1

EE 491 HW 9

1)

$$\mathcal{L} = f_1 + f_2 + \lambda(P_D - P_{G_1} - P_{G_2})$$

$$\frac{\partial \mathcal{L}}{\partial P_{G_1}} = 2 + 0.08 P_{G_1} - \lambda = 0 \Rightarrow \boxed{P_{G_1} = \frac{\lambda - 2}{0.08}} \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial P_{G_2}} = 5 + 0.04 P_{G_2} - \lambda = 0 \Rightarrow \boxed{P_{G_2} = \frac{\lambda - 5}{0.04}} \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = P_D - P_{G_1} - P_{G_2} = 0 \Rightarrow P_{G_1} + P_{G_2} = P_D$$

$$\Rightarrow \frac{\lambda - 2}{0.08} + \frac{\lambda - 5}{0.04} = P_D \Rightarrow \boxed{\lambda = \frac{0.08}{3} P_D + 4} \quad (3)$$

a) $P_D = 200$ MW

$$(3) \Rightarrow \lambda = \frac{0.08}{3} \times 200 + 4 = 9.33$$

$$(1) \Rightarrow P_{G_1} = \frac{9.33 - 2}{0.08} = 91.67 \text{ MW}$$

$$(2) \Rightarrow P_{G_2} = \frac{9.33 - 5}{0.04} = 108.33 \text{ MW}$$

$$f_1 = 2(91.67) + 0.04(91.67)^2 = 519.44 \ \$/h$$

$$f_2 = 5(108.33) + 0.02(108.33)^2 = 776.39 \ \$/h$$

$$\text{Unit Cost} = \frac{f_1 + f_2}{200} = 6.4792 \ \$/\text{Mwh}$$

b) $P_D = 400^{MW}$

③ $\Rightarrow \lambda = \frac{0.08}{3} \times 400 + 4 = 14.67$

① $\Rightarrow P_{G_1} = \frac{14.67 - 2}{0.08} = 158.33^{MW}$

② $\Rightarrow P_{G_2} = \frac{14.67 - 5}{0.04} = 241.67^{MW}$

$f_1 = 2(158.33) + 0.04(158.33)^2 = 1319.4 \ \$/h$

$f_2 = 5(241.67) + 0.02(241.67)^2 = 2376.4 \ \$/h$

$\text{unit cost} = \frac{f_1 + f_2}{400} = 9.2396 \ \$/_{Mwh}$

c) $P_D = 600$ MW

③ $\Rightarrow \lambda = \frac{0.08}{3} \times 600 + 4 = 20$

① $\Rightarrow P_{G_1} = \frac{20 - 2}{0.08} = 225^{MW}$

② $\Rightarrow P_{G_2} = \frac{20 - 5}{0.04} = 375^{MW}$

$f_1 = 2(225) + 0.04(225)^2 = 2475 \ \$/h$

$f_2 = 5(375) + 0.02(375)^2 = 4687.5 \ \$/h$

$\text{Unit Cost} = \frac{f_1 + f_2}{600} = 11.938 \ \$/_{Mwh}$

d) $P_D = 800^{MW}$

$$\begin{cases} P_{G_1} = 400^{MW} \\ P_{G_2} = 400^{MW} \end{cases}$$

$$f_1 = 2(400) + 0.04(400)^2 = 7200 \ \$/h$$

$$f_2 = 5(400) + 0.02(400)^2 = 5200 \ \$/h$$

$$\text{unit cost} = \frac{f_1 + f_2}{800} = 15.5 \ \$/_{Mwh}$$

2) $$\frac{\partial \mathcal{L}}{\partial P_{G_3}} = 6 + 0.02 P_{G_3} - \lambda = 0 \Rightarrow \boxed{P_{G_3} = \frac{\lambda - 6}{0.02}} \quad (4)$$

$$P_{G_1} + P_{G_2} + P_{G_3} = P_D \Rightarrow \frac{\lambda - 2}{0.08} + \frac{\lambda - 5}{0.04} + \frac{\lambda - 6}{0.02} = P_D$$

$$\Rightarrow \boxed{\lambda = \frac{1}{7}(0.08 P_D + 36)} \quad (5)$$

a) $P_D = 200^{MW}$

$$(5) \Rightarrow \lambda = \frac{1}{7}(0.08 \times 200 + 36) = 7.4286$$

① $\Rightarrow P_{G_1} = \dfrac{7.4286 - 2}{0.08} = 67.857$ MW

② $\Rightarrow P_{G_2} = \dfrac{7.4286 - 5}{0.04} = 60.714$ MW

④ $\Rightarrow P_{G_3} = \dfrac{7.4286 - 6}{0.02} = 71.429$ MW

$f_1 = 2(67.857) + 0.04(67.857)^2 = 319.9$ \$/h

$f_2 = 5(60.714) + 0.02(60.714)^2 = 377.3$ \$/h

$f_3 = 6(71.429) + 0.01(71.429)^2 = 479.59$ \$/h

Unit cost $= \dfrac{f_1 + f_2 + f_3}{200} = 5.8839$ \$/MWh

b) $P_D = 600$ MW

⑤ $\Rightarrow \lambda = \dfrac{1}{7}(0.08 \times 600 + 36) = 12$

① $\Rightarrow P_{G_1} = \dfrac{12 - 2}{0.08} = 125$ MW

② $\Rightarrow P_{G_2} = \dfrac{12 - 5}{0.04} = 175$ MW

④ $\Rightarrow P_{G_3} = \dfrac{12 - 6}{0.02} = 300$ MW

5

$$f_1 = 2(125) + 0.04(125)^2 = 875 \ \$/h$$

$$f_2 = 5(175) + 0.02(175)^2 = 1487.5 \ \$/h$$

$$f_3 = 6(300) + 0.01(300)^2 = 2700 \ \$/h$$

$$\text{unit Cost} = \frac{f_1 + f_2 + f_3}{600} = 8.4375 \ \$/Mwh$$

c) $P_D = 1000 \ MW$

$$⑤ \Rightarrow \lambda = \frac{1}{7}(0.08 \times 1000 + 36) = 16.571$$

$$① \Rightarrow P_{G_1} = \frac{16.571 - 2}{0.08} = 182.14 \ MW$$

$$② \Rightarrow P_{G_2} = \frac{16.571 - 5}{0.04} = 289.29 \ MW$$

$$④ \Rightarrow P_{G_3} = \frac{16.571 - 6}{0.02} = 528.57 \ MW > 400 \ MW$$

$$\Rightarrow \text{fix } P_{G_3} = 400 \ MW$$

$$P_D^{new} = P_D - P_{G_3} = 1000 - 400 = 600 \ MW$$

$$③ \Rightarrow \lambda = \frac{0.08}{3} \times 600 + 4 = 20$$

$$① \Rightarrow P_{G_1} = \frac{20 - 2}{0.08} = 225 \ MW$$

$$② \Rightarrow P_{G_2} = \frac{20 - 5}{0.04} = 375 \ MW$$

$$f_1 = 2(225) + 0.04(225)^2 = 2475 \ \$/h$$

$$f_2 = 5(375) + 0.02(375)^2 = 4687.5 \ \$/h$$

$$f_3 = 6(400) + 0.01(400)^2 = 4000 \ \$/h$$

$$\text{unit cost} = \frac{f_1 + f_2 + f_3}{1000} = 11.162 \ \$/\text{Mwh}$$

3) Lossless case :

a) $P_D = 400$ MW

from 1.b: $\lambda = 14.67$, $\begin{cases} P_{G_1} = 158.33 \text{ MW} \\ P_{G_2} = 241.67 \text{ MW} \end{cases}$

Lossy case:

$$\mathcal{L} = f_1 + f_2 + \lambda(P_D + P_{Loss} - P_{G_1} - P_{G_2})$$

$$\frac{\partial \mathcal{L}}{\partial P_{G_1}} = 2 + 0.08 P_{G_1} + \lambda(0.0004 P_{G_1} - 1) = 0 \Rightarrow \boxed{P_{G_1} = \frac{\lambda - 2}{0.08 + 0.0004\lambda}} \quad (6)$$

$$\frac{\partial \mathcal{L}}{\partial P_{G_2}} = 5 + 0.04 P_{G_2} + \lambda(0.0003 P_{G_2} - 1) = 0 \Rightarrow \boxed{P_{G_2} = \frac{\lambda - 5}{0.04 + 0.0003\lambda}} \quad (7)$$

Iteration -1

Assume $\lambda^{(-1)} = 14$

$$⑥ \Rightarrow P_{G_1}^{(-1)} = \frac{14-2}{0.08+0.0004(14)} = 140.19 \text{ MW}$$

$$⑦ \Rightarrow P_{G_2}^{(-1)} = \frac{14-5}{0.04+0.0003(14)} = 230.62 \text{ MW}$$

Iteration 0

$\lambda^{(0)} = 14.67$, $P_{G_1}^{(0)} = 158.33$ MW , $P_{G_2}^{(0)} = 241.67$ MW

$$P_{Loss}^{(0)} = 0.0002(158.33)^2 + 0.00015(241.67)^2 = 13.774 \text{ MW}$$

$$\Delta P^{(0)} = P_D + P_{Loss}^{(0)} - P_{G_1}^{(0)} - P_{G_2}^{(0)} = 13.774 \text{ MW}$$

$$\Delta\lambda^{(0)} = \frac{\lambda^{(0)} - \lambda^{(-1)}}{\sum P_{G_i}^{(0)} - \sum P_{G_i}^{(-1)}} \left[\Delta P^{(0)}\right] = 0.16342$$

Iteration 1

$$\lambda^{(1)} = \lambda^{(0)} + \Delta\lambda^{(0)} = 14.83$$

$$⑥ \Rightarrow P_{G_1}^{(1)} = \frac{14.83-2}{0.08+0.0004(14.83)} = 149.31 \text{ MW}$$

$$⑦ \Rightarrow P_{G_2} = \frac{14.83-5}{0.04+0.0003(14.83)} = 221.15 \text{ MW}$$

$$P_{Loss}^{(1)} = 0.0002(149.31)^2 + 0.00013(221.13)^2 = 11.793 \text{ MW}$$

$$\Delta P^{(1)} = P_D + P_{Loss}^{(1)} - P_{G_1}^{(1)} - P_{G_2}^{(1)} = 41.336 \text{ MW}$$

$$\Delta\lambda^{(1)} = \frac{\lambda^{(1)} - \lambda^{(0)}}{\sum P_{G_i}^{(1)} - \sum P_{G_i}^{(0)}}\left[\Delta P^{(1)}\right] = -0.22866$$

Iteration 2

$$\lambda^{(2)} = \lambda^{(1)} + \Delta\lambda^{(1)} = 14.601$$

$$6 \Rightarrow P_{G_1}^{(2)} = \frac{14.601 - 2}{0.08 + 0.0004(14.601)} = 146.8 \text{ MW}$$

$$7 \Rightarrow P_{G_2}^{(2)} = \frac{14.601 - 3}{0.04 + 0.0003(14.601)} = 216.34 \text{ MW}$$

$$P_{Loss}^{(2)} = 0.0002(146.8)^2 + 0.00013(216.34)^2 = 11.331 \text{ MW}$$

$$\Delta P^{(2)} = P_D + P_{Loss}^{(2)} - P_{G_1}^{(2)} - P_{G_2}^{(2)} = 48.187 \text{ MW}$$

$$\Delta\lambda^{(2)} = \frac{\lambda^{(2)} - \lambda^{(1)}}{\sum P_{G_i}^{(2)} - \sum P_{G_i}^{(1)}}\left[\Delta P^{(2)}\right] = 1.5063$$

Iteration 3

$$\lambda^{(3)} = \lambda^{(2)} + \Delta\lambda^{(2)} = 16.108$$

$$⑥ \Rightarrow P_{G_1}^{(3)} = 163.2 \text{ MW}$$

$$⑦ \Rightarrow P_{G_2}^{(3)} = 247.76 \text{ MW}$$

$$P_{Loss}^{(3)} = 0.0002(163.2)^2 + 0.00015(247.76)^2 = 14.538 \text{ MW}$$

$$\Delta P^{(3)} = P_D + P_{Loss}^{(3)} - P_{G_1}^{(3)} - P_{G_2}^{(3)} = 3.5725 \text{ MW}$$

$$\Delta \lambda^{(3)} = \frac{\lambda^{(3)} - \lambda^{(2)}}{\sum P_{G_i}^{(3)} - \sum P_{G_i}^{(2)}} \left[\Delta P^{(3)}\right] = 0.11253$$

Iteration 4

$$\lambda^{(4)} = \lambda^{(3)} + \Delta\lambda^{(3)} = 16.22$$

$$⑥ \Rightarrow P_{G_1}^{(4)} = 164.42 \text{ MW}$$

$$⑦ \Rightarrow P_{G_2}^{(4)} = 250.08 \text{ MW}$$

$$P_{Loss}^{(4)} = 0.0002(164.42)^2 + 0.00015(250.08)^2 = 14.788 \text{ MW}$$

$$\Delta P^{(4)} = P_D + P_{Loss}^{(4)} - P_{G_1}^{(4)} - P_{G_2}^{(4)} = 0.28764 \text{ MW}$$

$$\Delta \lambda^{(4)} = \frac{\lambda^{(4)} - \lambda^{(3)}}{\sum P_{G_i}^{(4)} - \sum P_{G_i}^{(3)}} \left[\Delta P^{(4)}\right] = 0.0091492$$

Iteration 5

$$\lambda^{(5)} = \lambda^{(4)} + \Delta\lambda^{(4)} = 16.229$$

$$⑥ \Rightarrow P_{G_1}^{(5)} = 164.52 \text{ MW}$$

$$⑦ \Rightarrow P_{G_2}^{(5)} = 250.27 \text{ MW}$$

$$P_{Loss}^{(5)} = 0.0002(164.52)^2 + 0.00015(250.27)^2 = 14.808 \text{ MW}$$

$$\Delta P^{(5)} = P_D + P_{Loss}^{(5)} - P_{G_1}^{(5)} - P_{G_2}^{(5)} = 0.020866 < \varepsilon \Rightarrow \boxed{\text{Stop}}$$

b) $P_D = 600$ MW

from I.C: $\lambda = 20$, $\begin{cases} P_{G_1} = 225 \text{ MW} \\ P_{G_2} = 375 \text{ MW} \end{cases}$

Iteration -1

Assume $\lambda^{(-1)} = 19.5$

$$⑥ \Rightarrow P_{G_1}^{(-1)} = \frac{19.5 - 2}{0.08 + 0.0004(19.5)} = 199.32 \text{ MW}$$

$$⑦ \Rightarrow P_{G_2}^{(-1)} = \frac{19.5 - 5}{0.04 + 0.0003(19.5)} = 316.25 \text{ MW}$$

Iteration 0

$$\lambda^{(0)} = 20, \quad P_{G_1}^{(0)} = 225^{MW}, \quad P_{G_2}^{(0)} = 375^{MW}$$

$$P_{Loss}^{(0)} = 0.0002(225)^2 + 0.00015(375)^2 = 31.219^{MW}$$

$$\Delta P^{(0)} = P_D + P_{Loss}^{(0)} - P_{G_1}^{(0)} - P_{G_2}^{(0)} = 31.219^{MW}$$

$$\Delta\lambda^{(0)} = \frac{\lambda^{(0)} - \lambda^{(-1)}}{\sum P_{G_i}^{(0)} - \sum P_{G_i}^{(-1)}}\left[\Delta P^{(0)}\right] = 0.18487$$

Iteration 1

$$\lambda^{(1)} = \lambda^{(0)} + \Delta\lambda^{(0)} = 20.185$$

$$⑥ \Rightarrow P_{G_1}^{(1)} = 206.67^{MW}$$

$$⑦ \Rightarrow P_{G_2}^{(1)} = 329.71^{MW}$$

$$P_{Loss}^{(1)} = 0.0002(206.67)^2 + 0.00015(329.71)^2 = 24.832^{MW}$$

$$\Delta P^{(1)} = P_D + P_{Loss} - P_{G_1}^{(1)} - P_{G_2}^{(1)} = 88.651^{MW}$$

$$\Delta\lambda^{(1)} = \frac{\lambda^{(1)} - \lambda^{(0)}}{\sum P_{G_i}^{(1)} - \sum P_{G_i}^{(0)}}\left[\Delta P^{(1)}\right] = -0.2368$$

Iteration 2

$$\lambda^{(2)} = \lambda^{(1)} + \Delta\lambda^{(1)} = 19.928$$

$$⑥ \Rightarrow P_{G_1}^{(2)} = 203.79 \text{ MW}$$

$$⑦ \Rightarrow P_{G_2}^{(2)} = 324.68 \text{ MW}$$

$$P_{Loss}^{(2)} = 0.0002(203.79)^2 + 0.00018(324.68)^2 = 24.119 \text{ MW}$$

$$\Delta P^{(2)} = P_D + P_{Loss}^{(2)} - P_{G_1}^{(2)} - P_{G_2}^{(2)} = 93.648 \text{ MW}$$

$$\Delta\lambda^{(2)} = \frac{\lambda^{(2)} - \lambda^{(1)}}{\sum P_{G_i}^{(2)} - \sum P_{G_i}^{(1)}} \left[\Delta P^{(2)}\right] = 3.1855$$

Iteration 3

$$\lambda^{(3)} = \lambda^{(2)} + \Delta\lambda^{(2)} = 23.114$$

$$⑥ \Rightarrow P_{G_1}^{(3)} = 236.58 \text{ MW}$$

$$⑦ \Rightarrow P_{G_2}^{(3)} = 383.94 \text{ MW}$$

$$P_{Loss}^{(3)} = 0.0002(236.58)^2 + 0.00018(383.94)^2 = 33.536 \text{ MW}$$

$$\Delta P^{(3)} = P_D + P_{Loss}^{(3)} - P_{G_1}^{(3)} - P_{G_2}^{(3)} = 11.022 \text{ MW}$$

$$\Delta\lambda^{(3)} = \frac{\lambda^{(3)} - \lambda^{(2)}}{\sum P_{G_i}^{(3)} - \sum P_{G_i}^{(2)}} \left[\Delta P^{(3)}\right] = 0.37334$$

Iteration 4

$$\lambda^{(4)} = \lambda^{(3)} + \Delta\lambda^{(3)} = 23.487$$

$$⑥ \Rightarrow P_{G_1}^{(4)} = 240.36 \text{ MW}$$

$$⑦ \Rightarrow P_{G_2}^{(4)} = 392.95 \text{ MW}$$

$$P_{Loss}^{(4)} = 0.0002(240.36)^2 + 0.00015(392.95)^2 = 34.716 \text{ MW}$$

$$\Delta P^{(4)} = P_D + P_{Loss}^{(4)} - P_{G_1}^{(4)} - P_{G_2}^{(4)} = 1.4046 \text{ MW}$$

$$\Delta\lambda^{(4)} = \frac{\lambda^{(4)} - \lambda^{(3)}}{\sum P_{G_i}^{(4)} - \sum P_{G_i}^{(3)}} \left[\Delta P^{(4)}\right] = 0.048563$$

Iteration 5

$$\lambda^{(5)} = \lambda^{(4)} + \Delta\lambda^{(4)} = 23.535$$

$$⑥ \Rightarrow P_{G_1}^{(5)} = 240.85 \text{ MW}$$

$$⑦ \Rightarrow P_{G_2}^{(5)} = 393.86 \text{ MW}$$

$$P_{Loss}^{(5)} = 0.0002(240.85)^2 + 0.00015(393.86)^2 = 34.871 \text{ MW}$$

$$\Delta P^{(5)} = P_D + P_{Loss}^{(5)} - P_{G_1}^{(5)} - P_{G_2}^{(5)} = 0.15806 \text{ MW}$$

$$\Delta\lambda^{(5)} = \frac{\lambda^{(5)} - \lambda^{(4)}}{\sum P_{G_i}^{(5)} - \sum P_{G_i}^{(4)}} \left[\Delta P^{(5)}\right] = 0.0034782$$

Iteration 6

$$\lambda^{(6)} = \lambda^{(5)} + \Delta\lambda^{(5)} = 23.541$$

$$(6) \Rightarrow P_{G_1}^{(6)} = 240.91 \text{ MW}$$

$$(7) \Rightarrow P_{G_2}^{(6)} = 393.97 \text{ MW}$$

$$P_{Loss}^{(6)} = 0.0002(240.91)^2 + 0.00013(393.97)^2 = 34.888 \text{ MW}$$

$$\Delta P^{(6)} = P_D + P_{Loss}^{(6)} - P_{G_1}^{(6)} - P_{G_2}^{(6)} = 0.017316 \text{ MW} < \varepsilon \Rightarrow \boxed{\text{Stop}}$$