ASSIGNMENT 04 of EE 523, SPRING 2023

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1. Type 1 Model

$$E = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \end{array} \begin{bmatrix} 5 \\ 6 \\ 11 \\ 10 \\ 1, 6 \\ 2, 5, 7 \\ 6, 8 \\ 7, 9 \\ 8, 10 \\ 4, 9, 11 \\ 3, 10 \end{bmatrix}$$

Adjacency list

$$\underset{12\times 1}{x} = \begin{bmatrix} \theta_2 \\ \omega_2 \\ E'_{q_2} \\ E'_{d_2} \\ \theta_3 \\ \omega_3 \\ E'_{q_3} \\ E'_{d_3} \\ \theta_4 \\ \omega_4 \\ E'_{q_4} \\ E'_{d_4} \end{bmatrix}_{12\times 1} \qquad \underset{6\times 1}{y} = \begin{bmatrix} \delta_2 \\ |V_2| \\ \delta_3 \\ |V_3| \\ \delta_4 \\ |V_4| \end{bmatrix}_{6\times 1}$$

Note: $\circ\!\!/$ implies that the variables are ~~either~~ given (P_{m_i}, X_{d_i}', X_{d_i}, X_{q_i}', X_{q_i}, Y_{ik}, γ_{ik}, ~~α_k~~ K_{D_i}, ω_s, E_{d_i}, M_i ~~or have been initialized, say using power flow~~ ~~(V_k, δ_k for nongen buses)~~) 1.2

$$P_{G_2} - P_{D_2} - P_2 = 0$$

or $$V_{d_2} I_{d_2} + V_{q_2} I_{q_2} - P_{D_2} - |V_2| \sum_{(2,k)\in E} |Y_{2k} V_k| \cos(\gamma_{2k} + \delta_k - \delta_2)$$

or $$|V_2| \sin(\theta_2 - \delta_2) \cdot \left\{ \frac{E_{q_2}' - V_2 \cos(\theta_2 - \delta_2)}{X_{d_2}'} \right\} + |V_2| \cos(\theta_2 - \delta_2) \cdot \left\{ \frac{E_{d_2}' - V_2 \sin(\theta_2 - \delta_2)}{-X_{q_2}'} \right\} - P_{D_2} - \left[G_{22} |V_2|^2 + |Y_{25} V_5 V_2| \cos(\gamma_{25} + \delta_5 - \delta_2) \right] = 0 \qquad g_1$$

$$Q_{G_2} - Q_{D_2} - Q_2 = 0$$

or $$|V_2| \cos(\theta_2 - \delta_2) \left\{ \frac{E_{q_2}' - V_2 \cos(\theta_2 - \delta_2)}{X_{d_2}'} \right\} - |V_2| \sin(\theta_2 - \delta_2) \left\{ \frac{E_{d_2}' - V_2 \sin(\theta_2 - \delta_2)}{-X_{q_2}'} \right\} - Q_{D_2} - \left[-B_{22} |V_2|^2 - |Y_{25} V_5 V_2| \sin(\gamma_{25} + \delta_5 - \delta_2) \right] = 0 \qquad g_2$$

~~Similarly, for buses~~

$$
\begin{aligned}
& |V_3| \sin(\theta_3 - \delta_3) \cdot \left\{ \frac{E'_{q3} - V_3 \cos(\theta_3 - \delta_3)}{x_{d3}} \right\} \\
& + |V_3| \cos(\theta_3 - \delta_3) \cdot \left\{ \frac{\cancel{E_q}\ E'_{d3} - V_3 \sin(\theta_3 - \delta_3)}{-x'_{q3}} \right\} \\
& - P_{D_3} \\
& - \left[G_{33} |V_3|^2 + |Y_{3,11} V_{11} V_3| \cos(\gamma_{3,11} + \delta_{11} - \delta_3) \right] \\
& = 0
\end{aligned}
\tag{g3}
$$

$$
\begin{aligned}
& |V_3| \cos(\theta_3 - \delta_3) \cdot \left\{ \frac{E'_{q3} - V_3 \cos(\theta_3 - \delta_3)}{x'_{d3}} \right\} \\
& - |V_3| \sin(\theta_3 - \delta_3) \cdot \left\{ \frac{E'_{d3} - V_3 \sin(\theta_3 - \delta_3)}{-x'_{q3}} \right\} \\
& - Q_{D_3} \\
& - \left[- B_{33} |V_3|^2 - |Y_{3,11} V_{11} V_3| \sin(\gamma_{3,11} + \delta_{11} - \delta_3) \right] \\
& = 0
\end{aligned}
\tag{g4}
$$

$$|V_4| \sin(\theta_4 - \delta_4) \cdot \left\{ \frac{E'_{q4} - V_4 \cos(\theta_4 - \delta_4)}{x'_{d4}} \right\} + |V_4| \cos(\theta_4 - \delta_4) \cdot \left\{ \frac{E'_{d4} - V_4 \sin(\theta_4 - \delta_4)}{-x'_{q4}} \right\} - P_{D4} - \left[G_{44} |V_4|^2 - |Y_{4,10} V_{10} V_4| \cos(\gamma_{4,10} + \delta_{10} - \delta_4) \right] = 0 \qquad (g5)$$

$$|V_4| \cos(\theta_4 - \delta_4) \cdot \left\{ \frac{E'_{q4} - V_4 \cos(\theta_4 - \delta_4)}{x'_{d4}} \right\} - |V_4| \sin(\theta_4 - \delta_4) \cdot \left\{ \frac{E'_{d4} - V_4 \sin(\theta_4 - \delta_4)}{-x'_{q4}} \right\} - Q_{D4} - \left[-B_{44} |V_4|^2 - |Y_{4,10} V_{10} V_4| \sin(\gamma_{4,10} + \delta_{10} - \delta_4) \right] = 0 \qquad (g6)$$

$\boxed{1.5}$

$$P_{G5}^{\,0} - P_{D5} - P_5 = 0$$

or

$$\boxed{-P_{D5} - \left[G_{55}|V_5|^2 + |Y_{51} V_1 V_5| \cos(\gamma_{51} + \delta_1 - \delta_5) + |Y_{56} V_6 V_5| \cos(\gamma_{56} + \delta_6 - \delta_5) \right] = 0}$$

(g7)

$$Q_{G5} - Q_{D5} - Q_5 = 0$$

or

$$\boxed{-Q_{D5} - \left[-B_{55}|V_5|^2 - |Y_{51} V_1 V_5| \sin(\gamma_{51} + \delta_1 - \delta_5) - |Y_{56} V_6 V_5| \sin(\gamma_{56} + \delta_6 - \delta_5) \right] = 0}$$

(g8)

$$\boxed{\begin{aligned} -P_{D6} - \Big[G_{66}|V_6|^2 &+ |Y_{62} V_2 V_6| \cos(\gamma_{62} + \delta_2 - \delta_6) \\ &+ |Y_{65} V_5 V_6| \cos(\gamma_{65} + \delta_5 - \delta_6) \\ &+ |Y_{67} V_7 V_6| \cos(\gamma_{67} + \delta_7 - \delta_6) \Big] = 0 \end{aligned}}$$

(g9)

$$\boxed{\begin{aligned} -Q_{D6} - \Big[-B_{66}|V_6|^2 &- |Y_{62} V_2 V_6| \sin(\gamma_{62} + \delta_2 - \delta_6) \\ &- |Y_{65} V_5 V_6| \sin(\gamma_{65} + \delta_5 - \delta_6) \\ &- |Y_{67} V_7 V_6| \sin(\gamma_{67} + \delta_7 - \delta_6) \Big] = 0 \end{aligned}}$$

(g10)

1-6

$$-P_{D_7} - \left[G_{77}|V_7|^2 + |Y_{76}V_6V_7|\cos(\gamma_{76}+\delta_6-\delta_7) + |Y_{78}V_8V_7|\cos(\gamma_{78}+\delta_8-\delta_7) \right] = 0$$

g11

$$-Q_{D_7} - \left[-B_{77}|V_7|^2 - |Y_{76}V_6V_7|\sin(\gamma_{76}+\delta_6-\delta_7) - |Y_{78}V_8V_7|\sin(\gamma_{78}+\delta_8-\delta_7) \right] = 0$$

g12

$$-P_{D_8} - \left[G_{88}|V_8|^2 + |Y_{87}V_7V_8|\cos(\gamma_{87}+\delta_7-\delta_8) + |Y_{89}V_9V_8|\cos(\gamma_{89}+\delta_9-\delta_8) \right] = 0$$

g13

$$-Q_{D_8} - \left[-B_{88}|V_8|^2 - |Y_{87}V_7V_8|\sin(\gamma_{87}+\delta_7-\delta_8) - |Y_{89}V_9V_8|\sin(\gamma_{89}+\delta_9-\delta_8) \right] = 0$$

g14

$$-P_{D_9} - \left[G_{99}|V_9|^2 + |Y_{98}V_8V_9|\cos(\gamma_{98}+\delta_8-\delta_9) + |Y_{9,10}V_{10}V_9|\cos(\gamma_{9,10}+\delta_{10}-\delta_9) \right] = 0$$

g15

$$-Q_{D_9} - \left[-B_{99}|V_9|^2 - |Y_{98}V_8V_9|\sin(\gamma_{98}+\delta_8-\delta_9) - |Y_{9,10}V_{10}V_9|\sin(\gamma_{9,10}+\delta_{10}-\delta_9) \right] = 0$$

g16

$$-P_{D_{10}} - \Big[\cancel{G_{10,10}}\; G_{10,10}|V_{10}|^2 + |Y_{10,4}V_4V_{10}|\cos(\gamma_{10,4}+\delta_4-\delta_{10}) + |Y_{10,9}V_9V_{10}|\cos(\gamma_{10,9}+\delta_9-\delta_{10}) + |Y_{10,11}V_{11}V_{10}|\cos(\gamma_{10,11}+\delta_{11}-\delta_{10})\Big] = 0 \qquad g17$$

$$-Q_{D_{10}} - \Big[-B_{10,10}|V_{10}|^2 - |Y_{10,4}V_4V_{10}|\sin(\gamma_{10,4}+\delta_4-\delta_{10}) - |Y_{10,9}V_9V_{10}|\sin(\gamma_{10,9}+\delta_9-\delta_{10}) - |Y_{10,11}V_{11}V_{10}|\sin(\delta_{10,11}+\delta_{11}-\delta_{10})\Big] = 0 \qquad g18$$

$$-P_{D_{11}} - \Big[G_{11,11}|V_{11}|^2 + |Y_{11,3}V_3V_{11}|\cos(\gamma_{11,3}+\delta_3-\delta_{11}) + |Y_{11,10}\cdot V_{10}V_{11}|\cos(\gamma_{11,10}+\delta_{10}-\delta_{11})\Big] = 0 \qquad g19$$

$$-Q_{D_{11}} - \Big[-B_{11,11}|V_{11}|^2 - |Y_{11,3}V_3V_{11}|\sin(\gamma_{11,3}+\delta_3-\delta_{11}) - |Y_{11,10}V_{10}V_{11}|\sin(\gamma_{11,10}+\delta_{10}-\delta_{11})\Big] = 0 \qquad g20$$

$\forall\ i = 2, 3$ and 4

$$\dot{\theta}_i = (\omega_i - 1)\,\omega_s$$

$$\dot{\omega}_i = \frac{1}{2H_i}\left[P_{m_i} - \left[V_i \sin(\theta_i - \delta_i) \cdot \left\{ \frac{E'_{q_i} - V_i\cos(\theta_i - \delta_i)}{X'_{d_i}} \right\} + V_i\cos(\theta_i - \delta_i) \cdot \left\{ \frac{E'_{d_i} - V_i \sin(\theta_i - \delta_i)}{-X'_{q_i}} \right\} \right] - K_{D_i}(\omega_i - 1) \right]$$

$$\dot{E}'_{q_i} = \frac{1}{T'_{do_i}}\left[-E'_{q_i} - (X_{d_i} - X'_{d_i})\left\{ \frac{E'_{q_i} - V_i\cos(\theta_i - \delta_i)}{X'_{d_i}} \right\} + E_{fd_i} \right]$$

$$\dot{E}'_{d_i} = \frac{1}{T'_{qo_i}}\left[-E'_{d_i} - (X_{q_i} - X'_{q_i})\left\{ \frac{E'_{d_i} - V_i\sin(\theta_i - \delta_i)}{-X'_{q_i}} \right\} \right]$$

Again, $i = 2, 3, 4$.

f_1 to f_{12}

where

$f_1 + f_4$ are for $i = 2$

$f_5 + f_8$ are for $i = 3$

f_9 to f_{12} are for $i = 4$

Thus,

$$\left[g(x, y) = 0 \right]_{20\times1} \equiv \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \\ g_5 \\ g_6 \\ g_7 \\ \vdots \\ g_{20} \end{bmatrix}_{20\times1}$$

gen. buss

represent the Type 1 model

Load buss

for Kundur's 11 bus system

$$\dot{x} = f(x, y)$$

$$\dot{x} \equiv \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_{12} \end{bmatrix}_{12\times1}$$