Problem 1

You pick two cards at random (without replacement) from a standard deck of 52 cards. Please answer the following questions:

a) What is the probability that both cards show the same value (e.g., both are 4's, or both are Kings, etc)?

b) Given that the two cards have the same color, what is the probability that both cards show the same value? Is this probability larger or smaller than the probability in part a? Conceptually, why does this make sense?

c) Are the events that the two cards have the same color and the two cards have the same value independent?

Problem 2

a) If two events A and B are independent, is it necessarily true that A and $\bar{B}$ are independent? Please prove or give a counterexample.

b) If events A and B are independent, and events A and C are independent, is it always true that events B and C are independent? Please prove or give a counterexample.

c) If two events are disjoint, are they always, sometimes, or never independent? Please explain.

Problem 3

A randomly-selected student in an engineering class is diligent with probability 0.3, and lazy with probability 0.7. A diligent student completes all of her homework with probability 0.9, and does not do so with probability 0.1. Meanwhile, a lazy student completes all homework with probability 0.5, and does not do so with probability 0.5. A student who completes all homework receives an A grade with probability 0.6, and receives a B grade with probability 0.4 (regardless of whether she is lazy or diligent). A student who doesn't complete all homework receives an A grade with probability 0.3, a B grade with probability 0.4, and an F grade with probability 0.3 (again regardless of whether she is lazy or diligent). Please answer the following questions:

a) What is the probability that a randomly selected student is diligent, completes all homework, and receives an A grade?

b) What is the probability that a student completes all her homework?

c) What is the probability that a student receives an A grade?

d) Given that a randomly-selected student completed all her homework, what is the probability that the student is lazy?

e) Given that a randomly-selected student received an A grade, what is the probability that the student is lazy?

f) Is the event that a student passes the course (receives a grade of A or B) independent of the event that the student is lazy?

g) Is the event that a student receives a B grade independent of the event that the student is lazy?

h) What's the probability that a student completes all homework given that she is either lazy or receives a grade of A?

Problem 4

An experiment has three outcomes A, B, and C, which have probabilities P(A)=0.5, P(B)=0.3, and P(C)=0.2. You repeat this experiment 100 times, independently. Please answer the following questions.

a) What is the probability that outcome A occurs on exactly 50 trials.

b) What is the probability that the outcome A occurs on 50 trials, outcome B occurs on 25 trials, and outcome C occurs on 25 trials?

c) Given that outcome A occurred on exactly 60 trials, what is the probability that A occurred on the first trial? How about B?

Problem 5

a) Please define a random variable, and explain why the concept is important.

b) Please define the notion of a cumulative distribution function (CDF).

Problem 6

An experiment has four outcomes A, B, C, and D, which have probabilities $P(A)=0.4$, $P(B)=0.3$, $P(C)=0.2$, and $P(D)=0.1$. We define a random variable X as follows: $X(A)=1$, $X(B)=5$, $X(C)=2$, $X(D)=1$. Also, we define the event Z={A,B,C}. Please answer the following questions:

a) Find and plot the CDF of X.

c) What is the probability that $2<X<6$?

b) Find and plot the probability mass function (PMF) of X.

c) Find the CDF of X given Z has occurred.

d) What is the probability of Z, given than $X>1.5$?

Problem 7

You throw a dart a circular dartboard with unit radius, and have an equal probability of hitting each point on the dartboard. Let R be the distance of the dart from the center of the dartboard. Please answer the following questions.

a) Please argue that R is a random variable.

b) Please find the CDF of R.

c) What is the probability that $0.2<R<0.5$?

d) Please find the probability density function (pdf) of R.

e) Now let Q be the vertical distance of the dart from the bottom of the dartboard. Please find the pdf and CDF of Q.