WASHINGTON STATE UNIVERSITY

Power-Flow Analysis

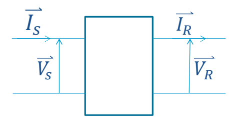
Admittance Matrix Representation

1

WASHINGTON STATE UNIVERSITY

Radial Network Analysis

Individual transmission line models



$$\begin{bmatrix} \vec{V_S} \\ \vec{I_S} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \vec{V_R} \\ \vec{I_R} \end{bmatrix}$$

 2

2

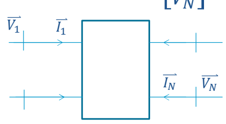
WASHINGTON STATE UNIVERSITY

Meshed Network Analysis

Admittance representation easier.
Ybus formulation.
Main difference: Current injection at all buses.

$$\overrightarrow{I_{bus}} = \begin{bmatrix} \vec{I_1} \\ \vdots \\ \vec{I_N} \end{bmatrix} \quad \text{injection} \quad \overrightarrow{V_{bus}} = \begin{bmatrix} \vec{V_1} \\ \vdots \\ \vec{V_N} \end{bmatrix}$$

$$\overrightarrow{I_{bus}} = \overrightarrow{Y_{bus}}\,\overrightarrow{V_{bus}}$$

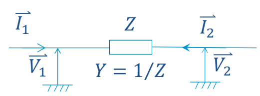


 3

3

WASHINGTON STATE UNIVERSITY

Admittance Analysis



$$\vec{I_1} = \frac{\vec{V_1} - \vec{V_2}}{Z} = \frac{\vec{V_1}}{Z} - \frac{\vec{V_2}}{Z}$$

$$\vec{I_2} = \frac{\vec{V_2} - \vec{V_1}}{Z} = \frac{\vec{V_2}}{Z} - \frac{\vec{V_1}}{Z}$$

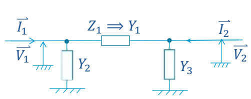
$$\begin{bmatrix} \vec{I_1} \\ \vec{I_2} \end{bmatrix} = \begin{bmatrix} Y & -Y \\ -Y & Y \end{bmatrix} \begin{bmatrix} \vec{V_1} \\ \vec{V_2} \end{bmatrix}$$

 4

4

WASHINGTON STATE UNIVERSITY

Network Analysis



$$\vec{I_1} = (\vec{V_1} - \vec{V_2})Y_1 + \vec{V_1}Y_2$$
$$\vec{I_2} = (\vec{V_2} - \vec{V_1})Y_1 + \vec{V_2}Y_3$$

$$\vec{I_1} = \vec{V_1}(Y_1 + Y_2) + \vec{V_2}(-Y_1)$$
$$\vec{I_2} = \vec{V_1}(-Y_1) + \vec{V_2}(Y_1 + Y_3)$$

$$\begin{bmatrix} \vec{I_1} \\ \vec{I_2} \end{bmatrix} = \begin{bmatrix} Y_1 + Y_2 & -Y_1 \\ -Y_1 & Y_1 + Y_3 \end{bmatrix} \begin{bmatrix} \vec{V_1} \\ \vec{V_2} \end{bmatrix}$$

 5

5

WASHINGTON STATE UNIVERSITY

Network Analysis

$$\overrightarrow{I_{bus}} = \overrightarrow{Y_{bus}}\,\overrightarrow{V_{bus}}$$

$$\overrightarrow{Y_{ij}} = Off-diagonal\ entry$$
$$= -Net\,admittance\ between\ Buses\ i\ and\ j$$

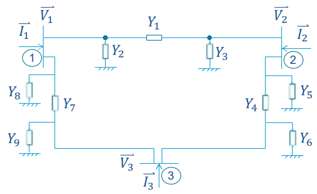
$$\overrightarrow{Y_{ii}} = Diagonal\ entry$$
$$= Sum\ of\ all\ admittances\ connected\ to\ Bus\ i$$

$$\overrightarrow{Y_{bus}} = \begin{bmatrix} Y_1 + Y_2 & -Y_1 \\ -Y_1 & Y_1 + Y_3 \end{bmatrix}$$

 6

6

Example 1



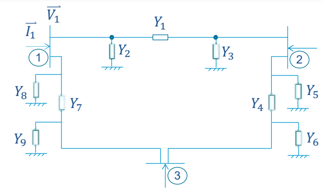
Step 1: Convert all impedances to Admittances.

Step 2: use the rule to write out the Y_{bus} matrix.

 7

7

Example 1



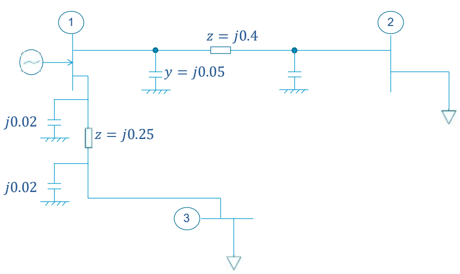
$$\begin{bmatrix}\vec{I_1}\\ \vec{I_2}\\ \vec{I_3}\end{bmatrix} = \begin{bmatrix}\vec{Y_{bus}}\end{bmatrix}\begin{bmatrix}\vec{V_1}\\ \vec{V_2}\\ \vec{V_3}\end{bmatrix}$$

$$\vec{Y_{bus}} = \begin{bmatrix} Y_1+Y_7+Y_2+Y_8 & -Y_1 & -Y_7 \\ -Y_1 & Y_1+Y_4+Y_3+Y_5 & -Y_4 \\ -Y_7 & -Y_4 & Y_4+Y_7+Y_6+Y_9 \end{bmatrix}$$

 8

8

Example 2

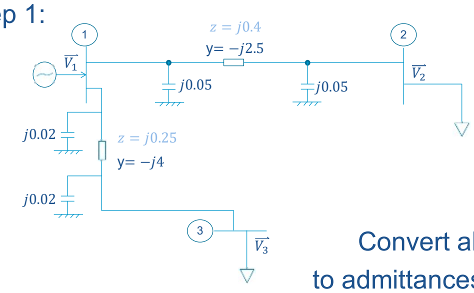


 9

9

Example 2

Step 1:

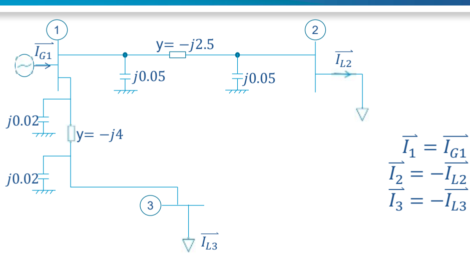


Convert all to admittances

 10

10

Example 2



$$\vec{I_1} = \vec{I_{G1}}$$
$$\vec{I_2} = -\vec{I_{L2}}$$
$$\vec{I_3} = -\vec{I_{L3}}$$

$$\vec{Y_{bus}} = \begin{bmatrix} -j2.5 - j4 + j0.05 + j0.02 & j2.5 & j4 \\ j2.5 & -j2.5 + j0.05 & 0 \\ j4 & 0 & -j4 + j0.02 \end{bmatrix}$$

 11

11