WASHINGTON STATE
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Scalable Multi-Period Optimal Power Flow
for Active Power Distribution Systems

or simply, Scalable MP-OPF in ADS

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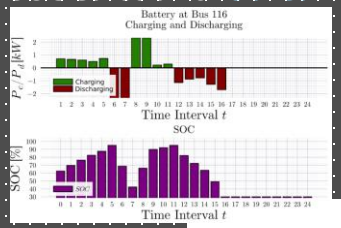
$IEEE123A_{\{1ph\}}$, $T = 24$

DDP Trajectory vs #Forward Passes $[k]$: LinDistFlow Model

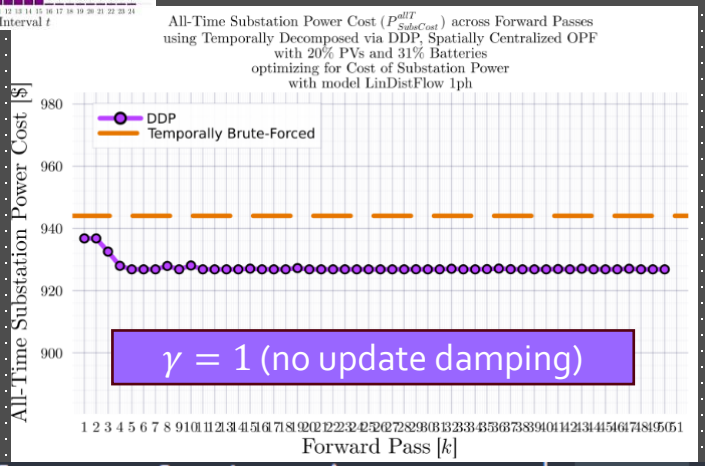
Terminal SOC Constraint Relaxed

DDP

BruteForced (BF)




```
*******
FP45: 42_0 42_0 42_0 42_0 42_0 44_0 42_0 42_0 45_0 42_0 40_0 57_0 57_0 57_0 57_0 57_0 42_0 41_0 40_0 40_0 40_0 39_0 39_0 38_0
FP45: Objective function value update is under the threshold.
FP45: All SOC updates are under the threshold.
FP46: 42_0 42_0 42_0 42_0 42_0 43_0 42_0 44_0 45_0 41_0 42_0 57_0 57_0 57_0 57_0 57_0 42_0 41_0 40_0 40_0 40_0 40_0 39_0 38_0
FP46: Objective function value update is under the threshold.
FP46: All SOC updates are under the threshold.
FP47: 42_0 42_0 42_0 42_0 42_0 42_0 43_0 45_0 42_0 41_0 57_0 57_0 57_0 57_0 57_0 57_0 42_0 41_0 41_0 41_0 41_0 40_0 39_0 39_0
*******
FP48: 42_0 42_0 42_0 42_0 42_0 43_0 44_0 43_0 41_0 50_0 42_0 57_0 57_0 57_0 57_0 57_0 42_0 41_0 41_0 41_0 40_0 40_0 39_0 38_0
*******
FP49: 42_0 42_0 42_0 42_0 42_0 44_0 44_0 41_0 45_0 50_0 42_0 57_0 57_0 57_0 57_0 57_0 42_0 42_0 42_0 41_0 40_0 40_0 39_0 38_0
FP49: Objective function value update is under the threshold.
FP49: All SOC updates are under the threshold.
FP50: 42_0 42_0 42_0 42_0 42_0 44_0 42_0 43_0 50_0 42_0 42_0 57_0 57_0 57_0 57_0 57_0 42_0 42_0 41_0 40_0 40_0 39_0 38_0 38_0
Maximum iterations reached: 50
Maximum iterations reached!
```



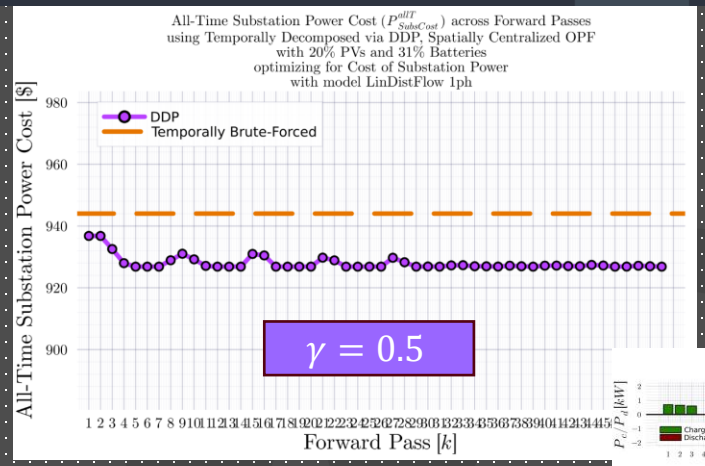
$\gamma = 1$ (no update damping)

```
μ[3, 1] = 42_26
μ[3, 2] = 42_26
μ[3, 3] = 42_26
μ[3, 4] = 42_26
μ[3, 5] = 42_26
μ[3, 6] = 42_27
μ[3, 7] = 42_27
μ[3, 8] = 42_26
μ[3, 9] = 42_26
μ[3, 10] = 42_26
μ[3, 11] = 42_26
μ[3, 12] = 56_85
μ[3, 13] = 56_85
μ[3, 14] = 56_85
μ[3, 15] = 56_85
μ[3, 16] = 56_85
μ[3, 17] = 41_73
μ[3, 18] = 41_12
μ[3, 19] = 40_66
μ[3, 20] = 40_25
μ[3, 21] = 39_86
μ[3, 22] = 39_45
μ[3, 23] = 39_0
μ[3, 24] = 38_39
```

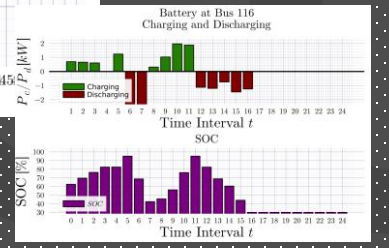
```
10. Horizon Total Cost of Substation Power: $ 926.85
```

$\gamma = 1$ (no update damping)

```
*******
FP45: 42_0 42_0 42_0 42_0 42_0 42_0 42_0 42_0 42_0 57_0 42_0 57_0 57_0 57_0 57_0 57_0 42_0 41_0 38_0 39_0 38_0 38_0 38_0 38_0
*******
FP46: 42_0 42_0 42_0 42_0 42_0 42_0 42_0 42_0 57_0 42_0 42_0 57_0 57_0 57_0 57_0 57_0 41_0 39_0 39_0 41_0 41_0 41_0 41_0 40_0
*******
FP47: 42_0 42_0 42_0 42_0 42_0 42_0 42_0 57_0 42_0 42_0 42_0 57_0 57_0 57_0 57_0 57_0 41_0 41_0 42_0 42_0 41_0 41_0 40_0 38_0
*******
FP48: 42_0 42_0 42_0 42_0 42_0 42_0 57_0 42_0 42_0 42_0 42_0 57_0 57_0 57_0 57_0 57_0 42_0 42_0 42_0 42_0 42_0 40_0 38_0 38_0
*******
FP49: 42_0 42_0 42_0 42_0 42_0 57_0 42_0 42_0 42_0 42_0 42_0 57_0 57_0 57_0 57_0 57_0 42_0 42_0 42_0 42_0 41_0 38_0 38_0 38_0
*******
FP50: 42_0 42_0 42_0 42_0 42_0 42_0 42_0 42_0 42_0 42_0 42_0 57_0 57_0 57_0 57_0 57_0 42_0 42_0 42_0 41_0 38_0 39_0 38_0 38_0
Maximum iterations reached: 50
Maximum iterations reached!
```



$\gamma = 0.5$



$\gamma = 0.5$

$k_{Max} = 25$

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$IEEE123A_{\{1ph\}}$, $T = 3$

DDP Trajectory vs #Forward Passes $[k]$

Terminal SOC Constraint Relaxed

DDP

$FP(k): \mu_k(t=1), \mu_k(t=2), \mu_k(t=3)$

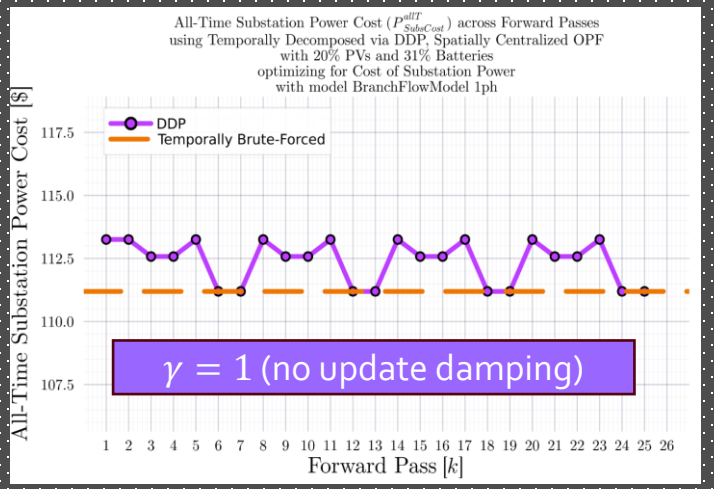
BruteForced (BF)

```
FP01: 0_0001 59_0 42_0
FP2: α_fpi = 1.0
FP02: 30_0 59_0 42_0
FP3: α_fpi = 1.0
FP03: 59_0 42_0 0_0009
*******
FP4: α_fpi = 1.0
FP04: 50_0 21_0 0_0003
FP5: α_fpi = 1.0
FP05: 31_0 59_0 42_0
*******
FP6: α_fpi = 1.0
FP06: 40_0 21_0 39_0
*******
FP7: α_fpi = 1.0
FP07: 40_0 41_0 39_0
FP8: α_fpi = 1.0
FP08: 31_0 59_0 42_0
*******
FP9: α_fpi = 1.0
FP09: 50_0 41_0 0_0003
```

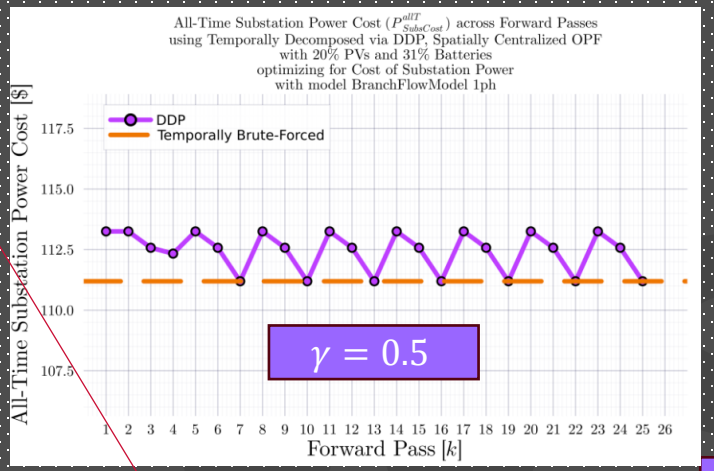
$\gamma = 1$ (no update damping)

```
FP01: 0_0001 59_0 42_0
FP2: α_fpi = 1.0
FP02: 30_0 59_0 42_0
FP3: α_fpi = 0.5
FP03: 59_0 42_0 0_0003
*******
FP4: α_fpi = 0.25
FP04: 45_0 8_4 0_0008
*******
FP5: α_fpi = 0.125
FP05: 12_0 59_0 42_0
*******
FP6: α_fpi = 0.0625
FP06: 56_0 39_0 0_0003
*******
FP7: α_fpi = 0.03125
FP07: 40_0 1_3 39_0
*******
FP8: α_fpi = 0.015625
FP08: 1_9 59_0 42_0
*******
FP9: α_fpi = 0.0078125
FP09: 59_0 42_0 0_0003
```

$\gamma = 0.5$



$\gamma = 1$ (no update damping)



$\gamma = 0.5$

```
μ[3, 1] = 38_08
μ[3, 2] = 38_08
μ[3, 3] = 38_08
```

$$\mu_{used} = \frac{\mu^{k-1} + \alpha * \mu^{k-2}}{1 + \alpha}$$

$$\alpha^k = \alpha_0 * \gamma^{k-2}$$

$$\alpha_0 = 1.0$$

Fluctuation of μ_k^T value between 0 and a 'regular' non-zero value

$k_{Max} = 25$

DDP Trajectory vs #Forward Passes $[k]$

Terminal SOC Constraint Relaxed

$IEEE123A_{\{1ph\}}$, $T = 6$

DDP

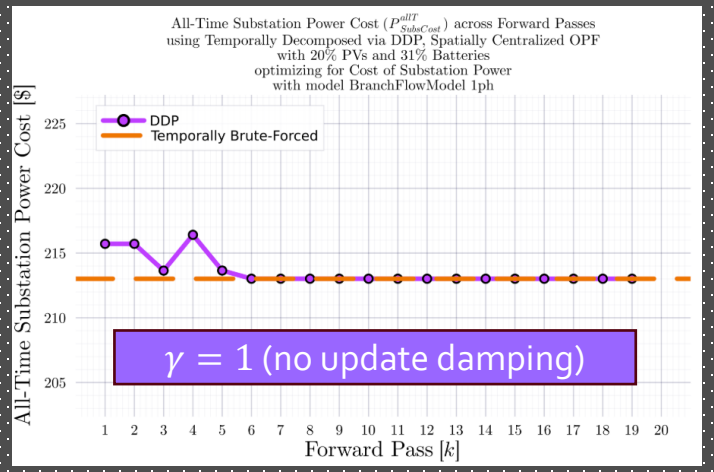
BruteForced (BF)

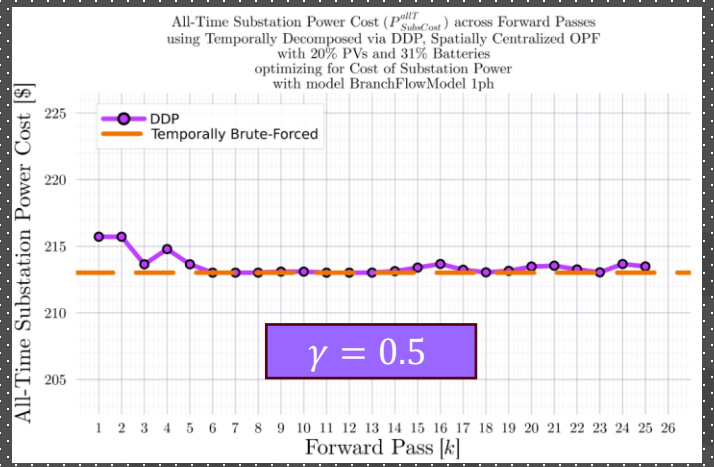
```
FP01: 0_0001 39_0 63_0 42_0 42_0 42_0
FP2: α_fpi = 1.0
FP02: 20_0 39_0 63_0 42_0 42_0 42_0
FP3: α_fpi = 1.0
FP03: 39_0 63_0 42_0 42_0 42_0 38_0
*******
FP4: α_fpi = 1.0
FP04: 51_0 44_0 42_0 42_0 40_0 0_0001
*******
FP5: α_fpi = 1.0
FP05: 54_0 42_0 42_0 41_0 19_0 39_0
*******
FP6: α_fpi = 1.0
FP06: 43_0 42_0 41_0 39_0 42_0 42_0
*******
FP7: α_fpi = 1.0
FP07: 42_0 42_0 40_0 39_0 43_0 41_0
FP8: α_fpi = 1.0
FP08: 42_0 41_0 39_0 42_0 42_0 39_0
FP9: α_fpi = 1.0
FP09: 41_0 40_0 41_0 42_0 40_0 39_0
```

$\gamma = 1$ (no update damping)

```
FP01: 0_0001 39_0 63_0 42_0 42_0 42_0
FP2: α_fpi = 1.0
FP02: 20_0 39_0 63_0 42_0 42_0 42_0
FP3: α_fpi = 0.5
FP03: 39_0 63_0 42_0 42_0 42_0 38_0
*******
FP4: α_fpi = 0.25
FP04: 58_0 44_0 42_0 42_0 39_0 38_0
*******
FP5: α_fpi = 0.125
FP05: 46_0 42_0 42_0 39_0 38_0 40_0
*******
FP6: α_fpi = 0.0625
FP06: 42_0 42_0 40_0 39_0 42_0 41_0
*******
FP7: α_fpi = 0.03125
FP07: 42_0 40_0 39_0 42_0 41_0 39_0
FP8: α_fpi = 0.015625
FP08: 40_0 39_0 42_0 41_0 40_0 40_0
FP9: α_fpi = 0.0078125
FP09: 39_0 42_0 41_0 40_0 40_0 39_0
```

$\gamma = 0.5$






```
μ[3, 1] = 38_1
μ[3, 2] = 38_1
μ[3, 3] = 38_1
μ[3, 4] = 38_1
μ[3, 5] = 38_1
μ[3, 6] = 38_1
```

$k_{Max} = 25$

DDP Trajectory vs #Forward Passes $[k]$

Terminal SOC Constraint Relaxed

$IEEE123A_{\{1ph\}}$, $T = 24$

DDP

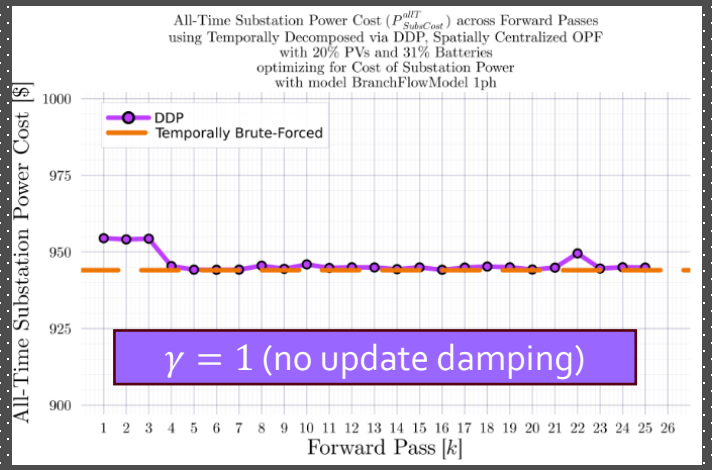
```
FP01: 0_0001 39_0 41_0 41_0 42_0 62_0 62_0 42_0 42_0 42_0 42_0 63_0 63_0 63_0 63_0 63_0 42_0 42_0 42_0 43_0 43_0 42_0 42_0 42_0
FP2: α_fpi = 1.0
FP02: 20_0 39_0 42_0 41_0 42_0 62_0 62_0 42_0 42_0 42_0 42_0 63_0 63_0 63_0 63_0 63_0 42_0 42_0 42_0 43_0 43_0 42_0 42_0 42_0
*******
FP3: α_fpi = 1.0
FP03: 39_0 41_0 41_0 42_0 62_0 62_0 42_0 42_0 42_0 42_0 63_0 63_0 63_0 63_0 63_0 42_0
 42_0 42_0 43_0 43_0 42_0 42_0 42_0 39_0
*******
FP4: α_fpi = 1.0
FP04: 40_0 42_0 42_0 52_0 44_0 52_0 42_0 42_0 42_0 52_0 63_0 63_0 63_0 63_0 52_0 42_0 42_0 43_0 43_0 43_0 42_0 42_0 40_0 39_0
*******
FP5: α_fpi = 1.0
FP05: 41_0 41_0 47_0 44_0 41_0 42_0 42_0 42_0 47_0 63_0 44_0 61_0 62_0 58_0 42_0 59_0 43_0 43_0 43_0 43_0 43_0 43_0 43_0 41_0
*******
FP6: α_fpi = 1.0
FP04: 40_0 42_0 42_0 52_0 44_0 52_0 42_0 42_0 42_0 52_0 63_0 63_0 63_0 63_0 52_0 42_0 42_0 43_0 43_0 43_0 42_0 42_0 40_0 39_0
FP05: 41_0 41_0 47_0 44_0 41_0 42_0 42_0 42_0 47_0 63_0 44_0 61_0 62_0 58_0 42_0 59_0 43_0 43_0 43_0 43_0 43_0 43_0 43_0 41_0
*******
FP6: α_fpi = 1.0
FP06: 42_0 44_0 48_0 43_0 44_0 42_0 42_0 45_0 58_0 45_0 42_0 61_0 60_0 47_0 50_0 59_0 44_0 43_0 43_0 43_0 43_0 42_0 41_0 40_0
*******
FP7: α_fpi = 1.0
FP07: 43_0 47_0 43_0 43_0 42_0 42_0 43_0 52_0 54_0 43_0 45_0 60_0 52_0 46_0 59_0 59_0 44_0 44_0 44_0 43_0 42_0 42_0 41_0 40_0
*******
FP8: α_fpi = 1.0
FP08: 46_0 44_0 41_0 41_0 40_0 43_0 48_0 56_0 44_0 43_0 44_0 56_0 47_0 59_0 63_0 62_0 44_0 44_0 44_0 43_0 42_0 41_0 43_0 41_0
*******
FP9: α_fpi = 1.0
FP09: 46_0 42_0 42_0 41_0 42_0 46_0 54_0 49_0 43_0 45_0 58_0 50_0 53_0 61_0 61_0 59_0 44_0 44_0 43_0 42_0 42_0 42_0 40_0 40_0
```

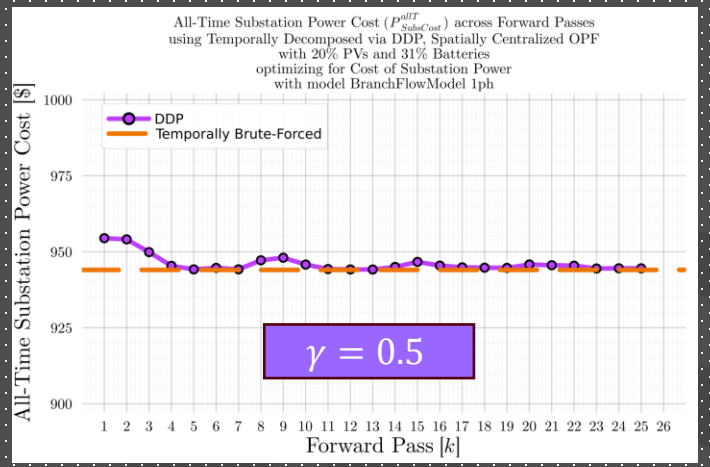
$\gamma = 1$ (no update damping)

```
FP01: 0_0001 39_0 41_0 41_0 42_0 62_0 62_0 42_0 42_0 42_0 42_0 63_0 63_0 63_0 63_0 63_0 42_0 42_0 42_0 43_0 43_0 42_0 42_0 42_0
FP2: α_fpi = 1.0
FP02: 20_0 39_0 42_0 41_0 42_0 62_0 62_0 42_0 42_0 42_0 42_0 63_0 63_0 63_0 63_0 63_0 42_0 42_0 42_0 43_0 43_0 42_0 42_0 42_0
*******
FP3: α_fpi = 0.5
FP03: 39_0 41_0 41_0 42_0 62_0 62_0 42_0 42_0 42_0 42_0 63_0 63_0 63_0 63_0 63_0 42_0 42_0 42_0 43_0 43_0 42_0 42_0 42_0 39_0
*******
FP4: α_fpi = 0.25
FP04: 41_0 41_0 42_0 58_0 44_0 46_0 42_0 42_0 42_0 59_0 63_0 63_0 63_0 63_0 46_0 42_0 42_0 43_0 43_0 42_0 42_0 42_0 40_0 40_0
*******
FP5: α_fpi = 0.125
FP05: 41_0 42_0 56_0 44_0 43_0 42_0 42_0 42_0 57_0 63_0 44_0 61_0 61_0 48_0 42_0 59_0 43_0 43_0 43_0 43_0 43_0 42_0 42_0 41_0
*******
FP6: α_fpi = 0.0625
FP06: 42_0 55_0 44_0 41_0 41_0 42_0 42_0 56_0 63_0 44_0 42_0 60_0 49_0 42_0 59_0 61_0 44_0 44_0 43_0 43_0 42_0 43_0 42_0 40_0
*******
FP7: α_fpi = 0.03125
FP07: 55_0 44_0 41_0 40_0 41_0 42_0 55_0 62_0 45_0 43_0 44_0 49_0 43_0 59_0 62_0 60_0 44_0 44_0 44_0 43_0 44_0 43_0 41_0 41_0
*******
FP8: α_fpi = 0.015625
FP08: 44_0 41_0 40_0 41_0 42_0 55_0 62_0 45_0 43_0 44_0 44_0 43_0 58_0 62_0 60_0 58_0 44_0 44_0 43_0 44_0 43_0 41_0 41_0 40_0
*******
FP9: α_fpi = 0.0078125
FP09: 41_0 40_0 41_0 42_0 55_0 62_0 45_0 43_0 44_0 44_0 43_0 58_0 62_0 60_0 58_0 59_0 44_0 43_0 44_0 44_0 41_0 42_0 40_0 40_0
```

$\gamma = 0.5$

BruteForced (BF)






```
μ[3, 1] = 42_54
μ[3, 2] = 42_54
μ[3, 3] = 42_54
μ[3, 4] = 42_54
μ[3, 5] = 42_54
μ[3, 6] = 42_61
μ[3, 7] = 42_61
μ[3, 8] = 42_61
μ[3, 9] = 42_61
μ[3, 10] = 42_61
μ[3, 11] = 42_61
μ[3, 12] = 57_13
μ[3, 13] = 57_13
μ[3, 14] = 57_13
μ[3, 15] = 57_13
μ[3, 16] = 57_13
μ[3, 17] = 42_02
μ[3, 18] = 41_42
μ[3, 19] = 40_96
μ[3, 20] = 40_56
μ[3, 21] = 40_16
μ[3, 22] = 39_75
μ[3, 23] = 39_28
μ[3, 24] = 38_65
```

$k_{Max} = 25$

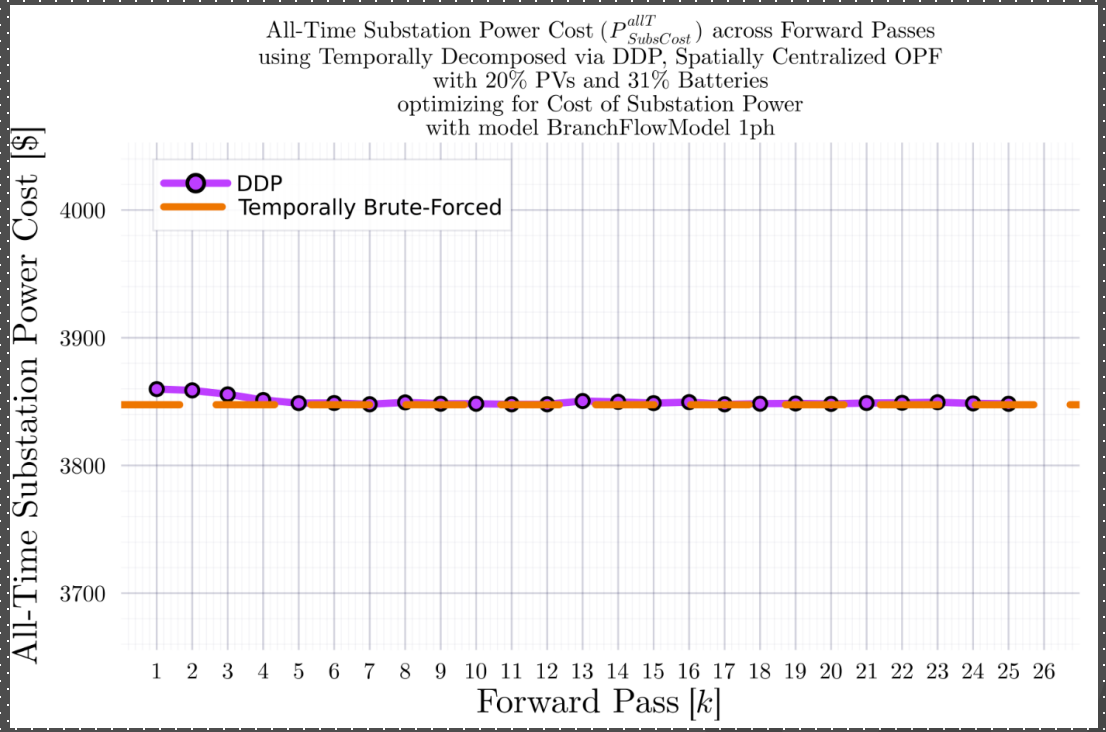
$IEEE123A_{\{1ph\}}$, $T = 96$

DDP Trajectory vs #Forward Passes $[k]$

Terminal SOC Constraint Relaxed

DDP

BruteForced (BF)



$k_{Max} = 25$

$IEEE123B_{\{1ph\}}$, $T = 96$

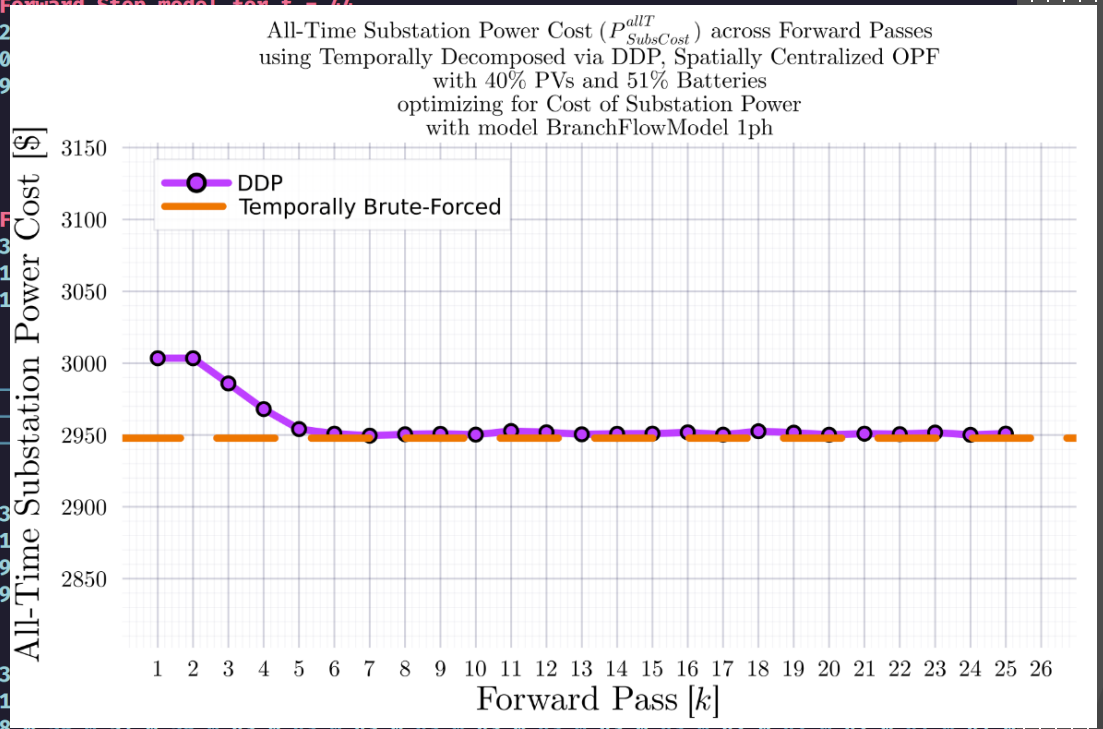
DDP Trajectory vs #Forward Passes $[k]$

Terminal SOC Constraint Relaxed

DDP

BruteForced (BF)

```
FP21: α_fpi = 1.0
Forward Pass k_ddp = 21 : Optimal solution not found for Forward Step model for t = 44
FP21: 41_0 41_0 41_0 41_0 41_0 41_0 42_0 43_0 43_0 42_0 42
 42_0 42_0 42_0 42_0 43_0 46_0 48_0 44_0 41_0 39_0 39_0 40
 58_0 56_0 54_0 55_0 59_0 61_0 60_0 59_0 58_0 58_0 58_0 59
 41_0 41_0 41_0 41_0 40_0 40_0 40_0
 41_0 42_0 41_0 41_0 40_0 39_0
*******
FP22: α_fpi = 1.0
Forward Pass k_ddp = 22 : Optimal solution not found for F
FP22: 41_0 41_0 41_0 41_0 41_0 42_0 42_0 43_0 43_0 42_0 43
 43_0 42_0 42_0 43_0 45_0 47_0 46_0 42_0 40_0 39_0 40_0 41
 57_0 54_0 55_0 58_0 61_0 60_0 59_0 58_0 58_0 58_0 58_0 61
 41_0 41_0 41_0 41_0 40_0 41_0 41_0
 41_0 41_0 41_0 40_0 40_0 39_0
43_0 42_0 42_0 43_0 44_0 47_0 47_0 43_0 40_0 39_0 40_0 40_
57_0 55_0 54_0 57_0 60_0 60_0 59_0 58_0 58_0 58_0 58_0 60_
41_0 41_0 41_0 41_0 40_0 40_0 41_0 41_0 41_0 41_0 40_0 40_
*******
FP24: α_fpi = 1.0
FP24: 41_0 41_0 41_0 41_0 42_0 43_0 43_0 42_0 42_0 43_0 43
 42_0 43_0 44_0 46_0 47_0 43_0 40_0 39_0 39_0 40_0 41_0 41
 54_0 56_0 59_0 61_0 60_0 59_0 58_0 58_0 58_0 59_0 61_0 59
 41_0 41_0 40_0 40_0 41_0 41_0 41_0 41_0 41_0 40_0 39_0 39
*******
FP25: α_fpi = 1.0
FP25: 41_0 41_0 41_0 42_0 42_0 43_0 43_0 42_0 43_0 43_0 43
 43_0 43_0 45_0 47_0 45_0 42_0 40_0 39_0 40_0 41_0 41_0 41
 55_0 58_0 60_0 60_0 59_0 58_0 58_0 58_0 58_0 60_0 60_0 58_0
 41_0 41_0 40_0 41_0 41_0 41_0 41_0 41_0 40_0 40_0 39_0 39_0 39_0
```




```
μ[3, 1] = 42_39
μ[3, 2] = 42_39
μ[3, 3] = 42_39
μ[3, 4] = 42_39
μ[3, 5] = 42_39
μ[3, 6] = 42_39
μ[3, 7] = 42_39
μ[3, 8] = 42_39
μ[3, 9] = 42_39
μ[3, 10] = 42_39
μ[3, 11] = 42_39
μ[3, 12] = 42_4
μ[3, 13] = 42_4
μ[3, 14] = 42_4
μ[3, 15] = 42_4
μ[3, 16] = 42_4
μ[3, 17] = 42_4
μ[3, 18] = 42_4
μ[3, 19] = 42_4
μ[3, 20] = 42_4
μ[3, 21] = 42_4
μ[3, 22] = 57_09
μ[3, 23] = 57_09
μ[3, 24] = 57_09
μ[3, 25] = 57_09
μ[3, 26] = 42_37
μ[3, 27] = 42_37
μ[3, 28] = 42_37
μ[3, 29] = 42_37
μ[3, 30] = 42_37
μ[3, 31] = 42_37
μ[3, 32] = 42_37
μ[3, 33] = 42_36
μ[3, 34] = 42_36
μ[3, 35] = 42_36
μ[3, 36] = 42_36
μ[3, 37] = 42_36
μ[3, 38] = 42_36
μ[3, 39] = 42_36
μ[3, 40] = 42_36
μ[3, 41] = 42_36
μ[3, 42] = 42_36
μ[3, 43] = 42_36
μ[3, 44] = 42_36
μ[3, 45] = 42_36
μ[3, 46] = 42_36
μ[3, 47] = 57_13
μ[3, 48] = 57_13
```

```
μ[3, 49] = 57_13
μ[3, 50] = 57_14
μ[3, 51] = 57_14
μ[3, 52] = 57_14
μ[3, 53] = 57_14
μ[3, 54] = 57_14
μ[3, 55] = 57_14
μ[3, 56] = 57_14
μ[3, 57] = 57_14
μ[3, 58] = 57_14
μ[3, 59] = 57_14
μ[3, 60] = 57_14
μ[3, 61] = 57_14
μ[3, 62] = 57_14
μ[3, 63] = 57_14
μ[3, 64] = 57_14
μ[3, 65] = 57_14
μ[3, 66] = 57_14
μ[3, 67] = 57_14
μ[3, 68] = 57_14
μ[3, 69] = 57_14
μ[3, 70] = 57_14
μ[3, 71] = 42_13
μ[3, 72] = 41_86
μ[3, 73] = 41_66
μ[3, 74] = 41_5
μ[3, 75] = 41_35
μ[3, 76] = 41_22
μ[3, 77] = 41_09
μ[3, 78] = 40_97
μ[3, 79] = 40_85
μ[3, 80] = 40_74
μ[3, 81] = 40_63
μ[3, 82] = 40_52
μ[3, 83] = 40_41
μ[3, 84] = 40_3
μ[3, 85] = 40_19
μ[3, 86] = 40_07
μ[3, 87] = 39_96
μ[3, 88] = 39_84
μ[3, 89] = 39_72
μ[3, 90] = 39_6
μ[3, 91] = 39_47
μ[3, 92] = 39_33
μ[3, 93] = 39_18
μ[3, 94] = 39_0
μ[3, 95] = 38_8
μ[3, 96] = 38_49
```

$\gamma = 1$ (no update damping)

$\gamma = 1$ (no update damping)

$k_{Max} = 25$

DDP Trajectory vs #Forward Passes $[k]$

Terminal SOC Constraint Relaxed

$IEEE123B_{\{1ph\}}$, $T = 24$

DDP

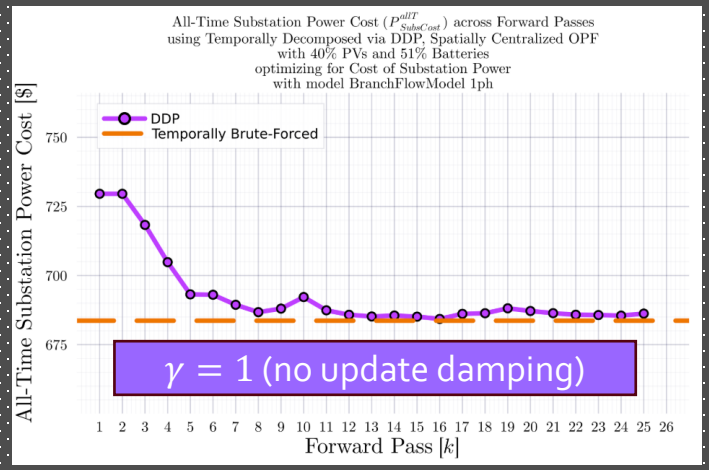
```
FP01: 0_0 39_0 41_0 41_0 41_0 61_0 61_0 41_0 41_0 41_0 41_0 61_0 61_0 61_0 61_0 61_0 41_0 41_0 42_0 42_0 42_0 42_0 41_0 41_0
FP2: α_fpi = 1.0
FP02: 19_0 39_0 41_0 41_0 41_0 60_0 61_0 41_0 41_0 41_0 41_0 61_0 61_0 60_0 61_0 61_0 41_0 41_0 42_0 42_0 42_0 42_0 41_0 41_0
FP3: α_fpi = 1.0
FP03: 39_0 41_0 41_0 41_0 61_0 61_0 41_0 41_0 41_0 41_0 61_0 61_0 61_0 61_0 61_0 41_0 41_0 42_0 42_0 42_0 42_0 41_0 41_0 39_0
*******
FP4: α_fpi = 1.0
FP04: 40_0 41_0 41_0 51_0 44_0 51_0 41_0 41_0 41_0 51_0 61_0 61_0 61_0 61_0 51_0 41_0 41_0 42_0 42_0 42_0 41_0 41_0 40_0 39_0
*******
FP5: α_fpi = 1.0
Forward Pass k_ddp = 5 : Optimal solution not found for Forward Step model for t = 23
FP05: 41_0 41_0 46_0 44_0 41_0 41_0 41_0 41_0 46_0 61_0 43_0 59_0 60_0 56_0 41_0 41_0 42_0 42_0 42_0 42_0 42_0 41_0 39_0 39_0
*******
FP6: α_fpi = 1.0
FP06: 41_0 44_0 44_0 40_0 42_0 41_0 41_0 43_0 56_0 52_0 43_0 59_0 58_0 46_0 41_0 42_0 42_0 42_0 42_0 42_0 41_0 40_0 39_0 39_0
*******
FP7: α_fpi = 1.0
FP07: 42_0 45_0 42_0 41_0 41_0 41_0 42_0 51_0 57_0 43_0 43_0 58_0 51_0 41_0 41_0 58_0 42_0 42_0 42_0 42_0 41_0 40_0 41_0 40_0
*******
FP8: α_fpi = 1.0
Forward Pass k_ddp = 8 : Optimal solution not found for Forward Step model for t = 3
FP08: 44_0 43_0 41_0 41_0 41_0 41_0 47_0 56_0 47_0 43_0 43_0 55_0 44_0 41_0 58_0 60_0 43_0 42_0 42_0 41_0 40_0 41_0 41_0 40_0
*******
FP9: α_fpi = 1.0
FP09: 44_0 41_0 41_0 41_0 41_0 45_0 58_0 52_0 43_0 43_0 56_0 47_0 41_0 50_0 59_0 58_0 43_0 43_0 42_0 42_0 41_0 42_0 41_0 39_0
```

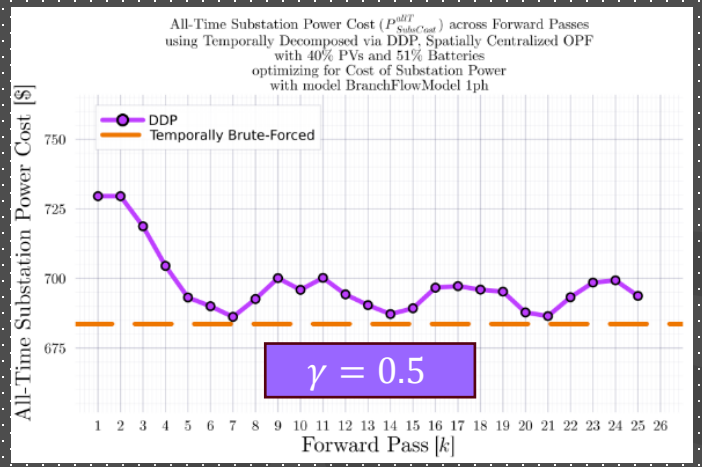
$\gamma = 1$ (no update damping)

```
FP01: 0_0 39_0 41_0 41_0 41_0 61_0 61_0 41_0 41_0 41_0 41_0 61_0 61_0 61_0 61_0 61_0 41_0 41_0 42_0 42_0 42_0 42_0 41_0 41_0
FP2: α_fpi = 1.0
FP02: 19_0 39_0 41_0 41_0 41_0 60_0 61_0 41_0 41_0 41_0 41_0 61_0 61_0 60_0 61_0 61_0 41_0 41_0 42_0 42_0 42_0 42_0 41_0 41_0
FP3: α_fpi = 0.5
FP03: 39_0 41_0 41_0 41_0 61_0 61_0 41_0 41_0 41_0 41_0 61_0 61_0 61_0 61_0 61_0 41_0 41_0 42_0 42_0 42_0 42_0 41_0 41_0 39_0
*******
FP4: α_fpi = 0.25
FP04: 41_0 41_0 41_0 57_0 44_0 45_0 41_0 41_0 41_0 57_0 61_0 61_0 61_0 61_0 45_0 41_0 41_0 42_0 42_0 42_0 41_0 41_0 39_0 39_0
*******
FP5: α_fpi = 0.125
FP05: 41_0 41_0 55_0 44_0 42_0 41_0 41_0 41_0 55_0 61_0 43_0 59_0 60_0 47_0 41_0 41_0 42_0 42_0 42_0 42_0 41_0 39_0 39_0 39_0
*******
FP6: α_fpi = 0.0625
FP06: 41_0 54_0 44_0 41_0 41_0 41_0 41_0 54_0 61_0 43_0 43_0 59_0 48_0 41_0 41_0 58_0 42_0 43_0 42_0 41_0 42_0 40_0 40_0 40_0
*******
FP7: α_fpi = 0.03125
FP07: 54_0 44_0 41_0 41_0 41_0 41_0 54_0 61_0 44_0 43_0 43_0 48_0 41_0 41_0 57_0 58_0 43_0 42_0 42_0 42_0 40_0 42_0 40_0 39_0
*******
FP8: α_fpi = 0.015625
FP08: 44_0 41_0 41_0 41_0 41_0 54_0 61_0 44_0 43_0 43_0 48_0 42_0 41_0 57_0 58_0 57_0 42_0 42_0 42_0 41_0 42_0 41_0 40_0 40_0
*******
FP9: α_fpi = 0.0078125
FP09: 41_0 41_0 41_0 41_0 54_0 60_0 44_0 43_0 43_0 48_0 42_0 41_0 57_0 58_0 57_0 58_0 43_0 43_0 41_0 43_0 41_0 40_0 40_0 39_0
```

$\gamma = 0.5$

BruteForced (BF)





$k_{Max} = 25$

```
μ[3, 1] = 42_45
μ[3, 2] = 42_45
μ[3, 3] = 42_45
μ[3, 4] = 42_45
μ[3, 5] = 42_45
μ[3, 6] = 42_45
μ[3, 7] = 42_45
μ[3, 8] = 42_45
μ[3, 9] = 42_45
μ[3, 10] = 42_45
μ[3, 11] = 42_45
μ[3, 12] = 57_0
μ[3, 13] = 57_0
μ[3, 14] = 57_0
μ[3, 15] = 57_0
μ[3, 16] = 57_0
μ[3, 17] = 41_84
μ[3, 18] = 41_29
μ[3, 19] = 40_88
μ[3, 20] = 40_51
μ[3, 21] = 40_14
μ[3, 22] = 39_76
μ[3, 23] = 39_32
μ[3, 24] = 38_73
```

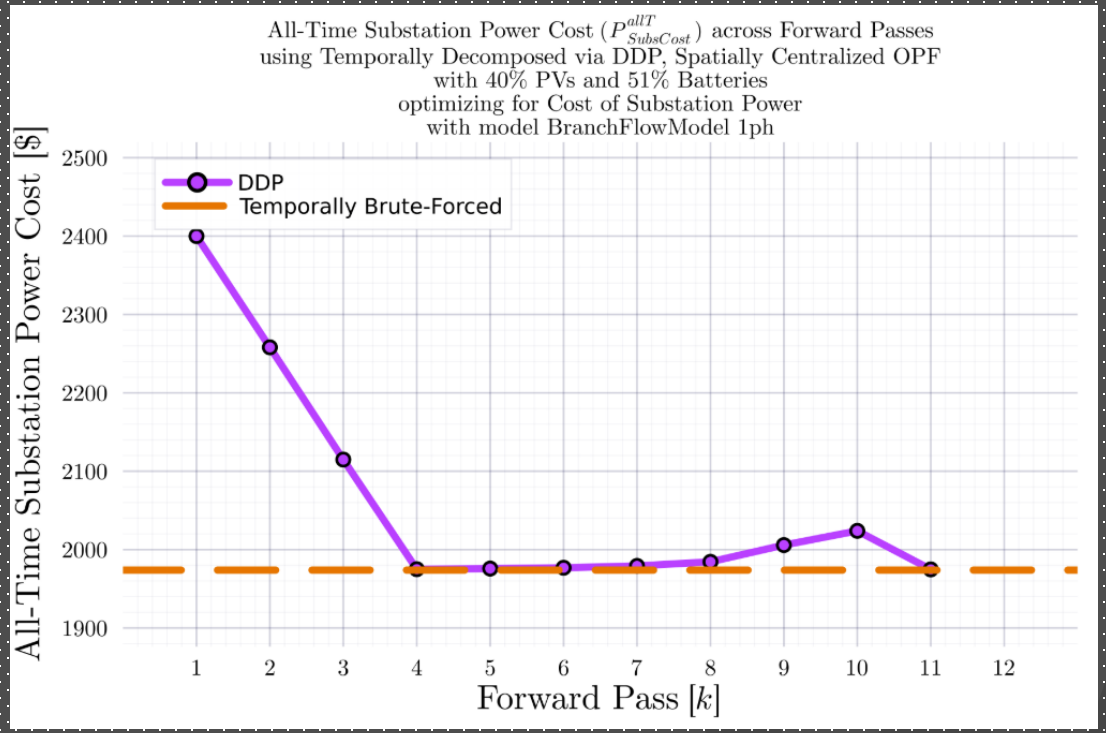
$IEEE123B_{\{1ph\}}$, $T = 24$

DDP Trajectory vs #Forward Passes $[k]$

Terminal SOC Constraint Relaxed

DDP

```
**************
∇L_{B_j^t} for [61, 1]: 0_0649
∇L_{B_j^t} for [61, 2]: 0_0695
∇L_{B_j^t} for [61, 3]: 0_0257
∇L_{B_j^t} for [61, 4]: -0_01
∇L_{B_j^t} for [61, 5]: -258_3461
∇L_{B_j^t} for [61, 6]: 255_2
∇L_{B_j^t} for [61, 7]: 0_7125
∇L_{B_j^t} for [61, 8]: 0_3203
∇L_{B_j^t} for [61, 9]: -0_6333
∇L_{B_j^t} for [61, 10]: 0_003
∇L_{B_j^t} for [61, 11]: -0_1014
∇L_{B_j^t} for [61, 12]: 1_774
∇L_{B_j^t} for [61, 13]: 0_3056
∇L_{B_j^t} for [61, 14]: -2_1936
∇L_{B_j^t} for [61, 15]: -0_1915
∇L_{B_j^t} for [61, 16]: 1_479
∇L_{B_j^t} for [61, 17]: 1_605
∇L_{B_j^t} for [61, 18]: 1_241
∇L_{B_j^t} for [61, 19]: 1_549
∇L_{B_j^t} for [61, 20]: 0_2987
∇L_{B_j^t} for [61, 21]: -1_1867
∇L_{B_j^t} for [61, 22]: -0_2987
∇L_{B_j^t} for [61, 23]: 2_219
∇L_{B_j^t} for [61, 24]: 0_0
**************
Total KKT balance for B_61: 22_08
**************
```



$k_{Max} = 11$

BruteForced (BF)

```
**************
∇L_{B_j^t} for [61, 1]: 0_0
∇L_{B_j^t} for [61, 2]: 0_0
∇L_{B_j^t} for [61, 3]: 0_0
∇L_{B_j^t} for [61, 4]: -0_0
∇L_{B_j^t} for [61, 5]: 0_0
∇L_{B_j^t} for [61, 6]: 0_0
∇L_{B_j^t} for [61, 7]: 0_0
∇L_{B_j^t} for [61, 8]: 0_0
∇L_{B_j^t} for [61, 9]: 0_0
∇L_{B_j^t} for [61, 10]: 0_0
∇L_{B_j^t} for [61, 11]: -0_0
∇L_{B_j^t} for [61, 12]: 0_0
∇L_{B_j^t} for [61, 13]: 0_0
∇L_{B_j^t} for [61, 14]: 0_0
∇L_{B_j^t} for [61, 15]: -0_0
∇L_{B_j^t} for [61, 16]: 0_0
∇L_{B_j^t} for [61, 17]: 0_0
∇L_{B_j^t} for [61, 18]: 0_0
∇L_{B_j^t} for [61, 19]: 0_0
∇L_{B_j^t} for [61, 20]: 0_0
∇L_{B_j^t} for [61, 21]: -0_0
∇L_{B_j^t} for [61, 22]: 0_0
∇L_{B_j^t} for [61, 23]: 0_0
∇L_{B_j^t} for [61, 24]: -0_0
**************
Total KKT balance for B_61: 0_0
**************
```

$IEEE123B_{\{1ph\}}$, $T = 24$

DDP Trajectory vs #Forward Passes $[k]$

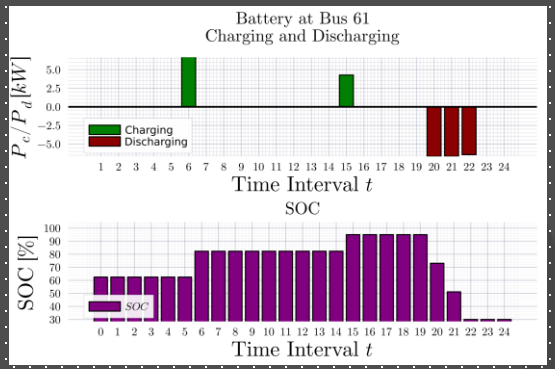
Terminal SOC Constraint Relaxed

DDP

$k_{Max} = 11$

```
**************
∇L_{B_j^t} for [61, 1]: 0_0649
∇L_{B_j^t} for [61, 2]: 0_0695
∇L_{B_j^t} for [61, 3]: 0_0257
∇L_{B_j^t} for [61, 4]: -0_01
∇L_{B_j^t} for [61, 5]: -258_3461
∇L_{B_j^t} for [61, 6]: 255_2
∇L_{B_j^t} for [61, 7]: 0_7125
∇L_{B_j^t} for [61, 8]: 0_3203
∇L_{B_j^t} for [61, 9]: -0_6333
∇L_{B_j^t} for [61, 10]: 0_003
∇L_{B_j^t} for [61, 11]: -0_1014
∇L_{B_j^t} for [61, 12]: 1_774
∇L_{B_j^t} for [61, 13]: 0_3056
∇L_{B_j^t} for [61, 14]: -2_1936
∇L_{B_j^t} for [61, 15]: -0_1915
∇L_{B_j^t} for [61, 16]: 1_479
∇L_{B_j^t} for [61, 17]: 1_605
∇L_{B_j^t} for [61, 18]: 1_241
∇L_{B_j^t} for [61, 19]: 1_549
∇L_{B_j^t} for [61, 20]: 0_2987
∇L_{B_j^t} for [61, 21]: -1_1867
∇L_{B_j^t} for [61, 22]: -0_2987
∇L_{B_j^t} for [61, 23]: 2_219
∇L_{B_j^t} for [61, 24]: 0_0
**************
Total KKT balance for B_61: 22_08
**************
```

```
λ_lb[61, 1] = 0_0 | λ_ub[61, 1] = 0_0
λ_lb[61, 2] = 0_0 | λ_ub[61, 2] = 0_0
λ_lb[61, 3] = 0_0 | λ_ub[61, 3] = 0_0
λ_lb[61, 4] = 0_0 | λ_ub[61, 4] = 0_0
λ_lb[61, 5] = 0_0 | λ_ub[61, 5] = 0_0
λ_lb[61, 6] = 0_0 | λ_ub[61, 6] = 0_0001
λ_lb[61, 7] = 0_0 | λ_ub[61, 7] = 0_0
λ_lb[61, 8] = 0_0 | λ_ub[61, 8] = 0_0
λ_lb[61, 9] = 0_0 | λ_ub[61, 9] = 0_0
λ_lb[61, 10] = 0_0 | λ_ub[61, 10] = 0_0
λ_lb[61, 11] = 0_0 | λ_ub[61, 11] = 0_0
λ_lb[61, 12] = 0_0 | λ_ub[61, 12] = 0_0
λ_lb[61, 13] = 0_0 | λ_ub[61, 13] = 0_0
λ_lb[61, 14] = 0_0 | λ_ub[61, 14] = 0_0001
λ_lb[61, 15] = 0_0 | λ_ub[61, 15] = 242_6
λ_lb[61, 16] = 0_0 | λ_ub[61, 16] = 3_98
λ_lb[61, 17] = 0_0 | λ_ub[61, 17] = 0_9011
λ_lb[61, 18] = 0_0 | λ_ub[61, 18] = 0_5681
λ_lb[61, 19] = 0_0 | λ_ub[61, 19] = 0_0649
λ_lb[61, 20] = 0_0 | λ_ub[61, 20] = 0_0
λ_lb[61, 21] = 0_0 | λ_ub[61, 21] = 0_0
λ_lb[61, 22] = 244_1 | λ_ub[61, 22] = 0_0
λ_lb[61, 23] = 1_809 | λ_ub[61, 23] = 0_0
λ_lb[61, 24] = 47_34 | λ_ub[61, 24] = 0_0
```




```
μ[61, 1, 11] = 49_05
μ[61, 2, 11] = 48_99
μ[61, 3, 11] = 48_92
μ[61, 4, 11] = 48_89
μ[61, 5, 11] = 48_9
μ[61, 6, 11] = 307_2
μ[61, 7, 11] = 52_03
μ[61, 8, 11] = 51_32
μ[61, 9, 11] = 51_0
μ[61, 10, 11] = 51_63
μ[61, 11, 11] = 51_63
μ[61, 12, 11] = 51_73
μ[61, 13, 11] = 49_95
μ[61, 14, 11] = 49_65
μ[61, 15, 11] = 51_84
μ[61, 16, 11] = 294_6
μ[61, 17, 11] = 297_1
μ[61, 18, 11] = 296_4
μ[61, 19, 11] = 295_7
μ[61, 20, 11] = 294_2
μ[61, 21, 11] = 294_0
μ[61, 22, 11] = 295_1
μ[61, 23, 11] = 51_37
μ[61, 24, 11] = 47_34
```

$IEEE123B_{\{1ph\}}$, $T = 24$

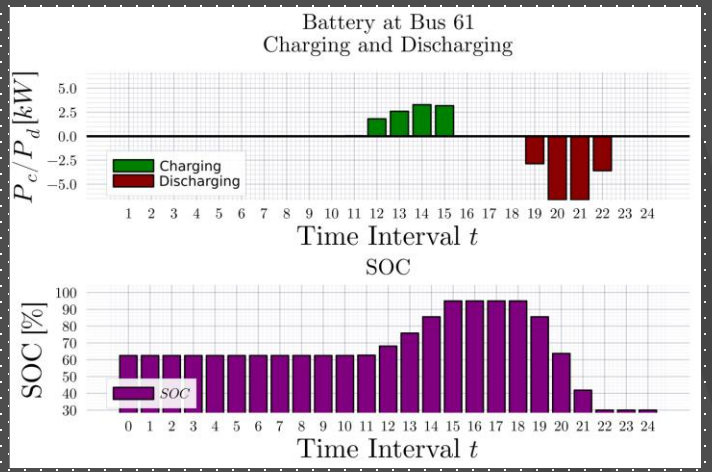
DDP Trajectory vs #Forward Passes $[k]$

Terminal SOC Constraint Relaxed

BruteForced (BF)

```
**************
∇L_{B_j^t} for [61, 1]: 0_0
∇L_{B_j^t} for [61, 2]: 0_0
∇L_{B_j^t} for [61, 3]: 0_0
∇L_{B_j^t} for [61, 4]: -0_0
∇L_{B_j^t} for [61, 5]: 0_0
∇L_{B_j^t} for [61, 6]: 0_0
∇L_{B_j^t} for [61, 7]: 0_0
∇L_{B_j^t} for [61, 8]: 0_0
∇L_{B_j^t} for [61, 9]: 0_0
∇L_{B_j^t} for [61, 10]: 0_0
∇L_{B_j^t} for [61, 11]: -0_0
∇L_{B_j^t} for [61, 12]: 0_0
∇L_{B_j^t} for [61, 13]: 0_0
∇L_{B_j^t} for [61, 14]: 0_0
∇L_{B_j^t} for [61, 15]: -0_0
∇L_{B_j^t} for [61, 16]: 0_0
∇L_{B_j^t} for [61, 17]: 0_0
∇L_{B_j^t} for [61, 18]: 0_0
∇L_{B_j^t} for [61, 19]: 0_0
∇L_{B_j^t} for [61, 20]: 0_0
∇L_{B_j^t} for [61, 21]: -0_0
∇L_{B_j^t} for [61, 22]: 0_0
∇L_{B_j^t} for [61, 23]: 0_0
∇L_{B_j^t} for [61, 24]: -0_0
**************
Total KKT balance for B_61: 0_0
**************
```

```
λ_lb[61, 1] = 0_0 | λ_ub[61, 1] = 0_0
λ_lb[61, 2] = 0_0 | λ_ub[61, 2] = 0_0
λ_lb[61, 3] = 0_0 | λ_ub[61, 3] = 0_0
λ_lb[61, 4] = 0_0 | λ_ub[61, 4] = 0_0
λ_lb[61, 5] = 0_0 | λ_ub[61, 5] = 0_0
λ_lb[61, 6] = 0_0 | λ_ub[61, 6] = 0_0
λ_lb[61, 7] = 0_0 | λ_ub[61, 7] = 0_0
λ_lb[61, 8] = 0_0 | λ_ub[61, 8] = 0_0
λ_lb[61, 9] = 0_0 | λ_ub[61, 9] = 0_0
λ_lb[61, 10] = 0_0 | λ_ub[61, 10] = 0_0
λ_lb[61, 11] = 0_0 | λ_ub[61, 11] = 0_0
λ_lb[61, 12] = 0_0 | λ_ub[61, 12] = 0_0
λ_lb[61, 13] = 0_0 | λ_ub[61, 13] = 0_0
λ_lb[61, 14] = 0_0 | λ_ub[61, 14] = 0_0001
λ_lb[61, 15] = 0_0 | λ_ub[61, 15] = 240_6
λ_lb[61, 16] = 0_0 | λ_ub[61, 16] = 1_425
λ_lb[61, 17] = 0_0 | λ_ub[61, 17] = 0_7012
λ_lb[61, 18] = 0_0 | λ_ub[61, 18] = 0_3623
λ_lb[61, 19] = 0_0 | λ_ub[61, 19] = 0_0001
λ_lb[61, 20] = 0_0 | λ_ub[61, 20] = 0_0
λ_lb[61, 21] = 0_0001 | λ_ub[61, 21] = 0_0
λ_lb[61, 22] = 243_9 | λ_ub[61, 22] = 0_0
λ_lb[61, 23] = 2_515 | λ_ub[61, 23] = 0_0
λ_lb[61, 24] = 48_58 | λ_ub[61, 24] = 0_0
```




```
μ[61, 1] = 51_84
μ[61, 2] = 51_84
μ[61, 3] = 51_84
μ[61, 4] = 51_84
μ[61, 5] = 51_84
μ[61, 6] = 51_84
μ[61, 7] = 51_84
μ[61, 8] = 51_84
μ[61, 9] = 51_84
μ[61, 10] = 51_84
μ[61, 11] = 51_84
μ[61, 12] = 51_84
μ[61, 13] = 51_84
μ[61, 14] = 51_84
μ[61, 15] = 51_84
μ[61, 16] = 292_5
μ[61, 17] = 293_9
μ[61, 18] = 294_6
μ[61, 19] = 295_0
μ[61, 20] = 295_0
μ[61, 21] = 295_0
μ[61, 22] = 295_0
μ[61, 23] = 51_09
μ[61, 24] = 48_58
```

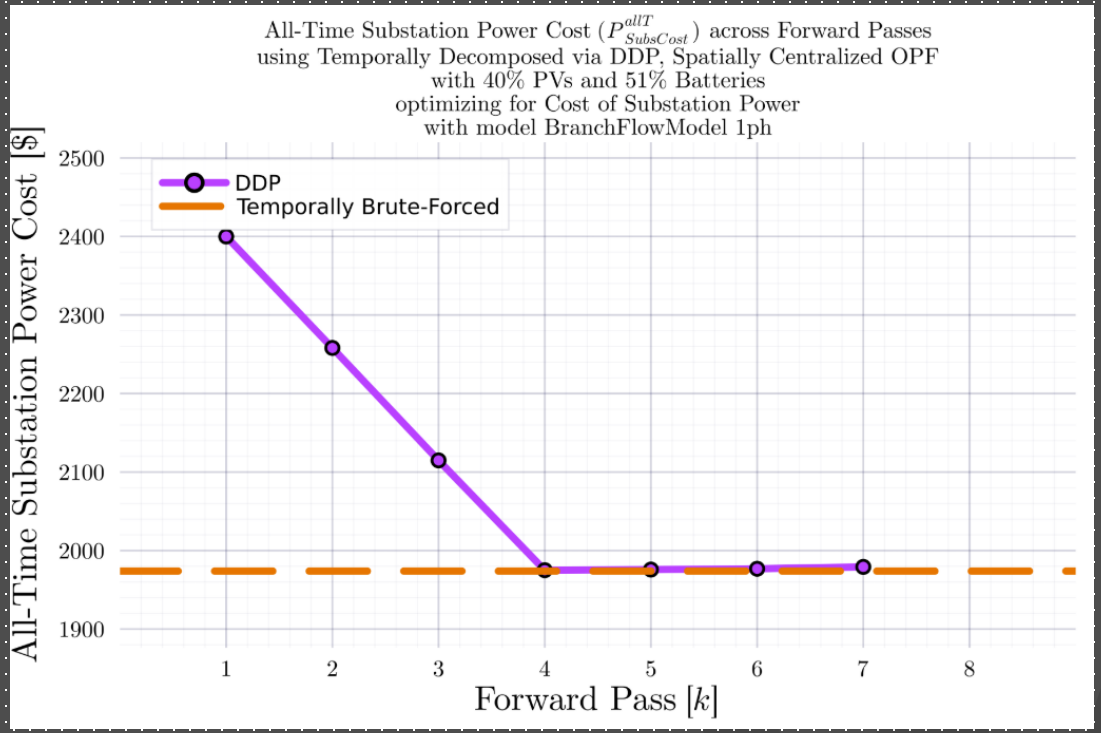
$IEEE123B_{\{1ph\}}$, $T = 24$

DDP Trajectory vs #Forward Passes $[k]$

Terminal SOC Constraint Relaxed

DDP

```
**************
∇L_{B_j^t} for [61, 1]: -0_058
∇L_{B_j^t} for [61, 2]: 0_2752
∇L_{B_j^t} for [61, 3]: -0_0761
∇L_{B_j^t} for [61, 4]: 0_2075
∇L_{B_j^t} for [61, 5]: 0_0649
∇L_{B_j^t} for [61, 6]: 0_0695
∇L_{B_j^t} for [61, 7]: 0_0257
∇L_{B_j^t} for [61, 8]: -0_01
∇L_{B_j^t} for [61, 9]: -258_3464
∇L_{B_j^t} for [61, 10]: 255_2
∇L_{B_j^t} for [61, 11]: 0_7125
∇L_{B_j^t} for [61, 12]: 0_3203
∇L_{B_j^t} for [61, 13]: -0_6333
∇L_{B_j^t} for [61, 14]: 0_0029
∇L_{B_j^t} for [61, 15]: 0_1537
∇L_{B_j^t} for [61, 16]: 250_4
∇L_{B_j^t} for [61, 17]: 0_3057
∇L_{B_j^t} for [61, 18]: -244_7698
∇L_{B_j^t} for [61, 19]: -4_1714
∇L_{B_j^t} for [61, 20]: 0_5775
∇L_{B_j^t} for [61, 21]: 1_291
∇L_{B_j^t} for [61, 22]: -0_0283
∇L_{B_j^t} for [61, 23]: 0_0327
∇L_{B_j^t} for [61, 24]: -0_0
**************
Total KKT balance for B_61: 42_41
**************
```



$k_{Max} = 7$

BruteForced (BF)

```
**************
∇L_{B_j^t} for [61, 1]: 0_0
∇L_{B_j^t} for [61, 2]: 0_0
∇L_{B_j^t} for [61, 3]: 0_0
∇L_{B_j^t} for [61, 4]: -0_0
∇L_{B_j^t} for [61, 5]: 0_0
∇L_{B_j^t} for [61, 6]: 0_0
∇L_{B_j^t} for [61, 7]: 0_0
∇L_{B_j^t} for [61, 8]: 0_0
∇L_{B_j^t} for [61, 9]: 0_0
∇L_{B_j^t} for [61, 10]: 0_0
∇L_{B_j^t} for [61, 11]: -0_0
∇L_{B_j^t} for [61, 12]: 0_0
∇L_{B_j^t} for [61, 13]: 0_0
∇L_{B_j^t} for [61, 14]: 0_0
∇L_{B_j^t} for [61, 15]: -0_0
∇L_{B_j^t} for [61, 16]: 0_0
∇L_{B_j^t} for [61, 17]: 0_0
∇L_{B_j^t} for [61, 18]: 0_0
∇L_{B_j^t} for [61, 19]: 0_0
∇L_{B_j^t} for [61, 20]: 0_0
∇L_{B_j^t} for [61, 21]: -0_0
∇L_{B_j^t} for [61, 22]: 0_0
∇L_{B_j^t} for [61, 23]: 0_0
∇L_{B_j^t} for [61, 24]: -0_0
**************
Total KKT balance for B_61: 0_0
**************
```

$IEEE123B_{\{1ph\}}$, $T = 24$

DDP Trajectory vs #Forward Passes $[k]$

Terminal SOC Constraint Relaxed

DDP

```
**************
∇L_{B_j^t} for [61, 1]: -0_058
∇L_{B_j^t} for [61, 2]: 0_2752
∇L_{B_j^t} for [61, 3]: -0_0761
∇L_{B_j^t} for [61, 4]: 0_2075
∇L_{B_j^t} for [61, 5]: 0_0649
∇L_{B_j^t} for [61, 6]: 0_0695
∇L_{B_j^t} for [61, 7]: 0_0257
∇L_{B_j^t} for [61, 8]: -0_01
∇L_{B_j^t} for [61, 9]: -258_3464
∇L_{B_j^t} for [61, 10]: 255_2
∇L_{B_j^t} for [61, 11]: 0_7125
∇L_{B_j^t} for [61, 12]: 0_3203
∇L_{B_j^t} for [61, 13]: -0_6333
∇L_{B_j^t} for [61, 14]: 0_0029
∇L_{B_j^t} for [61, 15]: 0_1537
∇L_{B_j^t} for [61, 16]: 250_4
∇L_{B_j^t} for [61, 17]: 0_3057
∇L_{B_j^t} for [61, 18]: -244_7698
∇L_{B_j^t} for [61, 19]: -4_1714
∇L_{B_j^t} for [61, 20]: 0_5775
∇L_{B_j^t} for [61, 21]: 1_291
∇L_{B_j^t} for [61, 22]: -0_0283
∇L_{B_j^t} for [61, 23]: 0_0327
∇L_{B_j^t} for [61, 24]: -0_0
**************
Total KKT balance for B_61: 42_41
**************
```

```
λ_lb[61, 1] = 0_0 | λ_ub[61, 1] = 0_0
λ_lb[61, 2] = 0_0 | λ_ub[61, 2] = 0_0
λ_lb[61, 3] = 0_0 | λ_ub[61, 3] = 0_0
λ_lb[61, 4] = 0_0 | λ_ub[61, 4] = 0_0
λ_lb[61, 5] = 0_0 | λ_ub[61, 5] = 0_0
λ_lb[61, 6] = 0_0 | λ_ub[61, 6] = 0_0
λ_lb[61, 7] = 0_0 | λ_ub[61, 7] = 0_0
λ_lb[61, 8] = 0_0 | λ_ub[61, 8] = 0_0
λ_lb[61, 9] = 0_0 | λ_ub[61, 9] = 0_0
λ_lb[61, 10] = 0_0 | λ_ub[61, 10] = 0_0001
λ_lb[61, 11] = 0_0 | λ_ub[61, 11] = 0_0
λ_lb[61, 12] = 0_0 | λ_ub[61, 12] = 0_0
λ_lb[61, 13] = 0_0 | λ_ub[61, 13] = 0_0
λ_lb[61, 14] = 0_0 | λ_ub[61, 14] = 0_0
λ_lb[61, 15] = 0_0 | λ_ub[61, 15] = 243_9
λ_lb[61, 16] = 0_0 | λ_ub[61, 16] = 4_984
λ_lb[61, 17] = 0_0 | λ_ub[61, 17] = 0_0
λ_lb[61, 18] = 0_0 | λ_ub[61, 18] = 0_0
λ_lb[61, 19] = 0_0 | λ_ub[61, 19] = 0_0
λ_lb[61, 20] = 0_0 | λ_ub[61, 20] = 0_0
λ_lb[61, 21] = 0_0001 | λ_ub[61, 21] = 0_0
λ_lb[61, 22] = 245_7 | λ_ub[61, 22] = 0_0
λ_lb[61, 23] = 2_532 | λ_ub[61, 23] = 0_0
λ_lb[61, 24] = 48_49 | λ_ub[61, 24] = 0_0
```

```
μ[61, 1, 7] = 49_4
μ[61, 2, 7] = 49_46
μ[61, 3, 7] = 49_18
μ[61, 4, 7] = 49_26
μ[61, 5, 7] = 49_05
μ[61, 6, 7] = 48_99
μ[61, 7, 7] = 48_92
μ[61, 8, 7] = 48_89
μ[61, 9, 7] = 48_9
μ[61, 10, 7] = 307_2
μ[61, 11, 7] = 52_03
μ[61, 12, 7] = 51_32
μ[61, 13, 7] = 51_0
μ[61, 14, 7] = 51_63
μ[61, 15, 7] = 51_63
μ[61, 16, 7] = 295_3
μ[61, 17, 7] = 49_95
μ[61, 18, 7] = 49_65
μ[61, 19, 7] = 294_4
μ[61, 20, 7] = 298_6
μ[61, 21, 7] = 298_0
μ[61, 22, 7] = 296_7
μ[61, 23, 7] = 51_05
μ[61, 24, 7] = 48_49
```

$k_{Max} = 11$

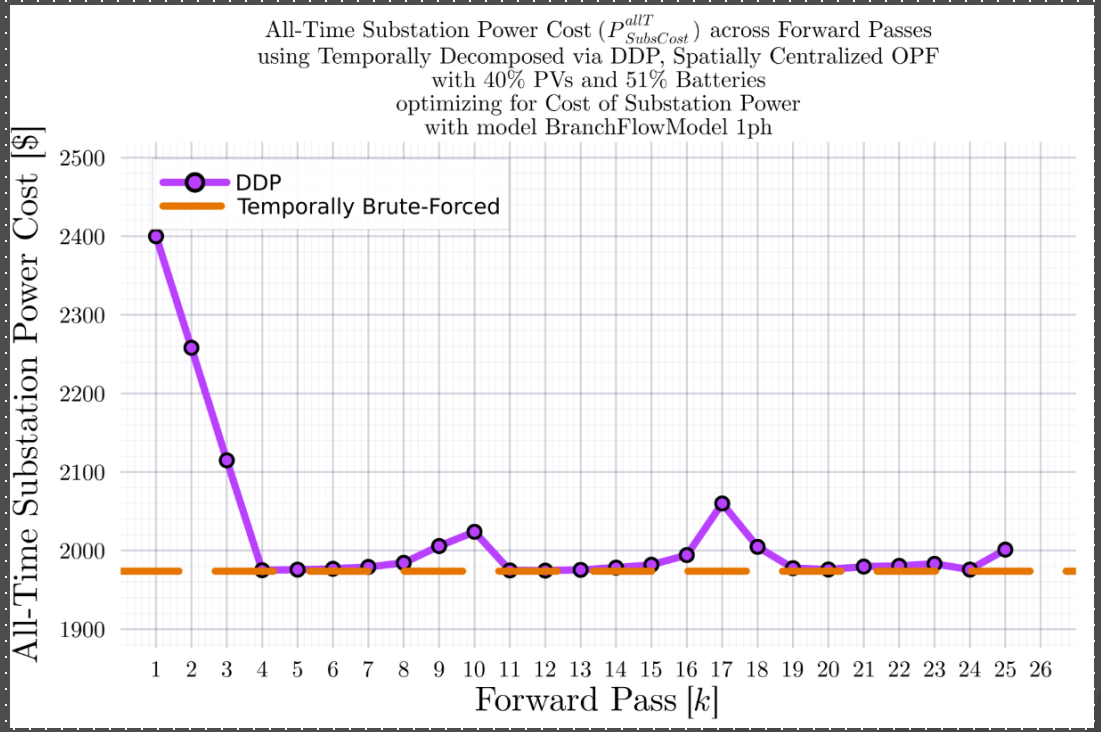
$IEEE123B_{\{1ph\}}$, $T = 24$

DDP Trajectory vs #Forward Passes $[k]$

Terminal SOC Constraint Relaxed

DDP

```
**************
∇L_{B_j^t} for [61, 1]: 0_2341
∇L_{B_j^t} for [61, 2]: 0_0087
∇L_{B_j^t} for [61, 3]: 0_0049
∇L_{B_j^t} for [61, 4]: -0_1182
∇L_{B_j^t} for [61, 5]: 0_0026
∇L_{B_j^t} for [61, 6]: -239_3876
∇L_{B_j^t} for [61, 7]: 239_1
∇L_{B_j^t} for [61, 8]: 0_2523
∇L_{B_j^t} for [61, 9]: 0_1387
∇L_{B_j^t} for [61, 10]: -0_0274
∇L_{B_j^t} for [61, 11]: -0_0082
∇L_{B_j^t} for [61, 12]: -0_0664
∇L_{B_j^t} for [61, 13]: -0_0043
∇L_{B_j^t} for [61, 14]: 0_1052
∇L_{B_j^t} for [61, 15]: -0_7692
∇L_{B_j^t} for [61, 16]: 0_103
∇L_{B_j^t} for [61, 17]: -0_3328
∇L_{B_j^t} for [61, 18]: -0_0635
∇L_{B_j^t} for [61, 19]: 0_4364
∇L_{B_j^t} for [61, 20]: 0_01
∇L_{B_j^t} for [61, 21]: 0_0276
∇L_{B_j^t} for [61, 22]: 0_0446
∇L_{B_j^t} for [61, 23]: 0_0227
∇L_{B_j^t} for [61, 24]: 0_0
**************
Total KKT balance for B_61: 20_05
**************
```



$k_{Max} = 25$

BruteForced (BF)

```
**************
∇L_{B_j^t} for [61, 1]: 0_0
∇L_{B_j^t} for [61, 2]: 0_0
∇L_{B_j^t} for [61, 3]: 0_0
∇L_{B_j^t} for [61, 4]: -0_0
∇L_{B_j^t} for [61, 5]: 0_0
∇L_{B_j^t} for [61, 6]: 0_0
∇L_{B_j^t} for [61, 7]: 0_0
∇L_{B_j^t} for [61, 8]: 0_0
∇L_{B_j^t} for [61, 9]: 0_0
∇L_{B_j^t} for [61, 10]: 0_0
∇L_{B_j^t} for [61, 11]: -0_0
∇L_{B_j^t} for [61, 12]: 0_0
∇L_{B_j^t} for [61, 13]: 0_0
∇L_{B_j^t} for [61, 14]: 0_0
∇L_{B_j^t} for [61, 15]: -0_0
∇L_{B_j^t} for [61, 16]: 0_0
∇L_{B_j^t} for [61, 17]: 0_0
∇L_{B_j^t} for [61, 18]: 0_0
∇L_{B_j^t} for [61, 19]: 0_0
∇L_{B_j^t} for [61, 20]: 0_0
∇L_{B_j^t} for [61, 21]: -0_0
∇L_{B_j^t} for [61, 22]: 0_0
∇L_{B_j^t} for [61, 23]: 0_0
∇L_{B_j^t} for [61, 24]: -0_0
**************
Total KKT balance for B_61: 0_0
**************
```

$IEEE123B_{\{1ph\}}$, $T = 24$

DDP Trajectory vs #Forward Passes $[k]$

Terminal SOC Constraint Relaxed

DDP

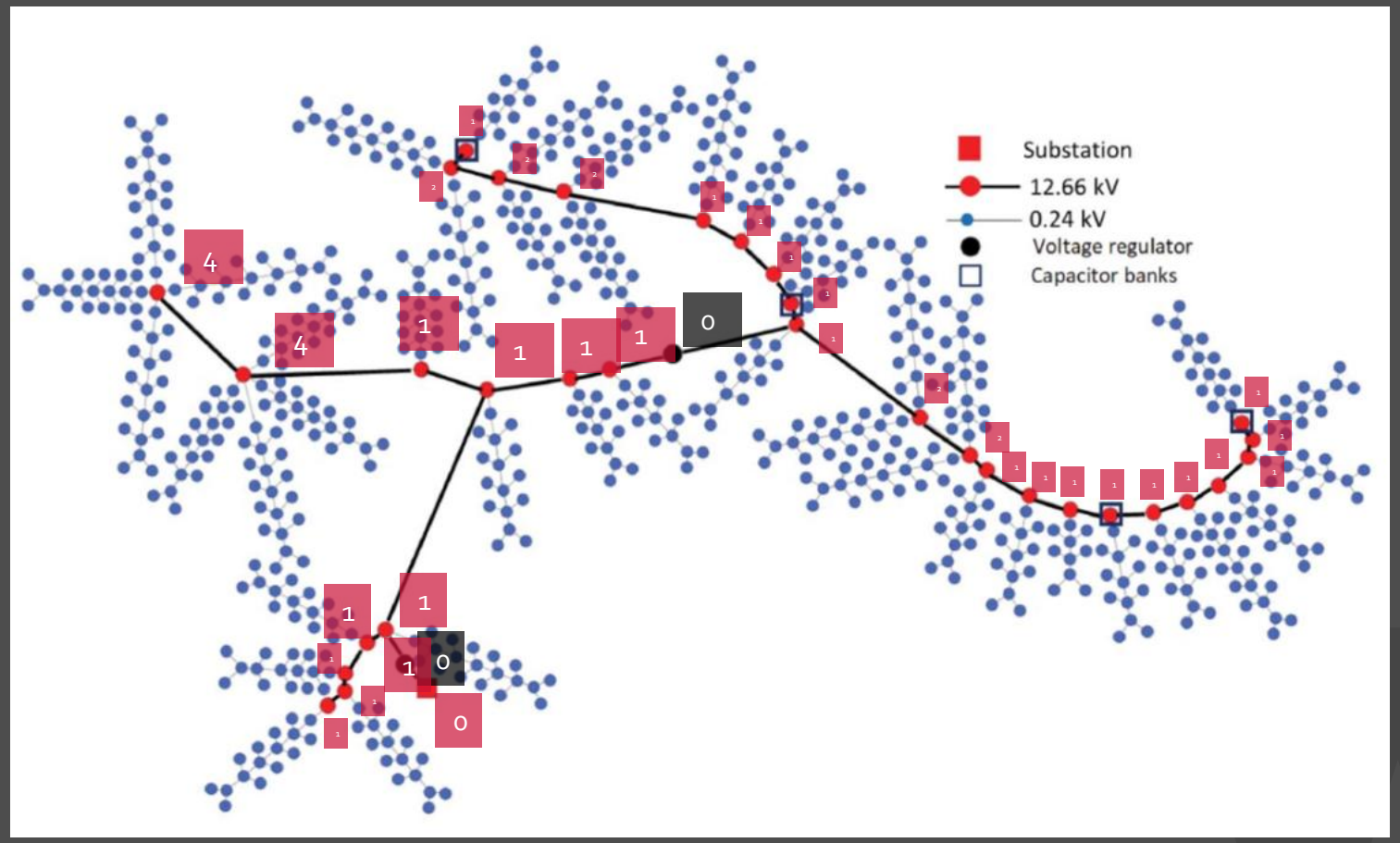
```
**************
∇L_{B_j^t} for [61, 1]: 0_2341
∇L_{B_j^t} for [61, 2]: 0_0087
∇L_{B_j^t} for [61, 3]: 0_0049
∇L_{B_j^t} for [61, 4]: -0_1182
∇L_{B_j^t} for [61, 5]: 0_0026
∇L_{B_j^t} for [61, 6]: -239_3876
∇L_{B_j^t} for [61, 7]: 239_1
∇L_{B_j^t} for [61, 8]: 0_2523
∇L_{B_j^t} for [61, 9]: 0_1387
∇L_{B_j^t} for [61, 10]: -0_0274
∇L_{B_j^t} for [61, 11]: -0_0082
∇L_{B_j^t} for [61, 12]: -0_0664
∇L_{B_j^t} for [61, 13]: -0_0043
∇L_{B_j^t} for [61, 14]: 0_1052
∇L_{B_j^t} for [61, 15]: -0_7692
∇L_{B_j^t} for [61, 16]: 0_103
∇L_{B_j^t} for [61, 17]: -0_3328
∇L_{B_j^t} for [61, 18]: -0_0635
∇L_{B_j^t} for [61, 19]: 0_4364
∇L_{B_j^t} for [61, 20]: 0_01
∇L_{B_j^t} for [61, 21]: 0_0276
∇L_{B_j^t} for [61, 22]: 0_0446
∇L_{B_j^t} for [61, 23]: 0_0227
∇L_{B_j^t} for [61, 24]: 0_0
**************
Total KKT balance for B_61: 20_05
**************
```

```
λ_lb[61, 1] = 0_0 | λ_ub[61, 1] = 0_0
λ_lb[61, 2] = 0_0 | λ_ub[61, 2] = 0_0
λ_lb[61, 3] = 0_0 | λ_ub[61, 3] = 0_0
λ_lb[61, 4] = 0_0 | λ_ub[61, 4] = 0_0
λ_lb[61, 5] = 0_0 | λ_ub[61, 5] = 0_0
λ_lb[61, 6] = 0_0 | λ_ub[61, 6] = 0_0
λ_lb[61, 7] = 0_0 | λ_ub[61, 7] = 0_0001
λ_lb[61, 8] = 0_0 | λ_ub[61, 8] = 0_0001
λ_lb[61, 9] = 0_0 | λ_ub[61, 9] = 0_0
λ_lb[61, 10] = 0_0 | λ_ub[61, 10] = 0_0
λ_lb[61, 11] = 0_0 | λ_ub[61, 11] = 0_0
λ_lb[61, 12] = 0_0 | λ_ub[61, 12] = 0_0
λ_lb[61, 13] = 0_0 | λ_ub[61, 13] = 0_0
λ_lb[61, 14] = 0_0 | λ_ub[61, 14] = 0_0
λ_lb[61, 15] = 0_0 | λ_ub[61, 15] = 239_9
λ_lb[61, 16] = 0_0 | λ_ub[61, 16] = 1_511
λ_lb[61, 17] = 0_0 | λ_ub[61, 17] = 1_149
λ_lb[61, 18] = 0_0 | λ_ub[61, 18] = 0_2085
λ_lb[61, 19] = 0_0 | λ_ub[61, 19] = 0_0001
λ_lb[61, 20] = 0_0 | λ_ub[61, 20] = 0_0
λ_lb[61, 21] = 0_0001 | λ_ub[61, 21] = 0_0
λ_lb[61, 22] = 244_0 | λ_ub[61, 22] = 0_0
λ_lb[61, 23] = 2_555 | λ_ub[61, 23] = 0_0
λ_lb[61, 24] = 48_45 | λ_ub[61, 24] = 0_0
```

```
μ[61, 1, 25] = 51_84
μ[61, 2, 25] = 51_6
μ[61, 3, 25] = 51_6
μ[61, 4, 25] = 51_59
μ[61, 5, 25] = 51_71
μ[61, 6, 25] = 51_71
μ[61, 7, 25] = 291_1
μ[61, 8, 25] = 52_02
μ[61, 9, 25] = 51_76
μ[61, 10, 25] = 51_62
μ[61, 11, 25] = 51_65
μ[61, 12, 25] = 51_66
μ[61, 13, 25] = 51_73
μ[61, 14, 25] = 51_73
μ[61, 15, 25] = 51_63
μ[61, 16, 25] = 292_3
μ[61, 17, 25] = 293_7
μ[61, 18, 25] = 295_2
μ[61, 19, 25] = 295_5
μ[61, 20, 25] = 295_1
μ[61, 21, 25] = 295_1
μ[61, 22, 25] = 295_0
μ[61, 23, 25] = 51_03
μ[61, 24, 25] = 48_45
```

$k_{Max} = 25$

IEEE 730 1ph Node System



34 12.66kV to 12.66kV branches (high impedance)

43 12.66kV to 0.24kV branches (low impedance)

$$kV_{B_{Sys}} = 0.24$$
$$kV_{B_{Sec}} = 0.24$$
$$kV_{B_{Pri}} = 12.66$$
$$MVA_{B_{Sys}} = 100$$
$$MVA_{B_{Xfm}} = 0.07$$

$$Z_{B_{Sys}} = Z_{B_{Sec}} = 0.000576$$
$$Z_{B_{Pri}} = 1.602756$$

Comparison of BFM-NL-1ph and LinDistFlow-1ph Optimization Results

LinDistFlow Optimal Actions used in OpenDSS

```
Full 24 Hour Horizon Validation Results
6. Horizon Total Substation Power Cost: $2798.4
7. Horizon Total Line Loss: 461.38 kW
8. Horizon Total Substation Power: 21065.89 kW + 12259.29 kVAr
9. Horizon Total Load: 21357.37 kW + 11751.08 kVAr
10. Horizon Total Generation: 752.65 kW + 399.75 kVAr
11. Horizon Total Static Capacitor Reactive Power Generation: 0.0 kVAr
12. Horizon Total PV Generation: 607.45 kW + 195.12 kVAr
13. Horizon Total Battery Generation: 145.21 kW + 204.63 kVAr
14. Horizon Total Battery Transaction Magnitude: 505.98 kW + 211.47 kVAr
15. Horizon Total SCD Observed: N/A
16. Horizon-end Battery Energy Deviation from Reference: 171.36 kWh
17. Horizon-Total All Time Substation Power Peak: 1156.22 kW
-----------------------------------------------------------
Discrepancies (Maximum All Time):
18. Maximum All Time Voltage Discrepancy: 0.002056 pu
19. Maximum All Time Line Loss Discrepancy: 1.807435 kW
20. Maximum All Time Substation Borrowed Real Power Discrepancy: 32.362217 kW
21. Maximum All Time Substation Borrowed Reactive Power Discrepancy: 64.402519
kVAr
-----------------------------------------------------------
```

BFM-NL Optimal Actions used in OpenDSS

```
-----------------------------------------------------------
Full 24 Hour Horizon Validation Results
6. Horizon Total Substation Power Cost: $2787.64
7. Horizon Total Line Loss: 379.83 kW
8. Horizon Total Substation Power: 20984.05 kW + 6812.67 kVAr
9. Horizon Total Load: 21357.41 kW + 11751.19 kVAr
10. Horizon Total Generation: 753.12 kW + 5681.79 kVAr
11. Horizon Total Static Capacitor Reactive Power Generation: 0.0 kVAr
12. Horizon Total PV Generation: 607.44 kW + 1972.26 kVAr
13. Horizon Total Battery Generation: 145.68 kW + 3709.53 kVAr
14. Horizon Total Battery Transaction Magnitude: 503.3 kW + 3709.53 kVAr
15. Horizon Total SCD Observed: N/A
16. Horizon-end Battery Energy Deviation from Reference: 171.06 kWh
17. Horizon-Total All Time Substation Power Peak: 1114.0 kW
-----------------------------------------------------------
Discrepancies (Maximum All Time):
18. Maximum All Time Voltage Discrepancy: 7.2e-5 pu
19. Maximum All Time Line Loss Discrepancy: 0.018184 kW
20. Maximum All Time Substation Borrowed Real Power Discrepancy: 0.431635 kW
21. Maximum All Time Substation Borrowed Reactive Power Discrepancy: 1.010155 kVAr
-----------------------------------------------------------
```

Comparison of BFM-NL-1ph and LinDistFlow-1ph Optimization Results

LinDistFlow Own Solutions

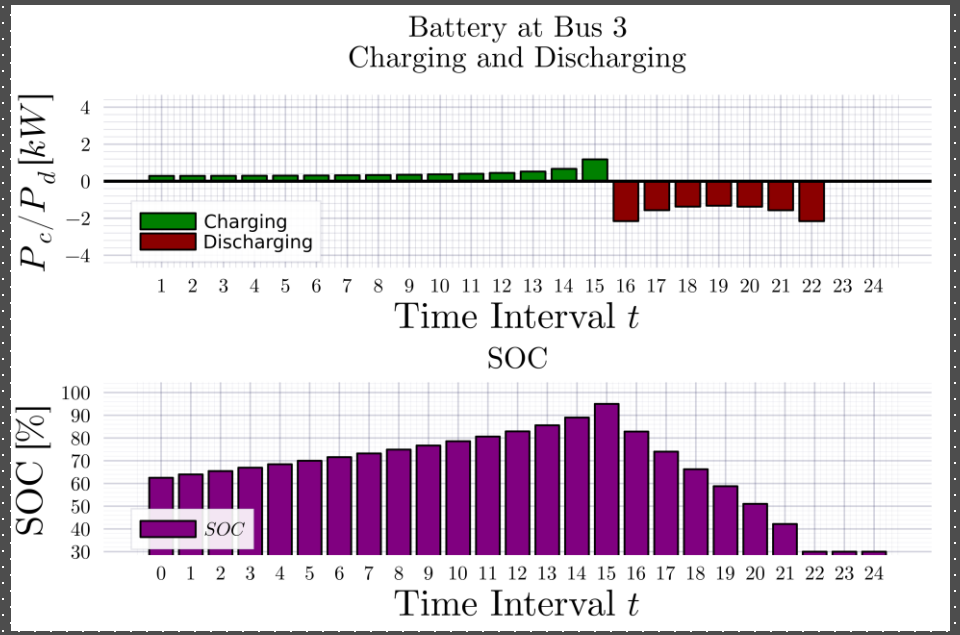
```
Full 24 Hour Horizon
9. Horizon Total Cost of Substation Power: $ 2731.34
10. Horizon Total Line Loss: 441.07 kW
11. Horizon Total Substation Power: 20604.8 kW + 11351.51 kVAr
12. Horizon Total Load: 21357.46 kW + 11751.26 kVAr
13. Horizon Total Generation: 752.66 kW + 399.75 kVAr
14. Horizon Total Static Capacitor Reactive Power Generation: 0.0 kVAr
15. Horizon Total PV Generation: 607.46 kW + 195.12 kVAr
16. Horizon Total Battery Generation: 145.21 kW + 204.63 kVAr
17. Horizon Total Battery Transaction Magnitude: 505.98 kW + 211.47 kVAr
18. Horizon Total SCD Observed: 0.02 kW
19. Horizon-end Battery Energy Deviation from Reference: 171.36 kWh
20. Horizon-Total All time Substation Power Peak: 1123.86 kW

21. Number of Macro-Iterations: 1
22. Simulation Time: 0.67 s
23. Time to solve with sequential (non-parallel) computation: 0.67 s
24. Time to solve if OPF computation parallelized: 0.67 s
```

BFM-NL Own Solutions

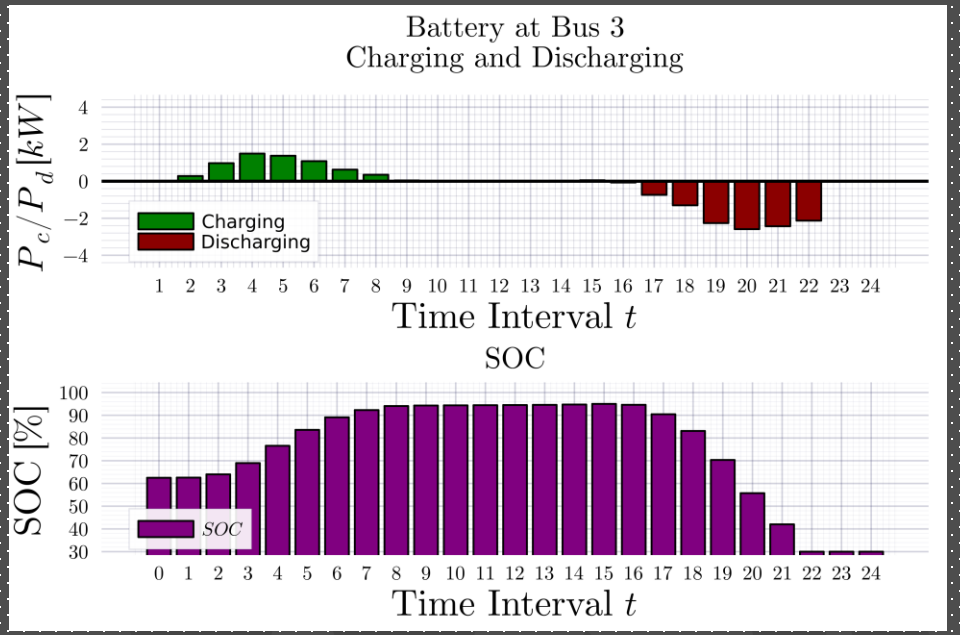
```
Full 24 Hour Horizon
9. Horizon Total Cost of Substation Power: $ 2787.44
10. Horizon Total Line Loss: 380.09 kW
11. Horizon Total Substation Power: 20984.89 kW + 6835.82 kVAr
12. Horizon Total Load: 21357.46 kW + 11751.26 kVAr
13. Horizon Total Generation: 752.66 kW + 5681.98 kVAr
14. Horizon Total Static Capacitor Reactive Power Generation: 0.0 kVAr
15. Horizon Total PV Generation: 607.46 kW + 1972.27 kVAr
16. Horizon Total Battery Generation: 145.21 kW + 3709.71 kVAr
17. Horizon Total Battery Transaction Magnitude: 505.98 kW + 3709.71 kVAr
18. Horizon Total SCD Observed: 0.02 kW
19. Horizon-end Battery Energy Deviation from Reference: 171.36 kWh
20. Horizon-Total All time Substation Power Peak: 1114.0 kW

21. Number of Macro-Iterations: 1
22. Simulation Time: 11.64 s
23. Time to solve with sequential (non-parallel) computation: 11.64 s
24. Time to solve if OPF computation parallelized: 11.64 s
```

Comparison of BFM-NL-1ph and LinDistFlow-1ph Optimization Results

LinDistFlow



BFM-NL



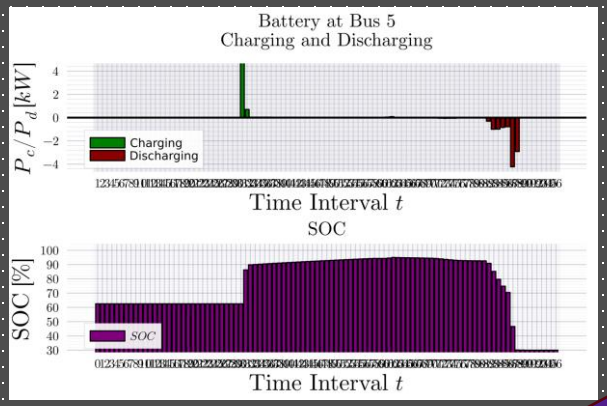
DDP Trajectory vs #Forward Passes $[k]$

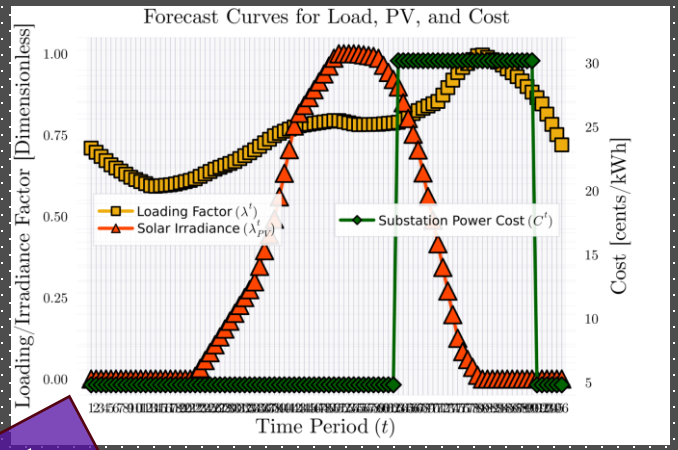
Terminal SOC Constraint Relaxed

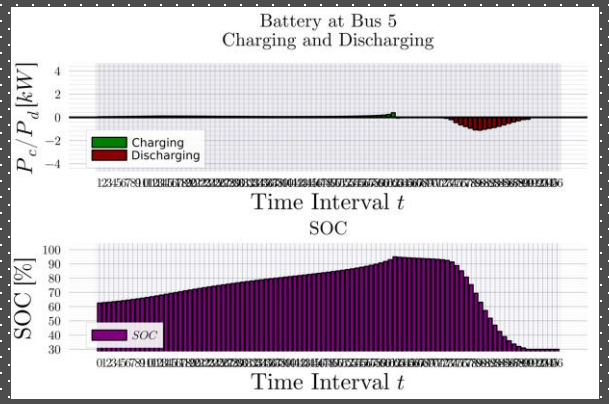
$ADS10_{1ph}$, $T = 96$

DDP

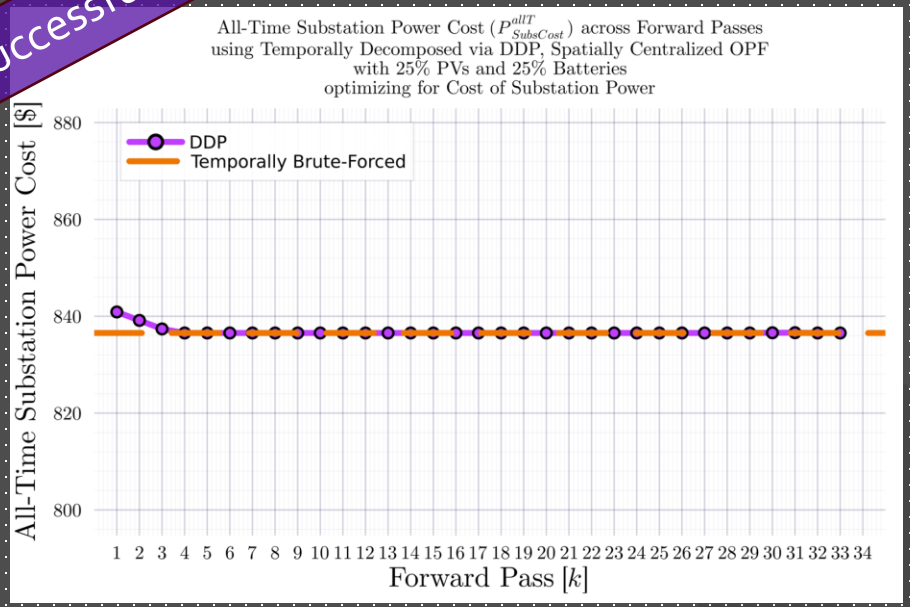
BruteForced (BF)

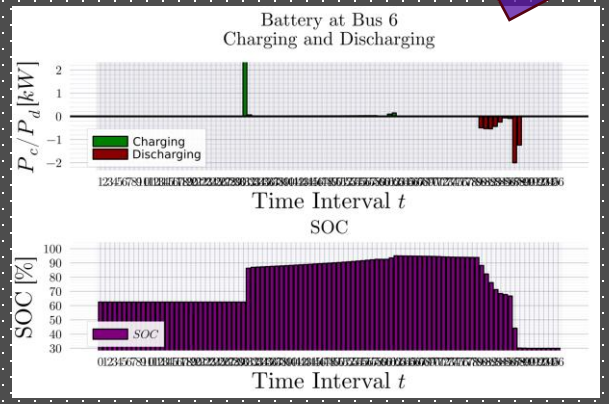


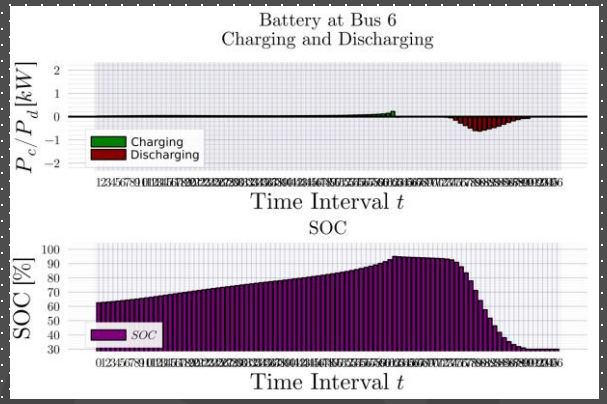




Looks 'successful'







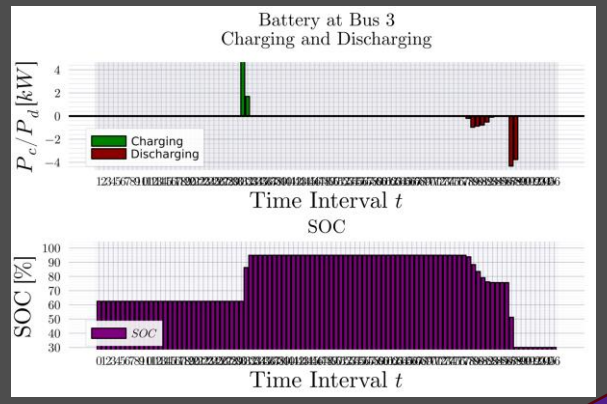
$k_{Max} = 33$

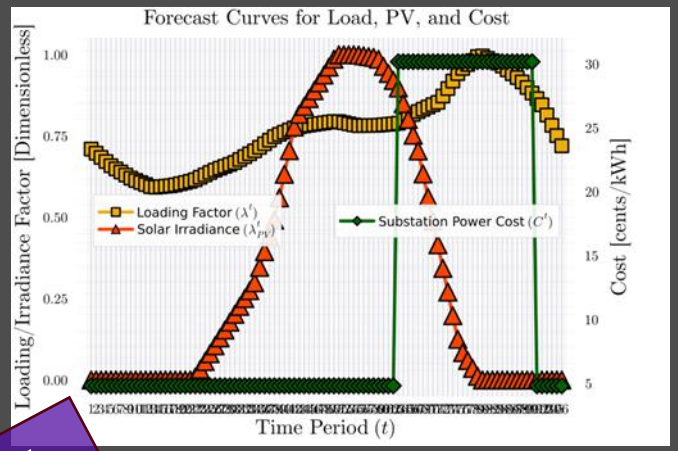
$IEEE123_{1ph}$, $T = 96$

DDP Trajectory vs #Forward Passes $[k]$

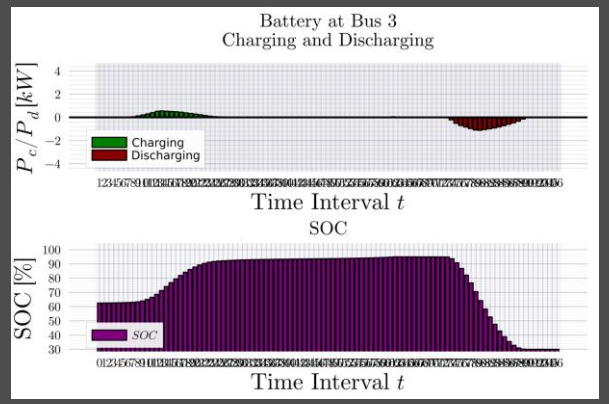
Terminal SOC Constraint Relaxed

DDP

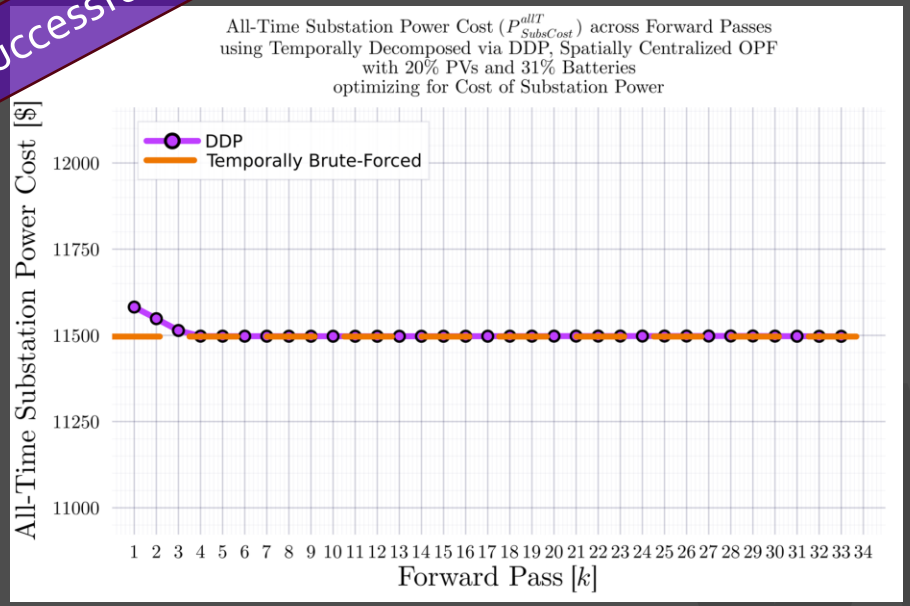


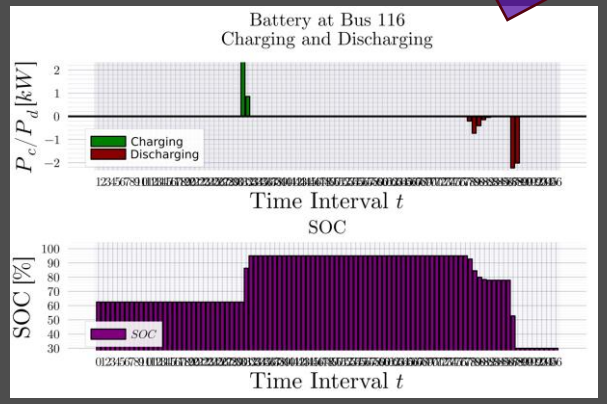


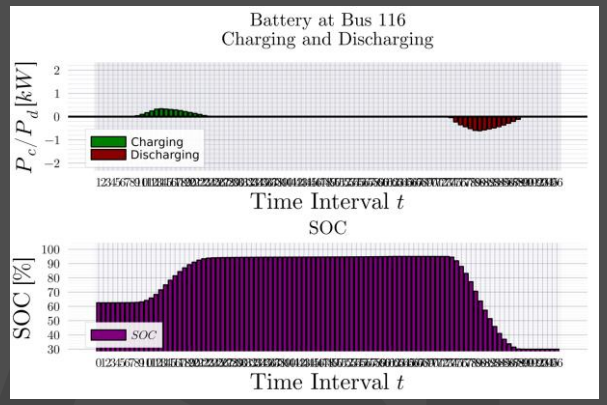
BruteForced (BF)



Looks 'successful'







$k_{Max} = 33$

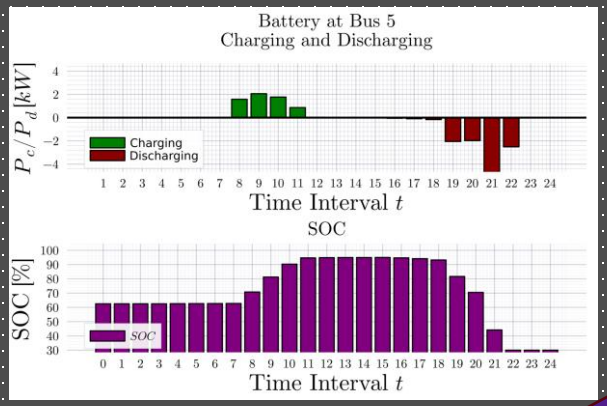
DDP Trajectory vs #Forward Passes $[k]$

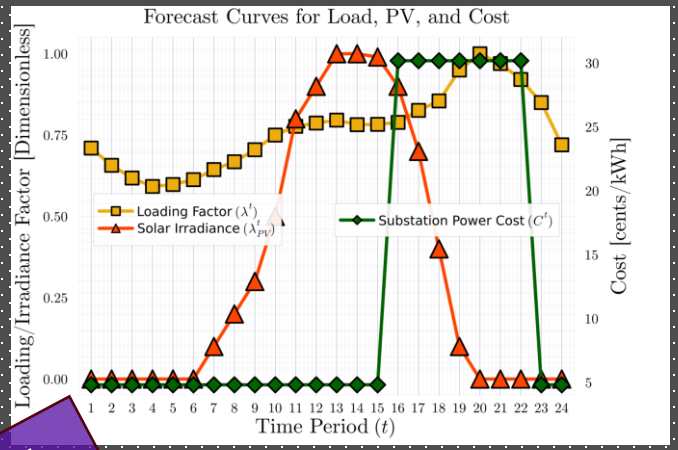
Terminal SOC Constraint Relaxed

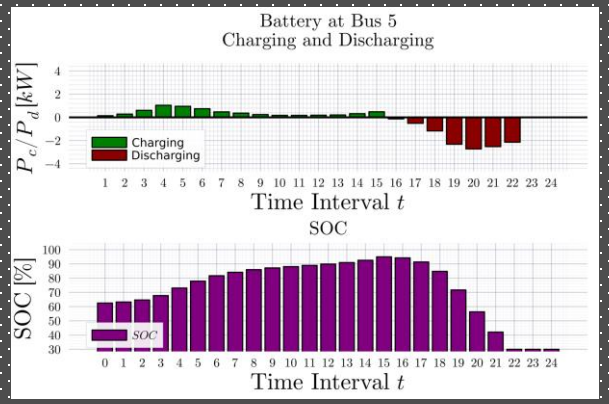
$ADS10_{1ph}$, $T = 24$

DDP

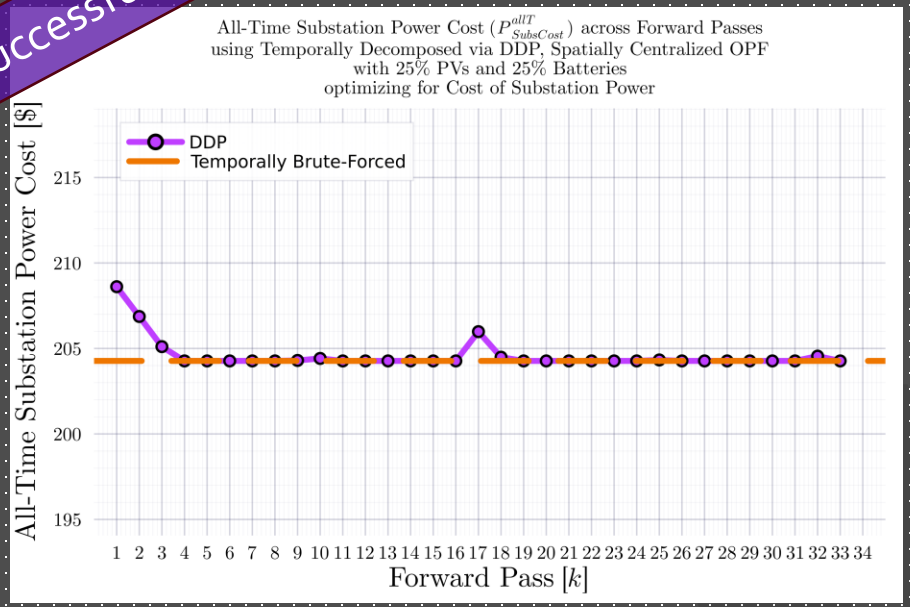
BruteForced (BF)

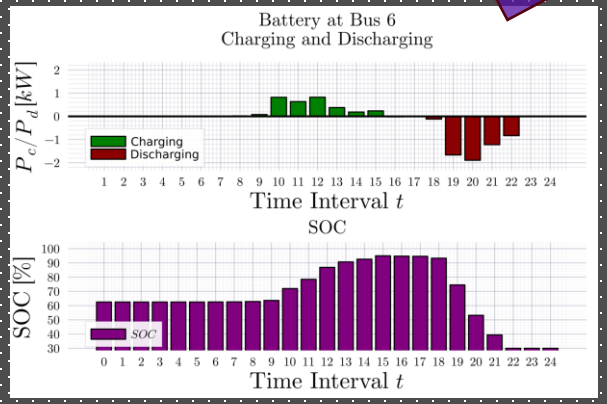


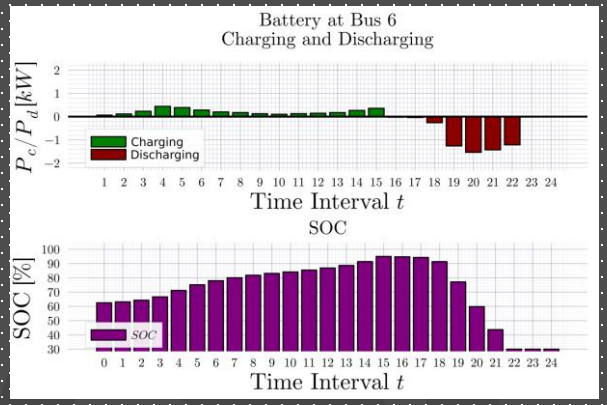




Looks 'successful'







$k_{Max} = 33$

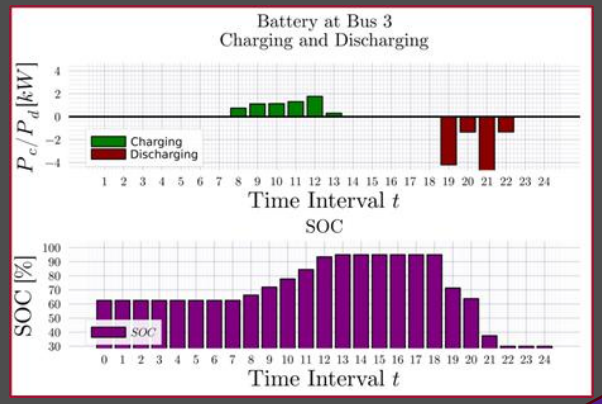
$IEEE123_{1ph}$, $T = 24$

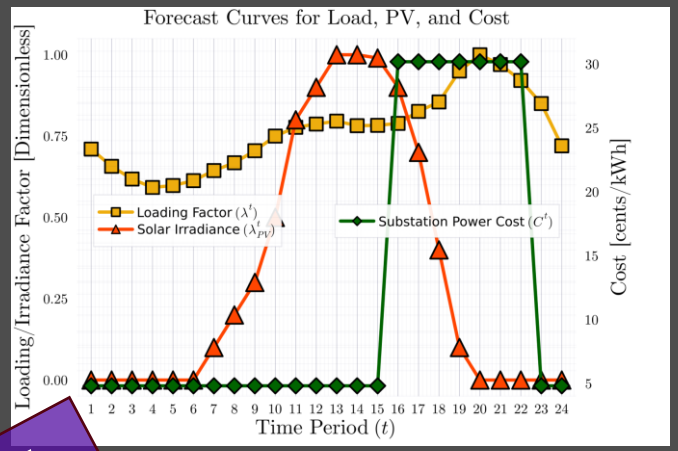
DDP Trajectory vs #Forward Passes $[k]$

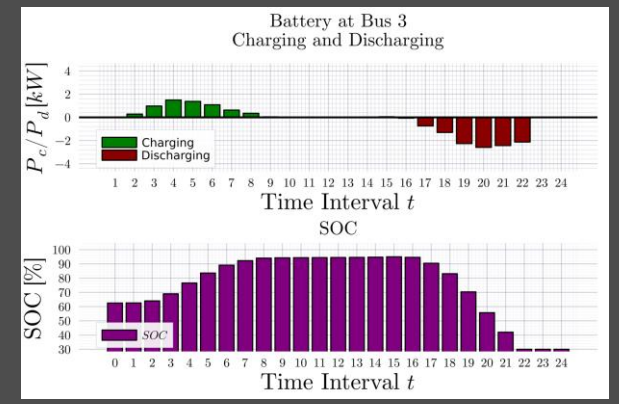
Terminal SOC Constraint Relaxed

DDP

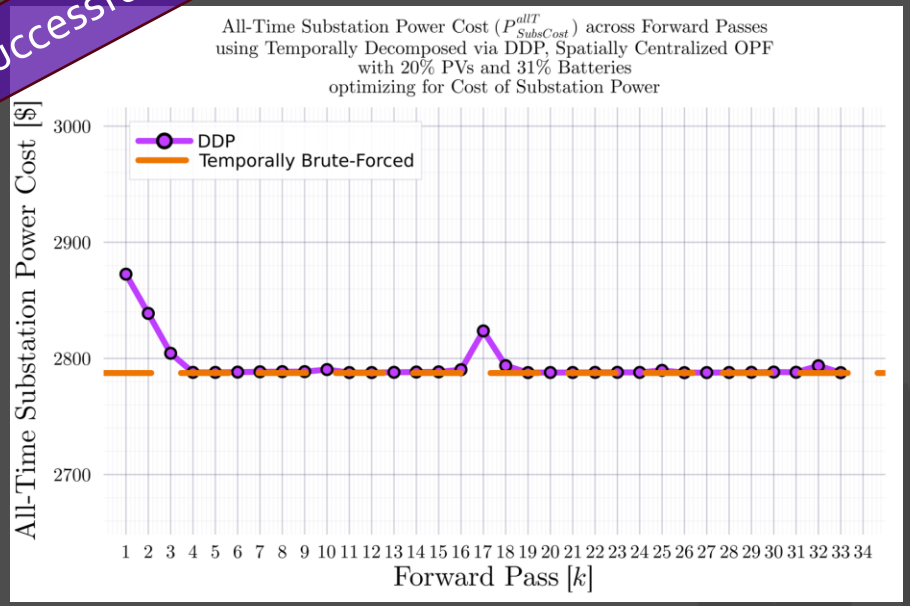
BruteForced (BF)

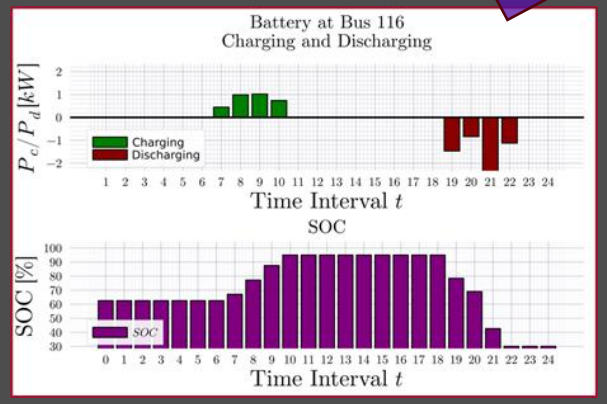


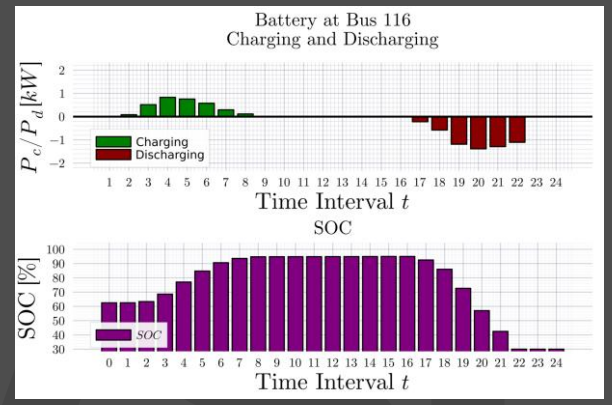




Looks 'successful'







$k_{Max} = 33$

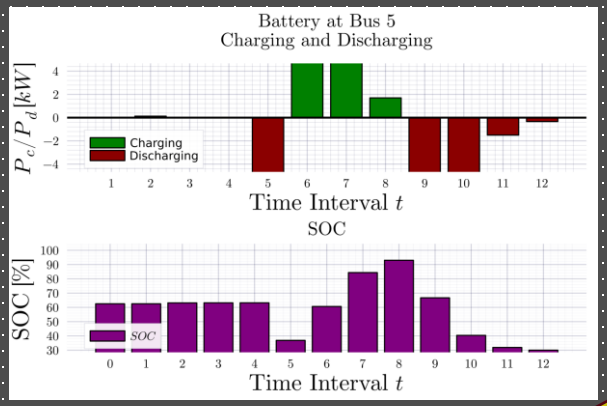
DDP Trajectory vs #Forward Passes $[k]$

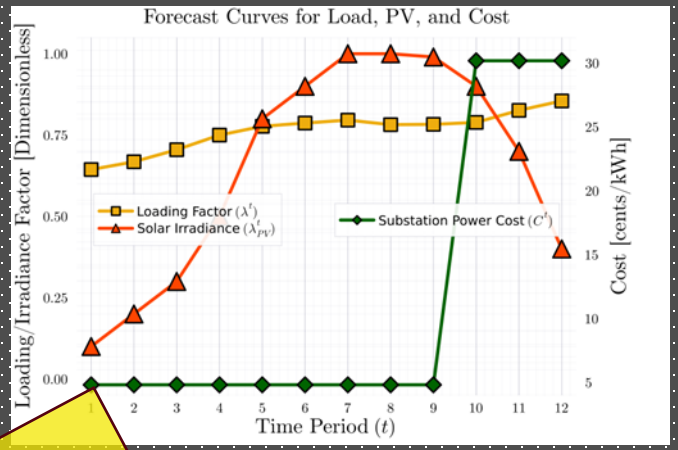
Terminal SOC Constraint Relaxed

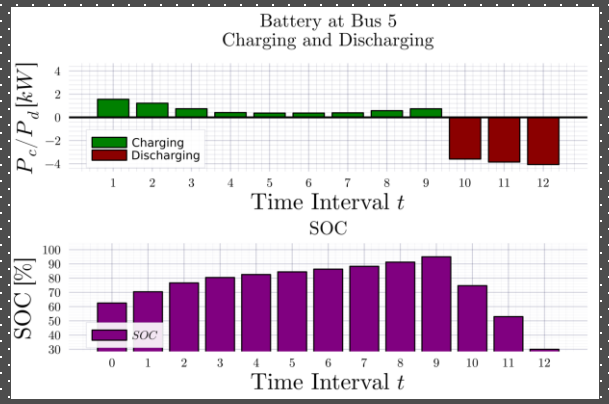
$ADS10_{1ph}$, $T = 12$

DDP

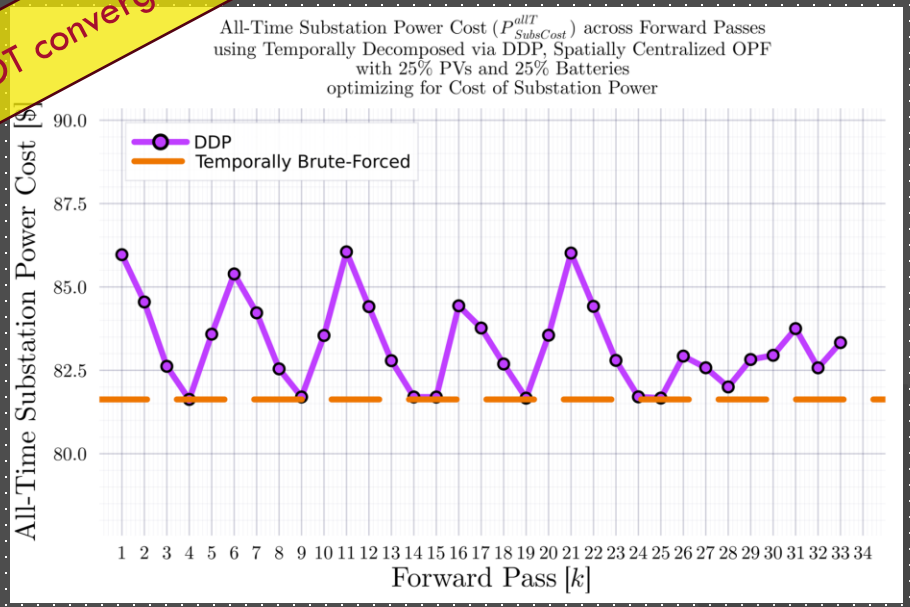
BruteForced (BF)

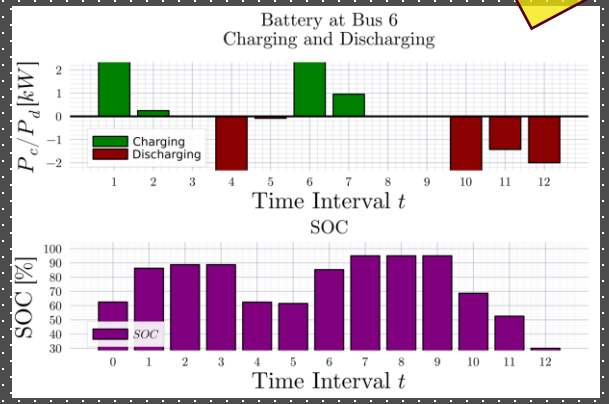


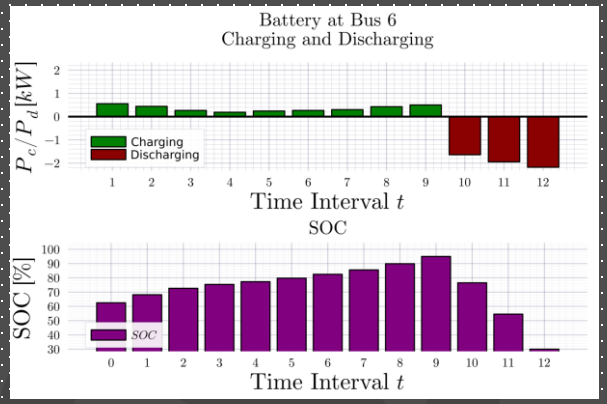




Will NOT converge







$k_{Max} = 33$

Even if not converging, it at least does 'hit' the optimal value periodically.

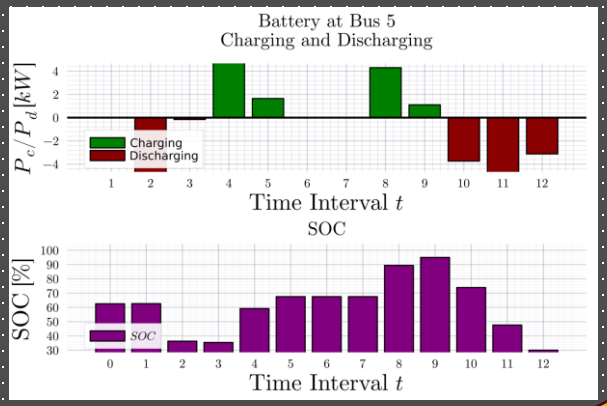
DDP Trajectory vs #Forward Passes $[k]$

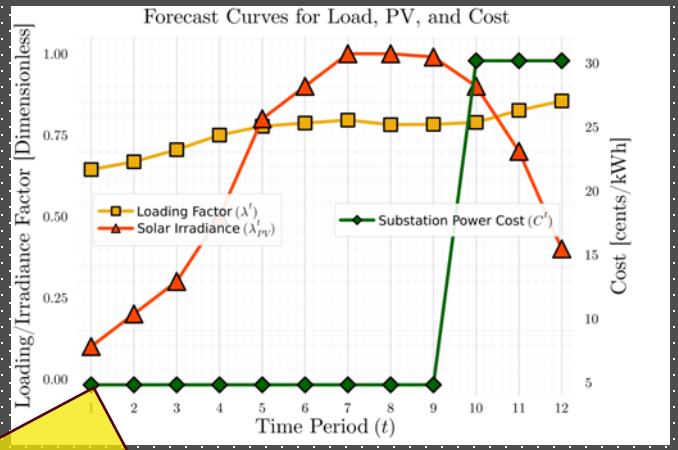
Terminal SOC Constraint Relaxed

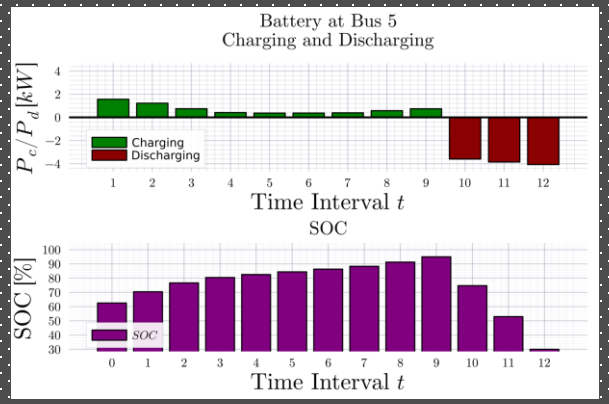
$ADS10_{1ph}$, $T = 12$

DDP

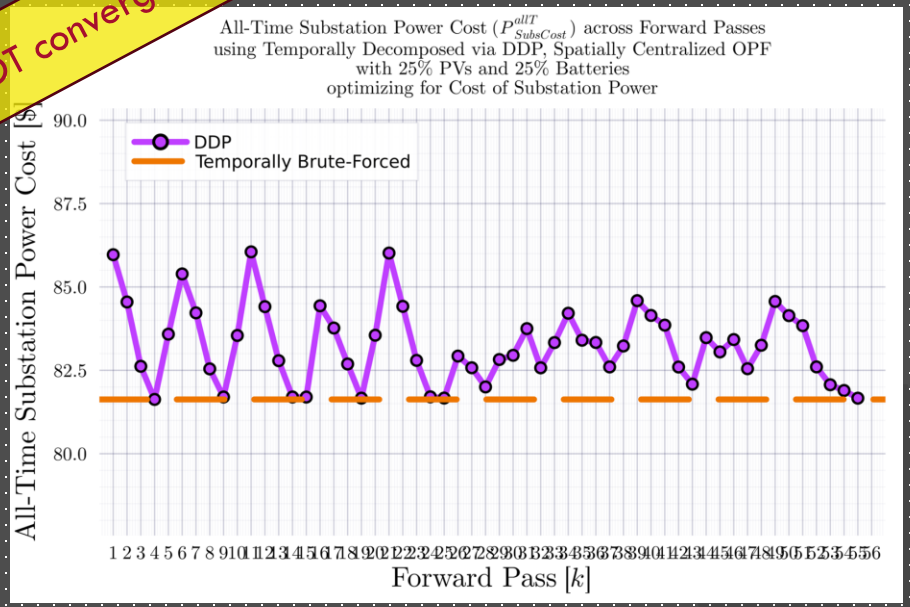
BruteForced (BF)

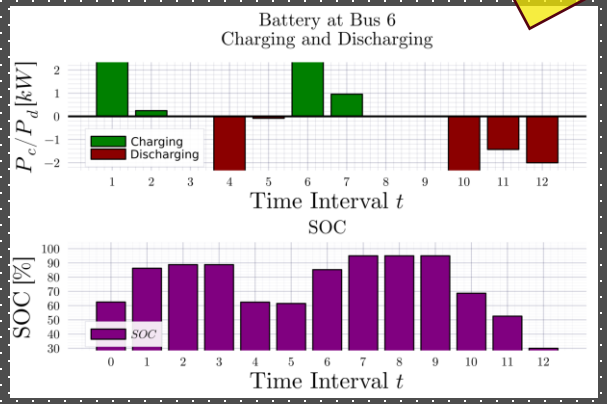


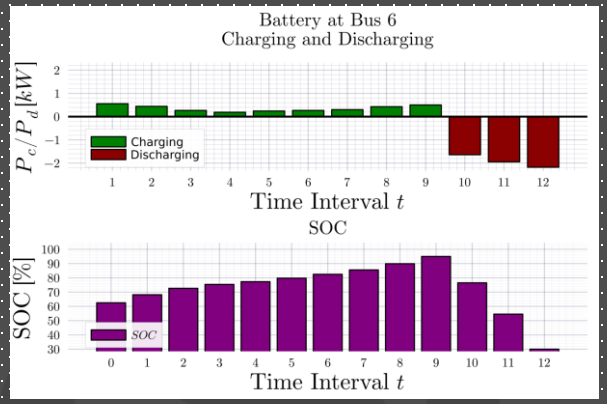




Will NOT converge







$k_{Max} = 55$

Even if not converging, it at least does 'hit' the optimal value periodically.

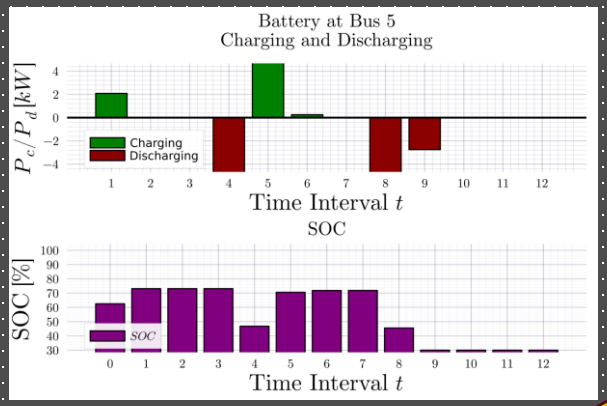
DDP Trajectory vs #Forward Passes $[k]$

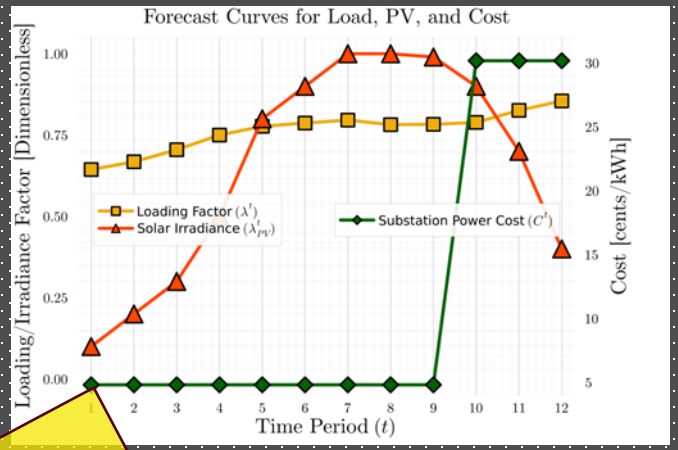
Terminal SOC Constraint Relaxed

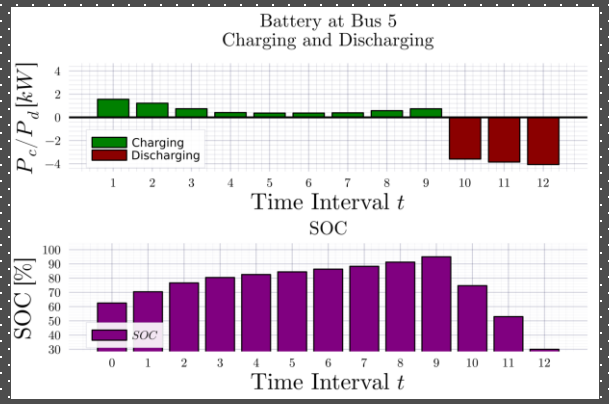
$ADS10_{1ph}$, $T = 12$

DDP

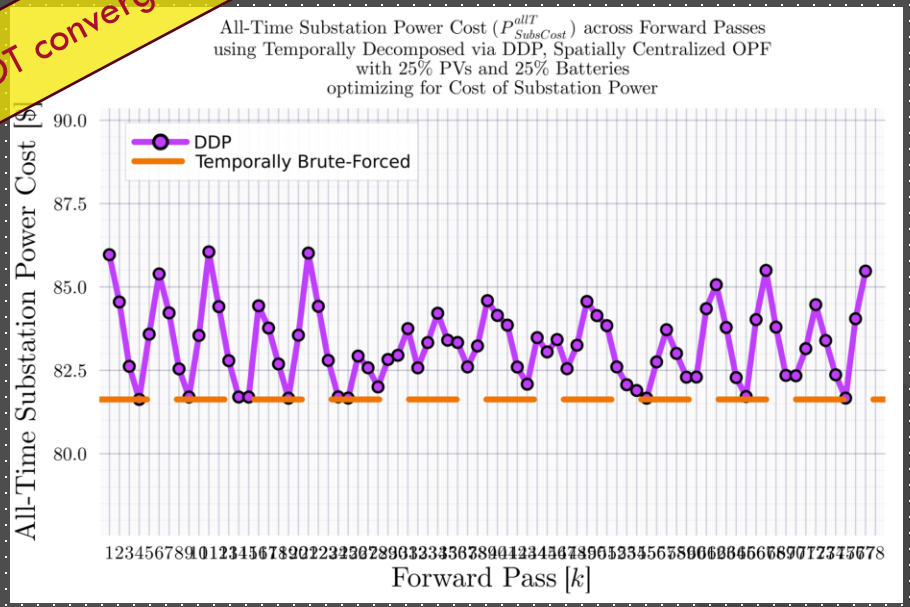
BruteForced (BF)

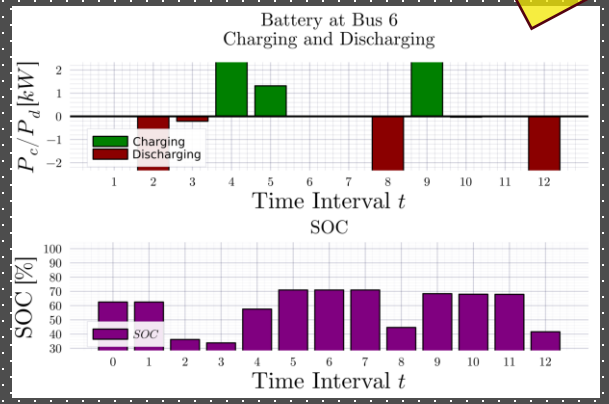


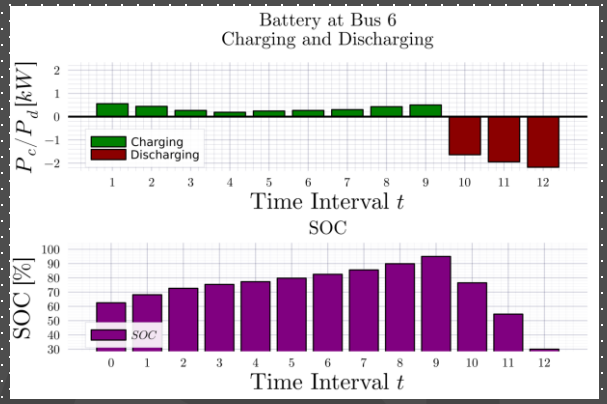




Will NOT converge







$k_{Max} = 77$

Even if not converging, it at least does 'hit' the optimal value periodically.

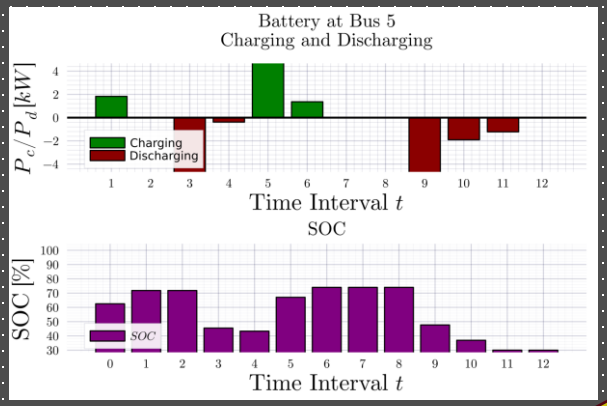
DDP Trajectory vs #Forward Passes $[k]$

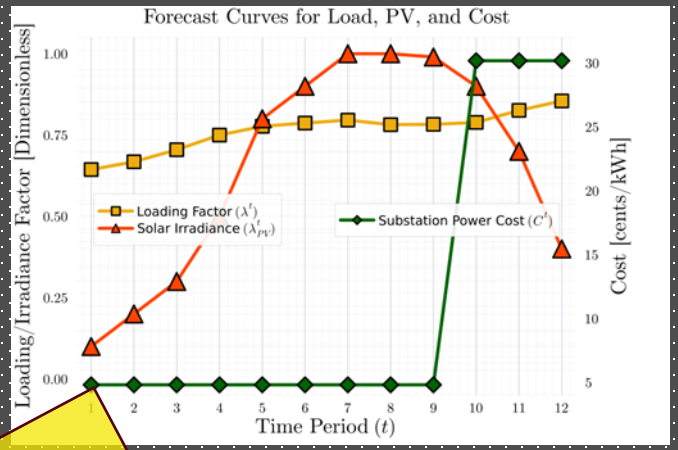
Terminal SOC Constraint Relaxed

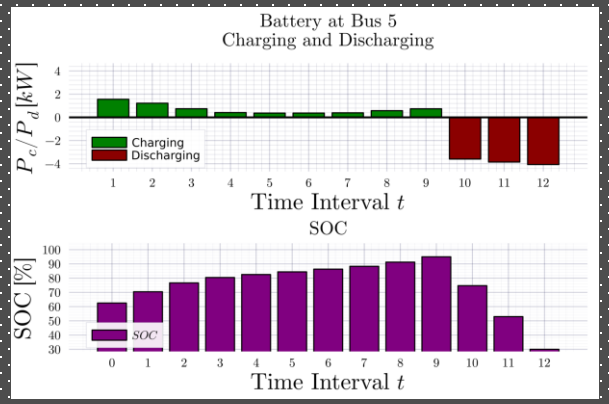
$ADS10_{1ph}$, $T = 12$

DDP

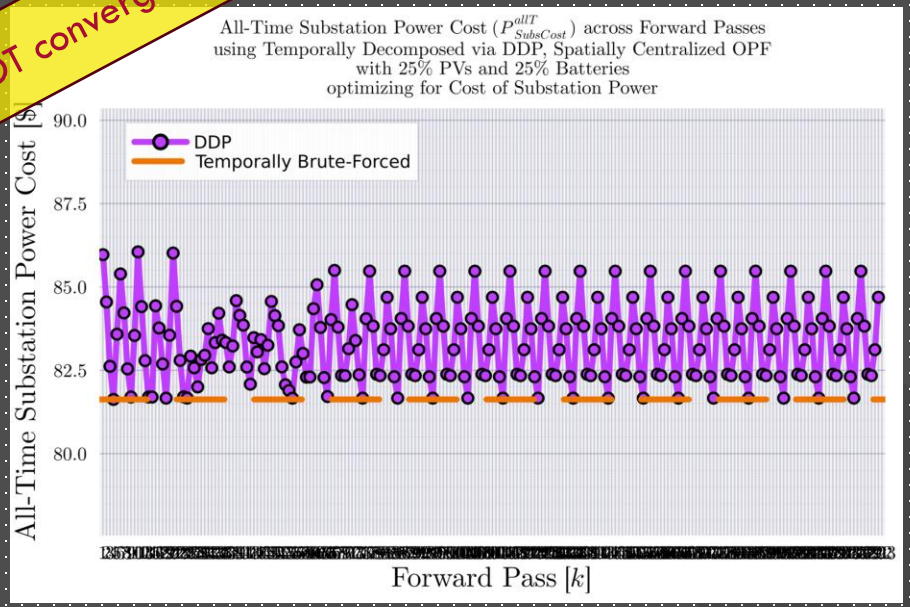
BruteForced (BF)

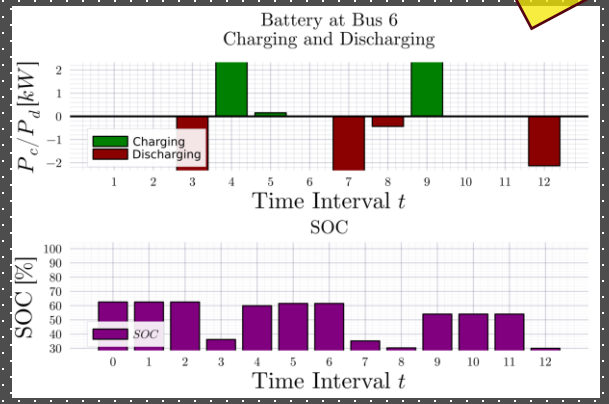


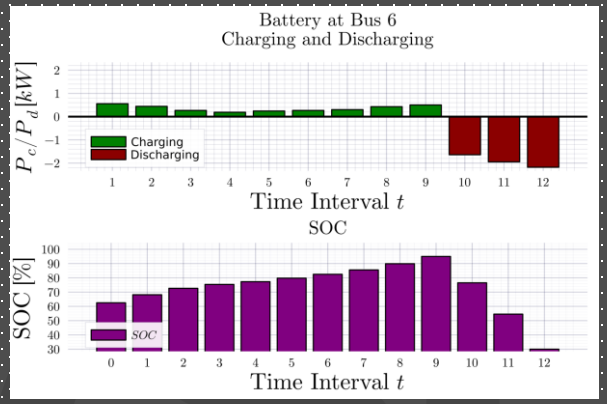




Will NOT converge







$k_{Max} = 222$

Even if not converging, it at least does 'hit' the optimal value periodically.

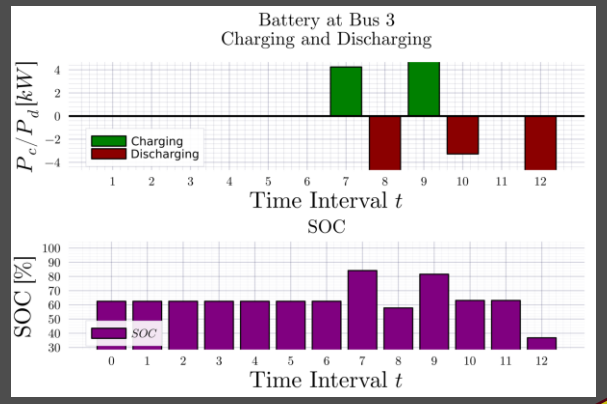
DDP Trajectory vs #Forward Passes $[k]$

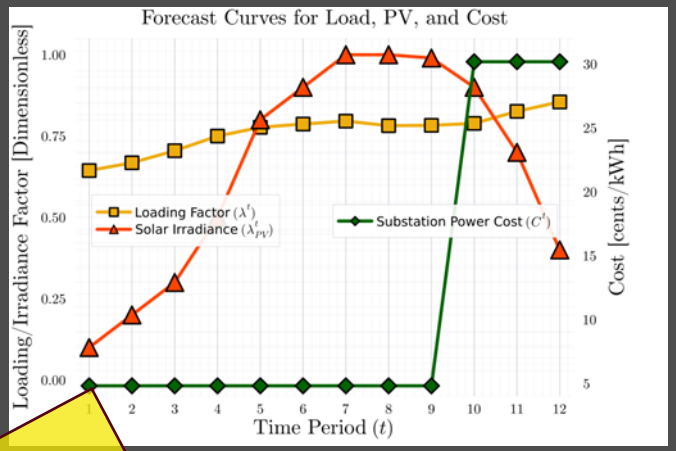
Terminal SOC Constraint Relaxed

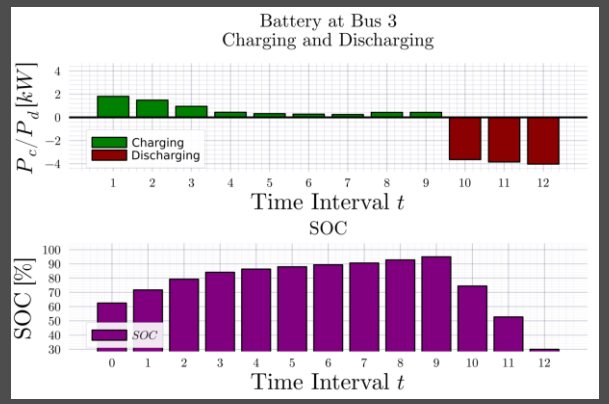
$IEEE123_{1ph}$, $T = 12$

DDP

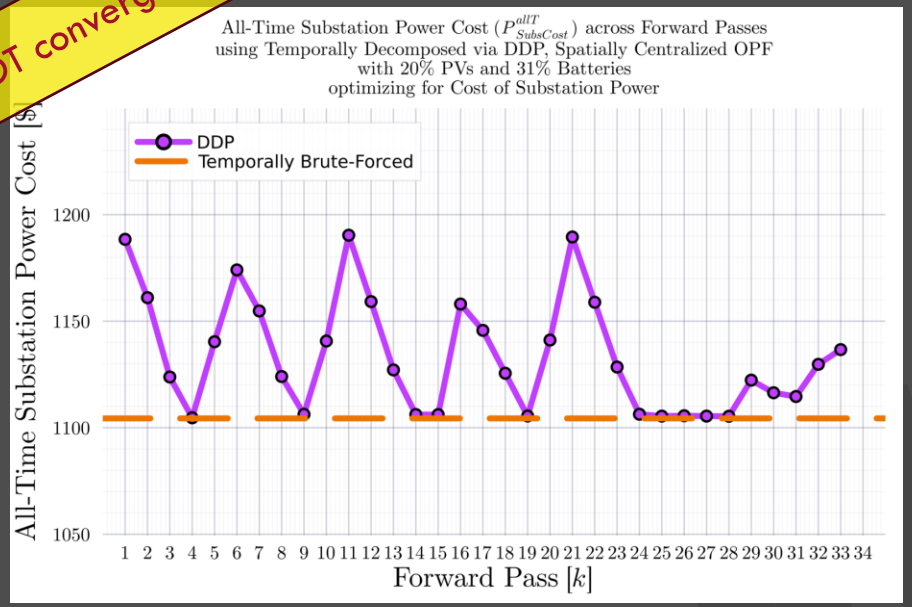
BruteForced (BF)

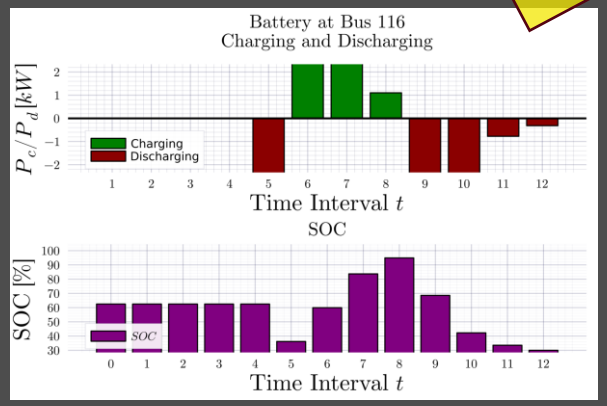


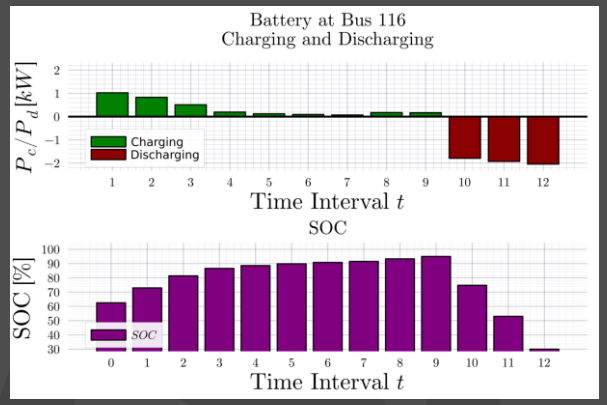




Will NOT converge







$k_{Max} = 33$

Even if not converging, it at least does 'hit' the optimal value periodically.