E_E 491
Review Session #4

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Power Flow Equations

Bus Type	Given Parameters	Unknown Parameters
Slack Bus	V, δ	P, Q
Generator Bus	P, |V|	Q, δ
Load Bus	P, Q	V, δ

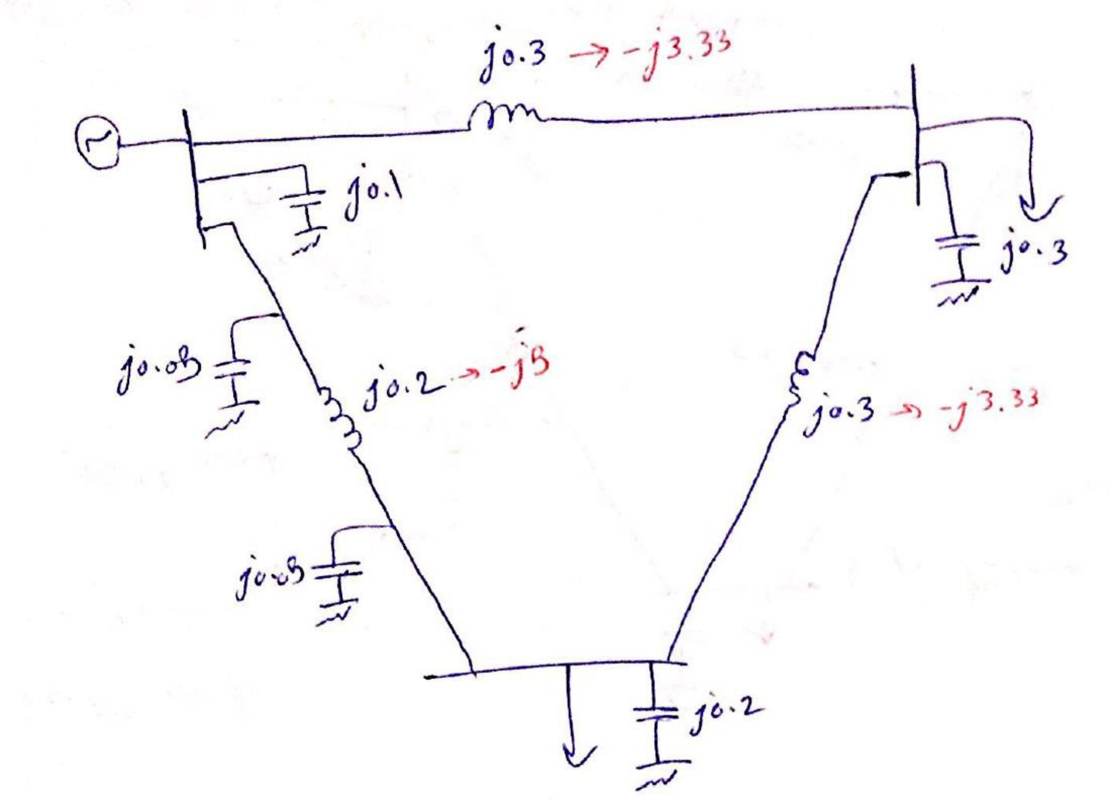
$$P_i = \sum_{j=1}^{n} |Y_{ij} V_i V_j| \cos(\delta_i - \delta_j - \theta_{ij})$$

$$Q_i = \sum_{j=1}^{n} |Y_{ij} V_i V_j| \sin(\delta_i - \delta_j - \theta_{ij})$$

Power Flow Equations (Ex. 1)

$V1 = 1, \delta 1 = 0, \; PL2 = 0.2, QL2 = 0.1, PL3 = 0.5, QL3 = 0.3$

(1) Slack Bus: $V1 = 1, \delta 1 = 0, P1 = ?, Q1 = ?$
(2) PQ Bus: $P2 = PG2 - Pl2 = 0 - 0.2 = -0.2, Q2 = -0.1, V2=?, \delta 2 = ?$
(3) PQ Bus: $P3 = -0.5, Q3 = -0.3, V3 = ?, \delta 3 = ?$



Power Flow Equations (Ex. 1)

$$\overline{Y_{BUS}} = \begin{bmatrix} 8.18\angle-90^\circ & 3.33\angle90^\circ & 5\angle90^\circ \\ 3.33\angle90^\circ & 6.36\angle-90^\circ & 3.33\angle90^\circ \\ 5\angle90^\circ & 3.33\angle90^\circ & 8.08\angle-90^\circ \end{bmatrix}$$

$P1 = 3.33*V1*V2*\cos(-\delta2-90) + 5*V1*V3*\cos(-\delta3-90)$

$Q1= 8.18*V1\text{^}2*\sin(90) + 3.33*V1*V2*\sin(\delta1-\delta2-90) + 5*V1*V3*\sin(\delta1-\delta3-90)$

Bus 2:

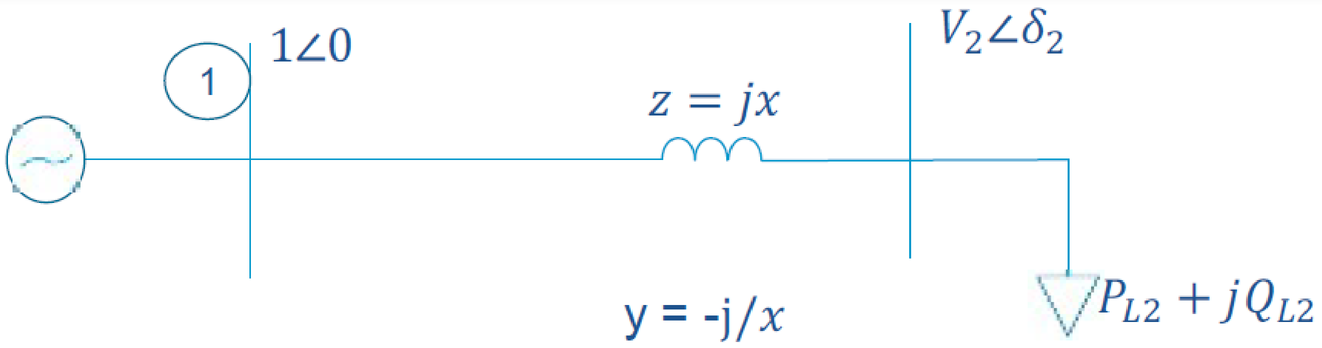
$P2 = 3.33*V2*V1*\cos(\delta2-\delta1-90) + 6.36*V2\text{^}2*\cos(90) +3.33*V2*V3*\cos(\delta2-\delta3-90)$

$Q2 = 3.33*V2*V1*\sin(\delta2-\delta1-90) + 6.36*V2\text{^}2*\sin(90) +3.33*V2*V3*\sin(\delta2-\delta3-90)$

Bus 3:

$P3 = 5*V3*V1*\cos(\delta3-\delta1-90) + 3.33*V3*V2*\cos(\delta3-\delta2-90) +8.08*V3\text{^}2*\cos(90)$

$Q3 = 5*V3*V1*\sin(\delta3-\delta1-90) + 3.33*V3*V2*\sin(\delta3-\delta2-90) +8.08*V3\text{^}2*\sin(90)$



$$\overrightarrow{Y_{Bus}} = \begin{bmatrix} \frac{1}{x}\angle -90^0 & \frac{1}{x}\angle 90^0 \\ \frac{1}{x}\angle 90^0 & \frac{1}{x}\angle -90^0 \end{bmatrix}$$

PV Diagram

PV Diagram

$$P_2 = V_2 V_1 y_{21} \cos(\delta_2 - \delta_1 - \theta_{21}) + V_2^2 y_{22} \cos(\theta_{22}) \quad (1)$$

$$P_2 = V_2 \frac{1}{x} \cos(\delta_2 - 90) + V_2^2 \frac{1}{x} \cos(-90) \quad (2)$$

$$P_2 = V_2 \frac{1}{x} \sin(\delta_2) = -P_{L_2} \quad (3)$$

$$Q_2 = V_2 V_1 y_{21} \sin(\delta_2 - \delta_1 - \theta_{21}) - V_2^2 y_{22} \sin(\theta_{22}) \quad (4)$$

$$Q_2 = V_2 \frac{1}{x} \sin(\delta_2 - 90) - V_2^2 \frac{1}{x} \sin(-90) \quad (5)$$

$$Q_2 = -V_2 \frac{1}{x} \cos(\delta_2) + V_2^2 \frac{1}{x} = -Q_{L_2} \quad (6)$$

$$V_2 \sin(\delta_2) = -x P_{L_2} \quad (7)$$

$$V_2^2 (\sin(\delta_2))^2 = (x P_{L_2})^2 \quad (8)$$

$$V_2^2 (\cos(\delta_2))^2 = (\mathrm{x} Q_{L_2} + V_2^2)^2 \quad (9)$$

$$V_2^2 = (x P_{L_2})^2 + (x Q_{L_2})^2 + 2\mathrm{x} Q_{L_2} V_2^2 + V_2^4 \quad (10)$$

$$V_2^4 + V_2^2 \left(2\mathrm{x} Q_{L_2} - 1\right) + x^2 \left(P_{L_2}^2 + Q_{L_2}^2\right) = 0 \quad (11)$$

PV Diagram (x=0.4, Unity PF)

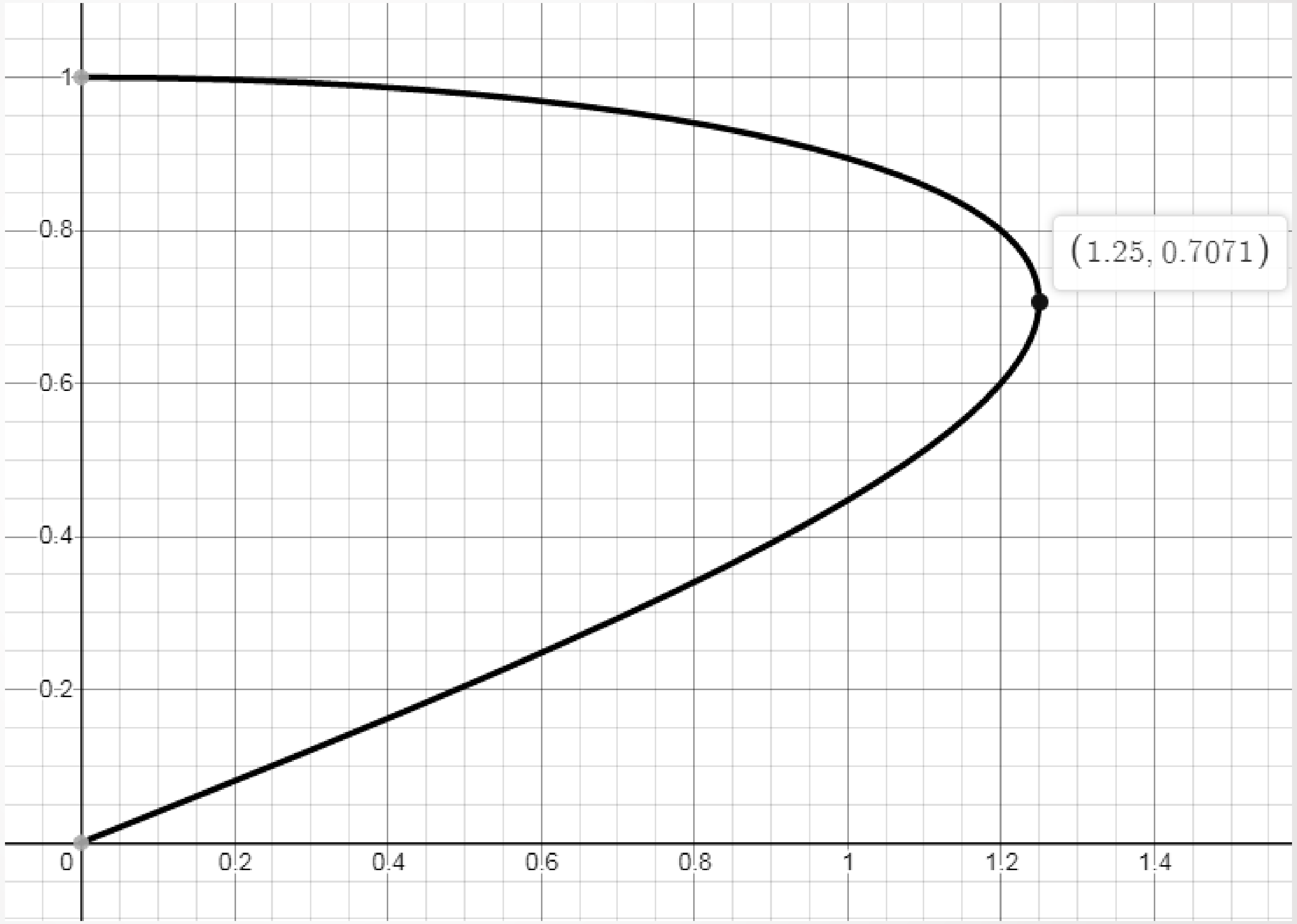
$$\cos(\delta_2) = 1 \Rightarrow \delta_2 = 0$$

$$Q_{L_2} = P_{L_2}\tan(\delta_2) = 0$$

$$V_2^4 - V_2^2 + 0.16P_{L_2}^2 = 0$$

$$V_2^2 = \frac{1 \pm \sqrt{1 - 0.64P_{L_2}^2}}{2}$$

$$V_2 = \begin{cases} 1, 0 & , P_{L_2} = 0 \\ \sqrt{\dfrac{1 \pm \sqrt{1 - 0.64P_{L_2}^2}}{2}} & , 0 < P_{L_2} < 1.25 \\ \dfrac{1}{\sqrt{2}} & , P_{L_2} = 1.25 \\ Undefined & , P_{L_2} > 1.25 \end{cases}$$



PV Diagram (x=0.4, 0.8 PF Lagging)

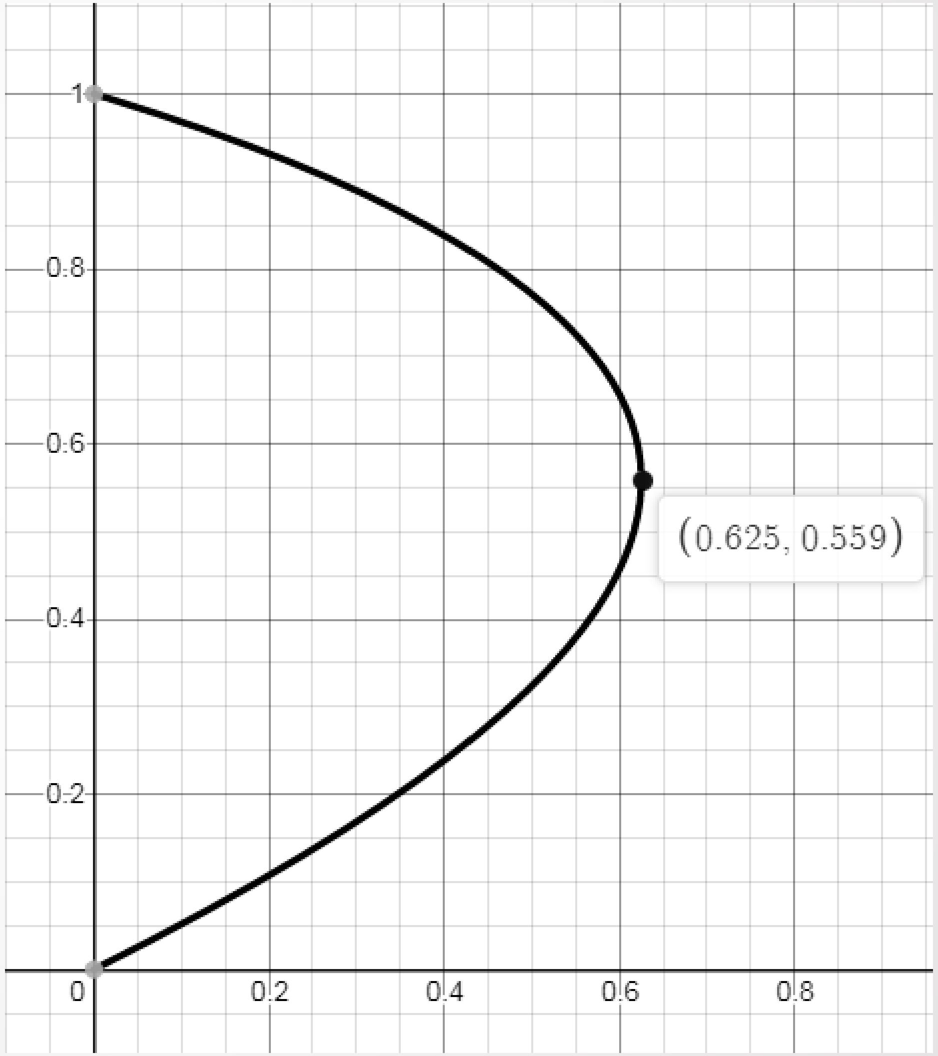
$$\cos(\delta_2) = 0.8 \Rightarrow \delta_2 = \arccos(0.8)$$

$$Q_{L_2} = P_{L_2}\tan(\delta_2) = P_{L_2}\tan(\arccos(0.8)) = 0.75P_{L_2}$$

$$V_2^4 + V_2^2\left(0.6P_{L_2} - 1\right) + 0.25P_{L_2}^2 = 0$$

$$V_2^2 = \frac{(1 - 0.6P_{L_2}) \pm \sqrt{1 - 1.2P_{L_2} - 0.64P_{L_2}^2}}{2}$$

$$V_2 = \begin{cases} 1, 0 & , P_{L_2} = 0 \\ \sqrt{\dfrac{(1 - 0.6P_{L_2}) \pm \sqrt{1 - 1.2P_{L_2} - 0.64P_{L_2}^2}}{2}} & , 0 < P_{L_2} < 0.625 \\ 0.559 & , P_{L_2} = 0.625 \\ Undefined & , P_{L_2} > 0.625 \end{cases}$$



PV Diagram (x=0.4, 0.8 PF Leading)

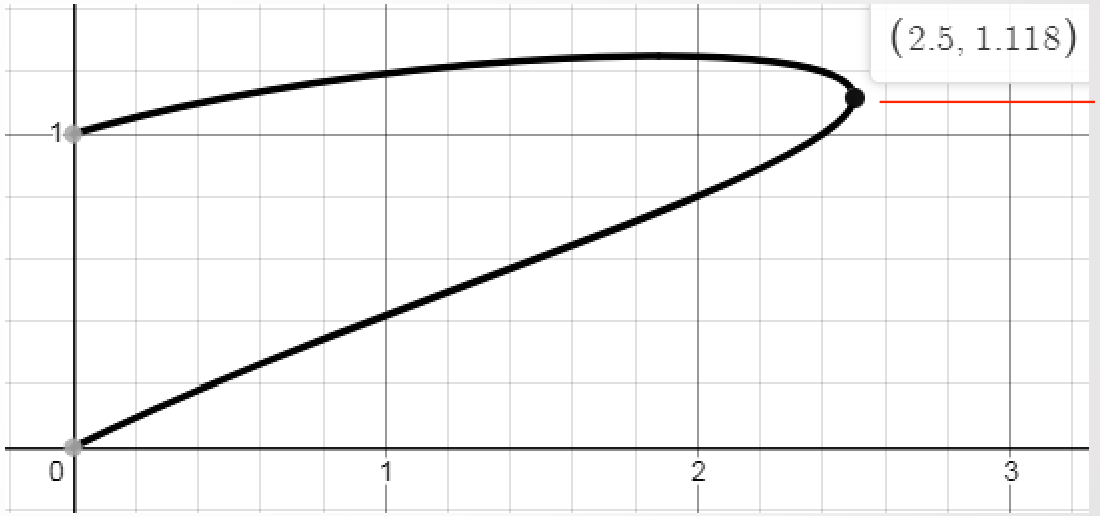
$$\cos(\delta_2) = 0.8 \Rightarrow \delta_2 = \arccos(0.8)$$

$$Q_{L_2} = -P_{L_2}\tan(\delta_2) = -P_{L_2}\tan(\arccos(0.8)) = -0.75P_{L_2}$$

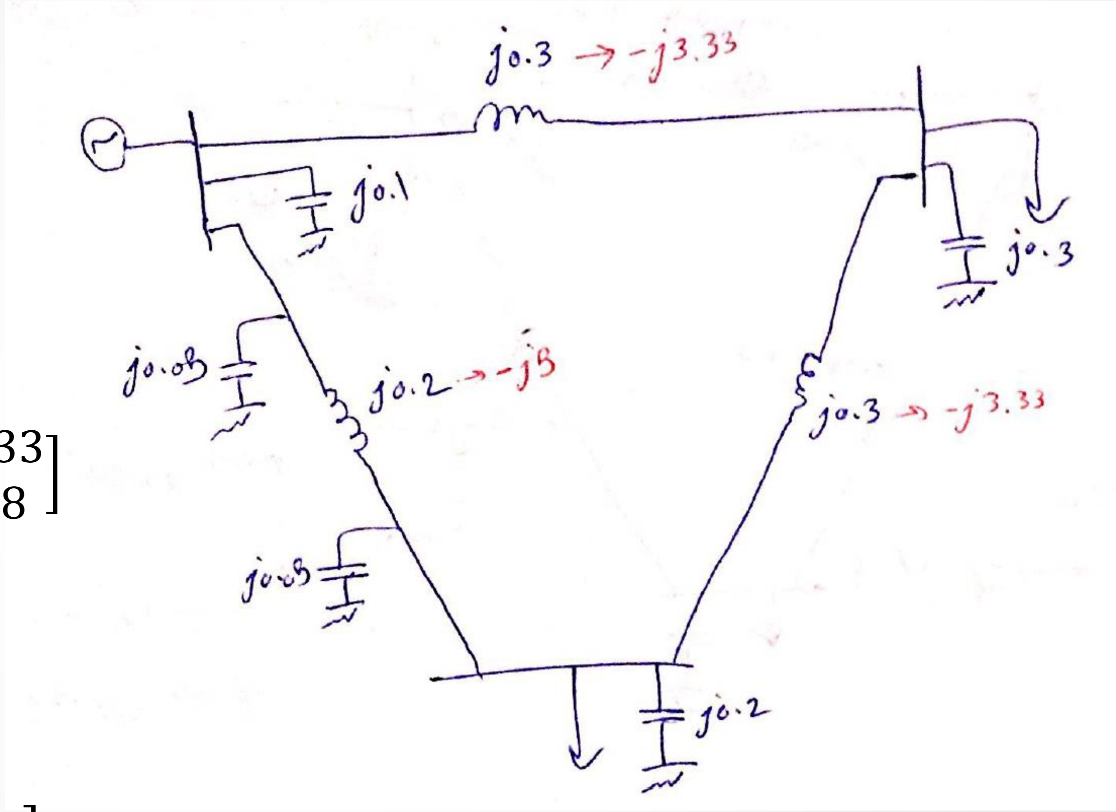
$$V_2^4 - V_2^2\left(0.6P_{L_2} + 1\right) + 0.25P_{L_2}^2 = 0$$

$$V_2^2 = \frac{(1 + 0.6P_{L_2}) \pm \sqrt{1 + 1.2P_{L_2} - 0.64P_{L_2}^2}}{2}$$

$$V_2 = \begin{cases} 1, 0 & , P_{L_2} = 0 \\ \sqrt{\dfrac{(1 + 0.6P_{L_2}) \pm \sqrt{1 + 1.2P_{L_2} - 0.64P_{L_2}^2}}{2}} & , 0 < P_{L_2} < 2.5 \\ 1.118 & , P_{L_2} = 2.5 \\ Undefined & , P_{L_2} > 2.5 \end{cases}$$



DC Power Flow



$$\overrightarrow{Y_{BUS}} = \begin{bmatrix} -j8.18 & j3.33 & j5 \\ j3.33 & -j6.36 & j3.33 \\ j5 & j3.33 & -j8.08 \end{bmatrix}$$

$$B_{DC} = -Imag[Y_{BUS}]_{\substack{2:3\\2:3}} = \begin{bmatrix} 6.36 & -3.33 \\ -3.33 & 8.08 \end{bmatrix}$$

$$P_{L_2} = 0.2, Q_{L_2} = 0.1$$
$$P_{L_3} = 0.5, Q_{L_3} = 0.3$$

$$\begin{bmatrix} P_2 \\ P_3 \end{bmatrix} = B_{DC} \begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix} \Rightarrow \begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix} = B_{DC}^{-1} \begin{bmatrix} -P_{L_2} \\ -P_{L_3} \end{bmatrix} \Rightarrow \begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix} = \begin{bmatrix} 0.2005 & 0.0826 \\ 0.0826 & 0.1578 \end{bmatrix} \begin{bmatrix} -0.2 \\ -0.5 \end{bmatrix} = \begin{bmatrix} -0.0814 \\ -0.0954 \end{bmatrix}$$

$$\begin{bmatrix} \overrightarrow{V_1} \\ \overrightarrow{V_2} \\ \overrightarrow{V_3} \end{bmatrix} = \begin{bmatrix} 1\angle 0 \\ 1\angle -0.0814 \\ 1\angle -0.0954 \end{bmatrix}$$

Questions?