Economic Dispatch

Lossless Economic Dispatch
Lossy Economic Dispatch

1

Optimization

Minimize total costs (minimize losses)

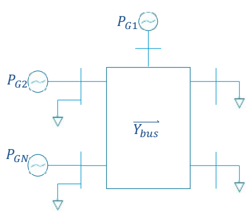
- Generation costs – Economic dispatch
- Generation scheduling – Unit commitment
- Market economics
  - Bidding, Market structure
  - Maximize profits

 2

2

Economic dispatch

$$f_i = \frac{a_i}{2} P_{Gi}{}^2 + b_i P_{Gi} + c_i \quad (\$/h)$$



How to change $P_{G1}, \ldots, P_{GN}$ so that generation meets the demand

$$P_{G1} + P_{G2} + \cdots + P_{GN}$$
$$= P_{L1} + P_{L2} + \cdots + P_{LN}$$
$$+ Line\ Losses$$

 3

3

Lossless Economic Dispatch

Case 1: No losses (lossless case)

$$\sum_{i=1}^{N} P_{Gi} = \sum P_{Li} = P_D$$

Optimization:

$$\boldsymbol{minimize} \ \sum_{i=1}^{N} f_i$$

$$subject\ to\colon\ P_D - \sum P_{Gi} = 0$$

 4

4

Constrained optimization

Lagrangian $\mathcal{L}(P_{G1}, \ldots, P_{GN}, \lambda)$

$$= \sum_{i=1}^{N} f_i(P_{Gi}) + \lambda\left(P_D - \sum P_{Gi}\right)$$

$$\frac{\partial \mathcal{L}}{\partial P_{Gi}} = \frac{\partial f_i}{\partial P_{Gi}} - \lambda = IC_i - \lambda = 0, \qquad i = 1, \ldots, \mathrm{N}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = P_D - \sum P_{Gi} = 0$$

$\boldsymbol{\lambda = IC_1 = IC_2 = \cdots = IC_N}$ for optimality

 5

5

Example:

$$f_1 = 9P_{G1} + 0.01P_{G1}{}^2 \quad \$/h$$
$$f_2 = 5P_{G2} + 0.02P_{G2}{}^2 \quad \$/h$$
$$P_D = 500MW; \quad P_{G1} + P_{G2} = 500$$
$$\Longrightarrow IC_1 = 9 + 0.02P_{G1}$$
$$IC_2 = 5 + 0.04P_{G2}$$
$$\mathcal{L}(P_{G1}, P_{G2}, \lambda) = f_1 + f_2 + \lambda(500 - (P_{G1} + P_{G2}))$$
$$\frac{\partial \mathcal{L}}{\partial P_{G1}} = IC_1 - \lambda = 0 \Longrightarrow 9 + 0.02P_{G1} - \lambda = 0$$
$$\Longrightarrow \lambda = 9 + 0.02P_{G1}$$

 6

6

Example:

$$\frac{\partial \mathcal{L}}{\partial P_{G2}} = IC_2 - \lambda = 0 \Longrightarrow 5 + 0.04P_{G2} - \lambda = 0$$
$$\Longrightarrow \lambda = 5 + 0.04P_{G2}$$
$$P_{G1} = \frac{\lambda - 9}{0.02} = 50(\lambda - 9)$$
$$P_{G2} = \frac{\lambda - 5}{0.04} = 25(\lambda - 5)$$
$$P_{G1} + P_{G2} = 500 \Longrightarrow 75\lambda - 450 - 125 = 500$$
$$\Longrightarrow \lambda = \frac{1075}{75} = 14.33$$

 7

7

Example:

$$\Downarrow$$
$$P_{G1} = 50(\lambda - 9) = 266.67\ MW$$
$$\Longrightarrow f_1 = 3111\ \$/h$$
$$P_{G2} = 25(\lambda - 5) = 233.33\ MW$$
$$\Longrightarrow f_2 = 2256\ \$/h$$

$$500\ MW \Longrightarrow 5367\,\$/h$$
$$\Longrightarrow 10.73\ \$/MWh$$
$\Longrightarrow 1.073\ cents/kWh$ unit cost

 8

8

Generation Limits

$$P_{Gimin} \le P_{Gi} \le P_{Gimax}$$
$$\Longrightarrow When\ limit\ reached, freeze\ P_{Gi}\ at\ the\ limit$$
$$\Longrightarrow Optimize\ the\ rest\ of\ the\ problem$$

 9

9

Example (continued)

$$0 \le P_{G1} \le 500$$
$$0 \le P_{G2} \le 500$$
$$P_D = 900 \Longrightarrow 75\lambda = 575 + 900$$
$$\lambda = 19.67$$
$$\Longrightarrow P_{G1} = 533.33\ \text{MW}$$
$$P_{G2} = 366.67\ \text{MW}$$
$$P_{G1} > 500\ \text{MW} \Longrightarrow set\ \ P_{G1} = 500\ MW$$
$$\Longrightarrow P_{G2} = 400\ MW$$

 10

10

Example

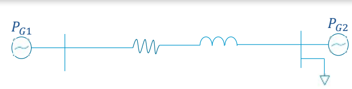
Dispatch Table:

P_D	P_{G1}	P_{G2}
100	67	33
⋮		
500	267	233
⋮		
900	500	400
1000	500	500

 11

11

Lossy case



$$P_{G2} \Longrightarrow No\ effect\ on\ losses$$
$$P_{G1} \Longrightarrow Impacts\ on\ losses$$
$$P_{Gi}\ Close\ to\ loads \Longrightarrow Less\ effect\ on\ losses$$
$$Far\ from\ loads \Longrightarrow More\ effect\ on\ losses$$

Usually cheap generation away from load centers $\Longrightarrow$ Effective cost higher due to transmission losses

 12

12

Loss Representation

$$\text{Losses} \quad P_L(P_{G1}, \dots, P_{GN})$$

$$= \underbrace{B_{00}}_{\text{Constant}} + \underbrace{\sum_{i=1}^{N} B_{0i} P_{Gi}}_{\text{Linear}} + \underbrace{\sum_{i=1}^{N}\sum_{j=1}^{N} B_{ij} P_{Gi} P_{Gj}}_{\text{Quadratic}}$$

13

Optimization

$$min \sum_{i=1}^{N} f_i(P_{Gi})$$

$$Subject\ to \quad P_D + P_L - \sum_{i=1}^{N} P_{Gi} = 0$$

$$min\, \mathcal{L}(P_{G1}, \dots, P_{GN}, \lambda)$$

$$= \sum f_i(P_{Gi}) + \lambda \left(P_D + P_L(P_{G1}, \dots, P_{GN}) - \sum P_{Gi} \right)$$

14

Lossy Economic Dispatch

$$\frac{\partial \mathcal{L}}{\partial P_{Gi}} = \frac{\partial f_i}{\partial P_{Gi}} + \lambda \left(\frac{\partial P_L}{\partial P_{Gi}} - 1 \right) = 0$$

$$\Rightarrow \lambda = \frac{IC_i}{1 - \frac{\partial P_L}{\partial P_{Gi}}} \qquad i = 1, \dots, N$$

$$P_D + P_L - \sum P_{Gi} = 0$$

N+1 equations, N+1 unknowns

15

Lossy Case

$$P_L = B_{00} + \sum_{i=1}^{N} B_{0i} P_{Gi} + \sum_{i=1}^{N}\sum_{j=1}^{N} B_{ij} P_{Gi} P_{Gj}$$

$$\frac{\partial P_L}{\partial P_{Gi}} = B_{0i} + 2 \sum_{j=1}^{N} B_{ij} P_{Gj}$$

$$\left(1 - \frac{\partial P_L}{\partial P_{Gi}}\right) \lambda = a_i P_{Gi} + b_i \quad ① $$

$$\sum P_{Gi} = P_D + P_L(P_{G1}, \dots, P_{GN}) \quad ②$$

N+1 equations

N+1 unknowns

16

Iterative Method

$$\lambda^0 \overset{k=0}{\Longrightarrow} \text{N equations in N unknowns}$$

Solve for P_{Gi} from ①,

$\Rightarrow$ Update $\lambda^{k+1} = \lambda^k + \Delta\lambda^k$ using

$$\Delta\lambda^k = \frac{\lambda^k - \lambda^{k-1}}{\sum P_{Gi}{}^k - \sum P_{Gi}{}^{k-1}} \left(P_D + P_L{}^k - \sum P_{Gi}{}^k \right)$$

17

Example

$$f_1 = 9P_{G1} + 0.01P_{G1}{}^2 \quad \$/h$$
$$f_2 = 5P_{G2} + 0.02P_{G2}{}^2 \quad \$/h$$
$$\Rightarrow IC_1 = 9 + 0.02P_{G1}$$
$$IC_2 = 5 + 0.04P_{G2}$$
$$P_D = 500MW;$$

$$P_L = 0.0001P_{G1}{}^2 + 0.0002P_{G2}{}^2$$

$$\Rightarrow \frac{\partial P_L}{\partial P_{G1}} = 0.0002P_{G1}, \quad \frac{\partial P_L}{\partial P_{G2}} = 0.0004P_{G2}$$

18

Example

$$\min \quad f_1 + f_2$$
$$Subject\ to \quad P_{G1} + P_{G2} = P_D + P_L$$
$$\left(1 - \frac{\partial P_L}{\partial P_{Gi}}\right)\lambda = IC_i$$
$$\Longrightarrow (1 - 0.0002P_{G1})\lambda = 9 + 0.02P_{G1}$$
$$(1 - 0.0004P_{G2})\lambda = 5 + 0.04P_{G2}$$
$$P_{G1} + P_{G2} = P_D + P_L$$

3 equations, 3 unknowns.

19

Lossy Economic Dispatch

$$f_i(P_{Gi}) = \frac{a_i}{2}P_{Gi}^{\ 2} + b_iP_{Gi} + c_i \ \ \$/h$$
$$IC_i = \frac{df_i}{dP_{Gi}} = a_iP_{Gi} + b_i \ \ \$/MWh$$
$$P_{Loss} = B_{00} + \sum_{i=1}^{N} B_{0i}P_{Gi} + \sum_{i=1}^{N}\sum_{j=1}^{N} B_{ij}P_{Gi}P_{Gj}$$

generator = N, P_D = total demand

20

Derivation

Objectives:

$$min \sum_{i=1}^{N} f_i(P_{Gi})$$
$$Subject\ to \quad \sum_{i=1}^{N} P_{Gi} = P_D + P_L$$
$$\mathcal{L}(P_{G1}, \dots, P_{GN}, \lambda) = \sum f_i(P_{Gi}) + \lambda(P_D + P_{Loss} - \sum P_{Gi})$$

21

Algorithm derivation

$$\frac{\partial \mathcal{L}}{\partial P_{Gi}} = IC_i + \lambda\left(\frac{\partial P_L}{\partial P_{Gi}} - 1\right) = 0 \quad (1)$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = P_D + P_L(P_{G1}, \dots, P_{GN}) - \sum P_{Gi} = 0$$
$$Here, \quad \frac{\partial P_L}{\partial P_{Gi}} = B_{0i} + 2\sum_{j=1}^{N} B_{ij}P_{Gj}$$

22

Derivation (continued)

$$\frac{\partial \mathcal{L}}{\partial P_{Gi}} = IC_i + \lambda\left(\frac{\partial P_L}{\partial P_{Gi}} - 1\right) = 0 \quad (1)$$

(1) becomes

$$a_iP_{Gi} + b_i + \lambda\left(B_{i0} + 2\sum_{j=1}^{N} B_{ij}P_{Gj} - 1\right) = 0$$
$$for\ i = 1, \dots, N$$

23

Derivation (continued)

Grouping terms and rearranging,

$$\begin{bmatrix} a_1 + 2\lambda B_{11} & 2\lambda B_{12} & \cdots & 2\lambda B_{1N} \\ 2\lambda B_{21} & a_2 + 2\lambda B_{22} & & 2\lambda B_{2N} \\ \vdots & & \ddots & \vdots \\ 2\lambda B_{N1} & 2\lambda B_{N2} & \cdots & a_N + 2\lambda B_{NN} \end{bmatrix} \begin{bmatrix} P_{G1} \\ P_{G2} \\ \vdots \\ P_{GN} \end{bmatrix} = \begin{bmatrix} \lambda - \lambda B_{10} - b_1 \\ \lambda - \lambda B_{20} - b_2 \\ \vdots \\ \lambda - \lambda B_{N0} - b_N \end{bmatrix}$$

24

Intermediate step

$$or\quad [A]\begin{bmatrix} P_{G1} \\ \vdots \\ P_{GN} \end{bmatrix} = [B] \qquad (2)$$

$$where$$

$$A_{ii} = a_i + 2\lambda B_{ii} \quad (diagonal\ terms)$$
$$A_{ij} = 2\lambda B_{ij} \quad (off\text{-}diagonal\ terms)$$
$$B_i = \lambda - \lambda B_{i0} - b_i$$

25

Lossy Algorithm:

Solve lossless economic dispatch.

Lossless solution: $\lambda^0, {P_{G1}}^0, \dots, {P_{GN}}^0$

Let $\lambda^{-1} = \lambda^0 + \tilde{\lambda}$. (Say $\tilde{\lambda} = 0.5$).

Solve (2) to get the corresponding schedule ${P_{G1}}^{-1}, \dots, {P_{GN}}^{-1}$.

26

Flowchart

$$\begin{matrix} \lambda^{-1} & & \lambda^0 \\ {P_{G1}}^{-1} & , & {P_{G1}}^0 \\ \vdots & & \vdots \\ {P_{GN}}^{-1} & & {P_{GN}}^0 \end{matrix} \xRightarrow{k=0} Solve\ (2)\ to\ get\ {P_{Gi}}^K$$

$$\Downarrow$$

$${P_{Loss}}^k = B_{00} + \sum_{i=1}^N B_{0i}\,{P_{Gi}}^k + \sum_{i=1}^N \sum_{j=1}^N B_{ij}{P_{Gi}}^k{P_{Gj}}^k$$

$$\Downarrow$$

$$\Delta P^k = P_D + {P_{Loss}}^k - \sum {P_{Gi}}^k$$

$$\Downarrow$$

$$|\Delta P^k| < \epsilon\,? \xRightarrow{Yes} Stop$$

$$\Downarrow No$$

$$\Delta\lambda^k = \frac{\lambda^k - \lambda^{k-1}}{\sum {P_{Gi}}^k - \sum {P_{Gi}}^{k-1}}\Delta P^k$$

$$\Downarrow$$

$$\lambda^{k+1} = \lambda^k + \Delta\lambda^k$$

$k = k + 1$

27

Example:

$$f_1 = 9P_{G1} + 0.01{P_{G1}}^2 \quad \$/h$$
$$f_2 = 5P_{G2} + 0.02{P_{G2}}^2 \quad \$/h$$
$$\Longrightarrow IC_1 = 9 + 0.02P_{G1}$$
$$IC_2 = 5 + 0.04P_{G2}$$
$$P_D = 500\ MW;$$
$$P_{Loss} = 0.0001{P_{G1}}^2 + 0.0002{P_{G2}}^2$$

28

Example:

Lossless:

$$\lambda = 9 + 0.02P_{G1}$$
$$\Longrightarrow P_{G1} = \frac{\lambda - 9}{0.02} = 50(\lambda - 9)$$
$$\lambda = 5 + 0.04P_{G2}$$
$$\Longrightarrow P_{G2} = \frac{\lambda - 5}{0.04} = 25(\lambda - 5)$$
$$P_{G1} + P_{G2} = P_D = 500 = 75\lambda - 575$$
$$\Longrightarrow \lambda = \frac{1075}{75} = 14.33$$

29

Example:

$$\Downarrow$$
$$P_{G1} = 50(\lambda - 9) = 266.67\ MW$$
$$P_{G2} = 25(\lambda - 5) = 233.33\ MW$$
$$\Downarrow$$

Setting up Lossy Equations:

$$\frac{\partial P_L}{\partial P_{G1}} = 0.0002P_{G1}$$
$$\frac{\partial P_L}{\partial P_{G2}} = 0.0004P_{G2}$$

30

Example:

Lagrangian Derivatives:

$$\frac{\partial \mathcal{L}}{\partial P_{G1}} = IC_1 + \lambda\left(\frac{\partial P_{Loss}}{\partial P_{G1}} - 1\right) = 0$$

$$= 9 + 0.02P_{G1} + \lambda(0.0002P_{G1} - 1) = 0$$

$$\frac{\partial \mathcal{L}}{\partial P_{G2}} = 5 + 0.04P_{G2} + \lambda(0.0004P_{G2} - 1) = 0$$

$$\begin{bmatrix} 0.02 + \lambda 0.0002 & 0 \\ 0 & 0.04 + \lambda 0.0004 \end{bmatrix} \begin{bmatrix} P_{G1} \\ P_{G2} \end{bmatrix} = \begin{bmatrix} \lambda - 9 \\ \lambda - 5 \end{bmatrix} \quad - ③$$

31

Example:

$\lambda^0 = 14.33$ say $\lambda^{-1} = 14$

$P_{G1}{}^0 = 266.67$ $P_{G1}{}^{-1} = 219$

$P_{G2}{}^0 = 233.33$ $P_{G2}{}^{-1} = 197$ $(from\ ③)$

$$\overset{k=0}{\Longrightarrow} \Delta P^0 = P_D + P_{Loss}{}^0 - P_{G1}{}^0 - P_{G2}{}^0 = 18 > \epsilon$$

$$\Downarrow Continue$$

$$\Delta\lambda^0 = \frac{\lambda^0 - \lambda^{-1}}{\sum P_{Gi}{}^0 - \sum P_{Gi}{}^{-1}} \Delta P^0 = 0.0720$$

32

Example:

$$\Downarrow$$

$$\lambda^1 = 14.4053$$

$$\Downarrow$$

$$P_{G1}{}^1 = 236.24 \quad (from\ ③)$$

$$P_{G2}{}^1 = 205.53$$

$$\Downarrow$$

$$P_{Loss}{}^1 = 14.03\ MW$$

$$\Downarrow$$

$$\Delta P^1 = P_D + P_{Loss}{}^1 - P_{G1}{}^1 - P_{G2}{}^1 = 72.2 > \epsilon$$

33

Example:

$$\Downarrow Continue$$

$$\Delta\lambda^1 = -0.0893$$

$$\Downarrow$$

$$\lambda^2 = 14.316$$

$$\Downarrow$$

$$P_{G1}{}^2 = 232.51$$

$$P_{G2}{}^2 = 203.73$$

$$\Downarrow$$

$$P_{Loss}{}^2 = 13.7\ MW$$

34

Example:

$$\Downarrow$$

$$\Delta P^2 = 77.46\ MW > \epsilon$$

$$\Downarrow Continue$$

$$\Delta\lambda^2 = 1.2546$$

$$\Downarrow$$

$$\lambda^3 = 15.57$$

$$\Downarrow$$

$$P_{G1}{}^3 = 284.26$$

$$P_{G2}{}^3 = 228.66$$

35

Example:

$$\Downarrow$$

$$P_{Loss}{}^3 = 18.54\ MW$$

$$\Downarrow$$

$$\Delta P^3 = 5.6\ MW > \epsilon$$

$$\Downarrow Continue$$

$$\Delta\lambda^3 = 0.918$$

$$\Downarrow$$

$$\lambda^4 = 15.66$$

$$\Downarrow$$

$$P_{G1}{}^4 = 288, \quad P_{G2}{}^4 = 230.5$$

36

Example:

$$\Downarrow$$
$$P_{Loss}{}^{4} = 18.9\ MW$$
$$\Downarrow$$
$$\Delta P^4 = 0.445\ MW > \epsilon$$
$$\Downarrow Continue$$
$$\Delta\lambda^4 = 0.0074$$
$$\Downarrow$$
$$\lambda^5 = 15.6697$$
$$\Downarrow$$
$$P_{G1}{}^{5} = 288.3, \qquad P_{G2}{}^{5} = 230.6$$

Example:

$$\Downarrow$$
$$P_{Loss}{}^{5} = 18.95\ MW$$
$$\Downarrow$$
$$\Delta P^5 = 0.03\ MW < \epsilon = 0.1\ MW$$
$$\Downarrow Stop$$

$P_{G1} = 288.3\ MW$
$P_{G2} = 230.6\ MW$ is the optimal solution for
$\lambda = 15.67$
our Lossy Economic Dispatch.