ADDITIONAL THOUGHTS ON THE EXISTENCE OF GLOBAL minimizers

We discussed in class that none of the six properties below is sufficient for guaranteeing the existence of a global minimizer of $f: \mathbb{R}^n \to \mathbb{R}$.

1. Continuous
2. Bounded Below
3. Convex
4. Quasi-convex
5. Coercive
6. existence of a local minimizer

However, some combinations of two properties are sufficent.

Theorem: Let $f: \mathbb{R}^n \to \mathbb{R}$ be convex. If x^* is a local minimizer of f, then x^* is a global minimizer of f.

Theorem: Let $f: \mathbb{R}^n \to \mathbb{R}$ be coercive. If f is either continuous or bounded below or quasi-convex, then some $x^* \in \mathbb{R}^n$ exists which is a global minimizer of f.

(note that the quasi convex condition allows convex as well).

Another way to think about coercive functions:

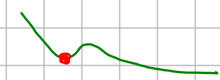
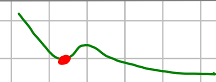
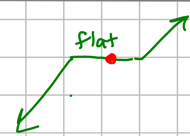
Fact: A function $f: \mathbb{R}^n \to \mathbb{R}$ is coercive if and only if every sublevel set of f is bounded.

All other combinations of two properties from the list above do not guarantee the existence of a global minimizer. See the chart on the next page.

Some cautions:

- Convexity or quasiconvexity is not necessary for the existence of a global minimizer.
- Convexity $\Rightarrow$ Quasiconvex not the other way.
- We will see examples of discontinuous convex functions (normally this cannot happen!)

This table shows counterexamples where two properties hold and no global min exists

	Bounded Below	convex	quasiconvex	coercive	local min exists
Continuous	e^{-x}	e^{-x}	e^{-x}	works!	 $(x^2+\frac{1}{2})e^{-x}$
Bounded Below		e^{-x}	e^{-x}	works!	 $(x^2+\frac{1}{2})e^{-x}$
convex			e^{-x}	works!	works!
quasiconvex				works!	
coercive