

## A Simple Example of Continuation Power Flow

An example is provided here for illustration of the detailed steps of continuation power flow. The approach used here is based on the method reported in [1].

Consider the following 2-bus power system. The generator is modelled as a voltage source with a constant voltage  $E$  magnitude but a variable angle  $\delta$ . The transmission line is modelled as a reactance  $X$ . The load is a constant MW and MVAR load, i.e.,  $P+jQ$ . For simplicity, we assume that  $E=1$  p.u. and  $X=1$  p.u.

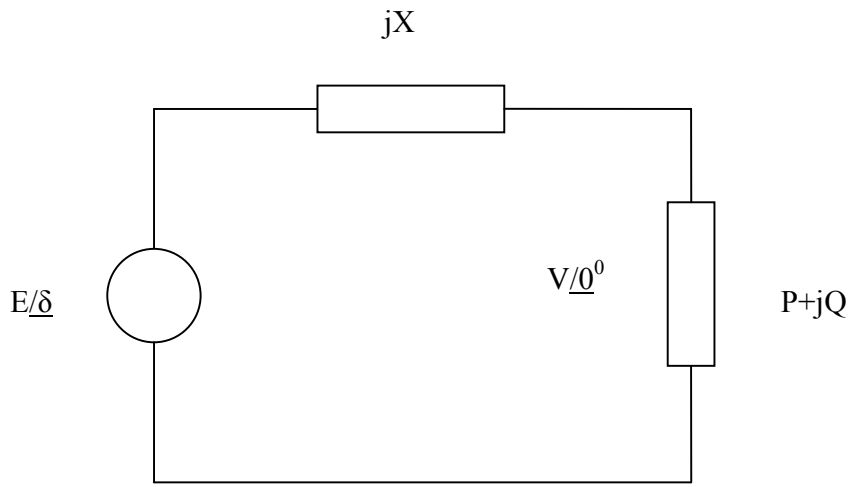


Figure 1 2-Bus Power System

The power flow equations for this simple system can be obtained as

$$P = V \sin \delta \quad (1)$$

$$Q = -V^2 + V \cos \delta \quad (2)$$

where  $P$  and  $Q$  are known and  $V$  and  $\delta$  are unknowns to be determined by the power flow computation.

The power flow equations are parameterized with a factor  $\lambda$ , which represents the variation of load demand. That is,

$$f_1(\delta, V, \lambda) = V \sin \delta - \lambda P = 0 \quad (3)$$

$$f_2(\delta, V, \lambda) = -V^2 + V \cos \delta - \lambda Q = 0 \quad (4)$$

The purpose of continuation power flow is to trace the solutions as the parameter  $\lambda$  varies. There are essentially two phases in the CPF computation – Predictor and Corrector.

### ***Predictor – Tangent Method***

This step is to find a next point  $(\delta', V', \lambda')$  starting from an initial point  $(\delta, V, \lambda)$ . We will use the tangent method for this example. Given an initial point  $(\delta, V, \lambda)$ , we would like to obtain the next point by the following iterative method:

$$\delta' = \delta + h (d\delta/ds) \quad (5)$$

$$V' = V + h (dV/ds) \quad (6)$$

$$\lambda' = \lambda + h (d\lambda/ds) \quad (7)$$

where  $h$  is a step size and the three derivative terms are to be found.

To find the three derivative terms in Eqs. (5-7), we consider the power flow functions  $f_1(\delta, V, \lambda)$  and  $f_2(\delta, V, \lambda)$ . The partial derivatives can be obtained as follows:

$$\partial f_1/\partial \delta * d\delta/ds + \partial f_1/\partial V * dV/ds + \partial f_1/\partial \lambda * d\lambda/ds = 0 \quad (8)$$

$$\partial f_2/\partial \delta * d\delta/ds + \partial f_2/\partial V * dV/ds + \partial f_2/\partial \lambda * d\lambda/ds = 0 \quad (9)$$

We need to find the derivative terms  $(d\delta/ds, dV/ds, d\lambda/ds)$  but there are only two equations (8-9). Therefore, we will first find two derivatives in terms of the third one and then find a third equation. Now suppose the term  $dV/ds$  is non-zero. Then we will find  $d\delta/ds$  and  $d\lambda/ds$  in terms of  $dV/ds$ . This is straightforward from Eqs. (8-9). That is,

$$\partial f_1/\partial \delta * d\delta/ds + \partial f_1/\partial \lambda * d\lambda/ds = - \partial f_1/\partial V * dV/ds \quad (10)$$

$$\partial f_2/\partial \delta * d\delta/ds + \partial f_2/\partial \lambda * d\lambda/ds = - \partial f_2/\partial V * dV/ds \quad (11)$$

Note that from Eqs. (3-4), we can evaluate the partial derivatives in Eqs. (10-11) at the initial point  $(\delta, V, \lambda)$ . The following partial derivatives are obtained:

$$\partial f_1/\partial \delta = V \cos \delta$$

$$\partial f_1/\partial \lambda = -P$$

$$\partial f_1/\partial V = \sin \delta$$

$$\partial f_2/\partial \delta = -V \sin \delta$$

$$\partial f_2/\partial \lambda = -Q$$

$$\partial f_2/\partial V = -2V + \cos \delta$$

Once Eqs. (10-11) are solved, the two derivative terms,  $d\delta/ds$  and  $d\lambda/ds$ , can be written in terms of  $dV/ds$ . That is,

$$d\delta/ds = \beta_1 * dV/ds \quad (12)$$

$$d\lambda/ds = \beta_3 * dV/ds \quad (13)$$

As mentioned previously, we need a third equation, in addition to Eqs. (12-13), to find the three derivatives. This third equation can be obtained by the arc length of the curve, which is:

$$(d\delta/ds)^2 + (dV/ds)^2 + (d\lambda/ds)^2 = 1 \quad (14)$$

In view of Eqs. (12-14), once we find the ratios  $\beta_1$  and  $\beta_3$ , the third derivative is given by

$$(dV/ds)^2 = 1 / (1 + \beta_1^2 + \beta_3^2) \quad (15)$$

Now that the three derivatives are found, one can use Eqs. (5-7) to calculate the next point  $(\delta', V', \lambda')$  based on the initial point  $(\delta, V, \lambda)$ .

### ***Corrector – Newton's Method***

The predictor step leads to the next point  $(\delta', V', \lambda')$  which is generally not on the PV curve. However, it serves as the initial values for the computation of the power flow solution. Since the power flow equations have been parameterized, we have the two equations involving three unknowns  $(\delta^*, V^*, \lambda^*)$ .

$$V^* \sin \delta^* = \lambda^* P \quad (16)$$

$$-V^{*2} + V^* \cos \delta^* = \lambda^* Q \quad (17)$$

Note that P and Q are known constants from the load data of the 2-bus system. In order to solve the power flow, we need to find a third equation again. The arc length will again provide the third equation. If the arc length  $\Delta s$  is specified, then the third equation can be obtained by:

$$(\delta^* - \delta')^2 + (V^* - V')^2 + (\lambda^* - \lambda')^2 = \Delta s^2 \quad (18)$$

With the three equations, Eqs. (16-18), one can use  $(\delta', V', \lambda')$  obtained from the predictor step as the initial point for the solution of the nonlinear equations (16-18). Newton's method is a natural choice of the numerical method. The solution  $(\delta^*, V^*, \lambda^*)$  must be on the PV curve. One can then go back to the predictor step and advance toward the point of voltage collapse.

### **Numerical Example**

The given parameters are  $E = 1$  p.u. and  $X = 1$  p.u.

To trace the upper part of the PV curve the initial points are selected as follows:

$$\lambda = 0.5 \text{ and } P = 0.4, P_0 = \lambda P = 0.2 \text{ p.u.}$$

$$V_0 = 0.9789 \text{ and } \delta_0 = 0.2058$$

To trace the lower part of the PV curve the initial points are selected as follows:

$$\lambda = 0.5 \text{ and } P = 0.4, P_0 = \lambda P = 0.2 \text{ p.u.}$$

$$V_0 = 0.2043 \text{ and } \delta_0 = 1.3650$$

Step size  $h$  is set to be 0.001.

Arc length  $\Delta s$  is set to a value of 0.04.

***Using Tangent method to calculate the next point (Predictor)***

Based on Eqs. (10-13),  $\beta_1$  and  $\beta_3$  can be derived as follows:

$$\beta_1 = ( \partial f_2 / \partial V * \partial f_1 / \partial \lambda - \partial f_1 / \partial V * \partial f_2 / \partial \lambda ) / ( \partial f_1 / \partial \delta * \partial f_2 / \partial \lambda - \partial f_2 / \partial \delta * \partial f_1 / \partial \lambda )$$

$$\beta_3 = ( \partial f_2 / \partial V * \partial f_1 / \partial \delta - \partial f_1 / \partial V * \partial f_2 / \partial \delta ) / ( \partial f_1 / \partial \lambda * \partial f_2 / \partial \delta - \partial f_1 / \partial \delta * \partial f_2 / \partial \lambda )$$

The derivatives of  $\delta$ ,  $V$  and  $\lambda$  with respect of  $s$  can be calculated based on Eqs. (12), (13) and (15).

The next point  $c$  can then be calculated from Eqs. (5-7).

***Using Newton's method to calculate the next point on the PV curve (Corrector)***

Newton's method is used here to solve the nonlinear equations (16-18). Note that if the next point  $(\delta', V', \lambda')$  is used as the initial value in the Newton's method, the last row of Jacobian will be a zero row. (The elements on the last row are  $2(\delta_0 - \delta')$ ,  $2(V_0 - V')$  and  $2(\lambda_0 - \lambda')$ .)

The technique here is to modify Eqs. (5-7) to allow the next point to be 1% further on the tangent direction.

$$\delta' = \delta + 1.01 * h (d\delta/ds) \quad (5')$$

$$V' = V + 1.01 * h (dV/ds) \quad (6')$$

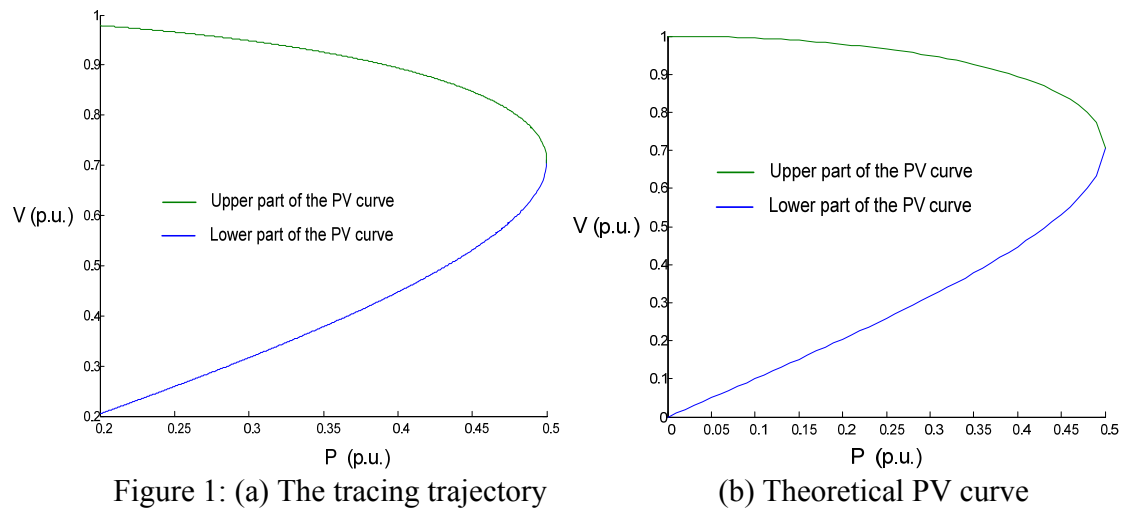
$$\lambda' = \lambda + 1.01 * h (d\lambda/ds) \quad (7')$$

Then we could simply follow the Newton iteration formula to find the next point on the PV curve.

$$\begin{bmatrix} V^{n+1} \\ \delta^{n+1} \\ \lambda^{n+1} \end{bmatrix} = \begin{bmatrix} V^n \\ \delta^n \\ \lambda^n \end{bmatrix} - [J(V^n, \delta^n, \lambda^n)]^{-1} f(V^n, \delta^n, \lambda^n)$$

As seen in Figure 1 (a) and (b), the PV curve calculated by the numerical method described above is almost identical to the theoretical PV curve derived from the closed form solutions of the power flow equations. The closed form solution of power flow

equations for a simple system such as the one used here can be found in a standard text book.



## 1.2 Computational Procedure

The computational procedure consists of two major steps, i.e., predictor and corrector, as shown in Figure 2. The predictor and corrector steps are further expanded in Figures 3 and 4.

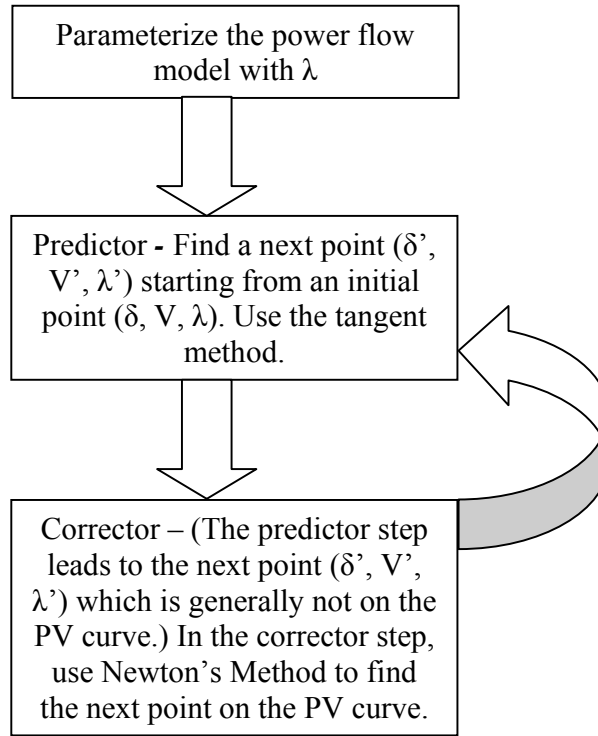


Figure 2: Predictor and Corrector Procedure

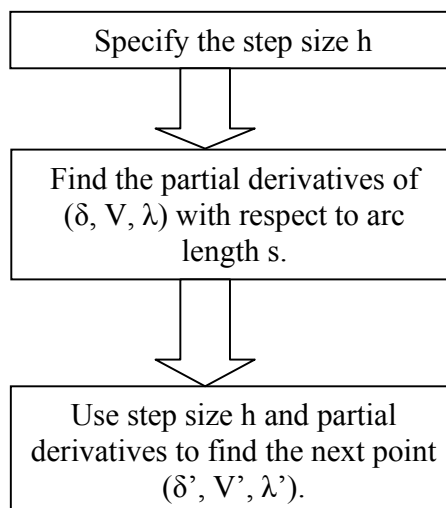


Figure 3: Predictor Step

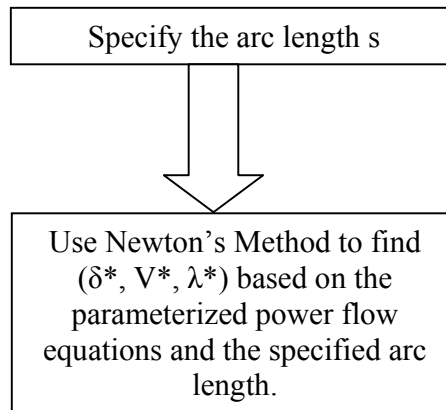


Figure 4: Corrector Step

The above procedure is the basic form of Predictor-Corrector method for continuation power flow. Various techniques and extensions have been proposed in the literature. Excellent papers by Professors Hsiao-Dong Chiang (Cornell University), Claudio Canizares (University of Waterloo) and Venkat Ajjarapu (Iowa State University) can be found in the literature.

## Reference

- [1] H. D. Chiang, A. J. Flueck, K. S. Shah, and N. Balu, "CPFLOW: A Practical Tool for Tracing Power System Steady-State Stationary Behavior Due to Load and Generation Variation.," *IEEE Trans. Power Systems*, May 1995, pp. 623-634.