

Counterexample to the conjecture:

For $f : \mathbb{R} \rightarrow \mathbb{R}$ $f''(x) > 0 \quad \forall x \in \mathbb{R}$, then $f(x)$ must be bounded below.

[Counterexample PDF](#)

Q: How did Tom come up with a reflection function for $f(x) = -\ln(1+x)$ in order to cover for the full domain $D = \mathbb{R}$ for negative x ?

A: By 'reflecting' the function across the line $y = x$, which meant using the inverse function of $f(x) = -\ln(1+x)$, which would be $f(x) = e^{-x} - 1$.

Q: How is the reflection across $y = x$ of $f(x)$ equal to the inverse function?

ChatGPT: The idea behind the reflection across the line ($y = x$) is that this line acts as a "mirror". Every point on one side of this "mirror" will have a corresponding point on the other side such that the line ($y = x$) is the perpendicular bisector of the line segment connecting the two points.

To show this:

1. Perpendicular Bisector:

- Consider two points $A(a, b)$ and $B(b, a)$.
- The midpoint M of segment (AB) is given by $(M(\frac{a+b}{2}, \frac{a+b}{2}))$, which lies on the line ($y = x$).
- The slope of segment (AB) is $(m_{AB} = \frac{a-b}{b-a} = -1)$. (Notice we're just computing the slope between two arbitrary points, one of which is our original point and the other is its reflected point.)
- The slope of the line ($y = x$) is 1.
- Since the product of the slopes of two perpendicular lines is -1, this proves that (AB) is perpendicular to the line ($y = x$).

2. Equal Distance:

- The distance (d) between $(A(a, b))$ and the line ($y = x$) can be found using the point-to-line distance formula.
- The equation of the line ($y = x$) can be written as $(x - y = 0)$.
- The distance (d) is:
[$d = \frac{|a-b|}{\sqrt{2}}$]
- Similarly, the distance from $(B(b, a))$ to the line ($y = x$) is:
[$d = \frac{|b-a|}{\sqrt{2}}$]

- These distances are equal, meaning (A) and (B) are equidistant from the line ($y = x$).

Combine these two pieces of information: segment (AB) is perpendicular to the line ($y = x$), and both (A) and (B) are equidistant from the line ($y = x$). This means the line ($y = x$) bisects segment (AB) perpendicularly, and thus point (B) is the reflection of point (A) across the line ($y = x$).