EE507 Fall 2022, Exam 1

The exam has eight problems, which have multiple parts. You will have 3 hours (180 minutes) to complete the exam. Please write the solutions on your own paper, and send me the solutions within 5 minutes of the end time of the exam (you can scan and send, or take photos and send). Please explain your answers carefully: I will score the problems based on your logic, not only whether or not the answer is correct.

You may use any inanimate references that you would like for the exam, including books and notes. You may also access my taped lectures if you wish, although I discourage you from doing this (it likely will take up too much time). Please do not communicate with other people about exam questions, and also don't post or request answers on forums and websites.

Problem 1

A rock that is being tested in a laboratory is either copper (C) with probability 0.9, or gold (G) with probability 0.1. A chemical test is used to determine whether or not the material is gold. If the material is copper, it passes (P) the test with probability 0.2, and fails (F) with probability 0.8. If the material is gold, it passes (P) with probability 0.8 and fails (F) with probability 0.2. **(10 points)**

a. If the rock passes the test, what is the probability that it is gold?
b. You repeat the test three times on a particular rock, and it passes all three times. What is the probability this the rock is gold? (You may assume that the three test outcomes are independent, given the type of the material.)

Problem 2 (10 points)

a. Consider an experiment with three events A, B, and C which satisfy $A + B + C = \Omega$, where Ω is the sample space. Please show that $P(A) + P(B) + P(C) + P(\overline{AB + AC}) \geq 2$.
b. We say two events X and Y are *positively dependent* if $P(XY) > P(X)P(Y)$. For a particular experiment, Two events D and E have probabilities $P(D) = 0.6$ and $P(E) = 0.7$. Also, $P(D + E) = 0.8$. Are D and E independent, positively dependent, or neither?

Problem 3

An experiment has five outcomes a, b, c, d, and e, which have equal probability of 0.2. We define a random variable X as follows: $X(a) = 1$, $X(b) = 2$, $X(c) = 3$, $X(d) = 2$, and $X(e) = 1$. Also, we define the event A as $A = \{a, b, d\}$. Please answer the following questions **(15 points in total, 8 for part a)**

a. Find and plot the pmf and CDF of X.
b. What is $P(A \,|\, X \leq 2)$.
c. What is $P(X \leq 2 \,|\, A)$

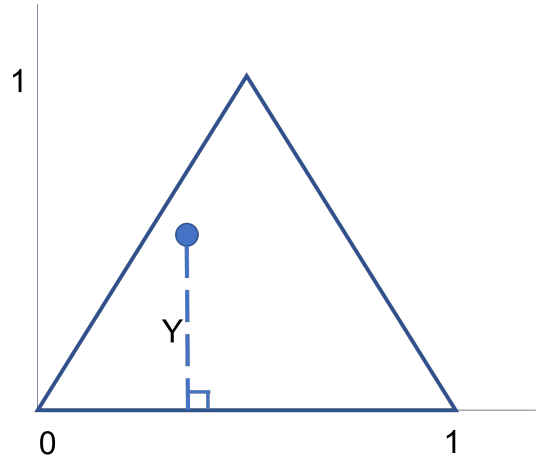
Problem 4

A random variable X has pdf of the form $f_X(x) = Ce^{-x}$ for $0 \leq x \leq T$ (and $f_X(x) = 0$ otherwise). **(15 points)**

a. Find the constant C. (Please leave your answer in terms of the parameter T.)
b. Find the CDF $F_X(x)$ of the random variable X, again leaving your answer in terms of T.
c. What is the mean of the random variable X, in terms of T?

Problem 5

A probabilistic experiment involves selecting a point at random on the triangle shown below. Each point on the triangle is equally likely. Let Y be the distance of the selected point from the base of the triangle, as shown below. **(15 points)**



a. Please find the pdf of Y.
b. Please find the pdf of $Z = \sqrt{Y}$.

Problem 6

You flip a fair coin. When the coin shows heads (H), the random variable X is a Gaussian random variable with distribution $\mathcal{N}(m = 0, \sigma^2 = 4)$. When the coil shows tails (T), X is uniform over the interval $[0, 2]$. **(15 points**)

a. Find the pdf of X.
b. Find $P(1 \leq X \leq 3)$. Please leave your answer in terms of the standard Gaussian CDF.
c. What is the probability that the coin flip was Heads, given that $X = 1$?

Problem 7

A discrete random variable X has pmf $p_X(x) = \frac{x}{21}$ for $x = 1, 2, \ldots, 6$, and $p_X(x) = 0$ otherwise. Please answer the following questions. **(10 points)**

a. What is $P(2 \leq X < 5)$?
b. What is $E(X)$?

Problem 8

Please briefly define the following concepts. **(10 points)**

a. Event
b. Random variable