Scalable Multi-Period Optimal Power Flow
for Active Power Distribution Systems

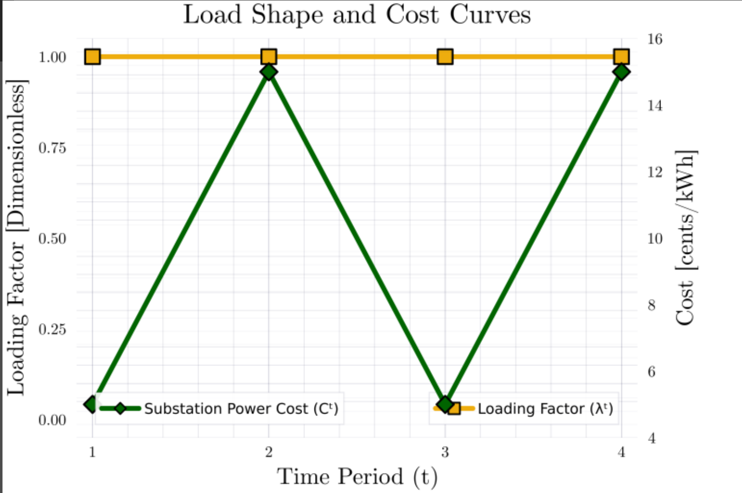
or simply, Scalable MP-OPF in ADS

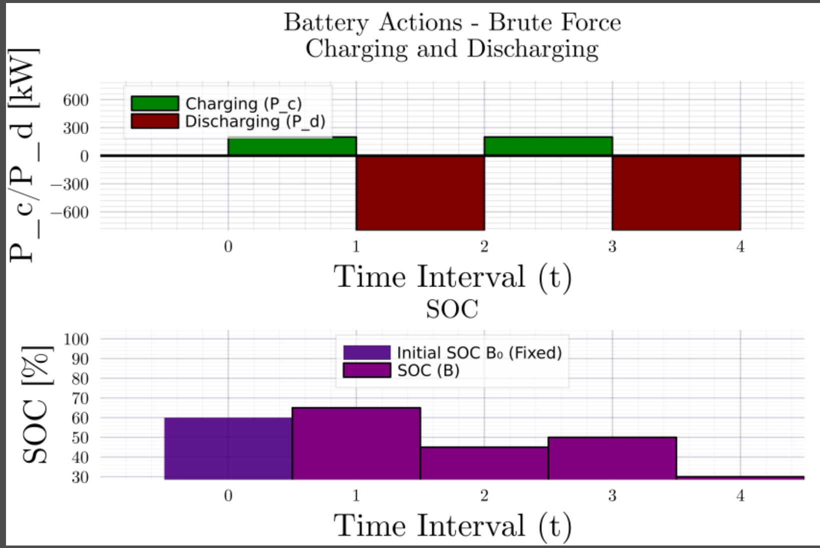
Latest Updates:

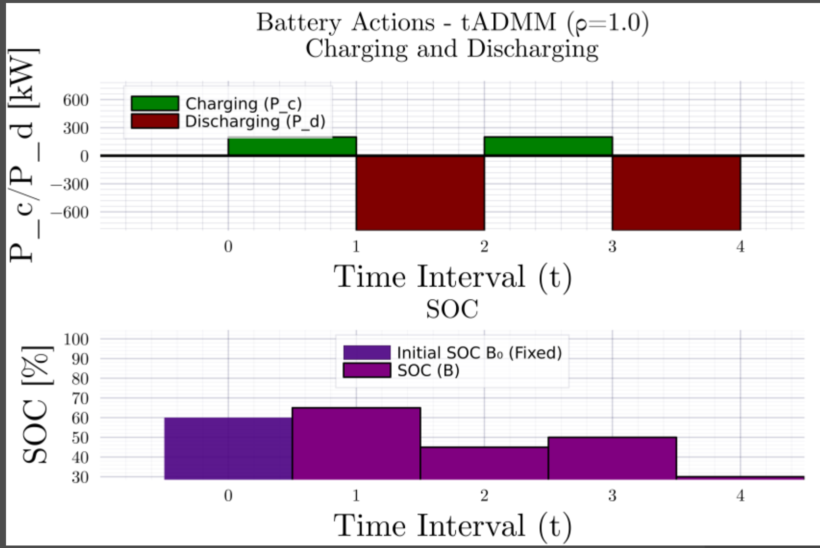
tADMM convergence for Copper Plate MPOPF - Results
tADMM formulation - What was changed
Battery action plots improved for clarity
Convergence plots improved for clarity

Aryan Ritwajeet Jha
Pursuing PhD ECE (Power Systems)
Washington State University

Consensus ADMM for MPOPF $T = 4$







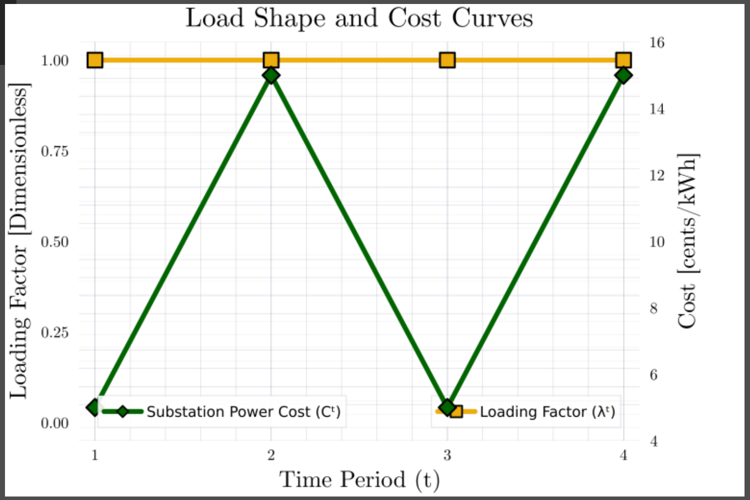
Implementational correction [Courtesy: Rahul]
It was found that my subproblem primal updates were overwriting each other instead of being compiled as part of a set. Fixed.

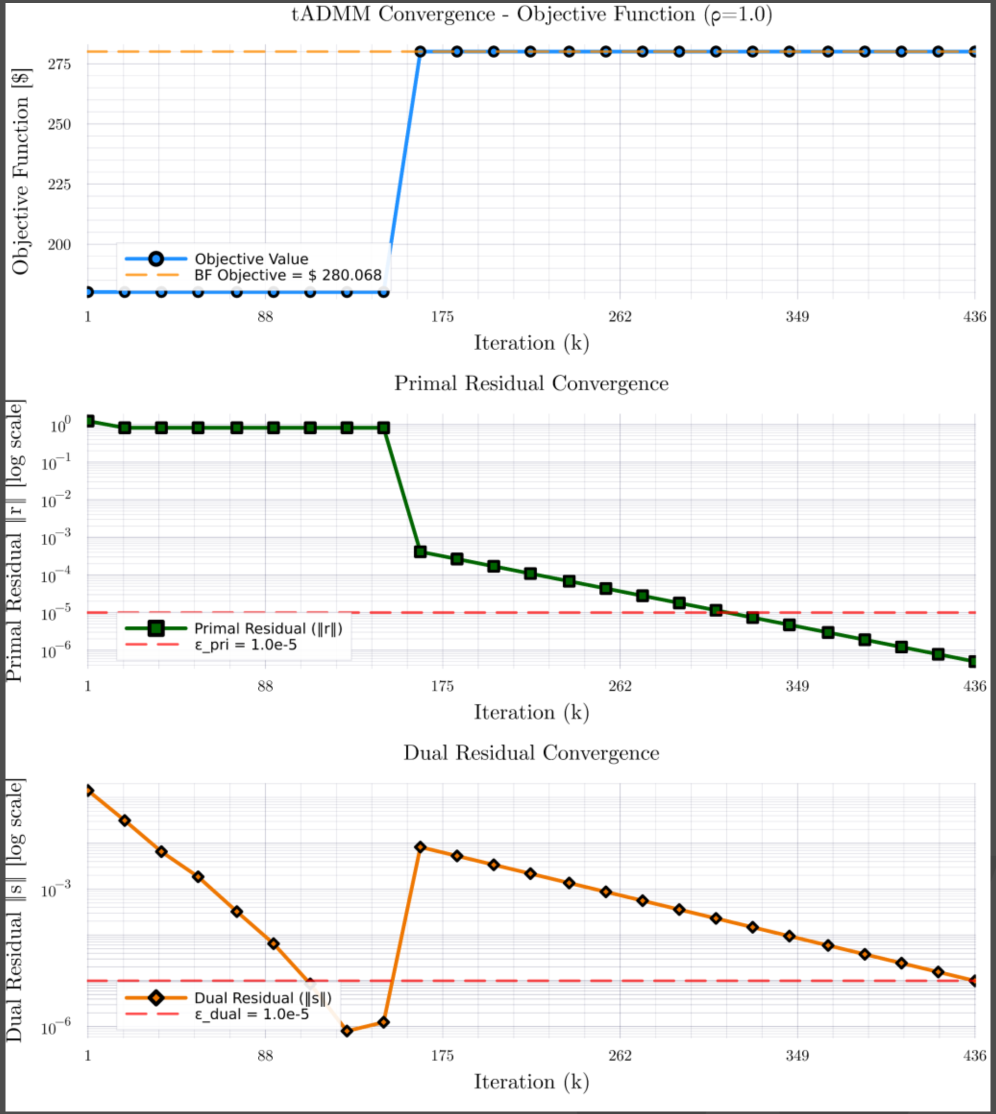
Description

Improved battery action plots (now B_0 - fixed variable is clearly differentiated from B^1 to B^T - the optimization variables)

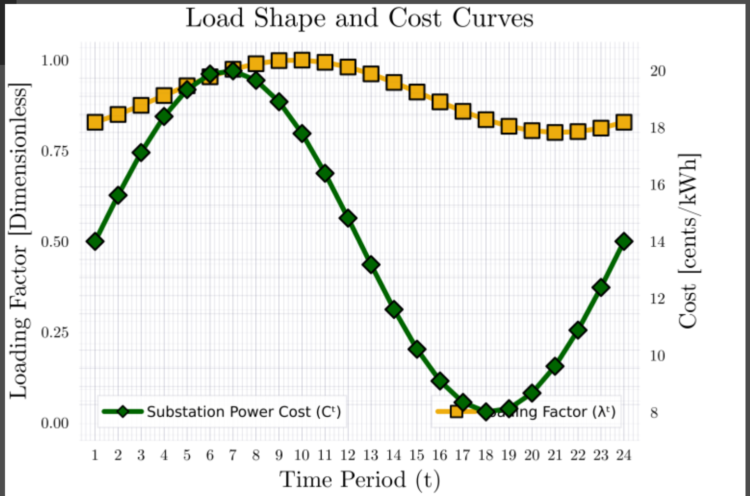
SOC Dynamics Now added for entire trajectory for each subproblem, not just for that subproblem's time-step t_0

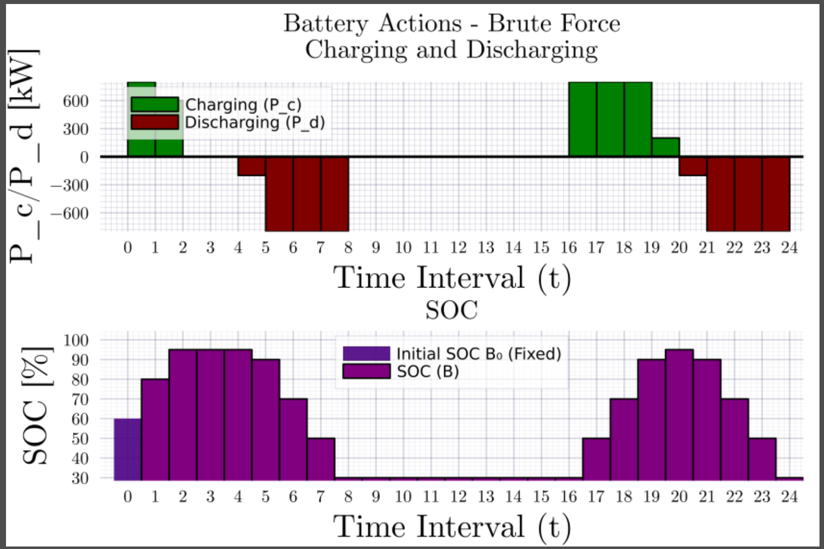
Consensus ADMM for MPOPF $T = 4$

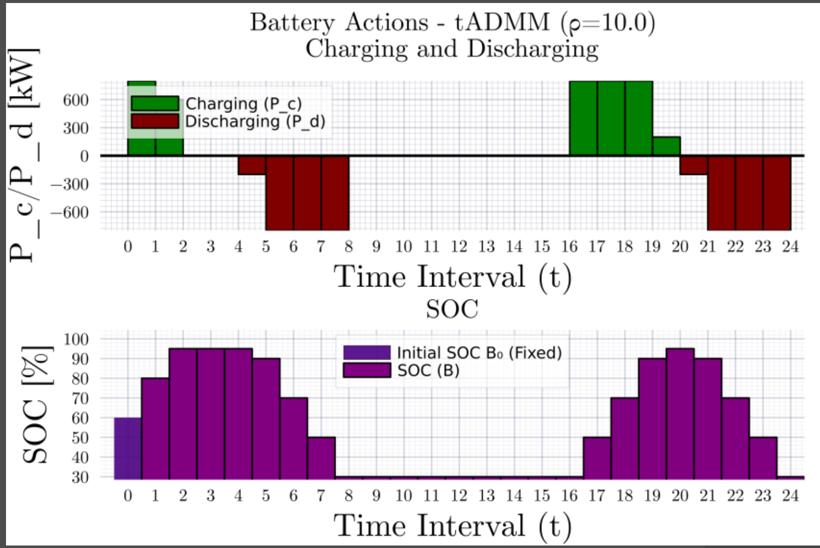




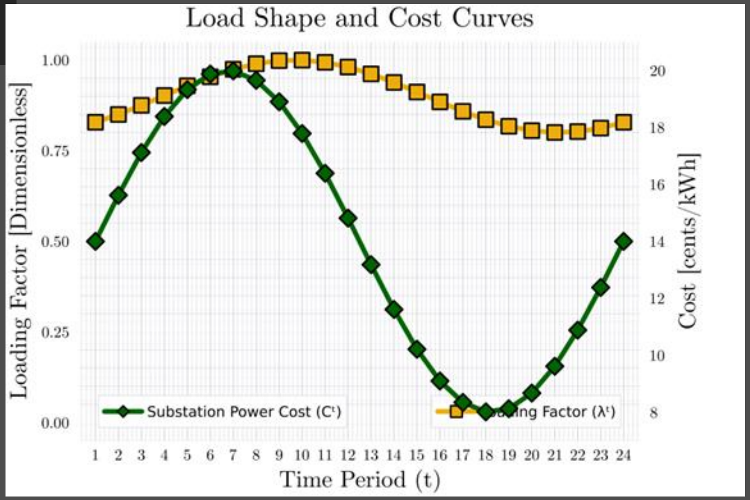
Consensus ADMM for MPOPF $T = 24$

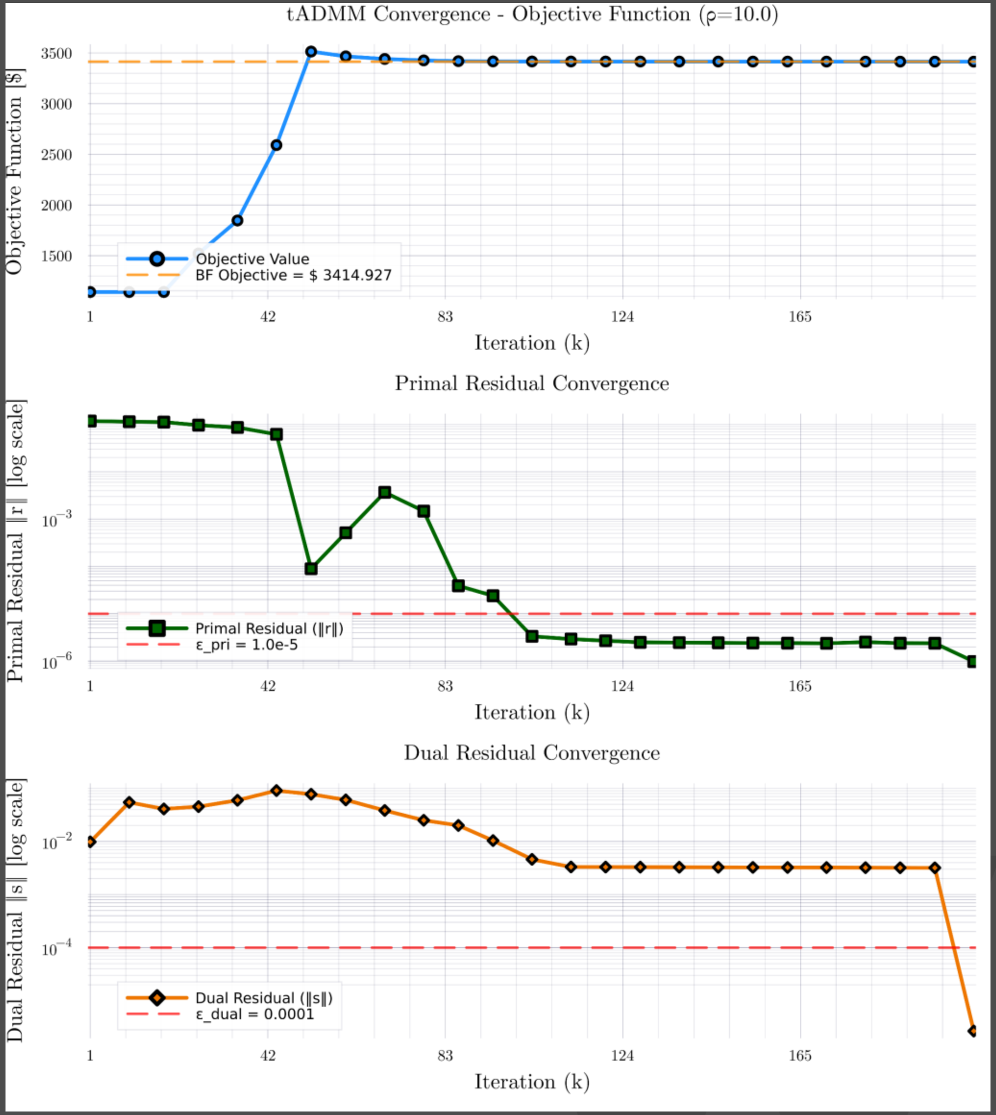






Consensus ADMM for MPOPF $T = 24$





ADMM Optimization Formulation for Copper Plate
(1 Substation, 1 Load, 1 Battery, T timesteps of cost C^t, loading λ^t)

1.3 Step 1: Primal Update (Blue Variables) tADMM Optimization Model

For each subproblem $t_0 \in \{1, 2, \ldots, T\}$:

$$\min_{P_{\text{subs}}^{t_0}, P_B^{t_0}, \mathbf{B}^{t_0}} \quad C^{t_0} \cdot P_{\text{subs}}^{t_0} + C_B \cdot \left(P_B^{t_0}\right)^2 + \frac{\rho}{2} \left\| \mathbf{B}^{t_0} - \hat{\mathbf{B}} + \mathbf{u}^{t_0} \right\|_2^2 \tag{1}$$

Subject to SOC Dynamics for Entire Trajectory:

$$\mathbf{B}^{t_0}[1] = B_0 - P_B^{t_0} \cdot \Delta t \tag{2}$$

$$\mathbf{B}^{t_0}[\mathrm{t}] = \mathbf{B}^{t_0}[\mathrm{t}-1] - P_B^{t_0} \cdot \Delta t, \quad \forall t \in \{2, \ldots, T\} \tag{3}$$

$$P_{\text{subs}}^{t_0} + P_B^{t_0} = P_L[t_0] \tag{4}$$

$$-P_{B,R} \leq P_B^{t_0} \leq P_{B,R} \tag{5}$$

$$\underline{B} \leq \mathbf{B}^{t_0}[\mathrm{t}] \leq \overline{B}, \quad \forall t \in \{1, \ldots, T\} \tag{6}$$

Step 2: Consensus Update (Red Variables)

$$\hat{\mathbf{B}}[\mathrm{t}] = \operatorname{clamp}\left(\frac{1}{T} \sum_{t_0=1}^{T} \left(\mathbf{B}^{t_0}[\mathrm{t}] + \mathbf{u}^{t_0}[\mathrm{t}]\right), \underline{B}, \overline{B} \right) \tag{7}$$

$$\forall t \in \{1, 2, \ldots, T-1\} \tag{8}$$

$$\hat{\mathbf{B}}[\mathrm{T}] = B_{T,\text{target}} \quad \text{(if terminal constraint exists)} \tag{9}$$

Step 3: Dual Update (Green Variables)

$$\mathbf{u}^{t_0}[\mathrm{t}] := \mathbf{u}^{t_0}[\mathrm{t}] + \left(\mathbf{B}^{t_0}[\mathrm{t}] - \hat{\mathbf{B}}[\mathrm{t}]\right) \tag{10}$$

$$\forall t_0 \in \{1, \ldots, T\}, \forall t \in \{1, \ldots, T\} \tag{11}$$

Note that now **only** the **augmented Lagrangian term has** the consensus variable $\widehat{\boldsymbol{B}}$
For battery SOC trajectory, the local subproblem variables are used so that **now each** optimal **subproblem output** $\boldsymbol{B}^{\{t_0\}}$ is **inherently consistent** for every primal update.
Side effect: P_B^t terms now also have to be bounded $\forall t = 1{:}T$ instead of just $t = t_0$

Technically this means that the entire set of battery constraints is solved for by every subproblem.

Q. Isn't putting the entire battery problem in every single subproblem a concern regarding scalability?

A. Not really. All battery constraints are just linear*, specifically their combined strength is $O(n_B * T)$ linear constraints. For my goal of nonlinear MPOPF optimization, this is not a bottleneck.

*Also when the problem does become nonlinear $P_B^2 + q_B^2 \leq S_B^2$ I'll do some thinking on how to use only neighbouring time-step variables in my augmented Lagrangian. The problem wasn't not using all time-step variables, it was solving for all of them despite not accounting for their coupling constraints

Latest slides end here.
Previous slides and/or
References may follow.