

Linear vs Nonlinear Optimal Power Flow Modelling - Tradeoffs and Takeaways

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KEY POINTS

A **Substation Operator's** key **objective** when operating a Power Distribution System is **minimizing Cost of Operation**, subject to safety and reliability limits. **This problem** of determining an Optimal Control Schedule to achieve the same **is called the Optimal Power Flow (OPF)** problem. **When the problem extends across a horizon of multiple time-steps**, it **is called the Multi-Period OPF (MPOPF)** problem.

In our paper [1], **we compared a nonlinear model (Branch Flow Model – Nonlinear) and a linear model (LinDistFlow) on several optimization benchmarks** for the MPOPF problem.

TEST SYSTEM

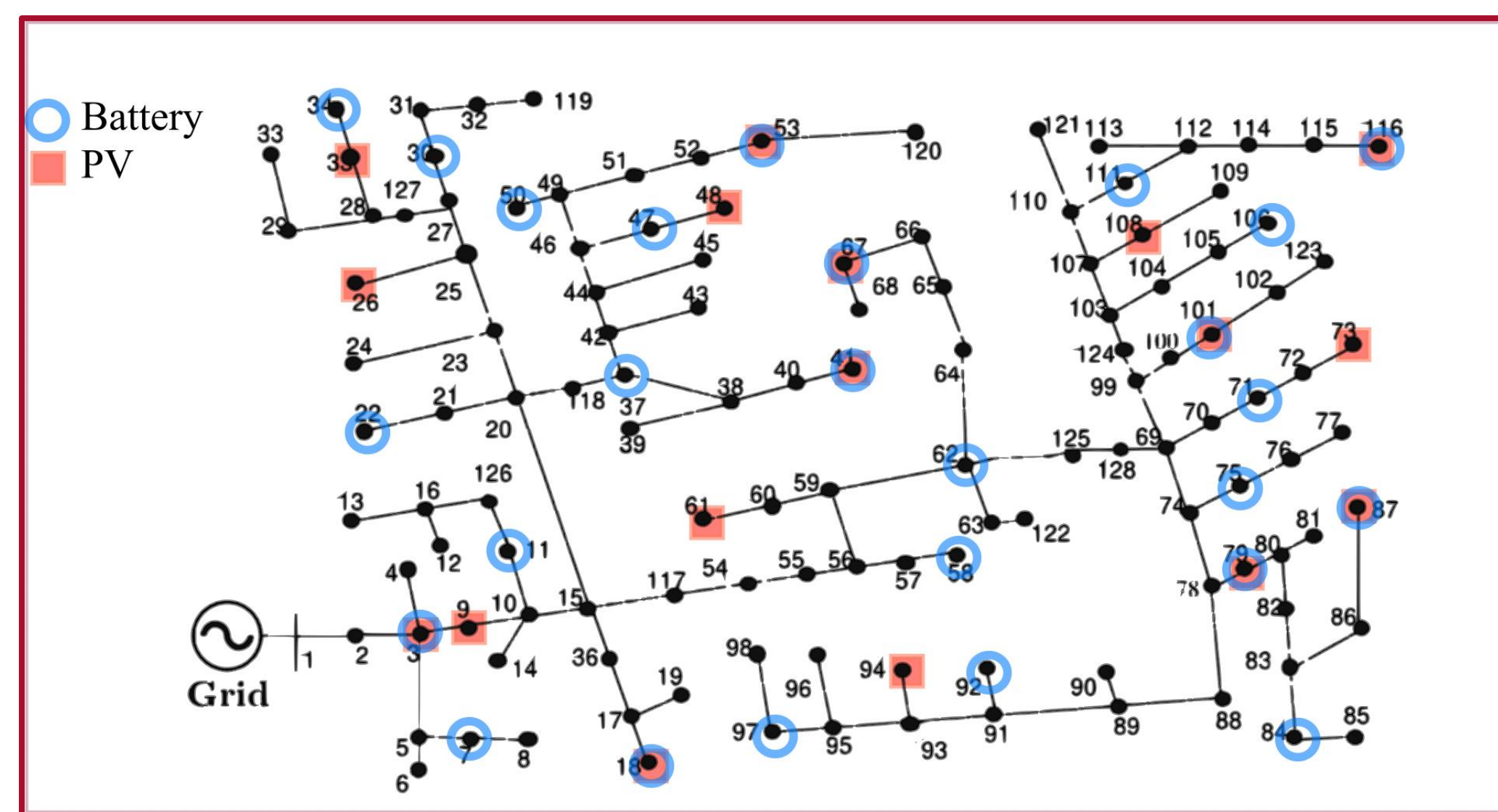


Fig. 1: IEEE 123 Node System A - 20% PVs + 30% Batteries

SIMULATION PROFILE

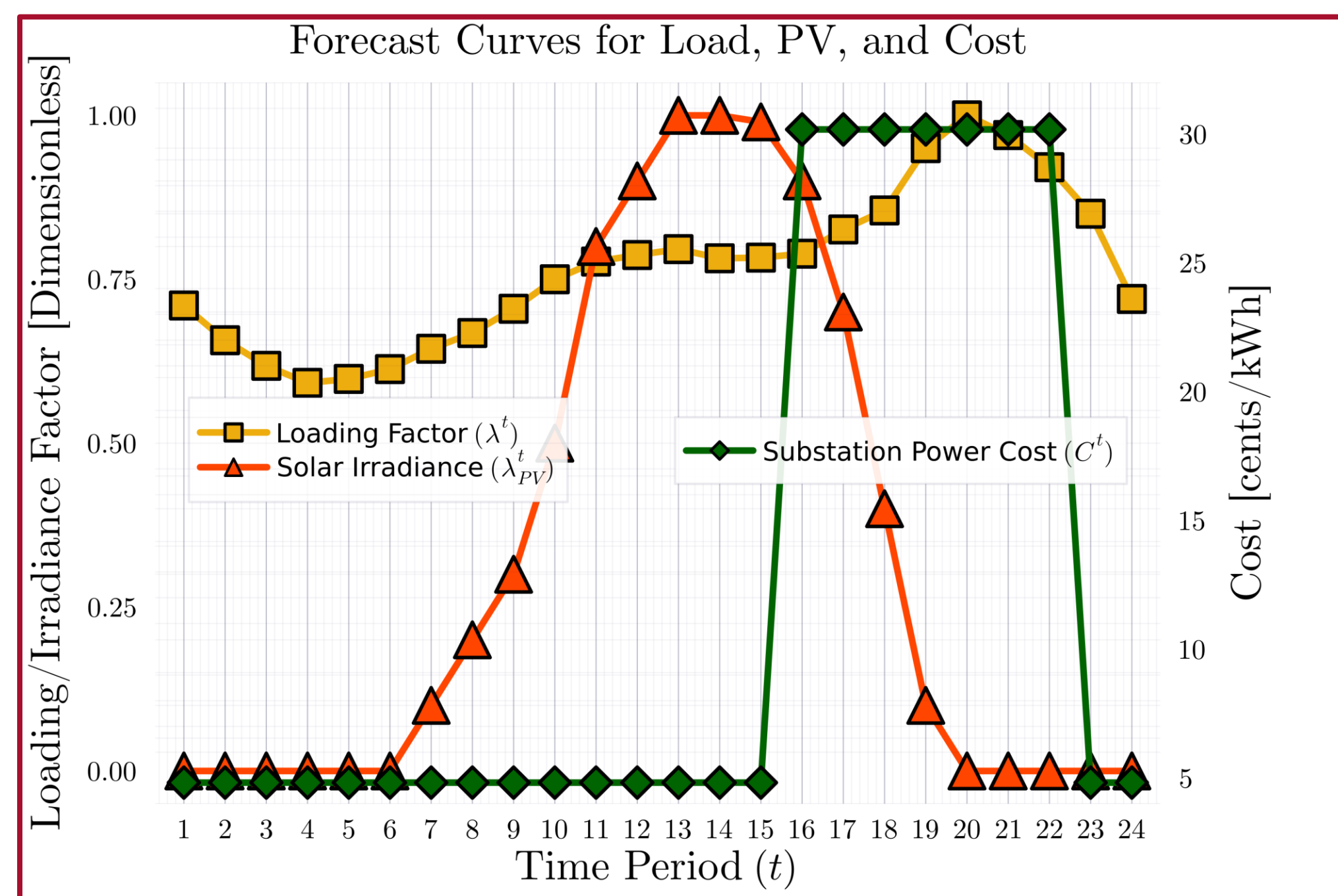


Fig. 2: Forecast of Demand, Solar and Cost of Substation Power

OPTIMIZATION SETUP

Horizon Duration: 24h
Time Period: 1h
Programming Language: Julia
Optimization Modelling Language: JuMP [2]
Optimization Solver: Ipopt
Plant Simulator: OpenDSS
Decision Variables: Upto 17k
Linear Constraints: Upto 30k
Nonlinear Constraints: Upto 14k

OPTIMIZATION MODEL

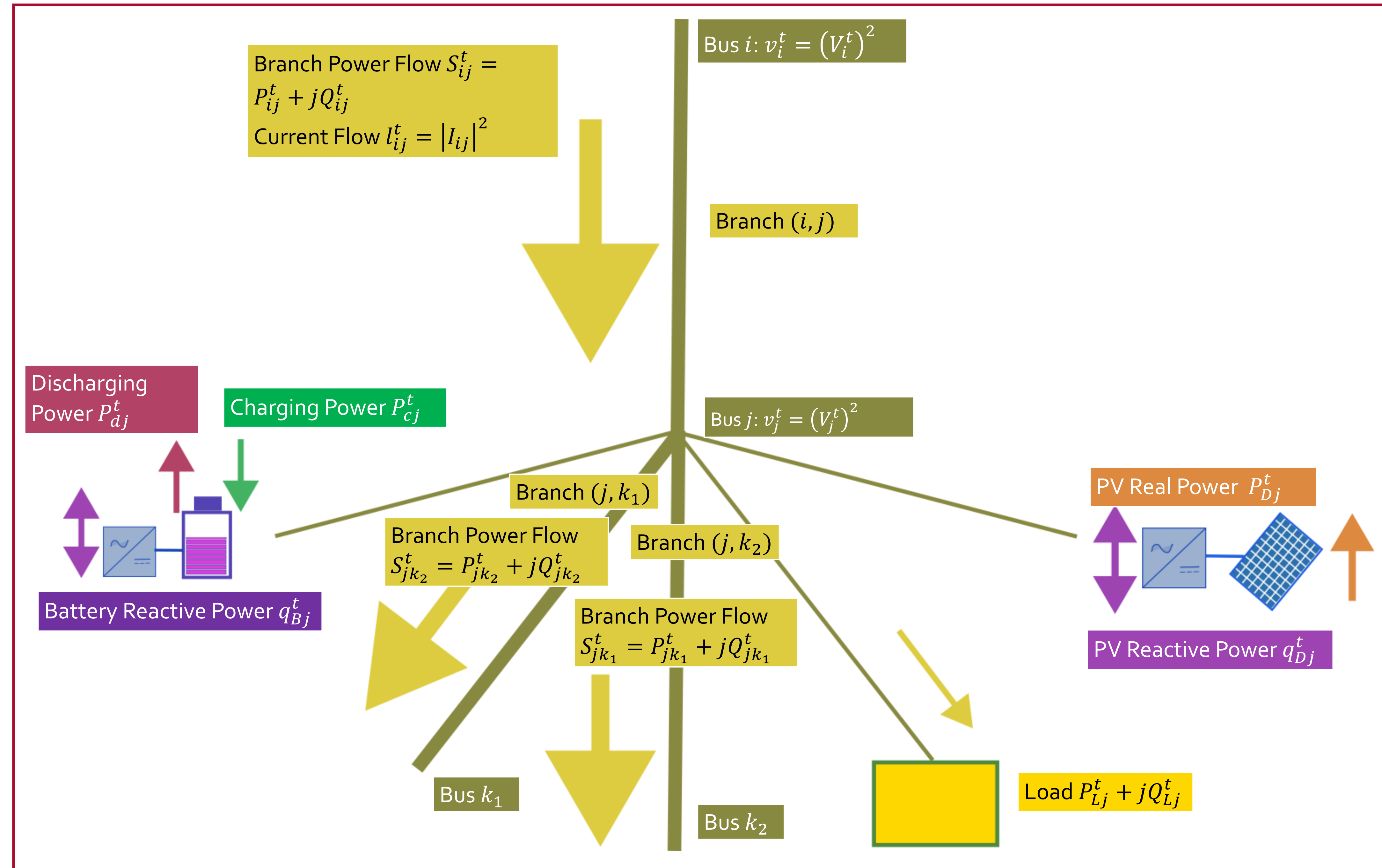


Fig. 3: A Schematic Representing all the Components in the System

BFM-NL

$$\sum_{(j,k) \in \mathcal{L}} \{P_{jk}^t\} - (P_{ij}^t - r_{ij}l_{ij}^t) = p_j^t$$

$$\sum_{(j,k) \in \mathcal{L}} \{Q_{jk}^t\} - (Q_{ij}^t - x_{ij}l_{ij}^t) = q_j^t$$

$$v_j^t = v_i^t - 2(r_{ij}P_{ij}^t + x_{ij}Q_{ij}^t) + \{r_{ij}^2 + x_{ij}^2\} l_{ij}^t$$

Only BFM-NL

$$(P_{ij}^t)^2 + (Q_{ij}^t)^2 = l_{ij}^t v_i^t$$

$$(P_{Bj}^t)^2 + (q_{Bj}^t)^2 \leq S_{B,R,j}^2$$

Common

$$B_j^t = B_j^{t-1} + \Delta t \eta_c P_c^t - \Delta t \frac{1}{\eta_d} P_d^t$$

Node j Real Power Balance

Node j Reactive Power Balance

KVL across branch (i, j)

Current Magnitude across branch (i, j)

Battery Reactive Power Limits

Battery SOC Equation

Remaining constraints are identical..

LinDistFlow

$$\sum_{(j,k) \in \mathcal{L}} \{P_{jk}^t\} - (P_{ij}^t) = p_j^t$$

$$\sum_{(j,k) \in \mathcal{L}} \{Q_{jk}^t\} - (Q_{ij}^t) = q_j^t$$

$$v_j^t = v_i^t - 2(r_{ij}P_{ij}^t + x_{ij}Q_{ij}^t)$$

$$q_{Bj}^t \in \left[-\sqrt{3}(P_{Bj}^t + S_{B,R,j}), -\sqrt{3}(P_{Bj}^t - S_{B,R,j}) \right]$$

$$q_{Bj}^t \in \left[-\frac{\sqrt{3}}{2} S_{B,R,j}, \frac{\sqrt{3}}{2} S_{B,R,j} \right]$$

$$q_{Bj}^t \in \left[\sqrt{3}(P_{Bj}^t - S_{B,R,j}), \sqrt{3}(P_{Bj}^t + S_{B,R,j}) \right]$$

$$B_j^t = B_j^{t-1} + \Delta t \eta_c P_c^t - \Delta t \frac{1}{\eta_d} P_d^t$$

Common

RESULTS

Table 1: Discrepancy comparison between linear and nonlinear models for IEEE123 for 24h

Metric	BFM-NL	LinDistFlow
Max. all-time discrepancy		
Voltage (pu)	0.00007	0.00206
Line loss (kW)	0.01818	1.8074
Substation power (kW)	0.43164	32.362
Substation reactive power (kVAR)	1.0102	64.403

Significant Deviation from true values of **Substation Real and Reactive Powers** for Linear Approximation

Table 2: Performance Comparison between linear and nonlinear models for IEEE123 for 24h

Metric	BFM-NL	LinDistFlow ^①
Full horizon		
Substation power cost (\$)	2787.44	2798.4
Substation real power (kW)	20984.89	21065.89
Line loss (kW)	380.09	461.38
Substation reactive power (kVAR)	6835.82	12259.29
PV reactive power (kVAR)	1972.27	195.12
Battery reactive power (kVAR)	3709.71	204.63
Computation		
Total Simulation Time (s)	17.44	0.85

Reactive Power allocated by linear model somewhat arbitrary, under-utilizing PV and BESS reactive powers

Low Optimality Gap and Fast Computation Time for Linear Model

TAKEAWAYS

Higher BESS and PV penetration or Bigger System Size \rightarrow Higher optimality gap of Linear approximation

Linear approximation control variables need to be passed through a true nonlinear simulator such as OpenDSS for correct state variables

Thus, depending on the use case (real-time constraints and infeasibility tolerance limits) a hybrid approach could be employed for best results

REFERENCES

- [1] Jha, A. R., Paul, S., & Dubey, A. (2025, July). Analyzing the performance of linear and nonlinear multi-period optimal power flow models for active distribution networks. Paper presented at the 2025 IEEE North-East India International Energy Conversion Conference and Exhibition (NE-IECC 2025), National Institute of Technology Silchar, Assam, India.
- [2] MultiPeriodDistOPF. (2025, July 27). Retrieved from <https://github.com/Realife-Brahmin/MultiPeriodDistOPF>
- [3] Jha, A. R., Paul, S., & Dubey, A. . Spatially Distributed Multi-Period Optimal Power Flow with Battery Energy Storage Systems. 2024 56th North American Power Symposium (NAPS). IEEE. doi: 10.1109/NAPS61145.2024.10741846