Voltage Regulation in Distribution Network Using Battery Storage Units via Distributed Optimization

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***Abstract*— This paper proposes a voltage regulation method by using battery energy storage (BES) units in distribution networks with abundant solar photovoltaic (PV) resources. The proposed method utilizes sparse optimization techniques to find the optimal actions of BES with *minimal involved unit numbers* and *minimal active power output variation*. Furthermore, a distributed Lagrangian primal-dual sub-gradient (DLPDS) algorithm is applied to solve the proposed method via local decision making with limited communication with neighbors. Finally, case studies are conducted on modified 15-bus and 43-bus radial distribution system to verify the performance of proposed method.**

***Keywords*— BES, distributed optimization, PV, sensitivity matrix, sparsity, voltage regulation**

I. INTRODUCTION

Solar photovoltaic (PV) is under rapid development in the last decades. There has experienced a large growth in the PV market in Australia with total installed capacity reached 5.4 gigawatts until June 2016 [1]. PV serves part of local load that relieves the stress on distribution feeders and improves system performance. However, due to the stochastic characteristic, the increasing penetration level of PV may cause temporary voltage rises problem during high generation period (e.g. 12:00 pm) which threaten system stable operation and cause damage to system equipment [2].

The voltage sudden rise issue has been considered as one of the urgent problems especially in medium-voltage and low-voltage distribution networks (DN) [2, 3]. Extensive studies have been conducted in [4-7]. Traditional method adopts conventional devices, such as capacitor banks, on load tap changers to maintain voltage within given range. However, the frequent operation would definitely influence the lifespan of these devices [4]. Battery energy storage technology is considered as one substitute solution with its distinct advantages such as fast response, easy deployment and flexible operation. Voltage rise problem can be effectively mitigated by absorbing the excess PV generation in the BES units [5]. Different control schemes have been applied for optimize BES units output power in localized [6] or centralized [7] way. However, each scheme had its own limitations. Centralized control method may encounter difficulty in information exchange while the number of BES units increases; localized control method may not guarantee global optimal solution since each BES units works individually. In this paper we propose a distributed optimization method for multi-BES units based on centralized control scheme. The DLPDS algorithm is applied to find the global optimal solution.

The other limitation of above control approaches is that the optimal solution usually requires power output variation in large amount of dispersed BES units. In practical, voltage regulation is supposed to be achieved through limited number of involved BES units. Moreover, from the perspective of the DN operator, large variations of BES units power output would not only disturb the system operation but also increase the communication and computation burdens. To overcome these difficulties, the sparse regularization theory has been applied for finding the optimal solution which brings system bus voltage back in acceptable range with minimum numbers of involved BES units as well as minimum variation in all BES units power output. Sparse regularization is a powerful optimization technique which has being widely studied in signal and image processing areas [8, 9].

The major contributions of this paper include: 1) a voltage regulation framework is proposed for DN operator using multiple dispersed BES units via distributed optimization; 2) sparse regulation technique is applied to bring voltage back to acceptable range with *minimal actions* on limited BES units.

This paper is organized as following. Section II gives the brief description of proposed framework, the linear relationship between generation output power and system voltage output profile is explained. Section III introduced the efficient distributed algorithm to solve the objective function. Section IV gives a simple case study to verify the effectiveness and accuracy of proposed method. Section V concludes the paper.

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II. MATHEMATICAL MODELLING

To mitigate the negative effects of voltage rise issues, BES units are adopted for ensuring voltage magnitude within the acceptable ranges. Generally, the normal range of voltage profile is set as ±6% of the nominal value. Once some of the voltages fall out of the normal value, the DN operator wants to bring voltage back to normal range. Since the power outputs of BES units are constrained by their power rating and capacity, sometimes it is difficult to bring all the voltage profiles back into normal range in one step. Thus, a relaxation factor ξ has been imposed to all the voltage constraints. The relaxed voltage profile are referred as acceptable range [10].

Furthermore, the ℓ_1 norm is utilized in the objective function to ensure that the solution is with the minimal numbers of involved BES units and minimal amount of BES units output power variation.

A. Problem Formulation

The mathematical formulation is as following:

$$\min_{\Delta \mathrm{p}} \|\Delta \mathrm{p}\|_1 \tag{1a}$$

$$\text{s.t. } \mathrm{p}_c - \mathrm{p}_0 = \Delta \mathrm{p} \tag{1b}$$

$$\mathrm{v}_c - \mathrm{v}_0 = \Delta \mathrm{v} \tag{1c}$$

$$\mathrm{v}_{\min} - \xi \times \vec{1} \le \mathrm{v}_c \le \mathrm{v}_{\max} + \xi \times \vec{1} \tag{1d}$$

$$-\mathrm{p}_{bat}^{\max} \le \mathrm{p}_c \le \mathrm{p}_{bat}^{\max} \tag{1e}$$

$$\xi \ge 0 \tag{1f}$$

where decision variable $\Delta \mathrm{p}$ denotes active power output variation of BES units. Constraint (1b) denotes the changes in power output at pre-state and current-state, respectively. Here we have $\mathrm{p}_0 = \left(\mathrm{p}_{1,0}, \mathrm{p}_{2,0}, \ldots \mathrm{p}_{I,0}\right)^T$ and $\mathrm{p}_c = \left(\mathrm{p}_{1,c}, \mathrm{p}_{2,c}, \ldots \mathrm{p}_{I,c}\right)^T$. I denotes the set of BES units, with index $i \in I$. Constraint (1c) denotes the changes in voltage profile at pre-state and current-state, where $\mathrm{v}_0 = \left(\mathrm{v}_{1,0}, \mathrm{v}_{2,0}, \ldots \mathrm{v}_{N,0}\right)^T$ and $\mathrm{v}_c = \left(\mathrm{v}_{1,c}, \mathrm{v}_{2,c}, \ldots \mathrm{v}_{N,c}\right)^T$. N denotes the set of system buses, with index $n \in N$. Constraint (1d) denotes the relaxed bound constraint of bus voltage profile v_c at current-state. Relaxation factor ξ is applied to all the system buses, where $\vec{1} = (1,1,\ldots 1)^T$. Both bus voltage profile v_0 and BES unit output power p_0 at pre-state are known. Constraint (1e) denotes BES units power capacity limitation, where $\mathrm{p}_{bat}^{\max} = \left(\mathrm{p}_{bat,1}^{\max}, \mathrm{p}_{bat,2}^{\max}, \ldots \mathrm{p}_{bat,I}^{\max}\right)^T$. Constraint (1f) indicates that ξ is non-negative.

B. Sensitivity Matrix

It is acknowledged that the changes in active/reactive power injections may strongly induce bus voltage profiles variation especially in DN. The linear relationships between generation output and voltage values can be obtained through the Jacobian sensitivity matrix which derived from Newton-Raphson power flow calculation [11]. The general conversion form between generation output (active and reactive power) and voltage profile is given as follows:

$$\begin{pmatrix} \Delta\theta \\ \Delta V \end{pmatrix} = \begin{pmatrix} \Lambda_{\theta P}\Lambda_{\theta Q} \\ \Lambda_{VP}\Lambda_{VQ} \end{pmatrix} \begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix} \tag{2}$$

with

$$\Lambda = \begin{pmatrix} \Lambda_{\theta P}\Lambda_{\theta Q} \\ \Lambda_{VP}\Lambda_{VQ} \end{pmatrix} \tag{3}$$

It is obvious that the approximate value of bus voltage profile after BES units actions can be calculated based on voltage profile v_0, Jacobian sensitivity matrix and BES units active power output change $\Delta \mathrm{p}$ (For simplicity, we assume BES units only release active power). Therefore, the voltage profile v_c at current-state can be expressed as:

$$\mathrm{v}_c = \mathrm{v}_0 + \Lambda_{VP} \times \Delta p \tag{4}$$

C. Reformulation of the Proposed Optimization Problem

The key innovation of the proposed model in (1) is the employment of ℓ_1 sparsity optimization in the objective function. (1a) indicates the difference between BES units power output at pre-state and current-state. The ideal solution of (1a) should comprise the least number of non-zero entries in the vector $\Delta \mathrm{p}$ which is known as sparse vector. Combined (1b) with (1e), the upper-lower limit of $\Delta \mathrm{p}$ can be expressed as (5b). Therefore, the model (1) can be transformed as the following equivalent form:

$$\min_{\Delta \mathrm{p}} \|\Delta \mathrm{p}\|_1 \tag{5a}$$

$$\text{s.t. } -\mathrm{p}_{bat}^{\max} - \mathrm{p}_0 \le \Delta \mathrm{p} \le \mathrm{p}_{bat}^{\max} - \mathrm{p}_0 \tag{5b}$$

$$\mathrm{v}_{\min} - \xi \times \vec{1} \le \mathrm{v}_0 + \Lambda_{VP} \times \Delta \mathrm{p} \le \mathrm{v}_{\max} + \xi \times \vec{1} \tag{5c}$$

Furthermore, the optimization problem (5) can be expressed as the general form:

$$\begin{cases} \min\limits_{\Delta \mathrm{p}} \sum\limits_{i=1}^{I} f_i(\Delta \mathrm{p}), \text{ s.t. } g(\Delta \mathrm{p}) \le 0, \\ \Delta \mathrm{p}_i \in \mathbb{P}_i \end{cases} \tag{6}$$

where $\sum_{i=1}^{I} f_i(\Delta \mathrm{p})$ is the ℓ_1 norm value of objective function in (5a), $f_i(\Delta \mathrm{p}) = |\Delta p_i|$, $\mathbb{P}_i = \left[-\mathrm{p}_{bat,i}^{\max} - \mathrm{p}_0, \mathrm{p}_{bat,i}^{\max} - \mathrm{p}_0\right]$ represents ith BES units power capacity limitation; $g(\Delta \mathrm{p}) = \left(g_1(\Delta \mathrm{p}), g_2(\Delta \mathrm{p})\right)^T$ represents the bus voltage constraints of (5c), where:

$$\begin{cases} g_1(\Delta \mathrm{p}) = \Lambda_{VP} \times \Delta \mathrm{p} - \mathrm{v}_{\max} + \mathrm{v}_0 - \xi \times \vec{1} \\ g_2(\Delta \mathrm{p}) = -\Lambda_{VP} \times \Delta \mathrm{p} - \mathrm{v}_0 + \mathrm{v}_{\min} - \xi \times \vec{1} \end{cases} \tag{7}$$

After the equivalent transformation, the problem (6) becomes a non-smooth convex problem with coupling linear inequality constraints, which is ready for applying the distributed optimization algorithm.

III. DISTRIBUTED METHOD BASED COMMUNICATION NETWORK

Generally speaking, there exits two types of control schemes to manage the output power of BES units: 1) centralized control; 2) localized control.

1) In centralized control: the charging/discharging actions of each BES units are determined in controller center. This control scheme mainly depends on reliable communication channels between the controller center and all distributed BES units, which requires high standard of communication system. However, the centralized architecture is very vulnerable to cyber-attacks or external unexpected events. Moreover, the computational processing time may increase when large number of BES units involved.

2) In localized control scheme, each BES units using its local measurements to control its charging/discharging action. This type of control scheme does not require information exchange. However, BES units cannot support each other due to the lack of communication.

To overcome these difficulties, the distributed optimization algorithm based centralized control scheme is established for voltage regulation in this section. DLPDS is a distributed optimization algorithm which was first proposed in [12]. Recently, it has been applied to power system optimal economic dispatch analysis [13] and power flow calculation in [14]. The optimal distributed decisions can be obtained without collecting the information of all dispersed BES units for a central controller. Information exchange is only needed with neighboring BES units. The significant advantage is that the convergence of the method has provable guarantee in theory comparing with most of existing algorithms. Detail description of the proposed methodology is explained as follows.

A. Communication Networks

At first, we randomly generated the topology to depict the communication networks of BES units as weighted adjacency matrix Γ. The matrix Γ is a time varying matrix, which consists of non-negative elements and can be expressed as: $\Gamma = \{a_{ij}(k)\} \in R^{I \times I}$ at time $k \geq 0$. The matrix Γ generally obeys the following characteristics [12]:

1) *Strong connectivity*

If there exits direct connection path between BES unit i and j $(i \neq j)$, $0 < a_{ij}(k) \leq 1$, otherwise $a_{ij}(k) = 0$. And we always have $a_{ij}(k) = a_{ji}(k)$ and $a_{ij}(k) \neq 0 (i = j)$.

2) *Balanced communication*

All the elements in weight matrix satisfy:

$$\sum\nolimits_{j=1}^{I} a_{ij}(k) = \sum\nolimits_{i=1}^{I} a_{ij}(k) = 1$$

The strong connectivity is to ensure that every BES unit sufficiently communicates with each other, so that each individual unit influence the states of all units in a long run. The balance communication implies that each unit receives and calculates a weighted average of the neighboring unit states.

B. Distributed Scheme

The Lagrangian relaxation method is being applied to the inequality constraints in (6) to obtain the following primal-dual Lagrangian function:

$$L(\Delta \mathrm{p}; \mu) = \sum_{i=1}^{I} L_i(\Delta \mathrm{p}; \mu) \tag{8}$$

where $L_i(\Delta \mathrm{p};\mu) = f_i(\Delta \mathrm{p}) + \mu^T g(\Delta \mathrm{p})$, μ is the Lagrange multiplier, $\Delta p \in \Re^I$ and $\mu \in \Re_{\geq 0}^{2N}$.

The DLPDS based control scheme is described as follows. Firstly, we define the initial value of each BES unit output and Lagrangian multiplier as $\Delta p_i(0)$, $\mu_i(0)$, respectively. Let $\Delta p_i(k)$, $\mu_i(k)$ (k=0,1,…) denote the values of Δp_i and μ_i, at iteration k. Each BESS unit calculates $v_{\Delta p}^i$, v_μ^i based on its own decision $v_{\Delta p}^i$, μ^i and neighboring BESS units decisions $v_{\Delta p}^j$, μ^j at each iteration k according to the following formulas:

$$\begin{aligned} v_{\Delta p}^i(k) &= \sum\nolimits_{j=1}^{N} a_j^i(k) \Delta p^j(k) \\ v_\mu^i(k) &= \sum\nolimits_{j=1}^{N} a_j^i(k) \mu^j(k) \end{aligned} \tag{9}$$

Then, each BES unit makes a local optimization based on its own objective L_i. The updates procedures are as follows:

$$\begin{aligned} \Delta p_i(k+1) &= \Pi_{\mathbb{P}_i}\left[v_{\Delta p}^i(k) - \alpha(k) D_{\Delta p}^i(k)\right] \\ \mu_i(k+1) &= \Pi_{\mathbb{M}_i}\left[v_\mu^i(k) + \alpha(k) D_\mu^i(k)\right] \end{aligned} \tag{10}$$

where $D_{\Delta p}^i = \partial L_i(\Delta p, v_\mu^i) / \partial \Delta p$ and $D_\mu^i = \partial(v_{\Delta p}^i, \mu) / \partial \mu$, $\Pi_{\mathbb{P}_i}$ and $\Pi_{\mathbb{M}_i}$ represents the projection operations onto the constraint of Δp_i, μ_i, respectively, here $\mathbb{M}_i = \{\mu_i \,|\, \mu_i \geq 0\}$. $\alpha(k)$ is a step-size factor satisfied the condition in [12].

It should be noticed that in the iteration process, without centralized control scheme, each BES unit can make its own decisions via limited information exchange between its neighbors to optimize the objective function. The convergence of DLPDS algorithm has been proved in [12]. Since problem (6) is a convex optimization problem with coupling linear inequality constraints, it is validated that under certain conditions, our proposed method also converges to a pair of primal and dual optimal solution of problem (6). Similar to the result of Theorem 3.2 in [12], if we take the step-size sequence which satisfies:

$$\lim_{k\to+\infty}\alpha(k)=0,\sum_{k=0}^{+\infty}\alpha(k)=+\infty,\sum_{k=0}^{+\infty}\alpha(k)^2<+\infty \quad (11)$$

Then we have the following convergence result:

$$\lim_{k\to+\infty}\left|\Delta p_i(k)-\Delta p^*\right|=0,\lim_{k\to+\infty}\mu_i(k)-\mu^*=0,\ \forall i \quad (12)$$

where Δp^* and μ^* are the optimal primal and dual solutions of the problem (6), respectively. The solutions can be obtained by the centralized control methods in our case study later.

IV. CASE STUDY

The developed voltage regulation method was conducted on a modified IEEE 15-bus and a 43-bus distribution network systems, respectively. The experiments were executed by Mat-lab (version R2014a).

A. *IEEE 15 bus distribution test system*

The network system topology of 15 bus system with PVs and BES units are given in Fig.1. The parameters of the system, as well as the rated active and reactive power levels of the buses and the branch parameters are available in [15]. PV penetration level is assumed to be 30% and we assume all PV cells have equal capacity. Three PV systems are located at bus 6, 9, and 13, respectively, and can operate up to 0.9 power factor. Five BES units are incorporated at bus 4, 8, 10, 11 and 14, respectively. The BES units' technical data is given in Table. I [16].

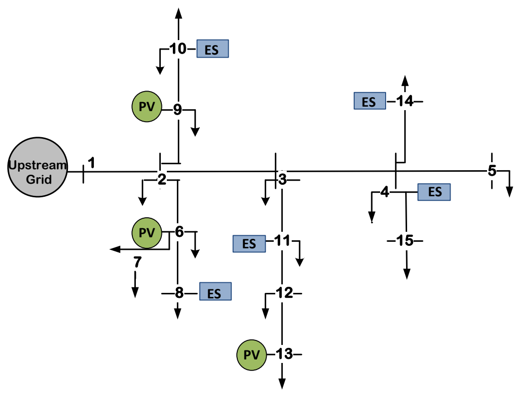

Fig. 1. Modified 15-bus distribution system structure

TABLE I BESU TECHNICAL DATA

BESU	Power Capacity	Energy Capacity	Technology	SOC limit
1	0.2MW	0.8MWh	Lead-acid	0.2~0.8
2	0.2MW	0.8MWh		
3	0.25MW	1MWh		
4	0.5MW	2MWh		
5	0.3MW	1.2MWh		

In order to analyze the proposed method itself and avoid influence from inadequate capacity of the involved BES units, it is assumed that every BES unit has enough capacity of for absorbing all the excess renewable energy. Normal range of all voltage profiles (excluding slack bus) are set as [0.94 1.06] p.u.

Table II shows the solutions for the proposed distributed voltage regulation method with different value ξ. We can easily find that when the value of ξ changes, the output power of BES units changes, correspondingly. The BES units have to absorb more power when the constraints become tighter. A simple case study was illustrated in Fig.2 - Fig.4 when parameter ξ takes the value of 0.016. Fig. 2 presents the voltage profile over all the system buses after BES units power output change. Fig.3 presents the BES units output power over computational times. As illustrated, the results find convergence through 2175 iterations and the computation times is 12.124s. Fig.4 depicts the solution error versus the iteration number, where solution error is defined as $\varepsilon(k)=\left|\Delta p_i(k)-x^*\right|/x^*$. x^* is obtained by using the centralized optimal control scheme. In addition, it's obvious to find that the voltage in bus 11 is more sensitive to the change of BES unit output power. That is because BES unit installed at bus 11 has larger power capacity than others.

TABLE II RESULTS OF BES OUTPUT POWER FOR 15 BUS SYSTEM

		BESU pre-state power (MW)	BESU current-state power (MW)	Iteration number	CPU times
ξ =0.005	1	0.1	0.1	3017	13.71s
	2	0	0.14		
	3	0	0		
	4	0.1	0.485		
	5	0	0.26		
ξ =0.01	1	0.1	0.1	2175	12.12s
	2	0	0		
	3	0	0		
	4	0.1	0.25		
	5	0	0.13		

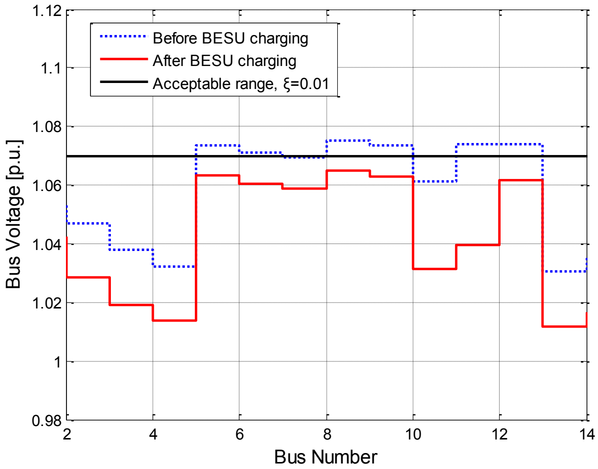

Fig. 2. Voltage profile at all the system bus

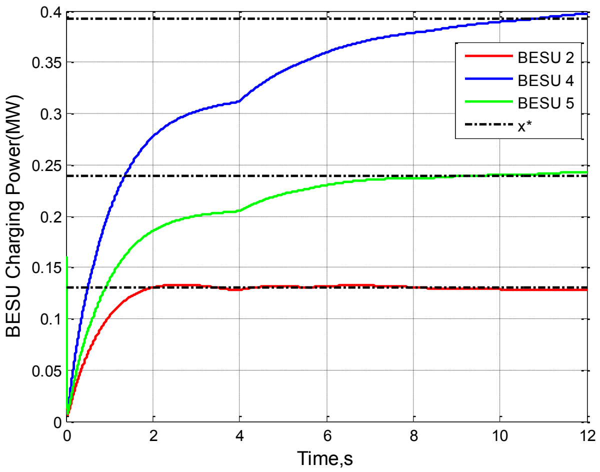

Fig. 3. Distributed results versus iteration number for 15 bus system

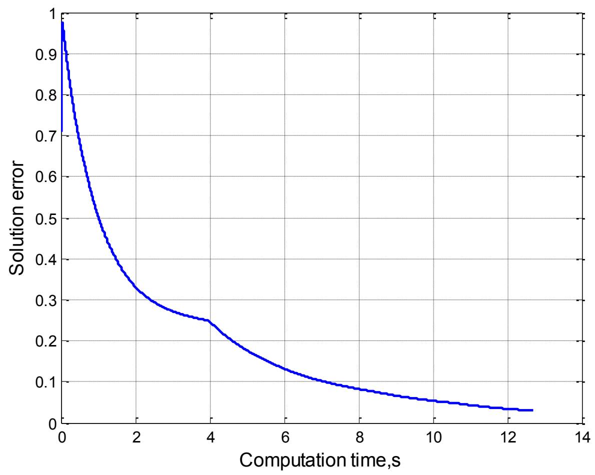

Fig. 4. Solution error versus iteration number for 15 bus system

B. IEEE 43 bus distribution test system

In the second case, a 43-bus radical distribution system is adopted to verify the proposed method. Seven PV panels are installed at bus 5, 13, 18, 28, 31, 35 and 42, respectively. Nine BES units are incorporated at bus 4, 9, 12, 19, 26, 24, 34, 41 and 42, respectively. The solutions are shown in Table. III.

Similar to case A, Fig.5 and Fig.6 present the BES units output power and the solution error versus the iteration number, as either. From the results we can find that each BES units communicated with their neighbors and reached its optimal decisions finally. The decision variables and Lagrangian multipliers for all BES units converged after dynamic iteration gradually.

TABLE III RESULTS OF BES OUTPUT POWER FOR 43 BUS SYSTEM

	BESU	pre-state power (MW)	BESU current-state power (MW)	Iteration number	CPU times
	1	0	1.11		
	2	0.2	0.89		
	3	0.5	3.86		
ξ =0	4	0.6	1.75		
	5	0	0	50271	95.26s
	6	0	0		
	7	0.2	0.2		
	8	0.1	0.1		
	9	0.3	3.10		

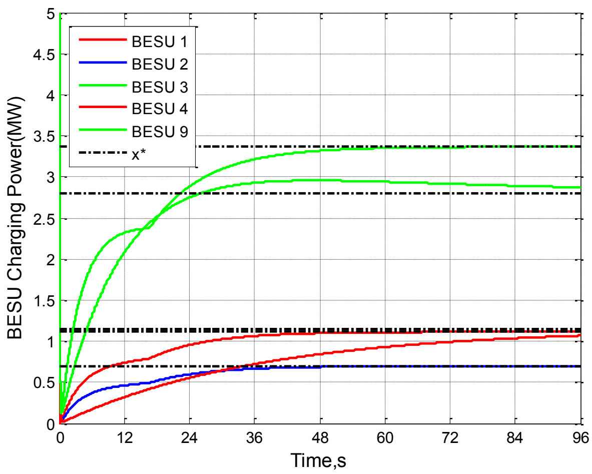

Fig. 5. Distributed results versus iteration number for 43 bus system

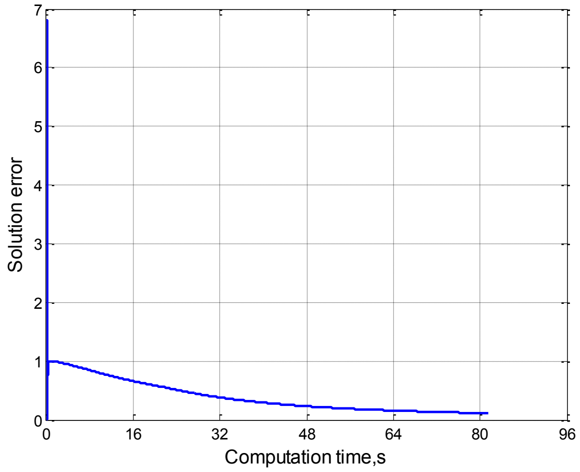

Fig. 6. Solution error versus iteration number for 43 bus system

V. CONCLUSION

In this paper, we proposed a distributed optimization framework for voltage regulation in distribution networks. The ℓ^1 regularization has being utilized in the objective function. This method adjust the distributed BES units power for bringing the voltage profile back to an acceptable range within minimal numbers of BES units as well as minimal amount of BES units output power variation. Modified IEEE 15 bus and 43-bus radial system were applied to verify the effectiveness of the proposed method, respectively. Future work includes the decision making process in the multi-time period optimization for the DN operator to smoothly bring system voltage back to normal range.

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