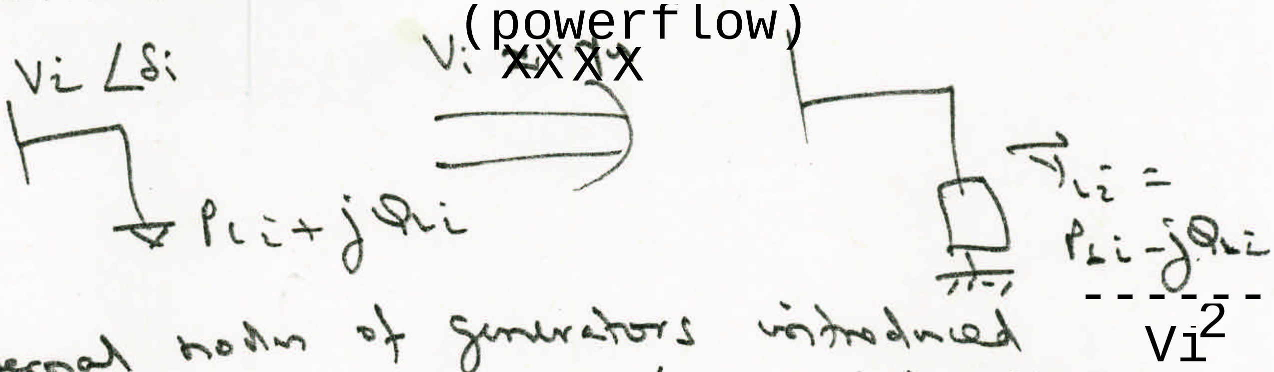
EE521 Handout!

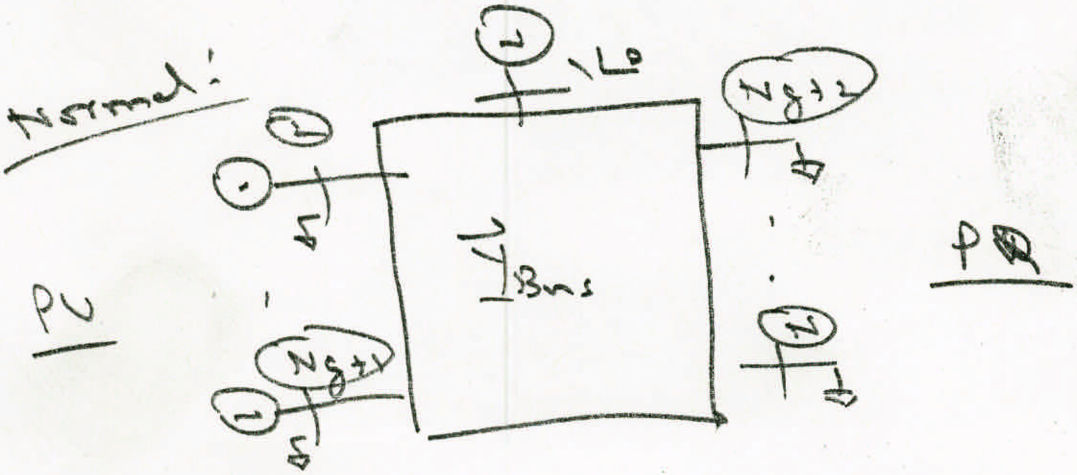
Network Reduction:

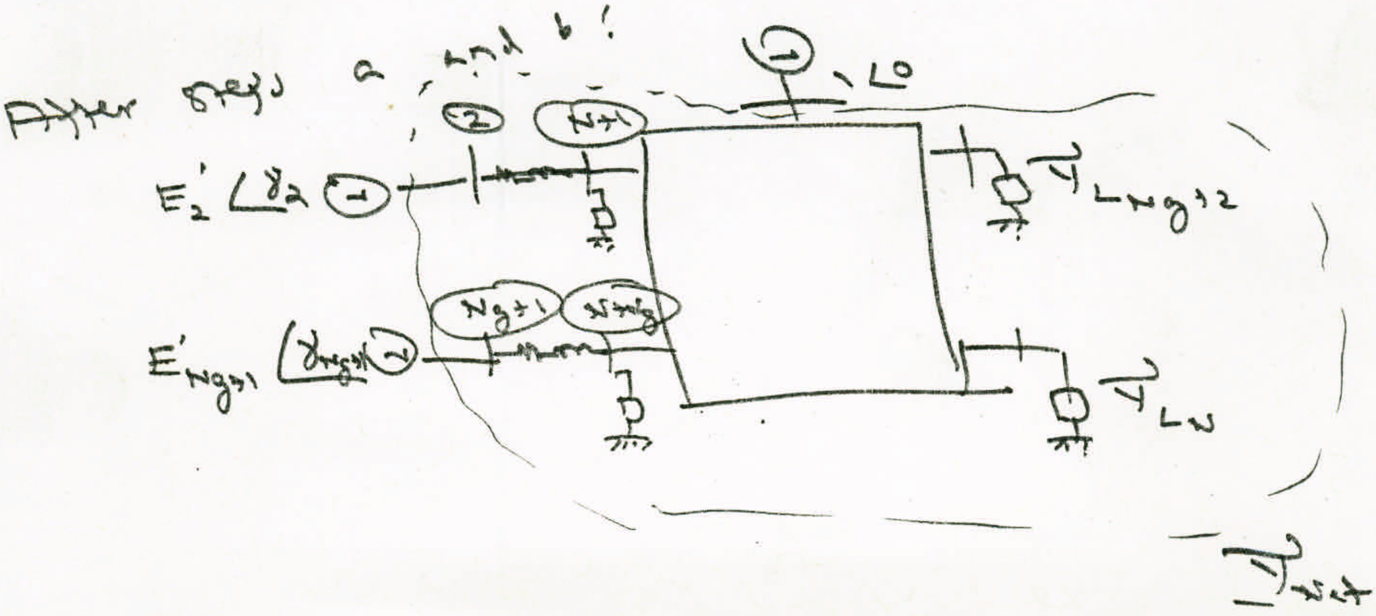
a) Assume all loads are represented by equivalent admittances



b) Internal nodes of generators introduced and buses renumbered from 1 to N+Ng.

c) Reduction principle.





$$E'_i \angle \delta_i = (E'_{di} + j E'_{qi}) \angle \delta_i - \frac{\pi}{2}$$

$$E'_1 \angle \delta_1 = 1 \angle 0$$

$$\vec{V}_{gen} = \begin{bmatrix} E'_1 \angle \delta_1 \\ E'_2 \angle \delta_2 \\ \vdots \\ E'_{N_g+1} \angle \delta_{N_g+1} \end{bmatrix} \qquad \vec{V}_{rest} = \begin{bmatrix} \vec{V}_{N_g+2} \\ \vdots \\ \vec{V}_{N+N_g} \end{bmatrix}$$

$$\vec{I}_{gen} = \begin{bmatrix} \vec{I}_{G1} \\ \vdots \\ \vec{I}_{GN} \end{bmatrix} \qquad \vec{I}_{rest} = \begin{bmatrix} \vec{I}_{N_g+2} \\ \vdots \\ \vec{I}_{N+N_g} \end{bmatrix} = \underline{0}$$

(No injections after loads moved to $\vec{Y}_{net}$ as equivalent admittances)

$$\begin{bmatrix} \vec{I}_{gen} \\ \cancel{\vec{I}_{rest}}^{\,0} \end{bmatrix} = \begin{bmatrix} \vec{Y}_{11} & \vec{Y}_{12} \\ \vec{Y}_{21} & \vec{Y}_{22} \end{bmatrix} \begin{bmatrix} \vec{V}_{gen} \\ \vec{V}_{rest} \end{bmatrix}$$

$$\Rightarrow \vec{I}_{gen} = \left(\vec{Y}_{11} - \vec{Y}_{12} \vec{Y}_{22}^{-1} \vec{Y}_{21} \right) \vec{V}_{gen}$$

$$= \vec{Y}_{gen} \vec{V}_{gen}$$

Define $\vec{Y}_{gen} = \vec{Y}_{11} - \vec{Y}_{12} \vec{Y}_{22}^{-1} \vec{Y}_{21}$.

$$= \left(Y_{gij} \angle \theta_{gij} \right) \quad i = 1, \ldots N_g+1, \; j = 1, \ldots N_g+1$$

Then $P_{ei} + j\,\cancel{Q_{ei}}\,Q_{ei} = \vec{E}_i' \angle \delta_i \; \vec{I}_{Gi}^{*}$

$$= E_i' \angle \delta_i \left[\sum \vec{Y}_{Gij} E_j' \angle \delta_j \right]^{*}$$

$$= \sum_{j=1}^{N_g+1} E_i' E_j' Y_{Gij} \angle \delta_i - \delta_j - \theta_{Gij}$$

$$\Rightarrow P_{ei} = \sum_{j=1}^{N_g+1} Y_{Gij} E_i' E_j' \cos(\delta_i - \delta_j - \theta_{Gij})$$

$$I_{di} + j I_{qi} = \vec{I}_{Gi} \angle \frac{\pi}{2} - \delta_i$$

$$= \sum_{j=1}^{N_g+1} Y_{Gij} E_j' \angle \delta_j + \frac{\pi}{2} - \delta_i + \theta_{Gij}$$

$$I_{di} = \sum_{j=1}^{N_g+1} Y_{Gij} E_j' \cos\left(\frac{\pi}{2} + \delta_j - \delta_i + \theta_{Gij}\right)$$

$$= \sum_{j=1}^{N_g+1} Y_{Gij} E_j' \sin(\delta_i - \delta_j - \theta_{Gij})$$

$$I_{qi} = \sum_{j=1}^{N_g+1} Y_{Gij} E_j' \cos(\delta_i - \delta_j - \theta_{Gij})$$