WASHINGTON STATE
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Scalable Multi-Period Optimal Power Flow
for Active Power Distribution Systems

or simply, Scalable MP-OPF in ADS

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Multi Period Optimality KKT Necessary Conditions

For a 'solution' to be even considered to be MP-optimal, it must meet this mathematical condition.
[NOT dependent on the optimization method.]

Note: All following results are for the terminal SOC constraint relaxed condition. This will be the case until DDP is corrected.

Multi Period Optimality
KKT necessary condition

$$\Rightarrow \boxed{\frac{\partial L_M}{\partial B_j^{t=1}} = 0 \Rightarrow -\lambda_{B_j}^{t=1} + \lambda_{\bar{B}_j}^{t=1} + \mu_{soc_j}^{t=1} - \mu_{soc_j}^{t=2} = 0}$$

$\boxed{\forall\, j \in B}$

$$\boxed{\frac{\partial L_M}{\partial B_j^{t=t_0}} = 0 \Rightarrow -\lambda_{B_j}^{t=t_0} + \lambda_{\bar{B}_j}^{t=t_0} + \mu_{soc_j}^{t=t_0} - \mu_{soc_j}^{t=(t_0+1)} = 0 \quad t_0 \neq \{1, T\}}$$

$\boxed{\forall\, j \in B}$

$$\boxed{\frac{\partial L_M}{\partial B_j^{t=T}} = 0 \Rightarrow \boxed{\gamma\,(B_j^{t=T} - B_{ref_j})} - \lambda_{B_j}^{t=T} + \lambda_{\bar{B}_j}^{t=T} + \mu_{soc_j}^{t=T} = 0}$$

$\boxed{\forall\, j \in B}$

Not present if terminal SOC condition relaxed ($\gamma = 0$)

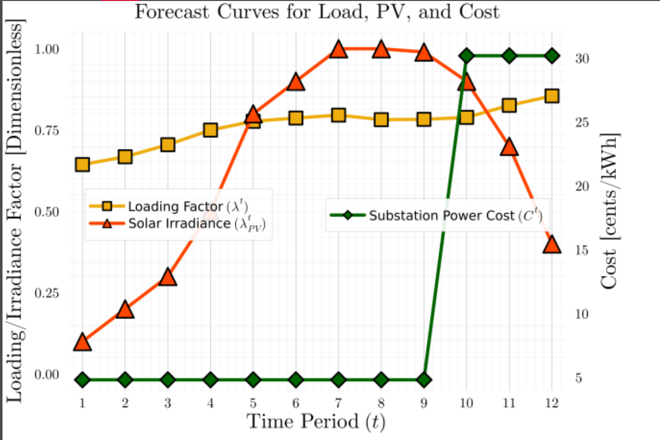
$ADS10_{1ph}$, $T = 12$

Dual Variables Comparison

Terminal SOC Constraint Relaxed

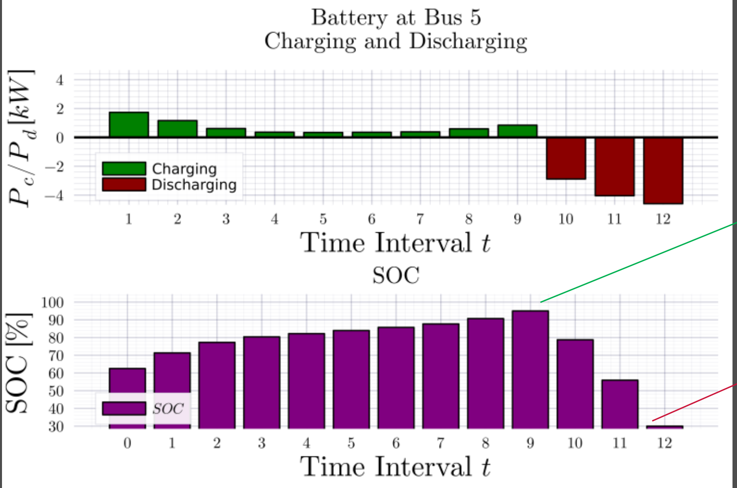
Note: Barely any difference between batteries (So results for only 1 battery given here, wlog)

BruteForced (BF)




```
Dual Variables (mu) for SOC Constraints:
mu[5, 1] = -51.32619604444343
mu[5, 2] = -51.3262312901566
mu[5, 3] = -51.3262973197369
mu[5, 4] = -51.32638597805104
mu[5, 5] = -51.32649200163958
mu[5, 6] = -51.32661933167931
mu[5, 7] = -51.32677696123401
mu[5, 8] = -51.326983615719065
mu[5, 9] = -51.32735860111123
mu[5, 10] = -286.25015874502935
mu[5, 11] = -286.2502208161647
mu[5, 12] = -286.25020241788184
```

```
Dual Variables (lambda) for SOC Limits:
lambda_lower[5, 1] = -3.263538340425111e-5
lambda_upper[5, 1] = -6.788109657727584e-5
lambda_lower[5, 2] = -2.784895835889937e-5
lambda_upper[5, 2] = -9.387853866423375e-5
lambda_lower[5, 3] = -2.640980970675628e-5
lambda_upper[5, 3] = -0.00011506812384869328
lambda_lower[5, 4] = -2.5885533908337213e-5
lambda_upper[5, 4] = -0.00013190912244960535
lambda_lower[5, 5] = -2.544334908661004 2e-5
lambda_upper[5, 5] = -0.00015277338881313946
lambda_lower[5, 6] = -2.500173934892876 6e-5
lambda_upper[5, 6] = -0.00018263129405474802
lambda_lower[5, 7] = -2.454107427717885e-5
lambda_upper[5, 7] = -0.00023119555932828552
lambda_lower[5, 8] = -2.371467359027666e-5
lambda_upper[5, 8] = -0.0003987000657502746
lambda_lower[5, 9] = -2.2618498893528976e-5
lambda_upper[5, 9] = -234.92282276241704
lambda_lower[5, 10] = -3.005489164962771e-5
lambda_upper[5, 10] = -9.212602697621761e-5
lambda_lower[5, 11] = -5.628888119088512 6e-5
lambda_upper[5, 11] = -3.789059831586417e-5
lambda_lower[5, 12] = -286.2502250363803 6
lambda_upper[5, 12] = -2.2618498503044483e-5
```



SOC hits Upper Limit

SOC hits Lower Limit

```
Checking for KKT Necessary Condition (∇L_{B_j^t} = 0 ∀ j ∈ B, t ∈ τ):
∇L_{B_j^t} for [5, 1]: 0.0
∇L_{B_j^t} for [5, 2]: 0.0
∇L_{B_j^t} for [5, 3]: 0.0
∇L_{B_j^t} for [5, 4]: -7.105427357601002e-15
∇L_{B_j^t} for [5, 5]: 7.105427357601002e-15
∇L_{B_j^t} for [5, 6]: -7.105427357601002e-15
∇L_{B_j^t} for [5, 7]: 7.105427357601002e-15
∇L_{B_j^t} for [5, 8]: 0.0
∇L_{B_j^t} for [5, 9]: 0.0
∇L_{B_j^t} for [5, 10]: 0.0
∇L_{B_j^t} for [5, 11]: 0.0
∇L_{B_j^t} for [5, 12]: 0.0
```

KKT Necessary Conditions for Multi Period Optimality Hold!

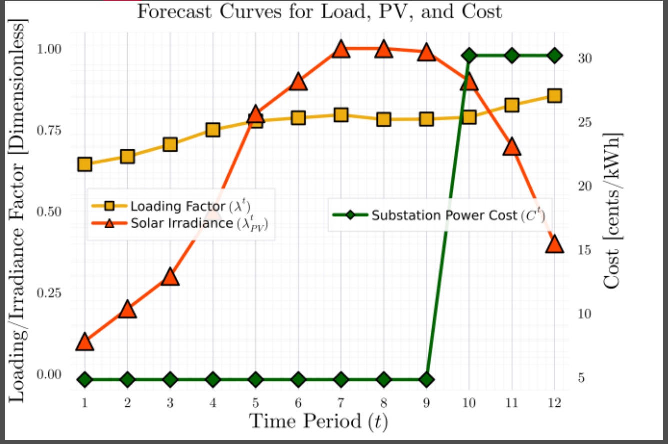
$ADS10_{1ph}$, $T = 12$

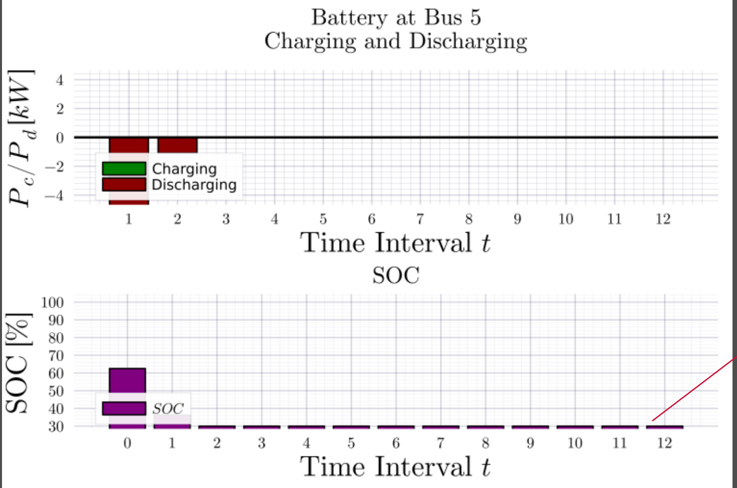
Dual Variables Comparison

Terminal SOC Constraint Relaxed

Note: Barely any difference between batteries (So results for only 1 battery given here, wlog)

DDP






```
Dual Variables (mu) for SOC Constraints:
mu[5, 1, 5] = 45.26340407950657
mu[5, 2, 5] = -45.26347456842723
mu[5, 3, 5] = -48.24228516448881
mu[5, 4, 5] = -48.24445293457344
mu[5, 5, 5] = -48.2446145901697
mu[5, 6, 5] = -48.244688417050966
mu[5, 7, 5] = -48.24474227101628
mu[5, 8, 5] = -48.24444408477721
mu[5, 9, 5] = -48.276539554167734
mu[5, 10, 5] = -301.6742767752171
mu[5, 11, 5] = -301.8354675397394
mu[5, 12, 5] = -302.0534881205564
```

```
Dual Variables (lambda) for SOC Limits:
lambda_lower[5, 1] = -7.877127945239953e-5
lambda_upper[5, 1] = -8.282358791502784e-6
lambda_lower[5, 2] = -93.50576722727318
lambda_upper[5, 2] = -7.494368422792878e-6
lambda_lower[5, 3] = -96.4867459345247
lambda_upper[5, 3] = -7.494368404391746e-6
lambda_lower[5, 4] = -96.48907501910134
lambda_upper[5, 4] = -7.4943684043728156e-6
lambda_lower[5, 5] = -96.48931050157213
lambda_upper[5, 5] = -7.494368404371317e-6
lambda_lower[5, 6] = -96.48943818244867
lambda_upper[5, 6] = -7.494368404371064e-6
lambda_lower[5, 7] = -96.48919385020083
lambda_upper[5, 7] = -7.494368404376815e-6
lambda_lower[5, 8] = -96.52099113324216
lambda_upper[5, 8] = -7.494368404186083e-6
lambda_lower[5, 9] = -349.9508389435478
lambda_upper[5, 9] = -2.2613566825548438e-5
lambda_lower[5, 10] = -603.5097669411317
lambda_upper[5, 10] = -2.262628456616248e-5
lambda_lower[5, 11] = -603.8889783036826
lambda_upper[5, 11] = -2.2643386776897727e-5
lambda_lower[5, 12] = -302.0535107390548
lambda_upper[5, 12] = -2.2618498409570715e-5
```

SOC hits Lower Limit (and stays there)

```
Checking for KKT Necessary Condition (∇L_{B_j^t} = 0 ∀ j ∈ B, t ∈ τ):
∇L_{B_j^t} for [5, 1]: 90.52694913685445
∇L_{B_j^t} for [5, 2]: 96.48457032896636
∇L_{B_j^t} for [5, 3]: 96.48890586916869
∇L_{B_j^t} for [5, 4]: 96.48922918032919
∇L_{B_j^t} for [5, 5]: 96.489376834085
∇L_{B_j^t} for [5, 6]: 96.48948454204557
∇L_{B_j^t} for [5, 7]: 96.48888816959337
∇L_{B_j^t} for [5, 8]: 96.55307910826428
∇L_{B_j^t} for [5, 9]: 603.3485535510304
∇L_{B_j^t} for [5, 10]: 603.6709350793694
∇L_{B_j^t} for [5, 11]: 604.1069762411129
∇L_{B_j^t} for [5, 12]: 0.0
```

KKT Necessary Conditions for Multi Period Optimality Violated!

KKT Conditions for $t = T$ is implicitly taken care of.

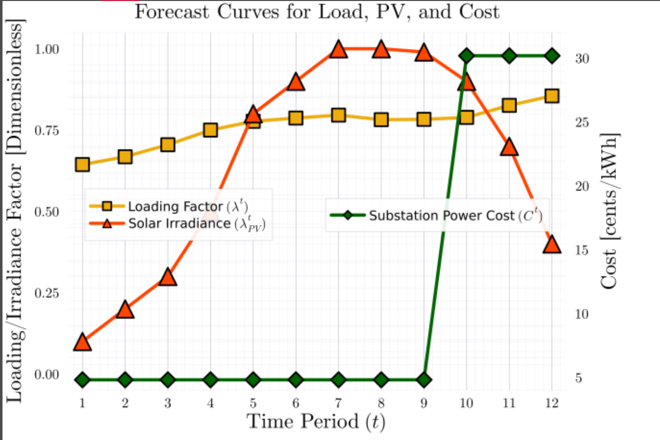
$IEEE123_{1ph}$, $T = 12$

Dual Variables Comparison

Terminal SOC Constraint Relaxed

Note: Barely any difference between batteries (So results for only 1 battery given here, wlog)

BruteForced (BF)

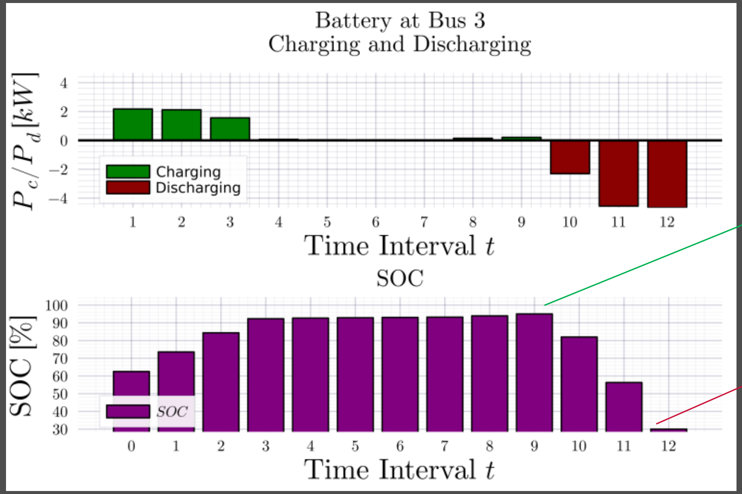



```
Dual Variables (mu) for SOC Constraints:
mu[3, 1] = -51.47912927835636
mu[3, 2] = -51.47916410870037
mu[3, 3] = -51.47927547171397
mu[3, 4] = -51.47979639288445
mu[3, 5] = -51.480402702928195
mu[3, 6] = -51.48106844789614
mu[3, 7] = -51.48179472081694
mu[3, 8] = -51.48258643895679
mu[3, 9] = -51.48394320815712
mu[3, 10] = -288.1055274967737
mu[3, 11] = -288.10561230031226
mu[3, 12] = -288.10559440070483
```

```
Dual Variables (lambda) for SOC Limits:
lambda_lower[3, 1] = -3.375000973885286e-5
lambda_upper[3, 1] = -6.858035375130018e-5
lambda_lower[3, 2] = -2.7049990227478815e-5
lambda_upper[3, 2] = -0.00013841300381912304
lambda_lower[3, 3] = -2.360072201052958e-5
lambda_upper[3, 3] = -0.0005445218924953236
lambda_lower[3, 4] = -2.346329942324204 5e-5
lambda_upper[3, 4] = -0.0006297733431677343
lambda_lower[3, 5] = -2.338843064904000 2e-5
lambda_upper[3, 5] = -0.0006891333985970901
lambda_lower[3, 6] = -2.332449989677511 7e-5
lambda_upper[3, 6] = -0.0007495974206967307
lambda_lower[3, 7] = -2.326573252280361 6e-5
lambda_upper[3, 7] = -0.0008149838723687847
lambda_lower[3, 8] = -2.29966815478793 1e-5
lambda_upper[3, 8] = -0.001379765881875642
lambda_lower[3, 9] = -2.261849887789704e-5
lambda_upper[3, 9] = -236.62160690711545
lambda_lower[3, 10] = -2.8275277261864143e-5
lambda_upper[3, 10] = -0.00011307881582905362
lambda_lower[3, 11] = -5.589309240391531e-5
lambda_upper[3, 11] = -3.799348496793565e-5
lambda_lower[3, 12] = -288.1056170192033
lambda_upper[3, 12] = -2.2618498491535494e-5
```

SOC hits Upper Limit

SOC hits Lower Limit




```
Checking for KKT Necessary Condition (∇L_{B_j^t} = 0 ∀ j ∈ B, t ∈ τ):
∇L_{B_j^t} for [3, 1]: -7.105427357601002e-15
∇L_{B_j^t} for [3, 2]: 7.105427357601002e-15
∇L_{B_j^t} for [3, 3]: 0.0
∇L_{B_j^t} for [3, 4]: 0.0
∇L_{B_j^t} for [3, 5]: 0.0
∇L_{B_j^t} for [3, 6]: 0.0
∇L_{B_j^t} for [3, 7]: 0.0
∇L_{B_j^t} for [3, 8]: 7.105427357601002e-15
∇L_{B_j^t} for [3, 9]: 0.0
∇L_{B_j^t} for [3, 10]: 0.0
∇L_{B_j^t} for [3, 11]: 0.0
∇L_{B_j^t} for [3, 12]: -5.684341886080802e-14
```

KKT Necessary Conditions for Multi Period Optimality Hold!

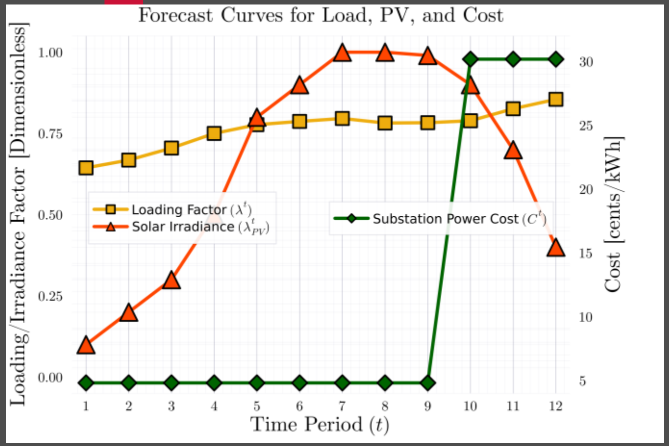
$IEEE123_{1ph}$, $T = 12$

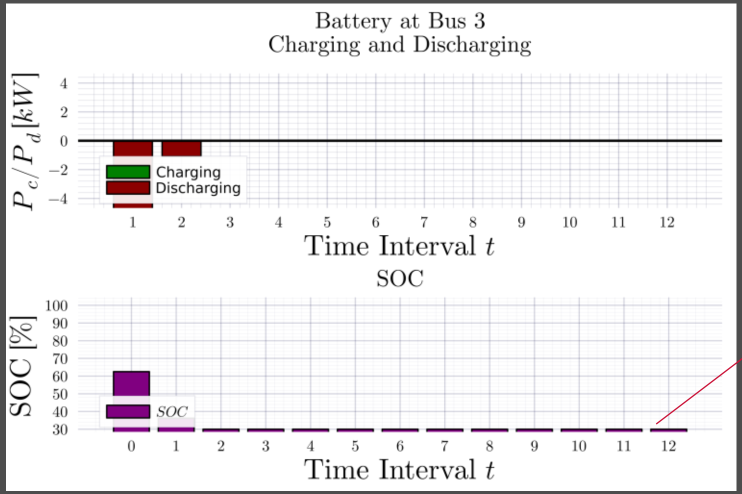
Dual Variables Comparison

Terminal SOC Constraint Relaxed

Note: Barely any difference between batteries (So results for only 1 battery given here, wlog)

DDP






```
Dual Variables (mu) for SOC Constraints:
mu[3, 1, 42] = 45.67098937560637
mu[3, 2, 42] = -45.671059864528
mu[3, 3, 42] = -48.533699256943954
mu[3, 4, 42] = -48.55005260668177
mu[3, 5, 42] = -48.553829375243176
mu[3, 6, 42] = -48.55561281864647
mu[3, 7, 42] = -48.556974007694414
mu[3, 8, 42] = -48.55035688451668
mu[3, 9, 42] = -48.58150624855195
mu[3, 10, 42] = -303.64677168059126
mu[3, 11, 42] = -303.9466748419695
mu[3, 12, 42] = -304.2643864796311
```

```
Dual Variables (lambda) for SOC Limits:
lambda_lower[3, 1] = -7.87712804120792e-5
lambda_upper[3, 1] = -8.282358780893312e-6
lambda_lower[3, 2] = -94.20476661583947
lambda_upper[3, 2] = -7.494368418451643e-6
lambda_lower[3, 3] = -97.08375935799877
lambda_upper[3, 3] = -7.494368400771288e-6
lambda_lower[3, 4] = -97.1038947625516
lambda_upper[3, 4] = -7.494368400651356e-6
lambda_lower[3, 5] = -97.1094496830422
lambda_upper[3, 5] = -7.494368400618239e-6
lambda_lower[3, 6] = -97.1125943206834
lambda_upper[3, 6] = -7.494368400599511e-6
lambda_lower[3, 7] = -97.107338386586
lambda_upper[3, 7] = -7.4943684006308146e-6
lambda_lower[3, 8] = -97.1318706274352
lambda_upper[3, 8] = -7.494368400484733e-6
lambda_lower[3, 9] = -352.2283015430748
lambda_upper[3, 9] = -2.3613899613003058e-5
lambda_lower[3, 10] = -607.5934702271825
lambda_upper[3, 10] = -2.370357607050999e-5
lambda_lower[3, 11] = -608.211085115043
lambda_upper[3, 11] = -2.3794156211883116e-5
lambda_lower[3, 12] = -304.2644090981295
lambda_upper[3, 12] = -2.2618498397230813e-5
```

SOC hits Lower Limit (and stays there)

```
Checking for KKT Necessary Condition (∇L_{B_j^t} = 0 ∀ j ∈ B, t ∈ τ):
∇L_{B_j^t} for [3, 1]: 91.342119729056
∇L_{B_j^t} for [3, 2]: 97.06739851388701
∇L_{B_j^t} for [3, 3]: 97.10010521336818
∇L_{B_j^t} for [3, 4]: 97.10765875044817
∇L_{B_j^t} for [3, 5]: 97.11122563733912
∇L_{B_j^t} for [3, 6]: 97.11394801536298
∇L_{B_j^t} for [3, 7]: 97.10071376904017
∇L_{B_j^t} for [3, 8]: 97.16301249710209
∇L_{B_j^t} for [3, 9]: 607.2935433612146
∇L_{B_j^t} for [3, 10]: 607.8933496849847
∇L_{B_j^t} for [3, 11]: 608.5287729585486
∇L_{B_j^t} for [3, 12]: -5.684341886080802e-14
```

KKT Necessary Conditions for Multi Period Optimality Violated!

KKT Conditions for $t = T$ is implicitly taken care of.

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