WASHINGTON STATE UNIVERSITY

Multiple Source Optimal Power Flow

Aryan Ritwajeet Jha

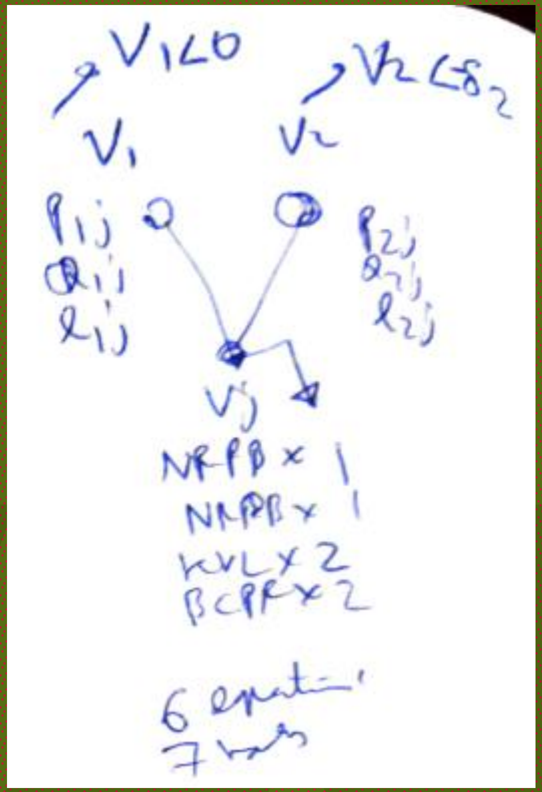
Pursuing PhD ECE (Power Systems)

Washington State University

Multiple Sources = 2 Substations*
Network = 2 Substations + 2 Branches + 1 Load*

(*for now..)

- Problem: Given a network with two substations with voltages $V_1\angle 0°$ and $V_2\angle -\delta°$:
- Determine the optimal value of δ for any given objective function, say cost of substation power $C_1 * P_{Subs_1} + C_2 * P_{Subs_2}$ s.t. other usual constraints of an active distribution network and **NO backflow**!



Optimization using BFM-NL

- Using Branch Flow Model (Nonlinear) I can optimize for the optimal power flow of the network – note that BFM does not take into consideration (relative) voltage angles of the substations.

- Is the modelling (and thus the solution) even correct?

```
# Constraints
# 1. NRealPB
@constraint(model, (P_1j - r1j * l_1j) + (P_2j - r2j * l_2j)
== P_L_j)
# 2. NReacPB
@constraint(model, (Q_1j - x1j * l_1j) + (Q_2j - x2j * l_2j)
== Q_L_j)
# 3. KVL1
@constraint(model, v_j == v_1 - 2 * (r1j * P_1j + x1j * Q_1j)
+ (r1j^2 + x1j^2) * l_1j)
# 4. KVL2
@constraint(model, v_j == v_2 - 2 * (r2j * P_2j + x2j * Q_2j)
+ (r2j^2 + x2j^2) * l_2j)
# 5. BCPF1
@constraint(model, P_1j^2 + Q_1j^2 ≥ v_1 * l_1j)
# 6. BCPF2
@constraint(model, P_2j^2 + Q_2j^2 ≥ v_2 * l_2j)
# 7. Power sharing
@constraint(model, P_2j == alpha * P_1j)
# 8. Voltage limits (already in variable bounds)        =, 2 mo
# 9. P_1j, P_2j ≥ 0 (already in variable bounds)

# Objective
@objective(model, Min, C_1 * P_1j + C_2 * P_2j)
```

Optimization using BFM-NL

Is the modelling (and thus the solution) even correct?

- BFM-NL-reference-paper-SHLow
-
- Technically the branch flow model *just* allows for multiple 'parent' nodes for each node. So my **current formulation shouldn't be incorrect**.
- But only listing $P_{ij}, Q_{ij}, v_j, l_{ij}$ isn't enough any longer, need to tell angles as well, in particular the δ value assumed for substation 2 for this solution (if this is even a non-fake solution!)

```
# Constraints
# 1. NRealPB
@constraint(model, (P_1j - r1j * l_1j) + (P_2j - r2j * l_2j)
== P_L_j)
# 2. NReacPB
@constraint(model, (Q_1j - x1j * l_1j) + (Q_2j - x2j * l_2j)
== Q_L_j)
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@constraint(model, v_j == v_1 - 2 * (r1j * P_1j + x1j * Q_1j)
+ (r1j^2 + x1j^2) * l_1j)
# 4. KVL2
@constraint(model, v_j == v_2 - 2 * (r2j * P_2j + x2j * Q_2j)
+ (r2j^2 + x2j^2) * l_2j)
# 5. BCPF1
@constraint(model, P_1j^2 + Q_1j^2 >= v_1 * l_1j)
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@constraint(model, P_2j == alpha * P_1j)
# 8. Voltage limits (already in variable bounds)      =, 2 mo
# 9. P_1j, P_2j >= 0 (already in variable bounds)

# Objective
@objective(model, Min, C_1 * P_1j + C_2 * P_2j)
```

III. Relaxations and Solution Strategy

A. Relaxed Branch Flow Model

Substituting (2) into (1) yields $V_j = V_i - z_{ij} S_{ij}^* / V_i^*$. Taking the magnitude squared, we have $v_j = v_i + |z_{ij}|^2 \ell_{ij} - (z_{ij} S_{ij}^* + z_{ij}^* S_{ij})$. Using (3) and (2) and in terms of real variables, we therefore have

$$p_j = \sum_{k:j\to k} P_{jk} - \sum_{i:i\to j} (P_{ij} - r_{ij}\ell_{ij}) + g_j v_j, \quad \forall j \quad (13)$$

$$q_j = \sum_{k:j\to k} Q_{jk} - \sum_{i:i\to j} (Q_{ij} - x_{ij}\ell_{ij}) + b_j v_j, \quad \forall j \quad (14)$$

$$v_j = v_i - 2(r_{ij}P_{ij} + x_{ij}Q_{ij}) + (r_{ij}^2 + x_{ij}^2)\ell_{ij} \quad \forall (i,j) \in E \quad (15)$$

$$\ell_{ij} = \frac{P_{ij}^2 + Q_{ij}^2}{v_i}, \quad \forall (i,j) \in E. \quad (16)$$

We will refer to (13)–(16) as the *relaxed (branch flow) model/ equations* and a solution a *relaxed (branch flow) solution*. These

Optimization using BFM-NL

It looks like I need to study part II of the paper to understand the angle recovery process – nonunique and nontrivial for 'meshed' systems

Given $z := (z_{ij}, (i,j) \in E,\ z_i, i \in N)$, V_0 and bus power injections s, the variables $(S, I, V, s_0) := (S_{ij}, I_{ij}, (i,j) \in E,\ V_i, i = 1, \ldots, n,\ s_0)$ satisfy the Ohm's law:

$$V_i - V_j = z_{ij} I_{ij}, \quad \forall (i,j) \in E \tag{1}$$

the definition of branch power flow:

$$S_{ij} = V_i I_{ij}^*, \quad \forall (i,j) \in E \tag{2}$$

and power balance at each bus: for all $j \in N$,

$$\sum_{k:j \to k} S_{jk} - \sum_{i:i \to j} (S_{ij} - z_{ij}|I_{ij}|^2) + y_j^* |V_j|^2 = s_j. \tag{3}$$

We will refer to (1)–(3) as the *branch flow model/equations*.

III. Relaxations and Solution Strategy

A. Relaxed Branch Flow Model

Substituting (2) into (1) yields $V_j = V_i - z_{ij} S_{ij}^* / V_i^*$. Taking the magnitude squared, we have $v_j = v_i + |z_{ij}|^2 \ell_{ij} - (z_{ij} S_{ij}^* + z_{ij}^* S_{ij})$. Using (3) and (2) and in terms of real variables, we therefore have

$$p_j = \sum_{k:j \to k} P_{jk} - \sum_{i:i \to j} (P_{ij} - r_{ij}\ell_{ij}) + g_j v_j, \quad \forall j \tag{13}$$

$$q_j = \sum_{k:j \to k} Q_{jk} - \sum_{i:i \to j} (Q_{ij} - x_{ij}\ell_{ij}) + b_j v_j, \quad \forall j \tag{14}$$

$$v_j = v_i - 2(r_{ij} P_{ij} + x_{ij} Q_{ij}) + (r_{ij}^2 + x_{ij}^2)\ell_{ij} \quad \forall (i,j) \in E \tag{15}$$

$$\ell_{ij} = \frac{P_{ij}^2 + Q_{ij}^2}{v_i}, \quad \forall (i,j) \in E. \tag{16}$$

We will refer to (13)–(16) as the *relaxed (branch flow) model/equations* and a solution a *relaxed (branch flow) solution*. These

For a connected mesh network, $m = |E| > |N| - 1 = n$, in which case there are more variables than equations for the relaxed model (13)–(16), and therefore the solution is generally nonunique. Moreover, some of these solutions may be spurious, i.e., they do not correspond to a solution of the original branch flow equations (1)–(3).

the distance of the point from the origin. It is therefore not surprising that a relaxed solution of (13)–(16) may not correspond to any solution of (1)–(3). The key is whether, given a relaxed solution, we can recover the angles $\angle V_i, \angle I_{ij}$ correctly from it. It is then remarkable that, when G is a tree, indeed the solutions of (13)–(16) coincide with those of (1)–(3). Moreover *for a general networks*, (13)–(16) *together with the angle recovery condition in Theorem 2 below are indeed equivalent to* (1)–(3), as explained in Remark 5 of Section V.

C. Solution Strategy

In the rest of this paper, we will prove the following:

1) OFP-cr is convex. Moreover, if there are no upper bounds on loads, then the conic relaxation is exact so that *any* optimal solution $(\hat{y}_{\text{cr}}, s_{\text{cr}})$ of OPF-cr is also optimal for OPF-ar for mesh as well as radial networks (Section IV, Theorem 1). OPF-cr is an SOCP when the objective function is linear.
2) Given a solution $(\hat{y}_{\text{ar}}, s_{\text{ar}})$ of OPF-ar, if the network is radial, then we can always recover the phase angles $\angle V_i, \angle I_{ij}$ uniquely to obtain an optimal solution (x_*, s_*) of the original OPF through an inverse projection (Section V, Theorems 2 and 4).
3) For a mesh network, an inverse projection may not exist to map the given $(\hat{y}_{\text{ar}}, s_{\text{ar}})$ to a feasible solution of OPF. Our characterization can be used to determined if $(\hat{y}_{\text{ar}}, s_{\text{ar}})$ is globally optimal.

These results motivate the algorithm in Fig. 2.

In Part II of this paper, we show that a mesh network can be convexified so that $(\hat{y}_{\text{ar}}, s_{\text{ar}})$ can always be mapped to an optimal solution of OPF for the convexified network. Moreover, convexification requires phase shifters only on lines outside an arbitrary spanning tree of the network graph.

- Meanwhile I've created the same system in OpenDSS where I am running powerflows for different values for δ – it does look like our solution should have a feasible value associated with it.

```
--------------------------------------------------------------------
Total loading: 50000.0 kW, 37500.0 kVAr
Substation voltages (pu): grid1=1.07, grid2=1.07
Substation BasekV: grid1=11.5470, grid2=11.5470
--------------------------------------------------------------------
delta   | PSubs1 [kW]   | PSubs2 [kW]   | QSubs1 [kVAr] | QSubs2 [kVAr]
--------------------------------------------------------------------
-10.0   | -111097.19    | 170886.48     | 109388.77     | -69928.60
-5.0    | -44422.26     | 97120.57      | 64022.41      | -25982.91
-2.0    | -2982.79      | 53696.05      | 36842.94      | 801.97
-1.5    | 4015.26       | 46528.31      | 32320.92      | 5287.74
-1.0    | 11040.58      | 39384.45      | 27804.09      | 9780.82
-0.5    | 18090.70      | 32263.18      | 23291.32      | 14279.33
-0.5    | 18090.52      | 32263.01      | 23291.16      | 14279.17
-0.4    | 19503.51      | 30841.55      | 22389.15      | 15179.53
-0.3    | 20917.43      | 29420.99      | 21487.29      | 16080.06
-0.2    | 22332.31      | 28001.37      | 20585.62      | 16980.78
-0.1    | 23748.15      | 26582.68      | 19684.12      | 17881.70
0.0     | 25164.94      | 25164.94      | 18782.82      | 18782.82
0.1     | 26582.68      | 23748.15      | 17881.70      | 19684.12
0.2     | 28001.36      | 22332.31      | 16980.78      | 20585.61
0.3     | 29420.99      | 20917.42      | 16080.05      | 21487.29
0.4     | 30841.54      | 19503.50      | 15179.52      | 22389.14
0.5     | 32263.03      | 18090.54      | 14279.19      | 23291.18
0.6     | 33685.44      | 16678.55      | 13379.06      | 24193.39
1.1     | 40811.29      | 9633.37       | 8881.65       | 28707.04
1.6     | 47959.53      | 2613.01       | 4389.65       | 33224.58
5.0     | 97119.80      | -44423.08     | -25983.25     | 64022.07
10.0    | 170883.99     | -111100.13    | -69930.61     | 109386.89
'Optimal' | 28571.43      | 21428.57      | 12984.81      | 24515.19
--------------------------------------------------------------------
```

Given some papers by Subho to read

IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 38, NO. 4, JULY 2023 3707

Loop Analysis and Angle Recovery Based Reactive Power Optimization for Three-Phase Unbalanced Weakly-Meshed Active Distribution Networks

Tianrui Xu, *Student Member, IEEE*, Tao Ding, *Senior Member, IEEE*, Chenggang Mu, *Student Member, IEEE*, Yuntao Ju, *Member, IEEE*, Mohammad Shahidehpour, *Fellow, IEEE*, Chao Zhu, and Yiyang Zhang

IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 31, NO. 5, SEPTEMBER 2016

A Compensation-Based Conic OPF for Weakly Meshed Networks

Rabih A. Jabr, *Senior Member, IEEE*, and Izudin Džafić, *Senior Member, IEEE*

IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 30, NO. 1, JANUARY 2015 19

Convex Relaxation for Optimal Power Flow Problem: Mesh Networks

Ramtin Madani, *Graduate Student Member, IEEE*, Somayeh Sojoudi, and Javad Lavaei, *Senior Member, IEEE*