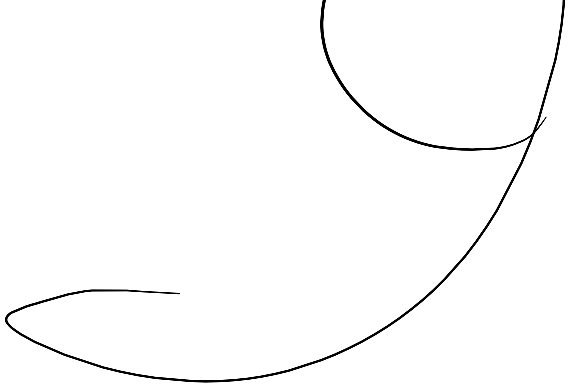
EE 507 Exam 1 Review



MIDTERM 1 REVIEW

I. Introduction to Probability Theory

A. Concept of probabilities (frequency, chance)

B. Framework | definition

Experiment

Outcomes

Events

Probabilities assigned to events

C. Axioms of Probabilities

$$P(A) \geq 0, \quad P(\Omega) = 1, \quad P(A+B) = P(A) + P(B)$$

if $AB = \phi$.

D. Derived relationships - set theory + axioms

$$P(A+B) = P(A) + P(B) - P(AB)$$

E. Conditional Probabilities: chances given a priori information

$$P(A|B) = \frac{P(AB)}{P(B)} \qquad \Leftrightarrow \text{independence}$$

$\Rightarrow$ Derived expressions: $\begin{cases} \text{law of total probability,} \\ \text{Baye's rule} \end{cases}$

F. Multiple experiments and Repeated Trials

II. Random Variables

A. Motivation and Definition

$\Rightarrow$ mapping of outcomes to numbers

B. Analysis constructs

1. Cumulative Distribution Function (CDF)

$$F_X(x) = P(\{X \leq x\}) = P(X \leq x)$$

$\Rightarrow$ learned to find from experiment description, properties

2. probability density function $f_X(x)$

$$\boxed{f_X(x) = \frac{d}{dx} F_X(x)} \qquad = \lim_{\delta \to 0} \frac{P(x \leq X \leq x+\delta)}{\delta}$$

properties of pdfs, work from them

3. probability mass function $P_X(\alpha) = P(X = \alpha)$ for all α.

C. Expectations: average value or mean

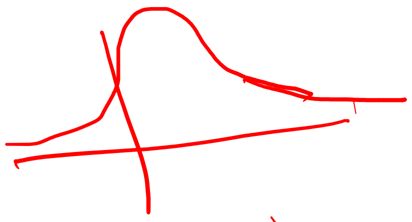
⇓

logic based on repeated trials

$$E(X) = \int_{-\infty}^{\infty} x\, f_X(x)\, dx$$

D. Several standard random variables

Gaussian, geometric, uniform, exponential, Bernoulli,



$\boxed{(1-\lambda)\lambda^{k-1}}$

$\lambda(1-\lambda)^{k-1}$

E. Random variables and conditioning

⇒ Basic definitions – get a bit complicated because thinking of densities (used limits)

$F_{X|A}$ $\qquad f_{X|A}$ ⇔ standard problem classes: $f_{X|A} + f_{X|\bar{A}}$

$P(A \mid X = x)$

III. Functions of Random variables

R.V. $\Rightarrow Y = g(X)$

A. Finding CDFs / pdfs

$\Rightarrow$ general approach was to find CDF and take a deriv.

$\Rightarrow$ alternate formula (works in special cases)

B. Expectations of functions of R.V.s

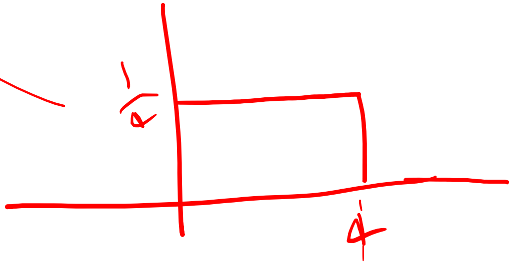
$$E(g(X)) = \int_{-\infty}^{\infty} g(x) f_X(x)\, dx$$

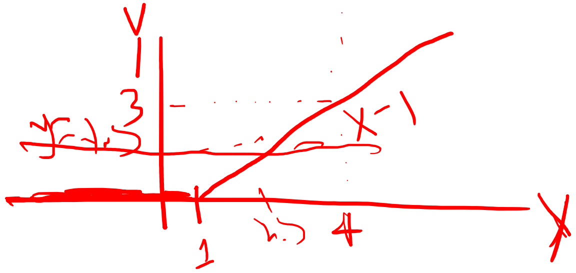
$\Rightarrow$ statistics: moments, variance, moment generating functions

HW 8

$X \sim \text{unif}(0,4)$ and $Y = \begin{cases} 0, & X \le 1 \\ X-1, & X \ge 1 \end{cases}$

$f_X(x)$





$X-1 \le y$

$X \le y+1$

$X-1 \le y$

$X \le y+1$

$f_Y(y)$

$F_Y(y) = P(Y \le y) = \begin{cases} 0 & y<0 \\ \frac{1}{4}(y+1) & 0 \le y < 3 \\ 1 & y \ge 3 \end{cases}$

$P(Y \le y) = P(0 \le X \le 1) + P(1 \le X \le y+1)$

$X-1 \le y$

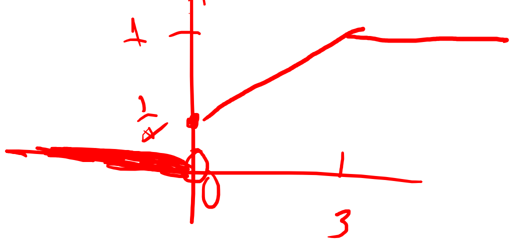
$X \le y+1$

$= P(0 \le X \le y+1)$

$= \int_0^{y+1} \frac{1}{4}\, dt$

$= \frac{1}{4}(y+1)$

$F_Y(y)$



$\boxed{f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{1}{4}\delta(y) + \frac{1}{4}, \quad 0 \le y \le 3}$

c. $P(A|B) = \frac{P(AB)}{P(B)}$

$P(AB) = \int_{-\infty}^{\infty} P(AB|X=x) f_X(x)\,dx$

$= \int_{-\infty}^{\infty} P(A|X=x) P(B|X=x) f_X(x)\,dx$

$\int_{0}^{2} \left(1-\frac{x}{2}\right)\left(\frac{x}{2}\right) \cdot \frac{1}{2}\,dx$

$\int_{1}^{2} \left(\frac{x}{2} - \frac{x^2}{4}\right)\frac{1}{2}\,dx = \frac{1}{2}\left[\left(\frac{x^2}{4} - \frac{x^3}{12}\right)\right]_{0}^{2}$

$\frac{1}{2}\left(1 - \frac{8}{12}\right) = \frac{1}{6}$

4. $f_{X|A}(x) = 2e^{-2x}, \; x \geq 0$

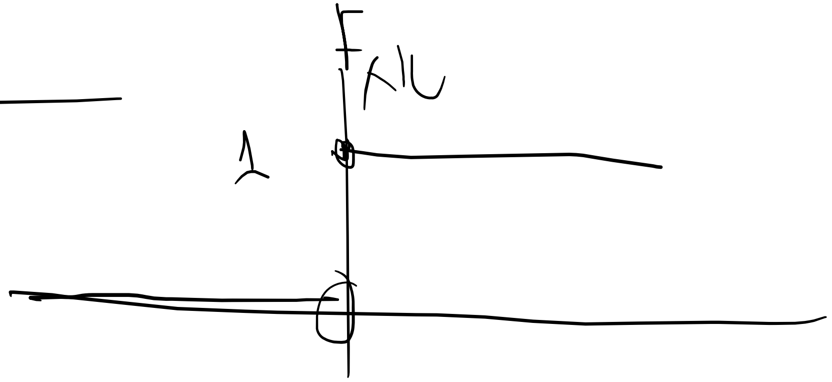
$f_{X|B}(x) = e^{-x}, \; x \geq 0$

$f_{X|C}(x) = \delta(x), \; x \geq 0$

$X = 0$ w.p. 1

pmf. $p_X(0) = 1$

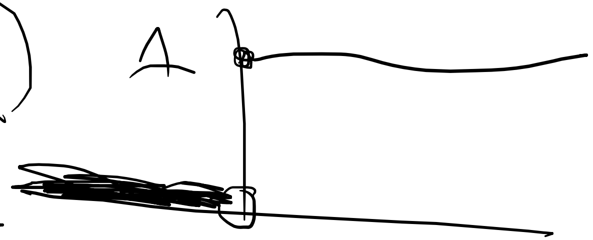
$p_X(x) = 0$, otherwise



$f_X(x)$

$$P(A \mid X=x) = \frac{f_{X|A}(x)\, P(A)}{f_X(x)}$$

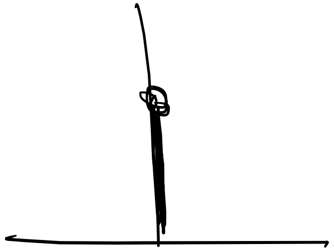
$F_{X|C}$



e^{-x}

$2e^{-2x}$

$$f_X(x) = f_{X|A}(x)\, P(A) + f_{X|B}(x)\, P(B) + f_{X|C}(x)\, P(C)$$



Given C, $X=0$ $\quad f_{X|C}(x)$ $\quad \delta(x)$

$$p_X(x) = \begin{cases} 1, & x=0 \\ 0, & \text{otherwise} \end{cases} \qquad f_X(x) = \delta(x)$$