EE 491 HW # 7 Solution

1) Solution:

$f_1 = 4P_{G1} + 0.04P_{G1}^2 \qquad 0 < P_{G1} < 401$

$f_2 = 5P_{G2} + 0.02P_{G2}^2 \qquad 0 < P_{G2} < 401$

$IC_1 = 4 + 0.08P_{G1} = \lambda \qquad P_{G1} = \frac{\lambda - 4}{0.08} = 12.5(\lambda - 4)$

$IC_2 = 5 + 0.04P_{G2} = \lambda \qquad P_{G2} = \frac{\lambda - 5}{0.04} = 25(\lambda - 5)$

$P_{G1} + P_{G2} = 37.5\lambda - 175$

1° $P_D = 200MW$

$P_{G1} + P_{G2} = 200$

$37.5\lambda - 175 = 200, \quad \lambda = 10$

$P_{G1} = 75MW \quad P_{G2} = 125MW$

2° $P_D = 400MW$

$P_{G1} + P_{G2} = 400$

$37.5\lambda - 175 = 400, \quad \lambda = 15.333$

$P_{G1} = 141.667\,MW \quad P_{G2} = 258.333\,MW$

3° $P_D = 600MW$

$P_{G1} + P_{G2} = 600$

$37.5\lambda - 175 = 600, \quad \lambda = 20.667$

$P_{G1} = 208.333MW \quad P_{G2} = 391.667\,MW$

2) Solution:

$f_3 = 6P_{G3} + 0.03P_{G3}^2 \quad 0 < P_{G3} \leq 401$

$IC_3 = 6 + 0.06P_{G3} = \lambda \quad P_{G3} = 16.667(\lambda - 6)$

$P_{G1} + P_{G2} + P_{G3} = 54.167\lambda - 275$

1° $P_D = 200MW$

$P_{G1} + P_{G2} + P_{G3} = 200$

$54.167\lambda - 275 = 200 \qquad \lambda = 8.769$

$P_{G1} = 59.613\,MW \quad P_{G2} = 74.225MW \quad P_{G3} = 46.162Mw$

$2^\circ\ P_D = 400\ MW$

$P_{G1} + P_{G2} + P_{G3} = 400\ MW$

$54.167\lambda - 275 = 400 \qquad \lambda = 12.461$

$P_{G1} = 105.763\ MW \qquad P_{G2} = 186.525\ MW \qquad P_{G3} = 107.712\ MW$

$3^\circ\ P_D = 600\ MW$

$P_{G1} + P_{G2} + P_{G3} = 600\ MW$

$54.167\lambda - 275 = 600 \qquad \lambda = 16.154$

$P_{G1} = 151.925\ MW \qquad P_{G2} = 278.85\ MW \qquad P_{G3} = 170.225\ MW$

$4^\circ\ P_D = 800\ MW$

$P_{G1} + P_{G2} + P_{G3} = 800\ MW$

$54.167\lambda - 275 = 800 \quad \lambda = 19.846$

$P_{G1} = 198.075 \quad P_{G2} = 371.15\ MW \quad P_{G3} = 230.775\ MW$

$5^\circ\ P_D = 1000\ MW$

$P_{G1} + P_{G2} + P_{G3} = 1000\ MW$

$54.167\lambda - 275 = 1000 \qquad \lambda = 23.538$

$P_{G2} = 463.458 > 401\ MW$, in this case, let $P_{G2} = 400\ MW$

$12.5(\lambda - 4) + 16.667(\lambda - 6) = 600 \qquad \lambda = 25.714$

$P_{G1} = 271.425\ MW \qquad P_{G3} = 328.575\ MW$

3) Solution:

$P_{losses} = 0.00025\,P_{G1}^2 + 0.0001\,P_{G2}^2 \qquad \varepsilon = 0.1\ MW$

$\partial P_{loss}/\partial P_{G1} = 0.0005\,P_{G1} \qquad \partial P_{losses}/\partial P_{G2} = 0.0002\,P_{G2}$

$\partial L/\partial P_{G1} = 4 + 0.08P_{G1} + \lambda(0.0005P_{G1} - 1) = (0.08 + 0.0005\lambda)P_{G1} + 4 - \lambda$

$\partial L/\partial P_{G2} = 5 + 0.04P_{G2} + \lambda(0.0002P_{G2} - 1) = (0.04 + 0.0002\lambda)P_{G2} + 5 - \lambda$

$$\Rightarrow \begin{bmatrix} 0.08 + 0.0005\lambda & 0 \\ 0 & 0.04 + 0.0002\lambda \end{bmatrix} \begin{bmatrix} P_{G1} \\ P_{G2} \end{bmatrix} = \begin{bmatrix} \lambda - 4 \\ \lambda - 5 \end{bmatrix}$$

1° $P_D = 200\,MW$

$\lambda^0 = 10 \quad P_{G_1}^0 = 75 \quad P_{G_2}^0 = 125$

let $\lambda^{-1} = 9, \quad P_{G_1}^{-1} = 59.1716 \quad P_{G_2}^{-1} = 95.6938$

$P_{losses}^0 = 2.9688\ MW$

$\Delta P^0 = 2.9688\,MW \quad > \varepsilon$ continue

$\Delta\lambda^0 = 0.0658$

$\lambda^1 = \Delta\lambda^0 + \lambda^0 = 10.0658$

$P_{G_1}^1 = 71.3345 \quad P_{G_2}^1 = 120.5759$

$P_{losses}^1 = 2.7260$

$\Delta P^1 = 10.8156 \quad > \varepsilon$ continue

$\Delta\lambda^1 = -0.0879$

$\lambda^2 = \lambda^1 + \Delta\lambda^1 = 9.9778$

$P_{G_1}^2 = 70.3366 \quad P_{G_2}^2 = 118.5324$

$P_{losses}^2 = 2.6418$

$\Delta P^2 = 13.7728 \quad > \varepsilon$ continue

$\Delta\lambda^2 = 0.3982$

$\lambda^3 = \lambda^2 + \Delta\lambda^2 = 10.3761$

$P_{G_1}^3 = 74.847 \quad P_{G_2}^3 = 127.7729$

$P_{losses}^3 = 3.0311$

$\Delta P^3 = 0.4132 \quad > \varepsilon$ continue

$\Delta\lambda^3 = 0.012$

$\lambda^4 = \lambda^3 + \Delta\lambda^3 = 10.388$

$P_{G_1}^4 = 74.9822 \quad P_{G_2}^4 = 128.05$

$P_{losses}^4 = 3.0453$

$\Delta P^4 = 0.013 \quad < \varepsilon$ converge

thus $\lambda = 10.388 \quad P_{G_1} = 74.9822\,MW \quad P_{G_2} = 128.05\,MW$

2. $P_D = 400MW$

$\lambda_0 = 15.333 \quad P_{G_1}^0 = 141.667 \quad P_{G_2}^0 = 258.333$

choose $\lambda^{-1} = 15$, then $P_{G_1}^{-1} = 125.7143 \quad P_{G_2}^{-1} = 232.5581$

$P_{losses}^0 = 11.691$

$\Delta P^0 = 11.691 \quad > \varepsilon$ continue

$\Delta \lambda^0 = 0.0933$

$\lambda^1 = \lambda^0 + \Delta\lambda^0 = 15.4263$

$P_{G_1}^1 = 130.2689 \quad P_{G_2}^1 = 241.9922$

$P_{losses}^1 = 10.0985$

$\Delta P^1 = 37.8374 \quad > \varepsilon$ continue

$\Delta\lambda^1 = -0.1273$

$\lambda^2 = \lambda^1 + \Delta\lambda^1 = 15.299$

$P_{G_1}^2 = 128.9115 \quad P_{G_2}^2 = 239.1798$

$P_{losses}^2 = 9.8152$

$\Delta P^2 = 41.7839 \quad > \varepsilon$ continue

$\Delta\lambda^2 = 1.2752$

$\lambda^3 = \lambda^2 + \Delta\lambda^2 = 16.5743$

$P_{G_1}^3 = 142.4248 \quad P_{G_2}^3 = 267.2126$

$P_{losses}^3 = 12.2115$

$\Delta P^3 = 2.5740 \quad > \varepsilon$ continue

$\Delta\lambda^3 = 0.079$

$\lambda^4 = \lambda^3 + \Delta\lambda^3 = 16.6533$

$P_{G_1}^4 = 143.2556 \quad P_{G_2}^4 = 268.9386$

$P_{losses}^4 = 12.3633$

$\Delta P^4 = 0.1691 \quad > \varepsilon$ continue

$\Delta\lambda^4 = 0.0052$

$\lambda^5 = \lambda^4 + \Delta\lambda^4 = 16.6585$

$P_{G_1}^5 = 143.3105 \quad P_{G_2}^5 = 269.0527$

$P_{losses}^5 = 12.3734$

$\Delta P^5 = 0.0101 \quad < \varepsilon$ converge

thus, $\lambda = 16.6585 \quad P_{G_1} = 143.3105 MW \quad P_{G_2} = 269.0527 MW$

3. $P_D = 600 MW$

choose $\lambda^{-1} = 20$, then $P_{G1}^{-1} = 177.7778$ $P_{G2}^{-1} = 340.9091$

$P_{losses}^{0} = 26.191$

$\Delta p^{0} = 26.191$ $> \varepsilon$ continue

$\Delta \lambda^{0} = 0.2148$

$\lambda^{1} = 20.8818$

$P_{G1}^{1} = 186.6615$ $P_{G2}^{1} = 359.5099$

$P_{losses}^{1} = 21.6354$

$\Delta p^{1} = 75.464$ $> \varepsilon$ continue

$\Delta \lambda^{1} = -0.3012$

$\lambda^{2} = 20.5806$

$P_{G1}^{2} = 183.6371$ $P_{G2}^{2} = 353.1735$

$P_{losses}^{2} = 20.9038$

$\Delta p^{2} = 20.9038$ $> \varepsilon$ continue

$\Delta \lambda^{2} = 2.7058$

$\lambda^{3} = 23.2864$

$P_{G1}^{3} = 210.4511$ $P_{G2}^{3} = 409.4833 > 401 MW$.

$P_{losses}^{3} = 27.8401$

$\Delta p^{3} = 7.9057$ $> \varepsilon$ continue

$\Delta \lambda^{3} = 0.2573$

$\lambda^{4} = 23.5437$

$P_{G1}^{4} = 212.9601$ $P_{G2}^{4} = 414.7678 > 401 MW$

$P_{losses}^{4} = 28.5412$

$\Delta p^{4} = 0.8133$ $> \varepsilon$ continue

$\Delta \lambda^{4} = 0.0269$

$\lambda^{5} = 23.5706$

$P_{G1}^{5} = 213.2216$ $P_{G2}^{5} = 415.3186 > 401 MW$

$P_{losses}^{5} = 28.6148$

$\Delta p^{5} = 0.0747$ $< \varepsilon$ converge

we could see that the final answer breaks the limit.
thus the optimal solution should be,

$P_{G_2} = 400\,MW$

$$P_{G_1} + P_{G_2} - 600 = 0.00025\,P_{G_1}^2 + 0.0001\,P_{G_2}^2$$

$$0.00025\,P_{G_1}^2 - P_{G_1} + 216 = 0$$

$\Rightarrow P_{G_1} = 229.1\,MW$