Power-flow Methods

Fast Decoupled Power-Flow Algorithm

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Power System Performance

- Viability/Acceptability
  - Voltages/currents within limits?
- Small-signal stability
  - Can damp out small disturbances
- Transient stability
  - Can recover after large disturbances?
- Economy
  - Cost minimal?

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Powerflow Analysis

- Power quantities specified
- Need to calculate voltages and currents and **ensure they are all within limits**
- Represent transmission by π circuits
- Formulate power-flow equations
- Solve for voltage phasors at all buses
- Currents by circuit analysis

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DC Method Summary

- Assume system lightly loaded
- Equivalent DC circuit
- Simplify powerflow equations to be linear
- Direct solution. Easy to solve.
- Only calculates phase angles
- Cannot check voltage magnitudes
- Not valid for systems with heavy loads

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Newton-Raphson Summary

- Can find accurate solutions
- Good initial condition needed
- Very fast convergence
- Poor initial conditions ⇒ Fast divergence
- Each iteration is time-consuming

$$(J^k)\Delta x^k = b - h(x^k)$$

- How to speed-up? Fast Decoupled Power-Flow.

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Fast Decoupled Derivation

$P \leftrightarrow \delta$ (DC power-flow method)

$Q \leftrightarrow V$

$$x = \begin{bmatrix} \delta \\ V \end{bmatrix} \Longrightarrow h(x) = \begin{bmatrix} p(x) \\ q(x) \end{bmatrix}$$

$$\Longrightarrow \frac{\partial h}{\partial x} = \begin{bmatrix} \frac{\partial p}{\partial \delta} & \frac{\partial p}{\partial V} \\ \frac{\partial q}{\partial \delta} & \frac{\partial q}{\partial V} \end{bmatrix}$$

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Decoupled Power-Flow

$$\frac{\partial p}{\partial V} = 0 \quad and \quad \frac{\partial q}{\partial \delta} = 0$$

$$J^k \approx \left.\begin{bmatrix} \frac{\partial p}{\partial \delta} & 0 \\ 0 & \frac{\partial q}{\partial V} \end{bmatrix}\right|_{x^k}$$

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Decoupled Power-Flow

$$(J^k)\Delta x^k = b - h(x^k)$$

$$\begin{bmatrix} \frac{\partial p}{\partial \delta} & 0 \\ 0 & \frac{\partial q}{\partial V} \end{bmatrix}\begin{bmatrix} \Delta\delta^k \\ \Delta V^k \end{bmatrix} = \begin{bmatrix} P - p(x^k) \\ Q - q(x^k) \end{bmatrix}$$

$$\frac{\partial p}{\partial \delta}\Delta\delta^k = \Delta P^k = Real\ power\ ?$$

$$\frac{\partial q}{\partial V}\Delta V^k = \Delta Q^k = Reactive\ power\ ?$$

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Real-Power Jacobian

$$\delta = \begin{bmatrix} \delta_2 \\ \vdots \\ \delta_N \end{bmatrix} \qquad V = \begin{bmatrix} V_{NG+2} \\ \vdots \\ V_N \end{bmatrix}$$

$$\frac{\partial p}{\partial \delta} = \begin{bmatrix} \frac{\partial p_2}{\partial \delta_2} & \cdots & \frac{\partial p_2}{\partial \delta_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial p_N}{\partial \delta_2} & \cdots & \frac{\partial p_N}{\partial \delta_N} \end{bmatrix}$$

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Jacobian Entries

$$p_i(x) = \sum_{k=1}^{N} Y_{ik}V_iV_k cos(\delta_i - \delta_k - \theta_{ik})$$

$$\frac{\partial p_i}{\partial \delta_j} = Y_{ij}V_iV_j sin(\delta_i - \delta_j - \theta_{ij})$$

$$\approx Y_{ij}V_iV_j\left[\cancel{sin(\delta_i - \delta_j)}^{0}\cos(\theta_{ij}) - \cancel{cos(\delta_i - \delta_j)}^{1}\sin(\theta_{ij})\right]$$

$$\approx -Y_{ij}V_iV_j\sin(\theta_{ij})$$

$$\approx B_{ij}V_iV_j \approx B_{ij}V_i(\text{since } V_j \approx 1)$$

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Jacobian Entry

$$\frac{\partial p_i}{\partial \delta_j} \approx B_{ij}V_i$$

where:

$$B_{ij} = -Imag(\overrightarrow{Y_{ij}}) = -Y_{ij}\sin(\theta_{ij})$$

$$\frac{\partial p_i}{\partial \delta_j} \approx B_{ij}V_i \qquad \frac{\partial p_i}{\partial \delta_i} \approx B_{ii}V_i$$

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Fast Decoupled Algorithm

$$\frac{\partial p}{\partial \delta} \approx \begin{bmatrix} B_{22}V_2 & \cdots & B_{2N}V_2 \\ \vdots & \ddots & \vdots \\ B_{N2}V_N & \cdots & B_{NN}V_N \end{bmatrix}$$

$$\begin{bmatrix} B_{22}{V_2}^k & \cdots & B_{2N}{V_2}^k \\ \vdots & \ddots & \vdots \\ B_{N2}{V_N}^k & \cdots & B_{NN}{V_N}^k \end{bmatrix}\begin{bmatrix} \Delta{\delta_2}^k \\ \vdots \\ \Delta{\delta_N}^k \end{bmatrix} = \begin{bmatrix} \Delta{P_2}^k \\ \vdots \\ \Delta{P_N}^k \end{bmatrix}$$

$$B_\delta\begin{bmatrix} \Delta{\delta_2}^k \\ \vdots \\ \Delta{\delta_N}^k \end{bmatrix} = \begin{bmatrix} \Delta{P_2}^k/{V_2}^k \\ \vdots \\ \Delta{P_N}^k/{V_N}^k \end{bmatrix} \qquad B_\delta[\Delta\delta] = [\overline{\Delta P}]$$

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Jacobian Entries

$$q_i(x) = \sum_{k=1}^{N} Y_{ik} V_i V_k sin(\delta_i - \delta_k - \theta_{ik})$$

$$\frac{\partial q_i}{\partial V_j} = Y_{ij} V_i sin(\delta_i - \delta_j - \theta_{ij})$$

$$\approx -V_i Y_{ij} \sin(\theta_{ij})$$

$$\approx V_i B_{ij}$$

$$\frac{\partial q_i}{\partial V_i} \approx V_i B_{ii}$$

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Fast Decoupled Algorithm

Similarly:

$$B_V \begin{bmatrix} \Delta V_{NG+2}{}^k \\ \vdots \\ \Delta V_N{}^k \end{bmatrix} = \begin{bmatrix} \Delta Q_{NG+2}{}^k / V_{NG+2}{}^k \\ \vdots \\ \Delta Q_N{}^k / V_N{}^k \end{bmatrix}$$

$$B_V = -\mathrm{Imag}(\overrightarrow{Y_{Bus}})_{\substack{NG+2,\dots,N \\ NG+2,\dots,N}}$$

$$B_V [\Delta V] = [\overline{\Delta Q}]$$

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FD Algorithm

$$x^0 = \begin{bmatrix} \delta_0 \\ V_0 \end{bmatrix} \qquad B_\delta = -Imag(\overrightarrow{Y_{Bus}})_{angles}$$

$$B_V = -Imag(\overrightarrow{Y_{Bus}})_{voltages}$$

⇓ k=0

Find $\Delta P^k = P - p(\delta^k, V^k)$, $\Delta Q^k = Q - q(\delta^k, V^k)$

⇓

$k = k + 1$ $\qquad \|\Delta P^k\| < \varepsilon \;\&\; \|\Delta Q^k\| < \varepsilon \xrightarrow{\text{Ye}} Stop$

⇓ No

Normalize ΔP^k and ΔQ^k to current injections $\overline{\Delta P^k}$ and $\overline{\Delta Q^k}$

⇓

$$\Delta\delta^k = B_\delta{}^{-1} \overline{\Delta P^k}, \; \Delta V^k = B_V{}^{-1} \overline{\Delta Q^k}$$

⇓

$$\delta^{k+1} = \delta^k + \Delta\delta^k, \; V^{k+1} = V^k + \Delta V^k$$

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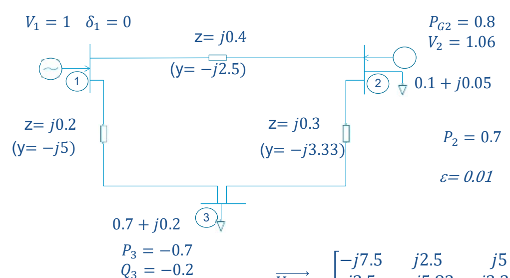
Example 1 (Same system as for NR)



$$\overrightarrow{Y_{Bus}} = \begin{bmatrix} -j7.5 & j2.5 & j5 \\ j2.5 & -j5.83 & j3.33 \\ j5 & j3.33 & -j8.33 \end{bmatrix}$$

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Example 1

$$p_i(x) = \sum_{j=1}^{3} Y_{ij}\, V_i\, V_j \cos(\delta_i - \delta_j - \theta_{ij})$$

$$q_i(x) = \sum_{j=1}^{3} Y_{ij}\, V_i\, V_j \sin(\delta_i - \delta_j - \theta_{ij})$$

$$x^0 = \begin{bmatrix} \delta_2{}^0 \\ \delta_3{}^0 \\ V_3{}^0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \qquad \begin{bmatrix} p_2{}^0 \\ p_3{}^0 \\ q_3{}^0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -0.1998 \end{bmatrix},$$

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Example 1

$$\begin{bmatrix} \Delta p_2{}^0 \\ \Delta p_3{}^0 \\ \Delta q_3{}^0 \end{bmatrix} = \begin{bmatrix} 0.7000 \\ -0.7000 \\ -0.0002 \end{bmatrix}$$

ε=0.01

$$B_\delta = -\mathrm{Imag}(\overrightarrow{Y_{Bus}})_{\substack{2,3 \\ 2,3}} = \begin{bmatrix} 5.83 & -3.33 \\ -3.33 & 8.33 \end{bmatrix}$$

$$B_V = -\mathrm{Imag}(\overrightarrow{Y_{Bus}})_{\substack{3 \\ 3}} = 8.33$$

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Example 1

$$\begin{bmatrix} \Delta\delta_2{}^0 \\ \Delta\delta_3{}^0 \end{bmatrix} = B_\delta{}^{-1} \begin{bmatrix} \Delta p_2{}^0/V_2{}^0 \\ \Delta p_3{}^0/V_3{}^0 \end{bmatrix} = \begin{bmatrix} 0.0846 \\ -0.0502 \end{bmatrix}$$

$$\Delta V_3{}^0 = B_V{}^{-1} \frac{\Delta q_3{}^0}{V_3{}^0} = 0$$

$$x^1 = x^0 + \Delta x^0 = \begin{bmatrix} \delta_2{}^0 + \Delta\delta_2{}^0 \\ \delta_3{}^0 + \Delta\delta_3{}^0 \\ V_3{}^0 + \Delta V_3{}^0 \end{bmatrix} = \begin{bmatrix} 0.0846 \\ -0.0502 \\ 1 \end{bmatrix},$$

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Example 1

$$\begin{bmatrix} p_2{}^1 \\ p_3{}^1 \\ q_3{}^1 \end{bmatrix} = \begin{bmatrix} 0.6983 \\ -0.7254 \\ -0.1617 \end{bmatrix}$$

$$\begin{bmatrix} \Delta p_2{}^1 \\ \Delta p_3{}^1 \\ \Delta q_3{}^1 \end{bmatrix} = \begin{bmatrix} 0.0017 \\ -0.0254 \\ -0.0383 \end{bmatrix}$$

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Example 1

$$\begin{bmatrix} \Delta\delta_2{}^1 \\ \Delta\delta_3{}^1 \end{bmatrix} = B_\delta{}^{-1} \begin{bmatrix} \Delta p_2{}^1/V_2{}^1 \\ \Delta p_3{}^1/V_3{}^1 \end{bmatrix} = \begin{bmatrix} 0.0026 \\ 0.0041 \end{bmatrix}$$

$$\Delta V_3{}^1 = B_V{}^{-1} \frac{\Delta q_3{}^1}{V_3{}^1} = -0.0046$$

$$x^2 = x^1 + \Delta x^1 = \begin{bmatrix} \delta_2{}^1 + \Delta\delta_2{}^1 \\ \delta_3{}^1 + \Delta\delta_3{}^1 \\ V_3{}^1 + \Delta V_3{}^1 \end{bmatrix} = \begin{bmatrix} 0.0872 \\ -0.0461 \\ 0.9954 \end{bmatrix},$$

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Example 1

$$\begin{bmatrix} p_2{}^2 \\ p_3{}^2 \\ q_3{}^2 \end{bmatrix} = \begin{bmatrix} 0.6983 \\ -0.6966 \\ -0.1999 \end{bmatrix}$$

$$\begin{bmatrix} \Delta p_2{}^2 \\ \Delta p_3{}^2 \\ \Delta q_3{}^2 \end{bmatrix} = \begin{bmatrix} 0.0021 \\ -0.0034 \\ -0.0001 \end{bmatrix}$$

$|max(|\Delta p^2|, |\Delta q^2|)| < 0.01 \Longrightarrow$ Stop

Converged in two iterations!

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Fast Decoupled Summary

- Summary
  - Fast computation per iteration
  - More number of iterations (slower convergence)
  - Better tolerance of initial conditions

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