Primal Dual Decomposition of the Constraints containing Variables from different time-steps

$$
\begin{aligned}
\mathcal{L}^t_{MPOPF} = \mathcal{L}^t_{ACOPF} &+ \overline{\mu_{P_{ramp}}}^{t} \cdot \left(P^t_G - P^{t-\Delta t}_G - \overline{P_{ramp}}\right) \\
&- \underline{\mu_{P_{ramp}}}^{t} \cdot \left(P^t_G - P^{t-\Delta t}_G - \underline{P_{ramp}}\right) \\
&- P^t_G \left(\overline{\mu_{P_{ramp}}}^{t+\Delta t} - \underline{\mu_{P_{ramp}}}^{t+\Delta t}\right) \\
&+ \lambda^t_B (B^t - (B^{t-\Delta t} \\
&+ (\eta_c P_c - \eta_d P_d)\,\Delta t) - \lambda^{t+\Delta t}_B B^t \qquad (47
\end{aligned}
$$

- Finally, from the full horizon problem, a ***t*-th time-step problem function** is carved out, which, outside the variables for that time-step, **only contains certain primal and dual variables outside the time-step**, including:
  - Past Primal Variables: $P_{\mathrm{G}}^{t-\Delta t}, B^{t-\Delta t}$
  - Future Dual Variables: $\mu_{P_{ramp}}^{t+\Delta t}$ (both lower and upper bounds), $\lambda_B^{t+\Delta t}$

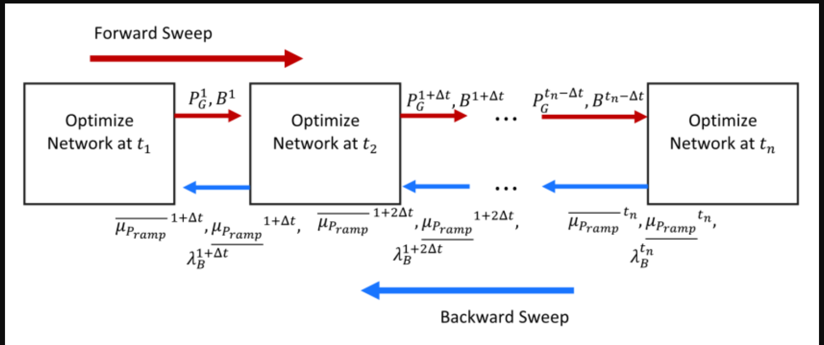
  - While the problem itself cannot be solved for without knowing both past and future variables, as part of their Differential Dynamic Programming (DDP) Paradigm, they do a Forwards Pass and a Backwards Pass, wherein in each pass, they assume one set of variables to be constant (initially guessed, or possibly known from a previous computation).

How do others 'scalably' solve it? – Some sort of Algorithms involving Primal Dual Decomposition of the Constraints containing Variables from different time-steps

$$\mathcal{L}^t_{MPOPF} = \mathcal{L}^t_{ACOPF} + \overline{\mu_{P_{ramp}}}^t \cdot (P^t_G - P^{t-\Delta t}_G - \overline{P_{ramp}}$$
$$- \underline{\mu_{P_{ramp}}}^t \cdot \left(P^t_G - P^{t-\Delta t}_G - \underline{P_{ramp}}\right)$$
$$- P^t_G \left(\overline{\mu_{P_{ramp}}}^{t+\Delta t} - \underline{\mu_{P_{ramp}}}^{t+\Delta t}\right)$$
$$+ \lambda^t_B (B^t - (B^{t-\Delta t}$$
$$+ (\eta_c P_c - \eta_d P_d) \Delta t) - \lambda^{t+\Delta t}_B B^t \quad (47$$

- Past Primal Variables: $P_G^{t-\Delta t}, B^{t-\Delta t}$
- Future Dual Variables: $\mu_{P_{ramp}}^{t+\Delta t}$ (both lower and upper bounds), $\lambda_B^{t+\Delta t}$

- Specifically, in Forward Pass, they use the solved-for values of the past primal variables i.e. from the $t - \Delta t$-th step as knowns for the problem at the t-th step, and assume the last-known/initially-guessed values of the dual variables from the future i.e. from the $t + \Delta t$-th step as constants, in order to fully solve the problem at the t-th step.
- Once the values of the primal variables values for the t-th step is known, they proceed one step *forward*, this time using those as the values of the past primal variables, again assuming the future dual variables as some constants.
- This forward pass process continues till the end-of-horizon (last time-step, let's call it T).

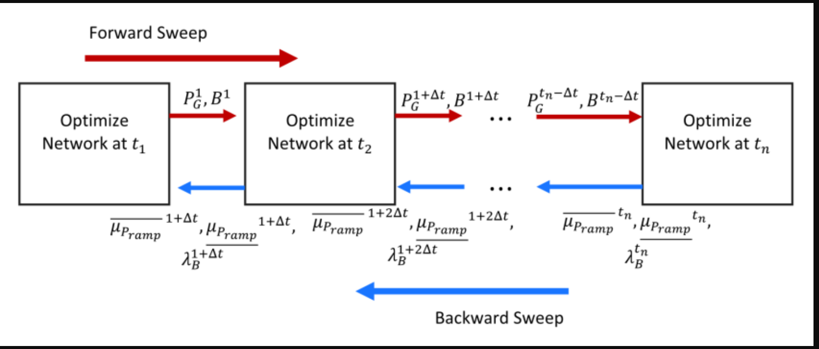


How do others ‘scalably’ solve it? – Some sort of Algorithms involving Primal Dual Decomposition of the Constraints containing Variables from different time-steps

- Past Primal Variables: $P_G^{t-\Delta t}, B^{t-\Delta t}$
- Future Dual Variables: $\mu_{P_{ramp}}^{t+\Delta t}$ (both lower and upper bounds), $\lambda_B^{t+\Delta t}$

$$\begin{aligned}\mathcal{L}^t_{MPOPF} = \mathcal{L}^t_{ACOPF} &+ \overline{\mu_{P_{ramp}}}^{t} \cdot \left(P_G^t - P_G^{t-\Delta t} - \overline{P_{ramp}}\right. \\ &- \underline{\mu_{P_{ramp}}}^{t} \cdot \left(P_G^t - P_G^{t-\Delta t} - \underline{P_{ramp}}\right) \\ &- P_G^t\left(\overline{\mu_{P_{ramp}}}^{t+\Delta t} - \underline{\mu_{P_{ramp}}}^{t+\Delta t}\right) \\ &+ \lambda_B^t(B^t - (B^{t-\Delta t} \\ &+ (\eta_c P_c - \eta_d P_d)\,\Delta t) - \lambda_B^{t+\Delta t} B^t \qquad (47\end{aligned}$$

- Once a forward pass is completed, i.e. *the cost of the forward trajectory is constructed.* It is time to solve for the gradients that will update the dual variables for the next forward pass.
- Thus after computation of these gradients, the next set of dual variables $\mu_{P_{ramp}}^{t+\Delta t}, \lambda_B^{t+\Delta t}$ is computed.
- The process of Forward Pass and Backward Pass updates is carried on until *convergence* is achieved (updates no longer being significant).



$$\nabla_{P_G^t} \mathcal{L}_{MPOPF} = \nabla_{P_G^t} \mathcal{L}^t_{APOPF} - \left(\overline{\mu_{P_{ramp}}}^{t+\Delta t} - \underline{\mu_{P_{ramp}}}^{t+\Delta t}\right) \qquad (49)$$

$$\nabla_{B^t} \mathcal{L}_{MPOPF} = \nabla_{B^t} \mathcal{L}^t_{ACOPF} - \lambda_B^{t+\Delta t} \qquad (50)$$

How do others 'scalably' solve it? – Some sort of Algorithms involving Primal Dual Decomposition of the Constraints containing Variables from different time-steps

- Another paper doing something like that [pinto2020] (not explained here).

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