# A Compensation-Based Conic OPF for Weakly Meshed Networks

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***Abstract*—This letter presents a method for solving the optimal power flow (OPF) problem in weakly meshed networks, using complex power compensation and the branch flow conic formulation. The method extends the practical applicability of the branch flow conic OPF and has been successfully tested on distribution networks with up to 3145 nodes.**

***Index Terms*—Load flow control, nonlinear programming, optimization methods.**

## I. Compensation Technique

The branch flow conic OPF solves a convex relaxation of the OPF problem [1]; for radial networks, there is empirical evidence together with sufficiency conditions which suggest that this relaxation is exact, i.e. it gives a global optimum of the OPF problem. However, when the network is meshed, the conic relaxation does not generally satisfy the network cycle constraints; Ref. [1] suggested the installation of phase shifters to recover a feasible and globally optimal solution.

This letter proposes a new method for handling meshes in the conic OPF solution, without requiring changes in the network infrastructure. First the meshed network is broken at a number of points to make it radial and the flows at the breakpoints are accounted for by injecting equal with opposite sign complex powers at their two end nodes (see Fig. 1); the OPF in the radial network is solved using the conic approach in [1]. To recover the solution of the original meshed network from the radial one, the complex voltages at the two end nodes of each breakpoint should be identical. This is accomplished by injecting complex compensation powers at the two end nodes of each breakpoint. The idea of using compensation currents in power flow has been originally proposed in [2] to handle meshes in the forward/backward sweep power flow.

With reference to the radial network in Fig. 1, the minimum-loss branch flow conic OPF can be formulated as the program in (1)–(7) where $E$ denotes the set of branches, $BP$ denotes the set of breakpoints, $r_{ij}/x_{ij}$ denotes the resistance/reactance of branch $ij$, $g_j/b_j$ denotes the shunt conductance/susceptance at node $j$, $P_j/Q_j$ is the real/reactive injection power at node $j$, $P_{ij}/Q_{ij}$ is the real/reactive power flow in branch $ij$, $\ell_{ij}$ is the squared current magnitude in branch $ij$, and $\upsilon_i$ is the squared voltage magnitude at node $i$.

$$\min \sum_{(i,j)\in E} r_{ij}\ell_{ij} \tag{1}$$

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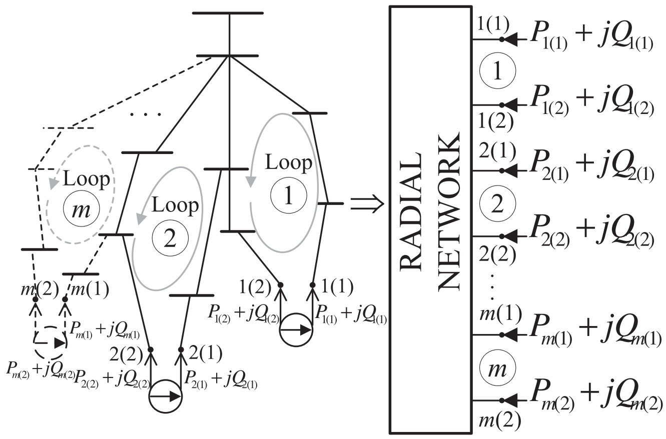


Fig. 1. Radial network with breakpoint ports.

$$P_j = \sum_{k:j\to k} P_{jk} - \sum_{i:i\to j}(P_{ij} - r_{ij}\ell_{ij}) + g_j\upsilon_j,\ \forall j \tag{2}$$

$$Q_j = \sum_{k:j\to k} Q_{jk} - \sum_{i:i\to j}(Q_{ij} - x_{ij}\ell_{ij}) + b_j\upsilon_j,\ \forall j \tag{3}$$

$$\upsilon_j = \upsilon_i - 2(r_{ij}P_{ij} + x_{ij}Q_{ij}) + \left(r_{ij}^2 + x_{ij}^2\right)\ell_{ij},\ \forall (i,j)\in E \tag{4}$$

$$\upsilon_i\ell_{ij} \geq P_{ij}^2 + Q_{ij}^2,\ \forall (i,j)\in E \tag{5}$$

$$P_{i(1)} + P_{i(2)} = 0,\ Q_{i(1)} + Q_{i(2)} = 0,\ \forall i \in BP \tag{6}$$

$$\upsilon_{i(1)} - \upsilon_{i(2)} = 0,\ \forall i \in BP \tag{7}$$

Limits are also placed on power generation, branch current magnitudes, and voltage magnitudes. Once the conic solution is obtained, the vector of nodal angles $\theta$ can be computed by solving the linear system $B\theta = \beta$, where $B$ is the reduced incidence matrix and $\beta$ is a vector whose elements are given by $\beta_{ij} = \angle(\upsilon_i - (r_{ij} - jx_{ij})(P_{ij} + jQ_{ij}))$ [1]. The complex voltages at the end nodes of the breakpoints ($p = 1, 2$) can be then computed from (8), where $\star$ denotes a solution from the conic program:

$$\begin{aligned}\underline{V}_{j(p)}^{\star} &= V_{j(p)}^{re\star} + jV_{j(p)}^{im\star}\\ &= \sqrt{\upsilon_{j(p)}^{\star}}\cos\left(\theta_{j(p)}^{\star}\right) + j\sqrt{\upsilon_{j(p)}^{\star}}\sin\left(\theta_{j(p)}^{\star}\right)\end{aligned} \tag{8}$$

If $|\underline{V}_{j(1)}^{\star} - \underline{V}_{j(2)}^{\star}| < \varepsilon = 10^{-5}$ pu for all breakpoints, then the radial conic solution is valid for the original meshed network, otherwise complex compensation powers are computed to reduce the difference between the breakpoint end node voltages. To achieve this, the real and imaginary breakpoint end node voltages are expressed in terms of the complex compensation powers $(\Delta P_i + j\Delta Q_i)$ as in (9), (10).

$$V_{j(p)}^{re} \cong V_{j(p)}^{re\star} + \sum_{i\in BP}\left(\frac{\partial V_{j(p)}^{re}}{\partial P_{i(1)}} - \frac{\partial V_{j(p)}^{re}}{\partial P_{i(2)}}\right)\Delta P_i$$

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$$+\sum_{i\in BP}\left(\frac{\partial V^{re}_{j(p)}}{\partial Q_{i(1)}}-\frac{\partial V^{re}_{j(p)}}{\partial Q_{i(2)}}\right)\Delta Q_i \tag{9}$$

$$V^{im}_{j(p)}\cong V^{im\star}_{j(p)}+\sum_{i\in BP}\left(\frac{\partial V^{im}_{j(p)}}{\partial P_{i(1)}}-\frac{\partial V^{im}_{j(p)}}{\partial P_{i(2)}}\right)\Delta P_i$$
$$+\sum_{i\in BP}\left(\frac{\partial V^{im}_{j(p)}}{\partial Q_{i(1)}}-\frac{\partial V^{im}_{j(p)}}{\partial Q_{i(2)}}\right)\Delta Q_i \tag{10}$$

Equating the real and imaginary voltages at the two end nodes of each breakpoint (11) results in a linear system of equations whose solution gives the compensation powers. The injection powers at the breakpoint ports are subsequently updated using (12). The solution of the radial conic OPF (1)–(7) and the computation of the compensation powers are iterated until the breakpoint end node voltages become equal within tolerance.

$$\begin{aligned}&V^{re}_{j(1)}-V^{re}_{j(2)}=0,\ \forall j\in BP\\&V^{im}_{j(1)}-V^{im}_{j(2)}=0,\ \forall j\in BP\end{aligned} \tag{11}$$

$$\begin{aligned}P_{i(1)}&=P^\star_{i(1)}+\Delta P_i,\quad P_{i(2)}=P^\star_{i(2)}-\Delta P_i\\Q_{i(1)}&=Q^\star_{i(1)}+\Delta Q_i,\quad Q_{i(2)}=Q^\star_{i(2)}-\Delta Q_i\end{aligned} \tag{12}$$

## II. Sensitivity Computation

To compute the voltage sensitivity to the real/reactive power injections at the breakpoint end nodes ($P_i/Q_i$—the subscript $(p)$ is dropped), the nodal equations are expressed in terms of the real $(re)$ and imaginary $(im)$ components of nodal voltages, injection currents, and bus admittance values:

$$\begin{bmatrix}Y^{re} & -Y^{im}\\ Y^{im} & Y^{re}\end{bmatrix}\begin{bmatrix}V^{re}\\ V^{im}\end{bmatrix}=\begin{bmatrix}I^{re}\\ I^{im}\end{bmatrix} \tag{13}$$

$$Y^{re}=\begin{bmatrix}1 & \cdots & 0 & \cdots & 0\\ Y^{re}_{21} & \cdots & Y^{re}_{2j} & \cdots & Y^{re}_{2n}\\ \vdots & & \vdots & & \vdots\\ Y^{re}_{i1} & \cdots & Y^{re}_{ij} & \cdots & Y^{re}_{in}\\ \vdots & & \vdots & & \vdots\\ Y^{re}_{n1} & \cdots & Y^{re}_{nj} & \cdots & Y^{re}_{nn}\end{bmatrix} \tag{14}$$

$$Y^{im}=\begin{bmatrix}0 & \cdots & 0 & \cdots & 0\\ Y^{im}_{21} & \cdots & Y^{im}_{2j} & \cdots & Y^{im}_{2n}\\ \vdots & & \vdots & & \vdots\\ Y^{im}_{i1} & \cdots & Y^{im}_{ij} & \cdots & Y^{im}_{in}\\ \vdots & & \vdots & & \vdots\\ Y^{im}_{n1} & \cdots & Y^{im}_{nj} & \cdots & Y^{im}_{nn}\end{bmatrix} \tag{15}$$

$$\begin{aligned}V^{re}&=[\,V^{re}_1\ \cdots\ V^{re}_j\ \cdots\ V^{re}_n\,]^T\\V^{im}&=[\,V^{im}_1\ \cdots\ V^{im}_j\ \cdots\ V^{im}_n\,]^T\\I^{re}&=[\,V_{slack}\ \ I^{re}_2\ \cdots\ I^{re}_i\ \cdots\ I^{re}_n\,]^T\\I^{im}&=[\,0\ \ I^{im}_2\ \cdots\ I^{im}_i\ \cdots\ I^{im}_n\,]^T\end{aligned} \tag{16}$$

$$I^{re}_i=\frac{P_iV^{re}_i+Q_iV^{im}_i}{V^2_i},\ I^{im}_i=\frac{P_iV^{im}_i-Q_iV^{re}_i}{V^2_i} \tag{17}$$

Taking the derivative of each of the nodal equations in (13) with respect to variable $x$ ($x=P_i$ or $Q_i$) gives the system of sensitivity equations in (18), where the $\underline{Y}^{(\bullet)}_D$ sub-matrices are diagonal with their elements (19) computed from the expressions for the injection currents (17) and the chain-rule for derivatives. The right-hand-side vector in (18) is all zeros except for two elements in (20) representing the sensitivity of the real/imaginary currents to $x$. The solution to (18) gives the sensitivity information used in forming (9), (10).

$$\begin{bmatrix}Y^{re}-Y^{(1)}_D & -Y^{im}-Y^{(2)}_D\\ Y^{im}-Y^{(3)}_D & Y^{re}-Y^{(4)}_D\end{bmatrix}\begin{bmatrix}\frac{\partial V^{re}}{\partial x}\\ \frac{\partial V^{im}}{\partial x}\end{bmatrix}=\begin{bmatrix}\frac{\partial I^{re}}{\partial x}\\ \frac{\partial I^{im}}{\partial x}\end{bmatrix} \tag{18}$$

$$\begin{aligned}Y^{(1)}_{Dii}&=\frac{P_i}{V^2_i}-\frac{2V^{re}_i\left(P_iV^{re}_i+Q_iV^{im}_i\right)}{V^4_i}\\Y^{(2)}_{Dii}&=\frac{Q_i}{V^2_i}-\frac{2V^{im}_i\left(P_iV^{re}_i+Q_iV^{im}_i\right)}{V^4_i}\\Y^{(3)}_{Dii}&=\frac{Q_i}{V^2_i}-\frac{2V^{im}_i\left(P_iV^{re}_i+Q_iV^{im}_i\right)}{V^4_i}\\Y^{(4)}_{Dii}&=\frac{P_i}{V^2_i}-\frac{2V^{im}_i\left(P_iV^{im}_i-Q_iV^{re}_i\right)}{V^4_i}\end{aligned} \tag{19}$$

$$\begin{aligned}x=P_i&\Rightarrow\frac{\partial I^{re}_i}{\partial P_i}=\frac{V^{re}_i}{V^2_i},\quad\frac{\partial I^{im}_i}{\partial P_i}=\frac{V^{im}_i}{V^2_i}\\x=Q_i&\Rightarrow\frac{\partial I^{re}_i}{\partial Q_i}=\frac{V^{im}_i}{V^2_i},\quad\frac{\partial I^{im}_i}{\partial Q_i}=\frac{-V^{re}_i}{V^2_i}\end{aligned} \tag{20}$$

## III. Numerical Results

A computer program was implemented in Matlab and tested on a modified version of a realistic Brazilian distribution system in addition to two test networks with 1463 and 3145 nodes at three loading levels (low, medium, high); the complete data sets of the test networks can be downloaded from [3]. The Brazilian distribution system has one loop, the 1463 node system has 11 loops (including nine parallel branches), and the 3145 node system has 12 loops (including 10 parallel branches). For all the networks and at the three loading levels, the program required 3 solutions of the branch flow conic model to make the breakpoint end node voltages equal within an error less than $10^{-5}$ pu. The number of loops in the 1463 and the 3145 node systems was then successively increased by closing a total of 7 open line switches in the original data sets, thus creating 7 new data sets for each network with 1, 2, 3, 4, 5, 6, and 7 additional loops [3]; the compensation solver required 3 iterations to solve each case. The total loss value coincided with that from MATPOWER [4].

## References


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