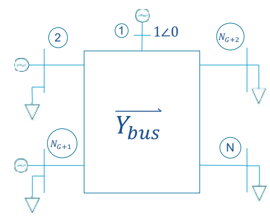
Power-flow Methods

Generator VAR Limits

1

Power-Flow Formulation

PV bus: $\delta_i =?, Q_i =?, P_i = \surd, V_i = \surd$
PQ bus: $\delta_i =?, V_i =?, P_i = \surd, Q_i = \surd$



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Newton-Raphson Formulation

$$x = \begin{pmatrix} \delta_2 \\ \vdots \\ \delta_N \\ V_{N_G+2} \\ \vdots \\ V_N \end{pmatrix}$$

all angles

PQ voltages

PV voltages known
V_i is specified for
i=1,…,N_G+1

$$f(x) = h(x) - b$$

$$= \begin{pmatrix} p_2(x) \\ \vdots \\ p_N(x) \\ q_{N_G+2}(x) \\ \vdots \\ q_N(x) \end{pmatrix} - \begin{pmatrix} P_2 \\ \vdots \\ P_N \\ Q_{N_G+2} \\ \vdots \\ Q_N \end{pmatrix} = 0$$

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Algorithm

$$x^0$$
$$\Downarrow k = 0$$
$k = k + 1$
$$Evaluate\ \ f(x^k) = h(x^k) - b$$
$$\Downarrow$$
Largest power mismatch $\quad |f(x^k)| < \varepsilon \overset{Yes}{\Longrightarrow} stop$
$$\Downarrow No$$
$$Evaluate\ J^k = \frac{\partial f}{\partial x}\Big|_{x^k} = \frac{\partial h}{\partial x}\Big|_{x^k}$$
$$\Downarrow$$
$$\Delta x^k = (J^k)^{-1}\left(-f(x^k)\right) \Longrightarrow x^{k+1} = x^k + \Delta x^k$$

x^0 can be flat initial conditions ($V_i{}^0 = 1, \delta_i{}^0 = 0$) or the DC power-flow solution or previous known solution

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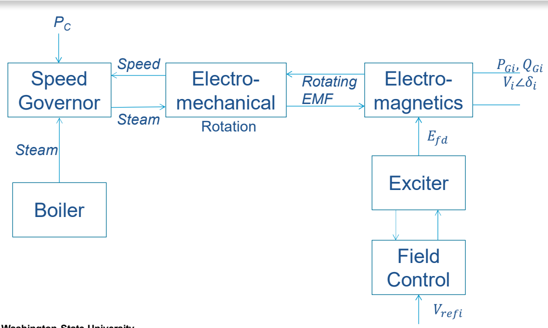
Generator Bus (PV Bus)



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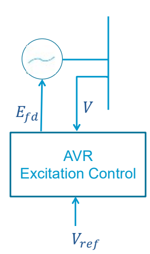
Reactive Power Limits



- Exciter Control
  - V < V_{ref} ⇒ Increase E_{fd}
  - V > V_{ref} ⇒ Decrease E_{fd}
- E_{fd} has limits
  - Too high => Damage to field coils
  - Too low => Under-excitation

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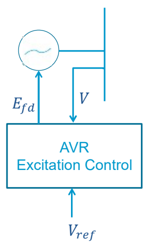
Reactive Power Limits



- Synchronous generator

$$E_{fdmin} < E_{fd} < E_{fdmax}$$
$$\Downarrow$$
$$Q_{Gmin} < Q_G < Q_{Gmax}$$
$$\Downarrow$$
$$V = V_{ref}$$

- If $Q_G > Q_{Gmax}$
  - Set $Q_G = Q_{Gmax}$
  - V becomes an unknown variable
- If $Q_G < Q_{Gmin}$
  - Set $Q_G = Q_{Gmin}$
  - V becomes an unknown variable

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Reactive Power Limits

- When limits are exceeded

PV $\Rightarrow$ PQ

P_{Gi} V_i $\quad$ P_{Gi} Q_{Gi} $\lessgtr$ Q_{Gimax} / Q_{Gimin}

- $x = \begin{pmatrix} \delta_2 \\ \vdots \\ \delta_N \\ V_2 \\ V_j \\ V_{N_g+2} \\ \vdots \\ V_N \end{pmatrix}$

Generators with Q limits exceeded

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NR Algorithm with Q Limits

$$f(x) = h(x) - b = 0$$

$$x = \begin{bmatrix} \delta \\ V \end{bmatrix} \qquad h(x) = \begin{bmatrix} p(x) \\ q(x) \end{bmatrix} \qquad b = \begin{bmatrix} P \\ Q \end{bmatrix}$$

Check if any Qi outside its Q limits and convert those buses to PQ buses. Adjust x, h(x) and b.

$$Evaluate\ \ f(x^k) = h(x^k) - b$$
$$\Downarrow$$
$$|f(x^k)| < \varepsilon \overset{Yes}{\Longrightarrow} stop$$
$$k = k + 1 \qquad \Downarrow No$$
$$Evaluate\ J^k = \frac{\partial f}{\partial x}|_{x^k} = \frac{\partial h}{\partial x}|_{x^k}$$
$$\Downarrow$$
$$\Delta x^k = -(J^k)^{-1} f(x^k) \Longrightarrow x^{k+1} = x^k + \Delta x^k$$

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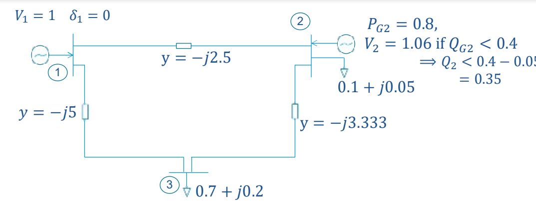
Example



$$\overrightarrow{Y_{Bus}} = \begin{bmatrix} -j7.5 & j2.5 & j5 \\ j2.5 & -j5.833 & j3.333 \\ j5 & j3.333 & -j8.333 \end{bmatrix}$$

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DC Solution

$$\begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix} = B^{-1} \begin{bmatrix} P_2 \\ P_3 \end{bmatrix}$$
$$= \begin{bmatrix} 5.833 & -3.333 \\ -3.333 & 8.333 \end{bmatrix}^{-1} \begin{bmatrix} 0.7 \\ -0.7 \end{bmatrix}$$
$$= \begin{bmatrix} 0.0933 \\ -0.0467 \end{bmatrix} rad$$

$$\overrightarrow{V_1} = 1\angle 0,\ \overrightarrow{V_2}^0 = 1.06\angle 0.0933,\ \overrightarrow{V_3}^0 = 1\angle -0.0467$$

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Newton-Raphson:

$$x = \begin{bmatrix} \delta_2 \\ \delta_3 \\ V_3 \end{bmatrix},\ b = \begin{bmatrix} P_2 \\ P_3 \\ Q_3 \end{bmatrix},\ p(x) = \begin{bmatrix} p_2(x) \\ p_3(x) \\ q_3(x) \end{bmatrix}$$

Calculate

$$q_2(x^0) = 2.5V_2V_1 \sin(\delta_2 - \delta_1 - 90^o)$$
$$+5.8333{V_2}^2 \sin(\delta_2 - \delta_2 + 90^o)$$
$$+3.3333V_2V_3 \sin(\delta_2 - \delta_3 - 90^o)$$
$$= 2.5(1.06)(1)\sin(0.0933 - 0 - 1.57)$$
$$+5.8333(1.06)^2 \sin(1.57)$$
$$+3.3333(1.06)(1)\sin(0.0933 + 0.0467 - 1.57)$$
$$= 0.4171 > 0.35 \Longrightarrow Set\ Q_2 = 0.35$$

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PV Bus changes to PQ Bus

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$\Longrightarrow$ Switch Bus 2 to be a PQ Bus.

$$x = \begin{bmatrix} \delta_2 \\ \delta_3 \\ V_2 \\ V_3 \end{bmatrix}, b = \begin{bmatrix} P_2 \\ P_3 \\ Q_2 \\ Q_3 \end{bmatrix}, \mathrm{h}(x) = \begin{bmatrix} p_2(x) \\ p_3(x) \\ q_2(x) \\ q_3(x) \end{bmatrix}$$

$$x^0 = \begin{bmatrix} 0.0933 \\ -0.0467 \\ 1.06 \\ 1 \end{bmatrix}, b = \begin{bmatrix} 0.7 \\ -0.7 \\ 0.35 \\ -0.2 \end{bmatrix}, \mathrm{h}(x) = \begin{bmatrix} p_2(x) \\ p_3(x) \\ q_2(x) \\ q_3(x) \end{bmatrix}$$

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Iteration 1

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$$\mathrm{h}(x^0) = \begin{bmatrix} 0.74 \\ -0.7263 \\ 0.4171 \\ -0.16 \end{bmatrix}$$

$$b - h(x^0) = \begin{bmatrix} -0.04 \\ 0.0263 \\ -0.0671 \\ -0.06 \end{bmatrix}$$

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NR with Q limits

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$$J^0 = J(x^0)$$
$$= \begin{bmatrix} 6.137 & -3.499 & 0.698 & 0.493 \\ -3.499 & 8.493 & -0.465 & -0.726 \\ 0.740 & -0.493 & 6.577 & -3.499 \\ 0.493 & -0.726 & -3.499 & 8.173 \end{bmatrix}$$

$$\Longrightarrow \Delta x^0 = (J^0)^{-1}\left(b - h(x^0)\right) = \begin{bmatrix} -0.004 \\ -0.0004 \\ -0.0156 \\ -0.0110 \end{bmatrix}$$

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End of Iteration 1

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$$\Longrightarrow x^1 = x^0 + \Delta x^0 = \begin{bmatrix} 0.0893 \\ -0.047 \\ 1.0444 \\ 0.9890 \end{bmatrix}$$

$$\Downarrow k = 1$$

Check $q_2(x^1)$ now

$q_2(x^1) = 0.3510 > 0.35$ keep as PQ bus

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Start of Iteration 2

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$$b - h(x^1) = \begin{bmatrix} -0.0006 \\ 0.0004 \\ -0.00096 \\ -0.00051 \end{bmatrix}$$

$|b - h(x^1)| = 0.00096 < 0.001 \Longrightarrow$ Stop

NR Converged in one iteration!

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NR Algorithm with Q Limits

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$k = k + 1$

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