Power System Stability

Power System Dynamic Models

Small-signal Stability

Transient Stability

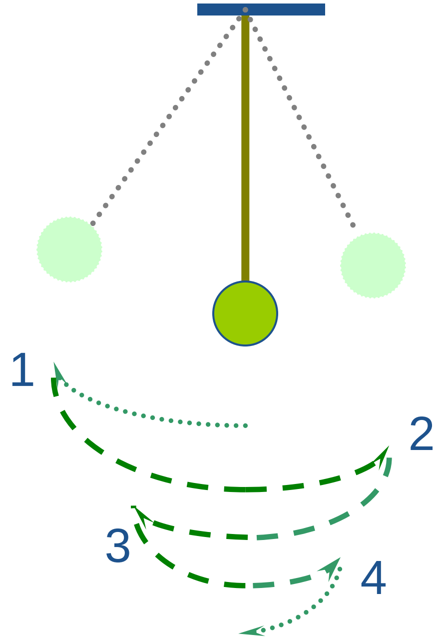
Stability Concepts

- Small-signal Stability
  - Ability to damp out small perturbations
  - Oscillations?
- Transient stability
  - Recovery from large disturbances
  - Islanding? Voltage collapse?

Stability concepts

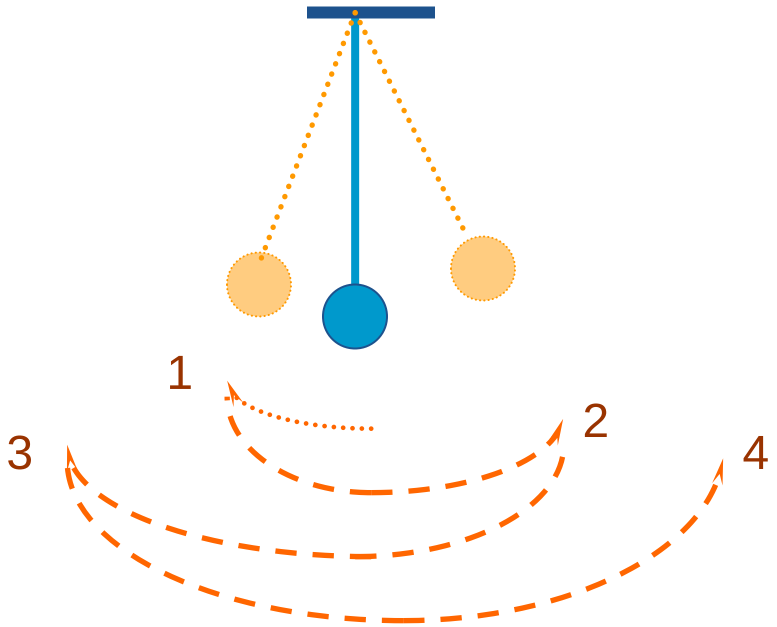
- Small signal stability
  - Load fluctuations
  - Generation changes
  - Oscillatory modes
    - well damped?

Small-signal stability



Positive damping

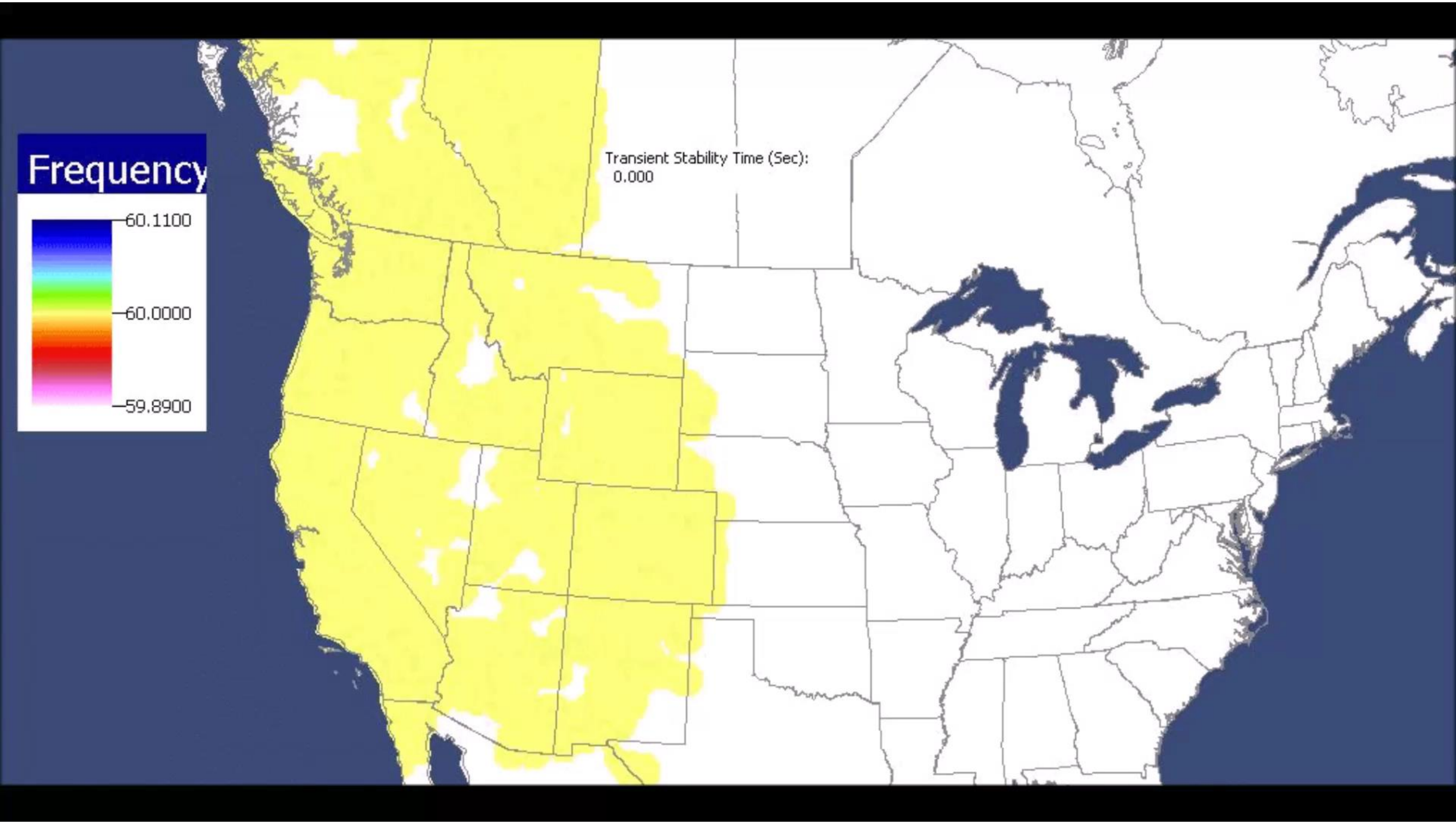
Oscillations damp out



Negative damping

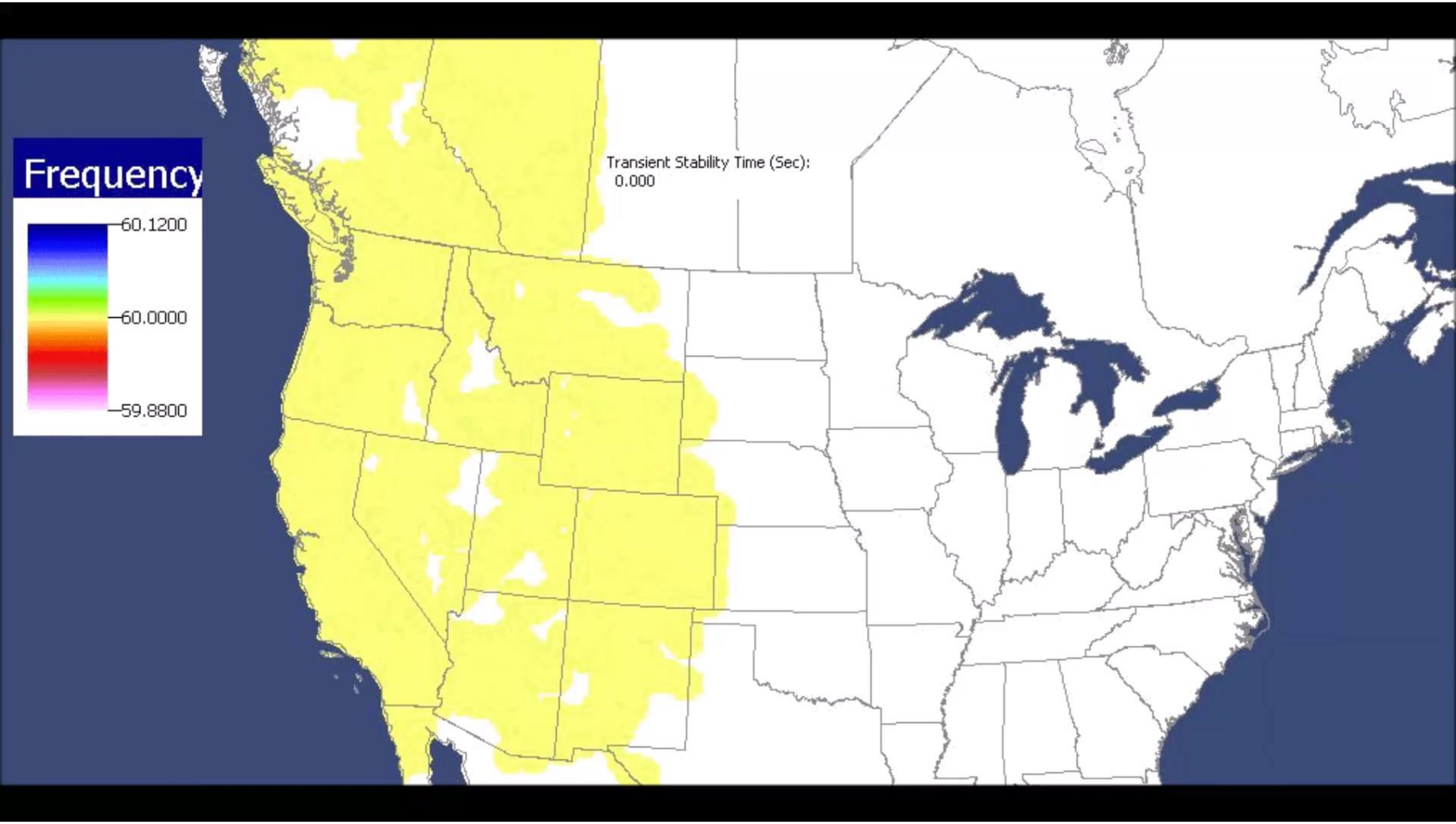
Growing oscillations

Well-damped oscillations



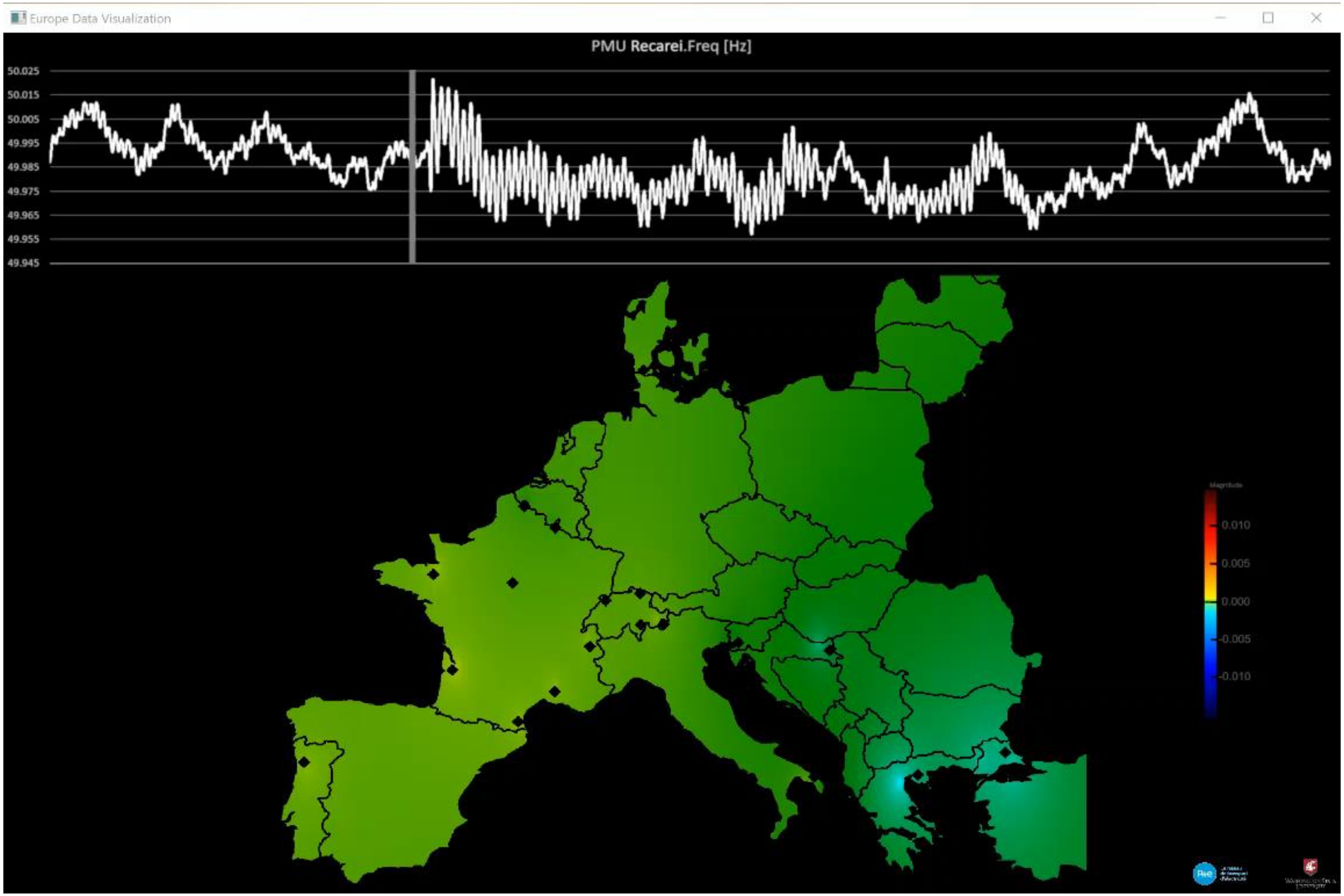
Powerworld Animation

Poorly Damped Oscillations

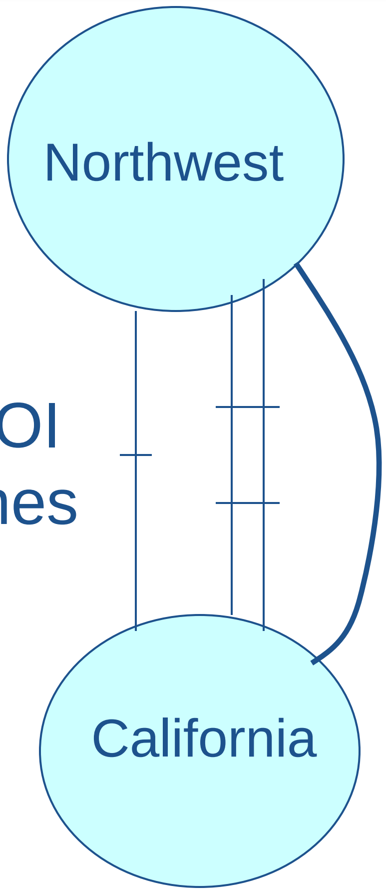


Powerworld Animation

Oct 29 2018 European Event



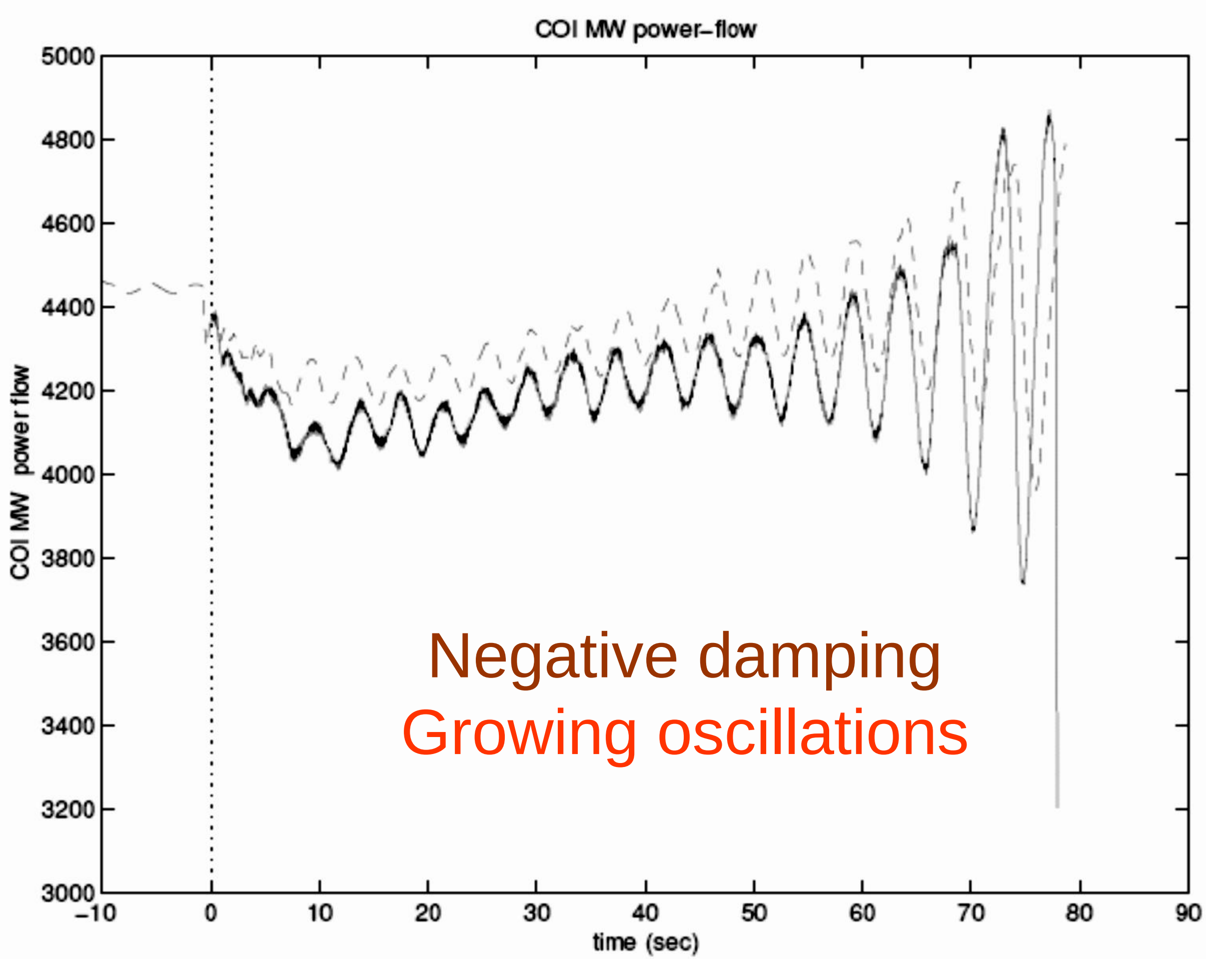
Small-signal instability in WECC



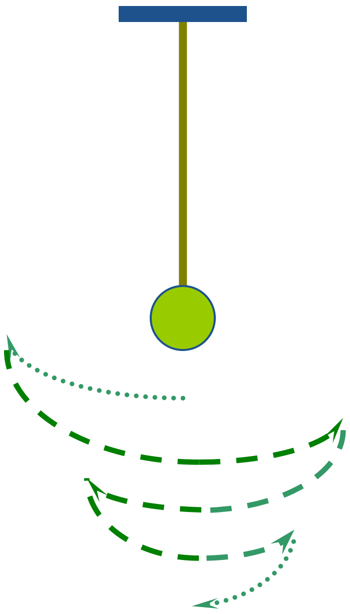
COI lines

Unstable 0.25 Hz Inter-area mode

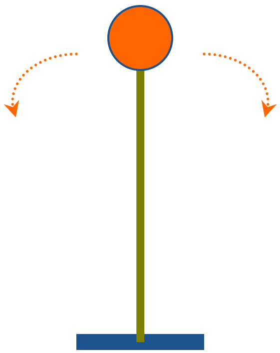
August 10, 1996 WECC blackout



Small-signal stability



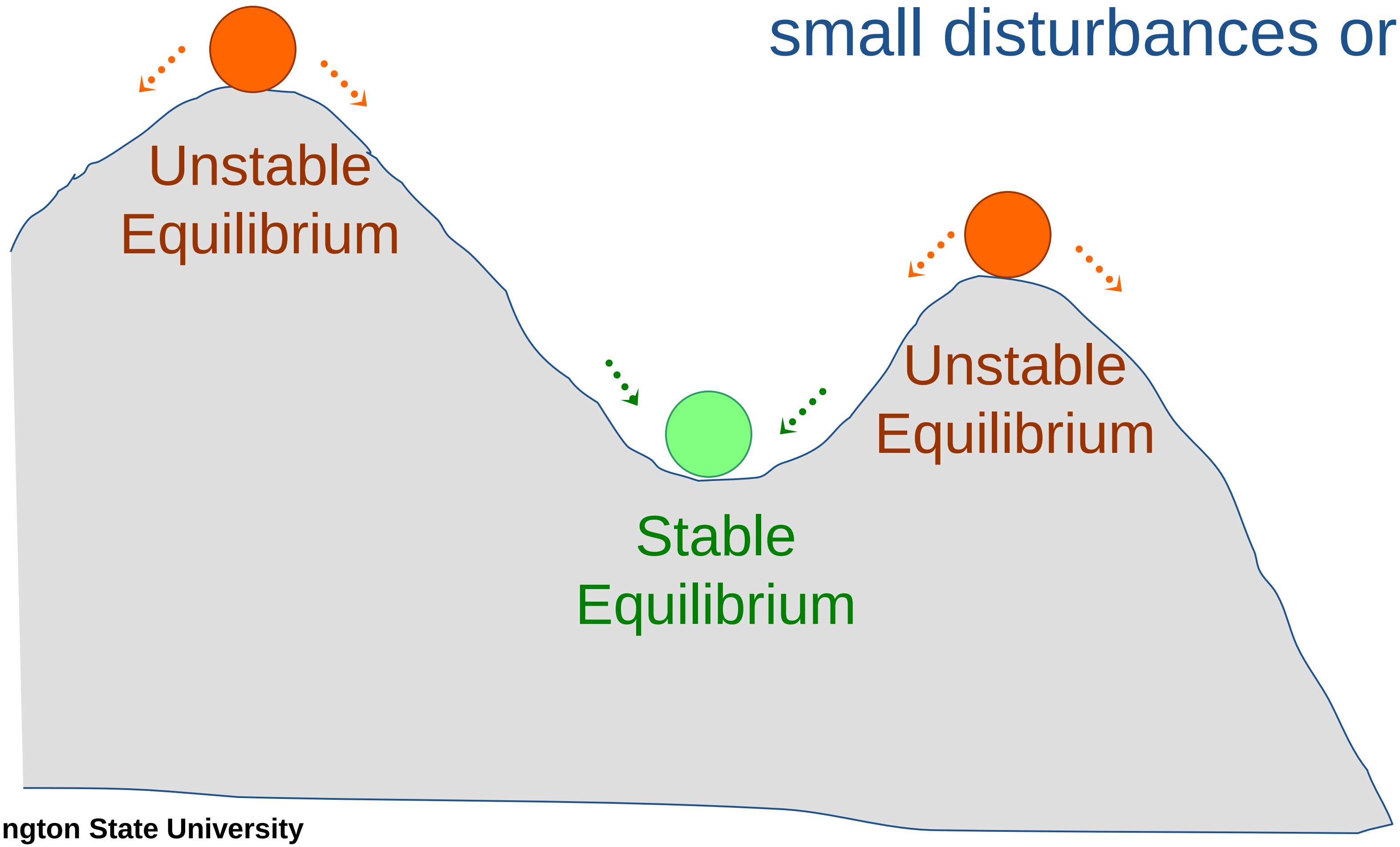
Small-signal stable equilibrium



Small-signal unstable equilibrium

Small-signal stability

Can withstand small disturbances or not?

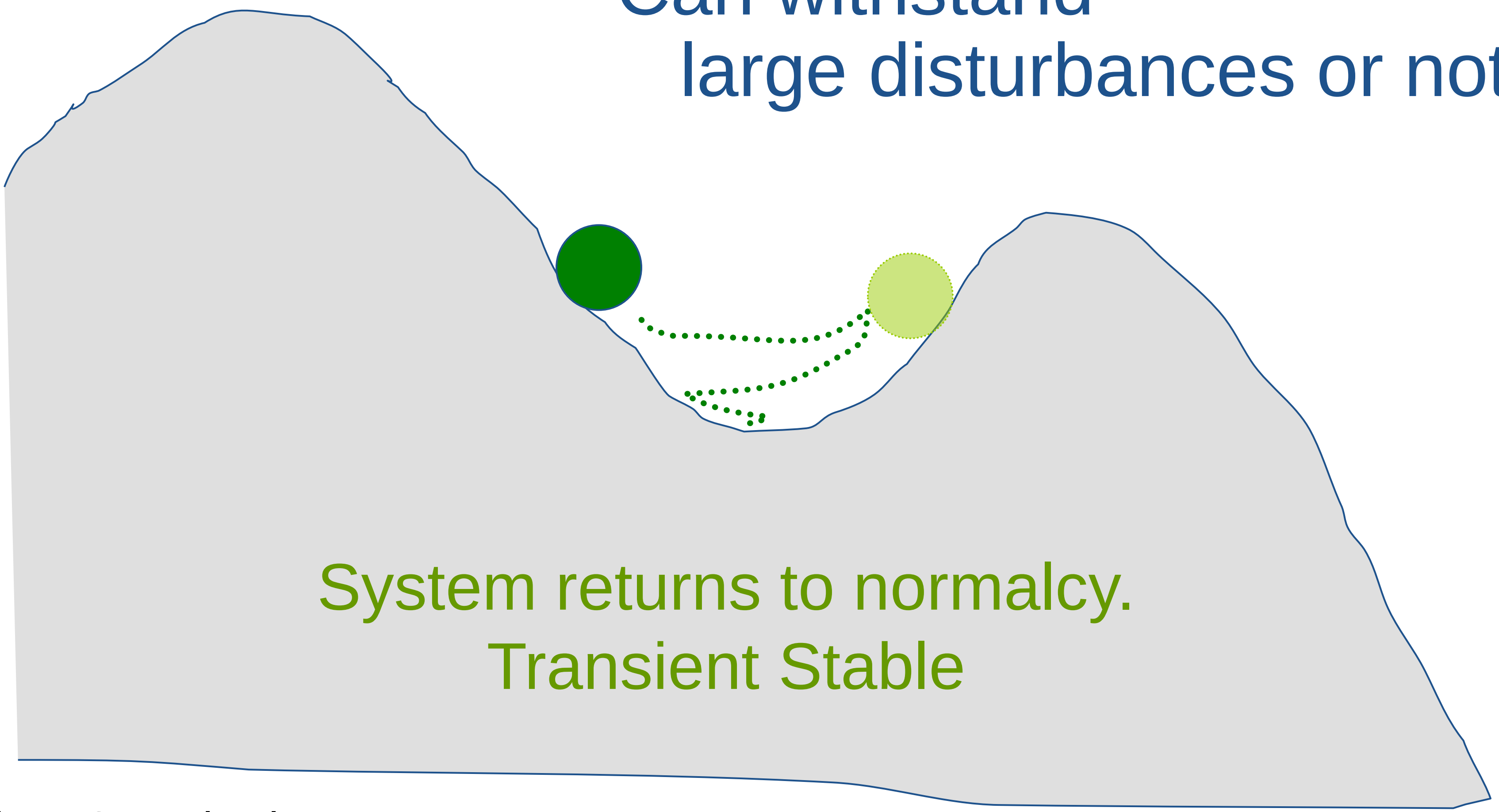


Stability concepts

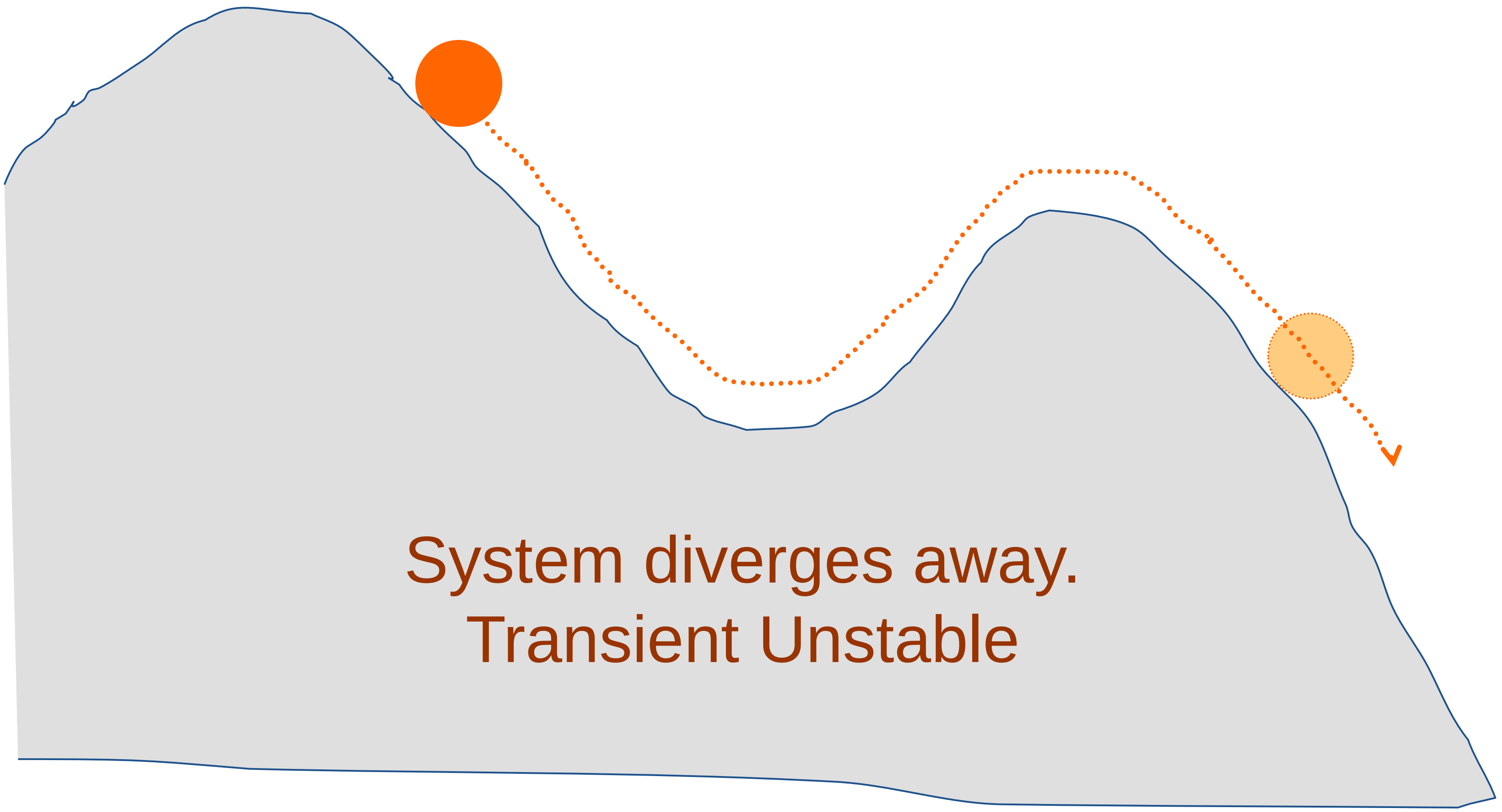
- Transient stability
  - Faults/line openings
  - Generator outages
  - Major disturbances
  - Loss of synchronization?
  - Voltage declines

Transient stability

Can withstand
large disturbances or not?

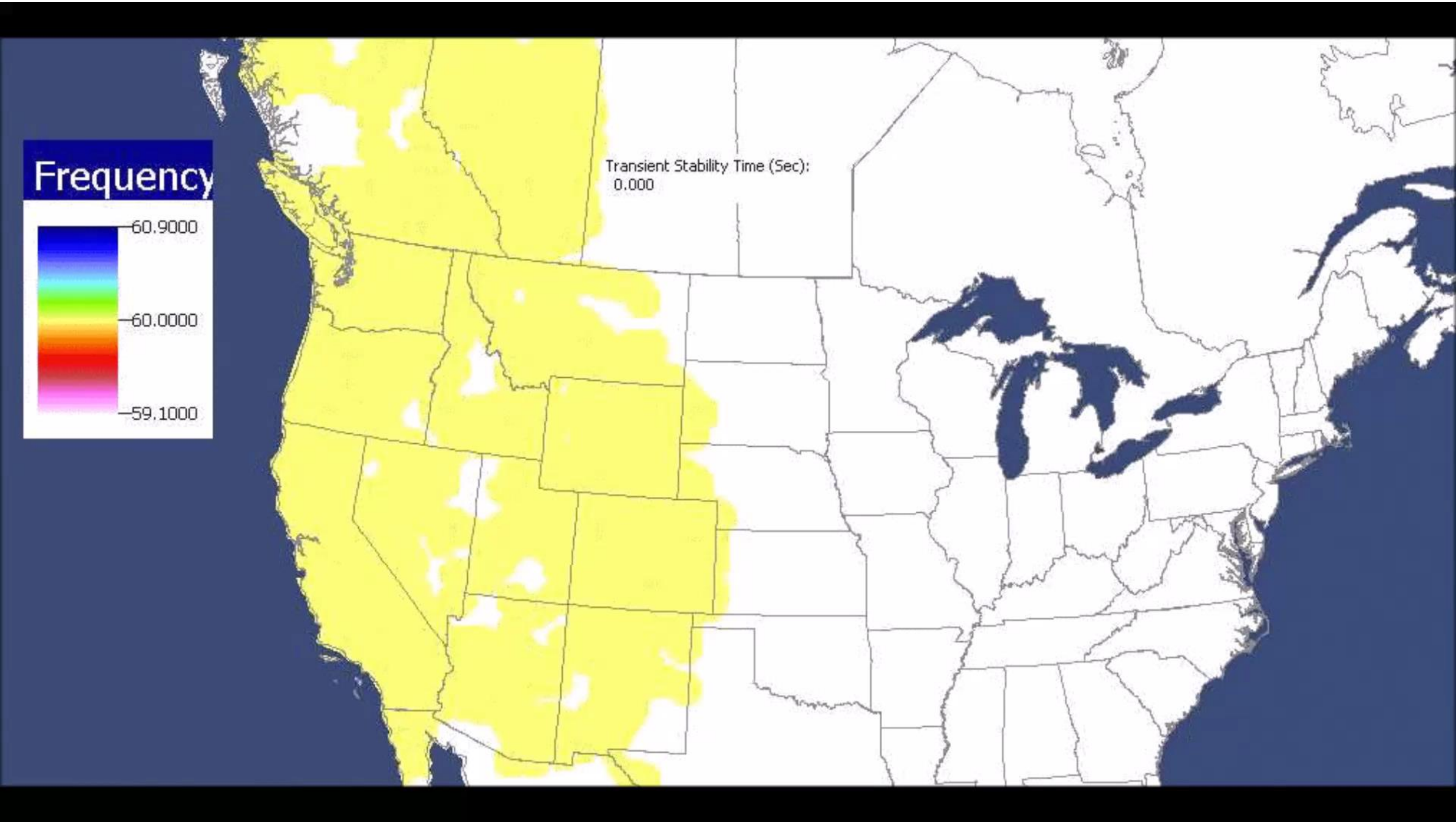


Transient instability



Loss of synchronism. Islanding.

Islanding

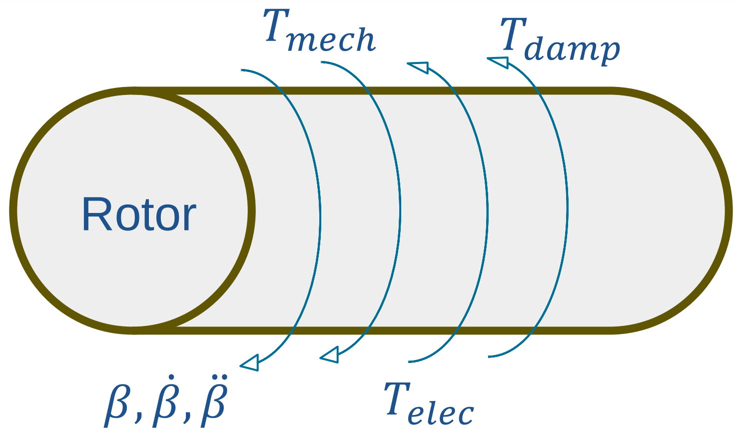


Frequency
60.9000
60.0000
59.1000
Transient Stability Time (Sec):
0.000

Stability Analysis

- Modeling
  - – Swing equations model
- Analysis
  - – Eigenvalues (Small signal stability)
  - – Numerical Integration (Transient Stability)
  - – Equal Area Criterion (Transient stability)
- Controls
  - – Governor controls/AGC

Rotor electromechanics



$\beta =$ actual angular displacement of the rotor

$\theta =$ relative angular displacement with respect to 60 Hz frame

$$\beta = \omega_s t + \theta - \pi/2,\ \dot{\theta} = \dot{\beta} - \omega_s = \omega - \omega_s$$

$$J\ddot{\beta} = T_m - T_e - T_d = J\ddot{\theta}$$

$\omega =$ actual angular speed of the rotor

Swing equations

$$J\ddot{\theta} = T_m - T_e - T_d$$

$$J\omega_s\ddot{\theta} = T_m\omega_s - T_e\omega_s - T_d\omega_s$$

$$\frac{J\omega_s\ddot{\theta}}{\text{S rating}} = \frac{T_m\omega_s}{\text{S rating}} - \frac{T_e\omega_s}{\text{S rating}} - \frac{T_d\omega_s}{\text{S rating}}$$

$$= P_m(p.u.) - P_e(p.u.) - P_d(p.u.)$$

$$H = \frac{Kinetic\ Energy}{MVA\ rating} = \frac{1/2J{\omega_s}^2}{\text{S rating}} = \text{Inertia time-constant}$$

$$\Longrightarrow \frac{J\omega_s}{\text{S rating}} = \frac{2H}{\omega_s}$$

Machine inertia

Stored energy in rotor interias

$$\sum_i P_{Mi} \Rightarrow \sum_i P_{Gi} = \sum_i P_{Li} + \sum_i \sum_j P_{Loss,ij}$$

Change very slow

Must change very fast

Change very fast

Swing equations

$$\Longrightarrow \frac{2H}{\omega_s}\ddot{\theta} = P_m(p.u.) - P_e(p.u.) - P_d(p.u.)$$

$\dot{\theta} = \omega_r - \omega_s,\qquad \omega_r =$ Speed of the rotor

$$\frac{2H}{\omega_s}\dot{\omega}_r = P_m - P_e - P_d,$$

Define $\omega(p.u.) = \omega_r/\omega_s$. Then,

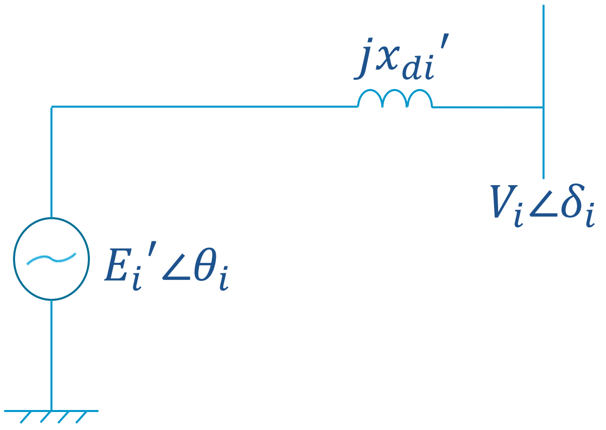
$$\boldsymbol{\dot{\theta} = (\omega - 1)\omega_s}$$
$$\boldsymbol{2H\dot{\omega} = P_m - P_e - P_d}$$

Classical machine model

Assume $P_d = K_D(\omega - 1)$

$$\dot{\theta} = (\omega - 1)\omega_s$$

$$2H\dot{\omega} = P_m - P_e - K_D(\omega - 1)$$

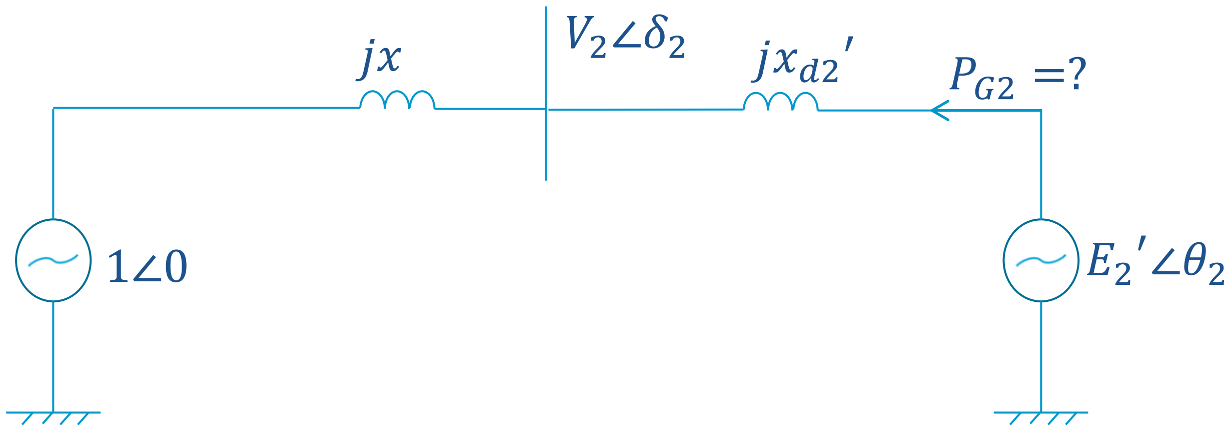


x_{di}' = Transient reactance of the stator coil

E_i'= Induced Voltage (Assume constant)

θ_i = Relative rotor angle

Example



$$\overrightarrow{I_{G2}} = \frac{E_2'\angle\theta_2 - 1\angle 0}{jx_{d2}' + jx}$$

$$\Longrightarrow P_{G2} + jQ_{G2} = E_2'\angle\theta_2 \overrightarrow{I_{G2}}^{*}$$

$$or\ P_{G2} = \frac{E_2' 1}{x_{d2}' + x}\sin(\theta_2 - 0) = \frac{E_2'}{x_{d2}' + x}\sin(\theta_2)$$

Swing equations model

$$\dot{\theta}_2 = (\omega_2 - 1)\omega_s$$

$$2H_2\dot{\omega}_2 = P_{T2} - P_{G2} - K_{D2}(\omega_2 - 1)$$

$$where \quad P_{G2} = \frac{E_2{}'}{x_{d2}{}' + x} \sin\theta_2$$

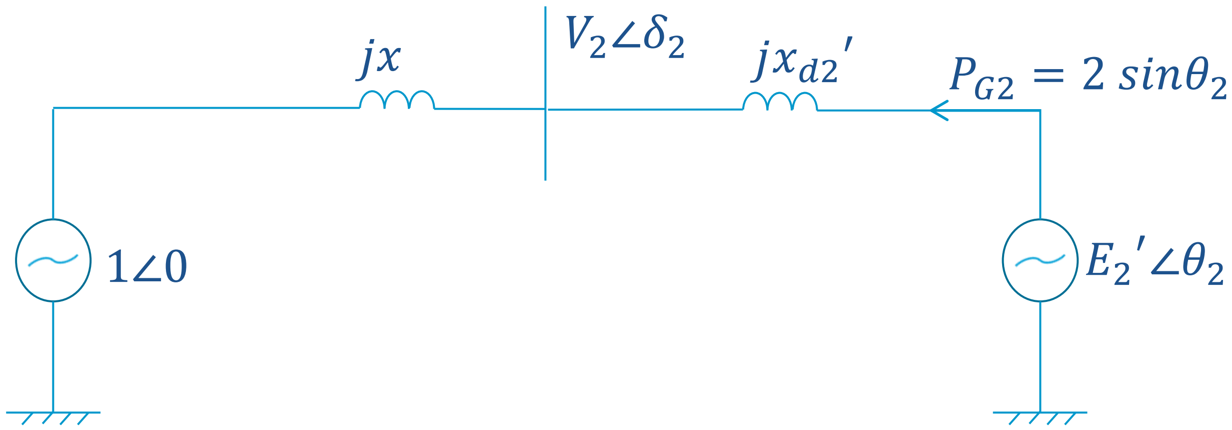
$$Suppose\ x = 0.5, x_{d2}{}' = 0.25, P_{T2} = 1,$$

$$K_{D2} = 1, \omega_s = 2\pi\, 60 = 377,$$

$$H_2 = 5, E_2{}' = 1.5$$

Then, $P_{G2} = \frac{1.5}{0.75}\sin\theta_2 = 2\sin\theta_2$

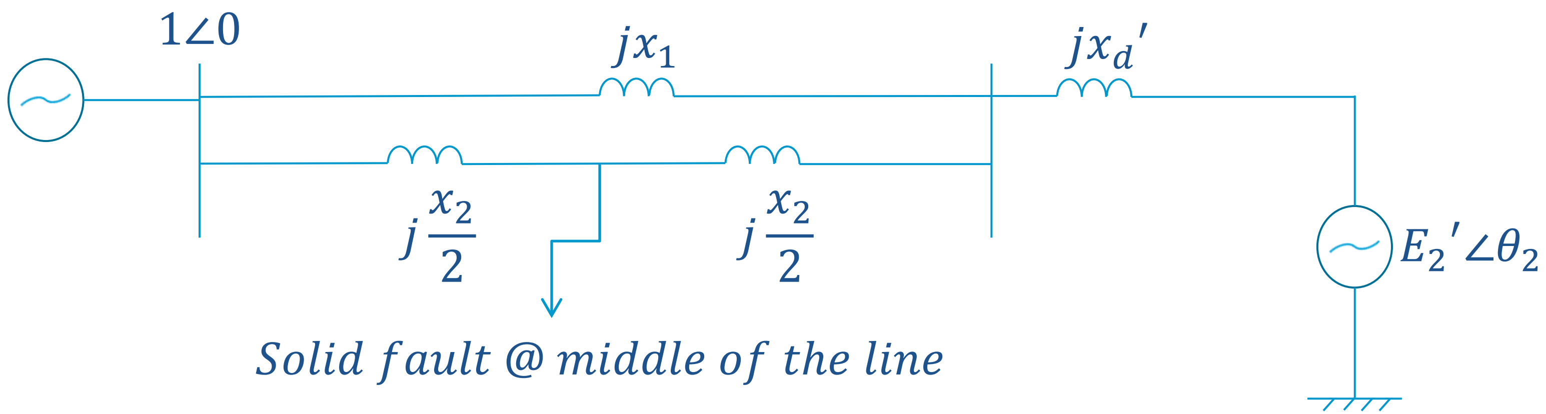
Swing equations for the system

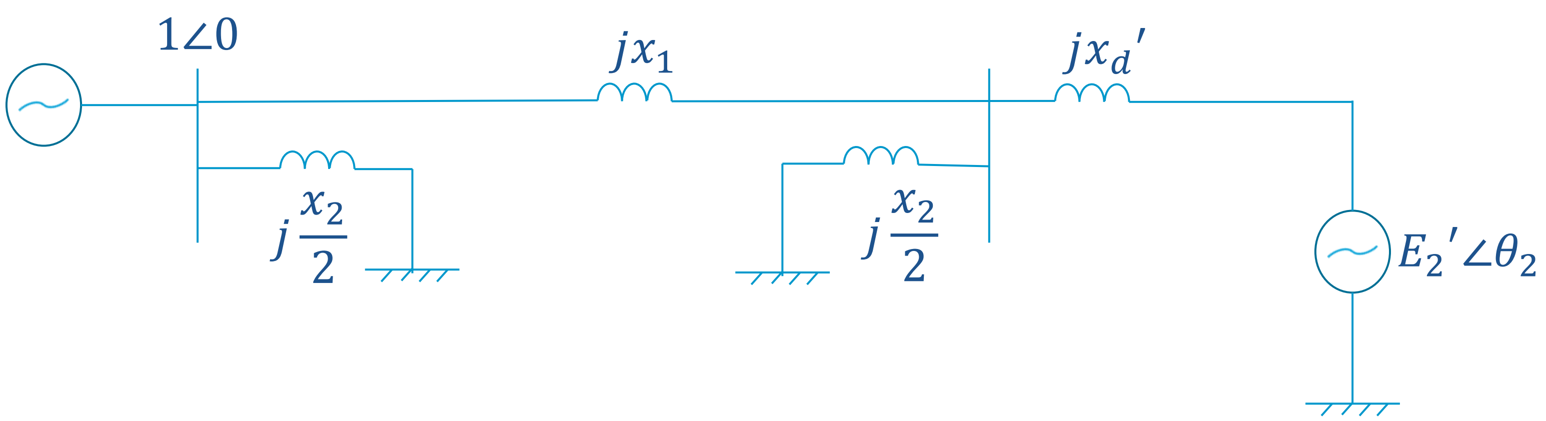


$$x = 0.5, x_{d2}' = 0.25, P_{T2} = 1, K_{D2} = 1,$$
$$\omega_s = 2\pi\ 60, H_2 = 5, E_2' = 1.5$$

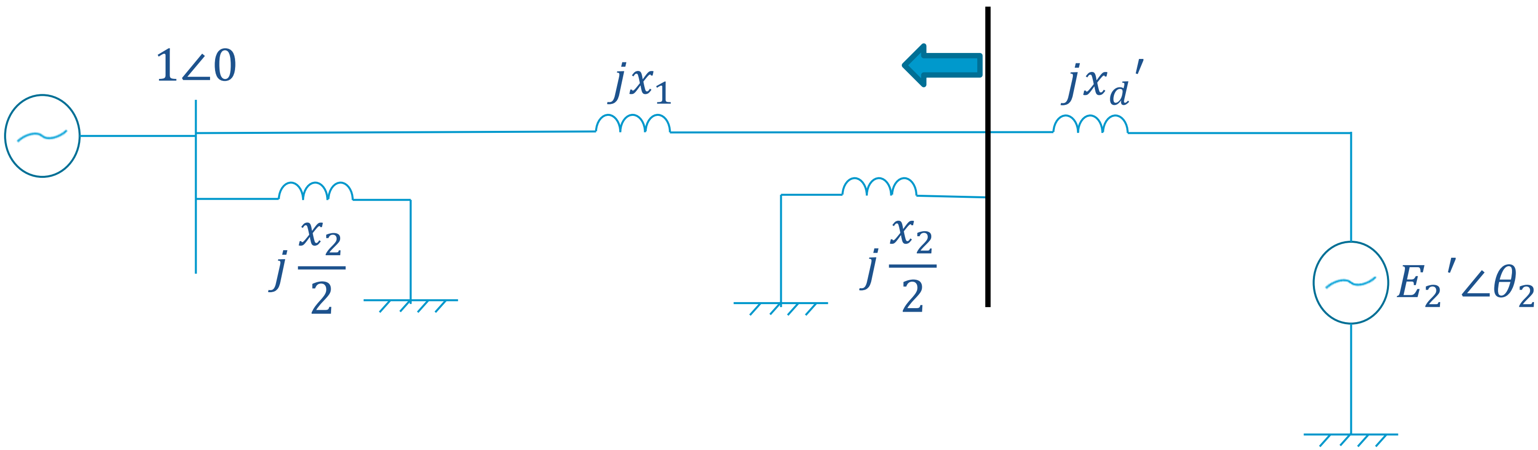
$$\dot{\theta}_2 = (\omega_2 - 1)377$$
$$10\dot{\omega}_2 = 1 - 2\sin\theta_2 - (\omega_2 - 1)$$

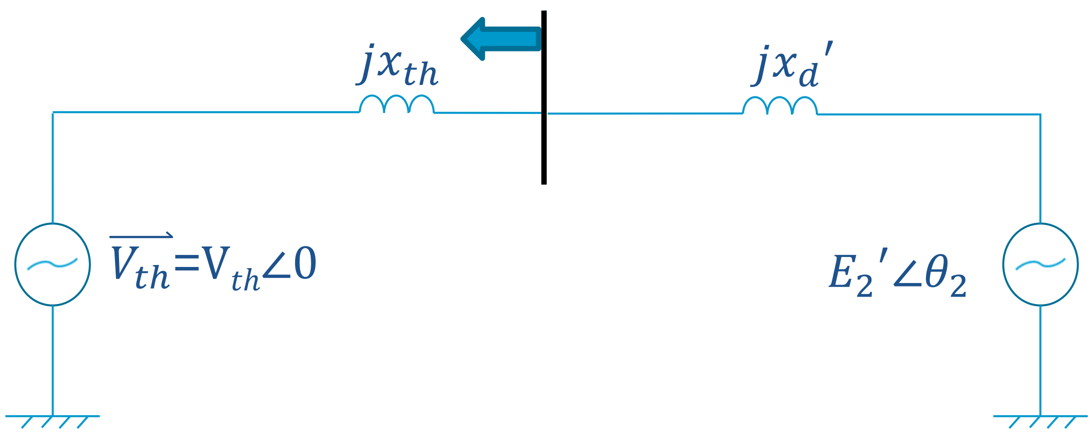
Faulted system





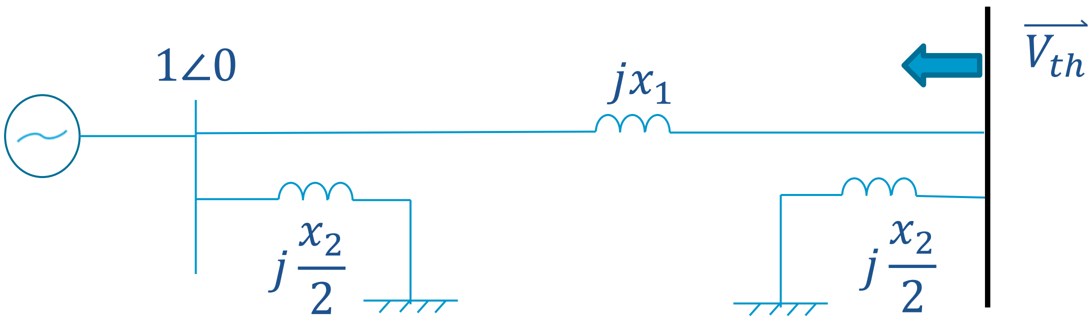
Thevenin equivalents



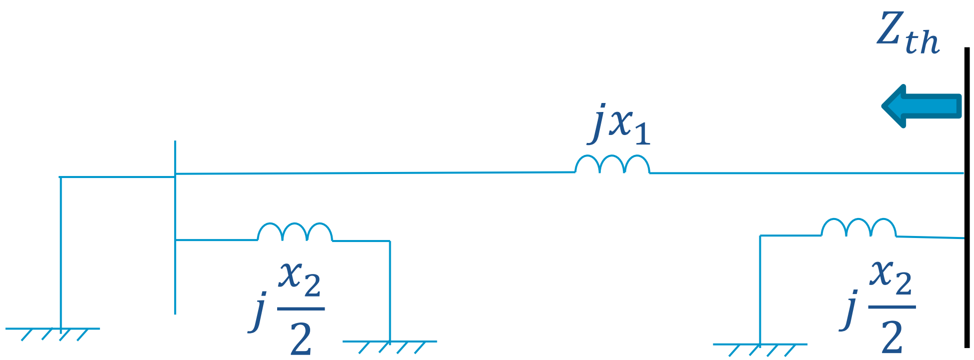


$$P_{G2} = \frac{E_2' V_{th}}{x_d' + x_{th}} \sin\theta_2$$

Thevenin equivalents

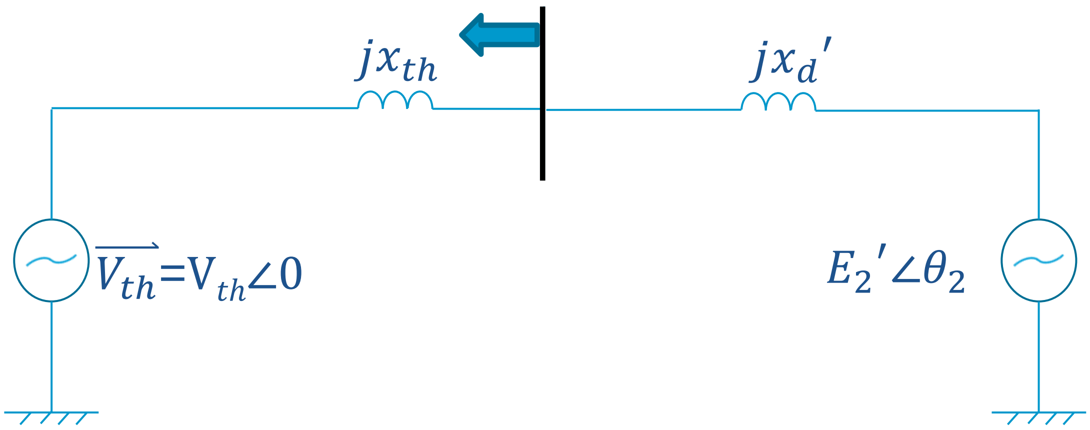


$$\overrightarrow{V_{th}} = (1\angle 0)\frac{j\frac{x_2}{2}}{jx_1 + j\frac{x_2}{2}}$$



$$Z_{th} = (jx_1) \parallel \left(j\frac{x_2}{2}\right)$$
$$= jx_{th}$$

Faulted system equations



$$\overrightarrow{V_{th}} = (1\angle 0)\frac{j\frac{x_2}{2}}{jx_1 + j\frac{x_2}{2}}$$

$$Z_{th} = (jx_1) \parallel \left(j\frac{x_2}{2}\right)$$
$$= jx_{th}$$

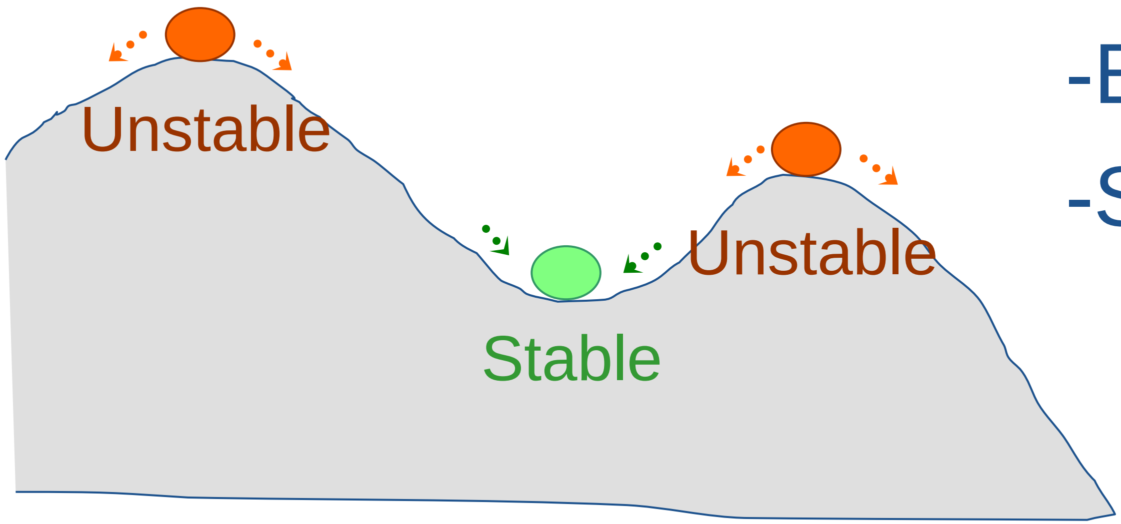
$$\dot{\theta}_2 = (\omega_2 - 1)\omega_s$$
$$2H_2\dot{\omega}_2 = P_{T2} - P_{G2} - K_{D2}(\omega_2 - 1)$$
$$where \quad P_{G2} = \frac{E_2' V_{th}}{x_d' + x_{th}}\sin\theta_2$$

Small-Signal Stability

Model: $\frac{dx}{dt} = f(x)$ $\qquad x \in \mathbb{R}^n$

Equilibrium point : $f(x_e) = 0$

Linearized system: $\Delta\dot{x} = \mathrm{J}\,\Delta x$ where $J = \left.\frac{\partial f}{\partial x}\right|_{x_e}$



-Equilibrium

-S.S.Stability?

Small-signal stability analysis

General: $$\dot{x} = f(x)$$

$$x_e \ is\ an\ equilibrium$$

$$\Delta x = x - x_e \Longrightarrow x = \Delta x + x_e$$

$$\dot{\Delta x} = \dot{x} = f(x_e + \Delta x)$$

$$= \cancel{f(x_e)} + \left.\frac{\partial f}{\partial x}\right|_{x_e} \Delta x + \cancel{\left.\frac{\partial^2 f}{\partial x^2}\right|_{x_e} \Delta x^2 + O(3) + \cdots}$$

$$\dot{\Delta x} = J\Delta x, \qquad where\ J = \left.\frac{\partial f}{\partial x}\right|_{x_e}$$

$$is\ the\ Jacobian\ matrix\ of\ \dot{x} = f(x)$$

Eigenvalues

Eigenvalues of J are the solutions of

$$\det(\lambda I - J) = 0$$

All eigenvalues have negative real parts $\Longrightarrow$ x_e is a small-signal stable eq. point

Example

$$\dot{x} = -\sin(x)$$

Equilibrium:

$$set \quad \sin(x) = 0 \Longrightarrow x = 0, \pm\pi, \pm 2\pi \ldots = \pm n\pi$$

Multiple Equilibrium points.

Small-Signal Model around an Equilibrium.

Linearization.

Compute eigenvalues.

Small-signal linearized model

$$\dot{x} = -\sin(x)$$

$x = 0$ equilibrium, J = -cos(0) = -1

$$\Delta\dot{x} = -\Delta x \Longrightarrow \text{Eigenvalue of } -1 \Longrightarrow stable$$

Equilibrium $x = 0$ is small-signal stable

Example (continued)

$$\dot{x} = -\sin(x)$$

$$x = 0 \Longrightarrow J = \left.\frac{\partial f}{\partial x}\right|_{x=0} = -\cos(x)\big|_{x=0} = -1$$

-1 has negative real part $\Longrightarrow x = 0$ is s.s.stable.

$$x = \pi \Longrightarrow J = \left.\frac{\partial f}{\partial x}\right|_{x=\pi} = -\cos(x)\big|_{x=\pi} = +1$$

+1 has positive real part $\Longrightarrow x = \pi$ is s.s.unstable.

Small perturbations can drive the system away from equilibrium $x = \pi$.

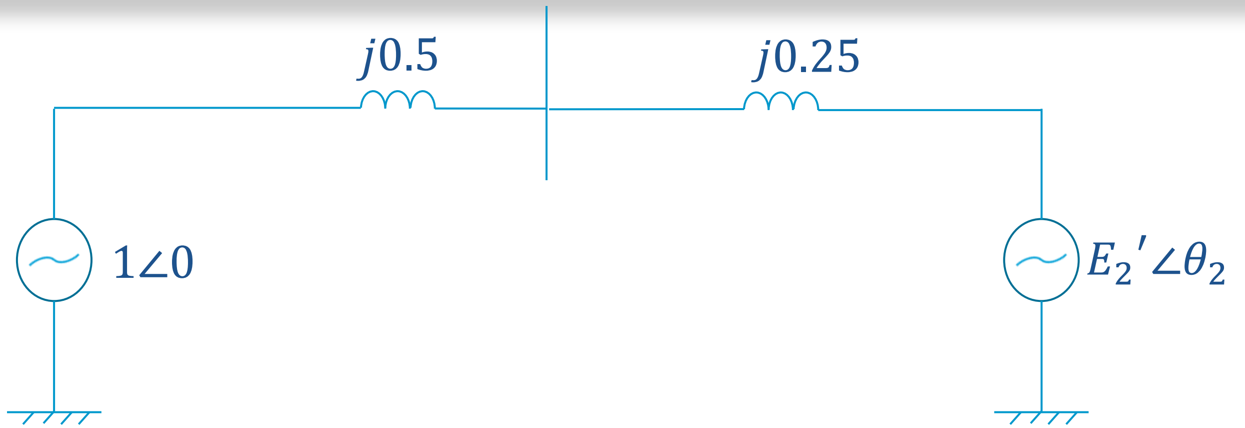
Example (continued)

$$x = 2\pi \Longrightarrow J = \left.\frac{\partial f}{\partial x}\right|_{x=2\pi} = -\cos(x)\big|_{x=2\pi} = -1$$

$\Longrightarrow x = 2\pi$ is s.s.stable.

$$Equilibria\ x = n\pi \begin{cases} s.s.stable\ if\ n\ even \\ s.s.unstable\ if\ n\ odd. \end{cases}$$

Example 2



$$x = 0.5, x_{d2}' = 0.25, P_m = 1p.u.,$$
$$P_e = 2\sin\theta_2$$
$$K_{D2} = 1p.u., \omega_s = 2\pi\, 60 = 377,$$
$$H_2 = 5, E_2' = 1.5$$

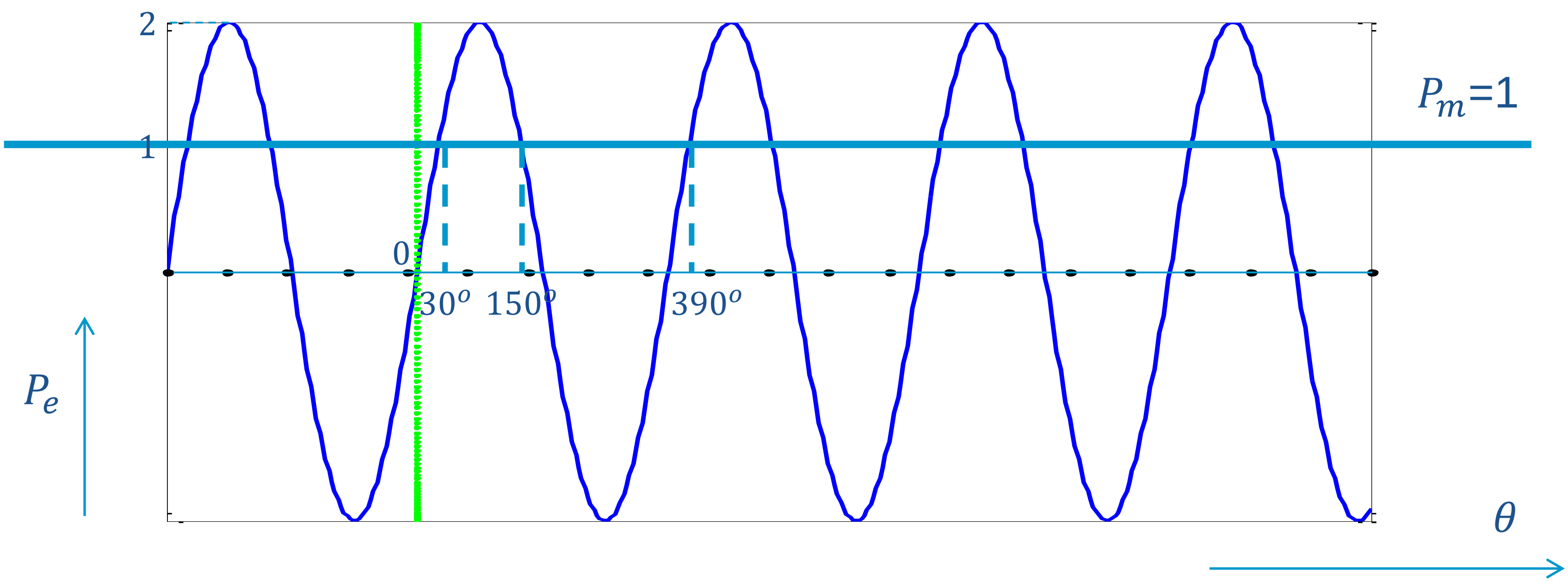
Equilibrium points

$$\dot{\theta}_2 = (\omega_2 - 1)377$$
$$\dot{\omega}_2 = \frac{1}{10}[1 - 2\sin\theta_2 - (\omega_2 - 1)]$$

Equilibrium points:

$$\dot{\theta}_2 = 0 \Longrightarrow \omega_2 = 1$$
$$\dot{\omega}_2 = 0 \Longrightarrow 1 - 2\sin\theta_2 - (\omega_2 - 1) = 0$$
$$2\sin\theta_2 = 1 \Longrightarrow \sin\theta_2 = \frac{1}{2}$$
$$\Longrightarrow \theta_2 = 30^o, 150^o, 390^o,$$

Power – Angle curve



Equilibrium point are

$$(\theta,\omega)^T = (30^o,1)^T or\ (150^o,1)^T or\ (390^o,1)^T$$
$$or\ (510^o,1)^T \ldots$$

Jacobian

$$\dot{\theta}_2 = (\omega_2 - 1)377 = f_1(\theta_2, \omega_2)$$

$$\dot{\omega}_2 = \frac{1}{10}[1 - 2\sin\theta_2 - (\omega_2 - 1)] = f_2(\theta_2, \omega_2)$$

$$\frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial \theta_2} & \frac{\partial f_1}{\partial \omega_2} \\ \frac{\partial f_2}{\partial \theta_2} & \frac{\partial f_2}{\partial \omega_2} \end{bmatrix} = \begin{bmatrix} 0 & \omega_s \\ -\frac{2}{10} cos\theta_2 & -\frac{1}{10} \end{bmatrix}$$

Jacobian

$$\frac{\partial f}{\partial x} = \begin{bmatrix} \dfrac{\partial f_1}{\partial \theta_2} & \dfrac{\partial f_1}{\partial \omega_2} \\ \dfrac{\partial f_2}{\partial \theta_2} & \dfrac{\partial f_2}{\partial \omega_2} \end{bmatrix} = \begin{bmatrix} 0 & \omega_s \\ -\dfrac{2}{10} cos\theta_2 & -\dfrac{1}{10} \end{bmatrix}$$

$$(30^o, 1)^T \Longrightarrow \left.\frac{\partial f}{\partial x}\right|_{(30^o,1)^T} = \begin{bmatrix} 0 & 377 \\ -\dfrac{2}{10} 0.866 & -0.1 \end{bmatrix}$$

$$\mathrm{J} = \begin{bmatrix} 0 & 377 \\ -0.17 & -0.1 \end{bmatrix}$$

Eigenvalues

$$J = \begin{bmatrix} 0 & 377 \\ -0.17 & -0.1 \end{bmatrix}$$

$$\det(\lambda I - J) = 0$$

$$det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 377 \\ -0.17 & -0.1 \end{bmatrix}\right) = 0$$

$$det\begin{bmatrix} \lambda & -377 \\ 0.17 & \lambda + 0.1 \end{bmatrix} = 0$$

$$\Longrightarrow \lambda^2 + 0.1\lambda + 65.3 = 0$$

$$\lambda = \frac{-0.1 \pm \sqrt{0.1^2 - 4(65.3)}}{2}$$

8

Eigenvalues

$$J = \begin{bmatrix} 0 & 377 \\ -0.17 & -0.1 \end{bmatrix} \Longrightarrow \det(\lambda I - J) = 0$$

$$\lambda^2 + 0.1\lambda + 65.3 = 0$$

$$\lambda = \frac{-0.1 \pm \sqrt{0.1^2 - 4(65.3)}}{2}$$

$$= \underline{-0.05} \pm j8.08 \Longrightarrow Freq = \frac{8}{2\pi} \approx 1.286\ Hz$$

$\Downarrow$

negative $\Longrightarrow$ Equilibrium $(30^o, 1)^T$is small-signal stable

Eigenvalues

Standard form: $-\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$

$$\Rightarrow \xi = \frac{|Real\ part|}{\sqrt{Real^2 + Imag^2}} = \frac{0.05}{\sqrt{0.05^2 + 8.08^2}}$$

$$= 0.006 = 0.6\% \Rightarrow low\ damping$$

Equilibrium point $(150^o, 1)^T \Rightarrow$

$$J = \left[\begin{matrix} 0 & \omega_s \\ -\frac{2}{10}cos\theta & -\frac{1}{10} \end{matrix}\right]\Bigg|_{(150^o,1)} = \begin{bmatrix} 0 & 377 \\ 0.17 & -0.1 \end{bmatrix}$$

Unstable equilibrium

$$\Longrightarrow \lambda^2 + 0.1\lambda - 65.3 = 0$$

$$\lambda = \underline{8.03}, -8.13$$

$$\Downarrow$$

$$\text{positive real part}$$

$$\Longrightarrow \text{Equilibrium } (150^o, 1)^T \text{s. s. unstable}$$

small perturbations $\Longrightarrow$ will drive system away.

cannot operate at $(150^o, 1)^T$.

Example 3

$$\dot{x}_1 = -x_1 + x_1 x_2$$
$$\dot{x}_2 = -x_2 + x_1 x_2$$
$$\Longrightarrow \begin{array}{l} \dot{x}_1 = -x_1(1 - x_2) \\ \dot{x}_2 = -x_2(1 - x_1) \end{array}$$

$\dot{x}_1 = 0 \; and \; \dot{x}_2 = 0$ for eq. point

$$\Longrightarrow \begin{array}{l} \dot{x}_1 = 0 \Longrightarrow x_1 = 0 \; or \; x_2 = 1 \\ \dot{x}_2 = 0 \Longrightarrow x_2 = 0 \; or \; x_1 = 1 \end{array}$$

$$\frac{\partial f}{\partial x} = \begin{bmatrix} -1 + x_2 & x_1 \\ x_2 & -1 + x_1 \end{bmatrix}$$

Analysis

eq. points: $(0,0)^T$ and $(1,1)^T$

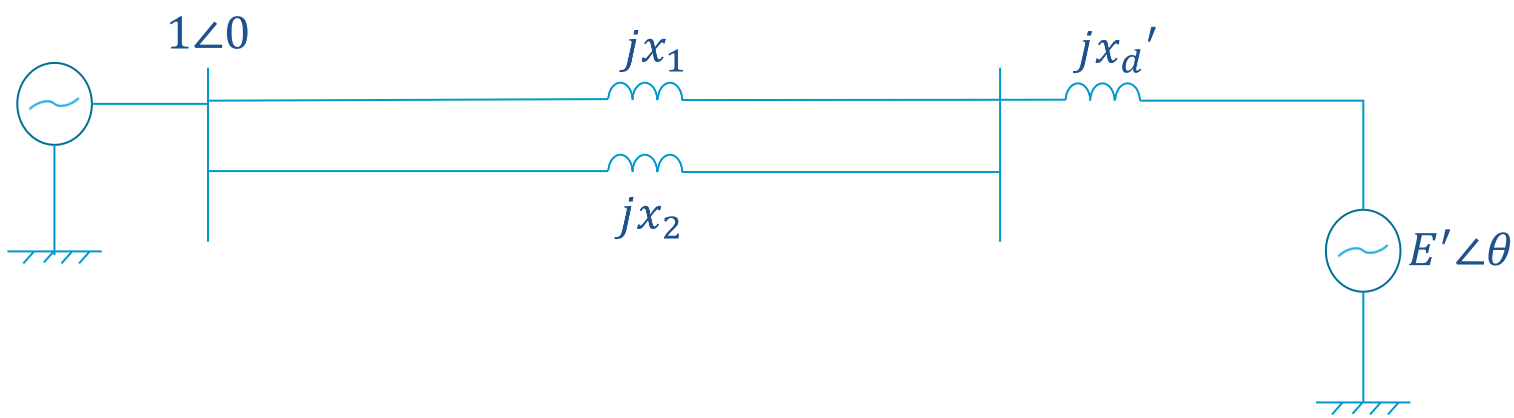
$$(0,0)^T\text{:}\ \ \mathrm{J} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \Longrightarrow (\lambda+1)^2 = 0$$

$$\Longrightarrow \lambda = -1, -1 \Longrightarrow S.S.Stable$$

$$(1,1)^T\text{:}\ \ \mathrm{J} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Longrightarrow \lambda^2 - 1 = 0$$

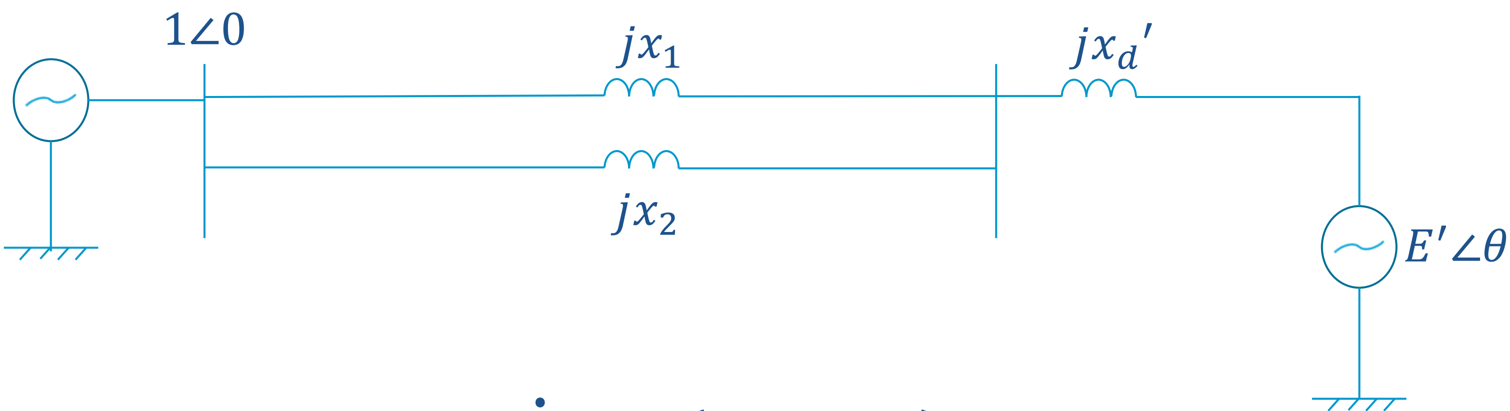
$$\Longrightarrow \lambda = 1, -1 \Longrightarrow S.S.Unstable$$

Transient Stability Analysis



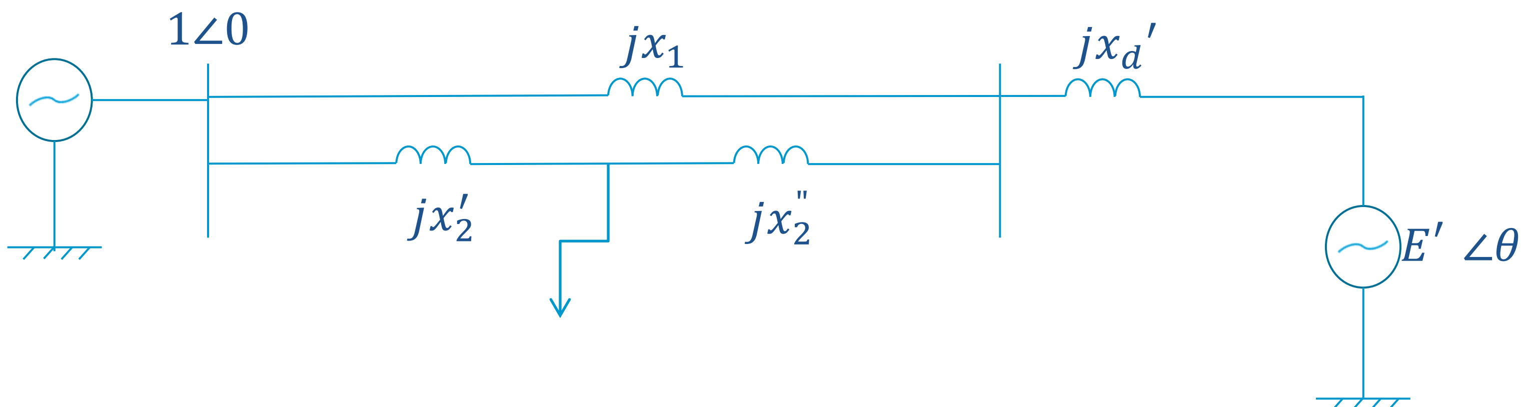
$$\begin{matrix}\text{Pre−fault} \\ t<0\end{matrix} \;\overset{\begin{pmatrix}\theta_s \\ 1\end{pmatrix}}{\Longrightarrow}\; \begin{matrix}\text{Fault−on} \\ t=0 \text{ to } t_c\end{matrix} \;\overset{\begin{pmatrix}\theta_c \\ \omega_c\end{pmatrix}}{\Longrightarrow}\; \begin{matrix}\text{Post−fault} \\ t>t_c\end{matrix}$$

Pre-fault



$$\dot{\theta} = (\omega - 1)\omega_s$$
$$2H\dot{\omega} = P_m - P_e^{pre} - P_d$$
$$P_m = P_e^{pre} = P_{max}^{pre}\sin\theta$$
$$P_{max} = \frac{E' \cdot 1}{x_d{}' + (x_1 \parallel x_2)} \qquad \mathrm{x} = \begin{pmatrix} \theta_s^{pre} \\ 1 \end{pmatrix}$$

Fault-on



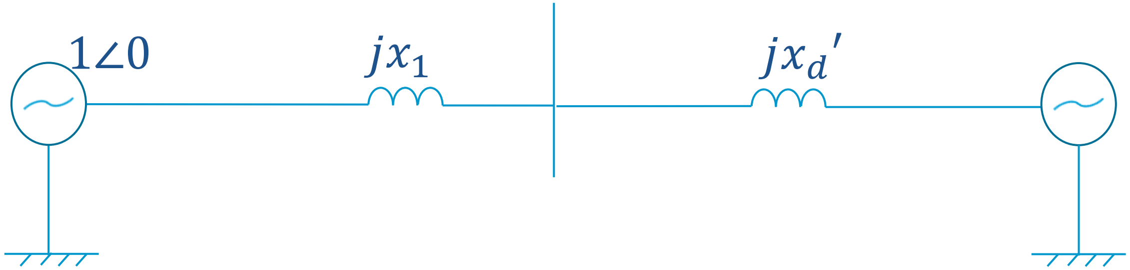
$$\dot{\theta} = (\omega - 1)\omega_s$$

$$2H\dot{\omega} = P_m - P_e^{fault} - P_d$$

$$P_e \downarrow \Longrightarrow \dot{\omega} > 0 \Longrightarrow \omega \uparrow \Longrightarrow \theta \uparrow$$

$\mathrm{t} = t_c$ fault cleared $\Longrightarrow$ $\mathrm{x} = \begin{pmatrix} \theta_c \\ \omega_c \end{pmatrix}$

Post-fault



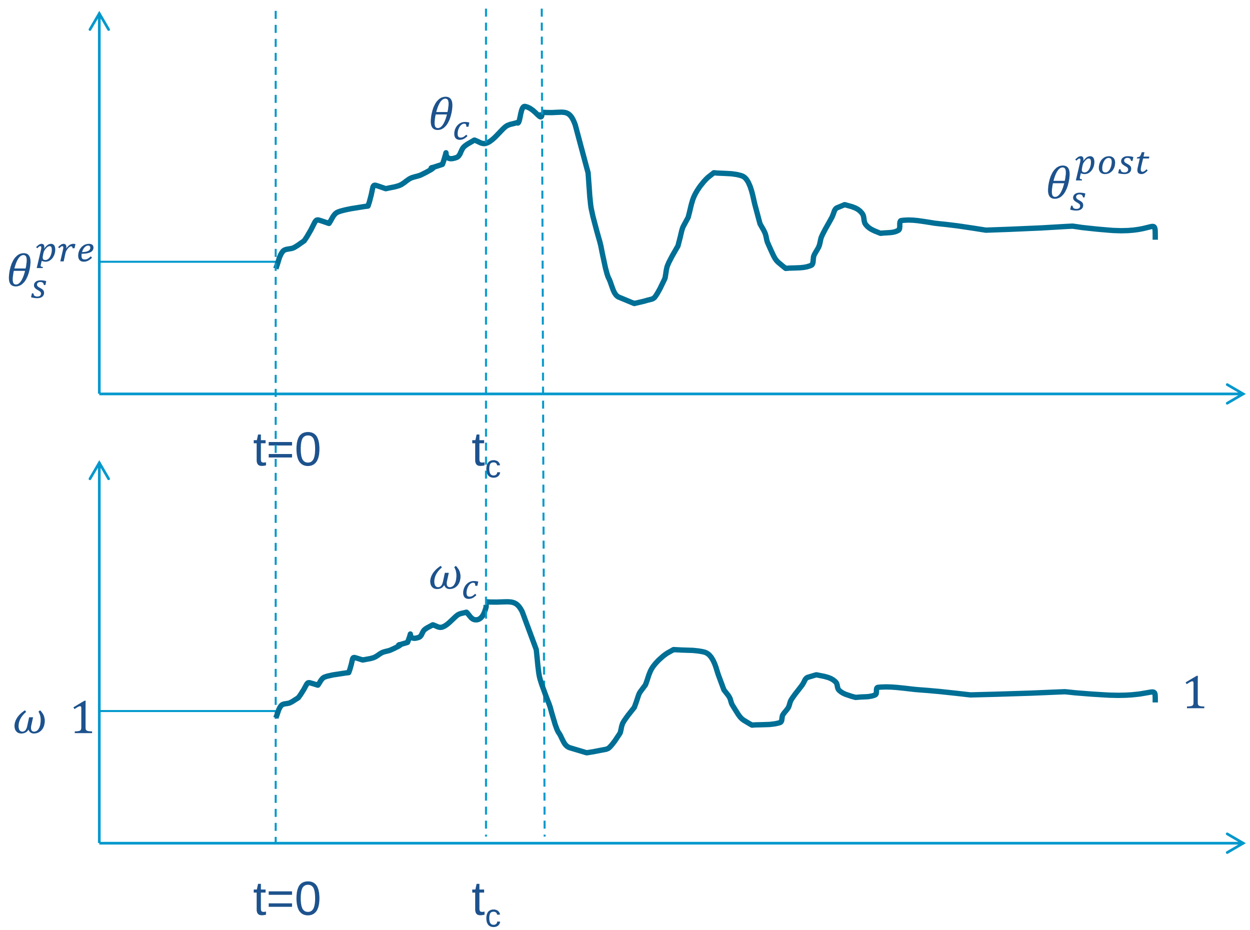
$$\dot{\theta} = (\omega - 1)\omega_s$$

$$2H\dot{\omega} = P_m - P_e^{post} - P_d$$

$$P_e^{post} = P_{max}^{post}\sin\theta,\ \ P_{max}^{post} = \frac{E'\cdot 1}{x_d{}' + x_1}$$

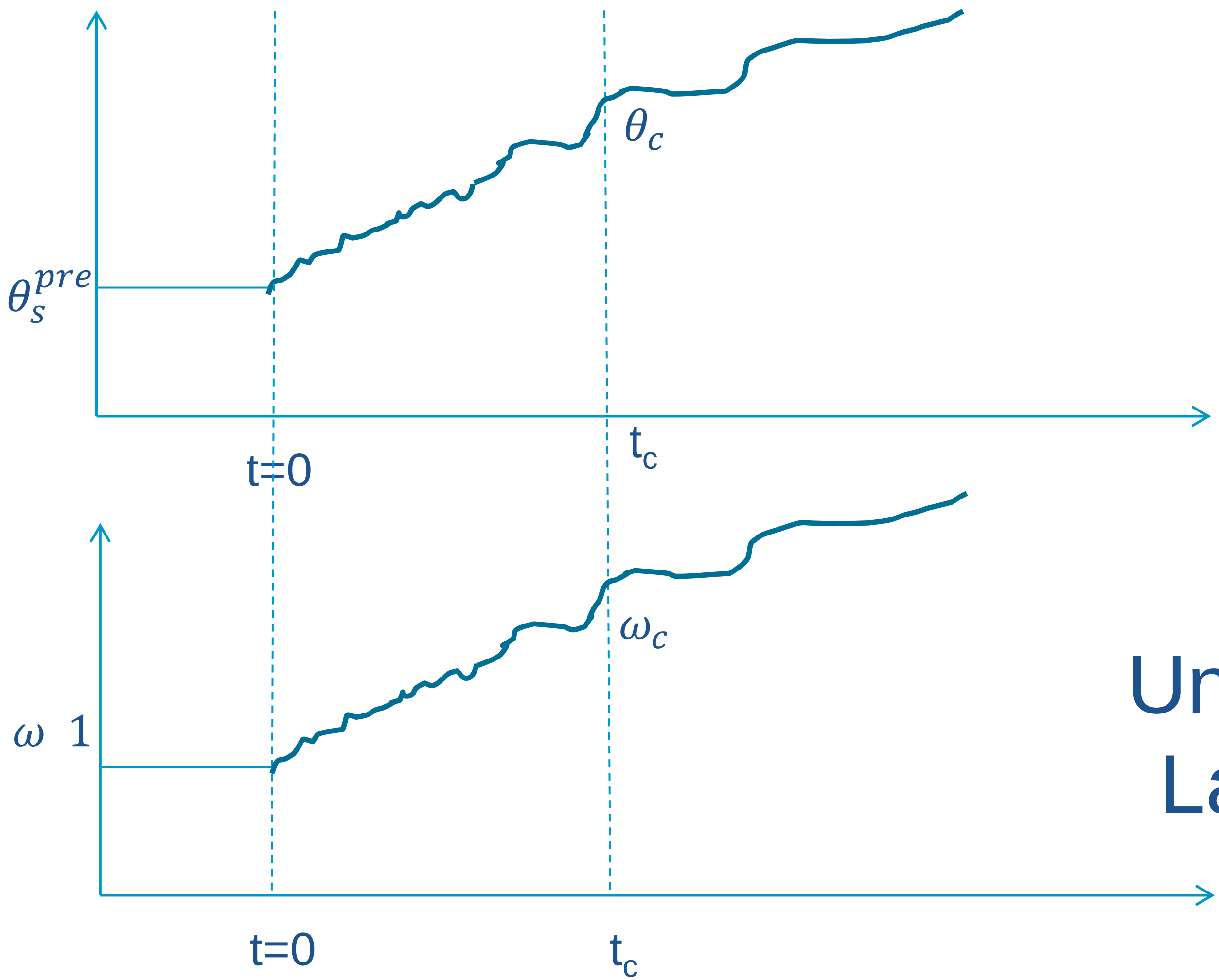
Can system recover?

Transient stable case



Stable.
Small t_c

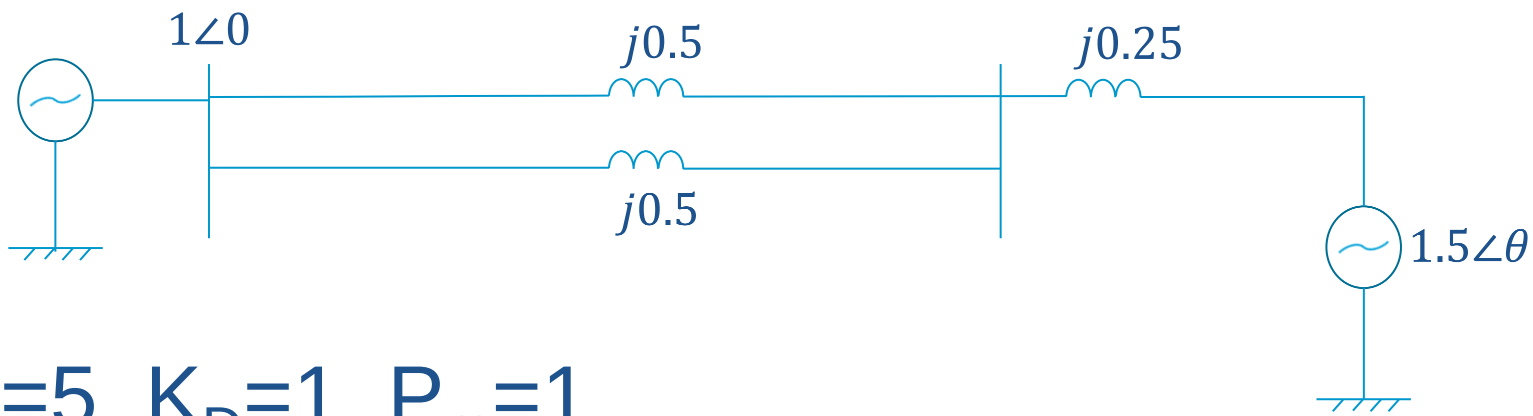
Transient unstable case



Unstable.

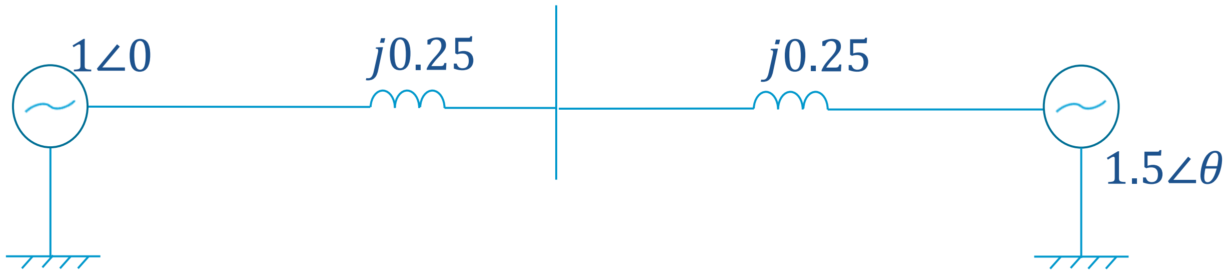
Large t_c

Example



H=5, K_D=1, P_m=1

Pre-fault:



Pre-fault system

$$P_e = \frac{1.5}{0.5}\sin\theta = 3\sin\theta$$

$$\dot{\theta} = (\omega - 1)\omega_s$$

$$10\dot{\omega} = 1 - 3\sin\theta - (\omega - 1)$$

Equilibria:

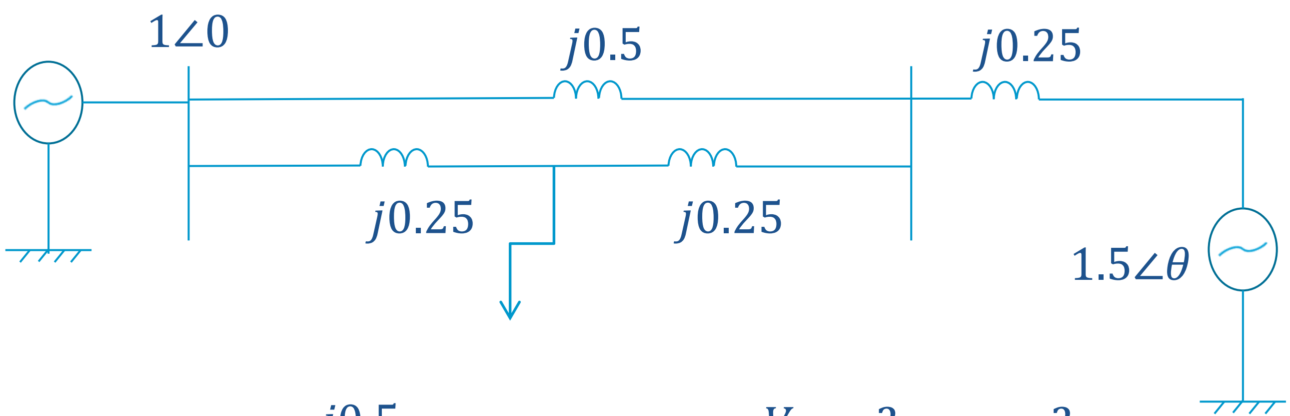
$$\omega = 1, \qquad 1 = 3\sin\theta \Longrightarrow \theta = \sin^{-1}\frac{1}{3} = 19.5^o$$

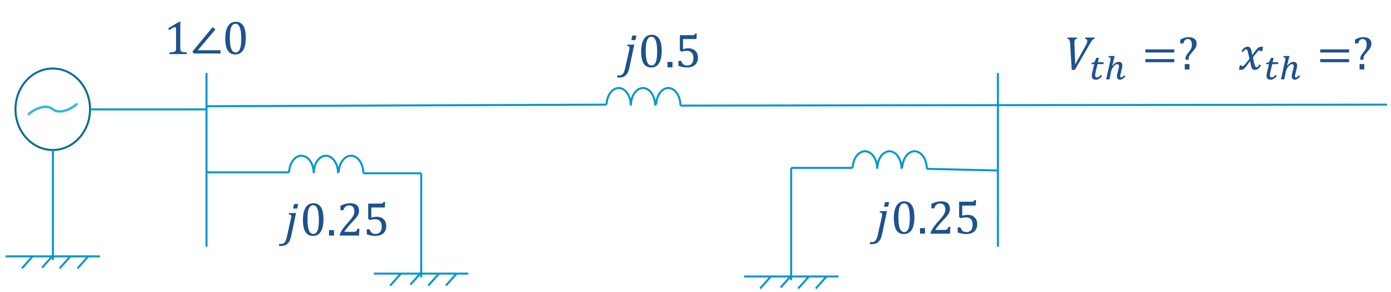
$$\mathrm{x}_s^{pre} = \begin{pmatrix} 19.5^o \\ 1 \end{pmatrix}$$

Fault-on system

Fault occurs $\Downarrow t = 0$

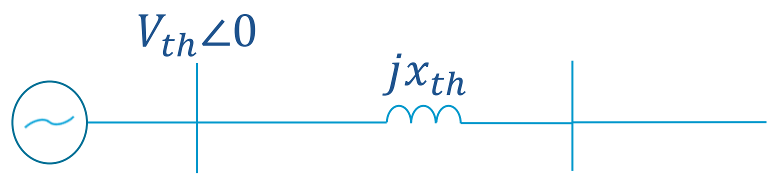
Fault-on: middle of lower line



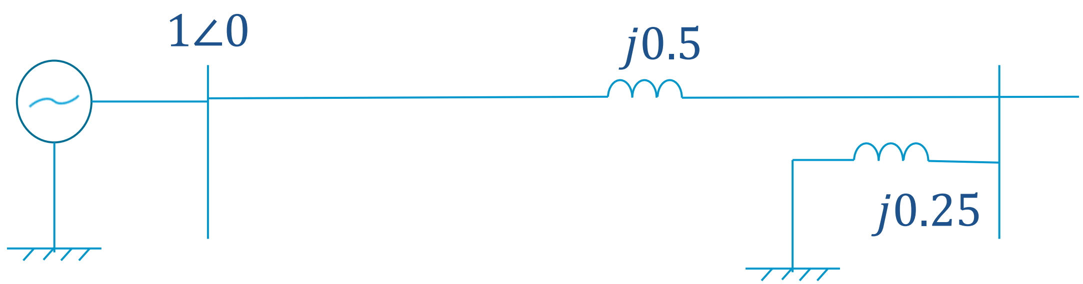


Thevenin equivalents

$$\Downarrow \textit{Thevenin}\ \text{Equivalent}$$



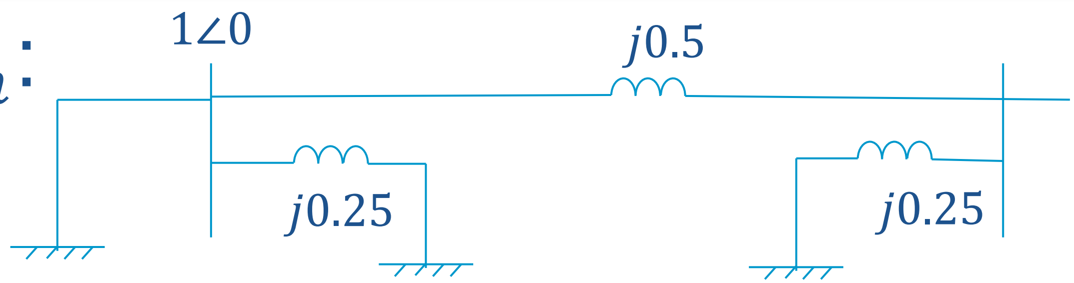
to find V_{th}:



$$V_{th} = \frac{0.25}{0.5+0.25} \cdot 1\angle 0 = \frac{1}{3}\angle 0$$

Thevenin equivalents

to find x_{th}:



$$x_{th} = 0.5 \parallel 0.25 = \frac{0.5 \cdot 0.25}{0.5 + 0.25} = \frac{0.5}{3}$$

$$P_e^{fault} = \frac{1.5 \cdot \frac{1}{3}}{0.25 + 0.166} \sin\theta = 1.2 \sin\theta$$

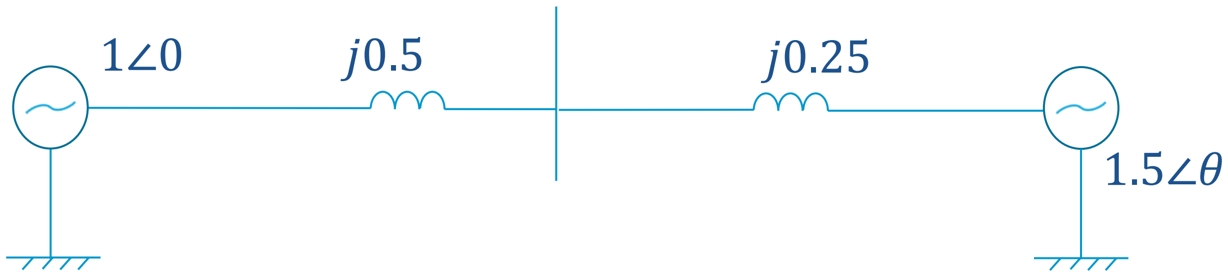
Fault-on system response

$$\dot{\theta} = (\omega - 1)\omega_s$$
$$10\dot{\omega} = 1 - 1.2\sin\theta - (\omega - 1)$$

Integrate from $\begin{pmatrix} 19.5^o \\ 1 \end{pmatrix}$ *or* $\begin{pmatrix} 0.3398 \\ 1 \end{pmatrix}$ *at t=0 to clearing time say t=6 cycles=0.1 sec.*

Post-fault system

Post-fault:



$$P_e^{post} = \frac{1.5}{0.5+0.25}\sin\theta = 2\sin\theta$$

$$\dot{\theta} = (\omega - 1)\omega_s$$

$$10\dot{\omega} = 1 - 2\sin\theta - (\omega - 1)$$

Integrate from $\begin{pmatrix}\theta_c \\ \omega_c\end{pmatrix}$ at $t = t_c$ onwards to see if frequency returns to 1 (Stable) or diverges (Unstable).

Euler Numerical integration

$$\dot{x} = f(x), x(t_0) = x_0, \mathrm{h} = \text{step size}$$

$$\frac{\Delta x}{\Delta t} = f(x) \Longrightarrow \Delta x = f(x) \cdot \Delta t$$

$$\Longrightarrow x(t_0 + h) - x(t_0) = f\big(x(t_0)\big) \cdot h$$

$$x(t_0 + h) = x(t_0) + f\big(x(t_0)\big) \cdot h$$

Euler Numerical integration

$$\dot{x} = f(x), x(t_0) = x_0, \mathrm{h} = \text{step size}$$
$$x(t_0 + h) = x(t_0) + f\big(x(t_0)\big) \cdot h$$

$$\overset{k=0}{\Longrightarrow}$$
$$t_k = t_0$$
$$x_k = x_0$$

$$x_{k+1} = x(t_{k+1}) = x_k + f(x_k) \cdot h$$
$$t_{k+1} = t_k + h$$

k=k+1

Fault-on trajectory

$$x_s^{pre} = \begin{pmatrix} 0.3398 \\ 1 \end{pmatrix}$$

$$\Downarrow t = 0$$

Integrate fault-on

$$\dot{\theta} = (\omega - 1)\omega_s$$

$$10\dot{\omega} = 1 - 1.2\sin\theta - (\omega - 1)$$

from $\begin{pmatrix} 0.3398 \\ 1 \end{pmatrix}$ at t=0

to $\begin{pmatrix} \theta_c \\ \omega \end{pmatrix}$ at t=t_c

Post-fault system

$$\Downarrow t = t_c$$

Integrate post-fault say for 30 seconds

$$\dot{\theta} = (\omega - 1)\omega_s$$

$$10\dot{\omega} = 1 - 2\sin\theta - (\omega - 1)$$

from $\begin{pmatrix} \theta_c \\ \omega_c \end{pmatrix}$ at $t=t_c$

onwards

Euler Algorithm

$$k = 0$$

$$t_k = t_0,\ x_k = x_0$$

$$\Downarrow$$

k=k+1

$$t_k = t_{final}? \overset{Yes}{\Longrightarrow} Stop$$

$$\Downarrow No$$

$$x(t_{k+1}) = x(t_k) + f(x(t_k)) \cdot h$$

$$t_{k+1} = t_k + h$$

Euler Iterations Example

$$x_s^{pre} = \begin{pmatrix} 0.3398 \\ 1 \end{pmatrix}$$

$$\Downarrow t = 0$$

Integrate fault-on

$$\dot{\theta} = (\omega - 1)\omega_s$$

$$10\dot{\omega} = 1 - 1.2\sin\theta - (\omega - 1)$$

from $\begin{pmatrix} 0.3398 \\ 1 \end{pmatrix}$ at t=0.

h = 0.002.

Euler Iterations Example

$$\dot{\theta}|_{t=0} = 0.0$$

$$\dot{\omega}|_{t=0} = 0.06$$

$$\theta\Big|_{0.002} = 0.3398 + 0.002 * 0.0 = 0.3398$$

$$\omega|_{0.002} = 1 + 0.002 * 0.06 = 1.00012$$

$$\Downarrow h = 2$$

Euler Iterations

$$\Downarrow k = 2$$

$$\dot{\theta}\big|_{t=0.002} = 0.0452$$

$$\dot{\omega}\big|_{t=0.002} = 0.06$$

$$\Downarrow$$

$$\theta\Big|_{0.004} = 0.3398 + 0.002 * 0.0452 = 0.3399$$

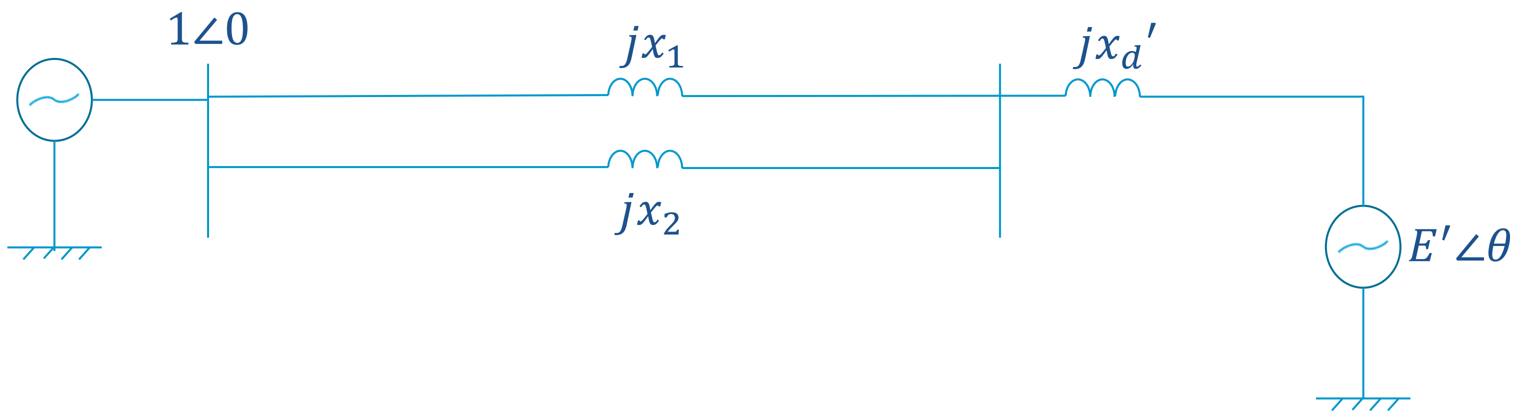
$$\omega\big|_{0.004} = 1.00012 + 0.002 * 0.06 = 1.00024$$

Continue till t=tc. Then, switch to post-fault equations and continue iterations till end time.

Final Exam

- ProctorU exam on Dec 14th and 15th
- 2 hour exam
- Schedule anytime during the two day window
- 3 formula sheets allowed
- Scientific calculator with no programs

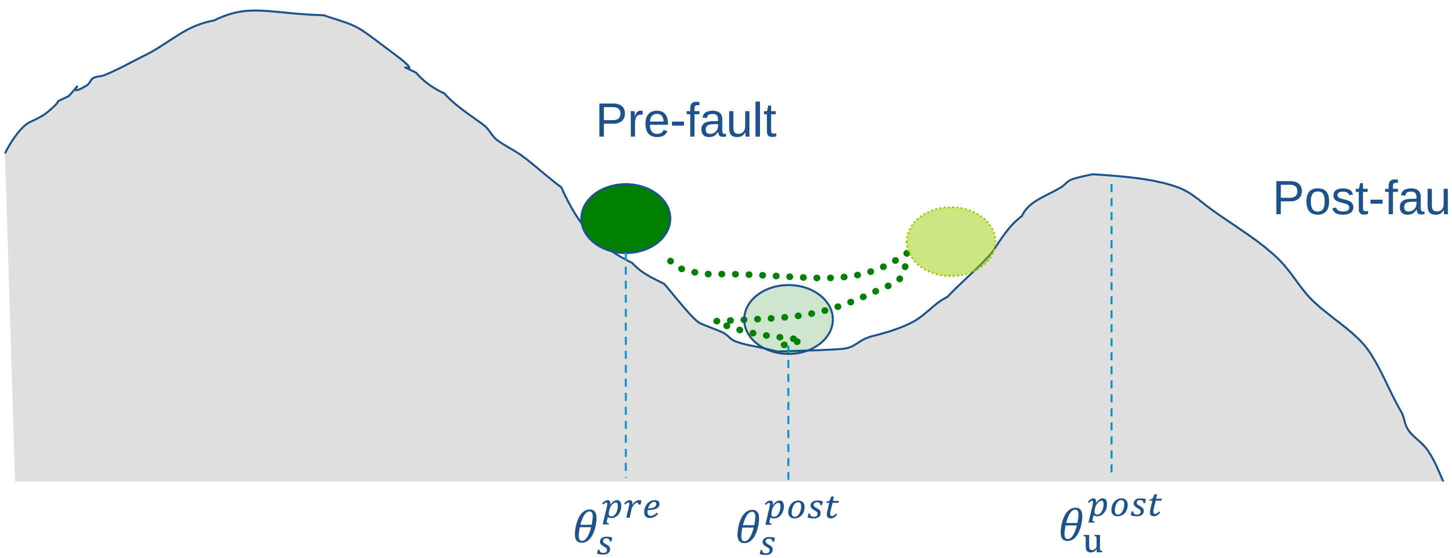
Transient Stability Analysis



Pre−fault $\xrightarrow{\begin{pmatrix}\theta_s \\ 1\end{pmatrix}}$ Fault−on $\xrightarrow{\begin{pmatrix}\theta_c \\ \omega_c\end{pmatrix}}$ Post−fault

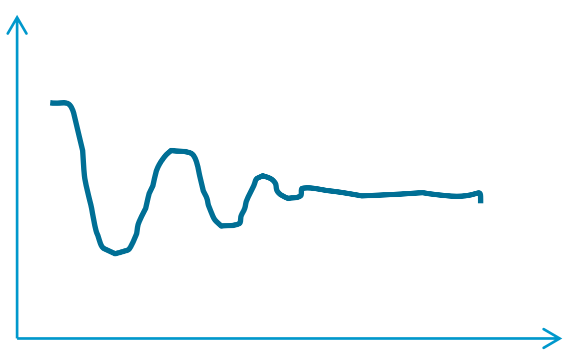
t < 0 | t=0 to t_c | t > t_c

Equal Area Criterion



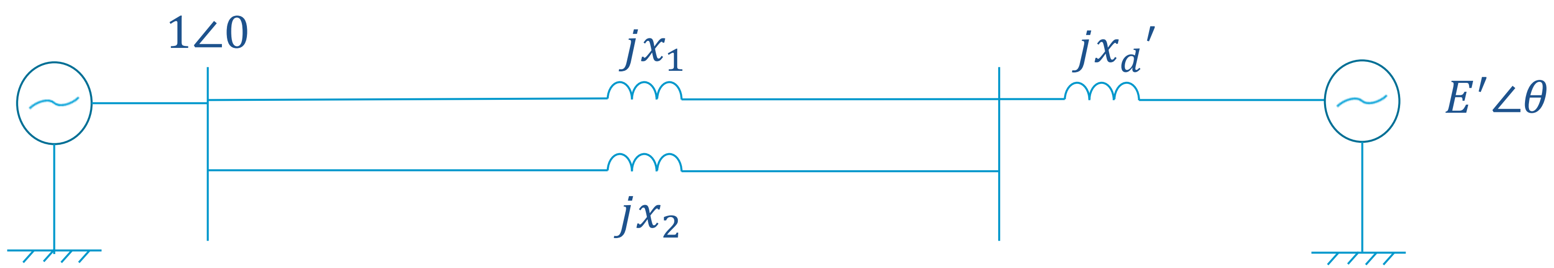
Assume

$t_c = 0$



Analytical criterion

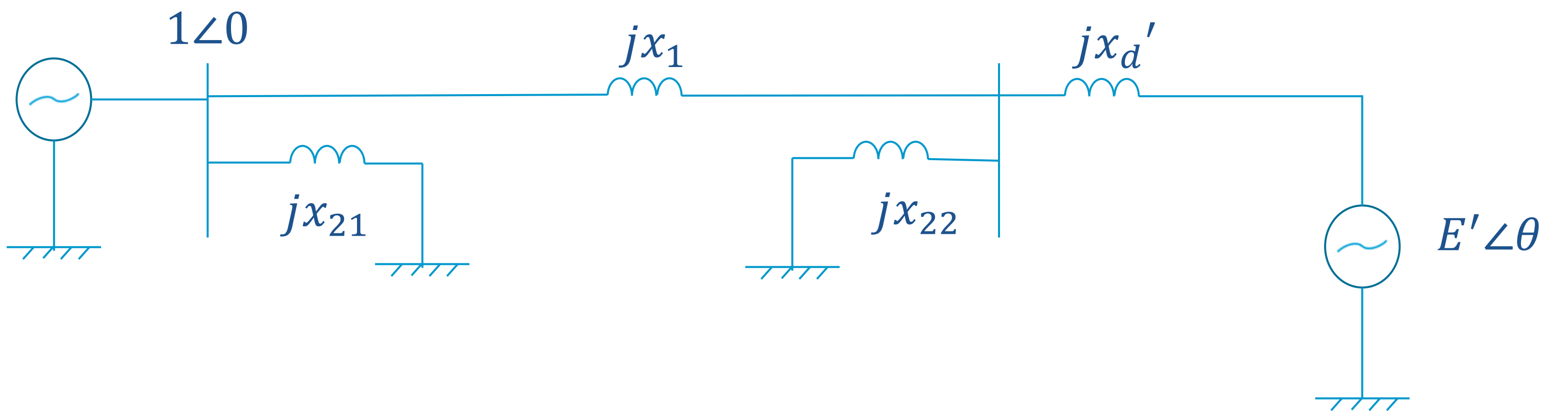
Pre-fault



$$P_e^{pre} = \frac{E' \cdot 1}{x_d{}' + (x_1 \parallel x_2)} \sin\theta$$

$$P_{max}^{pre} = \frac{E' \cdot 1}{x_d{}' + (x_1 \parallel x_2)}$$

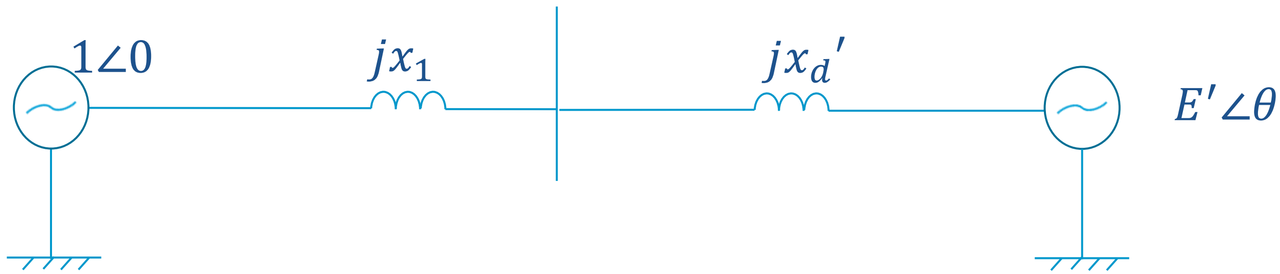
Fault-on system



$$P_e^{fault} = \frac{E' \cdot V_{Th}}{x_d{}' + x_{Th}} \sin\theta$$

$$P_{max}^{fault} = \frac{E' \cdot V_{Th}}{x_d{}' + x_{Th}}$$

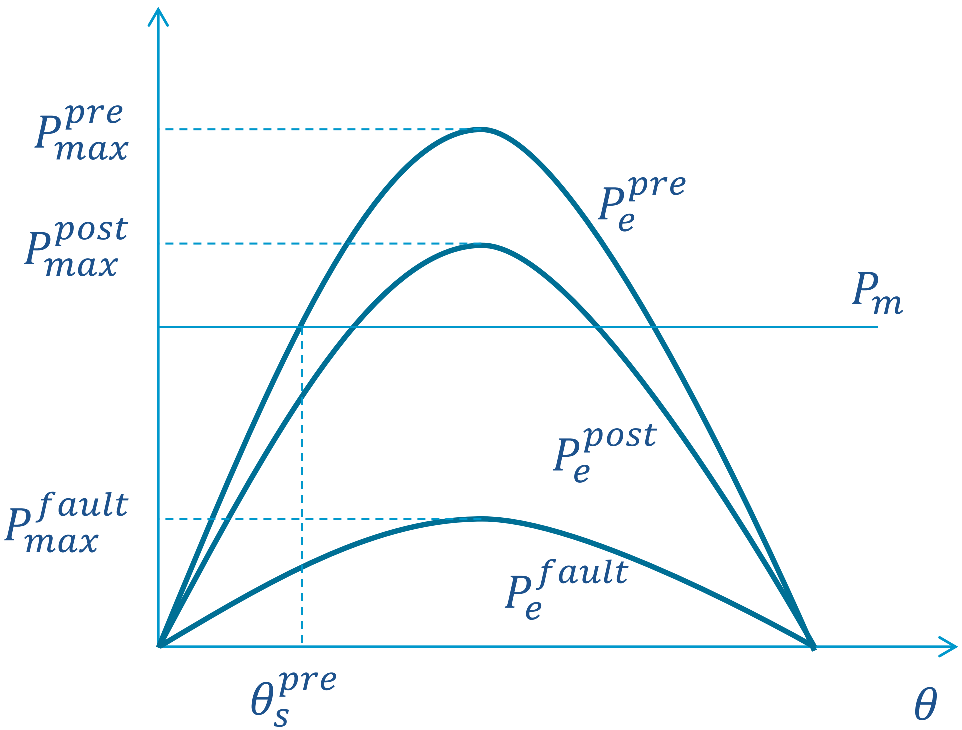
Post-fault system



$$P_e^{post} = \frac{E'\cdot 1}{x_d{}' + x_1}\sin\theta$$

$$P_{max}^{post} = \frac{E'\cdot 1}{x_d{}' + x_1}$$

Power-Angle curves



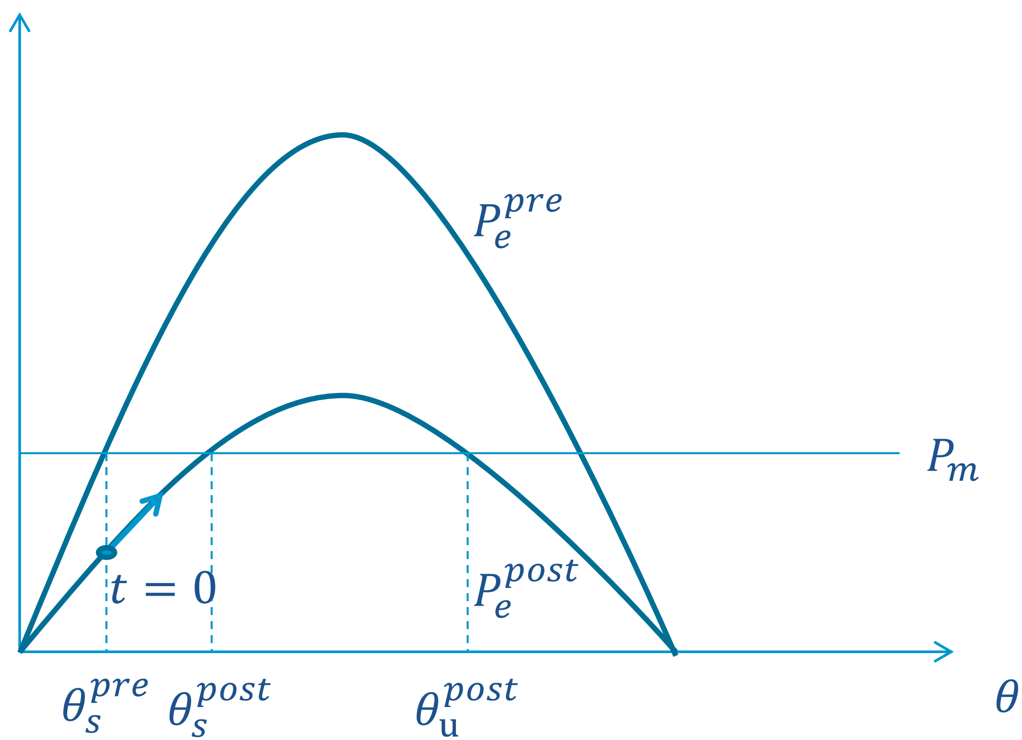
$$P_{max}^{pre} = \frac{E' \cdot 1}{x_d{}' + (x_1 \parallel x_2)}$$

$$P_{max}^{fault} = \frac{E' \cdot V_{Th}}{x_d{}' + x_{Th}}$$

$$P_{max}^{post} = \frac{E' \cdot 1}{x_d{}' + x_1}$$

Transient analysis

Fault clearing assumed instantaneous. K_D=0

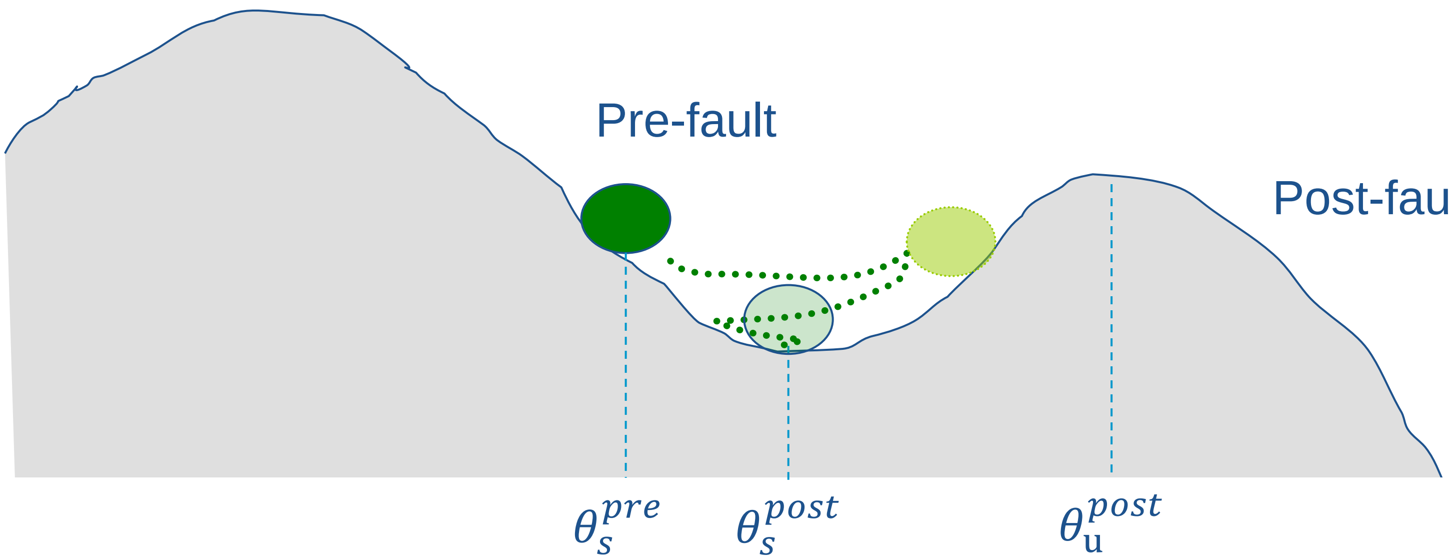


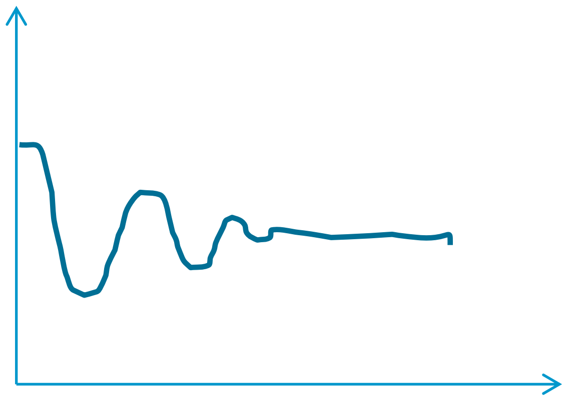
$$2H\dot{\omega} = P_m - P_e$$

1) $P_e^{post} < P_m \Rightarrow$
$\dot{\omega} > 0 \Longrightarrow \ \omega \uparrow$

2) $P_e^{post} > P_m \Rightarrow$
$\dot{\omega} < 0 \Longrightarrow \ \omega \downarrow$

Transient stable case

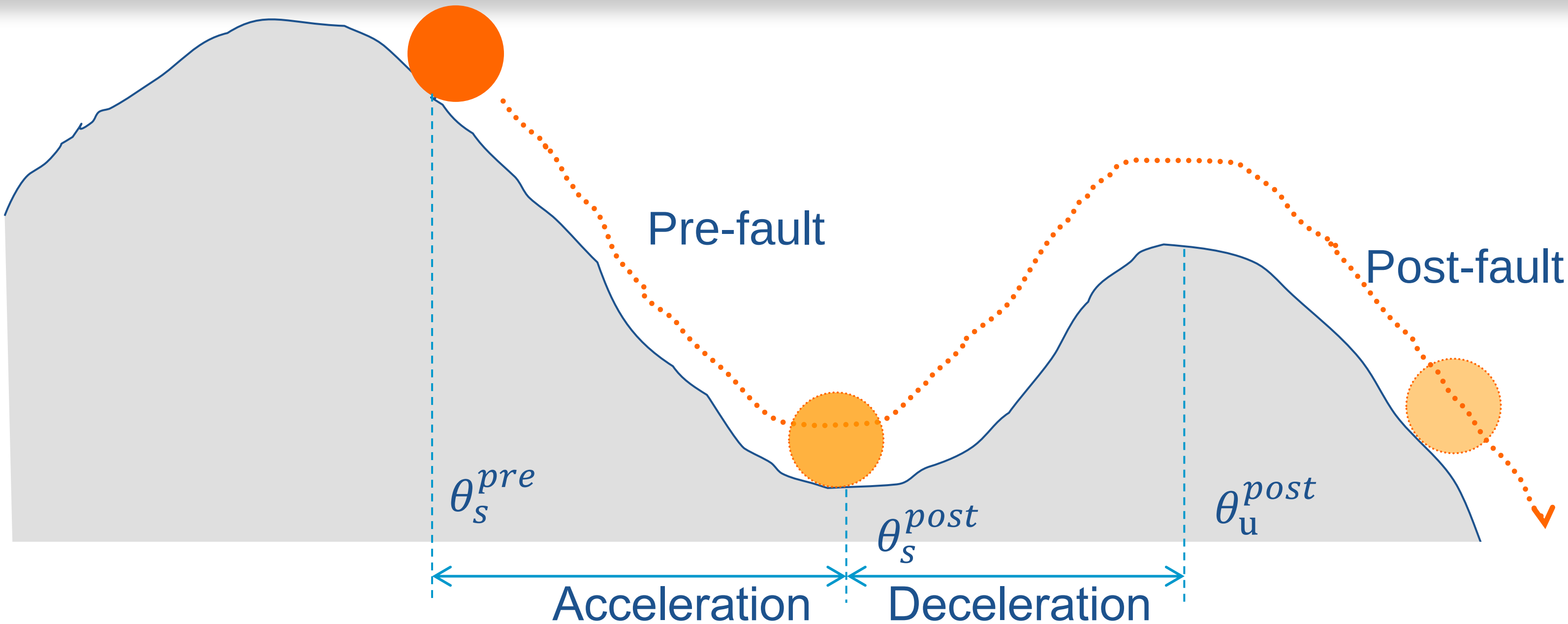


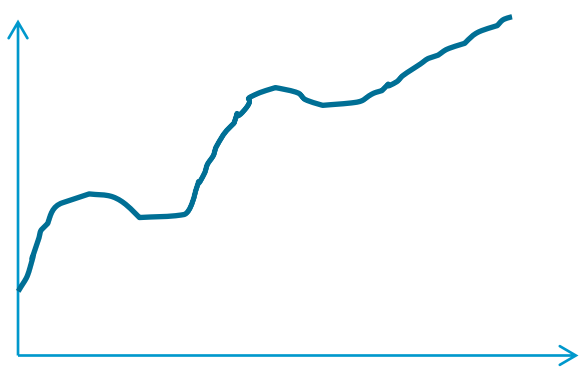


Net Acceleration < Maximum Deceleration

$\Longrightarrow$Stable

Transient unstable case

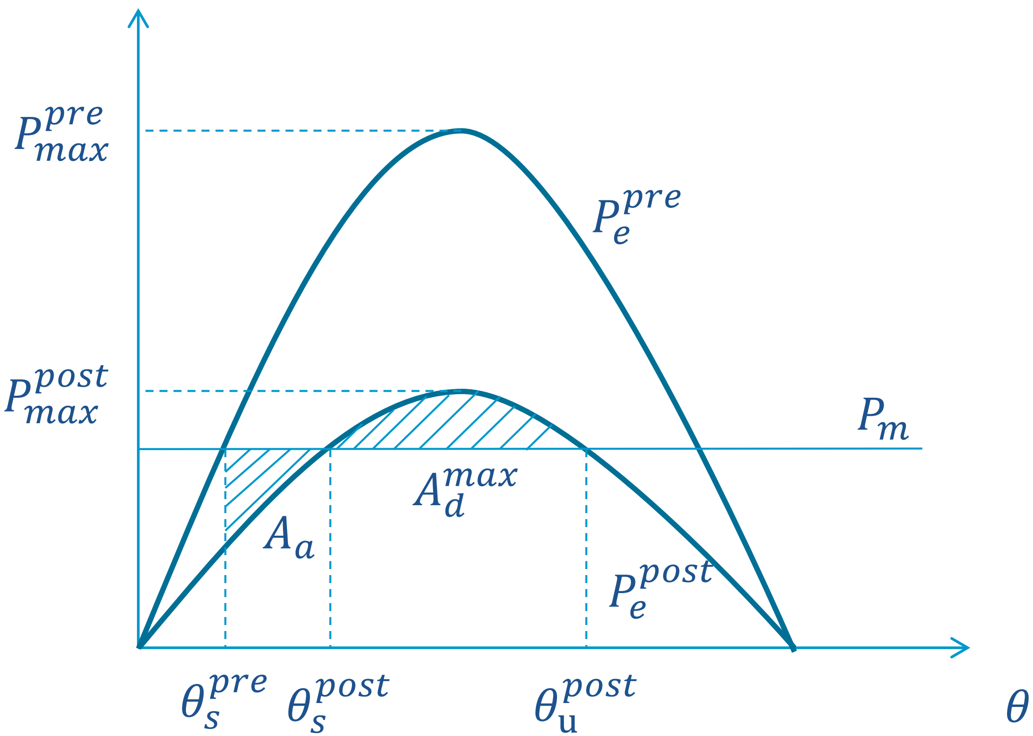




Net Acceleration > Maximum Deceleration

$\Longrightarrow$ Unstable

Equal Area Criterion



$$A_a < A_d^{max}$$
$$\Rightarrow Stable$$
$$A_a > A_d^{max}$$
$$\Rightarrow Unstable$$

Area Definitions

$$A_a = \int_{\theta_s^{pre}}^{\theta_s^{post}} \left(P_T - P_e^{post}\right) d\theta \propto KE_{acceleration}$$

$$A_d^{max} = \int_{\theta_s^{post}}^{\theta_\mathrm{u}^{post}} \left(P_e^{post} - P_T\right) d\theta \propto KE_{deceleration}^{max}$$

$$A_a < A_d^{max} \Longrightarrow Transient\ Stable$$

$$A_a > A_d^{max} \Longrightarrow Transient\ Unstable$$

Equal Area Criterion

$\Longrightarrow$ Pick up K.E. during acceleration from θ_s^{pre} to θ_s^{post}

$\Longrightarrow$ Loses K.E. between θ_s^{post} and θ_u^{post}

$\Longrightarrow$ When K.E. becomes zero, rotor angle turns back

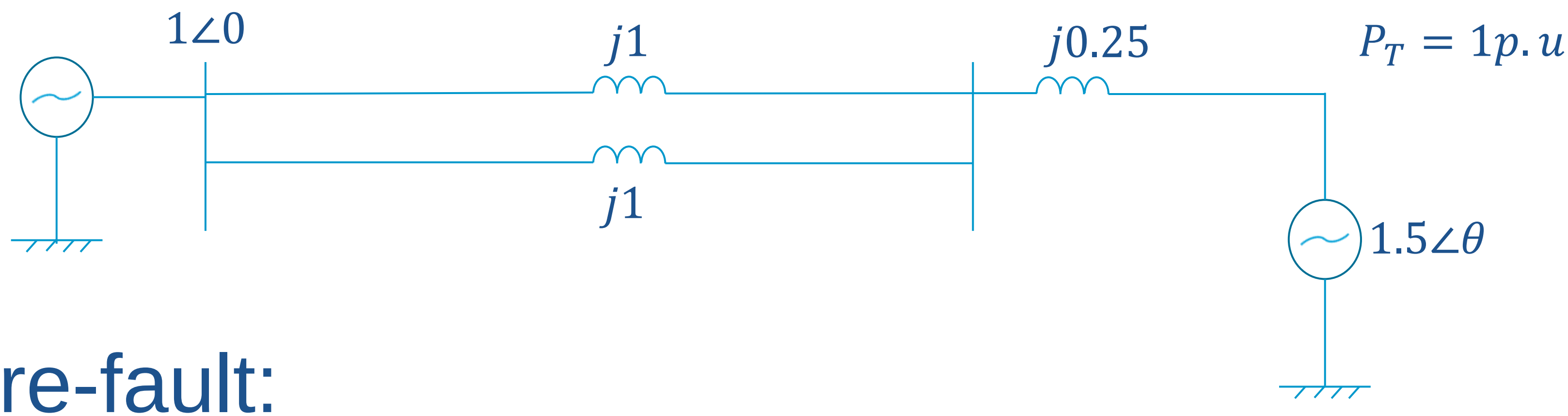
- $\Longrightarrow$ stable if $KE_{acc} < KE_{dmax}$, i.e. $A_a < A_{dmax}$

$\Longrightarrow$ $A_{dmax} < A_a \Longrightarrow$ rotor angle goes past θ_u^{post}

- $\Longrightarrow$ Continues to accelerate

θ_u^{post} $\Longrightarrow$ "point of no return" $\Longrightarrow$ Transient unstable

Example



Pre-fault:

$$P_e^{pre} = 2\sin\theta\,, P_m = 1 \Longrightarrow \theta_s^{pre} = 30^o$$

Post-fault:

$$P_e^{post} = \frac{1.5}{1.25}\sin\theta = 1.2\sin\theta \Longrightarrow \theta_s^{post} = 56.4^o$$

$$\theta_{\mathrm{u}}^{post} = 123.6^o$$

Area computations

$$A_a = \int_{30^o}^{56.4^o} (1 - 1.2\sin\theta)d\theta$$

$$= \int_{0.524}^{0.985} (1 - 1.2\sin\theta)d\theta$$

$$= (\theta + 1.2\cos\theta)\Big|_{0.524}^{0.985} = 0.0856$$

$$A_d^{max} = \int_{0.985}^{2.157} (1.2\sin\theta - 1)d\theta$$

$$= (-1.2\cos\theta - \theta)\Big|_{0.985}^{2.157} = 0.1553$$

$$A_d^{max} > A_a \Longrightarrow Transient\ Stable$$

Higher loading case

$$\text{Say } P_T = 1.1$$

$$\Longrightarrow \theta_s^{pre} = 0.582$$

$$\theta_s^{post} = 1.16$$

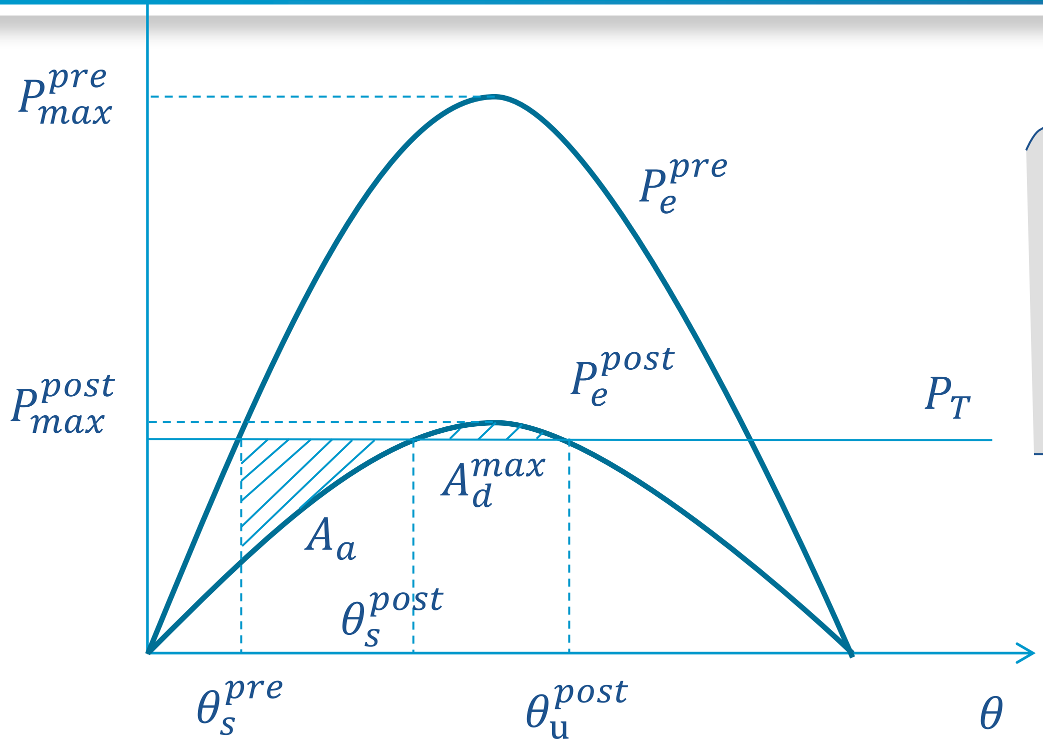
$$\theta_\mathrm{u}^{post} = 1.98$$

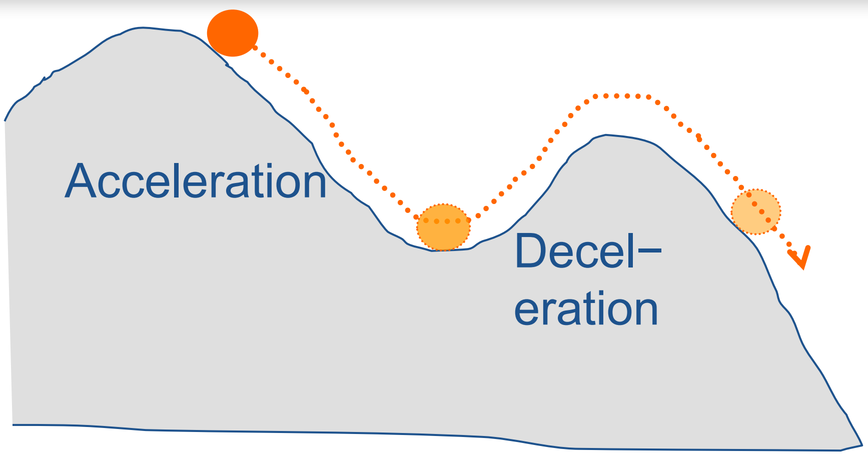
$$A_a = \int_{0.582}^{1.16} (1.1 - 1.2\sin\theta)d\theta = 0.1135$$

$$A_d^{max} = \int_{1.16}^{1.98} (1.2\sin\theta - 1.1)d\theta = 0.0555$$

$$A_d^{max} < A_a \Longrightarrow Unstable$$

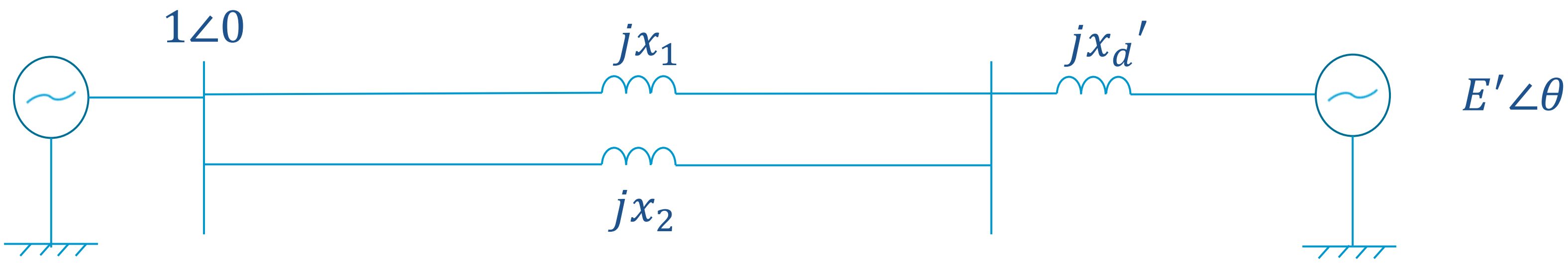
Transient Instability





$$A_a > A_d^{max} \Longrightarrow \begin{array}{c} KE\ keeps\ increasing \\ Rotor\ spins\ faster\ and\ faster \\ \Longrightarrow Instability \end{array}$$

Equal Area Criterion Summary

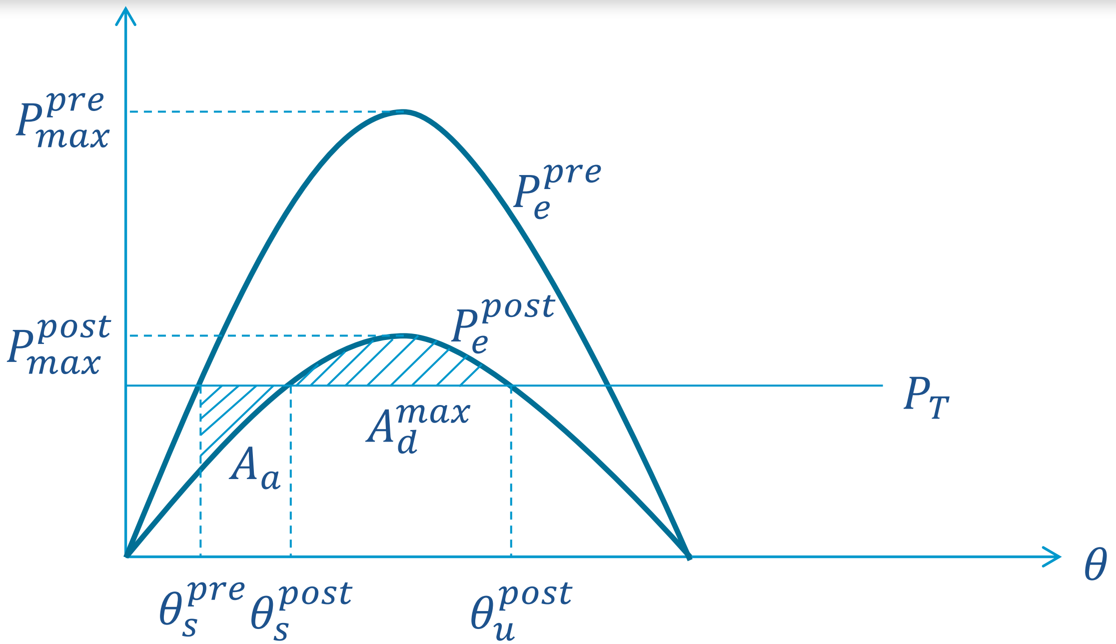


$$\dot{\theta} = (\omega - 1)\omega_s$$

$$2H\dot{\omega} = P_T - P_e - K_D(\omega - 1)$$

$$K_D = 0, t_c = 0$$

Analytical Criterion



$$A_a = \int_{\theta_s^{pre}}^{\theta_s^{post}} \left(P_T - P_e^{post}\right) d\theta \propto KE_{acc}$$

$$A_d^{max} = \int_{\theta_s^{post}}^{\theta_{\mathrm{u}}^{post}} \left(P_e^{post} - P_T\right) d\theta \propto KE_{dec}^{max}$$

Stability Concepts

- Small-signal Stability
  - Ability to damp out small perturbations
  - Oscillations?
- Transient stability
  - Recovery from large disturbances
  - Islanding? Voltage collapse?