Problem 1

Let be a continuous random variable with PDF

$$f_X(x) = \begin{cases} \frac{5}{32}x^4, & 0 < x \leq 2 \\ 0, & otherwise \end{cases}$$

and let $Y = X^2$

a. Find the CDF of Y

b. Find the PDF of Y

c. Find E[Y]

Problem 2

You are offered to play the following game. You roll a fair die once and observe the result which is shown by the random variable X. At this point, you can stop the game and win X dollars. You can also choose to roll the die for the second time to observe the value Y. In this case, you will win Y dollars. Let W be the value that you win in this game. What strategy do you use to maximize E[W] What is the maximum E[W] you can achieve using your strategy?

Problem 3

A casino offers a game of chance for a single player in which a fair coin is tossed at each stage. The pot starts at 1 dollar and is doubled every time a head appears. The first time a tail appears, the game ends and the player wins whatever is in the pot. Thus the player wins 1 dollar if a tail appears on the first toss, 2 dollars if a head appears on the first toss and a tail on the second, 4 dollars if a head appears on the first two tosses and a tail on the third, 8 dollars if a head appears on the first three tosses and a tail on the fourth, and so on. In short, the player wins 2^{k-1} dollars if the coin is tossed k times until the first tail appears. What would be a fair price to pay the casino for entering the game?

a. Let X be the amount of money (in dollars) that the player wins. Find E[X].

b. What is the probability that the player wins more than 65 dollars?

c. Now suppose that the casino only has a finite amount of money. Specifically, suppose that the maximum amount of the money that the casino will pay you is 2^{30} dollars (around 1.07 billion dollars). That is, if you win more than 2^{30} dollars, the casino is going to pay you only 2^{30} dollars. Let Y be the money that the player wins in this case. Find E[Y].

Problem 4

Let X $\sim Geometric(\frac{1}{3})$. , and let $Y = |X - 5|$. Find the pmf of Y.

Problem 5

Let X be a discrete random variable with the following PMF

$$p_X(k) = \begin{cases} 0.5, & k = 1 \\ 0.3, & k = 2 \\ 0.2, & k = 3 \\ 0, & otherwise \end{cases}$$

a. Find E[X].

b. Find Var(X) and SD(X)

c. If Y = $\frac{2}{X}$, find E[Y]

Problem 6

In this experiment, you flip a fair coin. If this first coin toss shows heads, you flip the coin 20 more times. If the first coin toss shows tails, you only flip the coin 10 more times. Assume X be the number of times that heads shows in all of your coin tosses (including the first one).

a. what is the probability that X = 5

b. what is the probability that the first coin toss showed heads, given X = 5

c. what is the probability that the last coin toss showed heads, given X = 5.

Problem 7

Consider a probabilistic experiment with three events A, B and C. If P(AB) = P(A)P(B), is it true that P(AB|C) = P(A|C)P(B |C)? You may assume that P(C) > 0

Problem 8

Consider a probabilistic experiment with three events A, B and C. If P(A+B) = P(A)+P(B), is it true that P(A+B |C) = P(A|C)+P(B|C)? You may assume that P(C) > 0