Spatially Distributed Multi Period Optimal Power Flow with Battery Energy Storage Systems

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Quick Overview of the contents of this presentation

- High level overview of a standard OPF problem for Distribution Systems and the MPOPF problem.
- Example describing spatial decomposition of the OPF problem (DOPF), and extension to MPDOPF.
- Modelling of the MPOPF problem
- A comparison of the proposed *Spatially Decomposed* MPDOPF results vs the *Centralized* MPCOPF algorithm

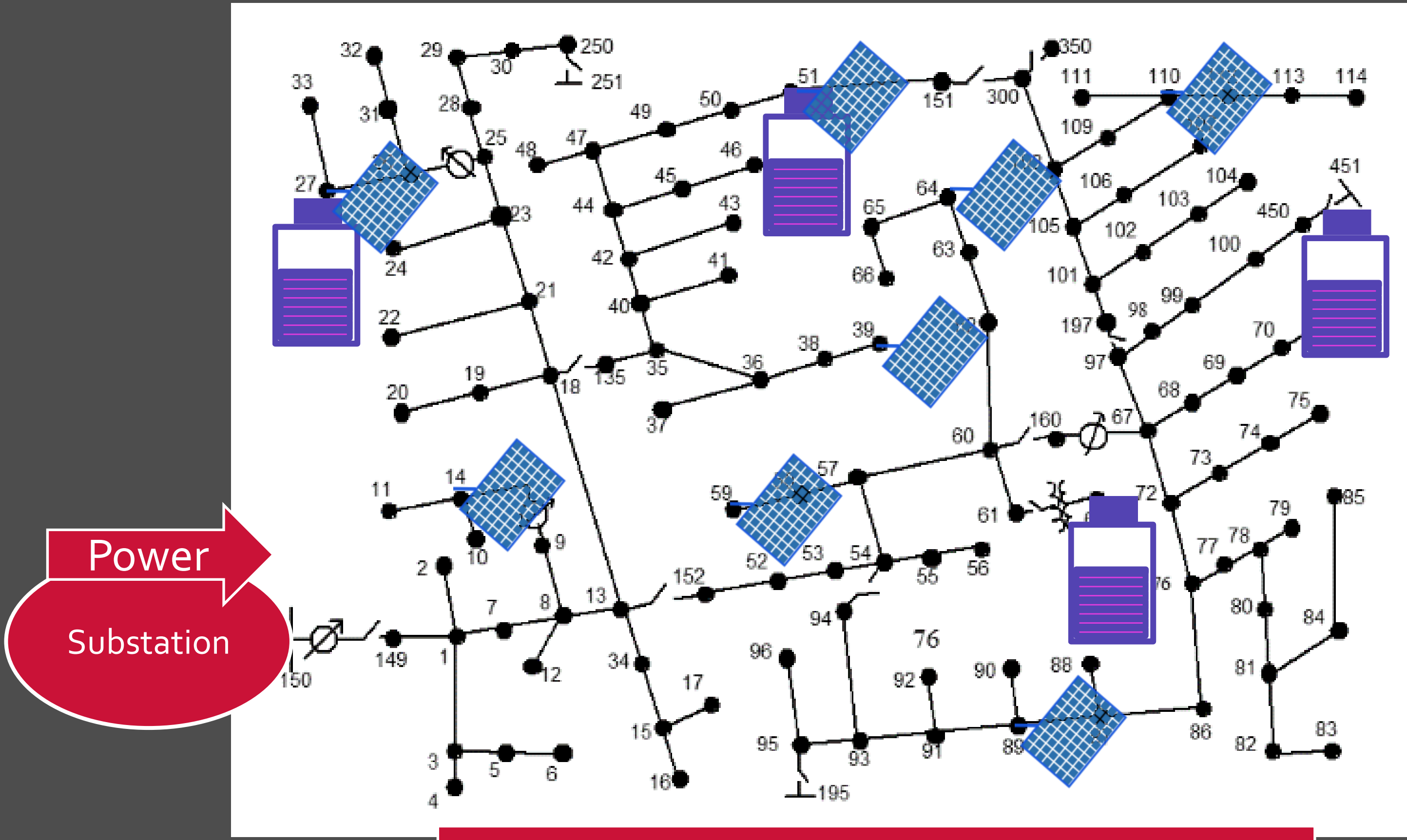
OPF, MPOPF, Scalability of the problem

The Optimal Power Flow (OPF) problem for Active Distribution Systems

Optimal?

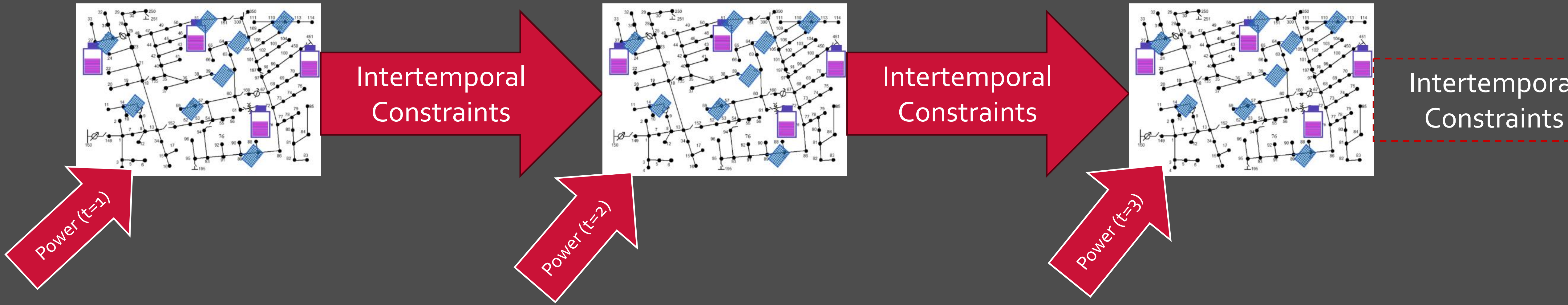
Feasible?

- $min.$ Desired Objective Function
- *subject to*
- Network Constraints
- Engineering Constraints
- Component Constraints (DERs, Batteries)



Loads and Controllable/Uncontrollable Components spread across a topology

The Multi Period Optimal Power Flow (MPOPF) problem for Active Distribution Systems



An example of **Intertemporal Constraints** is

Battery SOC Equation

$$B_j^t = B_j^{t-1} + \Delta t \eta_c P_{c_j}^t - \Delta t \frac{1}{\eta_d} P_{d_j}^t$$

Due to these intertemporal constraints, the optimization problem size becomes T times larger, becoming more difficult to solve.

Our MPOPF problem is unfortunately Nonlinear and Nonconvex – not scalable with respect to horizon duration (T)

Our proposed algorithm achieves the same objective function values as traditional methods in a fraction of the time, without using additional approximations or relaxations

Model to be explained in a few slides ..

$$\min \sum_{t=1}^{T} \Bigg[C^t P^t_{Subs} \Delta t + \alpha \sum_{j \in \mathcal{B}} \Big\{ (1-\eta_C) P^t_{C_j} + \left(\frac{1}{\eta_D} - 1 \right) P^t_{D_j} \Big\} \Bigg] \tag{1}$$

Subject to the constraints (2) to (12) as given below:

$$\sum_{(j,k)\in\mathcal{L}} \{P^t_{jk}\} - (P^t_{ij} - r_{ij} l^t_{ij}) = \left(P^t_{d_j} - P^t_{c_j}\right) + p^t_{D_j} - p^t_{L_j} \tag{2}$$

$$\sum_{(j,k)\in\mathcal{L}} \{Q^t_{jk}\} - (Q^t_{ij} - x_{ij} l^t_{ij}) = q^t_{D_j} + q^t_{B_j} - q^t_{L_j} \tag{3}$$

$$v^t_j = v^t_i - 2(r_{ij} P^t_{ij} + x_{ij} Q^t_{ij}) + \{r^2_{ij} + x^2_{ij}\} l^t_{ij} \tag{4}$$

$$(P^t_{ij})^2 + (Q^t_{ij})^2 = l^t_{ij} v^t_i \tag{5}$$

$$P^t_{Subs} \geq 0 \tag{6}$$

$$v^t_j \in [V^2_{min}, V^2_{max}] \tag{7}$$

$$q^t_{D_j} \in \left[-\sqrt{S_{D_{R,j}}{}^2 - p^t_{D_j}{}^2}, \sqrt{S_{D_{R,j}}{}^2 - p^t_{D_j}{}^2} \right] \tag{8}$$

$$B^t_j = B^{t-1}_j + \Delta t \eta_c P^t_{c_j} - \Delta t \frac{1}{\eta_d} P^t_{d_j} \tag{9}$$

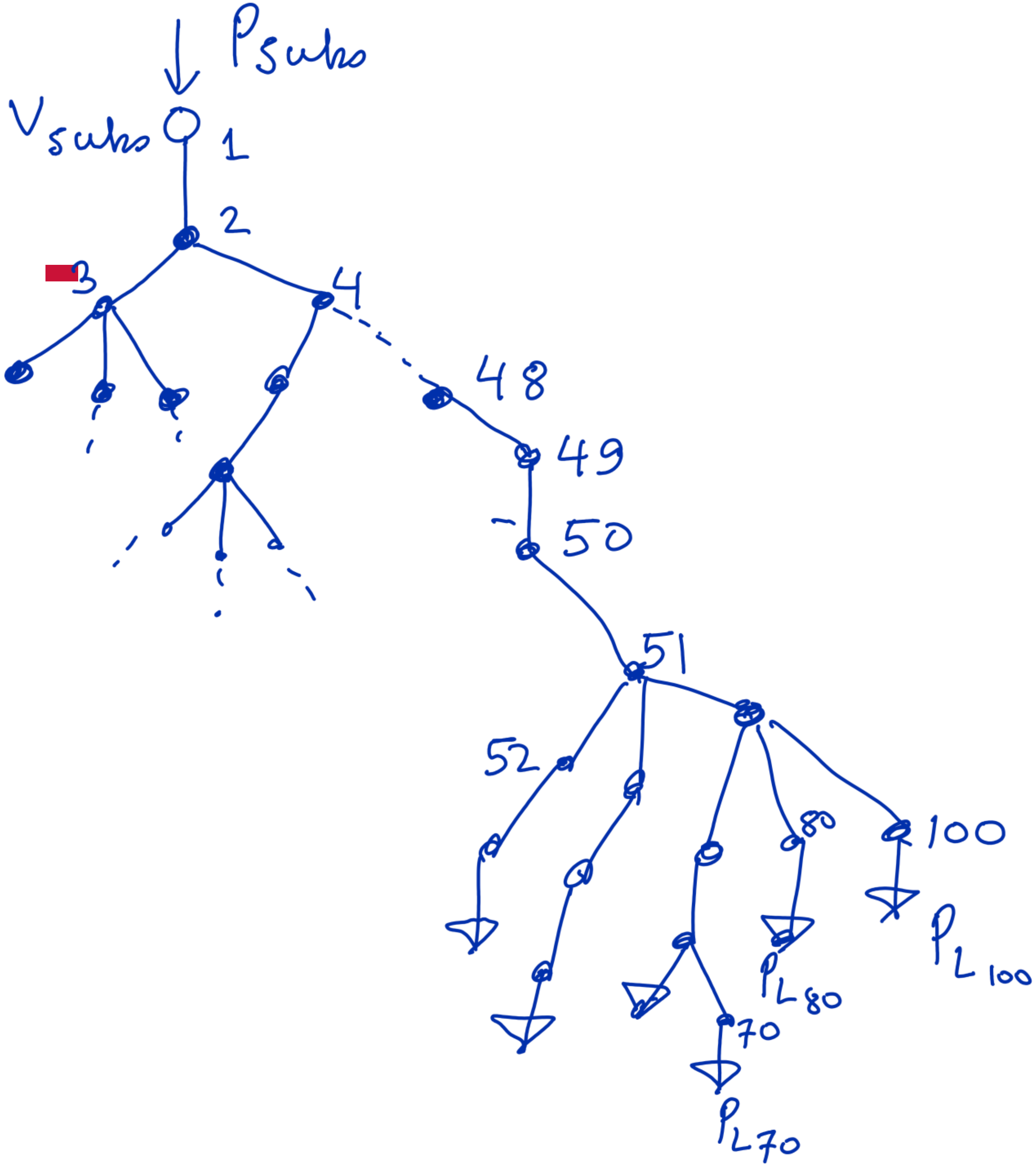
$$P^t_{c_j}, P^t_{d_j} \in \left[0, P_{B_{R_j}}\right], \quad B^0_j = B^T_j \tag{10}$$

$$q^t_{B_j} \in \left[-\sqrt{0.44} P_{B_{R,j}}, \sqrt{0.44} P_{B_{R,j}} \right] \tag{11}$$

$$B^t_j \in [soc_{min} B_{R,j}, soc_{max} B_{R,j}] \tag{12}$$

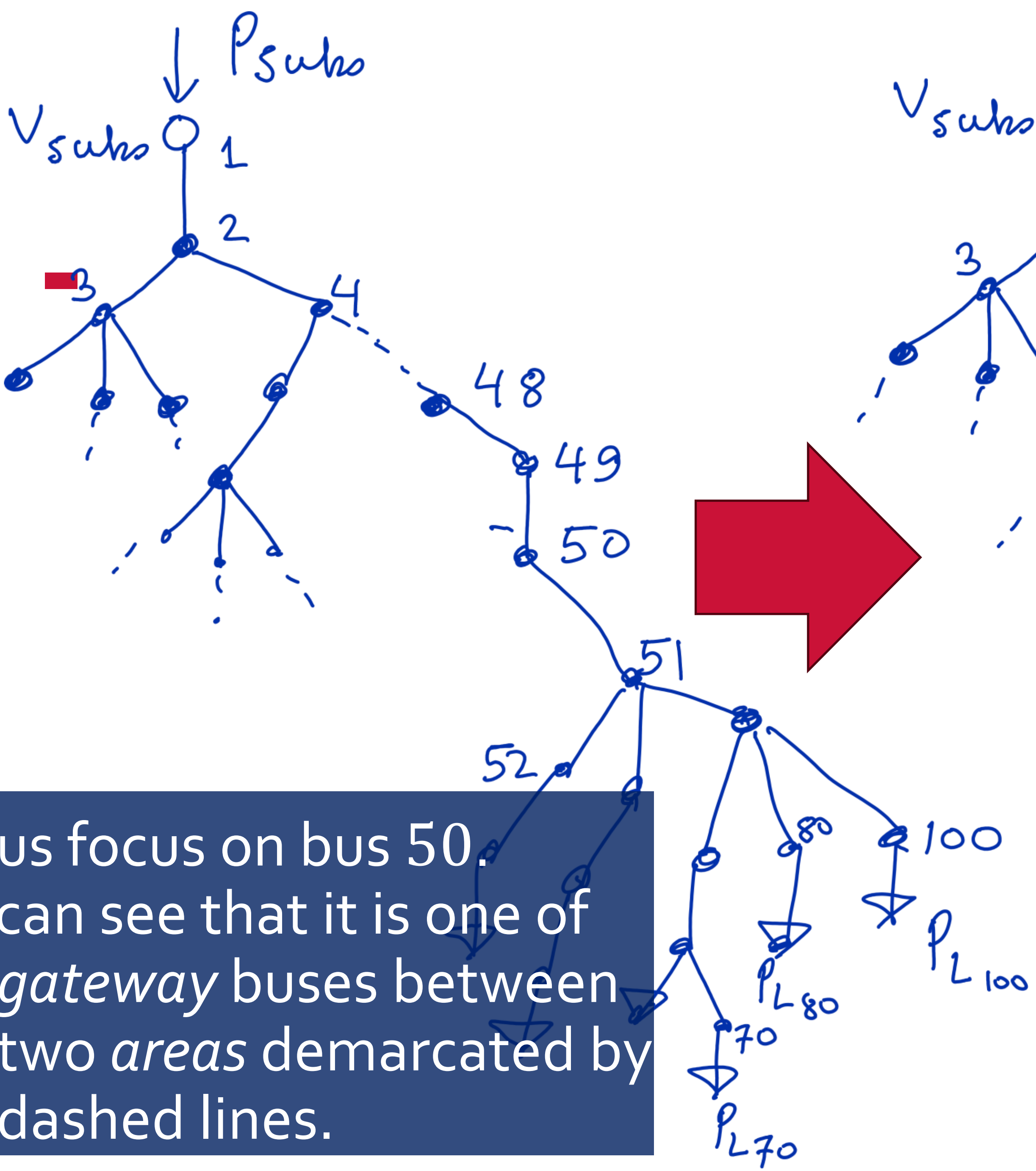
Spatially Decomposing the MPOPF Problem

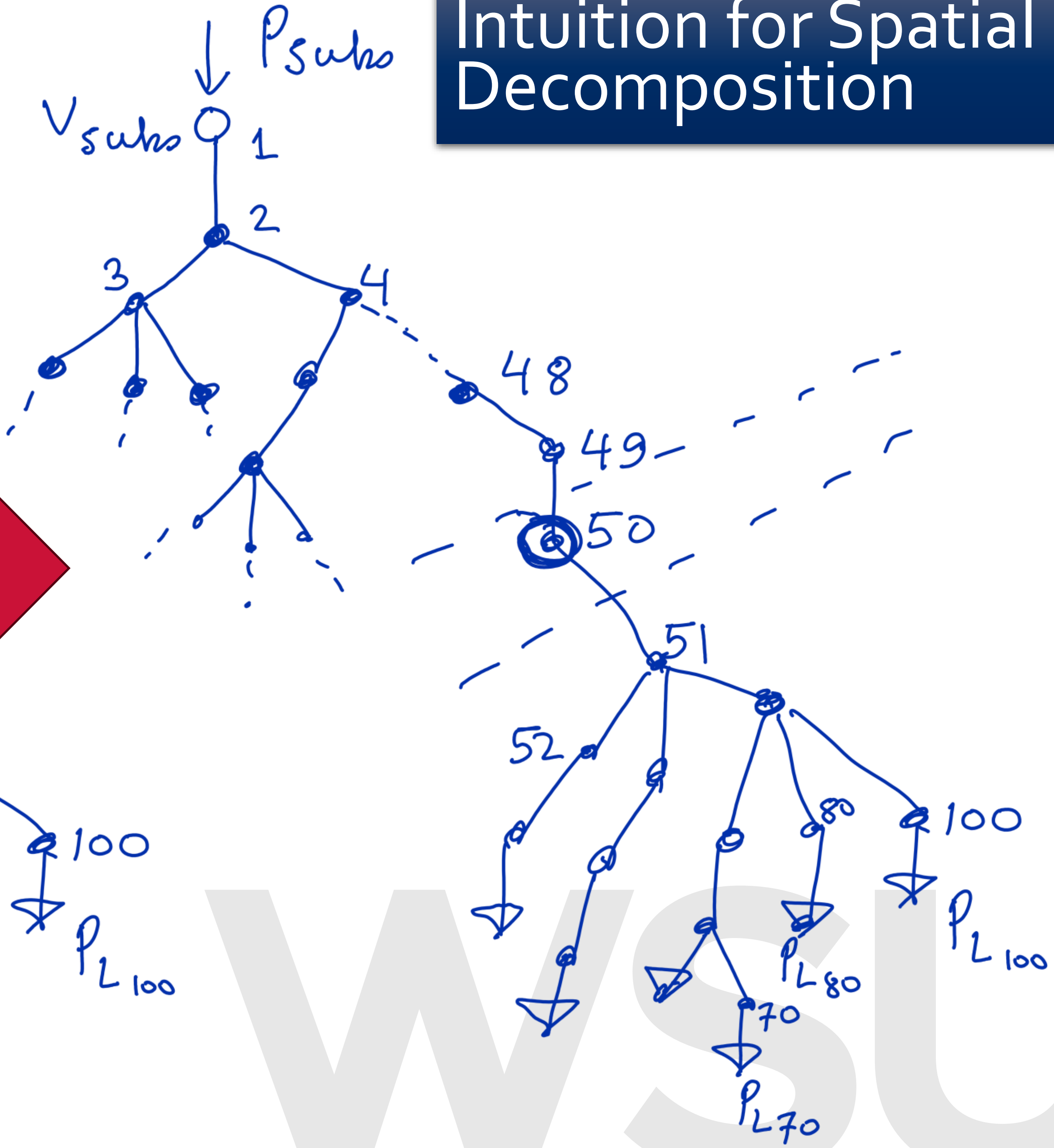
Intuition for Spatial Decomposition



Imagine we have a 100 bus distribution system.
Substation bus is 1.
We wish to solve for its OPF, but would like to avoid solving the whole system in one go.
Let us focus on bus 50.

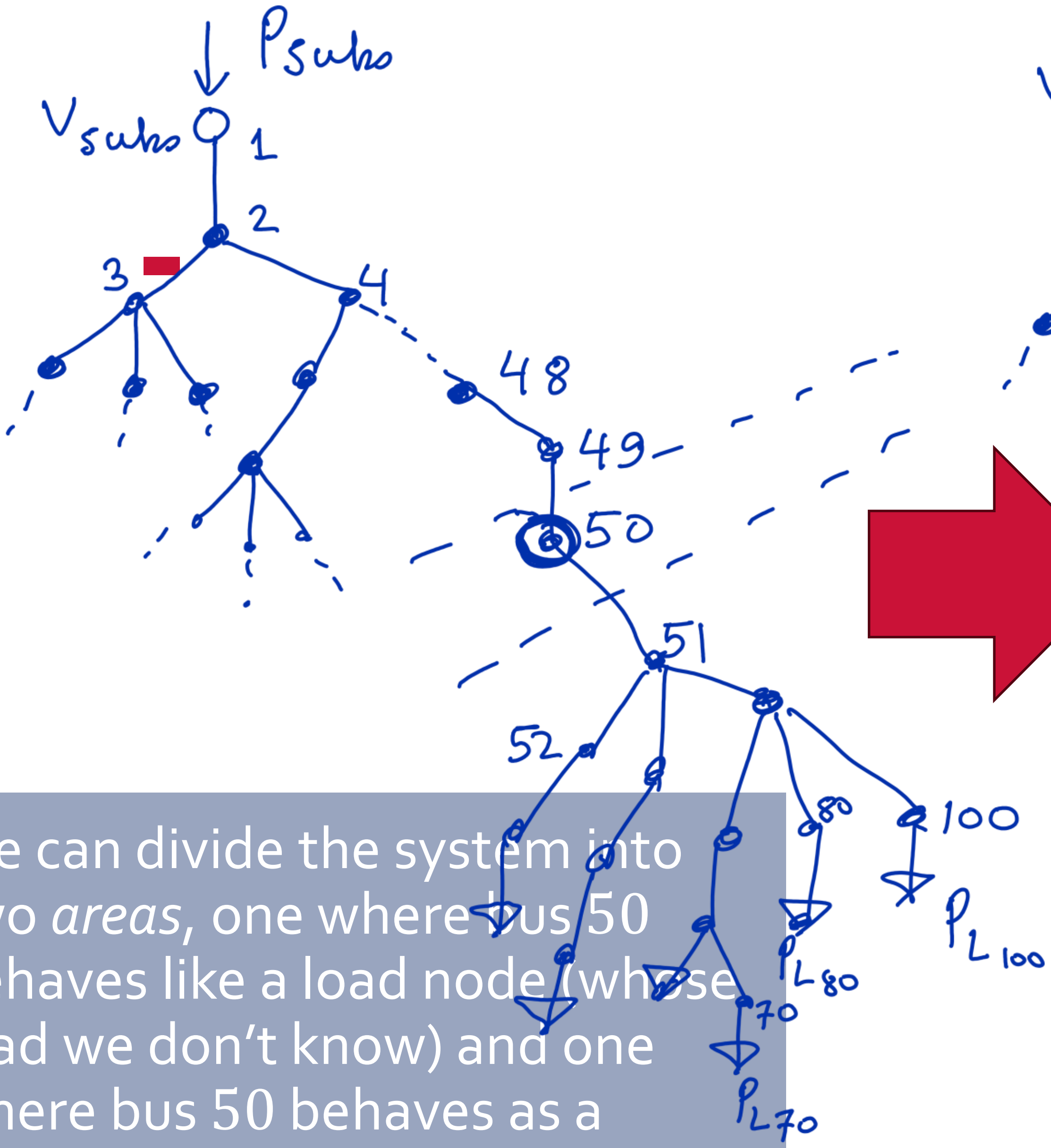
Intuition for Spatial Decomposition



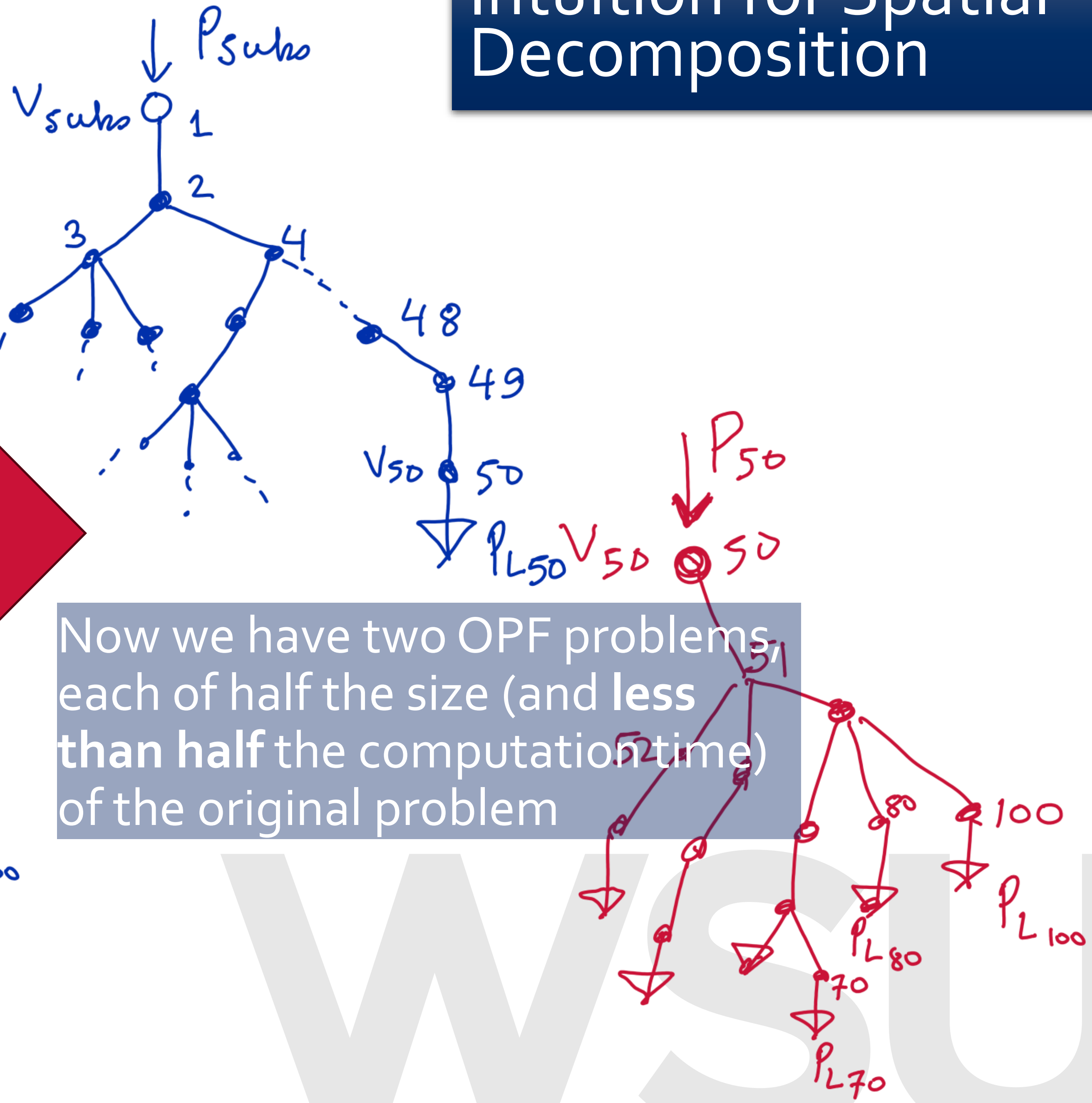


Let us focus on bus 50.
We can see that it is one of the *gateway* buses between the two *areas* demarcated by the dashed lines.

Intuition for Spatial Decomposition



We can divide the system into two *areas*, one where bus 50 behaves like a load node (whose load we don't know) and one where bus 50 behaves as a *substation* node (whose voltage we do don't know).



Now we have two OPF problems, each of half the size (and **less than half** the computation time) of the original problem

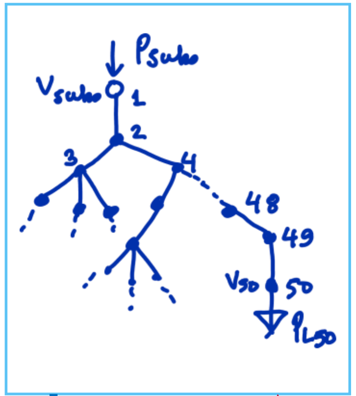
Intuition for Spatial Decomposition

We call this process DOPF where D stands for Spatially Distributed/Decomposed

Refer to [1] for details

Start

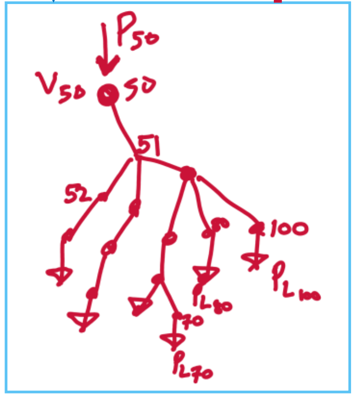
Each *macro-iteration* (k):



Area 1

Solve OPF to get for v_{50}
Set $v_{Subs} = v_{50}$

Solve OPF to get P_{50}
Set $P_{L50} = P_{50}$



Area 2

Are boundary variables *converged*?

No

Exchange boundary variables, and continue.

k++

Yes

Stop

Convergence means:

$$\left|v_{50}^{\{k+1\}} - v_{50}^{\{k\}}\right| < \epsilon$$

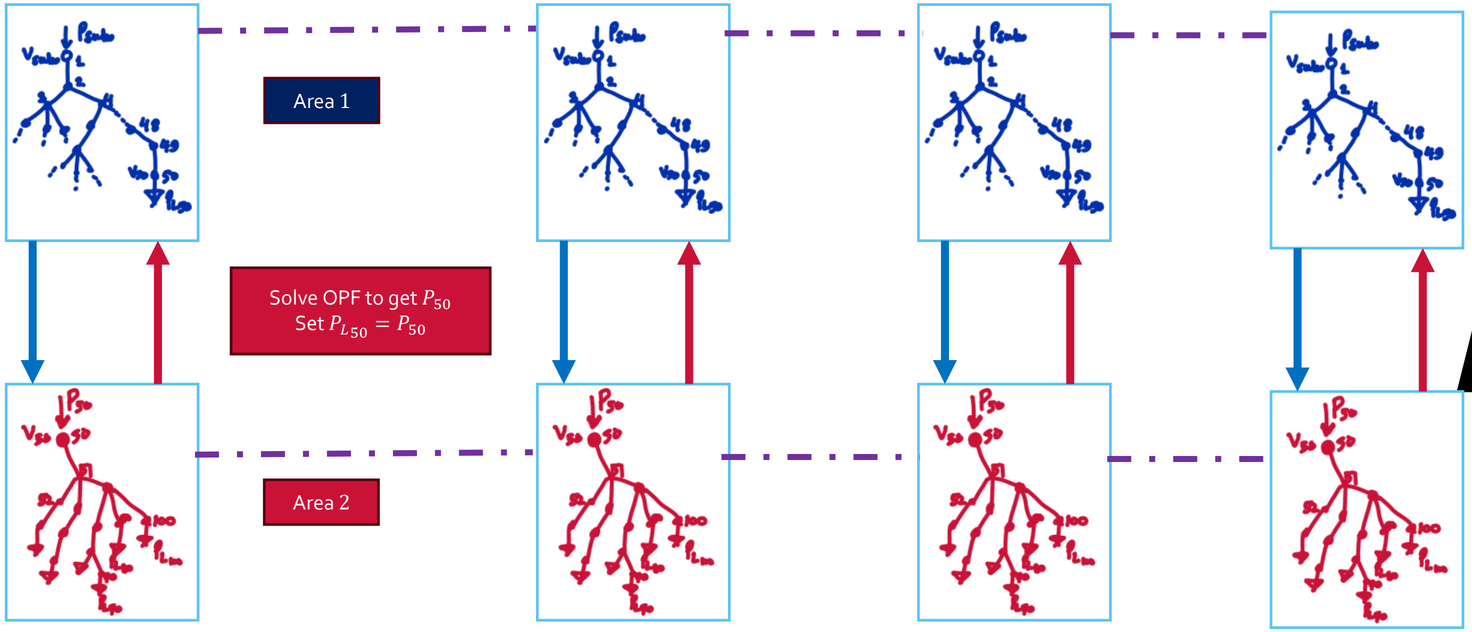
$$\left|S_{50}^{\{k+1\}} - S_{50}^{\{k\}}\right| < \epsilon$$

Intuition for Spatial Decomposition

We call this process MPDOPF

Start

Each *macro-iteration* (k):



Solve OPF to get for v_{50}
Set $v_{Subs} = v_{50}$

Solve OPF to get P_{50}
Set $P_{L50} = P_{50}$

$t = 1$ | $t = 2$ | $t = 3$ | $t = 4$

k++

Exchange boundary variables, and continue.

Are boundary variables *converged*?

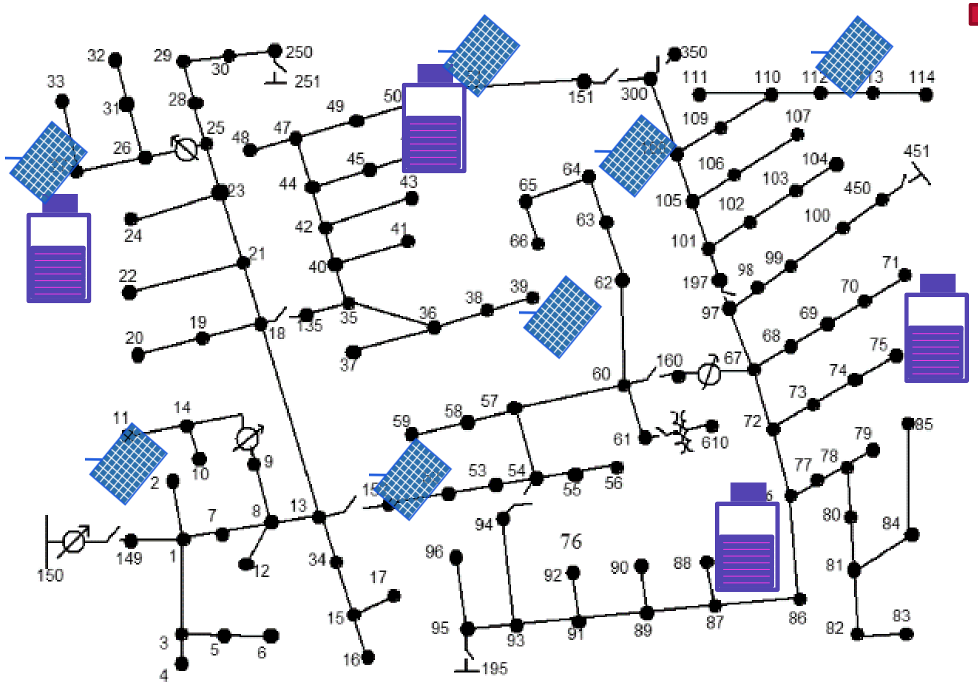
No

Yes

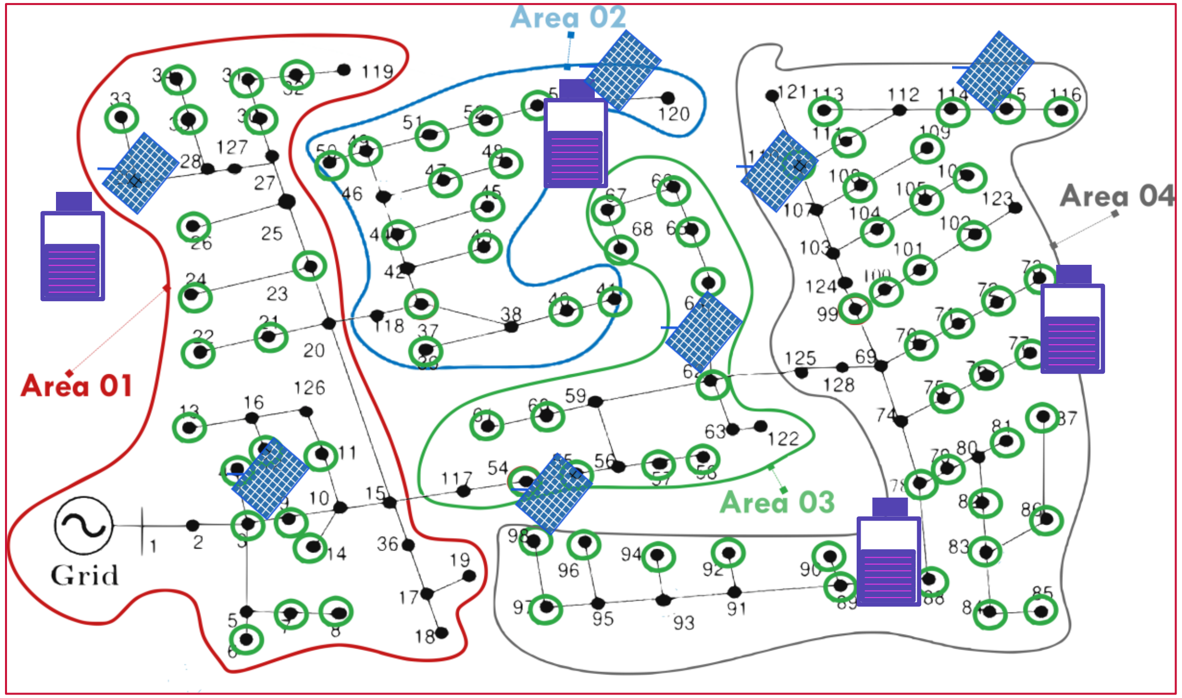
Stop

Modelling of our MPOPF problem

Network Topology and Decomposition



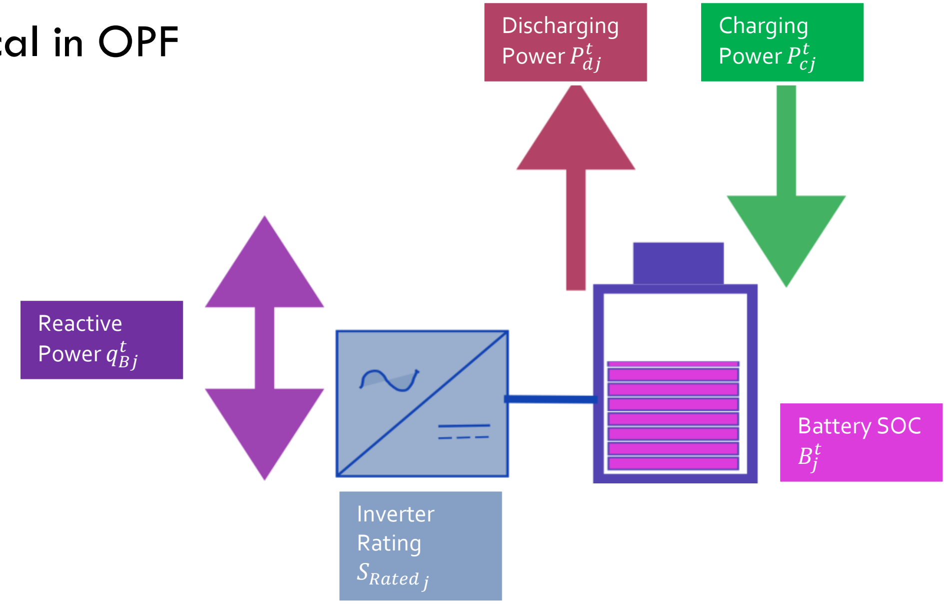
Original IEEE123 Bus Balanced Three Phase System With 20% PVs 30% Batteries



Same system but Decomposed into four areas for performing MPDOPF

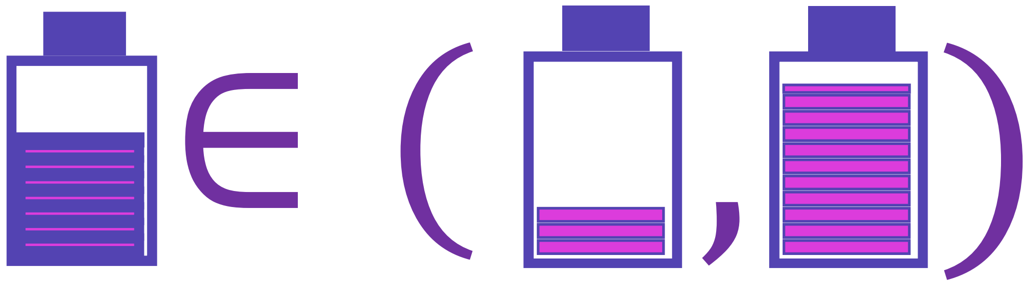
Modelling of Battery with Inverter

Note: Values here are typical in OPF literature [2, 3]



Battery SOC Equation

$$B_j^t = B_j^{t-1} + \Delta t \eta_c P_{c_j}^t - \Delta t \frac{1}{\eta_d} P_{d_j}^t$$



$$B_j^t \in [soc_{min}, soc_{max}] * E_{Rated,j}$$

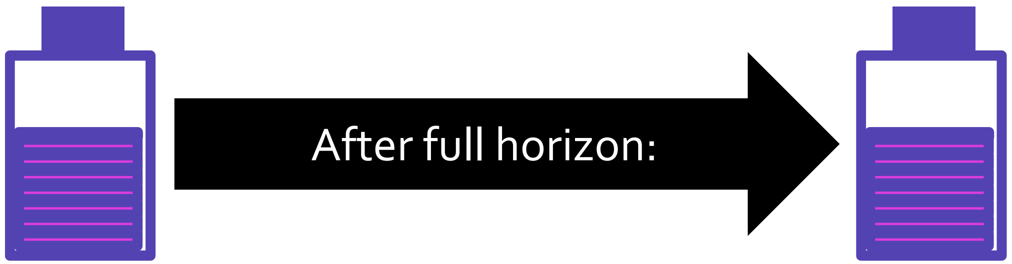
Battery SOC Limits

$$soc_{min}, soc_{max} = 0.30, 0.95$$

Battery SOC Limits

$$\eta_c, \eta_d = 0.95, 0.95$$

Charging/Discharging Efficiencies



$$B_j^0 = 0.625 E_{Rated,j}$$

Initial SOC

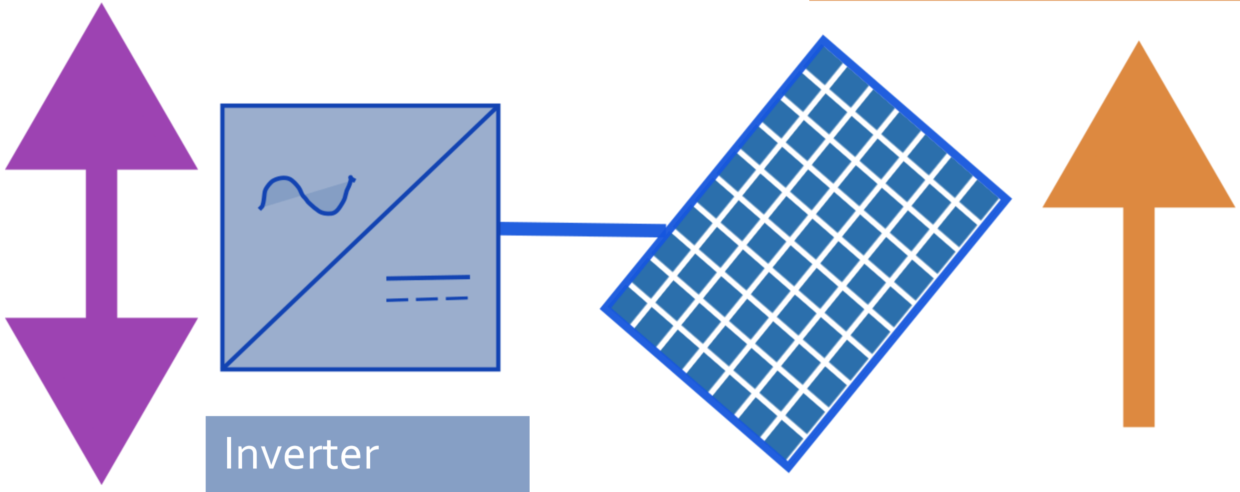
$$B_j^T = B_j^0$$

Terminal SOC constraint

Solar PVs with Inverters

PV Real Power P_{Dj}^{t}

Reactive Power q_{Dj}^{t}



Inverter Rating $S_{Rated\,j}$

$$S_{Rated,j} = 1.2P_{Rated,j}$$

$$q_{Dj}^{t} \in [-\sqrt{0.44}, \sqrt{0.44}] * S_{Rated,j}$$

Relationships between the rated powers

$$(p_{Dj}^{t})^2 + (q_{Dj}^{t})^2 \leq (S_{Rated,j})^2$$

Solar Inverter Rating Limit

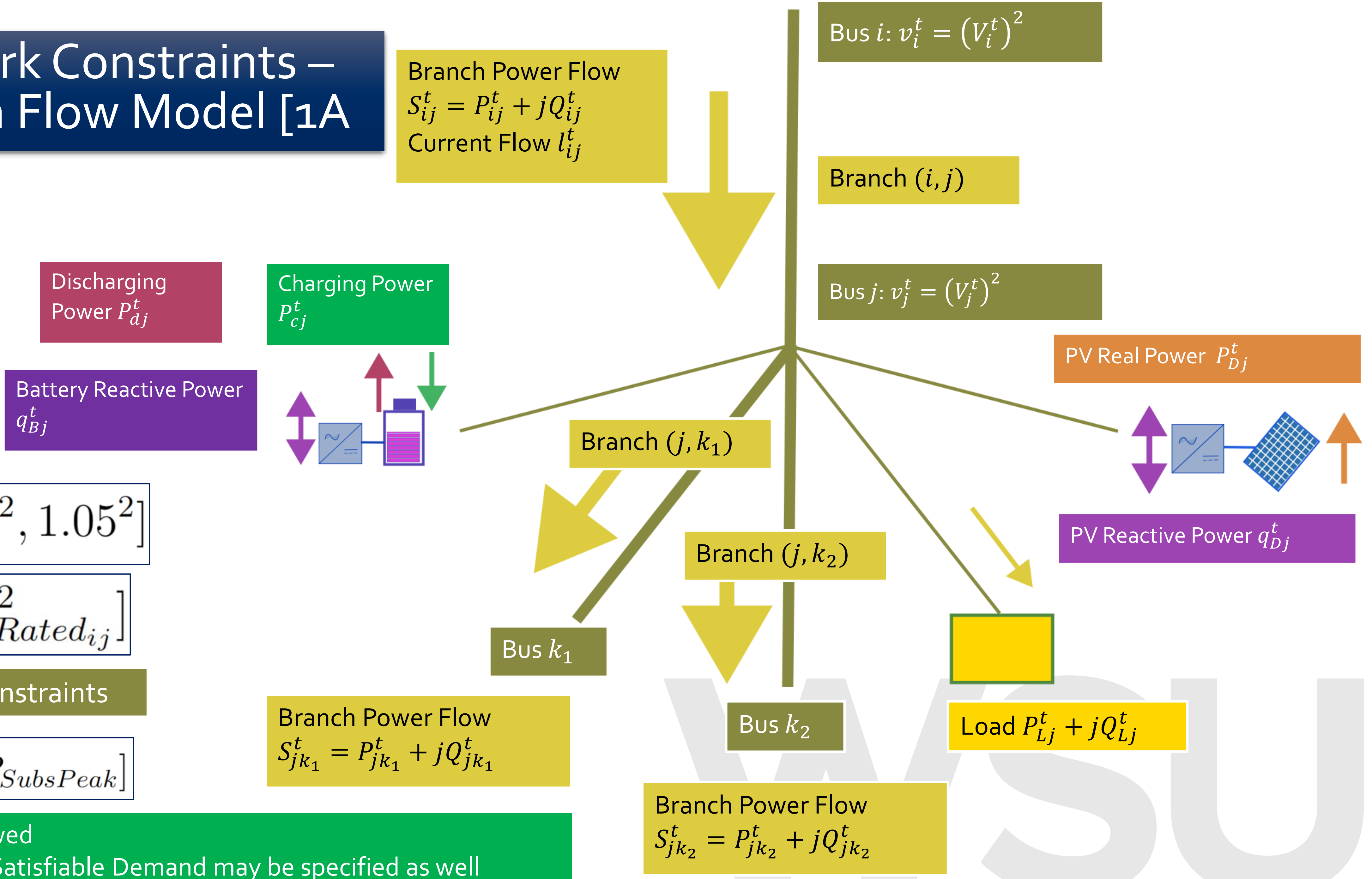
Typical $P_{Rated\,j} \in [5, 40]$ kW

Typically, $P_{Rated\,j} = \frac{1}{3} * P_{Load\,j}$

Typical $q_{Rated\,j} \in [1, 10]$ kVAr

Typically, $q_{Rated\,j} = \frac{1}{3} * Q_{Load\,j}$

Network Constraints – Branch Flow Model [1A



$$v_j^t \in [0.95^2, 1.05^2]$$

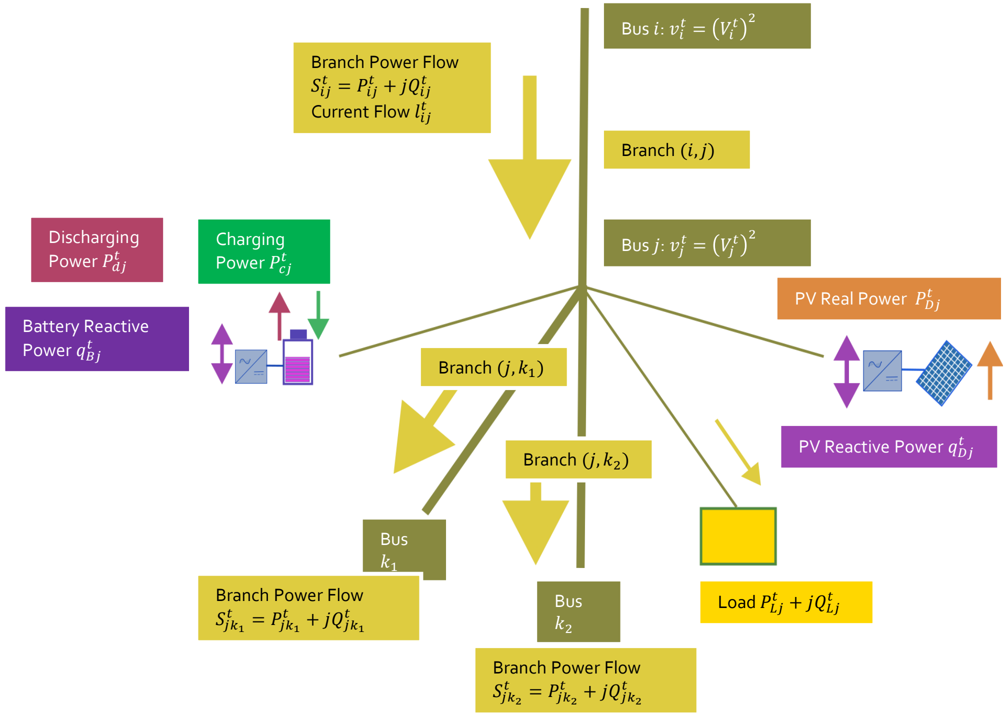
$$l_{ij}^t \in [0, I_{Rated_{ij}}^2]$$

Engineering Constraints

$$P_{Subs}^t \in [0, P_{SubsPeak}]$$

No backflow allowed
Optionally, Peak Satisfiable Demand may be specified as well

Network Constraints – Branch Flow Model [1A]



Node j Real Power Balance

$$p_j^t = \sum_{(j,k)\in\mathcal{L}} P_{jk}^t - \sum_{(i,j)\in\mathcal{L}} \{P_{ij}^t - r_{ij} l_{ij}^t\} - P_{d_j}^t + P_{c_j}^t$$

Node j Reactive Power Balance

$$q_j^t = \sum_{(j,k)\in\mathcal{L}} Q_{jk}^t - \sum_{(i,j)\in\mathcal{L}} \{Q_{ij}^t - x_{ij} l_{ij}^t\} - q_{D_j}^t - q_{B_j}^t$$

KVL across branch (i, j)

$$v_j^t = v_i^t + \{r_{ij}^2 + x_{ij}^2\} l_{ij}^t - 2(r_{ij} P_{ij}^t + x_{ij} Q_{ij}^t)$$

Current Magnitude across branch (i, j)

$$l_{ij}^t = \frac{(P_{ij}^t)^2 + (Q_{ij}^t)^2}{v_i^t}$$

Net Generation at Node j

$$p_j^t = {p_{Dj}}^t - {p_{Lj}}^t$$

$$q_j^t = -{q_{Lj}}^t$$

Full Optimization Model – Balanced Three-Phase Nonlinear OPF

min. Desired Objective Function, *(appended with an Battery Loss Function) [4]*

subject to

Network Constraints

Engineering Constraints

Component Constraints (DERs, Batteries)

$$\min \sum_{t=1}^{T} \left[C^t P^t_{Subs}\Delta t + \alpha \sum_{j\in\mathcal{B}} \left\{ (1-\eta_C) P^t_{C_j} + \left(\frac{1}{\eta_D} - 1\right) P^t_{D_j} \right\} \right] \tag{1}$$

Subject to the constraints (2) to (12) as given below:

$$\sum_{(j,k)\in\mathcal{L}} \{P^t_{jk}\} - (P^t_{ij} - r_{ij} l^t_{ij}) = \left(P^t_{d_j} - P^t_{c_j}\right) + p^t_{D_j} - p^t_{L_j} \tag{2}$$

$$\sum_{(j,k)\in\mathcal{L}} \{Q^t_{jk}\} - (Q^t_{ij} - x_{ij} l^t_{ij}) = q^t_{D_j} + q^t_{B_j} - q^t_{L_j} \tag{3}$$

$$v^t_j = v^t_i - 2(r_{ij}P^t_{ij} + x_{ij}Q^t_{ij}) + \{r^2_{ij} + x^2_{ij}\}\, l^t_{ij} \tag{4}$$

$$(P^t_{ij})^2 + (Q^t_{ij})^2 = l^t_{ij} v^t_i \tag{5}$$

$$P^t_{Subs} \geq 0 \tag{6}$$

$$v^t_j \in [V^2_{min}, V^2_{max}] \tag{7}$$

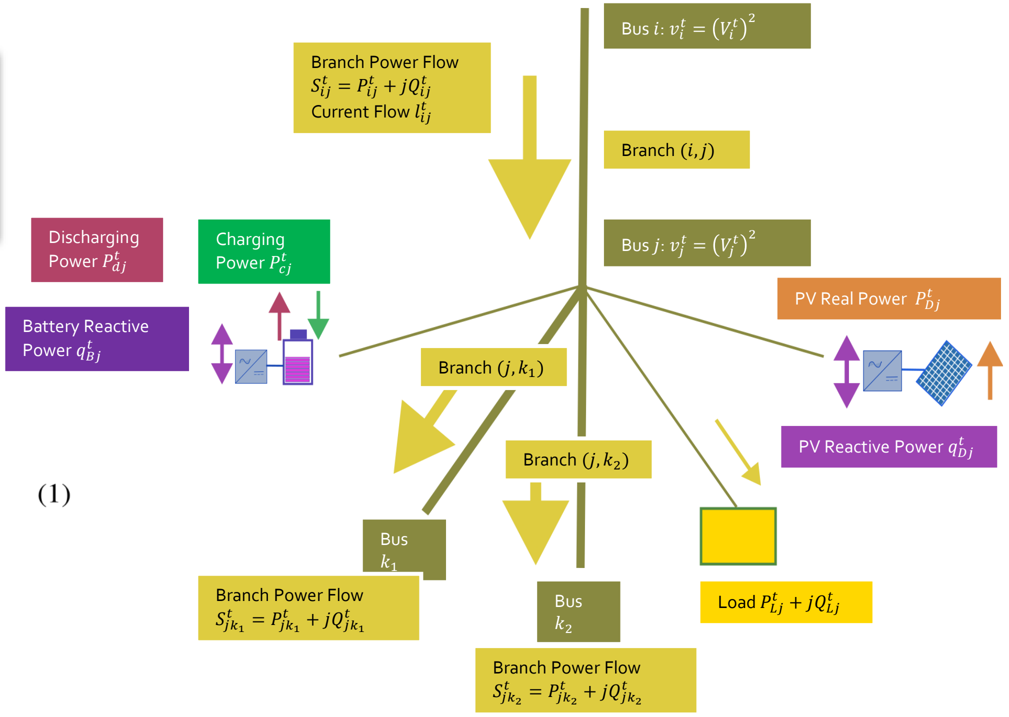
$$q^t_{D_j} \in \left[-\sqrt{{S_{D_{R,j}}}^2 - {p^t_{D_j}}^2}, \sqrt{{S_{D_{R,j}}}^2 - {p^t_{D_j}}^2}\right] \tag{8}$$

$$B^t_j = B^{t-1}_j + \Delta t \eta_c P^t_{c_j} - \Delta t \frac{1}{\eta_d} P^t_{d_j} \tag{9}$$

$$P^t_{c_j}, P^t_{d_j} \in \left[0, P_{B_{R_j}}\right], \quad B^0_j = B^T_j \tag{10}$$

$$q^t_{B_j} \in \left[-\sqrt{0.44} P_{B_{R,j}}, \sqrt{0.44} P_{B_{R,j}}\right] \tag{11}$$

$$B^t_j \in [soc_{min} B_{R,j}, soc_{max} B_{R,j}] \tag{12}$$



Results and Insights

Comparison between MPCOPF and MPDOPF for $T = 5$

Metric	MPCOPF	MPDOPF
Largest subproblem		
Decision variables	3150	1320
Linear constraints	5831	2451
Nonlinear constraints	635	265
Simulation results		
Substation power cost ($)	576.31	576.30
Substation real power (kW)	4308.28	4308.14
Line loss (kW)	75.99	76.12
Substation reactive power (kVAR)	574.18	656.24
PV reactive power (kVAR)	116.92	160.64
Battery reactive power (kVAR)	202.73	76.01
Computation		
Number of Iterations	-	5
Total Simulation Time (s)	521.25	49.87

Smaller Problem Size

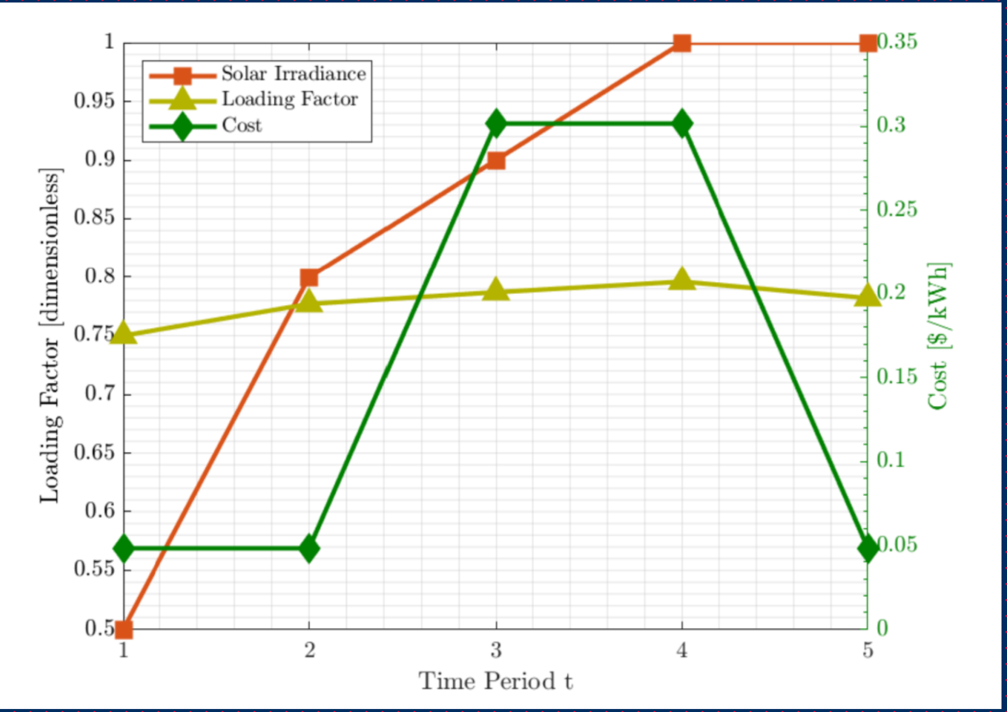
Same Converged Objective Value

Decision Variables could be different

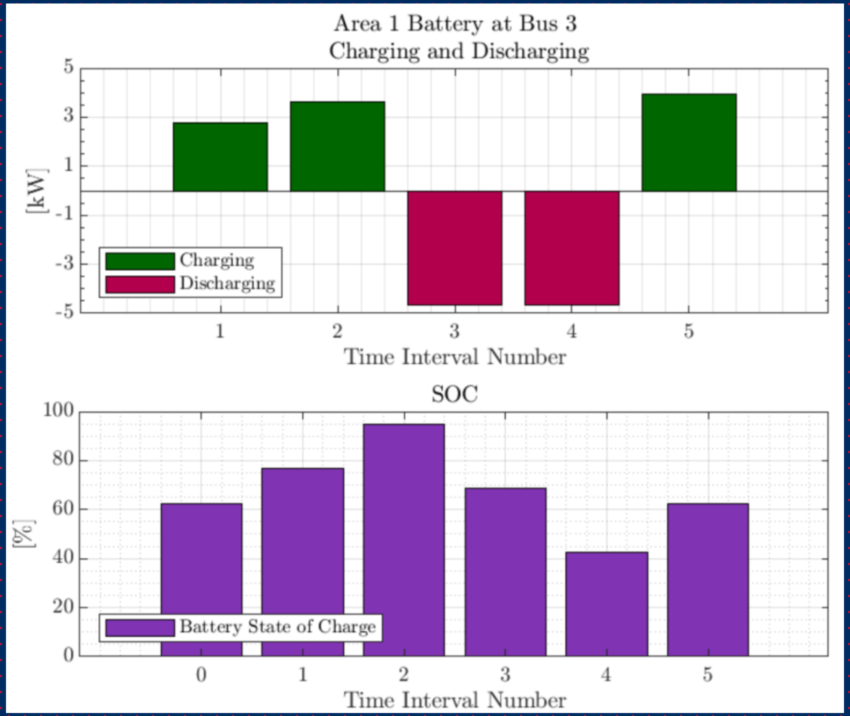
Appreciable Computation Speedup (10 times) !

Result: Proposed MPDOPF Method cuts down the computational complexity of the MPOPF problem, making way for faster solution!

Checking MPDOPF Solution to see if it makes sense



Forecasted timeseries



Optimal Battery Actions

Battery Dispatch Pattern Intutively tracks the cost curve for substation power. It charges up when cost is low, and discharges (injects power into the grid) when cost if higher

Complementarity of Charging and Discharging Operations ensured despite lack of Integer Constraint Modelling

Battery SOCs coming back to their original SOC values at the end of the horizon

Comparison between MPCOPF and MPDOPF for $T = 10$

Metric	MPCOPF	MPDOPF
Largest subproblem		
Decision variables	6300	2640
Linear constraints	11636	4891
Nonlinear constraints	1270	530
Simulation results		
Substation power cost ($)	1197.87	1197.87
Substation real power (kW)	8544.28	8544.04
Line loss (kW)	148.67	148.94
Substation reactive power (kVAR)	1092.39	1252.03
PV reactive power (kVAR)	222.59	139.81
Battery reactive power (kVAR)	388.52	310.94
Computation		
Number of Iterations	-	5
Total Simulation Time (s)	4620.73	358.69

Smaller Problem Size

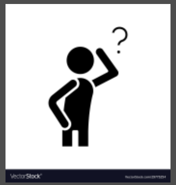
Same Converged Objective Value

Decision Variables could be different

Appreciable Computation Speedup (10 times) !

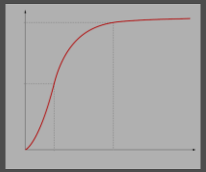
Result: Proposed MPDOPF Method cuts down the computational complexity of the MPOPF problem, making way for faster solution!

Review and Conclusions



MPOPF Problem (unrelaxed, unapproximated) can be hard to solve.

By Spatially Decomposing the system into smaller connected areas and solving for the MPOPF for each area in an iterative exchange manner, we can cut down the problem size and solving time.



But this does not truly *solve* the MPOPF problem, as in it *also* will not scale up for bigger horizon times (say, $T = 96$ for a medium system like IEEE123).

Currently we're working on development and testing of an algorithm which will *both* spatially and temporally decompose the MPOPF problem, making it scalable.

Thank You.

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LinkedIn

Appendix and References follow

References

- [1] Sadnan, R., & Dubey, A. (2021). Distributed Optimization Using Reduced Network Equivalents for Radial Power Distribution Systems. IEEE Trans. Power Syst., 36(4), 3645–3656. doi: 10.1109/TPWRS.2020.3049135

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[4] Nazir, N., & Almassalkhi, M. (2021). Guaranteeing a Physically Realizable Battery Dispatch Without Charge-Discharge Complementarity Constraints. IEEE Trans. Smart Grid, 14(3), 2473–2476. doi: 10.1109/TSG.2021.3109805

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