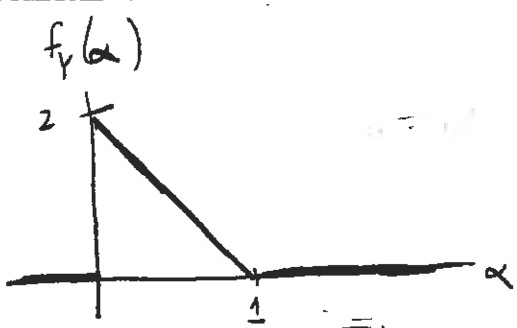
EE 507, Homework $2\pi + 1$

due. November 9

Problem 1

A random variable Y has the PDF shown below:



a. Please find the ~~maximum~~ probability estimate for Y.
b. Please find the MMSE estimate for Y.
c. Please find the minimum mean absolute error estimate for Y.

Problem 2

A random variable X is uniform on $[0, 1]$. Given $X = x$, the random variable Y is exponential with mean x^2.

a. Please find the MMSE estimate for Y given $X = x$. What is the expected error of the estimate, given $X = x$? What is the (overall) average estimation error?
b. Please find the MMSE estimate for Y^2 given $X = x$. Conceptually, why isn't this estimate just the square of the MMSE estimate of Y given $X = x$?
c. Please find the MMSE estimate of X given $Y = y$. What is the (overall) average estimation error?
d. What is the LMMSE estimate for Y given $X = x$?

EE 507, Homework 9
due December 4, 2015

Problem 1

a. What is a random process?

b. Why are random processes of interest to scientists and engineers?

c. Find a signal in the world around you that can be viewed as a random process. Actually measure a sample trajectory of this process.

d. When we defined random processes, we associated entire signals with outcomes of an experiment rather than using a different experiments to define the process values at particular times. What is the advantage of this approach?

Problem 2

An uncertain experiment has two equally likely outcomes, A and B. A random process $X(\omega, t)$, $t \in R$, is defined for this experiment as follows: $X(A, t) = cos(t)$ and $X(B, t) = sin(t)$.

a. Please find the first-order PDF for $X(t)$.

b. Please find the second-order PDF for $X(t)$.

c. Please find the nth-order PDF for $X(t)$. (Assume WLOG that the times in the joint PDF are increasing, i.e. $t_1 < t_2 < \ldots < t_n$).

d. Please find $E[X(t)]$ and $R_{XX}(t_1, t_2)$.

Problem 3

Consider a discrete-time random process $X[k]$, $k = 0, 1, 2, \ldots$, where each $X[k]$ is an independent random variable that equals 0 with probability 0.6 amd equals 1 with probability 0.4.

a. Please find the nth-order PMF for $X[k]$. (Assume without loss of generality that that the times in the joint PMF are increasing.)

b. Please find $E[X(t)]$, $R_{XX}(t,\tau)$, and $C_{XX}(t,\tau)$.

Now consider the process $Y[k] = \sum_{i=0}^{k-1} X[k]$, $k = 1, 2, 3, \ldots$.

c. Please find the first-order PMF for $Y[k]$.
d. Please find the nth-order PMF for $Y[k]$. (Assume without loss of generality that the times in the joint PMF are increasing.)
e. Please find $E[Y(t)]$, $R_{YY}(t,\tau)$, and $C_{YY}(t,\tau)$.
f. Please find $R_{XY}(t,\tau)$.

Problem 4

Let T be an exponential random variable with mean 1. We define a random process $X(t)$, $t \in R^+$, as follows: $X(t) = 1$ for $t \leq T$ and $X(t) = 0$ for $t \geq T$. Please find the second-order joint PDF of $X(t)$.