EE 491 HW 05

1)

$$\overrightarrow{Y_{BUS}} = \begin{bmatrix} 4.2\angle-90 & 2\angle90 & 2.5\angle90 \\ 2\angle90 & 5.03\angle-90 & 3.33\angle90 \\ 2.5\angle90 & 3.33\angle90 & 5.43\angle-90 \end{bmatrix}$$

DC power flow solution:

$$\begin{bmatrix} \overrightarrow{V_1} \\ \overrightarrow{V_2} \\ \overrightarrow{V_3} \end{bmatrix} = \begin{bmatrix} 1\angle0 \\ 1.04\angle0.11113 \\ 1\angle-0.042345 \end{bmatrix}$$

Newton Raphson:

1st iteration:

$$x^{(0)} = \begin{bmatrix} \delta_2^{(0)} \\ \delta_3^{(0)} \\ V_3^{(0)} \end{bmatrix} = \begin{bmatrix} 0.11113 \\ -0.042345 \\ 1 \end{bmatrix}, \quad b = \begin{bmatrix} P_{G_2} - P_{L_2} \\ -P_{L_3} \\ -Q_{L_3} \end{bmatrix} = \begin{bmatrix} 0.7 \\ -0.6 \\ -0.2 \end{bmatrix}, \quad h(x) = \begin{bmatrix} P_2(x) \\ P_3(x) \\ Q_3(x) \end{bmatrix}$$

$$P_2(x) = 2V_2\cos(\delta_2 - 90) + 3.33V_2V_3\cos(\delta_2 - \delta_3 - 90)$$

$$P_3(x) = 2.5V_3\cos(\delta_3 - 90) + 3.33V_3V_2\cos(\delta_3 - \delta_2 - 90)$$

$$Q_3(x) = 2.5V_3\sin(\delta_3 - 90) + 3.33V_3V_2\sin(\delta_3 - \delta_2 - 90) + 5.43V_3^2$$

$$h\left(x^{(0)}\right) = \begin{bmatrix} 0.7601 \\ -0.6353 \\ -0.4903 \end{bmatrix}$$

Elements of Jacobian matrix:

$$\frac{\partial P_2}{\partial \delta_2} = -2V_2\sin(\delta_2 - 90) - 3.33V_2V_3\sin(\delta_2 - \delta_3 - 90)$$

$$\frac{\partial P_2}{\partial \delta_3} = 3.33V_2V_3\sin(\delta_2 - \delta_3 - 90)$$

$$\frac{\partial P_2}{\partial V_3} = 3.33V_2\cos(\delta_2 - \delta_3 - 90)$$

$$\frac{\partial P_3}{\partial \delta_2} = 3.33V_3V_2\sin(\delta_3 - \delta_2 - 90)$$

$$\frac{\partial P_3}{\partial \delta_3} = -2.5V_3 \sin(\delta_3 - 90) - 3.33V_3V_2 \sin(\delta_3 - \delta_2 - 90)$$

$$\frac{\partial P_3}{\partial V_3} = 2.5 \cos(\delta_3 - 90) + 3.33V_2 \cos(\delta_3 - \delta_2 - 90)$$

$$\frac{\partial Q_3}{\partial \delta_2} = -3.33V_3V_2 \cos(\delta_3 - \delta_2 - 90)$$

$$\frac{\partial Q_3}{\partial \delta_3} = 2.5V_3 \cos(\delta_3 - 90) + 3.33V_3V_2 \cos(\delta_3 - \delta_2 - 90)$$

$$\frac{\partial Q_3}{\partial V_3} = 2.5 \sin(\delta_3 - 90) + 3.33V_2 \sin(\delta_3 - \delta_2 - 90) + 10.86V_3$$

$$J^{(0)} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial V_3} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial V_3} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \frac{\partial Q_3}{\partial V_3} \end{bmatrix} = \begin{bmatrix} 5.4897 & -3.4225 & 0.5294 \\ -3.4225 & 5.9203 & -0.6353 \\ 0.5294 & -0.6353 & 4.9397 \end{bmatrix}$$

$$\underline{\Delta x^{(0)} = \left[J^{(0)}\right]^{-1} \left(b - h\left(x^{(0)}\right)\right)} = \begin{bmatrix} -0.0141 \\ 0.0043 \\ 0.0608 \end{bmatrix}$$

$$\underline{x^{(1)} = x^{(0)} + \Delta x^{(0)}} = \begin{bmatrix} 0.0970 \\ -0.0380 \\ 1.0608 \end{bmatrix}$$

2nd iteration:

$$h\left(x^{(1)}\right) = \begin{bmatrix} 0.6960 \\ -0.5954 \\ -0.1799 \end{bmatrix}$$

$$J^{(1)} = \begin{bmatrix} 5.7106 & -3.6404 & 0.4662 \\ -3.6404 & 6.2906 & -0.5613 \\ 0.4946 & -0.5954 & 5.5907 \end{bmatrix}$$

$$\underline{\Delta x^{(1)} = \left[J^{(1)}\right]^{-1} \left(b - h\left(x^{(1)}\right)\right)} = \begin{bmatrix} 0.0005 \\ -0.0007 \\ -0.0037 \end{bmatrix}$$

$$\underline{x^{(2)} = x^{(1)} + \Delta x^{(1)}} = \begin{bmatrix} 0.0975 \\ -0.0388 \\ 1.0571 \end{bmatrix}$$

Checking the convergence criterion:

$$b - h\left(x^{(2)}\right) = \begin{bmatrix} 0.000017 \\ -0.000024 \\ -0.000077 \end{bmatrix}$$

$$max\left(\left|b - h\left(x^{(2)}\right)\right|\right) = 0.000077 < 0.001$$

The convergence criterion has been met.

Newton Raphson solution after 2 iterations:

$$\begin{bmatrix} \overrightarrow{V_1} \\ \overrightarrow{V_2} \\ \overrightarrow{V_3} \end{bmatrix} = \begin{bmatrix} 1\angle 0 \\ 1.04\angle 0.0975 \\ 1.0571\angle - 0.0388 \end{bmatrix}$$

2)

$$\overrightarrow{Y_{BUS}} = \begin{bmatrix} 5.6976\angle - 83.57 & 2.4807\angle 97.12 & 3.3168\angle 95.71 \\ 2.4807\angle 97.12 & 7.0578\angle - 83.46 & 4.9752\angle 95.71 \\ 3.3168\angle 95.71 & 4.9752\angle 95.71 & 8.2920\angle - 84.28 \end{bmatrix}$$

DC power flow solution:

$$\begin{bmatrix} \overrightarrow{V_1} \\ \overrightarrow{V_2} \\ \overrightarrow{V_3} \end{bmatrix} = \begin{bmatrix} 1\angle 0 \\ 1\angle - 0.20783 \\ 1\angle - 0.17318 \end{bmatrix}$$

Newton Raphson:

1st iteration:

$$x^{(0)} = \begin{bmatrix} \delta_2^{(0)} \\ \delta_3^{(0)} \\ V_2^{(0)} \\ V_3^{(0)} \end{bmatrix} = \begin{bmatrix} -0.20783 \\ -0.17318 \\ 1 \\ 1 \end{bmatrix}, \quad b = \begin{bmatrix} -P_{L_2} \\ -P_{L_3} \\ -Q_{L_2} \\ -Q_{L_3} \end{bmatrix} = \begin{bmatrix} -0.6 \\ -0.4 \\ -0.1 \\ -0.1 \end{bmatrix}, \quad h(x) = \begin{bmatrix} P_2(x) \\ P_3(x) \\ Q_2(x) \\ Q_3(x) \end{bmatrix}$$

$$P_2(x) = 2.4807V_2\cos(\delta_2 - 97.12) + 7.0578V_2^2\cos(83.46) + 4.9752V_2V_3\cos(\delta_2 - \delta_3 - 95.71)$$

$$P_3(x) = 3.3168V_3\cos(\delta_3 - 95.71) + 4.9752V_3V_2\cos(\delta_3 - \delta_2 - 95.71) + 8.2920V_3^2\cos(84.28)$$

$$Q_2(x) = 2.4807V_2\sin(\delta_2 - 97.12) + 7.0578V_2^2\sin(83.46) + 4.9752V_2V_3\sin(\delta_2 - \delta_3 - 95.71)$$

$$Q_3(x) = 3.3168V_3\sin(\delta_3 - 95.71) + 4.9752V_3V_2\sin(\delta_3 - \delta_2 - 95.71) + 8.2920V_3^2\sin(84.28)$$

$$h\left(x^{(0)}\right) = \begin{bmatrix} -0.6711 \\ -0.3905 \\ -0.2637 \\ 0.0919 \end{bmatrix}$$

Elements of Jacobian matrix:

$$\frac{\partial P_2}{\partial \delta_2} = -2.4807V_2 \sin(\delta_2 - 97.12) - 4.9752V_2V_3 \sin(\delta_2 - \delta_3 - 95.71)$$

$$\frac{\partial P_2}{\partial \delta_3} = 4.9752V_2V_3 \sin(\delta_2 - \delta_3 - 95.71)$$

$$\frac{\partial P_2}{\partial V_2} = 2.4807 \cos(\delta_2 - 97.12) + 14.1156V_2 \cos(83.46) + 4.9752V_3 \cos(\delta_2 - \delta_3 - 95.71)$$

$$\frac{\partial P_2}{\partial V_3} = 4.9752V_2 \cos(\delta_2 - \delta_3 - 95.71)$$

$$\frac{\partial P_3}{\partial \delta_2} = 4.9752V_3V_2 \sin(\delta_3 - \delta_2 - 95.71)$$

$$\frac{\partial P_3}{\partial \delta_3} = -3.3168V_3 \sin(\delta_3 - 95.71) - 4.9752V_3V_2 \sin(\delta_3 - \delta_2 - 95.71)$$

$$\frac{\partial P_3}{\partial V_2} = 4.9752V_3 \cos(\delta_3 - \delta_2 - 95.71)$$

$$\frac{\partial P_3}{\partial V_3} = 3.3168 \cos(\delta_3 - 95.71) + 4.9752V_2 \cos(\delta_3 - \delta_2 - 95.71) + 16.5840V_3 \cos(84.28)$$

$$\frac{\partial Q_2}{\partial \delta_2} = 2.4807V_2 \cos(\delta_2 - 97.12) + 4.9752V_2V_3 \cos(\delta_2 - \delta_3 - 95.71)$$

$$\frac{\partial Q_2}{\partial \delta_3} = -4.9752V_2V_3 \cos(\delta_2 - \delta_3 - 95.71)$$

$$\frac{\partial Q_2}{\partial V_2} = 2.4807 \sin(\delta_2 - 97.12) + 14.1156V_2 \sin(83.46) + 4.9752V_3 \sin(\delta_2 - \delta_3 - 95.71)$$

$$\frac{\partial Q_2}{\partial V_3} = 4.9752V_2 \sin(\delta_2 - \delta_3 - 95.71)$$

$$\frac{\partial Q_3}{\partial \delta_2} = -4.9752V_3V_2 \cos(\delta_3 - \delta_2 - 95.71)$$

$$\frac{\partial Q_3}{\partial \delta_3} = 3.3168 V_3 \cos(\delta_3 - 95.71) + 4.9752 V_3 V_2 \cos(\delta_3 - \delta_2 - 95.71)$$

$$\frac{\partial Q_3}{\partial V_2} = 4.9752 V_3 \sin(\delta_3 - \delta_2 - 95.71)$$

$$\frac{\partial Q_3}{\partial V_3} = 3.3168 \sin(\delta_3 - 95.71) + 4.9752 V_2 \sin(\delta_3 - \delta_2 - 95.71) + 16.5840 V_3 \sin(84.28)$$

$$J^{(0)} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial V_2} & \frac{\partial P_2}{\partial V_3} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial V_2} & \frac{\partial P_3}{\partial V_3} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial V_2} & \frac{\partial Q_2}{\partial V_3} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \frac{\partial Q_3}{\partial V_2} & \frac{\partial Q_3}{\partial V_3} \end{bmatrix} = \begin{bmatrix} 7.2756 & -4.9304 & 0.1327 & -0.6662 \\ -4.9647 & 8.1588 & -0.3232 & 0.4359 \\ -1.4750 & 0.6662 & 6.7482 & -4.9304 \\ 0.3232 & -1.2170 & -4.9647 & 8.3426 \end{bmatrix}$$

$$\Delta x^{(0)} = \left[J^{(0)}\right]^{-1} \left(b - h\left(x^{(0)}\right)\right) = \begin{bmatrix} 0.0145 \\ 0.0090 \\ 0.0182 \\ -0.0114 \end{bmatrix}$$

$$x^{(1)} = x^{(0)} + \Delta x^{(0)} = \begin{bmatrix} -0.1933 \\ -0.1642 \\ 1.0182 \\ 0.9886 \end{bmatrix}$$

2nd iteration:

$$h\left(x^{(1)}\right) = \begin{bmatrix} -0.5987 \\ -0.4003 \\ -0.0966 \\ -0.0976 \end{bmatrix}$$

$$J^{(1)} = \begin{bmatrix} 7.3657 & -4.9662 & 0.2305 & -0.6507 \\ -4.9953 & 8.1607 & -0.3465 & 0.4121 \\ -1.4321 & 0.6433 & 7.0445 & -5.0237 \\ 0.3528 & -1.2079 & -4.9061 & 8.0576 \end{bmatrix}$$

$$\Delta x^{(1)} = \left[J^{(1)}\right]^{-1} \left(b - h\left(x^{(1)}\right)\right) = \begin{bmatrix} -0.0004 \\ -0.0002 \\ -0.0014 \\ -0.0011 \end{bmatrix}$$

$$x^{(2)} = x^{(1)} + \Delta x^{(1)} = \begin{bmatrix} -0.1937 \\ -0.1644 \\ 1.0168 \\ 0.9874 \end{bmatrix}$$

Checking the convergence criterion:

$$b - h\left(x^{(2)}\right) = \begin{bmatrix} -3.8272 \times 10^{-6} \\ 9.4018 \times 10^{-7} \\ -4.8282 \times 10^{-6} \\ -3.1157 \times 10^{-6} \end{bmatrix}$$

$$max\left(\left|b - h\left(x^{(2)}\right)\right|\right) = 4.8282 \times 10^{-6} < 0.001$$

The convergence criterion has been met.

Newton Raphson solution after 2 iterations:

$$\begin{bmatrix} \overrightarrow{V_1} \\ \overrightarrow{V_2} \\ \overrightarrow{V_3} \end{bmatrix} = \begin{bmatrix} 1\angle 0 \\ 1.0168\angle - 0.1937 \\ 0.9874\angle - 0.1644 \end{bmatrix}$$

3)

$$\overrightarrow{Y_{BUS}} = \begin{bmatrix} 12.3010\angle - 87.30 & 4.9937\angle 92.86 & 3.3168\angle 95.71 & 4\angle 90 \\ 4.9937\angle 92.86 & 9.9906\angle - 88.57 & 5\angle 90 & 0 \\ 3.3168\angle 95.71 & 5\angle 90 & 13.3040\angle - 88.57 & 5\angle 90 \\ 4\angle 90 & 0 & 5\angle 90 & 8.7\angle - 90 \end{bmatrix}$$

DC power flow solution:

$$\begin{bmatrix} \overrightarrow{V_1} \\ \overrightarrow{V_2} \\ \overrightarrow{V_3} \\ \overrightarrow{V_4} \end{bmatrix} = \begin{bmatrix} 1\angle 0 \\ 1.05\angle 0.07569 \\ 1\angle - 0.048809 \\ 1\angle - 0.085523 \end{bmatrix}$$

Newton Raphson:

1st iteration:

$$x^{(0)}=\begin{bmatrix}\delta_2^{(0)}\\ \delta_3^{(0)}\\ \delta_4^{(0)}\\ V_3^{(0)}\\ V_4^{(0)}\end{bmatrix}=\begin{bmatrix}0.07569\\ -0.048809\\ -0.085523\\ 1\\ 1\end{bmatrix},\quad b=\begin{bmatrix}P_{G_2}\\ -P_{L_3}\\ -P_{L_4}\\ -Q_{L_3}\\ -Q_{L_4}\end{bmatrix}=\begin{bmatrix}1\\ -0.6\\ -0.5\\ -0.2\\ -0.1\end{bmatrix},\quad h(x)=\begin{bmatrix}P_2(x)\\ P_3(x)\\ P_4(x)\\ Q_3(x)\\ Q_4(x)\end{bmatrix}$$

$$P_2(x)=4.9937V_2\cos(\delta_2-92.86)+9.9906V_2^2\cos(88.57)+5V_2V_3\cos(\delta_2-\delta_3-90)$$

$$P_3(x)=3.3168V_3\cos(\delta_3-95.71)+5V_3V_2\cos(\delta_3-\delta_2-90)+13.3040V_3^2\cos(88.57)\\+5V_3V_4\cos(\delta_3-\delta_4-90)$$

$$P_4(x)=4V_4\cos(\delta_4-90)+5V_4V_3\cos(\delta_4-\delta_3-90)$$

$$Q_3(x)=3.3168V_3\sin(\delta_3-95.71)+5V_3V_2\sin(\delta_3-\delta_2-90)+13.3040V_3^2\sin(88.57)\\+5V_3V_4\sin(\delta_3-\delta_4-90)$$

$$Q_4(x)=4V_4\sin(\delta_4-90)+5V_4V_3\sin(\delta_4-\delta_3-90)+8.7V_4^2$$

$$h\left(x^{(0)}\right)=\begin{bmatrix}1.0619\\ -0.6270\\ -0.5252\\ -0.1865\\ -0.2820\end{bmatrix}$$

Elements of Jacobian matrix:

$$\frac{\partial P_2}{\partial \delta_2}=-4.9937V_2\sin(\delta_2-92.86)-5V_2V_3\sin(\delta_2-\delta_3-90)$$

$$\frac{\partial P_2}{\partial \delta_3}=5V_2V_3\sin(\delta_2-\delta_3-90)$$

$$\frac{\partial P_2}{\partial \delta_4}=0$$

$$\frac{\partial P_2}{\partial V_3}=5V_2\cos(\delta_2-\delta_3-90)$$

$$\frac{\partial P_2}{\partial V_4}=0$$

$$\frac{\partial P_3}{\partial \delta_2} = 5V_3V_2\sin(\delta_3 - \delta_2 - 90)$$

$$\frac{\partial P_3}{\partial \delta_3} = -3.3168V_3\sin(\delta_3 - 95.71) - 5V_3V_2\sin(\delta_3 - \delta_2 - 90) - 5V_3V_4\sin(\delta_3 - \delta_4 - 90)$$

$$\frac{\partial P_3}{\partial \delta_4} = 5V_3V_4\sin(\delta_3 - \delta_4 - 90)$$

$$\frac{\partial P_3}{\partial V_3} = 3.3168\cos(\delta_3 - 95.71) + 5V_2\cos(\delta_3 - \delta_2 - 90) + 26.6080V_3\cos(88.57) + 5V_4\cos(\delta_3 - \delta_4 - 90)$$

$$\frac{\partial P_3}{\partial V_4} = 5V_3\cos(\delta_3 - \delta_4 - 90)$$

$$\frac{\partial P_4}{\partial \delta_2} = 0$$

$$\frac{\partial P_4}{\partial \delta_3} = 5V_4V_3\sin(\delta_4 - \delta_3 - 90)$$

$$\frac{\partial P_4}{\partial \delta_4} = -4V_4\sin(\delta_4 - 90) - 5V_4V_3\sin(\delta_4 - \delta_3 - 90)$$

$$\frac{\partial P_4}{\partial V_3} = 5V_4\cos(\delta_4 - \delta_3 - 90)$$

$$\frac{\partial P_4}{\partial V_4} = 4\cos(\delta_4 - 90) + 5V_3\cos(\delta_4 - \delta_3 - 90)$$

$$\frac{\partial Q_3}{\partial \delta_2} = -5V_3V_2\cos(\delta_3 - \delta_2 - 90)$$

$$\frac{\partial Q_3}{\partial \delta_3} = 3.3168V_3\cos(\delta_3 - 95.71) + 5V_3V_2\cos(\delta_3 - \delta_2 - 90) + 5V_3V_4\cos(\delta_3 - \delta_4 - 90)$$

$$\frac{\partial Q_3}{\partial \delta_4} = -5V_3V_4\cos(\delta_3 - \delta_4 - 90)$$

$$\frac{\partial Q_3}{\partial V_3} = 3.3168\sin(\delta_3 - 95.71) + 5V_2\sin(\delta_3 - \delta_2 - 90) + 26.6080V_3\sin(88.57) + 5V_4\sin(\delta_3 - \delta_4 - 90)$$

$$\frac{\partial Q_3}{\partial V_4} = 5V_3 \sin(\delta_3 - \delta_4 - 90)$$

$$\frac{\partial Q_4}{\partial \delta_2} = 0$$

$$\frac{\partial Q_4}{\partial \delta_3} = -5V_4V_3 \cos(\delta_4 - \delta_3 - 90)$$

$$\frac{\partial Q_4}{\partial \delta_4} = 4V_4 \cos(\delta_4 - 90) + 5V_4V_3 \cos(\delta_4 - \delta_3 - 90)$$

$$\frac{\partial Q_4}{\partial V_3} = 5V_4 \sin(\delta_4 - \delta_3 - 90)$$

$$\frac{\partial Q_4}{\partial V_4} = 4\sin(\delta_4 - 90) + 5V_3 \sin(\delta_4 - \delta_3 - 90) + 17.4V_4$$

$$J^{(0)} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial \delta_4} & \frac{\partial P_2}{\partial V_3} & \frac{\partial P_2}{\partial V_4} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial \delta_4} & \frac{\partial P_3}{\partial V_3} & \frac{\partial P_3}{\partial V_4} \\ \frac{\partial P_4}{\partial \delta_2} & \frac{\partial P_4}{\partial \delta_3} & \frac{\partial P_4}{\partial \delta_4} & \frac{\partial P_4}{\partial V_3} & \frac{\partial P_4}{\partial V_4} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \frac{\partial Q_3}{\partial \delta_4} & \frac{\partial Q_3}{\partial V_3} & \frac{\partial Q_3}{\partial V_4} \\ \frac{\partial Q_4}{\partial \delta_2} & \frac{\partial Q_4}{\partial \delta_3} & \frac{\partial Q_4}{\partial \delta_4} & \frac{\partial Q_4}{\partial V_3} & \frac{\partial Q_4}{\partial V_4} \end{bmatrix} = \begin{bmatrix} 10.4510 & -5.2094 & 0 & 0.6519 & 0 \\ -5.2094 & 13.4863 & -4.9966 & -0.2950 & 0.1835 \\ 0 & -4.9966 & 8.9820 & -0.1835 & -0.5252 \\ 0.6519 & -0.9590 & -0.1835 & 13.1134 & -4.9966 \\ 0 & 0.1835 & -0.5252 & -4.9966 & 8.4180 \end{bmatrix}$$

$$\Delta x^{(0)} = \left[J^{(0)}\right]^{-1} \left(b - h\left(x^{(0)}\right)\right) = \begin{bmatrix} -0.0057 \\ 0.0017 \\ 0.0056 \\ 0.0101 \\ 0.0279 \end{bmatrix}$$

$$x^{(1)} = x^{(0)} + \Delta x^{(0)} = \begin{bmatrix} 0.0700 \\ -0.0471 \\ -0.0799 \\ 1.0101 \\ 1.0279 \end{bmatrix}$$

2nd iteration:

$$h\left(x^{(1)}\right) = \begin{bmatrix} 0.9996 \\ -0.6002 \\ -0.4987 \\ -0.1999 \\ -0.0946 \end{bmatrix}$$

$$J^{(1)} = \begin{bmatrix} 10.5090 & -5.2667 & 0 & 0.6133 & 0 \\ -5.2667 & 13.7697 & -5.1887 & -0.2588 & 0.1658 \\ 0 & -5.1887 & 9.2873 & -0.1687 & -0.4851 \\ 0.6195 & -0.9389 & -0.1704 & 13.2362 & -5.0478 \\ 0 & 0.1704 & -0.4987 & -5.1369 & 8.8509 \end{bmatrix}$$

$$\Delta x^{(1)} = \left[J^{(1)}\right]^{-1}\left(b - h\left(x^{(1)}\right)\right) = \begin{bmatrix} 3.0925 \times 10^{-5} \\ -5.3330 \times 10^{-5} \\ -2.2004 \times 10^{-4} \\ -3.1960 \times 10^{-4} \\ -8.0681 \times 10^{-4} \end{bmatrix}$$

$$x^{(2)} = x^{(1)} + \Delta x^{(1)} = \begin{bmatrix} 0.0700 \\ -0.0471 \\ -0.0801 \\ 1.0098 \\ 1.0271 \end{bmatrix}$$

Checking the convergence criterion:

$$b - h\left(x^{(2)}\right) = \begin{bmatrix} 1.4265 \times 10^{-7} \\ 6.7931 \times 10^{-7} \\ -1.6282 \times 10^{-6} \\ -1.0926 \times 10^{-7} \\ -4.4578 \times 10^{-6} \end{bmatrix}$$

$$max\left(\left|b - h\left(x^{(2)}\right)\right|\right) = 4.4578 \times 10^{-6} < 0.001$$

The convergence criterion has been met.

Newton Raphson solution after 2 iterations:

$$\begin{bmatrix} \overrightarrow{V_1} \\ \overrightarrow{V_2} \\ \overrightarrow{V_3} \\ \overrightarrow{V_4} \end{bmatrix} = \begin{bmatrix} 1\angle 0 \\ 1.05\angle 0.0700 \\ 1.0098\angle -0.0471 \\ 1.0271\angle -0.0801 \end{bmatrix}$$