ASSIGNMENT 04 of EE 523, SPRING 2023

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$\boxed{1.}$ Type 1 Model

$$
E = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \end{array}
\begin{bmatrix} 5 \\ 6 \\ 11 \\ 10 \\ 1, 6 \\ 2, 5, 7 \\ 6, 8 \\ 7, 9 \\ 8, 10 \\ 4, 9, 11 \\ 3, 10 \end{bmatrix}
$$

Adjacency list

$$
\underset{12\times 1}{x} = \begin{bmatrix} \theta_2 \\ \omega_2 \\ E'_{q2} \\ E'_{d2} \\ \theta_3 \\ \omega_3 \\ E'_{q3} \\ E'_{d3} \\ \theta_4 \\ \omega_4 \\ E'_{q4} \\ E'_{d4} \end{bmatrix}_{12\times 1}
\qquad
\underset{20\times 1}{y} = \begin{bmatrix} \delta_2 \\ |V_2| \\ \delta_3 \\ |V_3| \\ \delta_4 \\ |V_4| \\ \delta_5 \\ |V_5| \\ \vdots \\ \delta_{11} \\ |V_{11}| \end{bmatrix}_{20\times 1}
$$

{Note: ✓ implies that the variables are ~~either~~ given (P_{mi}, X_{di}', X_{di}, X_{qi}', X_{qi}, Y_{ik}, γ_{ik}, ~~δ_k~~ K_{Di}, ω_s, E_{di}', M_i) ~~or have been initialized, say using power flow~~ (~~V_k, δ_k for non-gen buses~~)} $\boxed{1.2}$

$$\cancel{P_2}\quad P_{G_2} - P_{D_2} - P_2 = 0$$

$$\text{or}\quad V_{d_2} I_{d_2} + V_{q_2} I_{q_2} - P_{D_2} - |V_2| \sum_{(2,k)\in E} |Y_{2k} V_k| \cos(\gamma_{2k} + \delta_k - \delta_2)$$

$$\boxed{\begin{aligned} \text{or}\quad & |V_2| \sin(\theta_2-\delta_2) \cdot \left\{ \frac{E_{q_2}' - V_2\cos(\theta_2-\delta_2)}{X_{d_2}'} \right\} \\ & + |V_2| \cos(\theta_2-\delta_2) \cdot \left\{ \frac{E_{d_2}' - V_2\sin(\theta_2-\delta_2)}{-X_{q_2}'} \right\} \\ & - P_{D_2} \\ & - \left[G_{22}|V_2|^2 + |Y_{25} V_5 V_2| \cos(\gamma_{25} + \delta_5 - \delta_2) \right] \\ & = 0 \end{aligned}} \qquad g_1$$

$$Q_{G_2} - Q_{D_2} - Q_2 = 0$$

$$\boxed{\begin{aligned} \text{or}\quad & |V_2| \cos(\theta_2-\delta_2) \left\{ \frac{E_{q_2}' - V_2\cos(\theta_2-\delta_2)}{X_{d_2}'} \right\} \\ & \cancel{+} - |V_2| \sin(\theta_2-\delta_2) \left\{ \frac{E_{d_2}' - V_2\sin(\theta_2-\delta_2)}{-X_{q_2}'} \right\} \\ & - Q_{D_2} \\ & - \left[-B_{22}|V_2|^2 - |Y_{25} V_5 V_2| \sin(\gamma_{25} + \delta_5 - \delta_2) \right] \\ & = 0 \end{aligned}} \qquad g_2$$

~~Similarly, for buses~~

$$|V_3| \sin(\theta_3 - \delta_3) \cdot \left\{ \frac{E'_{q3} - V_3 \cos(\theta_3 - \delta_3)}{x_{d3}} \right\}$$

$$+ |V_3| \cos(\theta_3 - \delta_3) \cdot \left\{ \frac{\cancel{E_q}\ E'_{d3} - V_3 \sin(\theta_3 - \delta_3)}{-x'_{q3}} \right\}$$

$$- P_{D3}$$

$$- \left[G_{33} |V_3|^2 + |Y_{3,11} V_{11} V_3| \cos(\gamma_{3,11} + \delta_{11} - \delta_3) \right]$$

$$= 0 \qquad g_3$$

$$|V_3| \cos(\theta_3 - \delta_3) \cdot \left\{ \frac{E'_{q3} - V_3 \cos(\theta_3 - \delta_3)}{x'_{d3}} \right\}$$

$$- |V_3| \sin(\theta_3 - \delta_3) \cdot \left\{ \frac{E'_{d3} - V_3 \sin(\theta_3 - \delta_3)}{-x'_{q3}} \right\}$$

$$- Q_{D3}$$

$$- \left[-B_{33} |V_3|^2 - |Y_{3,11} V_{11} V_3| \sin(\gamma_{3,11} + \delta_{11} - \delta_3) \right]$$

$$= 0 \qquad g_4$$

1.4

$$|V_4| \sin(\theta_4 - \delta_4) \cdot \left\{ \frac{E'_{q4} - V_4 \cos(\theta_4 - \delta_4)}{X'_{d4}} \right\}$$

$$+ |V_4| \cos(\theta_4 - \delta_4) \cdot \left\{ \frac{E'_{d4} - V_4 \sin(\theta_4 - \delta_4)}{-X'_{q4}} \right\}$$

$$- P_{D4}$$

$$- \left[G_{44} |V_4|^2 - |Y_{4,10} V_{10} V_4| \cos(\gamma_{4,10} + \delta_{10} - \delta_4) \right]$$

$$= 0$$

g5

$$|V_4| \cos(\theta_4 - \delta_4) \cdot \left\{ \frac{E'_{q4} - V_4 \cos(\theta_4 - \delta_4)}{X'_{d4}} \right\}$$

$$- |V_4| \sin(\theta_4 - \delta_4) \cdot \left\{ \frac{E'_{d4} - V_4 \sin(\theta_4 - \delta_4)}{-X'_{q4}} \right\}$$

$$- Q_{D4}$$

$$- \left[-B_{44} |V_4|^2 - |Y_{4,10} V_{10} V_4| \sin(\gamma_{4,10} + \delta_{10} - \delta_4) \right]$$

$$= 0$$

g6

$\boxed{\cancel{4.6}}$ $\boxed{1.5}$

$$\cancel{P_{G5}}^{\,0} - P_{D5} - P_5 = 0$$

or

$$\boxed{-P_{D5} - \left[G_{55}|V_5|^2 + |Y_{51}V_1V_5|\cos(\gamma_{51}+\delta_1-\delta_5) + |Y_{56}V_6V_5|\cos(\gamma_{56}+\delta_6-\delta_5)\right] = 0}$$

g7

$$Q_{G5} - Q_{D5} - Q_5 = 0$$

or

$$\boxed{-Q_{D5} - \left[-B_{55}|V_5|^2 - |Y_{51}V_1V_5|\sin(\gamma_{51}+\delta_1-\delta_5) - |Y_{56}V_6V_5|\sin(\gamma_{56}+\delta_6-\delta_5)\right] = 0}$$

g8

$$\boxed{-P_{D6} - \left[G_{66}|V_6|^2 + |Y_{62}V_2V_6|\cos(\gamma_{62}+\delta_2-\delta_6) + |Y_{65}V_5V_6|\cos(\gamma_{65}+\delta_5-\delta_6) + |Y_{67}V_7V_6|\cos(\gamma_{67}+\delta_7-\delta_6)\right] = 0}$$

g9

$$\boxed{-Q_{D6} - \left[-B_{66}|V_6|^2 - |Y_{62}V_2V_6|\sin(\gamma_{62}+\delta_2-\delta_6) - |Y_{65}V_5V_6|\sin(\gamma_{65}+\delta_5-\delta_6) - |Y_{67}V_7V_6|\sin(\gamma_{67}+\delta_7-\delta_6)\right] = 0}$$

g10

1-6

$$-P_{D_7} - \left[G_{77}|V_7|^2 + |Y_{76} V_6 V_7| \cos(\cancel{\delta}\,\gamma_{76} + \delta_6 - \delta_7) + |Y_{78} V_8 V_7| \cos(\gamma_{78} + \delta_8 - \delta_7) \right] = 0$$

g11

$$-Q_{D_7} - \left[-B_{77}|V_7|^2 - |Y_{76} V_6 V_7| \sin(\gamma_{76} + \delta_6 - \delta_7) - |Y_{78} V_8 V_7| \sin(\gamma_{78} + \delta_8 - \delta_7) \right] = 0$$

g12

$$-P_{D_8} - \left[G_{88}|V_8|^2 + |Y_{87} V_7 V_8| \cos(\gamma_{87} + \delta_7 - \delta_8) + |Y_{89} V_9 V_8| \cos(\gamma_{89} + \delta_9 - \delta_8) \right] = 0$$

g13

$$-Q_{D_8} - \left[-B_{88}|V_8|^2 - |Y_{87} V_7 V_8| \sin(\gamma_{87} + \delta_7 - \delta_8) - |Y_{89} V_9 V_8| \sin(\gamma_{89} + \delta_9 - \delta_8) \right] = 0$$

g14

$$-P_{D_9} - \left[G_{99}|V_9|^2 + |Y_{98} V_8 V_9| \cos(\gamma_{98} + \delta_8 - \delta_9) + |Y_{9,10} V_{10} V_9| \cos(\gamma_{9,10} + \delta_{10} - \delta_9) \right] = 0$$

g15

$$-Q_{D_9} - \left[\cancel{G_9} - B_{99}|V_9|^2 - |Y_{98} V_8 V_9| \sin(\gamma_{98} + \delta_8 - \delta_9) - |Y_{9,10} V_{10} V_9| \sin(\gamma_{9,10} + \delta_{10} - \delta_9) \right] = 0$$

g16

$$-P_{D_{10}} - \Big[G_{10,10}|V_{10}|^2 + |Y_{10,4} V_4 V_{10}| \cos(\gamma_{10,4} + \delta_4 - \delta_{10}) + |Y_{10,9} V_9 V_{10}| \cos(\gamma_{10,9} + \delta_9 - \delta_{10}) + |Y_{10,11} V_{11} V_{10}| \cos(\gamma_{10,11} + \delta_{11} - \delta_{10}) \Big] = 0$$

g17

$$-Q_{D_{10}} - \Big[-B_{10,10}|V_{10}|^2 - |Y_{10,4} V_4 V_{10}| \sin(\gamma_{10,4} + \delta_4 - \delta_{10}) - |Y_{10,9} V_9 V_{10}| \sin(\gamma_{10,9} + \delta_9 - \delta_{10}) - |Y_{10,11} V_{11} V_{10}| \sin(\delta_{10,11} + \delta_{11} - \delta_{10}) \Big] = 0$$

g18

$$-P_{D_{11}} - \Big[G_{11,11}|V_{11}|^2 + |Y_{11,3} V_3 V_{11}| \cos(\gamma_{11,3} + \delta_3 - \delta_{11}) + |Y_{11,10} \cdot V_{10} V_{11}| \cos(\gamma_{11,10} + \delta_{10} - \delta_{11}) \Big] = 0$$

g19

$$-Q_{D_{11}} - \Big[-B_{11,11}|V_{11}|^2 - |Y_{11,3} V_3 V_{11}| \sin(\gamma_{11,3} + \delta_3 - \delta_{11}) - |Y_{11,10} V_{10} V_{11}| \sin(\gamma_{11,10} + \delta_{10} - \delta_{11}) \Big] = 0$$

g20

$\forall\ i = \cancel{2}2, 3$ and 4

$$\dot{\theta}_i = (\omega_i - 1)\,\omega_s$$

$$\dot{\omega}_i = \frac{1}{2H_i}\left[P_{m_i} - \left[V_i \sin(\theta_i - \delta_i) \cdot \left\{ \frac{E'_{q_i} - V_i \cos(\theta_i - \delta_i)}{X'_{d_i}} \right\} + V_i \cos(\theta_i - \delta_i) \cdot \left\{ \frac{E'_{d_i} - V_i \sin(\theta_i - \delta_i)}{-X'_{q_i}} \right\} \right] - K_{D_i}(\omega_i - 1) \right]$$

$$\dot{E}'_{q_i} = \frac{1}{T'_{do_i}}\left[-E'_{q_i} - (X_{d_i} - X'_{d_i})\left\{ \frac{E'_{q_i} - V_i \cos(\theta_i - \delta_i)}{X'_{d_i}} \right\} + E_{fd_i} \right]$$

$$\dot{E}'_{d_i} = \frac{1}{T'_{qo_i}}\left[-E'_{d_i} - (X_{q_i} - X'_{q_i})\left\{ \frac{E'_{d_i} - V_i \sin(\theta_i - \delta_i)}{-X'_{q_i}} \right\} \right]$$

Again, $i = 2, 3, 4$.

$\rightarrow$ f_1 to f_{12}

where

f_1 + f_4 are for $i = 2$

f_5 + f_8 are for $i = 3$

f_9 to f_{12} are for $i = 4$

Thus,

$$[g(x, y) = 0]_{20\times 1} \equiv \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \\ g_5 \\ g_6 \\ g_7 \\ \vdots \\ g_{20} \end{bmatrix}_{20\times 1}$$

g_1 to g_6: gen. buss; represent the Type 1 model

g_7 to g_{20}: Load buss

for Kundur's 11 bus system

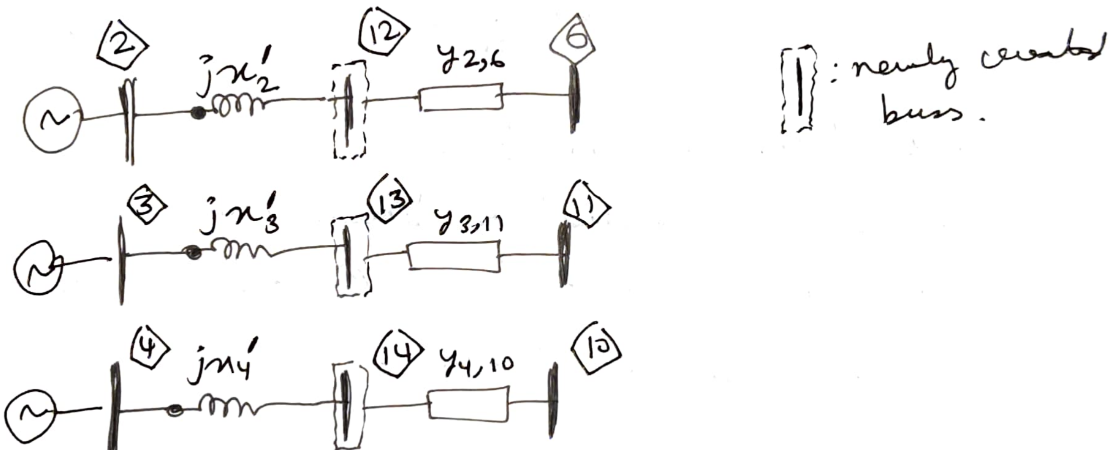
$$\dot{x} = f(x, y)$$

$$\dot{x} \equiv \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_{12} \end{bmatrix}_{12\times 1}$$

Type II Model

For Type II model we first need to create ~~Ygen~~ Ynet by: ~~adding~~ ~~node~~

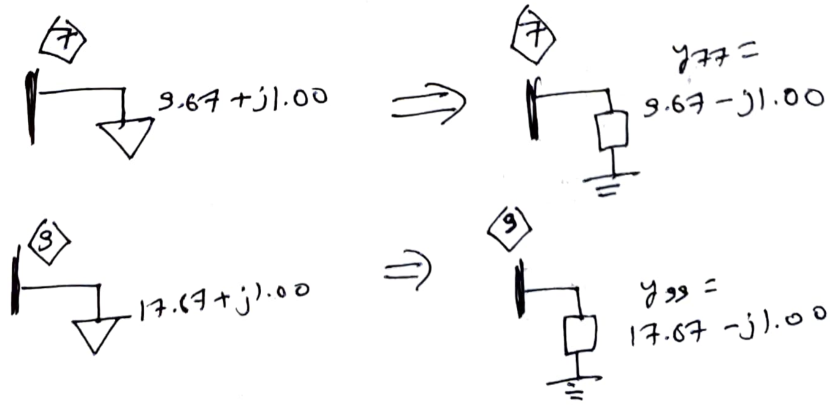
1. Adding extra buses at the terminals of every PV bus, and incorporating the generator impedance in the newly created branches.



Note: In Type II model, we ignore saliency,

so $x_{d_i}' = x_{q_i}' = x_i' = \dfrac{x_{d_i\,original}' + x_{q_i\,original}'}{2}$

2. Converting all ~~loads~~ loads into ~~an~~ constant impedances.



(3). Renumber/ ^Reorder our buses such that the slack bus is numbered $\langle 1 \rangle$, PV buses are numbered $\langle 2 \rangle$ to $\langle N_{pv}+1 \rangle$, PQ buses are numbered $\langle N_{pv}+2 \rangle$ to $\langle N \rangle$ and new buses at the terminals of the PV buses are numbered ~~to~~ $\langle 2+(N-1) \rangle$ to $\langle N_{pv}+1+(N-1) \rangle$

thus making a total of $N + N_{pv}$ buses.

N → original total count of all buses in the system.

N_{pv} → number of Non slack PV buses in the system.

Fortunately, for the Kundur 11 bus system, since the generator buses are already numbered ~~from~~ from $\langle 1 \rangle$ to $\langle 4 \rangle$, we need not do any renumbering.

(4). Finally ~~to~~ form the Ybus, called Ynet from the $N + N_{pv}$ buses:

$$
Y_{net} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ \vdots \\ 11 \\ 12 \\ 13 \\ 14 \end{array}
\begin{array}{c}
\begin{array}{cc} 1\ 2\ 3\ 4 & 5 \ \cdots \ 11\ 12\ 13\ 14 \end{array} \\
\left[\begin{array}{c|c}
\begin{matrix} Y_{gg} \\ (N_{pv}+1)\times(N_{pv}+1) \\ 4\times 4 \end{matrix} & \begin{matrix} Y_{gn} \\ (N_{pv}+1)\times(N-1) \\ 4\times 10 \end{matrix} \\ \hline
\begin{matrix} Y_{ng} \\ (N-1)\times(N_{pv}+1) \\ 10\times 4 \end{matrix} & \begin{matrix} Y_{nn} \\ (N-1)\times(N-1) \\ 10\times 10 \end{matrix}
\end{array}\right]
\end{array}
$$

$(N_{pv}+N)\times(N_{pv}+N)$

14×14

5. Identify Y_{gg}, Y_{gr}, Y_{rg} and Y_{rr} from Y_{net}.

$$\underset{4\times4}{Y_{gg}} = Y_{net}\,[1 : N_{pv}+1,\ 1 : N_{pv}+1]$$
$$\underset{4\times10}{Y_{gr}} = Y_{net}\,[1 : N_{pv}+1,\ N_{pv}+2 : \text{end}]$$
$$\underset{10\times4}{Y_{rg}} = Y_{net}\,[N_{pv}+2 : \text{end},\ 1 : N_{pv}+1]$$
$$\underset{10\times10}{Y_{rr}} = Y_{net}\,[N_{pv}+2 : \text{end},\ N_{pv}+2 : \text{end}].$$

6. Form Y_{gen}:

$$\underset{4\times4}{Y_{gen}} = \underset{4\times4}{Y_{gg}} - \underset{4\times10}{Y_{gr}}\ \underset{10\times10}{Y_{rr}^{-1}}\ \underset{10\times4}{Y_{rg}}$$

7. Now we can form the Type II model:

$$\dot{\theta}_i = (\omega_i - 1)\,\omega_s \qquad \forall \text{ buses } i = 1 \text{ to } N_{pv}+1$$
$$\underset{\cancel{4\times1}\ 3\times1}{\dot{\omega}_i} = \frac{1}{2H_i}\left\{ P_{m_i} - P_{a_i} - K_{D_i}(\omega_i - 1) \right\}$$
$$\underset{\cancel{4\times1}\ 3\times1}{\dot{E}'_{q_i}} = \frac{1}{T'_{do_i}}\left\{ E'_{q_i} - (x_{d_i} - x'_i)\, I_{d_i} + E_{fd_i} \right\}$$
$$\underset{\cancel{4\times1}\ 3\times1}{\dot{E}'_{d_i}} = \frac{1}{T'_{qo_i}}\left\{ -E'_{d_i} + (x_{q_i} - x'_i)\, I_{q_i} \right\}$$
$\cancel{4\times1}$ 3×1

where

$$P_{a_i} = \sum_{k=1}^{N_{pv}+1} Y_{gen,ik}\, E'_k E'_i \cos(\gamma_{ik} + \delta_k - \delta_i)$$

Note: Don't confuse internal voltage angle δ_i / δ_k with the $Y_{gen,ik}$ angle γ_{ik}.

where $E'_i = \sqrt{E'^2_{q_i} + E'^2_{d_i}}$

$$\delta_i = \tan^{-1}\left(\frac{E'_{q_i}}{E'_{d_i}}\right) + \left(\theta_i - \frac{\pi}{2}\right)$$

$$I_{d_i} = -\sum_{k=1}^{N_{pv}+1} Y_{gen,ik}\, E'_k \sin(\delta_k + \gamma_{ik} - \theta_i)$$

$$I_{q_i} = \sum_{k=1}^{N_{pv}+1} Y_{gen,ik}\, E'_k \cos(\gamma_{ik} + \delta_k - \theta_i)$$

$$x'_i = \frac{x_{q_i}' + x_{d_i}'}{2}$$

⑧ We may insert the other known algebraic variables into the differential equations to get the final set of equations: $\boxed{2.4}$

$$E_i \angle \gamma_i \angle (\tfrac{\pi}{2}-\theta_i) = E_{d_i} + j E_{q_i}$$

$$\gamma_i + \tfrac{\pi}{2} - \theta_i = \tan^{-1}\left(\frac{E_{q_i}}{E_{d_i}}\right)$$

$$\Rightarrow \boxed{\gamma_i = \tan^{-1}\left(\frac{E_{q_i}}{E_{d_i}}\right) + \left(\theta_i - \frac{\pi}{2}\right)}$$

$i = 2 \to N_{pv} + 1$

$$\underset{34\times 1}{\dot{\theta}_i} = (\omega_i - 1)\,\omega_s$$

$$\underset{34\times 1}{\dot{\omega}_i} = \frac{1}{2H_i}\left[P_{m_i} - \left\{ \sum_{k=1}^{N_{pv}+1} Y_{egen\,ik} \left(\sqrt{E_{q_k}'^2 + E_{d_k}'^2}\right)\left(\sqrt{E_{q_i}'^2 + E_{d_i}'^2}\right) \cos\left(\gamma_{ik} + \tan^{-1}\left(\frac{E_{q_k}'}{E_{d_k}'}\right) + \left(\theta_k - \frac{\pi}{2}\right) - \tan^{-1}\left(\frac{E_{q_i}'}{E_{d_i}'}\right) + \left(\theta_i - \frac{\pi}{2}\right)\right) \right\} - K_{D_i}(\omega_i - 1) \right]$$

$$\underset{34\times 1}{\dot{E}_{q_i}'} = \frac{1}{T_{do_i}'}\left[E_{q_i}' - (x_{d_i} - x_i')\left\{ - \sum_{k=1}^{N_{pv}+1} Y_{egen\,ik} \sqrt{E_{q_k}'^2 + E_{d_k}'^2}\, \sin\left(\gamma_{ik} + \tan^{-1}\left(\frac{E_{q_k}'}{E_{d_k}'}\right) - \theta_i\right) + \left(\theta_k - \frac{\pi}{2}\right) \right\} + E_{fd_i} \right]$$

$$\underset{34\times 1}{\dot{E}_{d_i}'} = \frac{1}{T_{qo_i}'}\left[-E_{d_i}' + (x_{q_i} - x_i')\left\{ \sum_{k=1}^{N_{pv}+1} Y_{egen\,ik} \sqrt{E_{q_k}'^2 + E_{d_k}'^2}\, \cos\left(\gamma_{ik} + \tan^{-1}\left(\frac{E_{q_k}'}{E_{d_k}'}\right) - \theta_i + \left(\theta_k - \frac{\pi}{2}\right)\right) \right\} \right]$$

$$x_i' = \frac{\underbrace{x_{q_i}' + x_{d_i}'}_{\text{original values.}}}{2}$$

$$\gamma_i = \tan^{-1}\left(\frac{E_{q_i}'}{E_{d_i}'}\right) + \left(\theta_i - \frac{\pi}{2}\right)$$

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