



WASHINGTON STATE
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Scalable Multi-Period Optimal Power Flow for Active Power Distribution Systems

or simply, Scalable MP-OPF in ADS

Aryan Ritwajeet Jha

Graduate Research Assistant

Washington State University

$IEEE123_{1ph}$,
 $T = 24$

Dual Variables Comparison

Terminal SOC Constraint Relaxed

$IEEE123_{1ph}$,
 $T = 3$

DDP

BruteForced (BF)

DDP

BruteForced (BF)

Dual Variables (μ) for SOC Constraints:

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mu[3, 1, 3] = 45.60864760521776
mu[3, 2, 3] = -45.60871809413895
mu[3, 3, 3] = -48.501845777200465
mu[3, 4, 3] = -48.49125943942271
mu[3, 5, 3] = -48.49400750612041
mu[3, 6, 3] = -48.50090569669647
mu[3, 7, 3] = -48.512206604238095
mu[3, 8, 3] = -48.520316096659364
mu[3, 9, 3] = -48.53452079210512
mu[3, 10, 3] = -48.54957735426274
mu[3, 11, 3] = -48.55335367549444
mu[3, 12, 3] = -48.5551370821374
mu[3, 13, 3] = -48.55649813697718
mu[3, 14, 3] = -48.54988031444939
mu[3, 15, 3] = -48.582414475949115
mu[3, 16, 3] = -303.64338615003595
mu[3, 17, 3] = -303.94432070460806
mu[3, 18, 3] = -304.0919188015224
mu[3, 19, 3] = -304.43789314137445
mu[3, 20, 3] = -304.6086406827508
mu[3, 21, 3] = -304.5149433341445
mu[3, 22, 3] = -304.4804277976777
mu[3, 23, 3] = -48.6613104154854
mu[3, 24, 3] = -183395.68396762005
```

```
mu[116, 1, 3] = 47.08876539391993
mu[116, 2, 3] = -47.088906345152424
mu[116, 3, 3] = -50.02028563871999
mu[116, 4, 3] = -49.939168077339765
mu[116, 5, 3] = -49.958082254667374
mu[116, 6, 3] = -50.005663401497095
mu[116, 7, 3] = -50.08286763445897
mu[116, 8, 3] = -50.138314593731565
mu[116, 9, 3] = -50.2366599987618
mu[116, 10, 3] = -50.341196614655466
mu[116, 11, 3] = -50.36721810258768
mu[116, 12, 3] = -50.380045203808876
mu[116, 13, 3] = -50.39026006222793
mu[116, 14, 3] = -50.343517862988904
mu[116, 15, 3] = -50.382407582723566
mu[116, 16, 3] = -315.08503717457677
mu[116, 17, 3] = -316.2826585511957
mu[116, 18, 3] = -317.31466316868733
mu[116, 19, 3] = -319.8278050520248
mu[116, 20, 3] = -321.1052878385193
mu[116, 21, 3] = -320.42375598320746
mu[116, 22, 3] = -319.4473164224873
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mu[3, 6] = -51.51278078349113
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mu[3, 8] = -51.51484563582417
mu[3, 9] = -51.517325758602176
mu[3, 10] = -51.52023804567281
mu[3, 11] = -51.523471880889936
mu[3, 12] = -51.52709555057574
mu[3, 13] = -51.53124886730688
mu[3, 14] = -51.53619815448398
mu[3, 15] = -51.54379936789264
mu[3, 16] = -288.35952695269947
mu[3, 17] = -288.5178563452341
mu[3, 18] = -288.6015637627998
mu[3, 19] = -288.65164393053993
mu[3, 20] = -288.65175671309373
mu[3, 21] = -288.6517442533275
mu[3, 22] = -288.6516450221879
mu[3, 23] = -51.63668986379404
mu[3, 24] = -51.636548882408114
```

```
mu[116, 1] = -53.173875250036204
mu[116, 2] = -53.173875309107316
mu[116, 3] = -53.17387539100572
mu[116, 4] = -53.17390469427558
mu[116, 5] = -53.17402363849502
mu[116, 6] = -53.17440092574151
mu[116, 7] = -53.1770914568914
mu[116, 8] = -53.188725859055204
mu[116, 9] = -53.20647159896489
mu[116, 10] = -53.227241502546654
mu[116, 11] = -53.25022356877818
mu[116, 12] = -53.27587912950017
mu[116, 13] = -53.30515202141375
mu[116, 14] = -53.3398261016505
mu[116, 15] = -53.39270701318736
mu[116, 16] = -300.2213698875003
mu[116, 17] = -301.38878058997847
mu[116, 18] = -302.01885852965825
mu[116, 19] = -302.40411917860666
mu[116, 20] = -302.4043103325418
mu[116, 21] = -302.40427745233603
mu[116, 22] = -302.40392181133296
mu[116, 23] = -54.050243374223136
mu[116, 24] = -54.04995997455812
```

Dual Variables (μ) for SOC Constraints:

```
mu[3, 1, 3] = 45.63893318054958
mu[3, 2, 3] = -45.639004920833266
mu[3, 3, 3] = -23288.521097996185
mu[116, 1, 3] = 47.301492040235075
mu[116, 2, 3] = -47.30164573295515
mu[116, 3, 3] = -11646.998034908327
```

Dual Variables (μ) for SOC Constraints:

```
mu[3, 1] = -51.59309339195896
mu[3, 2] = -51.593131240451726
mu[3, 3] = -51.593343229505564
mu[116, 1] = -53.683406950678155
mu[116, 2] = -53.68349448499136
mu[116, 3] = -53.68391677038888
```

DDP μ value for $t = 1$ is weirdly positive (similar magnitude as other timesteps), even though every other μ in the horizon in DDP as well as every μ in BF are all negative.

Note: It itself is not used in DDP workflow.

All batteries: μ values for all batteries for any particular time step t_0 are pretty much the same, and strongly correlated with the Substation Power Cost (\$/kWh)

$ADS10_{1ph}$
 $T = 24$

Dual Variables Comparison

Terminal SOC Constraint Relaxed

$ADS10_{1ph}$
 $T = 3$

DDP

BruteForced (BF)

DDP

BruteForced (BF)

Dual Variables (μ) for SOC Constraints:

```
mu[5, 1, 3] = 45.17473125550735
mu[5, 2, 3] = -45.174801744427846
mu[5, 3, 3] = -48.239942587228796
mu[5, 4, 3] = -48.24107478564388
mu[5, 5, 3] = -48.241196259159416
mu[5, 6, 3] = -48.241498954100564
mu[5, 7, 3] = -48.241996600606505
mu[5, 8, 3] = -48.242352832568955
mu[5, 9, 3] = -48.242971180373054
mu[5, 10, 3] = -48.243623170596926
mu[5, 11, 3] = -48.243784096347376
mu[5, 12, 3] = -48.243857925739185
mu[5, 13, 3] = -48.243911780436605
mu[5, 14, 3] = -48.243611647078
mu[5, 15, 3] = -48.27767043280479
mu[5, 16, 3] = -301.66935431134704
mu[5, 17, 3] = -301.83189287498675
mu[5, 18, 3] = -301.83806304872076
mu[5, 19, 3] = -301.8524462450138
mu[5, 20, 3] = -301.85952662140807
mu[5, 21, 3] = -301.85603642385746
mu[5, 22, 3] = -302.0049988523133
mu[5, 23, 3] = -48.254762967049885
mu[5, 24, 3] = -48.29407369993339
```

Dual Variables (μ) for SOC Constraints:

```
mu[5, 1] = -51.41802695151902
mu[5, 2] = -51.41802798371779
mu[5, 3] = -51.418031756159145
mu[5, 4] = -51.41804364258754
mu[5, 5] = -51.41808424654045
mu[5, 6] = -51.41815513995484
mu[5, 7] = -51.4182535283495
mu[5, 8] = -51.41837533403347
mu[5, 9] = -51.418522753357806
mu[5, 10] = -51.41869463350428
mu[5, 11] = -51.41889025242323
mu[5, 12] = -51.41911638738373
mu[5, 13] = -51.419386508564976
mu[5, 14] = -51.41972895515192
mu[5, 15] = -51.42029858759361
mu[5, 16] = -286.1783915481126
mu[5, 17] = -286.1841375032672
mu[5, 18] = -286.1869471474352
mu[5, 19] = -286.1883392241525
mu[5, 20] = -286.18841029994184
mu[5, 21] = -286.18839727378145
mu[5, 22] = -286.18829047003777
mu[5, 23] = -49.689599435704075
mu[5, 24] = -46.93556269016344
```

Dual Variables (μ) for SOC Constraints:

```
mu[5, 1, 3] = 45.16617199533143
mu[5, 2, 3] = -45.166242667641114
mu[5, 3, 3] = -301.88039226844836
mu[6, 1, 3] = 45.176080276280665
mu[6, 2, 3] = -45.17622177459524
mu[6, 3, 3] = -301.94542494114074
```

Dual Variables (μ) for SOC Constraints:

```
mu[5, 1] = -45.166820159899466
mu[5, 2] = -45.16681205294272
mu[5, 3] = -45.16679419074594
mu[6, 1] = -45.17720511742513
mu[6, 2] = -45.17718686319634
mu[6, 3] = -45.177151153601116
```

```
mu[6, 1, 3] = 45.185519186176556
mu[6, 2, 3] = -45.18566013739137
mu[6, 3, 3] = -48.25099019620321
mu[6, 4, 3] = -48.2516640313043
mu[6, 5, 3] = -48.25189015114355
mu[6, 6, 3] = -48.25245499800463
mu[6, 7, 3] = -48.253124259218936
mu[6, 8, 3] = -48.25352967582148
mu[6, 9, 3] = -48.25442486213601
mu[6, 10, 3] = -48.25512400205344
mu[6, 11, 3] = -48.25464693444468
mu[6, 12, 3] = -48.254525571498355
mu[6, 13, 3] = -48.25436727134667
mu[6, 14, 3] = -48.25382187859363
mu[6, 15, 3] = -48.287965604466486
mu[6, 16, 3] = -301.7358816667674
mu[6, 17, 3] = -301.9069334415277
mu[6, 18, 3] = -301.9232220706423
mu[6, 19, 3] = -301.9549648463871
mu[6, 20, 3] = -301.9698513713682
mu[6, 21, 3] = -301.9630767020347
mu[6, 22, 3] = -302.10671590494144
mu[6, 23, 3] = -48.26985199583276
mu[6, 24, 3] = -48.30691012037376
```

```
mu[6, 1] = -51.428826857098734
mu[6, 2] = -51.428829535387955
mu[6, 3] = -51.42883806734097
mu[6, 4] = -51.42885968075473
mu[6, 5] = -51.42890964706774
mu[6, 6] = -51.4289884202277
mu[6, 7] = -51.42909216187562
mu[6, 8] = -51.42921803595825
mu[6, 9] = -51.4293672642419
mu[6, 10] = -51.42953647611988
mu[6, 11] = -51.429724291983185
mu[6, 12] = -51.429941174608665
mu[6, 13] = -51.43020098856804
mu[6, 14] = -51.4305355483373
mu[6, 15] = -51.43114433845125
mu[6, 16] = -286.25232736477005
mu[6, 17] = -286.2697146831658
mu[6, 18] = -286.27910995633056
mu[6, 19] = -286.2848194607958
mu[6, 20] = -286.28504449442875
mu[6, 21] = -286.28502061342414
mu[6, 22] = -286.28461256432524
mu[6, 23] = -49.70443686618892
mu[6, 24] = -46.94839189883309
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mu[5, 11, 3] = -48.243784096347376
mu[5, 12, 3] = -48.243857925739185
mu[5, 13, 3] = -48.243911780436605
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mu[5, 21, 3] = -301.85603642385746
mu[5, 22, 3] = -302.0049526046895
mu[5, 23, 3] = -48.29845680868607
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```

```
mu[6, 1, 3] = 45.185519186176556
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mu[6, 5, 3] = -48.25189015114355
mu[6, 6, 3] = -48.25245499800463
mu[6, 7, 3] = -48.253124259218936
mu[6, 8, 3] = -48.25352967582148
mu[6, 9, 3] = -48.25442486213601
mu[6, 10, 3] = -48.25512400205344
mu[6, 11, 3] = -48.25464693444468
mu[6, 12, 3] = -48.254525571498355
mu[6, 13, 3] = -48.25436727134667
mu[6, 14, 3] = -48.25382187859363
mu[6, 15, 3] = -48.287965604466486
mu[6, 16, 3] = -301.7358816667674
mu[6, 17, 3] = -301.9069334415277
mu[6, 18, 3] = -301.9232220706423
mu[6, 19, 3] = -301.9549648463871
mu[6, 20, 3] = -301.9698513713682
mu[6, 21, 3] = -301.9630767020347
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mu[6, 23, 3] = -48.31416276384528
mu[6, 24, 3] = -88822.93735871241
```

Dual Variables (μ) for SOC Constraints:

```
mu[5, 1] = -51.41801881638614
mu[5, 2] = -51.41801979342437
mu[5, 3] = -51.418023465518246
mu[5, 4] = -51.41803524911316
mu[5, 5] = -51.41807640762771
mu[5, 6] = -51.418148149916966
mu[5, 7] = -51.418247324543884
mu[5, 8] = -51.41836979730944
mu[5, 9] = -51.41851776114211
mu[5, 10] = -51.41869015145029
mu[5, 11] = -51.4188863429248
mu[5, 12] = -51.419113183857114
mu[5, 13] = -51.41938428087563
mu[5, 14] = -51.41972842806792
mu[5, 15] = -51.42030280344431
mu[5, 16] = -286.1783903314517
mu[5, 17] = -286.18413492690695
mu[5, 18] = -286.1869461326777
mu[5, 19] = -286.18833870443524
mu[5, 20] = -286.1884103837522
mu[5, 21] = -286.1883973978632
mu[5, 22] = -286.1882908057603
mu[5, 23] = -51.424572090777126
mu[5, 24] = -51.424445284541065
```

```
mu[6, 1] = -51.428827793029235
mu[6, 2] = -51.42883047558444
mu[6, 3] = -51.42883904249744
mu[6, 4] = -51.42886076359697
mu[6, 5] = -51.428910924641634
mu[6, 6] = -51.428989919622026
mu[6, 7] = -51.42909387136665
mu[6, 8] = -51.429219943018055
mu[6, 9] = -51.429369368599055
mu[6, 10] = -51.42953880457677
mu[6, 11] = -51.42972686728392
mu[6, 12] = -51.4299440750174
mu[6, 13] = -51.43020441803826
mu[6, 14] = -51.43054027925132
mu[6, 15] = -51.43115597947386
mu[6, 16] = -286.25232257011146
mu[6, 17] = -286.2697067055625
mu[6, 18] = -286.2791023564704
mu[6, 19] = -286.28481722740173
mu[6, 20] = -286.285043875479
mu[6, 21] = -286.2850199931506
mu[6, 22] = -286.28460871257334
mu[6, 23] = -51.44038325614457
mu[6, 24] = -51.44010590105099
```

Dual Variables (μ) for SOC Constraints:

```
mu[5, 1, 3] = 45.16940459928702
mu[5, 2, 3] = -45.16947508820853
mu[5, 3, 3] = -22553.064658338142
mu[6, 1, 3] = 45.18525275674813
mu[6, 2, 3] = -45.185393707968935
mu[6, 3, 3] = -11279.18311886909
```

Dual Variables (μ) for SOC Constraints:

```
mu[5, 1] = -51.43296947278457
mu[5, 2] = -51.433014630013794
mu[5, 3] = -51.43322659704835
mu[6, 1] = -51.444417850802154
mu[6, 2] = -51.44449826072886
mu[6, 3] = -51.44492057842625
```

DDP μ value for $t = 1$ is weirdly positive (similar magnitude as other timesteps), even though every other μ in the horizon in DDP as well as every μ in BF are all negative.

Note: It itself is not used in DDP workflow.

All batteries: μ values for all batteries for any particular time step t_0 are pretty much the same, and strongly correlated with the Substation Power Cost (\$/kWh)