Linear vs Nonlinear Optimal Power Flow Modelling - Tradeoffs and Takeaways

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KEY POINTS

A **Substation Operator's** key **objective** when operating a Power Distribution System is **minimizing Cost of Operation**, subject to safety and reliability limits. **This problem** of determining an Optimal Control Schedule to achieve the same **is called** the **Optimal Power Flow** (**OPF**) problem. **When** the **problem extends across** a horizon of **multiple time-steps**, it **is** called the **Multi-Period OPF** (**MPOPF)** problem.

In our paper [1], **we compared** a **nonlinear** model (**Branch Flow Model – Nonlinear**) **and** a **linear** model (**LinDistFlow**) **on several** optimization **benchmarks** for the MPOPF problem.

TEST SYSTEM

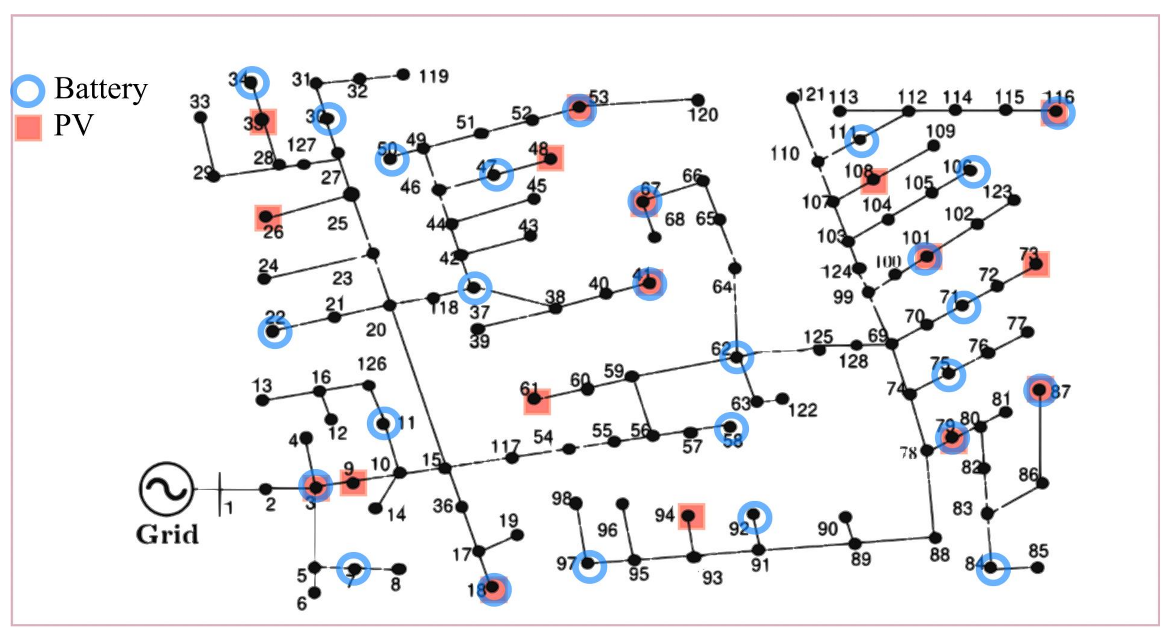

Fig. 1: IEEE 123 Node System A - 20% PVs +30% Batteries

SIMULATION PROFILE

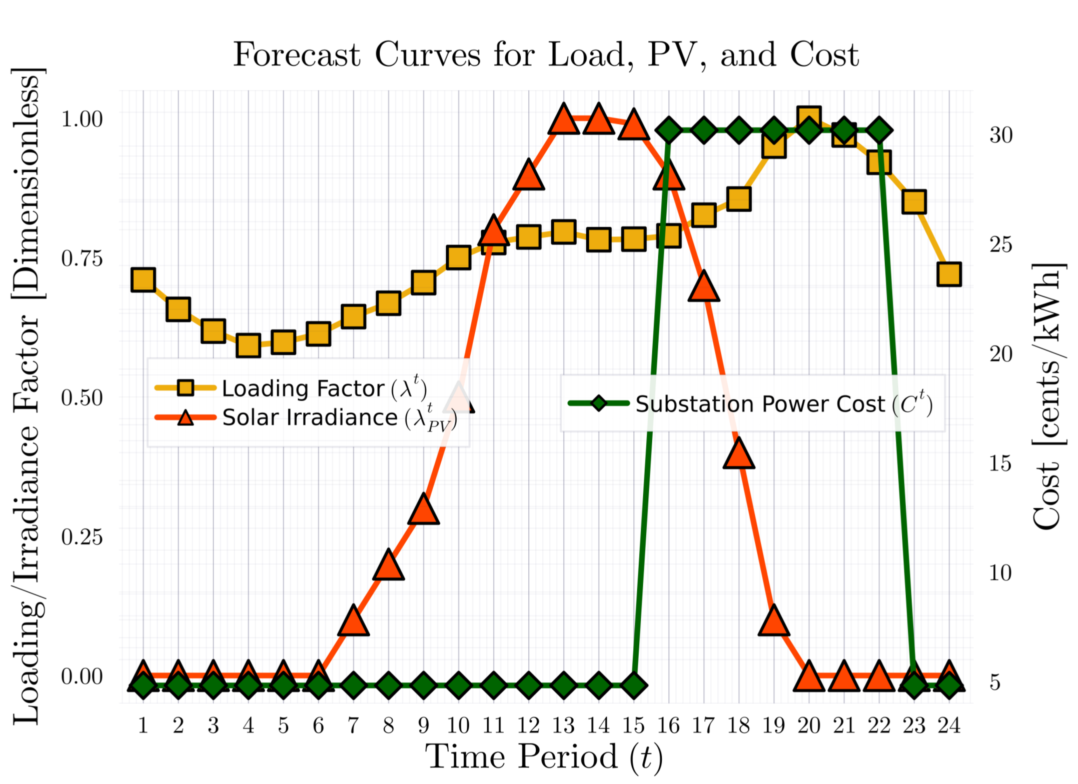

Fig. 2: Forecast of Demand, Solar and Cost of Substation Power

OPTIMIZATION SETUP

Horizon Duration: 24h
Time Period: 1h
Programming Language: Julia
Optimization Modelling Language: JuMP [2]

Optimization Solver: Ipopt
Plant Simulator: OpenDSS
Decision Variables: Upto 17k
Linear Constraints: Upto 30k
Nonlinear Constraints: Upto 14k

OPTIMIZATION MODEL

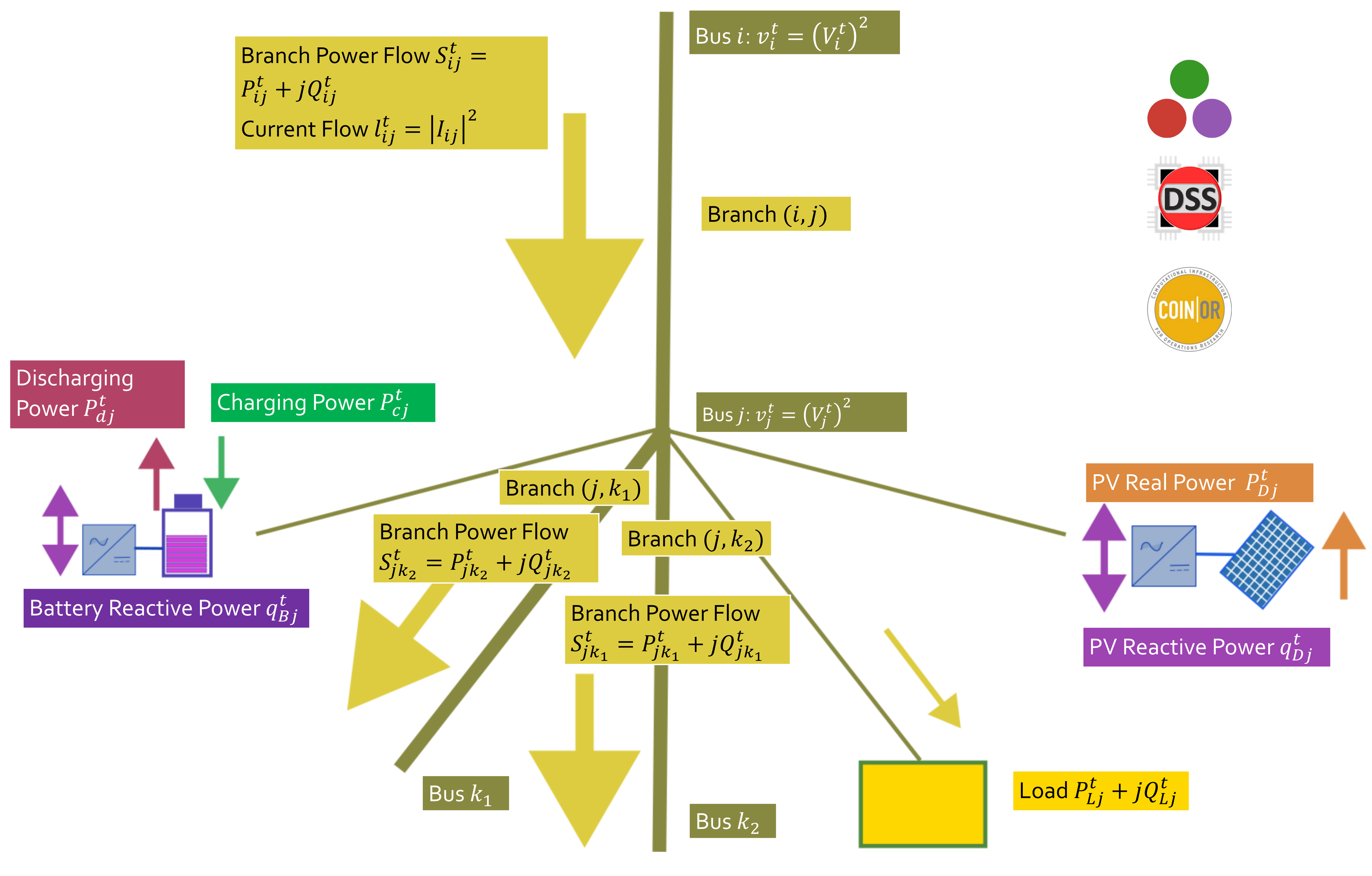

Fig. 3: A Schematic Representing all the Components in the System

BFM-NL		LinDistFlow
$\sum_{(j,k)\in\mathcal{L}} \{P_{jk}^t\} - (P_{ij}^t - r_{ij} l_{ij}^t) = p_j^t$	Node j Real Power Balance	$\sum_{(j,k)\in\mathcal{L}} \{P_{jk}^t\} - (P_{ij}^t) = p_j^t$
$\sum_{(j,k)\in\mathcal{L}} \{Q_{jk}^t\} - (Q_{ij}^t - x_{ij} l_{ij}^t) = q_j^t$	Node j Reactive Power Balance	$\sum_{(j,k)\in\mathcal{L}} \{Q_{jk}^t\} - (Q_{ij}^t) = q_j^t$
$v_j^t = v_i^t - 2(r_{ij}P_{ij}^t + x_{ij}Q_{ij}^t) + \{r_{ij}^2 + x_{ij}^2\} l_{ij}^t$	KVL across branch (i,j)	$v_j^t = v_i^t - 2(r_{ij}P_{ij}^t + x_{ij}Q_{ij}^t)$
Only BFM-NL: $(P_{ij}^t)^2 + (Q_{ij}^t)^2 = l_{ij}^t v_i^t$	Current Magnitude across branch (i,j)	
$(P_{B_j}^t)^2 + (q_{B_j}^t)^2 \le S_{B_{R,j}}^2$	Battery Reactive Power Limits	$q_{B_j}^t \in \left[-\sqrt{3}(P_{B_j}^t + S_{B_{R,j}}), -\sqrt{3}(P_{B_j}^t - S_{B_{R,j}})\right]$ $q_{B_j}^t \in \left[-\frac{\sqrt{3}}{2}S_{B_{R,j}}, \frac{\sqrt{3}}{2}S_{B_{R,j}}\right]$ $q_{B_j}^t \in \left[\sqrt{3}(P_{B_j}^t - S_{B_{R,j}}), \sqrt{3}(P_{B_j}^t + S_{B_{R,j}})\right]$
Common: $B_j^t = B_j^{t-1} + \Delta t \eta_c P_{c_j}^t - \Delta t \frac{1}{\eta_d} P_{d_j}^t$	Battery SOC Equation	$B_j^t = B_j^{t-1} + \Delta t \eta_c P_{c_j}^t - \Delta t \frac{1}{\eta_d} P_{d_j}^t$ Common

Remaining constraints are identical..

RESULTS

Table 1: Discrepancy comparison between linear and nonlinear models for IEEE123 for 24h

Metric	BFM-NL	LinDistFlow
Max. all-time discrepancy		
Voltage (pu)	0.00007	0.00206
Line loss (kW)	0.01818	1.8074
Substation power (kW)	0.43164	32.362
Substation reactive power (kVAR)	1.0102	64.403

Significant Deviation from true values of **Substation Real and Reactive Powers** for Linear Approximation

Table 2: Performance Comparison between linear and nonlinear models for IEEE123 for 24h

Metric	BFM-NL	LinDistFlow[0]
Full horizon		
Substation power cost ($)	2787.44	2798.4
Substation real power (kW)	20984.89	21065.89
Line loss (kW)	380.09	461.38
Substation reactive power (kVAR)	6835.82	12259.29
PV reactive power (kVAR)	1972.27	195.12
Battery reactive power (kVAR)	3709.71	204.63
Computation		
Total Simulation Time (s)	17.44	0.85

Reactive Power allocated by linear model somewhat arbitrary, under-utilizing PV and BESS reactive powers

Low Optimality Gap and Fast Computation Time for Linear Model

TAKEAWAYS

Higher BESS and PV penetration or Bigger System Size → Higher optimality gap of Linear approximation

Linear approximation control variables need to be passed through a true nonlinear simulator such as OpenDSS for correct state variables

Thus, depending on the use case (real-time constraints and infeasibility tolerance limits) a hybrid approach could be employed for best results

REFERENCES

[1] Jha, A. R., Paul, S., & Dubey, A. (2025, July). Analyzing the performance of linear and nonlinear multi-period optimal power flow models for active distribution networks. Paper presented at the 2025 IEEE North-East India International Energy Conversion Conference and Exhibition (NE-IECCE 2025), National Institute of Technology Silchar, Assam, India.

[2] MultiPeriodDistOPF. (2025, July 27). Retrieved from https://github.com/Realife-Brahmin/MultiPeriodDistOPF

[3] Jha, A. R., Paul, S., & Dubey, A. . Spatially Distributed Multi-Period Optimal Power Flow with Battery Energy Storage Systems. 2024 56th North American Power Symposium (NAPS). IEEE. doi: 10.1109/NAPS61145.2024.10741846