E_E 491
Review Session #9

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Linear State Estimation (Review)

$$\underline{z} = \underline{H}\,\underline{x} + \underline{e}$$
$$\underline{x} = States\ (known)$$
$$\underline{z} = Measurements$$
$$\underline{H} = Model\ representation$$
$$\underline{e} = Measurement\ errors$$
$$Estimate:\ \hat{x} = ?$$

Minimize x

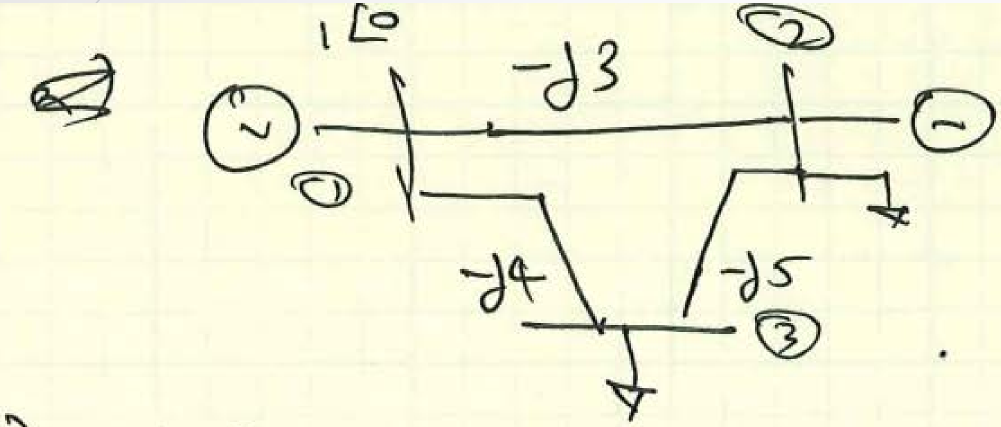
$$\sum_{i=1}^{N} \omega_i e_i^2 \Leftarrow weighted\ total\ error$$
$$= \underline{e}^T \underline{W}\,\underline{e}$$
$$\underline{e} = \underline{z} - \underline{H}\,\underline{x}$$

Minimize w.r.to x

$$(\underline{z} - \underline{H}\,\underline{x})^T \underline{W}(\underline{z} - \underline{H}\,\underline{x}) = g(\underline{x})$$

$$\frac{\partial g}{\partial \underline{x}} = -2\underline{H}^T\underline{W}(\underline{z} - \underline{H}\,\underline{x}) = 0$$
$$\underline{H}^T\underline{W}\,\underline{z} = \underline{H}^T\underline{W}\,\underline{H}\,\underline{x}$$
$$\boxed{\hat{x} = \underline{G}^{-1}\underline{H}^T\underline{W}\,\underline{z}} \Leftarrow Least\ Square\ Estimate$$
$$\underline{G} = \underline{H}^T\underline{W}\,\underline{H} = Gain\ Metrix$$

Linear State Estimation (Ex.)



a) $z_1 = P_{31} = \cancel{} -0.41$

$z_2 = P_{21} = 0.31$

$z_3 = P_{23} = 0.98$

$w_i = 1 \quad \forall i.$

b) $z_1 = P_{12} = -0.28, \quad z_2 = P_{32} = -1.05$

$z_3 = P_{23} = 0.99, \quad z_4 = P_{31} = -0.39$

All weights are equal to 1.

Linear State Estimation (Ex.)

(5)

(A)

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \overset{H}{\begin{bmatrix} 0 & 4 \\ 3 & 0 \\ 5 & -5 \end{bmatrix}} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

and

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} P_{31} \\ P_{21} \\ P_{23} \end{bmatrix} = \begin{bmatrix} -0.41 \\ 0.31 \\ 0.98 \end{bmatrix}$$

$$z_1 = \frac{\delta_3 - \delta_1}{0.25} + e_1 = \frac{x_2 - 0}{0.25} + e_1$$

$$z_2 = \frac{\delta_2 - \delta_1}{0.35} + e_2 = \frac{x_1 - 0}{0.35} + e_2$$

$$z_3 = \frac{\delta_2 - \delta_3}{0.20} + e_3 = \frac{x_1 - x_2}{0.20} + e_3$$

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$G = H^T W H = \begin{bmatrix} 34 & -25 \\ -25 & 41 \end{bmatrix}$$

$$\hat{x} = G^{-1} H^T W z = \begin{bmatrix} -0.0982 \\ -0.099 \end{bmatrix}$$

Linear State Estimation (Ex.)

(b)

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ -5 & 5 \\ 5 & -5 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} -0.28 \\ -1.05 \\ 0.99 \\ -0.39 \end{bmatrix}$$

$$z_1 = \frac{\delta_1 - \delta_2}{0.33} + e_1 = \frac{-x_1}{0.33} + e_1$$

$$z_2 = \frac{\delta_1 - \delta_2}{0.20} + e_2 = \frac{x_2 - x_1}{0.20} + e_2$$

$$z_3 = \frac{\delta_2 - \delta_3}{0.2} + e_3 = \frac{x_1 - x_2}{0.20} + e_3$$

$$z_4 = \frac{\delta_3 - \delta_1}{0.25} + e_4 = \frac{x_2}{0.25} + e_4$$

$$W = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now,

$$G = H^T W H = \begin{bmatrix} 59 & -50 \\ -50 & 66 \end{bmatrix}$$

$$\hat{x} = G^{-1} H^T W = \begin{bmatrix} 0.100889 \\ -0.10175 \end{bmatrix} \text{radians}$$

Questions?