

HW5, due 11/3/2022

Problem 1

- a. Consider three random variables X , Y , and Z . Let's say that $f_{X,Y,Z}(x,y,z) = f_X(x)f_Y(y)f_Z(z)$, i.e. that their joint PDF is the product of their individual PDFs. Is it necessarily true that X and Y are independent?
- b. Is your answer to part a different from the answer to the analogous problem for three events? If so, conceptually, why?

Problem 2

Consider a pair of jointly Gaussian random variables X and Y with parameters $m_x = m_y = 0$, $\sigma_x^2 = 1$, $\sigma_y^2 = 4$, and $r = 0.5$.

- a. Please sketch lines of equal probability for the joint PDF of X and Y .
- b. What are the marginal PDFs of X and Y ?
- c. Please find $f_{Y|X}(y | X = x)$.
- d. For what value of x is $E[Y | X = x] = -2$.
- e. What is the PDF of $Z = X + Y - 1$?
- f. Let $Z = 2X + 3Y$, and let $W = X - Y$. Please find $E[Z]$, $\text{var}(Z)$, $E[W]$, $\text{var}(W)$, and $\text{cov}(Z, W)$. Also, please find the joint PDF of Z and W .
- g. Consider a random variable $R = aX + bY$. For what constants a and b are R and Y independent?
- h. Let $Q = aX$, where a equals 1 with probability 0.5 and equals 2 with probability 0.5. Please find the PDF of Q .

Problem 3

- a. Consider two random variables X and Y that are independent, and consider $Z = X + Y$. Show that the PDF of Z is given by $f_Z(z) = \int_{-\infty}^{\infty} f_Y(z-x)f_X(x) dx$.
- b. For independent X and Y , show that $Z = X + Y$ has PDF $f_Z(z) = \int_{-\infty}^{\infty} f_Y(y)f_X(z-y) dy$.
- c. Say that X and Y are independently and identically distributed exponential random variables with mean 0.5. What is the PDF of $Z = X + Y$?

HW2 due 11/3/2025

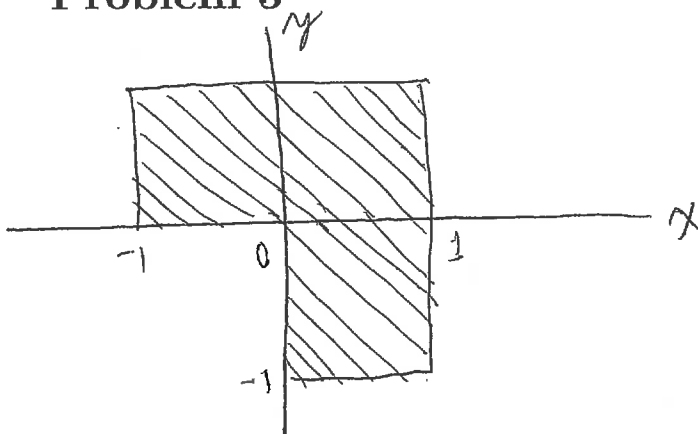
- d. Say that X and Y are independent random variables that are both uniform on $[0, 1]$. Please find the PDF of $Z = X + Y$.
- e. Consider the sum of 7 independent random variables that are each uniform on $[0, 1]$. Numerically find the PDF of this sum. What common density does this PDF remind you of?

Problem 4

Say that X and Y are independent random variables that are each uniform on $[0, 1]$. Let $Z = \frac{Y}{X}$.

- Please find the PDF of Z .
- Please find $E[X^2 + Y^2]$.
- Please find the joint PDF of X and Z , where $Z = \frac{Y}{X}$.
- Please find $E[XZ]$, $\text{cov}(X, Z)$, and $\rho_{X,Z}$.
- Please find the PDF of X given Z , and $E[X|Z]$.
- Verify that, in this example, $E[E[X|Z]] = E[X]$.
- Please find $E[XZ | Z = z]$.

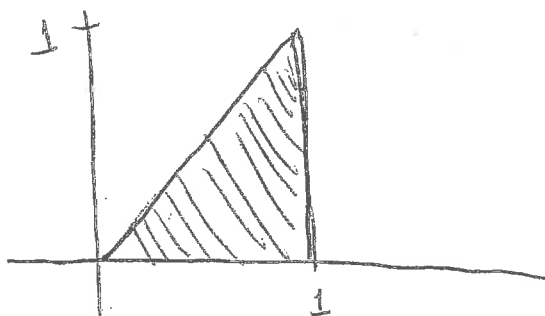
Problem 5



Consider a pair of random variables X and Y uniformly distributed in the region shown above. Please find $E[X]$, $E[Y]$, $E[XY]$, $\text{var}(X)$, $\text{var}(Y)$, $\text{cov}(X, Y)$, and $\rho_{X,Y}$.

Problem 6

Two random variables X and Y are uniformly distributed in the region shown below:



Please find: $f_{X,Y}(x,y)$, $F_{X,Y}(x,y)$, $f_X(x)$, $f_Y(y)$, and $f_{X|Y}(x|y=y)$.

Problem 7

Is it possible for two random variables X and Y to be each uniform on $[0,1]$, uncorrelated, and not independent?

