

# The Power Grid Library for Benchmarking

## AC Optimal Power Flow Algorithms

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The PGLib-OPF benchmarks are available at <https://github.com/power-grid-lib/pglib-opf>.

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**Abstract**—In recent years, the power systems research community has seen an explosion of novel methods for formulating the AC power flow equations. Consequently, benchmarking studies using the seminal AC Optimal Power Flow (AC-OPF) problem have emerged as the primary method for evaluating these emerging methods. However, it is often difficult to directly compare these studies due to subtle differences in the AC-OPF problem formulation as well as the network, generation, and loading data that are used for evaluation. To help address these challenges, this IEEE PES Task Force report proposes a standardized AC-OPF mathematical formulation and the PGLib-OPF networks for benchmarking AC-OPF algorithms. A motivating study demonstrates some limitations of the established network datasets in the context of benchmarking AC-OPF algorithms and a validation study demonstrates the efficacy of using the PGLib-OPF networks for this purpose. In the interest of scientific discourse and future additions, the PGLib-OPF benchmark library is open-access and all the of network data is provided under a creative commons license.

**Index Terms**—Nonlinear Optimization, Convex Optimization, AC Optimal Power Flow, Benchmarking

#### NOMENCLATURE

$N$	- The set of buses in the network
$G$	- The set of generators in the network
$E$	- The set of <i>from</i> branches in the network
$E^R$	- The set of <i>to</i> branches in the network
$i$	- Imaginary number constant
$e$	- Exponential constant
$S = p + iq$	- AC power
$V = v\angle\theta$	- AC voltage
$Z = r + ix$	- Branch impedance
$Y = g + ib$	- Branch admittance
$T = t\angle\theta^t$	- Branch transformer properties
$Y^s = g^s + ib^s$	- Bus shunt admittance
$\dot{v}$	- Nominal base voltage
$b^c$	- Line charging
$s^u$	- Branch apparent power limit
$I^u$	- Branch current magnitude limit
$\theta^\Delta$	- Voltage angle difference limit
$S^d = p^d + iq^d$	- AC power demand
$S^g = p^g + iq^g$	- AC power generation
$c_0, c_1, c_2$	- Generation cost coefficients
$\Re(\cdot), \Im(\cdot)$	- Real and imag. parts of a complex number
$ \cdot , \angle\cdot$	- Magnitude and angle of a complex number
$(\cdot)^*$	- Conjugate of a complex number
$x^l, x^u$	- Lower and upper bounds of $x$ , respectively
$\bar{x}$	- A constant value
$\hat{x}$	- An estimation of $x$
$\mu$	- Mean of a normal distribution
$\sigma$	- Standard deviation of a normal distribution
$\lambda$	- Rate of an exponential distribution

#### I. INTRODUCTION

Over the last decade, power systems research has experienced an explosion in variations of the steady-state AC

power flow equations. These include approximations such as the LPAC [1], IV-Flow [2] and relaxations such as the Second-Order Cone (SOC) [3], Convex-DistFlow (CDF) [4], Quadratic Convex (QC) [5], [6], Semidefinite Programming (SDP) [7], and Moment / Sum-of-Squares Hierarchies [8]–[11], just to name a few. Surveys of the power flow relaxation and approximation literature are provided in [12]–[14]. Much of the excitement underlying this line of research was ignited when [15] demonstrated that the SDP relaxation could provide globally optimal solutions to a variety of the transmission system networks distributed with MATPOWER [16]. Combining these results with industrial-strength convex optimization tools (e.g., Gurobi [17], Cplex [18], Mosek [19]) promises efficient and reliable algorithms for a wide variety of applications in power systems such as optimal power flow [20]–[23], optimal transmission switching [24], and network expansion planning [25], just to name a few.

Independent of the specific problem domain or the power flow model under consideration, all of these novel methods require AC power network data for experimental validation and it is important that suitable data are used to validate these emerging techniques [26]. However, due to the sensitive nature of critical infrastructure, detailed real-world network data is often difficult to obtain, even under non-disclosure agreements. Consequently, many of the available power network datasets are over thirty years old (e.g. [27]), were originally designed for testing AC Power Flow algorithms, and lack the parameters needed for testing the AC Optimal Power Flow algorithms, e.g. branch thermal limits and generator cost functions. The lack of comprehensive and modern AC power network data has been recognized by the Advanced Research Projects Agency-Energy (ARPA-e) and the GRID DATA program [28] has resulted in a number of new network datasets, which are synthetically generated to match the statistics of real-world networks and provided as open-access.

To help improve the evaluation of AC-OPF algorithms, this IEEE PES Task Force report introduces the Power Grid Library for benchmarking the AC-OPF problem, PGLIB-OPF. PGLIB-OPF is a comprehensive collection of creative-commons AC transmission system networks curated in the MATPOWER data format with all of the data required for modeling the proposed AC-OPF problem. This report is a companion document to the PGLIB-OPF data. Section II begins with a detailed specification of the AC-OPF problem. Section III provides a brief overview of the established AC-OPF cases provided with MATPOWER and highlights some limitations in using these cases for benchmarking AC-OPF algorithms. Section IV provides a survey of known creative-commons transmission system network data and highlights the missing information in each network data source. Section V briefly introduces methods for addressing the missing information in these datasets. Section VI introduces the PGLIB-

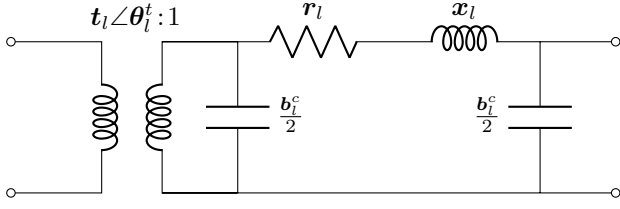


Fig. 1.  $\Pi$ -circuit branch model with an ideal transformer. This is the branch model used by MATPOWER [16].

OPF networks and conducts a baseline validation study to demonstrate that they are suitable for benchmarking the AC-OPF problem. Section VII provides concluding remarks. An appendix summarizes some variants of the proposed AC-OPF model and discusses the additional data that may be necessary in order to make the PGLIB-OPF test cases applicable to other classes of power system optimization and control problems.

## II. THE AC OPTIMAL POWER FLOW FORMULATION

Many variations of the AC-OPF problem are relevant to power system analysis. However, in accordance with PGLIB repository requirements, this section nominates a specific version of the AC-OPF problem for algorithmic benchmarking. The following mathematical model presents a variant of the AC-OPF problem that is often used in related AC-OPF publications (e.g. [1]–[5], [11], [15]) and is readily encoded in the MATPOWER network data format.

The proposed AC-OPF problem requires the following network parameters: a set of bus ids  $N$ ; a set of branch ids  $E$  with an arbitrary orientation; a set  $E^R$  that captures the reverse orientation of the branches; and a set of generator ids  $G$ . For each bus  $i \in N$ : the sets  $E_i$  and  $E_i^R$  indicate the subset of edges that are incident to that bus; the set  $G_i$  reflects the subset of generator ids that are connected to that bus;  $S_i^d$  is the constant power demand;  $Y_i^s$  is the bus shunt admittance; and  $v_i^l, v_i^u$  indicate the operating range of the bus' voltage magnitude. For each generator  $k \in G$ :  $S_k^{gl}, S_k^{gu}$  indicate the generator's power injection range; and  $c_{2k}, c_{1k}, c_{0k}$  provide the coefficients of a quadratic active power cost function. For each branch  $(l, i, j) \in E$ :  $i$  and  $j$  are the *from* and *to* buses respectively and  $l$  is the branch id; the series admittance, line charge, and transformer parameters are given by  $Y_l, b_l^c, T_l$  respectively; the branch's thermal limit is given by  $s_l^u$ ; and the branch voltage angle difference range is  $\theta_l^{\Delta l}, \theta_l^{\Delta u}$ . Lastly, a voltage angle reference bus  $ref \in N$  is specified. Most of these parameters are specified directly in a MATPOWER data file. However, the following parameters need to be computed from the raw data as follows,

$$Y_l = Z_l^{-1} = \frac{r_l}{r_l^2 + x_l^2} - i \frac{x_l}{r_l^2 + x_l^2} \quad (1a)$$

$$T_l = t_l \cos(\theta_l^t) + i t_l \sin(\theta_l^t) \quad (1b)$$

## Model 1 The AC Optimal Power Flow Problem (AC-OPF)

**variables:**

$$S_k^g \quad \forall k \in G$$

$$V_i \quad \forall i \in N$$

$$S_{lij} \quad \forall (l, i, j) \in E \cup E^R$$

$$\text{minimize: } \sum_{k \in G} c_{2k} (\Re(S_k^g))^2 + c_{1k} \Re(S_k^g) + c_{0k} \quad (2a)$$

**subject to:**

$$\angle V_{ref} = 0 \quad (2b)$$

$$S_k^{gl} \leq S_k^g \leq S_k^{gu} \quad \forall k \in G \quad (2c)$$

$$v_i^l \leq |V_i| \leq v_i^u \quad \forall i \in N \quad (2d)$$

$$\sum_{k \in G_i} S_k^g - S_i^d - Y_i^s |V_i|^2 = \sum_{(l, i, j) \in E_i \cup E_i^R} S_{lij} \quad \forall i \in N \quad (2e)$$

$$S_{lij} = \left( Y_l^* - i \frac{b_l^c}{2} \right) \frac{|V_i|^2}{|T_l|^2} - Y_l^* \frac{V_i V_j^*}{T_l} \quad \forall (l, i, j) \in E \quad (2f)$$

$$S_{lji} = \left( Y_l^* - i \frac{b_l^c}{2} \right) |V_j|^2 - Y_l^* \frac{V_i^* V_j}{T_l^*} \quad \forall (l, i, j) \in E \quad (2g)$$

$$|S_{lij}| \leq s_l^u \quad \forall (l, i, j) \in E \cup E^R \quad (2h)$$

$$\theta_l^{\Delta l} \leq \angle(V_i V_j^*) \leq \theta_l^{\Delta u} \quad \forall (l, i, j) \in E \quad (2i)$$

Figure 1 shows the circuit model used to represent each branch.

Model 1 presents the AC-OPF problem as a non-convex nonlinear mathematical program over complex values and variables. A detailed description of the model's notation and derivation can be found in [5]. The objective function (2a) strives to minimize the cost of active power injections. Constraint (2b) fixes the voltage angle of the reference bus. Constraint (2c) sets the generator injection limits and constraint (2d) sets the bus voltage magnitude limits. Constraint (2e) captures the nodal power balance and constraints (2f)–(2g) ensure that the branch power flows are consistent with Ohm's Law. Finally, constraints (2h) and (2i) capture the branch thermal and voltage angle difference limits. It is important to note that solving Model 1 is NP-Hard [29] in general, even if the network has a tree topology [30]. Consequently, it is expected that solution methods for Model 1 will exhibit a wide variety of quality-runtime tradeoffs and will be specialized to different classes of inputs.

## III. MOTIVATION

To motivate the need for a careful curation of the network data in PGLIB-OPF, this section conducts a preliminary study of thirty-five AC transmission system datasets that are distributed with MATPOWER v6.0 [16], [48]. The *optimality gap* measure is used as a simple and preliminary test of AC-OPF difficulty, as one expects that challenging cases will exhibit a large optimality gap. Given a feasible solution to the AC-OPF problem and the solution to a convex relaxation,

TABLE I  
AC-OPF OPTIMALITY GAPS ON NETWORK DATASETS DISTRIBUTED WITH MATPOWER V6.0.

Test Case	$ N $	$ E $	$\$/h$ AC	Gap (%) SOC	Runtime (seconds)	
					AC	SOC
case5	5	6	1.7552e+04	14.55	<1	<1
case6ww	6	11	3.1440e+03	0.63	<1	<1
case9	9	9	5.2967e+03	0.01	<1	<1
case9target	9	9	n.s.	inf.	<1	<1
case14	14	20	8.0815e+03	0.08	<1	<1
case24_ieee_rts	24	38	6.3352e+04	0.02	<1	<1
case30	30	41	5.7689e+02	0.58	<1	<1
case_ieee30	30	41	8.9061e+03	0.05	<1	<1
case39	39	46	4.1864e+04	0.03	<1	<1
case57	57	80	4.1738e+04	0.07	<1	<1
case89pegase	89	210	5.8198e+03	0.17	<1	<1
case118	118	186	1.2966e+05	0.25	<1	<1
case145	145	453	n.s.	inf.	14	7
case_illinois200	200	245	3.6748e+04	0.02	<1	<1
case300	300	411	7.1973e+05	0.15	<1	<1
case1354pegase	1354	1991	7.4069e+04	0.08	4	5
case1951rte	1951	2596	8.1738e+04	0.08	17	26
case2383wp	2383	2896	1.8685e+06	1.05	9	6
case2736sp	2736	3504	1.3079e+06	0.30	8	5
case2737sop	2737	3506	7.7763e+05	0.26	6	4
case2746wop	2746	3514	1.2083e+06	0.37	7	4
case2746wp	2746	3514	1.6318e+06	0.33	7	5
case2848rte	2848	3776	5.3022e+04	0.08	46	7
case2868rte	2868	3808	7.9795e+04	0.07	28	8
case2869pegase	2869	4582	1.3400e+05	0.09	9	58
case3012wp	3012	3572	2.5917e+06	0.78	12	6
case3120sp	3120	3693	2.1427e+06	0.54	11	6
case3375wp	3374	4161	7.4120e+06	0.26	13	98
case6468rte	6468	9000	8.6829e+04	0.23	67	226
case6470rte	6470	9005	9.8345e+04	0.17	61	105
case6495rte	6495	9019	1.0628e+05	0.45	44	239
case6515rte	6515	9037	1.0980e+05	0.38	45	36
case9241pegase	9241	16049	3.1591e+05	1.75	61	230
case13659pegase	13659	20467	3.8611e+05	1.52	228	215

the optimality gap is defined as the relative difference between the objective values of the feasible solution and the relaxation:

$$\frac{\text{AC Heuristic} - \text{AC Relaxation}}{\text{AC Heuristic}} \quad (3)$$

There are a wide variety of both AC heuristics (i.e., methods for obtaining feasible solutions to AC OPF problems [21]–[23]) and convex relaxation techniques [12]–[14]. In the interest of simplicity, this preliminary study will use a nonlinear optimization solver that converges to a KKT point as a heuristic for finding AC feasible solutions and a simple Second-Order Cone (SOC) relaxation [3] for providing objective bounds. All of the results were computed using IPOPT 3.12 [49] with the HSL [50] linear algebra library on a server with two 2.10GHz Intel CPU and 128GB of RAM. PowerModels.jl v0.9 [51] was

used to formulate and solve both mathematical programs.

The results of this study are presented in Table I. The data highlights two core points: (1) By-in-large the optimality gaps are less than 1%. Although a large optimality gap is not a necessary condition for AC-OPF hardness, it provides a good indication of a challenging instance. This work will demonstrate that much more significant gaps are possible, providing a significant increase in the variety of network cases for AC-OPF algorithm benchmarking; (2) No feasible solution was found in two cases, case9target and case145. This could suggest that these cases are challenging for AC heuristics. However, the SOC relaxation provides a numerical proof that

TABLE II  
A SURVEY OF TRANSMISSION SYSTEM DATA SOURCES\*

Name	Original Source	Generator Injection Limits	Generator Costs	Thermal Limits
Publication Test Cases				
3-Bus	[31]	[31]	[31]	[31]
case5	[32]	[32]	[32]	—
case30-as	[33]	[33]	[33]	[33]
case39	[35]	[35], [36]	[37]	[38]
IEEE Power Flow Test Cases				
14 Bus	[27]	—	—	—
30 Bus	[27]	—	—	—
57 Bus	[27]	—	—	—
118 Bus	[27]	—	—	—
300 Bus	[27]	—	—	—
IEEE Dynamic Test Cases				
17 Generator	[27]	—	—	—
IEEE Reliability Test Systems (RTS)				
RTS-79	[39]	[39]	[39], [40]	[39]
RTS-96	[41]	[39]	[40]	[39]
Polish Test Cases				
case2383wp	[16]	[16]	[16]	[16]
case2736sp	[16]	[16]	[16]	[16]
case2737sop	[16]	[16]	[16]	[16]
case2746wop	[16]	[16]	[16]	[16]
case2746wp	[16]	[16]	[16]	[16]
case3012wp	[16]	[16]	[16]	—
case3120sp	[16]	[16]	[16]	—
case3375wp	[16]	—	[16]	—
PEGASE Test Cases				
case89pegase	[42]	[42]	—	[42], partial
case1354pegase	[42]	[42]	—	[42], partial
case2869pegase	[42]	[42]	—	[42], partial
case9241pegase	[42]	[42]	—	[42], partial
case13659pegase	[42]	[42]	—	[42], partial
RTE Test Cases				
case1888rte	[42]	[42]	—	[42], partial
case1951rte	[42]	[42]	—	[42], partial
case2848rte	[42]	[42]	—	[42], partial
case2868rte	[42]	[42]	—	[42], partial
case6468rte	[42]	[42]	—	[42], partial
case6470rte	[42]	[42]	—	[42], partial
case6495rte	[42]	[42]	—	[42], partial
case6515rte	[42]	[42]	—	[42], partial
ACTIVSg Test Cases				
ACTIVSg200	[43]	[43]	[43]	[43]
Sustainable Data Evolution Technology Test Cases				
SDET 500	[44]	[44]	—	[44]
SDET 3000	[44]	[44]	—	[44]
SDET 4000	[44]	[44]	—	[44]
Power Systems Engineering Research Center Test Cases				
WECC 240 Bus	[46], [47]	[47]	[47]	[47]
Grid Optimization Competition Test Cases				
179 Bus	[45]	[45]	—	—
Network_02, 173 scenarios	[45]	[45]	[45]	[45]
Network_03, 200 scenarios	[45]	[45]	[45]	[45]
Network_06, 124 scenarios	[45]	[45]	[45]	[45]
Network_08, 373 scenarios	[45]	[45]	[45]	[45]
Network_09, 195 scenarios	[45]	[45]	[45]	[45]
Network_12, 50 scenarios	[45]	[45]	[45]	[45]
Network_13, 103 scenarios	[45]	[45]	[45]	[45]
Network_14, 50 scenarios	[45]	[45]	[45]	[45]
Network_20, 100 scenarios	[45]	[45]	[45]	[45]
Network_25, 60 scenarios	[45]	[45]	[45]	[45]
Network_30, 48 scenarios	[45]	[45]	[45]	[45]
Network_70, 422 scenarios	[45]	[45]	[45]	[45]
Network_75, 40 scenarios	[45]	[45]	[45]	[45]
Network_82, 10 scenarios	[45]	[45]	[45]	[45]
Network_82, 40 scenarios	[45]	[45]	[45]	[45]
Network_83, 130 scenarios	[45]	[45]	[45]	[45]
Network_86, 40 scenarios	[45]	[45]	[45]	[45]
Network_88, 30 scenarios	[45]	[45]	[45]	[45]

\* - only creative commons data sources were considered

these cases have no feasible AC-OPF solution,<sup>1</sup> suggesting that data quality is the source of infeasibility and not algorithmic difficulty. Overall, this simplistic study demonstrates some of the shortcomings of focusing exclusively on the network data that is distributed with MATPOWER v6.0 for benchmarking AC-OPF algorithms. The careful curation of AC network data developed in the following sections results in modified network datasets featuring significant optimality gaps, which will help to emphasize the differences between various AC-OPF solution methods.

#### IV. PUBLICLY AVAILABLE NETWORK DATA

In the interest of curating a comprehensive collection of AC-OPF networks, we begin with a survey of available network datasets. To the best of our knowledge, Table II summarizes all of the readily available transmission system datasets.<sup>2</sup> A careful investigation of these datasets reveals that very few networks include all of the data required to study Model 1. Table II highlights the source of, or lack of, generation capacity limits, generation cost functions, and branch thermal limits in these datasets. Cells containing “—” indicate missing data that must be added before the network will be suitable for benchmarking Model 1. To address the information that is missing in these datasets, the next section reviews a number of data-driven models that can be used to fill in these gaps.

##### A. Network Omissions

Some notable networks have not been included in Table II for the following reasons.

1) *IEEE 30 Bus “New England” Dynamic Test System:* This test case is nearly identical to the IEEE 30 test case and would not bring additional value to the proposed collection of cases.

2) *30 Bus System of Ferrero, Shahidehpour, Ramesh:* In PGLIB-OPF v20.07 the 30 Bus System of Ferrero, Shahidehpour and Ramesh [34] was removed due to being very similar to the 30 Bus System of Alsac and Stott [33] and would not bring additional value to the proposed collection of cases.

3) *IEEE 50 Generator Dynamic Test System:* In its specified state, this test case does not converge to an AC power flow solution. However, if the active generation upper bounds are increased to 1.5 times their given value and voltage bounds are set to  $1 \pm 0.16$ , then a solution can be obtained. This solution still exhibits significant voltage drops, atypical of other networks. Many of the lines in this network have negative  $r$  and  $x$  values, which is likely the result of a network reduction

procedure. Also, the size of the generating units are one or two orders of magnitude larger than any documented generation unit in the U.S., which suggests that these “generators” may actually be modeling imports and exports of power. Since many of these characteristics are atypical compared to the other test cases, this network is omitted.

4) *SDET 2000 System:* This system was omitted starting with PGLIB-OPF v20.07, due to being similar to the Network 70 systems from the Grid Optimization Competition [45].

5) *ACTIVGs 500, 2000 and 10000 Bus Systems:* These three systems were omitted starting with PGLIB-OPF v20.07, due to being similar to the Network 02, Network 06 and Network 13 systems from the Grid Optimization Competition [45].

#### V. DATA DRIVEN MODELS

Ideally, the data missing from Table II would be incorporated by returning to the original network design documents and extracting the required information, such as a generator’s nameplate capacity and line conductor specifications. Unfortunately, due to the age or the synthetic nature of these test cases, this approach to data completion is impractical. This work proposes to leverage publicly available data sources to build data-driven models that can complete the missing data. Such models may not reflect any specific real-world network, but at least they will reflect many of the statistical features found in realistic networks. As identified in Section IV, the key pieces of missing data are generator injection limits, generation costs functions, and branch thermal limits. The rest of this section reviews several data-driven models proposed in [52], which can be used to fill the gaps in these networks.

##### A. Generator Models

Most AC transmission datasets are brief in their description of the generation units. Typically, only the active power injection limits, reactive power injection limits, and a specific generation dispatch point are provided. A notable omission is an active power generation cost function, which is critical in formulating the objective in Model 1. Two key observations can be used to address the limited information on generation units: (1) The U.S. Energy Information Administration (EIA) collects extensive data on generation units throughout the United States. Two reports are particularly useful to this work, the detailed generator data (EIA-860 2012 [53]) and state fuel cost data (SEDS [54]); (2) The bulk of a generator’s properties are driven by its mechanical design, which is in turn significantly influenced by its fuel type. This work begins by developing a data-driven model for generators by assigning them a fuel category. Once a fuel category is determined, probabilistic models for both fuel costs and power injection limits can be derived from publicly available data sources. In the interest of simplicity, this work focuses on the four primary

<sup>1</sup>That is, there does not exist an assignment of the variables that can simultaneously satisfy all of the constraints in Model 1.

<sup>2</sup>Only creative commons datasets were considered to comply with PGLIB data requirements.

dispatchable fuel types, Petroleum (PEL), Natural Gas (NG), Coal (COW), and Nuclear Fuel (NUC).

1) *The Generation Fuel Category Model:* Assuming that a generator's active power injection limit (i.e.  $\Re(\mathbf{S}^{gu})$ ) is a sufficient proxy for its nameplate capacity, one can use the empirical distribution presented in Figure 2 for making a probabilistic guess of the fuel type of a given generation unit. The fuel category classifier is built by selecting the corresponding nameplate capacity bin in Figure 2 and rolling a weighted die to select the fuel type.

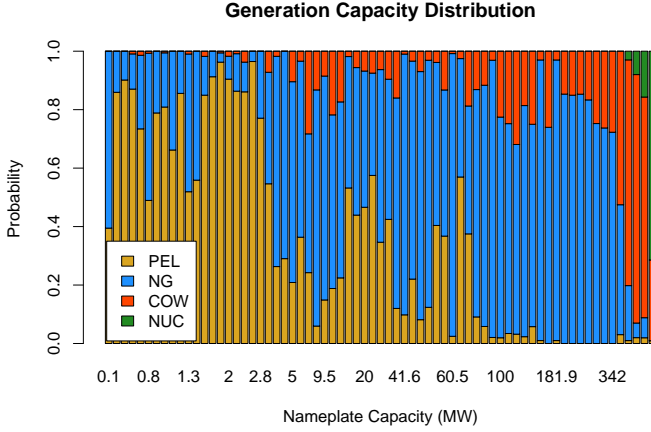


Fig. 2. An Empirical Distribution of Generation Fuel Categories by Nameplate Capacity.

One important point is the identification of synchronous condensers. In Model 1 such devices are not explicitly identified but are modeled as generators with no active generation capabilities. To identify such devices, one can introduce a new fuel category (e.g. SYNC), and any generator with active generation upper and lower bounds of 0 is assigned this category. The empirical distribution shown in Figure 2 combined with this special case forms the generation fuel classification model (**GF-Stat**).

2) *Generation Capacity Models:* Some AC transmission system datasets lack reasonable generation injection limits, especially in cases that were originally designed for benchmarking AC Power Flow algorithms. Two simple generation capability models are developed to address such cases.

a) *Active Generation Capability:* An investigation of the nameplate capacities of each fuel type in [52] revealed that an exponential distribution is a suitable model for PEL, NG, and COW generators, while a normal distribution is suitable for NUC generators. The parameters from maximum likelihood estimation of these distributions are presented in Table III. Using these distributions the active generation capacity model (**AG-Stat**) is constructed as follows. Given a fuel category  $f$  and an active generation upper limit or present output  $\Re(\mathbf{S}^g)$ , the fuel category nameplate capacity distribution is sampled as  $p^{gu}$ , until  $p^{gu} > \Re(\mathbf{S}^g)$ . Then the maximum

active power generation capacity is updated to the sampled value, i.e.  $\Re(\mathbf{S}^{gu}) = p^{gu}$ .

TABLE III  
DISTRIBUTION PARAMETERS FOR GENERATOR CAPACITY MODELS.

Fuel Type	Nameplate Capacity (MW)	
	$\hat{\lambda}$	
PEL	0.023254	
NG	0.009188	
COW	0.003201	

Fuel Type	Nameplate Capacity (MW)	
	$\hat{\mu}$	$\hat{\sigma}$
NUC	1044.56	219.27

b) *Reactive Generation Capabilities:* In synchronous machines, reactive generation capabilities are tightly coupled with active generation capabilities. Lacking detailed information about the generator's specifications it is observed in [52] that the reactive power capability of a synchronous machine is roughly  $\pm 50\%$  of its nameplate capacity leading to the **RG-AM50** model. This model assumes the given reactive power bounds are accurate, unless they exceed 50% of the nameplate capacity, in which case, they are reduced to  $\pm 50\%$  of the nameplate capacity. This provides a pessimistic model of the generator's capabilities.

3) *Generation Cost Model:* Observing the simplicity of the cost function in Model 1, this work proposes to focus on the marginal costs of power generation in an idealized non-competitive environment. Fuel price information is available in the "Primary Energy, Electricity, and Total Energy Price Estimates, 2012" in the SEDS dataset [54]. In [52] it was observed that for the fuel categories of interest, the fuel costs are roughly normally distributed, with the parameters specified in Table IV. Using these distributions the model for generation costs (**AC-Stat**) is built as follows. The fuel cost parameters from Table IV are assumed to be representative of the price variations across the various generating units, so that, given a fuel category, one simply draws a sample from the associated normal distribution to produce a linear fuel cost value (\$/BTU) for that generator. The conversion from heat energy input (BTU) to electrical energy output (MWh) depends on a generator's *heat rate*, which differs based on the efficiency of the generating plant. Representative heat rate values are adopted from 2016 EIA data [55].

## B. Branch Thermal Limit Models

Determining a transmission line's operational thermal rating is an intricate task that combines a wide variety of information such as the conductor's design, location, age and the season of the year. Unfortunately, this information is unavailable in all of the network datasets presented in Table II. The only available branch parameters are the impedance ( $Z = r + ix$

TABLE IV  
GENERATOR COST MODEL.

Fuel Category	SEDS Label	Cost (\$/MWh)	
		$\hat{\mu}$	$\hat{\sigma}$
PEL	Distillate Fuel Oil	111.3398	9.6736
NG	Natural Gas	34.2731	10.9810
COW	Coal	24.7919	8.0866
NUC	Nuclear Fuel	7.2504	0.7534

p.u.), line charge ( $b^c$  p.u.), and often the nominal voltage ( $\dot{v}$ ) on the connecting buses. This work leverages two models for producing reasonable thermal limits, a data-driven approach and an arithmetic approach leveraging other model parameters.

1) *A Statistical Model:* After reviewing a number of network datasets with realistic thermal limits, [52] concluded that the following exponential model was reasonable estimator of thermal limits when the impedance ( $Z = r + ix$  p.u.) and nominal voltage ( $\dot{v}$ ) is known,

$$s^u = \dot{v} e^{-5.0886 \left( \frac{x}{r} \right)^{0.4772}} \quad (4)$$

This model is referred to as **TL-Stat**. The intuition of the model is that the ratio of resistance to impedance provides some insight into the branch's conductor type and configuration, since this ratio should be independent of the line length.

2) *A Reasonable Upper Bound:* Although the statistical model for branch thermal limits is quite useful, it cannot be applied in cases where data for  $r$ ,  $x$ , or  $\dot{v}$  is missing. Notable examples include: transformers, where the nominal voltage value differs on both sides of the line, and ideal lines, which do not have an  $r$  value. For these cases, it is helpful to have an alternate method for producing reasonable thermal limits.

Given that Model 1 includes reasonable bounds on  $|V|$  and  $\theta^\Delta$ , a reasonable thermal limit can be computed as follows. For a branch  $(l, i, j) \in E$ , let  $\theta_l^{\Delta m} = \max(|\theta_l^{\Delta l}|, |\theta_l^{\Delta u}|)$ . A reasonable value for  $s_l^u$  is

$$(s_l^u)^2 = (v_i^u)^2 |Y_l|^2 ((v_i^u)^2 + (v_j^u)^2 - 2v_i^u v_j^u \cos(\theta_l^{\Delta m})) \quad (5)$$

This model is referred to as **TL-UB** and a more detailed discussion can be found in [52].

### C. Voltage Angle Difference Bounds

Voltage angle difference bounds are often used as a proxy for capturing voltage angle stability limits on long transmission lines [56]. Unfortunately, no available network datasets include detailed information for these bounds, which are specified in Model 1. To fill this gap all of the PGLIB-OPF models are given generous voltage angle difference bound of  $\theta^{\Delta l} = -30^\circ$ ,  $\theta^{\Delta u} = 30^\circ$ , which are easily justified by practical voltage stability requirements [56] and do not

impact the best-known solution of any available network. It is important to emphasize that a voltage angle difference bound of  $30^\circ$  is generous enough to be subsumed by the thermal limits provided with all of the networks considered here. Still, even this generous value has significant implications for the development of power system optimization methods (e.g. [1], [5], [6], [57], [58]).

## VI. THE PGLIB-OPF NETWORKS

Leveraging the proposed models for completing the missing data in Table II, the PGLIB-OPF networks are developed. Table V summarizes which models are used to convert the base network data into PGLIB-OPF networks.

### A. Results

To verify the usefulness of the proposed PGLIB-OPF networks for benchmarking AC-OPF algorithms, the study from Section III is revisited. PowerModels.jl v0.17 [51] was used to formulate Model 1 and its Second-Order Cone (SOC) relaxation [3], [5] and both models were solved with IPOPT 3.12 [49] using the HSL [50] linear algebra library on a server with two 2.10GHz Intel CPU and 128GB of RAM.

Table VI presents the results of the base PGLIB-OPF networks, which are called the Typical Operating Conditions (TYP) cases. Many of the optimality gaps have remained small (i.e. below 1%). However, a number of networks do exhibit significant optimality gaps, such as `pplib_opf_case5_pjm`, `pplib_opf_case30_ieee`, `pplib_opf_case162_ieee_dtc`, `pplib_opf_case6495_rte`, and `pplib_opf_case6515_rte`. This suggests that at least a subset of these cases are interesting for benchmarking AC-OPF algorithms.

### B. Building More Challenging Test Cases

The typical operating conditions networks presented in Table VI provide a suitable start for benchmarking AC-OPF algorithms. However, it is worthwhile to explore if even more challenging test cases can be devised. To that end, PGLIB-OPF includes two variants of the base PGLIB-OPF networks that exhibit even more extreme optimality gaps, the Active Power Increase (API) and Small Angle Difference (SAD) cases.

*Active Power Increase (API) Cases:* It was observed in [59], [60] that power flow congestion is a key feature of interesting AC Optimal Transmission Switching test cases. Inspired by this observation, the following Active Power Increase (API) PGLIB-OPF networks are proposed. For each of the standard PGLIB-OPF networks, an optimization problem is solved, which increases the active power demands proportionally throughout the network until the branch thermal limits are binding. Once a maximal increase in active power demand is determined, the statistical models are applied to appropriately



TABLE V  
PGLIB-OPF INSTANCE GENERATION DETAILS

PGLIB Name	Original Name	Model				Thermal Limit	Voltage Angle Diff. Bound
		Active Gen.	Reactive Gen.	Gen. Cost			
Publication Test Cases							
pglib_opf_case3_lmbd	3-Bus	—	—	—	—	30°	
pglib_opf_case5_pjm	case5	—	—	—	TL-Stat	30°	
pglib_opf_case30_as	30 Bus-as	—	—	—	—	30°	
pglib_opf_case39_epri	case39	—	—	—	—	30°	
IEEE Power Flow Test Cases							
pglib_opf_case14_ieee	14 Bus	AG-Stat	RG-AM50	AC-Stat	TL-Stat	30°	
pglib_opf_case30_ieee	30 Bus	AG-Stat	RG-AM50	AC-Stat	TL-Stat	30°	
pglib_opf_case57_ieee	57 Bus	AG-Stat	RG-AM50	AC-Stat	TL-Stat	30°	
pglib_opf_case118_ieee	118 Bus	AG-Stat	RG-AM50	AC-Stat	TL-Stat	30°	
pglib_opf_case300_ieee	300 Bus	AG-Stat	RG-AM50	AC-Stat	TL-UB	30°	
IEEE Dynamic Test Cases							
pglib_opf_case162_ieee_dtc	17 Generator	AG-Stat	RG-AM50	AC-Stat	TL-Stat	30°	
IEEE Reliability Test Systems (RTS)							
pglib_opf_case24_ieee_rts	RTS-79	—	—	—	—	30°	
pglib_opf_case73_ieee_rts	RTS-96	—	—	—	—	30°	
Polish Test Cases							
pglib_opf_case2383wp_k	case2383wp	—	—	—	—	30°	
pglib_opf_case2736sp_k	case2736sp	—	—	—	—	30°	
pglib_opf_case2737sop_k	case2737sop	—	—	—	—	30°	
pglib_opf_case2746wop_k	case2746wop	—	—	—	—	30°	
pglib_opf_case2746wp_k	case2746wp	—	—	—	—	30°	
pglib_opf_case3012wp_k	case3012wp	—	—	—	TL-Stat	30°	
pglib_opf_case3120sp_k	case3120sp	—	—	—	TL-Stat	30°	
pglib_opf_case3375wp_k	case3375wp	—	RG-AM50	—	TL-Stat	30°	
PEGASE Test Cases							
pglib_opf_case89_pegase	case89pegase	—	—	AC-Stat	TL-UB	30°	
pglib_opf_case1354_pegase	case1354pegase	—	—	AC-Stat	TL-UB	30°	
pglib_opf_case2869_pegase	case2869pegase	—	—	AC-Stat	TL-UB	30°	
pglib_opf_case9241_pegase	case9241pegase	—	—	AC-Stat	TL-UB	30°	
pglib_opf_case13659_pegase	case13659pegase	—	—	AC-Stat	TL-UB	30°	
RTE Test Cases							
pglib_opf_case1888_rte	case1888rte	—	—	AC-Stat	TL-UB	30°	
pglib_opf_case1951_rte	case1951rte	—	—	AC-Stat	TL-UB	30°	
pglib_opf_case2848_rte	case2848rte	—	—	AC-Stat	TL-UB	30°	
pglib_opf_case2868_rte	case2868rte	—	—	AC-Stat	TL-UB	30°	
pglib_opf_case6468_rte	case6468rte	—	—	AC-Stat	TL-UB	30°	
pglib_opf_case6470_rte	case6470rte	—	—	AC-Stat	TL-UB	30°	
pglib_opf_case6495_rte	case6495rte	—	—	AC-Stat	TL-UB	30°	
pglib_opf_case6515_rte	case6515rte	—	—	AC-Stat	TL-UB	30°	
ACTIVSg Test Cases							
pglib_opf_case200_activ	ACTIVSg200	—	—	—	—	30°	
Sustainable Data Evolution Technology Test Cases							
pglib_opf_case588_sdet	SDET 500	—	—	AC-Stat	—	30°	
pglib_opf_case2853_sdet	SDET 3000	—	—	AC-Stat	—	30°	
pglib_opf_case4661_sdet	SDET 4000	—	—	AC-Stat	—	30°	
Power Systems Engineering Research Center Test Cases							
pglib_opf_case240_pserc	WECC 240 Bus	—	—	AC-Stat	TL-UB	30°	
Grid Optimization Competition Test Cases							
pglib_opf_case179_goc	179 Bus	—	—	AC-Stat	TL-UB	30°	
pglib_opf_case500_goc	02O-173, scenario 72	—	—	—	—	30°	
pglib_opf_case793_goc	03O-200, scenario 72	—	—	—	—	30°	
pglib_opf_case2000_goc	06O-124, scenario 95	—	—	—	—	30°	
pglib_opf_case2312_goc	70O-422, scenario 335	—	—	—	—	30°	
pglib_opf_case2742_goc	75O-040, scenario 7	—	—	—	—	30°	
pglib_opf_case3022_goc	08O-373, scenario 261	—	—	—	—	30°	
pglib_opf_case3970_goc	82O-040, scenario 34	—	—	—	—	30°	
pglib_opf_case4020_goc	83O-130, scenario 47	—	—	—	—	30°	
pglib_opf_case4601_goc	82O-010, scenario 10	—	—	—	—	30°	
pglib_opf_case4619_goc	86O-040, scenario 17	—	—	—	—	30°	
pglib_opf_case4837_goc	88O-030, scenario 6	—	—	—	—	30°	
pglib_opf_case4917_goc	09O-195, scenario 40	—	—	—	—	30°	
pglib_opf_case9591_goc	12O-050, scenario 33	—	—	—	—	30°	
pglib_opf_case10000_goc	13O-103, scenario 81	—	—	—	—	30°	
pglib_opf_case10480_goc	14O-050, scenario 9	—	—	—	—	30°	
pglib_opf_case19402_goc	20O-100, scenario 42	—	—	—	—	30°	
pglib_opf_case24464_goc	25O-060, scenario 37	—	—	—	—	30°	
pglib_opf_case30000_goc	30O-048, scenario 36	—	—	—	—	30°	

TABLE VI  
AC-OPF BOUNDS ON PGLIB-OPF TYP NETWORKS.

Test Case	$ N $	$ E $	$\$/h$ AC	Gap (%) SOC	Runtime (sec.) AC	Runtime (sec.) SOC
Typical Operating Conditions (TYP)						
pglib_opf_case3_lmbd	3	3	5.8126e+03	1.32	<1	<1
pglib_opf_case5_pjm	5	6	1.7552e+04	14.55	<1	<1
pglib_opf_case14_ieee	14	20	2.1781e+03	0.11	<1	<1
pglib_opf_case24_ieee_rts	24	38	6.3352e+04	0.02	<1	<1
pglib_opf_case30_as	30	41	8.0313e+02	0.06	<1	<1
pglib_opf_case30_ieee	30	41	8.2085e+03	18.84	<1	<1
pglib_opf_case39_epri	39	46	1.3842e+05	0.56	<1	<1
pglib_opf_case57_ieee	57	80	3.7589e+04	0.16	<1	<1
pglib_opf_case73_ieee_rts	73	120	1.8976e+05	0.04	<1	<1
pglib_opf_case89_pegase	89	210	1.0729e+05	0.75	<1	<1
pglib_opf_case118_ieee	118	186	9.7214e+04	0.91	<1	<1
pglib_opf_case162_ieee_dtc	162	284	1.0808e+05	5.95	<1	<1
pglib_opf_case179_goc	179	263	7.5427e+05	0.16	<1	<1
pglib_opf_case200_activ	200	245	2.7558e+04	0.01	<1	<1
pglib_opf_case240_pserc	240	448	3.3297e+06	2.78	3	2
pglib_opf_case300_ieee	300	411	5.6522e+05	2.63	<1	<1
pglib_opf_case500_goc	500	733	4.5495e+05	0.25	<1	<1
pglib_opf_case588_sdet	588	686	3.1314e+05	2.14	<1	<1
pglib_opf_case793_goc	793	913	2.6020e+05	1.33	2	<1
pglib_opf_case1354_pegase	1354	1991	1.2588e+06	1.57	4	3
pglib_opf_case1888_rte	1888	2531	1.4025e+06	2.05	8	31
pglib_opf_case1951_rte	1951	2596	2.0856e+06	0.14	18	5
pglib_opf_case2000_goc	2000	3639	9.7343e+05	0.31	7	4
pglib_opf_case2312_goc	2312	3013	4.4133e+05	1.90	6	3
pglib_opf_case2383wp_k	2383	2896	1.8682e+06	1.04	7	5
pglib_opf_case2736sp_k	2736	3504	1.3080e+06	0.31	6	4
pglib_opf_case2737sop_k	2737	3506	7.7773e+05	0.27	5	3
pglib_opf_case2742_goc	2742	4673	2.7571e+05	1.35	28	6
pglib_opf_case2746wop_k	2746	3514	1.2083e+06	0.37	5	4
pglib_opf_case2746wp_k	2746	3514	1.6317e+06	0.33	6	5
pglib_opf_case2848_rte	2848	3776	1.2866e+06	0.13	16	7
pglib_opf_case2853_sdet	2853	3921	2.0524e+06	0.91	9	6
pglib_opf_case2868_rte	2868	3808	2.0096e+06	0.10	14	8
pglib_opf_case2869_pegase	2869	4582	2.4628e+06	1.01	12	7
pglib_opf_case3012wp_k	3012	3572	2.6008e+06	1.03	9	12
pglib_opf_case3022_goc	3022	4135	6.0138e+05	2.77	10	5
pglib_opf_case3120sp_k	3120	3693	2.1480e+06	0.56	9	5
pglib_opf_case3375wp_k	3374	4161	7.4382e+06	0.55	11	6
pglib_opf_case3970_goc	3970	6641	9.6099e+05	0.34	18	12
pglib_opf_case4020_goc	4020	6988	8.2225e+05	1.23	21	13
pglib_opf_case4601_goc	4601	7199	8.2624e+05	0.54	22	14
pglib_opf_case4619_goc	4619	8150	4.7670e+05	0.91	19	23
pglib_opf_case4661_sdet	4661	5997	2.2513e+06	1.99	15	11
pglib_opf_case4837_goc	4837	7765	8.7226e+05	0.47	20	11
pglib_opf_case4917_goc	4917	6726	1.3878e+06	2.50	17	9
pglib_opf_case6468_rte	6468	9000	2.0697e+06	1.13	63	27
pglib_opf_case6470_rte	6470	9005	2.2376e+06	1.76	33	23
pglib_opf_case6495_rte	6495	9019	3.0678e+06	15.11	67	24
pglib_opf_case6515_rte	6515	9037	2.8255e+06	6.40	54	22
pglib_opf_case9241_pegase	9241	16049	6.2431e+06	2.54	48	36
pglib_opf_case9591_goc	9591	15915	1.0617e+06	0.62	60	43
pglib_opf_case10000_goc	10000	13193	1.3540e+06	1.54	47	26
pglib_opf_case10480_goc	10480	18559	2.3146e+06	1.23	65	42
pglib_opf_case13659_pegase	13659	20467	8.9480e+06	1.39	54	61
pglib_opf_case19402_goc	19402	34704	1.9778e+06	1.19	139	110
pglib_opf_case24464_goc	24464	37816	2.6295e+06	0.97	118	98
pglib_opf_case30000_goc	30000	35393	1.1423e+06	2.89	258	119

update the other network parameters (e.g. generator capabilities and cost functions). The results of the API networks are presented in Table VII. As expected, the optimality gaps in these networks have increased significantly with 60% of cases having optimality gaps above 1% and eight cases with optimality gaps above 10%. This suggests that many of these cases will be useful for benchmarking AC-OPF algorithms.

*Small Angle Difference (SAD) Cases:* A second approach to modifying the PGLIB-OPF networks is inspired by recent lines of research [1], [5], [6], [57] that indicate voltage angle difference bounds can have significant impacts on power system optimization approaches. To emphasize these impacts, the following Small Angle Difference (SAD) PGLIB-OPF networks are proposed. For each of the standard PGLIB-OPF networks, an optimization problem is solved in order to find the minimum value of  $\theta^\Delta$  that can be applied on all of the branches in the network, while retaining a feasible AC power flow. Once this small value of  $\theta^\Delta$  is determined, the original test case is updated with this value, which introduces voltage angle difference congestion on the network branches. The results of the SAD networks are presented in Table VIII. Interestingly, this entirely different approach to modifying the base networks also leads to significant optimality gaps, with 75% of the SAD networks having optimality gaps above 1%. This suggest that many of these cases will be useful for benchmarking AC-OPF algorithms.

It is important to note that in all three result tables, the significant optimality gaps can be caused by two factors: (1) the heuristic for finding feasible AC-OPF solution fails to find the global optimum (e.g. see [61]); (2) the SOC convex relaxation is weak (e.g. see [5]) and does not provide a tight bound on the quality of the AC-OPF solution. Both factors present interesting avenues for research on AC-OPF algorithms.

## VII. CONCLUSIONS

This report has highlighted some of the shortcomings of using existing network datasets for benchmarking AC Optimal Power Flow algorithms and has proposed the PGLIB-OPF networks to mitigate them. Leveraging data-driven models, PGLIB-OPF ensures that all of the networks have reasonable values for key power network parameters, including generation injection limits, generation costs, and branch thermal limits. Furthermore, the active power increase and small angle difference network variants are developed to provide additional challenging cases for benchmarking. A detailed validation study demonstrates that the majority of the PGLIB-OPF networks exhibit significant optimality gaps and are therefore useful for benchmarking AC-OPF algorithms.

It is important to emphasize that while the primary challenge of this work has been to develop realistic and challenging network datasets for benchmarking the AC-OPF problem

presented in Model 1, there is still a significant gap in the models used for industry-grade AC optimal power flow studies [62]–[64]. Some of the key extensions include: information about configurable assets such as bus shunts, switches, and transformer tap settings; N-1 contingency cases; branch thermal limits for short- and long-term overloading; and generator capability curves. As the research community is able to address the challenges presented in the PGLIB-OPF networks, it is important to also consider more realistic extensions of Model 1 and to curate new PGLIB repositories for those model variants.

Although PGLIB-OPF has highlighted some advantages for AC-OPF benchmarking, its network datasets are still by-in-large synthetically generated. This highlights the continued need for industry engagement in the development of more detailed and realistic network datasets for benchmarking AC-OPF algorithms. We hope that the PGLIB-OPF networks proposed herein will be sufficient for the research community to benchmark AC-OPF algorithms, while more real-world network datasets are developed and contributed to the PGLIB-OPF repository in the years to come.

## APPENDIX

### EXTENSIONS AND ALTERNATIVE APPLICATIONS

While this report focuses on the AC-OPF formulation described in Model 1, there are a variety of possible modifications, extensions, and other problems that are relevant to many power system researchers. This appendix first presents common modifications of Model 1 and then summarizes several possible alternative uses of the PGLIB data.

#### A. Current Flow Limits

The OPF formulation in Model 1 considers line-flow limits that are based on apparent power (2h) and voltage angle differences (2i). A common modified problem formulation limits the magnitudes of the current flows on each line. Specifically, the following constraints either replace or augment the apparent power flow limits in (2h):

$$I_{lij} = \left( \mathbf{Y}_l + i \frac{\mathbf{b}_l^c}{2} \right) \frac{V_i}{|\mathbf{T}_l|^2} - \mathbf{Y}_l \frac{V_j}{\mathbf{T}_l^*} \quad \forall (l, i, j) \in E \quad (6a)$$

$$I_{lji} = \left( \mathbf{Y}_l + i \frac{\mathbf{b}_l^c}{2} \right) V_j - \mathbf{Y}_l \frac{V_i}{\mathbf{T}_l} \quad \forall (l, i, j) \in E \quad (6b)$$

$$|I_{lij}| \leq \mathbf{I}_l^u \quad \forall (l, i, j) \in E \cup E^R \quad (6c)$$

Note that the per unit value of  $\mathbf{I}_l^u$  is often assumed to be equivalent to the per unit value of  $\mathbf{s}_l^u$ .

#### B. Branch Charging Model

The II-circuit branch model of this work (i.e., Figure 1), only considers the susceptance impacts of line charging. After the inception of that MATPOWER data format, a number of

TABLE VII  
AC-OPF BOUNDS ON PGLIB-OPF API NETWORKS.

Test Case	$ N $	$ E $	\$/h AC	Gap (%) SOC	Runtime (sec.) AC	Runtime (sec.) SOC
Congested Operating Conditions (API)						
pglib_opf_case3_lmbd_api	3	3	1.1236e+04	9.27	<1	<1
pglib_opf_case5_pjm_api	5	6	7.6377e+04	4.09	<1	<1
pglib_opf_case14_ieee_api	14	20	5.9994e+03	5.13	<1	<1
pglib_opf_case24_ieee_rts_api	24	38	1.3494e+05	17.88	<1	<1
pglib_opf_case30_as_api	30	41	4.9962e+03	44.61	<1	<1
pglib_opf_case30_ieee_api	30	41	1.8044e+04	5.46	<1	<1
pglib_opf_case39_epri_api	39	46	2.4967e+05	1.72	<1	<1
pglib_opf_case57_ieee_api	57	80	4.9290e+04	0.08	<1	<1
pglib_opf_case73_ieee_rts_api	73	120	4.2263e+05	12.87	<1	<1
pglib_opf_case89_pegase_api	89	210	1.3017e+05	23.11	<1	<1
pglib_opf_case118_ieee_api	118	186	2.4224e+05	29.97	<1	<1
pglib_opf_case162_ieee_dtc_api	162	284	1.2099e+05	4.36	<1	<1
pglib_opf_case179_goc_api	179	263	1.9320e+06	9.88	<1	<1
pglib_opf_case200_activ_api	200	245	3.5701e+04	0.03	<1	<1
pglib_opf_case240_pserc_api	240	448	4.6406e+06	0.67	4	2
pglib_opf_case300_ieee_api	300	411	6.8499e+05	0.85	<1	<1
pglib_opf_case500_goc_api	500	733	6.9241e+05	3.44	2	<1
pglib_opf_case588_sdet_api	588	686	3.9476e+05	1.61	<1	<1
pglib_opf_case793_goc_api	793	913	3.1885e+05	13.43	2	<1
pglib_opf_case1354_pegase_api	1354	1991	1.4983e+06	0.55	4	3
pglib_opf_case1888_rte_api	1888	2531	1.9539e+06	0.23	7	6
pglib_opf_case1951_rte_api	1951	2596	2.4108e+06	0.57	8	5
pglib_opf_case2000_goc_api	2000	3639	1.4686e+06	2.07	8	4
pglib_opf_case2312_goc_api	2312	3013	5.7152e+05	13.09	11	3
pglib_opf_case2383wp_k_api	2383	2896	2.7913e+05	0.01	2	<1
pglib_opf_case2736sp_k_api	2736	3504	6.5394e+05	10.84	7	3
pglib_opf_case2737sop_k_api	2737	3506	3.6715e+05	5.88	6	2
pglib_opf_case2742_goc_api	2742	4673	6.4219e+05	24.45	22	5
pglib_opf_case2746wop_k_api	2746	3514	5.1166e+05	0.01	2	2
pglib_opf_case2746wp_k_api	2746	3514	5.8183e+05	0.00	4	2
pglib_opf_case2848_rte_api	2848	3776	1.4970e+06	0.25	26	7
pglib_opf_case2853_sdet_api	2853	3921	2.4578e+06	1.96	10	6
pglib_opf_case2868_rte_api	2868	3808	2.2946e+06	0.18	23	6
pglib_opf_case2869_pegase_api	2869	4582	2.9296e+06	1.00	13	8
pglib_opf_case3012wp_k_api	3012	3572	7.2887e+05	0.00	4	2
pglib_opf_case3022_goc_api	3022	4135	6.5189e+05	9.82	9	4
pglib_opf_case3120sp_k_api	3120	3693	9.3692e+05	23.96	12	4
pglib_opf_case3375wp_k_api	3374	4161	5.8478e+06	–	11	352
pglib_opf_case3970_goc_api	3970	6641	1.4557e+06	34.58	37	9
pglib_opf_case4020_goc_api	4020	6988	1.2979e+06	17.58	39	9
pglib_opf_case4601_goc_api	4601	7199	7.9253e+05	20.51	29	10
pglib_opf_case4619_goc_api	4619	8150	1.0299e+06	8.30	28	14
pglib_opf_case4661_sdet_api	4661	5997	2.6953e+06	2.64	16	105
pglib_opf_case4837_goc_api	4837	7765	1.1578e+06	8.15	36	12
pglib_opf_case4917_goc_api	4917	6726	1.5479e+06	8.54	16	8
pglib_opf_case6468_rte_api	6468	9000	2.3135e+06	0.82	81	178
pglib_opf_case6470_rte_api	6470	9005	2.6065e+06	1.20	52	80
pglib_opf_case6495_rte_api	6495	9019	2.9750e+06	2.04	54	22
pglib_opf_case6515_rte_api	6515	9037	3.0617e+06	1.89	59	21
pglib_opf_case9241_pegase_api	9241	16049	7.0112e+06	2.67	64	806
pglib_opf_case9591_goc_api	9591	15915	1.4259e+06	13.84	145	26
pglib_opf_case10000_goc_api	10000	13193	2.3728e+06	7.65	52	27
pglib_opf_case10480_goc_api	10480	18559	2.7627e+06	4.78	77	35
pglib_opf_case13659_pegase_api	13659	20467	9.2842e+06	1.85	55	76
pglib_opf_case19402_goc_api	19402	34704	2.3987e+06	4.75	195	100
pglib_opf_case24464_goc_api	24464	37816	2.4723e+06	4.00	137	151
pglib_opf_case30000_goc_api	30000	35393	1.3530e+06	24.79	439	119

TABLE VIII  
AC-OPF BOUNDS ON PGLIB-OPF SAD NETWORKS.

Test Case	$ N $	$ E $	\$/h AC	Gap (%) SOC	Runtime (sec.) AC	Runtime (sec.) SOC
Small Angle Difference Conditions (SAD)						
pglib_opf_case3_lmbd_sad	3	3	5.9593e+03	3.75	<1	<1
pglib_opf_case5_pjm_sad	5	6	2.6109e+04	3.62	<1	<1
pglib_opf_case14_ieee_sad	14	20	2.7768e+03	21.53	<1	<1
pglib_opf_case24_ieee_rts_sad	24	38	7.6918e+04	9.55	<1	<1
pglib_opf_case30_as_sad	30	41	8.9735e+02	7.88	<1	<1
pglib_opf_case30_ieee_sad	30	41	8.2085e+03	9.70	<1	<1
pglib_opf_case39_epri_sad	39	46	1.4834e+05	0.67	<1	<1
pglib_opf_case57_ieee_sad	57	80	3.8663e+04	0.71	<1	<1
pglib_opf_case73_ieee_rts_sad	73	120	2.2760e+05	6.73	<1	<1
pglib_opf_case89_pegase_sad	89	210	1.0729e+05	0.73	<1	<1
pglib_opf_case118_ieee_sad	118	186	1.0516e+05	8.17	<1	<1
pglib_opf_case162_ieee_dtc_sad	162	284	1.0869e+05	6.48	<1	<1
pglib_opf_case179_goc_sad	179	263	7.6253e+05	1.12	<1	<1
pglib_opf_case200_activ_sad	200	245	2.7558e+04	0.01	<1	<1
pglib_opf_case240_pserc_sad	240	448	3.4054e+06	4.93	4	2
pglib_opf_case300_ieee_sad	300	411	5.6570e+05	2.61	<1	<1
pglib_opf_case500_goc_sad	500	733	4.8740e+05	6.67	<1	<1
pglib_opf_case588_sdet_sad	588	686	3.2936e+05	6.67	<1	<1
pglib_opf_case793_goc_sad	793	913	2.8580e+05	7.97	2	<1
pglib_opf_case1354_pegase_sad	1354	1991	1.2588e+06	1.57	4	3
pglib_opf_case1888_rte_sad	1888	2531	1.4139e+06	2.82	9	19
pglib_opf_case1951_rte_sad	1951	2596	2.0924e+06	0.46	17	5
pglib_opf_case2000_goc_sad	2000	3639	9.9288e+05	1.52	7	4
pglib_opf_case2312_goc_sad	2312	3013	4.6235e+05	3.93	7	4
pglib_opf_case2383wp_k_sad	2383	2896	1.9112e+06	2.86	8	5
pglib_opf_case2736sp_k_sad	2736	3504	1.3266e+06	1.58	7	5
pglib_opf_case2737sop_k_sad	2737	3506	7.9095e+05	1.88	7	4
pglib_opf_case2742_goc_sad	2742	4673	2.7571e+05	1.36	28	5
pglib_opf_case2746wop_k_sad	2746	3514	1.2337e+06	2.32	7	4
pglib_opf_case2746wp_k_sad	2746	3514	1.6669e+06	2.19	7	5
pglib_opf_case2848_rte_sad	2848	3776	1.2890e+06	0.25	16	6
pglib_opf_case2853_sdet_sad	2853	3921	2.0692e+06	1.70	9	6
pglib_opf_case2868_rte_sad	2868	3808	2.0213e+06	0.59	16	6
pglib_opf_case2869_pegase_sad	2869	4582	2.4687e+06	1.12	11	7
pglib_opf_case3012wp_k_sad	3012	3572	2.6195e+06	1.56	10	14
pglib_opf_case3022_goc_sad	3022	4135	6.0143e+05	2.77	9	5
pglib_opf_case3120sp_k_sad	3120	3693	2.1749e+06	1.52	11	6
pglib_opf_case3375wp_k_sad	3374	4161	7.4382e+06	0.55	11	7
pglib_opf_case3970_goc_sad	3970	6641	9.6555e+05	0.76	20	12
pglib_opf_case4020_goc_sad	4020	6988	8.8969e+05	8.66	28	13
pglib_opf_case4601_goc_sad	4601	7199	8.7818e+05	6.37	29	14
pglib_opf_case4619_goc_sad	4619	8150	4.8435e+05	2.00	22	15
pglib_opf_case4661_sdet_sad	4661	5997	2.2610e+06	1.96	15	12
pglib_opf_case4837_goc_sad	4837	7765	8.7712e+05	0.85	22	11
pglib_opf_case4917_goc_sad	4917	6726	1.3890e+06	2.52	17	9
pglib_opf_case6468_rte_sad	6468	9000	2.0697e+06	1.12	63	30
pglib_opf_case6470_rte_sad	6470	9005	2.2416e+06	1.91	35	22
pglib_opf_case6495_rte_sad	6495	9019	3.0678e+06	15.11	69	23
pglib_opf_case6515_rte_sad	6515	9037	2.8698e+06	7.85	57	21
pglib_opf_case9241_pegase_sad	9241	16049	6.3185e+06	2.47	52	36
pglib_opf_case9591_goc_sad	9591	15915	1.1674e+06	9.56	83	42
pglib_opf_case10000_goc_sad	10000	13193	1.4902e+06	6.77	44	26
pglib_opf_case10480_goc_sad	10480	18559	2.3147e+06	1.23	65	41
pglib_opf_case13659_pegase_sad	13659	20467	9.0422e+06	1.68	62	43
pglib_opf_case19402_goc_sad	19402	34704	1.9838e+06	1.49	162	108
pglib_opf_case24464_goc_sad	24464	37816	2.6540e+06	1.84	145	101
pglib_opf_case30000_goc_sad	30000	35393	1.2866e+06	11.84	336	120

commercial power flow tools now support the following, more general model, of line charging:

$$S_{lij} = (\mathbf{Y}_l + \mathbf{Y}_{lij}^c)^* \frac{|V_i|^2}{|\mathbf{T}_l|^2} - \mathbf{Y}_l^* \frac{V_i V_j^*}{\mathbf{T}_l} \quad \forall (l, i, j) \in E \quad (7a)$$

$$S_{lji} = (\mathbf{Y}_l + \mathbf{Y}_{lji}^c)^* |V_j|^2 - \mathbf{Y}_l^* \frac{V_i^* V_j}{\mathbf{T}_l^*} \quad \forall (l, i, j) \in E \quad (7b)$$

where the values  $\mathbf{Y}_{lij}^c$  and  $\mathbf{Y}_{lji}^c$  represent the line charging admittance on the *from* and *to* sides of the branch respectively. This model generalizes the branch model from Figure 1 by incorporating line charge conductance effects and asymmetrical charging effects.

### C. Generator Capability Curves

Model 1 represents generators with box constraints that independently limit active and reactive power outputs. A more detailed model recognizes that the current flows inside of a generator are jointly dependent on both the active and reactive power outputs. The generator must be operated to limit the heating caused by these internal current flows. Thus, the more detailed “capability curve” generator model (also known as a “D-curve” model) forms generator limits that couple the active and reactive power outputs [65]. Estimates of the additional data needed for the generator capability curve model can be extracted from the box constraints on active and reactive power injections provided in typical datasets [66], [67]. Additionally, the MATPOWER data format is capable of representing a piecewise-linear approximation of the generator capability curve model [16].

### D. Voltage & Reactive Power Control

In practice, reactive power management devices, such as bus shunts and transformer taps, play a critical role in managing the voltage profile of an AC power network and improving power quality in congested parts of a network [68]. A notable limitation of the OPF formulation presented in Model 1 is the limited amount of reactive power control devices, which may bias the feasible solutions to a specific voltage profile provided with the network data. At this time, extensions of the Model 1 that consider reactive power controls are an active area of research with promising initial results [45], [69]–[71]. However, a variety of subtle challenges arise when incorporating such devices into an OPF problem formulation [63] and continued research is required to arrive at a broadly accepted generalization of Model 1 that incorporates more reactive control devices.

### E. Other Problem Formulations

The OPF formulation in Model 1 considers a single period of steady-state power system behavior. Power system engineers solve a wide variety of other optimization and control problems relevant to the design and operation of

power systems [72]–[74]. Formulating many of these problems requires augmenting the data available in PGLIB-OPF with other information. Table IX summarizes several other classes of optimization and control problems, indicates examples of information that may be required for these problems beyond the data provided in PGLIB-OPF, and provides selected references for each class of problems. While currently beyond the scope of this effort, extension of PGLIB to incorporate the data necessary for test cases that are applicable to these and other classes of problems is an important topic for future work.

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TABLE IX  
OTHER PROBLEM FORMULATIONS

Problem	Description	Additional Information Required	Refs.
Power Flow	Determine a voltage profile corresponding to a specified set of power injections and voltage magnitudes.	<ul style="list-style-type: none"> <li>Bus type specifications (PV, PQ, Slack) for each bus.</li> <li>Active power injections for PV and PQ buses.</li> <li>Reactive power injections for PQ buses.</li> <li>Voltage magnitudes for the slack bus and PV buses.</li> </ul>	[75], [76]
Multi-Period OPF	Extend the single-period OPF formulation of Model 1 to multiple periods with time-varying load demands and energy storage models.	<ul style="list-style-type: none"> <li>Time-varying load demands and renewable generation.</li> <li>Generator ramp rates.</li> <li>Models of energy storage devices.</li> </ul>	[41], [77]
Unit Commitment	Schedule generator on/off statuses and power outputs considering costs and constraints associated with operating, starting up, and shutting down generators.	<ul style="list-style-type: none"> <li>Time-varying load demands and renewable generation.</li> <li>Generator start-up and shut-down costs, ramp rates, minimum run times, and minimum down times.</li> <li>Models of energy storage devices.</li> </ul>	[78]–[81]
Network Reconfiguration	Optimize the topology of a transmission or distribution network by changing the statuses of switches.	<ul style="list-style-type: none"> <li>Locations of switches.</li> </ul>	[24], [82], [83]
State Estimation	Determine the operating point that minimizes a weighted error metric for a given set of measurements.	<ul style="list-style-type: none"> <li>Locations and values for the measurements.</li> <li>Characterizations of measurement noise.</li> </ul>	[84]–[86]
Voltage Stability Analyses	Compute various margins to the system's load-ability limit.	<ul style="list-style-type: none"> <li>Models for variations in the load demands and generator outputs.</li> <li>Models for the actions of voltage control devices such as switched shunt capacitors and tap-changing transformers.</li> </ul>	[87], [88]
Small-Signal Dynamic Stability Analyses	Simulate or analytically characterize the behavior of a power system subject to small perturbations.	<ul style="list-style-type: none"> <li>Dynamical models for devices such as generators and loads, often represented as a system of differential-algebraic equations.</li> <li>Models for the perturbations.</li> </ul>	[89]
Transient Stability Analyses	Simulate or analytically characterize the behavior of a power system following a large disturbance.	<ul style="list-style-type: none"> <li>Dynamical models for devices such as generators and loads, often represented as a system of differential-algebraic equations.</li> <li>Models for the disturbances.</li> </ul>	[89]–[91]
Cascading Failure Analyses	Simulate and analyze the processes by which failures of certain system devices can lead to cascades of subsequent failures.	<ul style="list-style-type: none"> <li>Models relating overload amounts to the failure probabilities of each device.</li> <li>System responses to failures of various devices.</li> </ul>	[92]
Reliability Analyses and Expansion Planning	Compute estimates of reliability metrics such as loss of load probability. Identify investments in new devices such as generators and transmission lines that facilitate reliable operation.	<ul style="list-style-type: none"> <li>Multiple snapshots of load demands representing time-series or scenario data.</li> <li>Failure rates for various devices such as generators and lines.</li> </ul>	[25], [41], [93]
Coupled Transmission / Distribution System Analyses	Analyses that jointly consider models of transmission and distribution systems.	<ul style="list-style-type: none"> <li>Distribution system models, possibly with three-phase unbalanced network representations.</li> <li>Model for the couplings between the transmission and distribution systems.</li> </ul>	[94]–[96]
Coupled Infrastructure Analyses	Analyses of electric power systems coupled with other networks such as natural gas, water, transportation, and communication systems.	<ul style="list-style-type: none"> <li>Models for the other infrastructures and their couplings with the electric grid.</li> </ul>	[97]–[99]
Stochastic Formulations	Problem variants that consider uncertainty, often with respect to the power injections.	<ul style="list-style-type: none"> <li>Models of the uncertainty characteristics.</li> <li>Recourse models describing how each device responds to the uncertainty realizations.</li> </ul>	[13], [80], [100]
Distributed Formulations	Problem variants that use distributed rather than centralized formulations.	<ul style="list-style-type: none"> <li>Models of the communications among the distributed agents that work together to solve the problem.</li> </ul>	[86], [101]
Contingency-Constrained Formulations	Problem variants that ensure the security of system operations after the occurrence of certain contingencies.	<ul style="list-style-type: none"> <li>A list of contingencies.</li> <li>Recourse models describing how each device responds to the contingency.</li> </ul>	[62]–[64]

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LA-UR-18-29054