EE 491 HW 08

a)

DC solution:

$$\begin{bmatrix} \overrightarrow{V_1} \\ \overrightarrow{V_2} \\ \overrightarrow{V_3} \end{bmatrix} = \begin{bmatrix} 1\angle 0 \\ 1.04\angle 0.11113 \\ 1\angle -0.042345 \end{bmatrix}$$

Linear state estimation:

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} P_{21} \\ P_{23} \\ P_{13} \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.5 \\ 0.1 \end{bmatrix}$$

$$z_1 = \frac{\delta_2 - \delta_1}{0.5} + e_1 = 2\delta_2 + e_1$$

$$z_2 = \frac{\delta_2 - \delta_3}{0.3} + e_2 = 3.33\delta_2 - 3.33\delta_3 + e_2$$

$$z_3 = \frac{\delta_1 - \delta_3}{0.4} + e_3 = -2.5\delta_3 + e_3$$

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 3.33 & -3.33 \\ 0 & -2.5 \end{bmatrix} \begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \Longrightarrow H = \begin{bmatrix} 2 & 0 \\ 3.33 & -3.33 \\ 0 & -2.5 \end{bmatrix}$$

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$G = H^T W H = \begin{bmatrix} 15.089 & -11.089 \\ -11.089 & 17.339 \end{bmatrix}$$

$$\hat{x} = G^{-1} H^T W Z = \begin{bmatrix} 0.11758 \\ -0.03525 \end{bmatrix}$$

b)

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} P_{21} \\ P_{23} \\ P_{13} \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.55 \\ 0.12 \end{bmatrix}$$

$$\hat{x} = G^{-1} H^T W Z = \begin{bmatrix} 0.10858 \\ -0.053491 \end{bmatrix}$$

c)

DC solution:

$$\begin{bmatrix} \overrightarrow{V_1} \\ \overrightarrow{V_2} \\ \overrightarrow{V_3} \end{bmatrix} = \begin{bmatrix} 1\angle 0 \\ 1\angle -0.20783 \\ 1\angle -0.17318 \end{bmatrix}$$

Linear state estimation:

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} P_{12} \\ P_{13} \\ P_{32} \\ P_{23} \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.6 \\ 0.22 \\ -0.2 \end{bmatrix}$$

$$z_1 = 2.4615(\delta_1 - \delta_2) + e_1 = -2.4615\delta_2 + e_1$$

$$z_2 = 3.3003(\delta_1 - \delta_3) + e_2 = -3.3003\delta_3 + e_2$$

$$z_3 = 4.9505(\delta_3 - \delta_2) + e_3 = 4.9505\delta_3 - 4.9505\delta_2 + e_3$$

$$z_4 = 4.9505(\delta_2 - \delta_3) + e_4 = 4.9505\delta_2 - 4.9505\delta_3 + e_4$$

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} -2.4615 & 0 \\ 0 & -3.3003 \\ -4.9505 & 4.9505 \\ 4.9505 & -4.9505 \end{bmatrix} \begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \Longrightarrow H = \begin{bmatrix} -2.4615 & 0 \\ 0 & -3.3003 \\ -4.9505 & 4.9505 \\ 4.9505 & -4.9505 \end{bmatrix}$$

$$W = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$G = H^T W H = \begin{bmatrix} 55.074 & -49.015 \\ -49.015 & 59.907 \end{bmatrix}$$

$$\hat{x} = G^{-1} H^T W Z = \begin{bmatrix} -0.21568 \\ -0.17482 \end{bmatrix}$$

d)

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} P_{21} \\ P_{31} \\ P_{32} \\ P_{23} \end{bmatrix} = \begin{bmatrix} -0.45 \\ -0.55 \\ 0.2 \\ -0.22 \end{bmatrix}$$

$$z_1 = 2.4615(\delta_2 - \delta_1) + e_1 = 2.4615\delta_2 + e_1$$

$$z_2 = 3.3003(\delta_3 - \delta_1) + e_2 = 3.3003\delta_3 + e_2$$

$$z_3 = 4.9505(\delta_3 - \delta_2) + e_3 = 4.9505\delta_3 - 4.9505\delta_2 + e_3$$

$$z_4 = 4.9505(\delta_2 - \delta_3) + e_4 = 4.9505\delta_2 - 4.9505\delta_3 + e_4$$

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} 2.4615 & 0 \\ 0 & 3.3003 \\ -4.9505 & 4.9505 \\ 4.9505 & -4.9505 \end{bmatrix} \begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \Rightarrow H = \begin{bmatrix} 2.4615 & 0 \\ 0 & 3.3003 \\ -4.9505 & 4.9505 \\ 4.9505 & -4.9505 \end{bmatrix}$$

$$W = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$G = H^T W H = \begin{bmatrix} 55.074 & -49.015 \\ -49.015 & 59.907 \end{bmatrix}$$

$$\hat{x} = G^{-1} H^T W Z = \begin{bmatrix} -0.19844 \\ -0.15796 \end{bmatrix}$$

e)

DC solution:

$$\begin{bmatrix} \overrightarrow{V_1} \\ \overrightarrow{V_2} \\ \overrightarrow{V_3} \\ \overrightarrow{V_4} \end{bmatrix} = \begin{bmatrix} 1\angle 0 \\ 1.05\angle 0.07569 \\ 1\angle -0.048809 \\ 1\angle -0.085523 \end{bmatrix}$$

Linear state estimation:

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \\ z_7 \end{bmatrix} = \begin{bmatrix} P_{21} \\ P_{13} \\ P_{31} \\ P_{14} \\ P_{23} \\ P_{32} \\ P_{34} \end{bmatrix} = \begin{bmatrix} 0.4 \\ 0.2 \\ -0.17 \\ 0.34 \\ 0.6 \\ -0.65 \\ 0.2 \end{bmatrix}$$

$$z_1 = 4.9875(\delta_2 - \delta_1) + e_1 = 4.9875\delta_2 + e_1$$

$$z_2 = 3.3003(\delta_1 - \delta_3) + e_2 = -3.3003\delta_3 + e_2$$

$$z_3 = 3.3003(\delta_3 - \delta_1) + e_3 = 3.3003\delta_3 + e_3$$

$$z_4 = 4(\delta_1 - \delta_4) + e_4 = -4\delta_4 + e_4$$

$$z_5 = 5(\delta_2 - \delta_3) + e_5 = 5\delta_2 - 5\delta_3 + e_5$$

$$z_6 = 5(\delta_3 - \delta_2) + e_6 = 5\delta_3 - 5\delta_2 + e_6$$

$$z_7 = 5(\delta_3 - \delta_4) + e_7 = 5\delta_3 - 5\delta_4 + e_7$$

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \\ z_7 \end{bmatrix} = \begin{bmatrix} 4.9875 & 0 & 0 \\ 0 & -3.3003 & 0 \\ 0 & 3.3003 & 0 \\ 0 & 0 & -4 \\ 5 & -5 & 0 \\ -5 & 5 & 0 \\ 0 & 5 & -5 \end{bmatrix} \begin{bmatrix} \delta_2 \\ \delta_3 \\ \delta_4 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ e_7 \end{bmatrix} \Longrightarrow H = \begin{bmatrix} 4.9875 & 0 & 0 \\ 0 & -3.3003 & 0 \\ 0 & 3.3003 & 0 \\ 0 & 0 & -4 \\ 5 & -5 & 0 \\ -5 & 5 & 0 \\ 0 & 5 & -5 \end{bmatrix}$$

$$W = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$G = H^T W H = \begin{bmatrix} 74.875 & -50 & 0 \\ -50 & 96.784 & -25 \\ 0 & -25 & 41 \end{bmatrix}$$

$$\hat{x} = G^{-1} H^T W Z = \begin{bmatrix} 0.076773 \\ -0.049932 \\ -0.088008 \end{bmatrix}$$