



# Spatially Distributed Multi Period Optimal Power Flow with Battery Energy Storage Systems

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# Quick Overview of the contents of this presentation



High level overview of a standard OPF problem for Distribution Systems and the MPOPF problem.



Example describing spatial decomposition of the OPF problem (DOPF), and extension to MPDOPF.



Modelling of the MPOPF problem



A comparison of the proposed *Spatially Decomposed* MPDOPF results vs the *Centralized* MPCOPF algorithm

# OPF, MPOPF, Scalability of the problem

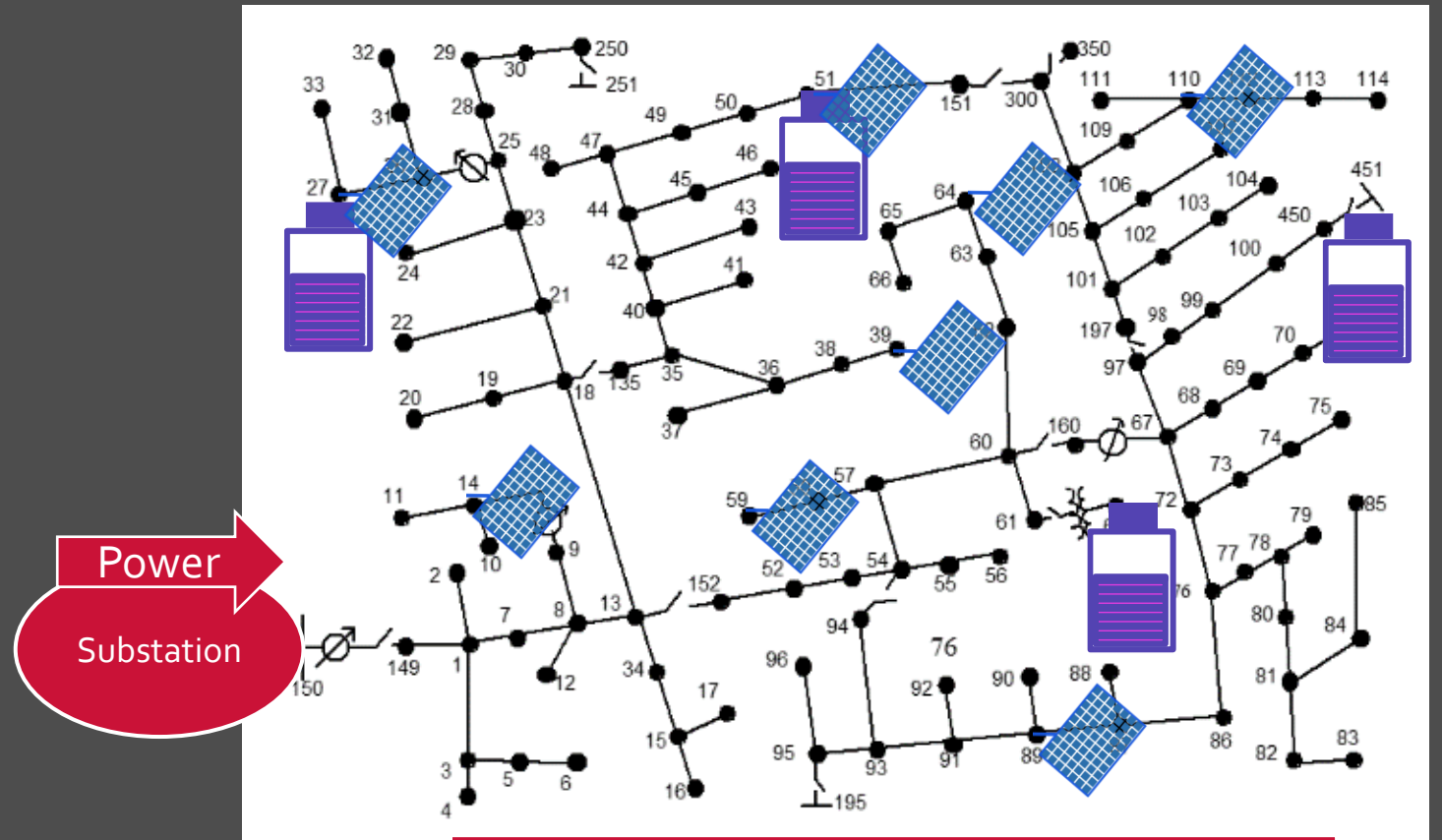


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# The Optimal Power Flow (OPF) problem for Active Distribution Systems

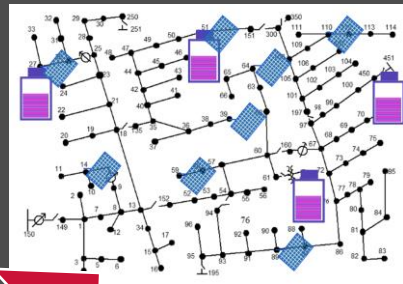
Optimal?  
Feasible?

<i>min.</i> Desired Objective Function
<i>subject to</i>
Network Constraints
Engineering Constraints
Component Constraints (DERs, Batteries)

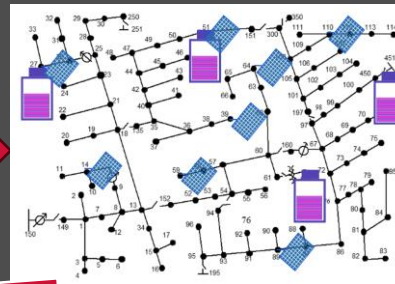


Loads and Controllable/Uncontrollable Components spread across a topology

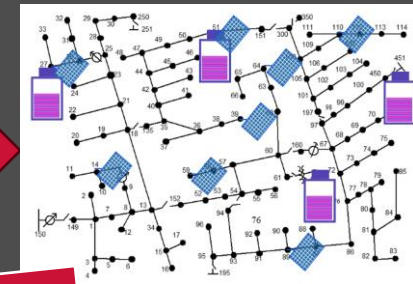
# The **Multi Period** Optimal Power Flow (**MPOPF**) problem for Active Distribution Systems



Intertemporal  
Constraints



Intertemporal  
Constraints



Intertemporal  
Constraints

An example  
of

Intertemporal  
Constraints

is

Battery SOC Equation

$$B_j^t = B_j^{t-1} + \Delta t \eta_c P_{c_j}^t - \Delta t \frac{1}{\eta_d} P_{d_j}^t$$

Due to these intertemporal constraints, the optimization problem size becomes  $T$  times larger, becoming more difficult to solve.

Our **MPOPF** problem is unfortunately **Nonlinear** and **Nonconvex** – **not scalable** with respect to horizon duration ( $T$ )

Our proposed algorithm achieves the same objective function values as traditional methods in a fraction of the time, without using additional approximations or relaxations

Model to be explained in a few slides ..

$$\min \sum_{t=1}^T \left[ C^t P_{Subs}^t \Delta t + \alpha \sum_{j \in \mathcal{B}} \left\{ (1 - \eta_C) P_{C_j}^t + \left( \frac{1}{\eta_D} - 1 \right) P_{D_j}^t \right\} \right] \quad (1)$$

Subject to the constraints (2) to (12) as given below:

$$\sum_{(j,k) \in \mathcal{L}} \{P_{jk}^t\} - (P_{ij}^t - r_{ij} l_{ij}^t) = (P_{d_j}^t - P_{c_j}^t) + p_{D_j}^t - p_{L_j}^t \quad (2)$$

$$\sum_{(j,k) \in \mathcal{L}} \{Q_{jk}^t\} - (Q_{ij}^t - x_{ij} l_{ij}^t) = q_{D_j}^t + q_{B_j}^t - q_{L_j}^t \quad (3)$$

$$v_j^t = v_i^t - 2(r_{ij} P_{ij}^t + x_{ij} Q_{ij}^t) + \{r_{ij}^2 + x_{ij}^2\} l_{ij}^t \quad (4)$$

$$(P_{ij}^t)^2 + (Q_{ij}^t)^2 = l_{ij}^t v_i^t \quad (5)$$

$$P_{Subs}^t \geq 0 \quad (6)$$

$$v_j^t \in [V_{min}^2, V_{max}^2] \quad (7)$$

$$q_{D_j}^t \in \left[ -\sqrt{S_{D_{R,j}}^2 - p_{D_j}^t{}^2}, \sqrt{S_{D_{R,j}}^2 - p_{D_j}^t{}^2} \right] \quad (8)$$

$$B_j^t = B_j^{t-1} + \Delta t \eta_C P_{c_j}^t - \Delta t \frac{1}{\eta_d} P_{d_j}^t \quad (9)$$

$$P_{c_j}^t, P_{d_j}^t \in [0, P_{B_{R,j}}], \quad B_j^0 = B_j^T \quad (10)$$

$$q_{B_j}^t \in \left[ -\sqrt{0.44 P_{B_{R,j}}}, \sqrt{0.44 P_{B_{R,j}}} \right] \quad (11)$$

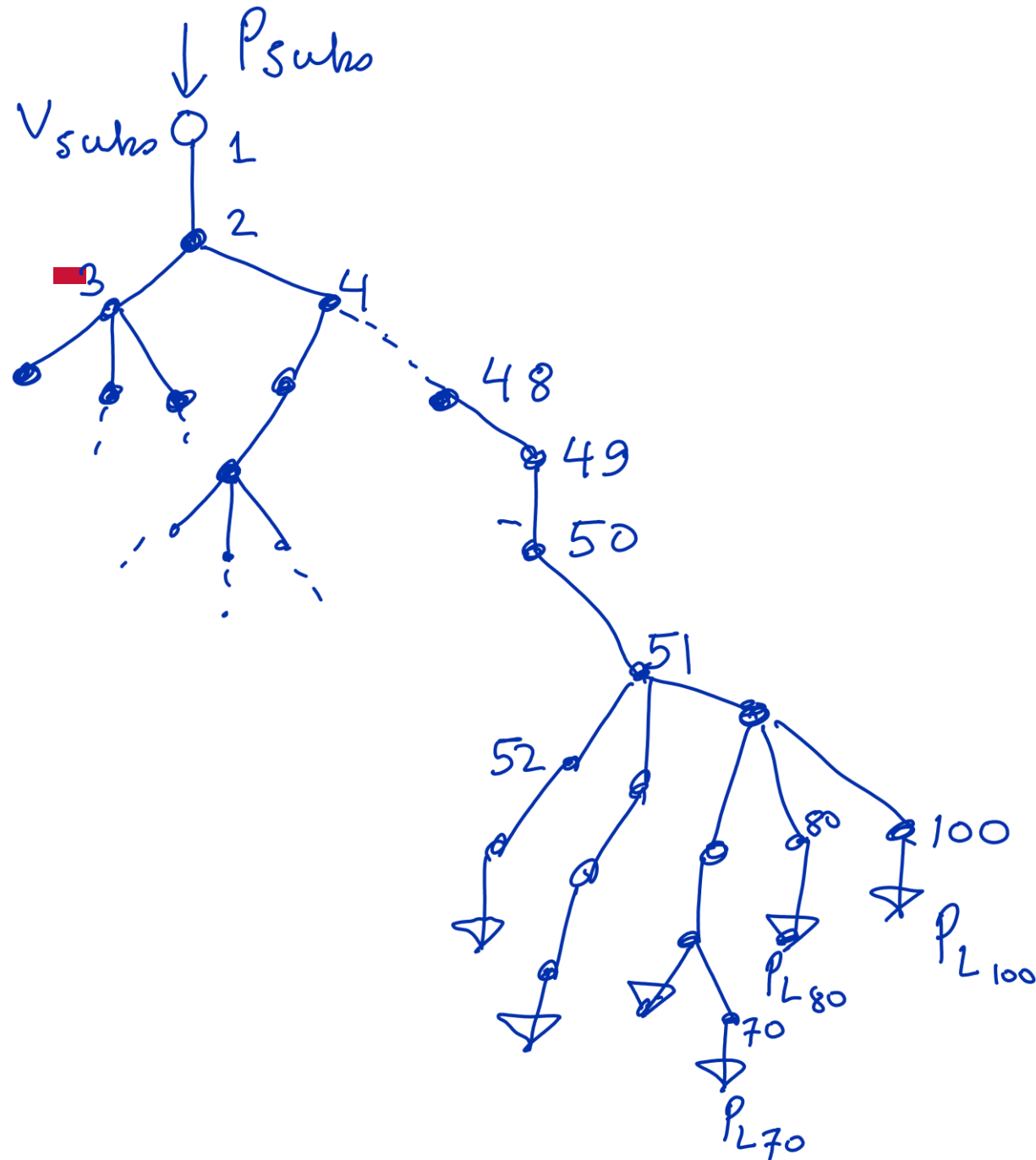
$$B_j^t \in [soc_{min} B_{R,j}, soc_{max} B_{R,j}] \quad (12)$$

# Spatially Decomposing the MPOPF Problem



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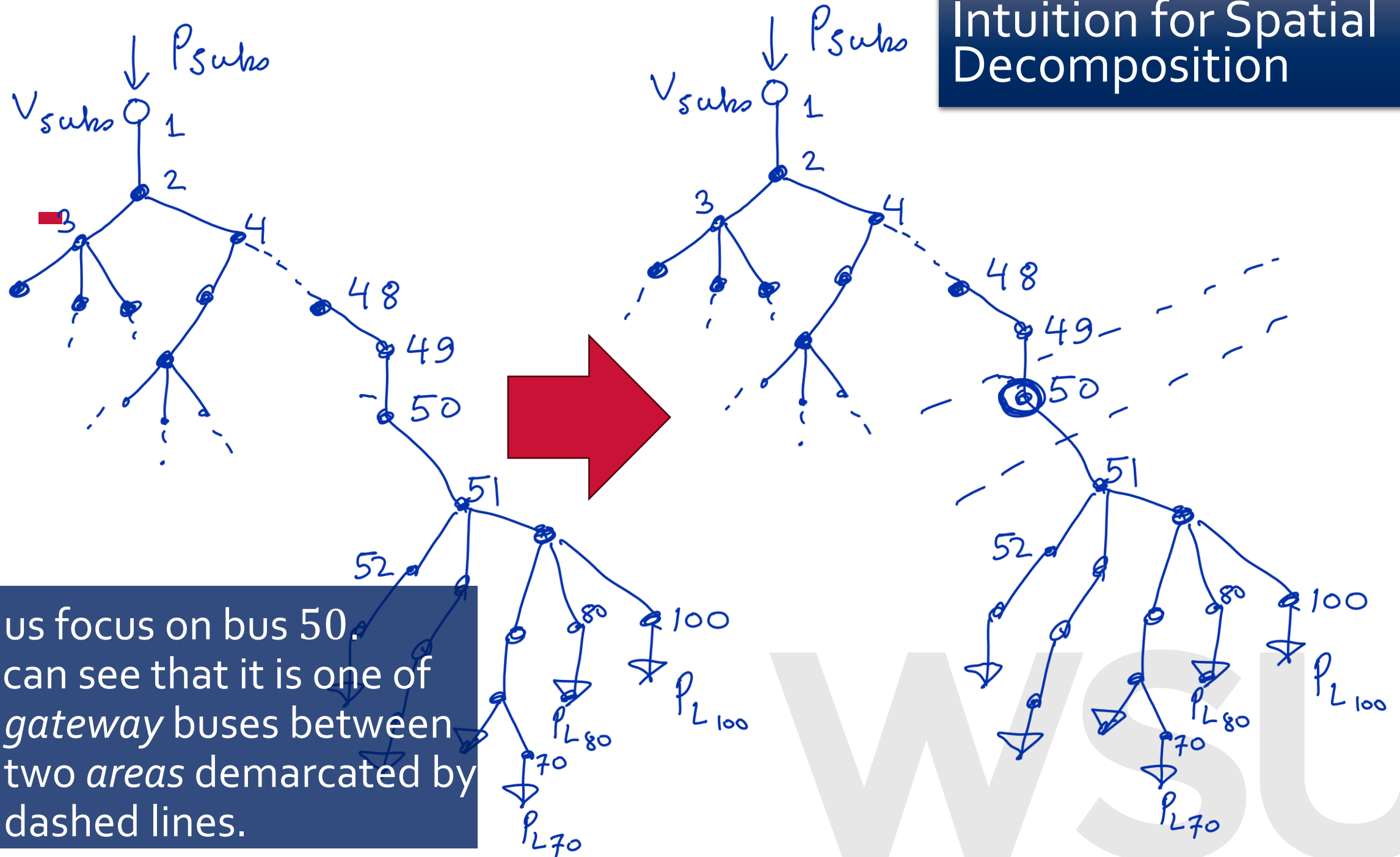
# Intuition for Spatial Decomposition



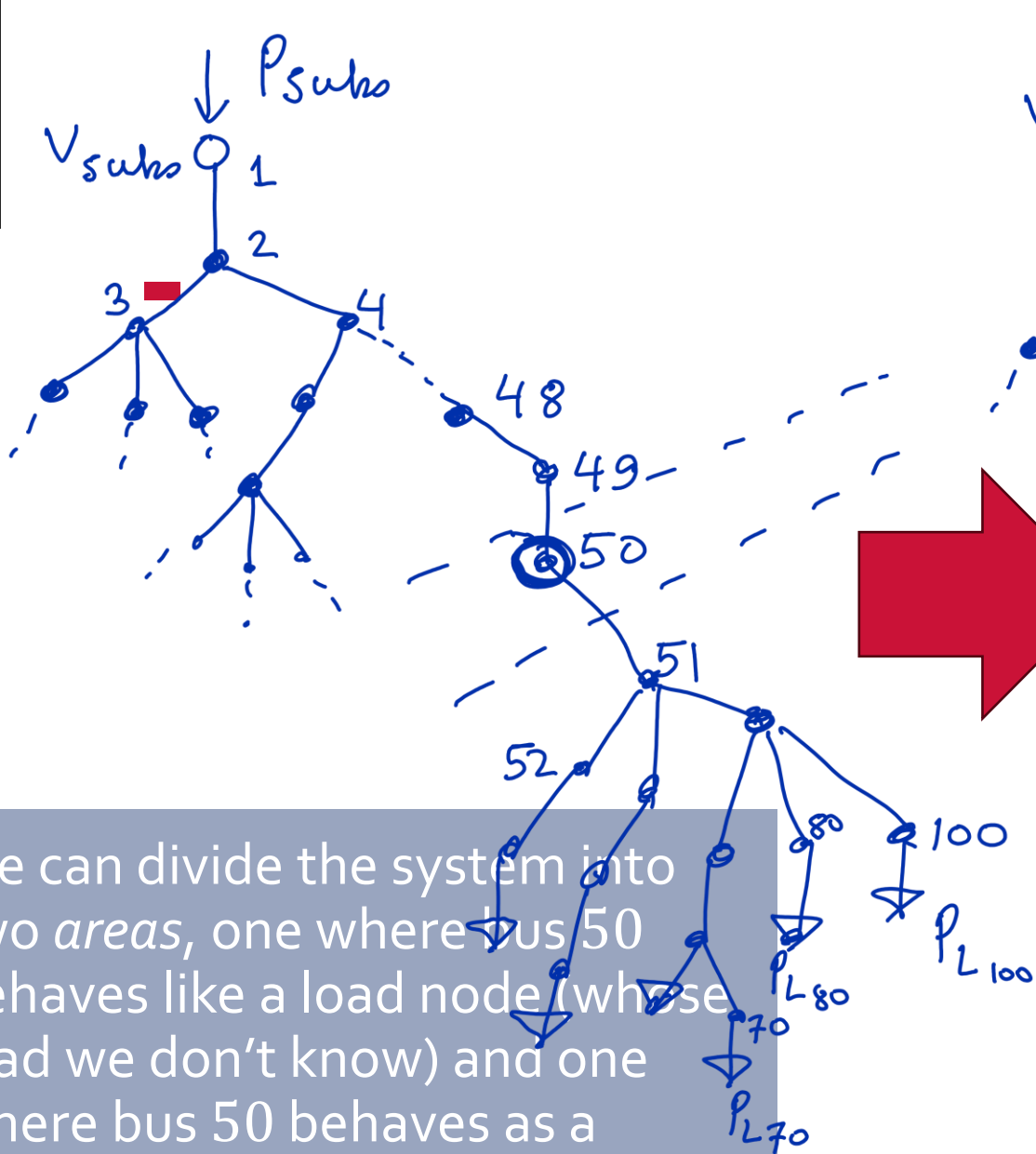
Imagine we have a 100 bus distribution system. Substation bus is 1. We wish to solve for its OPF, but would like to avoid solving the whole system in one go. Let us focus on bus 50.



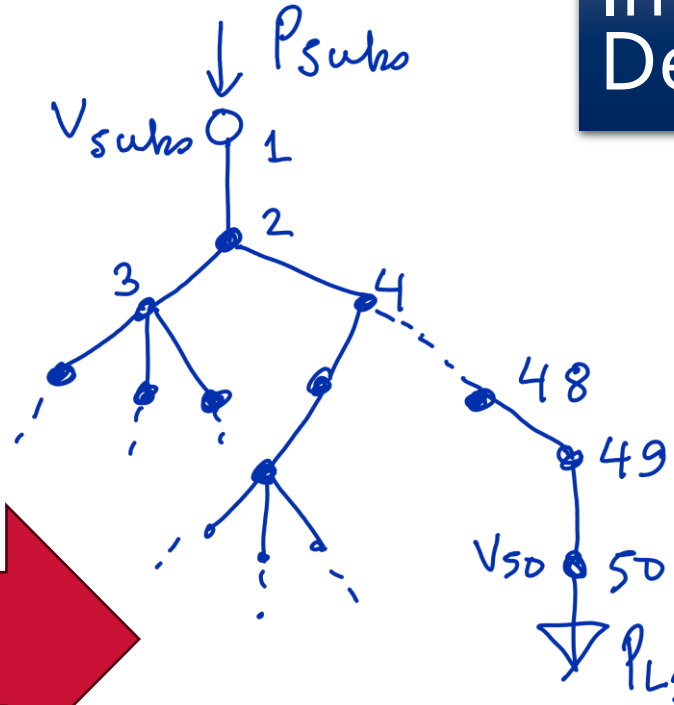
# Intuition for Spatial Decomposition



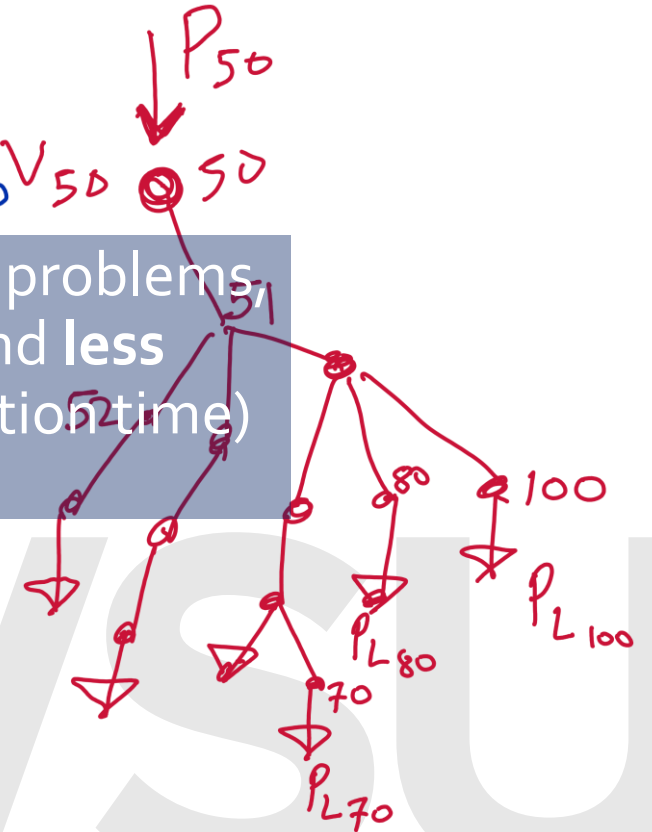
# Intuition for Spatial Decomposition



We can divide the system into two *areas*, one where bus 50 behaves like a load node (whose load we don't know) and one where bus 50 behaves as a *substation* node (whose voltage we don't know).



Now we have two OPF problems each of half the size (and **less than half** the computation time) of the original problem



# Intuition for Spatial Decomposition

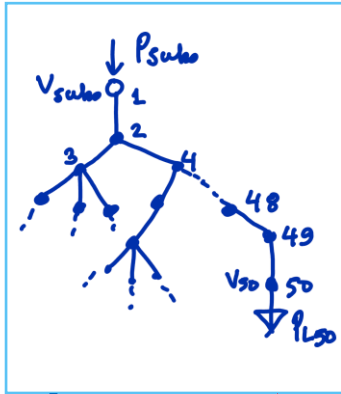
Start

We call this process DOPF where D stands for Spatially Distributed/Decomposed

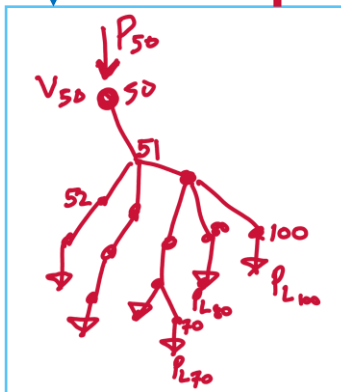
Refer to [1] for details

Each macro-iteration ( $k$ ):

Area 1



Solve OPF to get for  $v_{50}$   
Set  $v_{subs} = v_{50}$



Solve OPF to get  $P_{50}$   
Set  $P_{L50} = P_{50}$

Area 2

$k++$

Exchange boundary variables, and continue.

Convergence means:

$$\begin{aligned} |v_{50}^{\{k+1\}} - v_{50}^{\{k\}}| &< \epsilon \\ |S_{50}^{\{k+1\}} - S_{50}^{\{k\}}| &< \epsilon \end{aligned}$$

No

Yes

Are boundary variables converged?

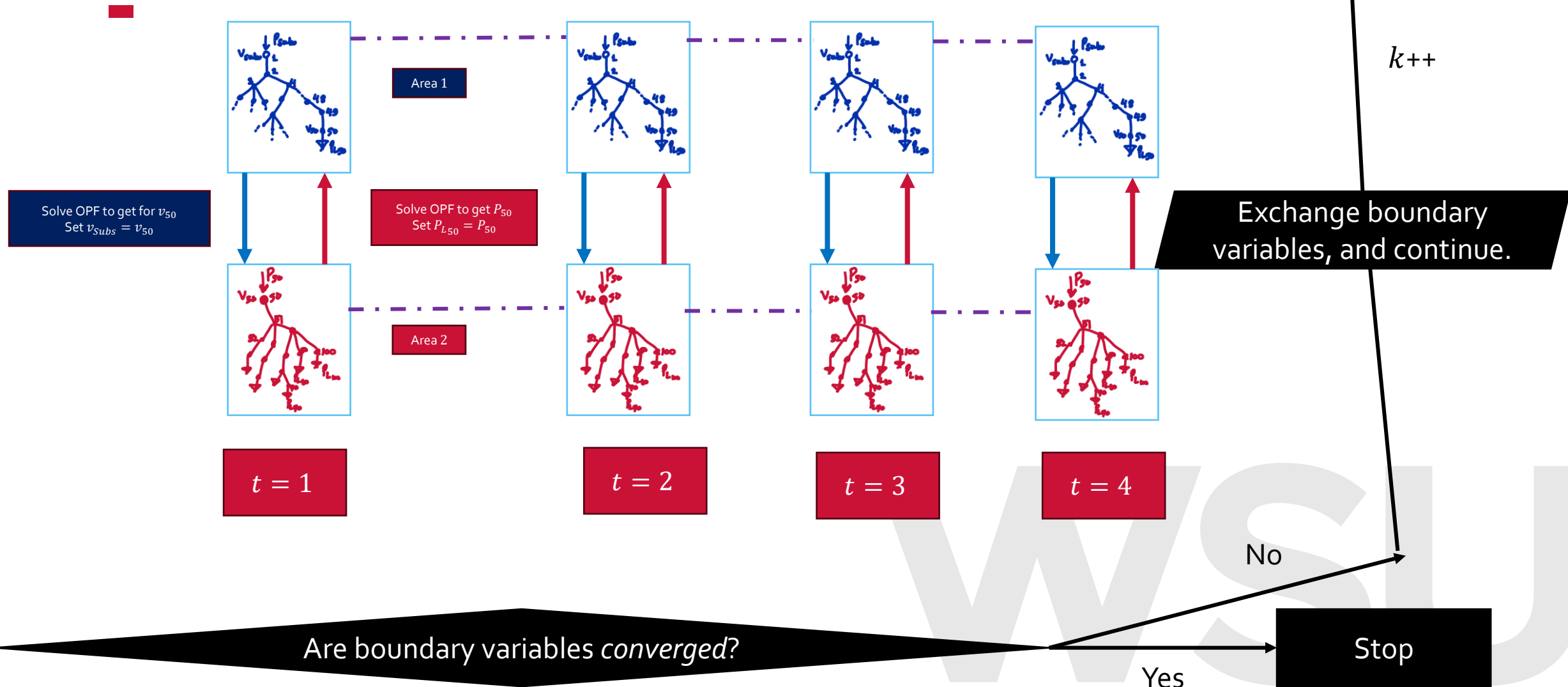
Stop

Start

We call this process MPDOPF

# Intuition for Spatial Decomposition

Each *macro-iteration* ( $k$ ):



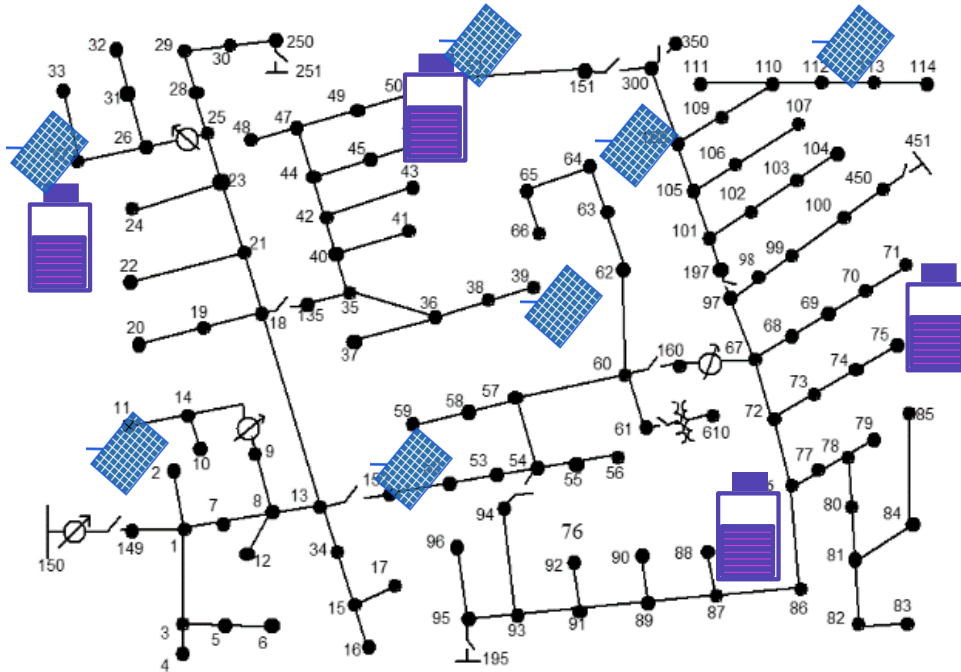
# Modelling of our MPOPF problem



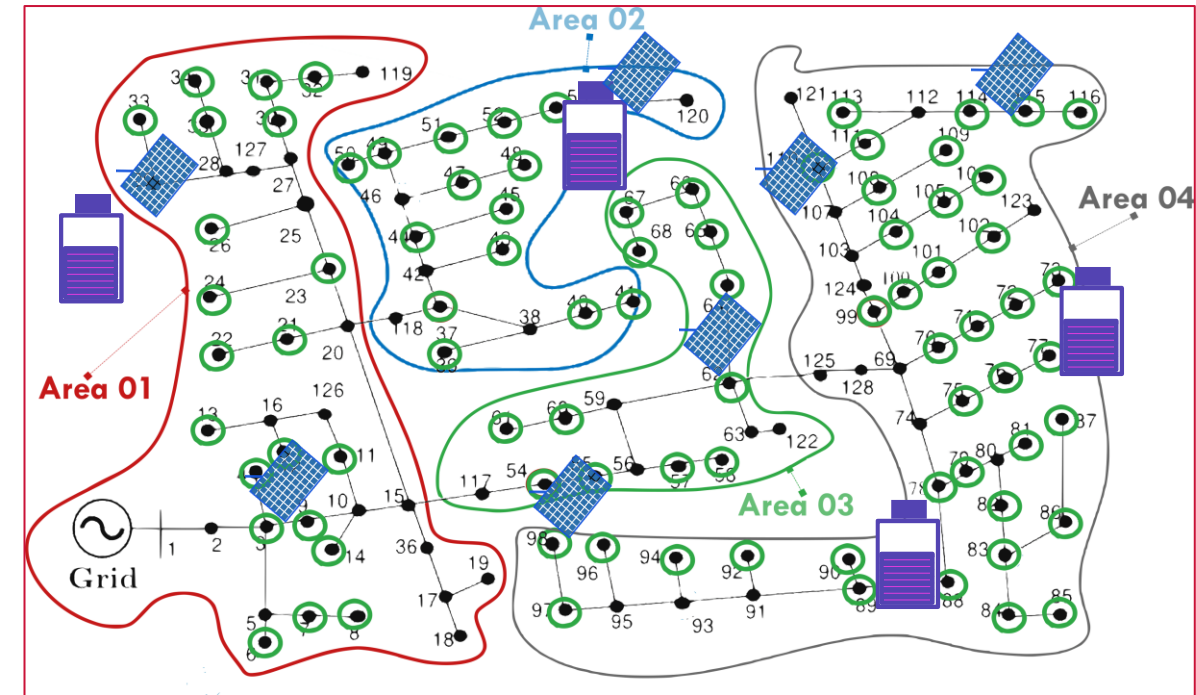
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## Network Topology and Decomposition



Original IEEE123 Bus Balanced  
Three Phase System  
With 20% PVs 30% Batteries



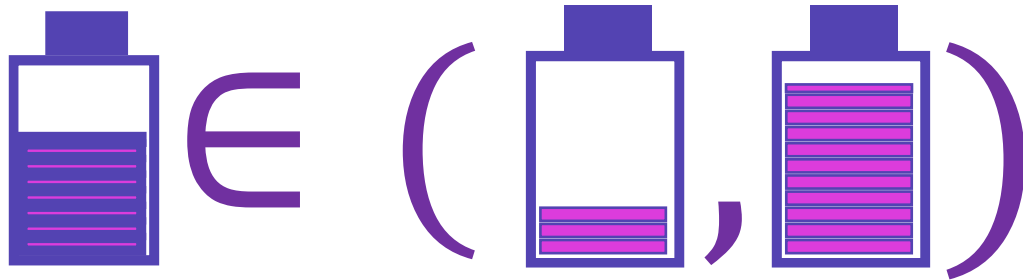
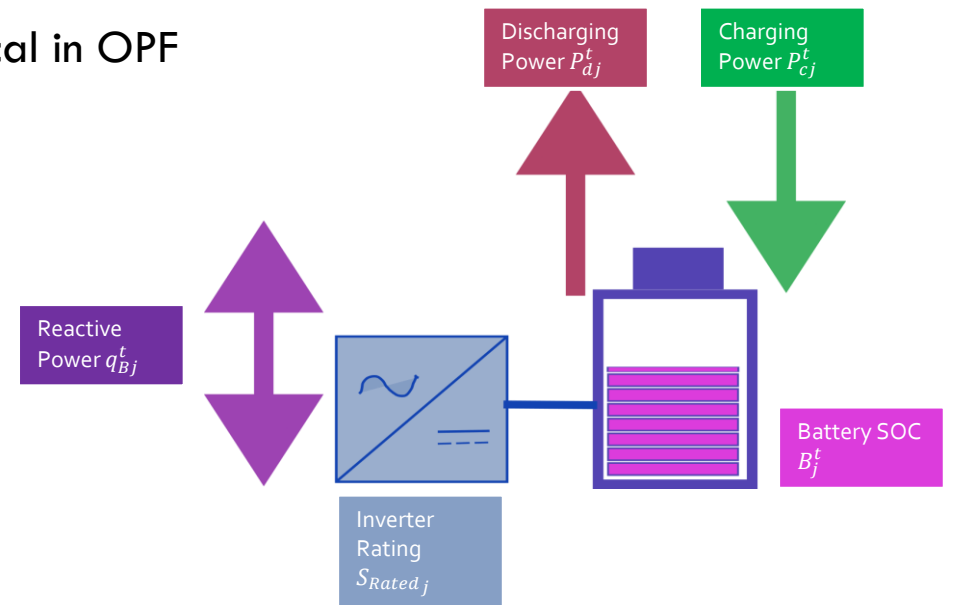
Same system but Decomposed  
into four areas for performing  
MPDOPF

# Modelling of Battery with Inverter

Note: Values here are typical in OPF literature [2, 3]

## Battery SOC Equation

$$B_j^t = B_j^{t-1} + \Delta t \eta_c P_{c_j}^t - \Delta t \frac{1}{\eta_d} P_{d_j}^t$$



$$B_j^t \in [SOC_{min}, SOC_{max}] * E_{Rated,j}$$

Battery SOC Limits

$$SOC_{min}, SOC_{max} = 0.30, 0.95$$

Battery SOC Limits

$$\eta_c, \eta_d = 0.95, 0.95$$

Charging/Discharging Efficiencies

$$B_j^0 = 0.625 E_{Rated,j}$$

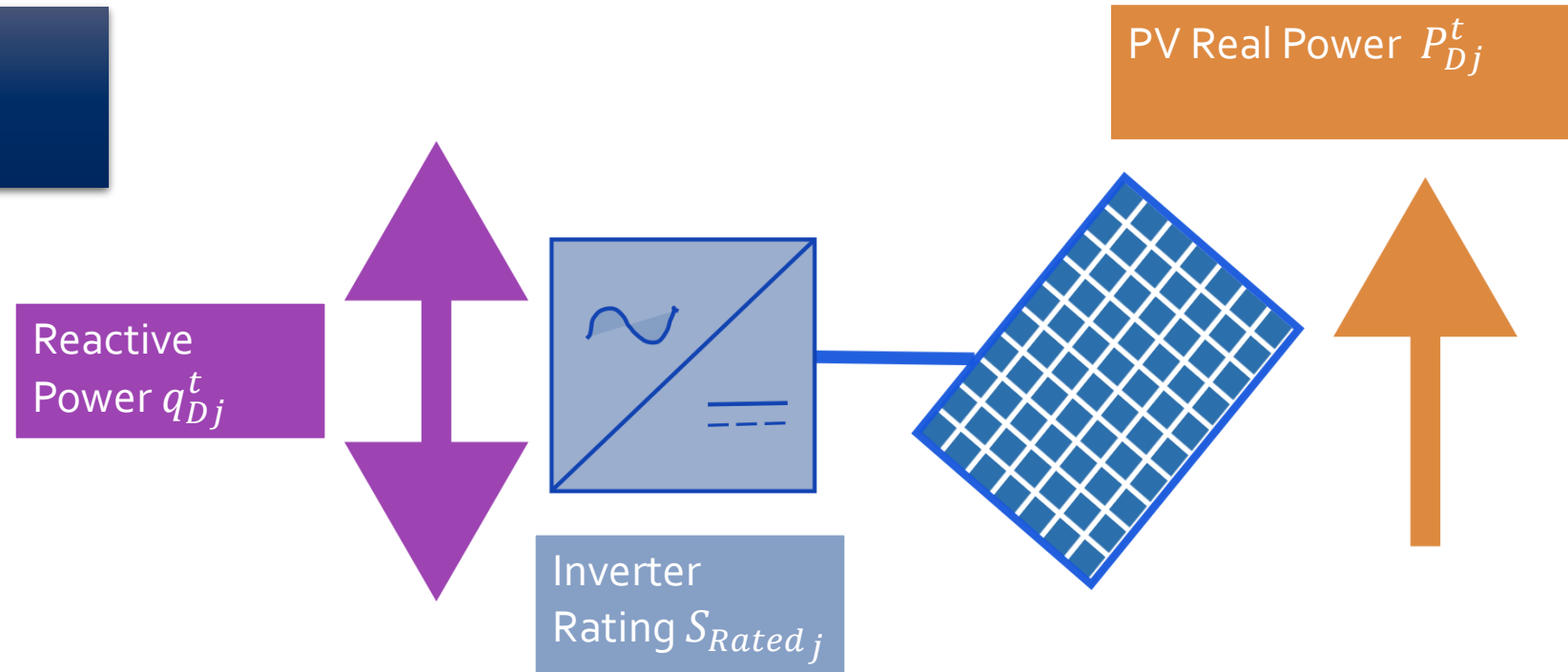
Initial SOC

$$B_j^T = B_j^0$$

Terminal SOC constraint



# Solar PVs with Inverters



$$S_{Rated,j} = 1.2 P_{Rated,j}$$

$$q_{Dj}^t \in [-\sqrt{0.44}, \sqrt{0.44}] * S_{Rated,j}$$

Relationships between the rated powers

$$(p_{Dj}^t)^2 + (q_{Dj}^t)^2 \leq (S_{Rated,j})^2$$

Solar Inverter Rating Limit

Typical  $P_{Rated,j} \in [5, 40]$  kW

Typically,  $P_{Rated,j} = \frac{1}{3} * P_{Load,j}$

Typical  $q_{Rated,j} \in [1, 10]$  kVAr

Typically,  $q_{Rated,j} = \frac{1}{3} * Q_{Load,j}$



# Network Constraints – Branch Flow Model [1A]

Branch Power Flow  
 $S_{ij}^t = P_{ij}^t + jQ_{ij}^t$   
 Current Flow  $l_{ij}^t$

Bus  $i$ :  $v_i^t = (V_i^t)^2$

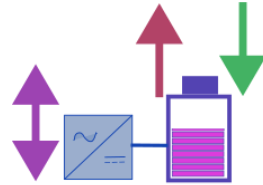
Branch  $(i, j)$

Bus  $j$ :  $v_j^t = (V_j^t)^2$

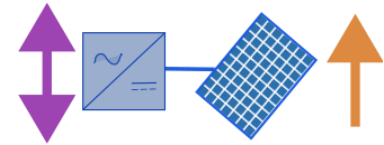
Discharging  
Power  $P_{dj}^t$

Charging Power  
 $P_{cj}^t$

Battery Reactive Power  
 $q_{Bj}^t$



PV Real Power  $P_{Dj}^t$



PV Reactive Power  $q_{Dj}^t$

$$v_j^t \in [0.95^2, 1.05^2]$$

$$l_{ij}^t \in [0, I_{Rated_{ij}}^2]$$

Engineering Constraints

$$P_{Subs}^t \in [0, P_{SubsPeak}]$$

No backflow allowed  
 Optionally, Peak Satisfiable Demand may be specified as well

Branch  $(j, k_1)$

Branch  $(j, k_2)$

Bus  $k_1$

Bus  $k_2$

Branch Power Flow  
 $S_{jk_1}^t = P_{jk_1}^t + jQ_{jk_1}^t$

Branch Power Flow  
 $S_{jk_2}^t = P_{jk_2}^t + jQ_{jk_2}^t$

Load  $P_{Lj}^t + jQ_{Lj}^t$

# Network Constraints – Branch Flow Model [1A]

## Node $j$ Real Power Balance

$$p_j^t = \sum_{(j,k) \in \mathcal{L}} P_{jk}^t - \sum_{(i,j) \in \mathcal{L}} \{P_{ij}^t - r_{ij} l_{ij}^t\} - P_{d_j}^t + P_{c_j}^t$$

## Node $j$ Reactive Power Balance

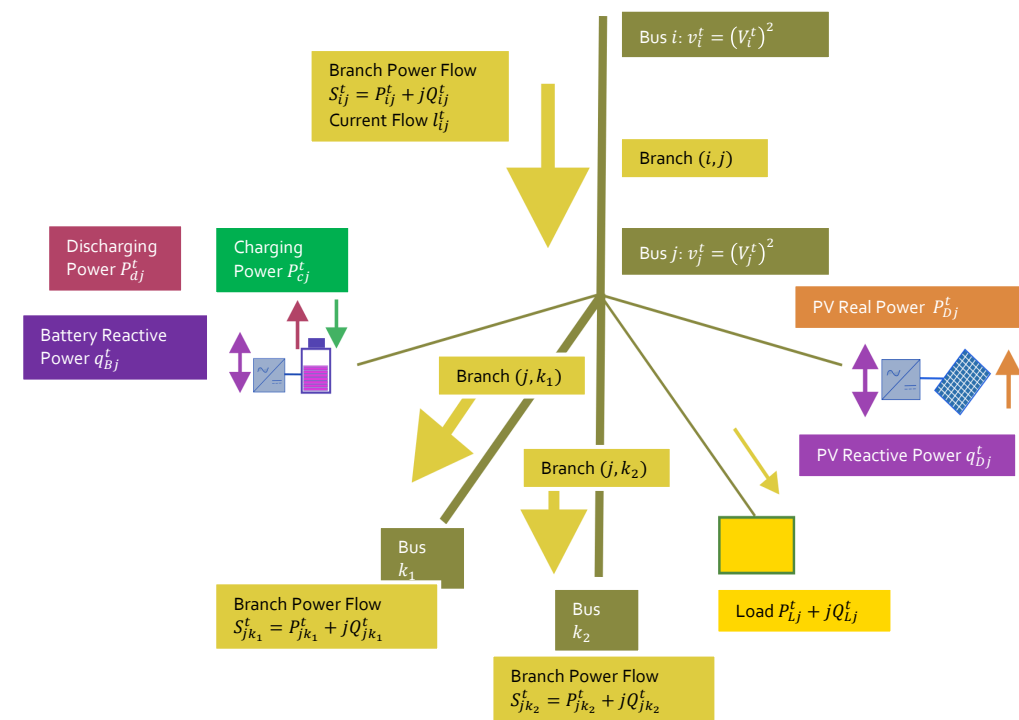
$$q_j^t = \sum_{(j,k) \in \mathcal{L}} Q_{jk}^t - \sum_{(i,j) \in \mathcal{L}} \{Q_{ij}^t - x_{ij} l_{ij}^t\} - q_{D_j}^t - q_{B_j}^t$$

## KVL across branch $(i, j)$

$$v_j^t = v_i^t + \{r_{ij}^2 + x_{ij}^2\} l_{ij}^t - 2(r_{ij} P_{ij}^t + x_{ij} Q_{ij}^t)$$

## Current Magnitude across branch $(i, j)$

$$l_{ij}^t = \frac{(P_{ij}^t)^2 + (Q_{ij}^t)^2}{v_i^t}$$



## Net Generation at Node $j$

$$p_j^t = p_{Dj}^t - p_{Lj}^t$$

$$q_j^t = -q_{Lj}^t$$

# Full Optimization Model – Balanced Three-Phase Nonlinear OPF

*min.* Desired Objective Function, (appended with an Battery Loss Function) [4]

*subject to*

Network Constraints

Engineering Constraints

Component Constraints (DERs, Batteries)

$$\min \sum_{t=1}^T \left[ C^t P_{Subs}^t \Delta t + \alpha \sum_{j \in \mathcal{B}} \left\{ (1 - \eta_C) P_{C_j}^t + \left( \frac{1}{\eta_D} - 1 \right) P_{D_j}^t \right\} \right] \quad (1)$$

Subject to the constraints (2) to (12) as given below:

$$\sum_{(j,k) \in \mathcal{L}} \{ P_{jk}^t \} - (P_{ij}^t - r_{ij} l_{ij}^t) = (P_{d_j}^t - P_{c_j}^t) + p_{D_j}^t - p_{L_j}^t \quad (2)$$

$$\sum_{(j,k) \in \mathcal{L}} \{ Q_{jk}^t \} - (Q_{ij}^t - x_{ij} l_{ij}^t) = q_{D_j}^t + q_{B_j}^t - q_{L_j}^t \quad (3)$$

$$v_j^t = v_i^t - 2(r_{ij} P_{ij}^t + x_{ij} Q_{ij}^t) + \{ r_{ij}^2 + x_{ij}^2 \} l_{ij}^t \quad (4)$$

$$(P_{ij}^t)^2 + (Q_{ij}^t)^2 = l_{ij}^t v_i^t \quad (5)$$

$$P_{Subs}^t \geq 0 \quad (6)$$

$$v_j^t \in [V_{min}^2, V_{max}^2] \quad (7)$$

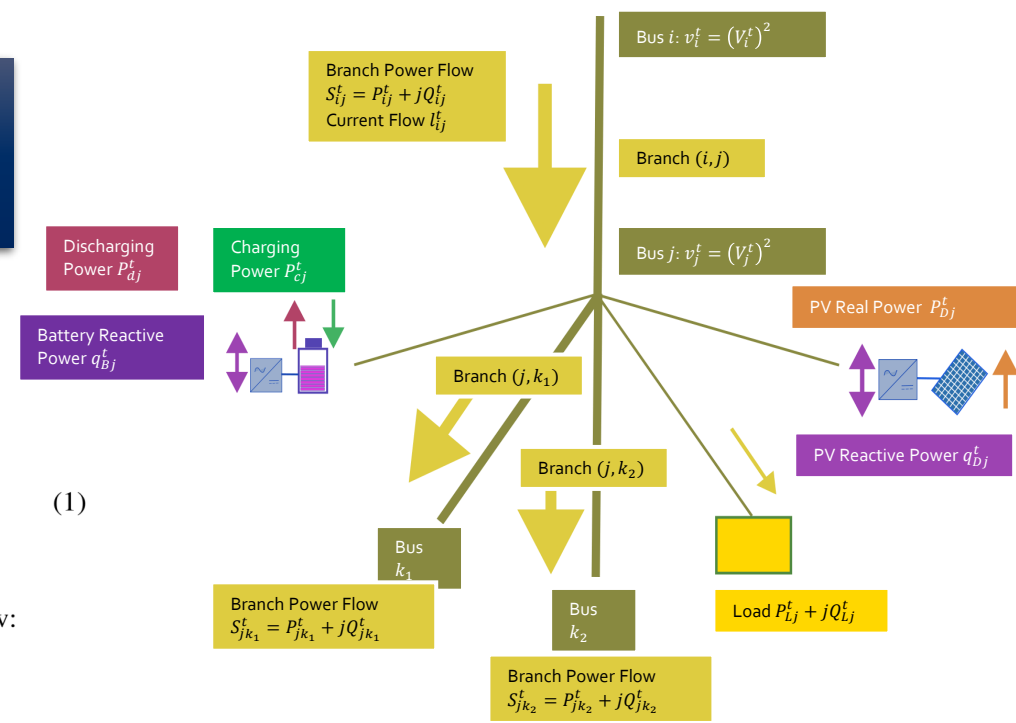
$$q_{D_j}^t \in \left[ -\sqrt{S_{DR,j}^2 - p_{D_j}^t{}^2}, \sqrt{S_{DR,j}^2 - p_{D_j}^t{}^2} \right] \quad (8)$$

$$B_j^t = B_j^{t-1} + \Delta t \eta_c P_{c_j}^t - \Delta t \frac{1}{\eta_d} P_{d_j}^t \quad (9)$$

$$P_{c_j}^t, P_{d_j}^t \in [0, P_{BR,j}], \quad B_j^0 = B_j^T \quad (10)$$

$$q_{B_j}^t \in [-\sqrt{0.44} P_{BR,j}, \sqrt{0.44} P_{BR,j}] \quad (11)$$

$$B_j^t \in [soc_{min} B_{R,j}, soc_{max} B_{R,j}] \quad (12)$$



# Results and Insights



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## Comparison between MPCOPF and MPDOPF for $T = 5$

Metric	MPCOPF	MPDOPF
Largest subproblem		
Decision variables	3150	1320
Linear constraints	5831	2451
Nonlinear constraints	635	265
Simulation results		
Substation power cost (\$)	576.31	576.30
Substation real power (kW)	4308.28	4308.14
Line loss (kW)	75.99	76.12
Substation reactive power (kVAR)	574.18	656.24
PV reactive power (kVAR)	116.92	160.64
Battery reactive power (kVAR)	202.73	76.01
Computation		
Number of Iterations	-	5
Total Simulation Time (s)	521.25	49.87

Smaller Problem Size

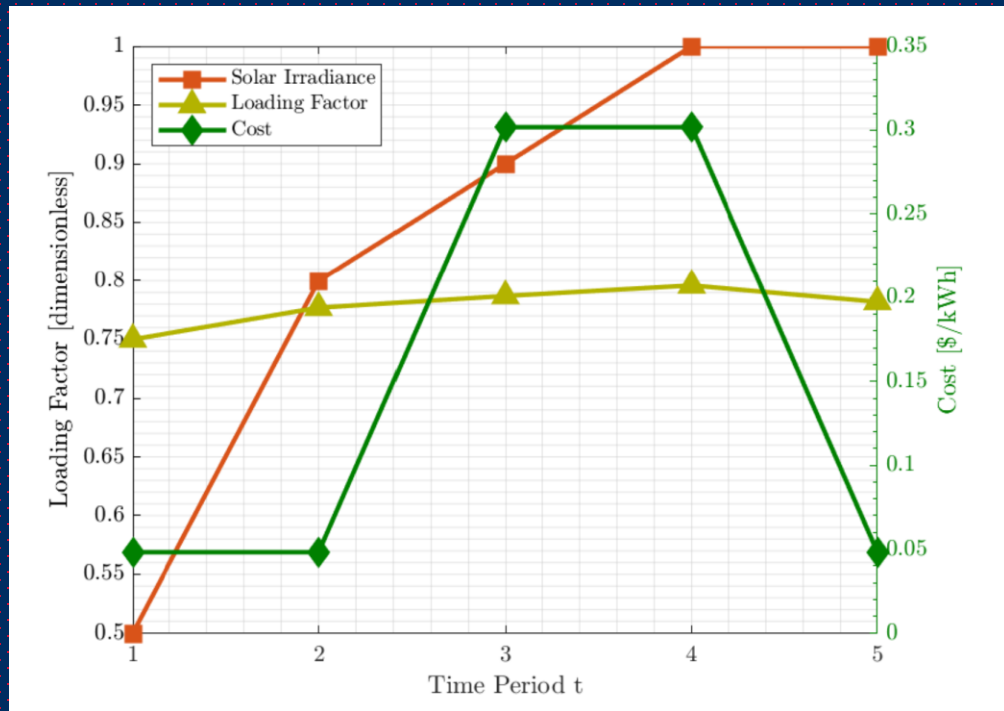
Same Converged Objective Value

Decision Variables could be different

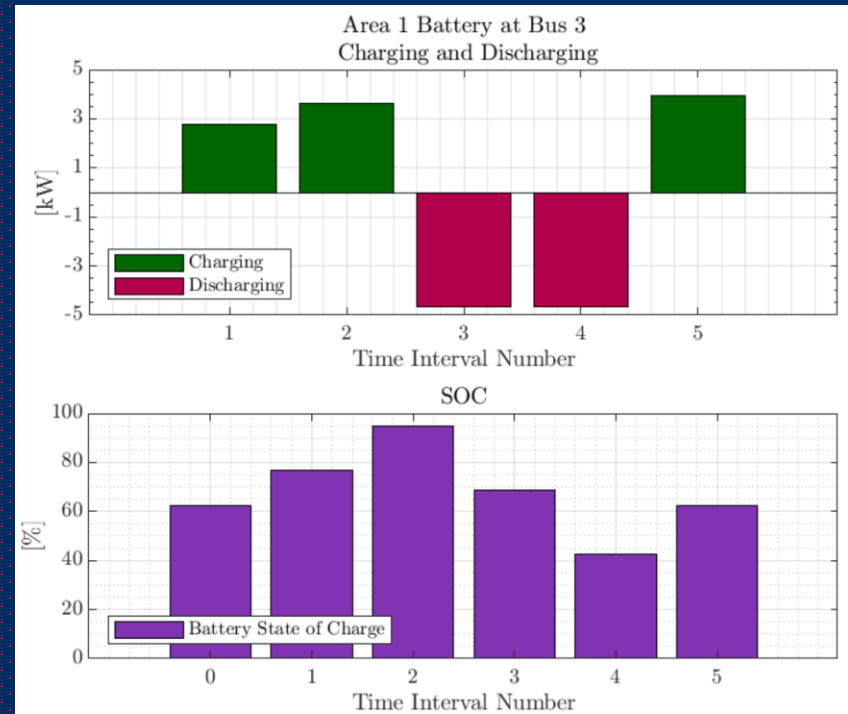
Appreciable Computation Speedup (10 times) !

Result: Proposed MPDOPF Method cuts down the computational complexity of the MPOPF problem, making way for faster solution!

# Checking MPDOPF Solution to see if it makes sense



Forecasted timeseries



Optimal Battery Actions

Battery Dispatch Pattern Intuitively tracks the cost curve for substation power. It charges up when cost is low, and discharges (injects power into the grid) when cost is higher

Complementarity of Charging and Discharging Operations ensured despite lack of Integer Constraint Modelling

Battery SOC's coming back to their original SOC values at the end of the horizon

## Comparison between MPCOPF and MPDOPF for $T = 10$

Metric	MPCOPF	MPDOPF
Largest subproblem		
Decision variables	6300	2640
Linear constraints	11636	4891
Nonlinear constraints	1270	530
Simulation results		
Substation power cost (\$)	1197.87	1197.87
Substation real power (kW)	8544.28	8544.04
Line loss (kW)	148.67	148.94
Substation reactive power (kVAR)	1092.39	1252.03
PV reactive power (kVAR)	222.59	139.81
Battery reactive power (kVAR)	388.52	310.94
Computation		
Number of Iterations	-	5
Total Simulation Time (s)	4620.73	358.69

Smaller Problem Size

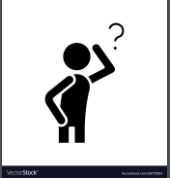
Same Converged Objective Value

Decision Variables could be different

Appreciable Computation Speedup (10 times) !

Result: Proposed MPDOPF Method cuts down the computational complexity of the MPOPF problem, making way for faster solution!

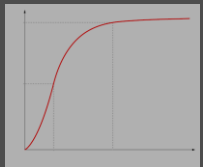
# Review and Conclusions



MPOPF Problem (unrelaxed, unapproximated) can be hard to solve.



By Spatially Decomposing the system into smaller connected areas and solving for the MPOPF for each area in an iterative exchange manner, we can cut down the problem size and solving time.



But this does not truly *solve* the MPOPF problem, as it *also* will not scale up for bigger horizon times (say,  $T = 96$  for a medium system like IEEE123).



Currently we're working on development and testing of an algorithm which will *both* spatially and temporally decompose the MPOPF problem, making it scalable.



# Thank You.



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LinkedIn

# Appendix and References follow



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# References

- [1] Sadnan, R., & Dubey, A. (2021). Distributed Optimization Using Reduced Network Equivalents for Radial Power Distribution Systems. IEEE Trans. Power Syst., 36(4), 3645–3656. doi: 10.1109/TPWRS.2020.3049135
- [1A] Farivar, M., & Low, S. H. (2012). Branch Flow Model: Relaxations and Convexification (Parts I, II). arXiv, 1204.4865. Retrieved from <https://arxiv.org/abs/1204.4865v4>
- [2] Eilyan Bitar's Papers. (2023, August 16). Coordinated Aggregation of Distributed Demand-Side Resources. Retrieved from <https://bitar.engineering.cornell.edu/papers.html>
- [3] Pandey, A., Agarwal, A., & Pileggi, L. (2020). Incremental Model Building Homotopy Approach for Solving Exact AC-Constrained Optimal Power Flow. arXiv, 2011.00587. Retrieved from <https://arxiv.org/abs/2011.00587v1>

# References



[4] Nazir, N., & Almassalkhi, M. (2021). Guaranteeing a Physically Realizable Battery Dispatch Without Charge-Discharge Complementarity Constraints. IEEE Trans. Smart Grid, 14(3), 2473–2476. doi: 10.1109/TSG.2021.3109805

[5] MultiPeriod-DistOPF-Benchmark. (2024, July 20). Retrieved from <https://github.com/Realife-Brahmin/MultiPeriod-DistOPF-Benchmark>

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