A MODIFICATION TO THE FAST DECOUPLED POWER FLOW FOR NETWORKS WITH HIGH R/X RATIOS

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Abstract. For solving power flow problems, the Fast Decoupled Method is probably the most popular because of its efficiency. Its reliability for most power systems is very high but it does have difficulties in convergence for systems with high ratios of branch resistance to reactance. Modifications, that retain the advantages of this method but can handle high r/x ratios, are of great interest and certain compensation techniques have been used for this purpose. Both the series and parallel compensation techniques, however, give mixed results and a new modification is presented here that performed better on several test systems. These test results show that this modified method not only converges very well for systems with high r/x ratios but is only slightly less efficient than the Fast Decoupled Method for systems with normal ratios.

I. INTRODUCTION

The power flow has become the work horse of the power system analyst today. The algorithms and programs to solve the power flow problem have steadily improved over the last three decades in speed and reliability. The Fast Decoupled Method (FDM) [1] is probably the most commonly used because it is computationally the fastest and for most power systems, is very reliable. The FDM, however, does have convergence difficulties [2] on systems with branches that have large resistance to reactance ratios. Modifications to the FDM to avoid such difficulties have been of interest to researchers.

The modification using series branch compensation [3] was suggested by DyLiacco and Ramarao. Deckman, et al. developed a similar modification using parallel branch compensation [4]. These compensation methods, however, give mixed results, that is, the improvement in convergence is not consistent.

A new modification is presented in this paper. It was tested on the IEEE test systems (modified to obtain high r/x ratios) of various sizes with very encouraging results. It consistently provided better convergence than the FDM for systems with high r/x ratios and performed much better than the compensation methods. The tests also show that this modification of the FDM converges only slightly slower than the FDM for normal r/x ratios. This property makes this modification reliable for use in those cases where the r/x ratios are marginal.

It should be pointed out that other methods [5,6] have been developed to handle high r/x ratios. However, these methods are significantly different from the FDM and do not lend themselves to simple modifications of an existing FDM based power flow program. In fact, the basic Newton's method is quite insensitive to branch r/x ratios. Since the FDM is still the best suited method for most systems, a simple modification of the same program to handle high r/x ratios would be easier to use for these exceptional systems. For this reason, the modification proposed in this paper is compared to the FDM and the compensation modifications but not these other methods.

In the next section, the notation used in the paper is described. In Section III, some comments about the FDM are made to provide the motivation for the proposed modification. Section IV describes the new modification itself. Section V presents the test results and Section VI concludes the paper.

II. NOTATION

$V_i \angle \theta_i$	voltage at bus i
$\Delta \underline{V}$	vector of voltage magnitude corrections
$\Delta \underline{\theta}$	vector of voltage angle corrections
$P_i + jQ_i$	complex power injection at bus i
$\Delta \underline{P}$	vector of real power mismatches
$\Delta \underline{Q}$	vector of reactive power mismatches
$r_{ij} + jx_{ij}$	impedance of branch from bus i to bus j
$g_{ij} + jb_{ij}$	admittance of branch from bus i to bus j
$G_{ij} + jB_{ij}$	element (i,j) of the admittance matrix
B', B'', B'''	constant, real matrices defined later

III. FAST DECOUPLED METHOD

For each bus in a power network the power injection equations can be written as

$$P_i/V_i = V_i G_{ii} + \sum_j V_j [G_{ij} \mathrm{Cos}(\theta_i - \theta_j) + B_{ij} \mathrm{Sin}(\theta_i - \theta_j)] \quad (1)$$

$$Q_i/V_i = -V_i B_{ii} + \sum_j V_j [G_{ij} \mathrm{Sin}(\theta_i - \theta_j) - B_{ij} \mathrm{Cos}(\theta_i - \theta_j)] \quad (2)$$

The FDM to solve these equations is basically the Newton's method where the P-θ subproblem is decoupled

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from the Q-V subproblem by neglecting the off-diagonal submatrices, and the remaining submatrices approximated with constant elements:

$$\underline{\Delta P} = B' \,\underline{\Delta\theta} \tag{3}$$

$$\underline{\Delta Q} = B'' \,\underline{\Delta V} \tag{4}$$

The assumption of $G_{ij} \ll B_{ij}$ allows decoupling. The assumptions $V_i = 1$, $\mathrm{Cos}(\theta_i-\theta_j) = 1$, and $\mathrm{Sin}(\theta_i-\theta_j) = 0$ produce the following constant Jacobian terms by differentiating (1) and (2):

$$\partial P_i/\partial\theta_j = -B_{ij} \qquad \partial P_i/\partial\theta_i = \sum_j B_{ij} \tag{5}$$

$$\partial Q_i/\partial V_j = -B_{ij} \qquad \partial Q_i/\partial V_i = -B_{ii} \tag{6}$$

Thus comparing (3) to (5) and (4) to (6) it would appear that

$$B'_{ij} = -B_{ij} \qquad B'_{ii} = \sum_j B_{ij} \tag{7}$$

and $$B''_{ij} = -B_{ij} \qquad B''_{ii} = -B_{ii} \tag{8}$$

are the matrices to be used in the FDM. However, B' given by (7) often produces slow convergence and this was observed in the original FDM paper [1]. Instead, FDM uses the approximation

$$\begin{aligned} B'_{ij} &= -1/x_{ij} = -B_{ij}-G^2_{ij}/B_{ij} \\ B'_{ii} &= -\sum_j B'_{ij} \end{aligned} \tag{9}$$

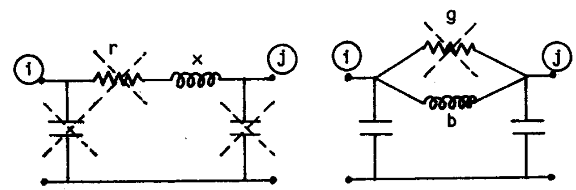
Table 1 shows the difference in convergence produced by the two different approximations of B'. The systems used for this test are a 5-bus system and the IEEE test systems with 14, 30, 57 and 118 buses. The difference in convergence is more pronounced when the r/x ratio is higher. For very small r/x ratios B' approximated by (7) works quite well, but when B' is approximated by (9) the convergence is much better for a greater range of r/x ratios. Thus (9) is used in the standard FDM. However, for r/x ratios greater than 1 even the standard FDM does not converge well, but the form of (9) provides the motivation for the modifications proposed in the next section. Figure 1 shows what is neglected in the representation of a branch of the B'(equation 9) and B"(equation 8) approximations in the standard FDM.

IV. A PROPOSED MODIFICATION

For most high voltage transmission networks the r/x ratio is quite small and the decoupling assumption of $G_{ij} \ll B_{ij}$ is usually true. In practice, however, it has been found that the FDM converges quite well as long as G_{ij} does not siginificantly exceed B_{ij}. This is also evident in the results shown in Table 1. Thus the decoupling assumption is not critical to the convergence of the FDM unless it is violated by a large margin. In such cases, it can be expected that the inclusion of the effects of the conductances in the iterations, even in an approximate way, would improve convergence.

TABLE 1. Convergence of FDM with different B'

System size (buses)	Biggest r/x ratio	Number of Iterations using .01 MW/MVAR tolerance: B' in (7)	B' in (9)
5	0.333	4.5	4
14	1.105	19.5	3.5
30	1.107	17	3.5
57	1.089	649	4.5
118	0.64	5.5	5.5



(a) (b)

Figure 1. The approximate branch representation in (a) B' and (b) B" as used in the standard FDM.

One way to include G_{ij} in the iterations is to add equations (1) and (2):

$$\begin{aligned}(P_i+Q_i)/V_i = (G_{ii}-B_{ii})V_i + \sum_j V_j[(G_{ij}-B_{ij})\mathrm{Cos}(\theta_i-\theta_j) \\ + (G_{ij}+B_{ij})\mathrm{Sin}(\theta_i-\theta_j)]\end{aligned} \tag{10}$$

Then equation (4) of the FDM can be replaced by

$$\underline{\Delta P} + \underline{\Delta Q} = B''' \,\underline{\Delta V} \tag{11}$$

where B"' can be approximated as

$$B'''_{ij} = G_{ij}-B_{ij} \qquad B'''_{ii} = G_{ii}-B_{ii} \tag{12}$$

Equation (3) of the FDM is not changed but the effect of large G_{ij} is reflected in the approximation for B'. In the selection of B', the improvement in convergence characteristics by going from (7) to (9) has been well known. In the cases where G_{ij} is large, the square of this term in (9) may exaggerate the effects of G_{ij}. Thus an alternative expression that moderates this effect is suggested for B':

$$\begin{aligned} B'_{ij} &= -B_{ij} - 0.4\, G_{ij} - 0.3\, G^2_{ij}/B_{ij} \\ B'_{ii} &= -\sum_j B'_{ij} \end{aligned} \tag{13}$$

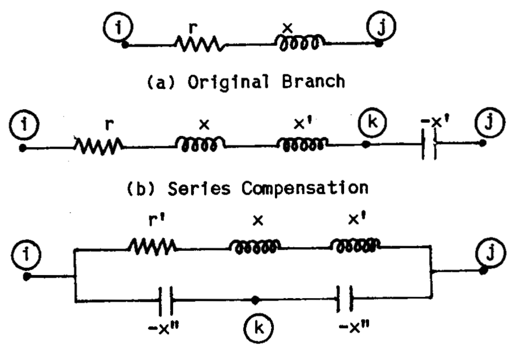
The coefficients 0.4 and 0.3 were found experimentally but seemed to work on all the power systems tested.

This is similar to the fact that for compensation methods a particular level of compensation can be experimentally found to be the best.

Thus, the modified FDM consists of iterations defined by (3) and (11), whose matrices are defined by (13) and (12) respectively.

V. TEST RESULTS

Exhaustive testing was done on a 5-bus system and the IEEE systems of 14, 30, 57, and 118 buses to compare the proposed modification with the standard FDM, the series compensation method, and the parallel compensation method. Tens of thousands of power flow cases were run to obtain the conclusions but only a small set of summarized typical results could be presented in this paper. All of the results clearly show that the proposed method is better than the others in handling high r/x ratio branches. In addition, the proposed method is only marginally less efficient than the standard FDM for normal r/x ratios.



(a) Original Branch

(b) Series Compensation

(c) Parallel Compensation

Figure 2. Series and Parallel Compensation of Branches

First, brief descriptions of the series [3] and parallel [4] compensation methods are given. The idea, as shown in Figure 2, is to replace the high r/x ratio branch with two or three low r/x ratio branches so that their total effect is the same as the original branch. In the series compensation method (Figure 2b), x' is chosen such that the $r/(x + x')$ ratio is small. In the parallel compensation method (Figure 2c), r', x' and x'' are chosen such that the $r'/(x+x')$ ratio is small. Both methods introduce one extra bus for each branch compensated.

It is known that the convergence, when using the compensation methods, is much slower than the FDM. Thus, power flow programs are set up to try compensation only when the standard FDM has failed and high r/x ratio is suspected as the cause of failure. It is also known that the parallel compensation works more reliably than the series compensation. However, for both methods, the chosen level of compensation, that is, the r/x ratio after compensation, has a significant effect on the convergence characteristics. This effect has not been adequately reported in the published literature but in the tests conducted here it was found that an after compensation r/x ratio of 0.9 provided the best convergence. This was true for both series and parallel methods on all of the test systems. Since 0.9 is higher than the usual transmission line r/x ratio, it is suspected that most compensation programs today are overcompensating. Thus, some of the cases that are reported today to be not solvable by compensation methods may actually be solvable if a more optimal compensation level is used. However, as shown in the following results, even when the optimal compensation level is used not all cases of high r/x ratios can be solved by this approach.

TABLE 2. Convergence Results for cases when one branch has high r/x ratio

Used Method	Number of buses in system	Number of cases solved in N iterations when N is: N<10	10≤N<50	N≥50 or diverged	Number of cases
Fast De-coupled	5	28	7	0	35
	14	55	25	5	85
	30	150	43	7	200
	57	223	90	2	315
	118	455	409	21	885
Series Compen-sation	5	22	2	11	35
	14	50	17	18	85
	30	137	23	40	200
	57	239	43	33	315
Parallel Compen-sation	5	33	2	0	35
	14	72	10	3	85
	30	175	16	9	200
	57	250	32	33	315
Proposed Modifi-cation	5	35	0	0	35
	14	79	4	2	85
	30	190	6	4	200
	57	310	5	0	315
	118	832	45	8	885

The comparison of the proposed method with the compensation methods and the FDM is shown in Table 2. For each test power system, the resistance of each transmission line was made 1, 2, 3, 4 and 5 times as much as its reactance. Changes were made to only one branch at a time. Thus for the 57 bus system which has 63 transmission lines, 315 different cases of 5 different levels of r/x ratio for each line are studied. The FDM could solve only 223 of these under 10 iterations whereas the series compensation method solved 239, the parallel compensation solved 250 and the proposed method solve 310. In the results shown, the compensation methods both use the best level of compensation obtained when r/x ratio is 0.9 after compensation. The proposed method uses the best coefficients shown in equation (13) to calculate B'. The convergence tolerance for the results in Table 2 was 0.01 MW/MVAR but the results were very similar at other tolerance levels.

The results clearly indicate that the proposed method to handle high r/x ratios converges better than the compensation methods, which are the only remedies available today other than using a power flow algorithm different from the FDM. The results also indicate that although the compensation methods speeded up convergence in many high r/x ratio cases, they also failed to obtain a solution in many moderately high r/x ratio cases where even the FDM obtained a solution. The proposed method, on the other hand, consistently had better convergence than the FDM on high r/x ratio cases and also solved most cases that the FDM could not.

Table 3 shows a comparison of the proposed method with the FDM when all branches of the power system have high r/x ratios. The tests are done by changing all line resistances multiplying them by 0.5, 1, 2, 3, 4 and 5. The compensation methods are not included in this comparison as they are practical when only a few of the branches have abnormal ratios. The results

TABLE 3. Number of iterations for cases when all line resistances are multiplied by a factor

Used Method	Number of buses in system	0.5	1	2	3	4	5
		Number of iterations when all line resistances are multiplied by the factor					
Fast De-coupled	5	3	4	4	5	7	8
	14	3.5	3.5	8.5	17.5	41.5	Div
	30	3	3.5	9.5	19.5	>50	Div
	57	3.5	4.5	7.5	14.5	Div	Div
	118	4	5.5	11	19.5	44.5	Div
Proposed Modifi-cation	5	4	5	5	5.5	6	7.5
	14	4.5	4.5	5.5	6.5	10.5	Div
	30	4.5	5	5.5	6	37.5	Div
	57	5	5	6	7.5	Div	Div
	118	5	5.5	7.5	8.5	14	Div

show that as the r/x ratio rises the proposed method obtains much better convergence. More significantly, however, the proposed method has nearly the same convergence properties when the r/x ratio is marginal (r multiplier is 0.5 or 1). This implies that the proposed method can be safely used in the power flow if marginal or high r/x ratio is detected in the input data. This cannot be done with the compensation methods because of their unreliability and they are only tried after the standard FDM has failed to converge.

VI. CONCLUSIONS

A modification to the FDM is proposed to handle high r/x branch ratios. The only other modifications known today to do this are the series and parallel compensation methods. Of course, algorithms different from the FDM can handle high r/x branch ratios but the FDM is probably the most used algorithm today and building in a small modification to handle high r/x ratios is more attractive than switching to a different algorithm.

This proposed modification is shown to be more efficient and reliable than both the series and parallel compensation methods. In addition, it is shown that the proposed method is almost as efficient as the FDM for marginal r/x ratios thus making it attractive to switch to the modification whenever an abnormal r/x ratio is encountered. Since the compensation methods are, in general, less reliable than the FDM, the present practice is to switch to them only when the FDM has failed. Thus, for the situations where computation time is important the proposed modification is much more attractive.

VII. ACKNOWLEDGEMENTS

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