We have considered various derivative free methods:

- Nelder Mead Simplex Method
- Genetic Algorithms
- Simulated Annealing
- Coordinate Search

For each of these methods we can employ a simple strategy for enforcing constraints: Extreme Barrier Method.

$$(*) \quad \boxed{\begin{array}{ll} \min & f(x) \\ \text{s.t.} & x \in \Omega \end{array}} \Rightarrow \boxed{\begin{array}{l} \min \ \bar{f}(x) \\ \text{s.t. } x \in \mathbb{R}^n \\ \bar{f}(x) := \begin{cases} f(x) & x \in \Omega \\ \infty & x \notin \Omega \end{cases} \end{array}}$$

$\bar{f}(x)$ is an extended value function with range $\mathbb{R} \cup \{\infty\}$.

Important Observation

If $f(x)$ is convex and bounded below and Ω is convex then $\bar{f}(x)$ is convex and bounded below.

And, thus a global minimizer of (*) exists.

Suppose $f: \mathbb{R}^n \to \mathbb{R}$ convex and Ω convex

$$\bar{f}(x) := \begin{cases} f(x) & \text{if } x \in \Omega \subseteq \mathbb{R}^n \\ \infty & \text{otherwise} \end{cases}$$

let $x, y \in \mathbb{R}^n$, $z = \lambda x + (1-\lambda)y$, $0 \le \lambda \le 1$.

Consider $\bar{f}(z) = \bar{f}(\lambda x + (1-\lambda)y)$. If $x, y \in \Omega$ then

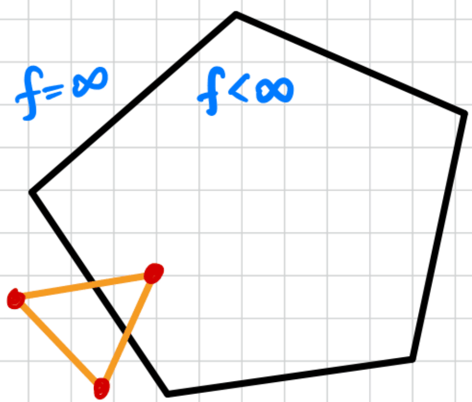
$$\bar{f}(z) = f(z) \le \lambda f(x) + (1-\lambda) f(y) = \lambda \bar{f}(x) + (1-\lambda)\bar{f}(y)$$

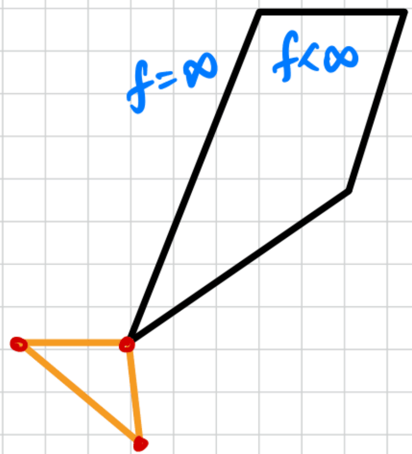
If either $x \notin \Omega$ or $y \notin \Omega$ or both, then

$$\bar{f}(z) = \infty \le \lambda \bar{f}(x) + (1-\lambda)\bar{f}(y).$$

Thus $\bar{f}: \mathbb{R}^n \to \mathbb{R} \cup \{\infty\}$ is convex.

As long as SA, GA, NM start with at least one feasible point then you can expect convergence to a local minimizer. The exact behavior for NM is complicated, depending on the nature of the constraints and the choice of initial simplex.





Filter Methods

(We will not employ filter methods this semester unless we have lots of time)

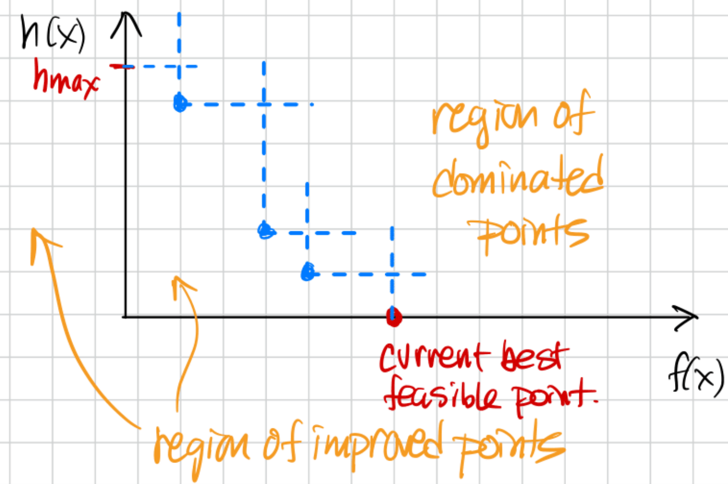
Consider the problem

$$\boxed{\begin{aligned} \min\ & f(x) \\ \text{s.t.}\ & c_1(x) = 0 \\ & c_2(x) \geq 0 \\ & x \in \mathbb{R}^n \end{aligned}}$$

And define the constraint violation function

$$h(x) = |c_1(x)| - \min\{0, c_2(x)\}$$

Then, for various methods we can track and utilize pareto optimal points relative to f and h.



Filter methods are particularly useful in traversing a discontinuous feasible region.