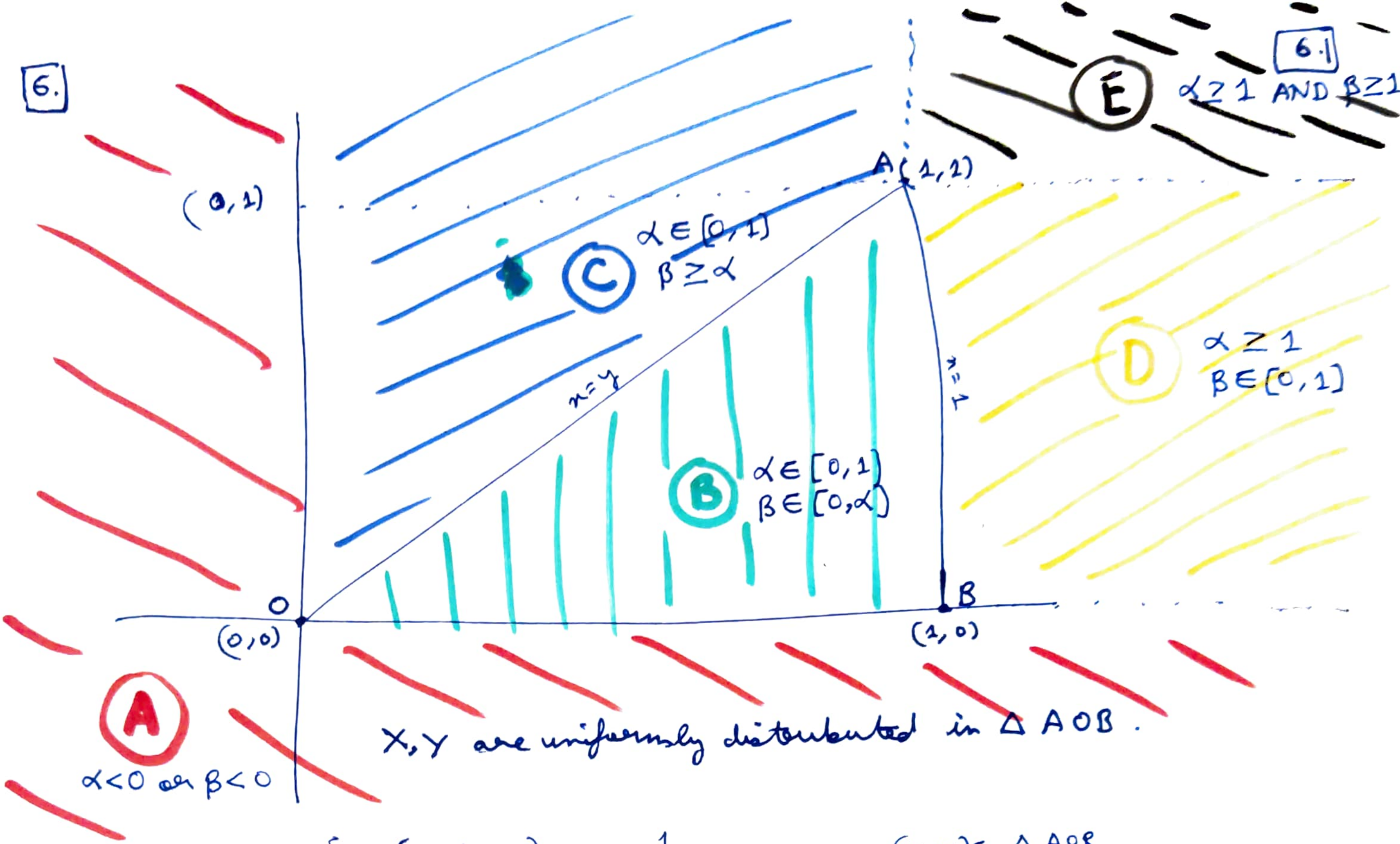


X, Y are uniformly distributed in Δ AOB.

So $f_{X,Y}(x,y) = \dfrac{1}{\text{Area of } \Delta AOB}$ $\quad (x,y) \in \Delta AOB$

or $f_{X,Y}(x,y) = \dfrac{1}{\frac{1}{2}\times 1 \times 1}$ $\quad y \in [0,1]$ or $x \in [y,1]$

6(a)

or $f_{X,Y}(x,y) = \begin{cases} 2 & y \in [0,1],\ x \in [y,1] \\ 0 & \text{else} \end{cases}$

Ans

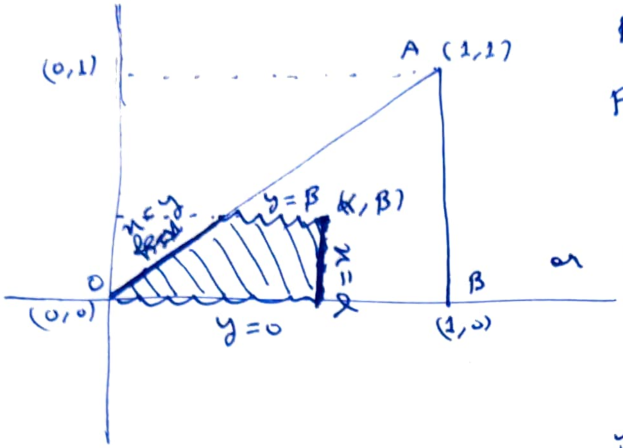
6(b) (Calculations on next pages.)

$$F_{X,Y}(x=\alpha, y=\beta) = \begin{cases} 0 & \alpha < 0 \text{ or } \beta < 0 \quad \text{A} \\ 2\alpha\beta - \beta^2 & \alpha \in [0,1],\ \beta \in [0,\alpha] \quad \text{B} \\ \alpha^2 & \alpha \in [0,1],\ \beta \geq \alpha \quad \text{C} \\ 2\beta - \beta^2 & \alpha \geq 1,\ \beta \in [0,\alpha] \quad \text{D} \\ 1 & \alpha \geq 1,\ \beta \geq 1 \quad \text{E} \end{cases}$$

Ans

For (α,β) in B:

i.e. $\alpha \in [0,1]$, $\beta \in [0,\alpha]$



~~$F_{X,Y}(x,y)$~~

$$F_{X,Y}(x=\alpha, y=\beta) = \iint_{\text{shaded region}} 2\,dx\,dy \qquad (\alpha,\beta)\in B$$

or $$F_{X,Y}(x=\alpha, y=\beta) = \int_{y=0}^{y=\beta}\int_{x=y}^{x=\alpha} 2\,dx\,dy \qquad (\alpha,\beta)\in B$$

or $$F_{X,Y}(\alpha,\beta) = \int_{y=0}^{y=\beta} 2x\Big|_{y}^{\alpha} dy \qquad (\alpha,\beta)\in B$$

or $$F_{X,Y}(\alpha,\beta) = \int_{y=0}^{y=\beta} 2(\alpha-y)\,dy \qquad (\alpha,\beta)\in B$$

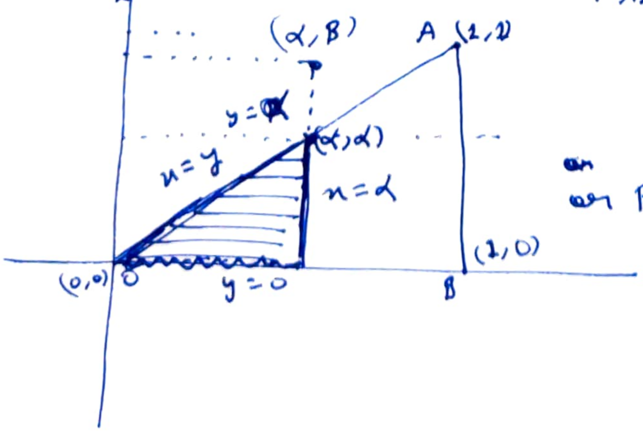
or $$F_{X,Y}(\alpha,\beta) = 2\alpha y - y^2\Big|_0^{\beta} \qquad (\alpha,\beta)\in B$$

or $$\boxed{F_{X,Y}(\alpha,\beta) = 2\alpha\beta - \beta^2 \qquad (\alpha,\beta)\in B}$$

$\boxed{\text{or } \alpha\in[0,1),\ \beta\in[0,\alpha]}$

For (α,β) in C:

i.e. $\alpha \in [0,1]$, $\beta \geq \alpha$



$$F_{X,Y}(x=\alpha, y=\beta) = \iint_{\text{shaded region}} 2\,dx\,dy \qquad (\alpha,\beta)\in C$$

or $$F_{X,Y}(x=\alpha, y=\beta) = \int_{y=0}^{y=\alpha}\int_{x=y}^{x=\alpha} 2\,dx\,dy \qquad (\alpha,\beta)\in C$$

or $F_{X,Y}(x=\alpha, y=\beta) = \int_{y=0}^{y=\alpha} 2(\alpha - y)\,dy \qquad (\alpha,\beta) \in C$

or $F_{X,Y}(x=\alpha, y=\beta) = 2\alpha^2 - \alpha^2 \qquad (\alpha,\beta) \in C$

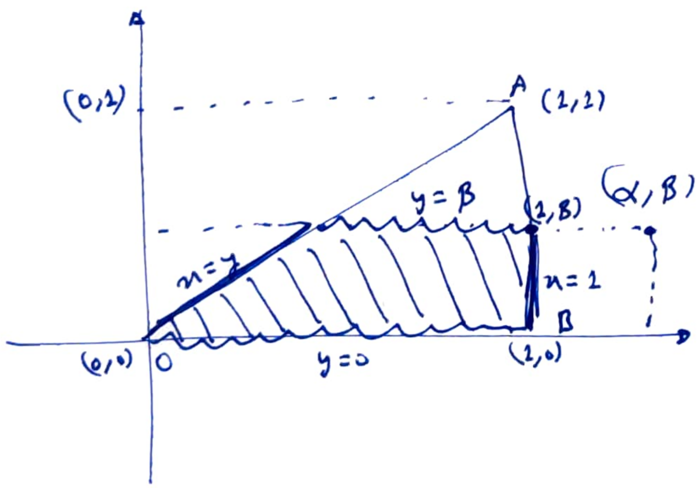
or $$\boxed{F_{X,Y}(x=\alpha, y=\beta) = \alpha^2 \qquad (\alpha,\beta) \in C \quad \text{or} \quad \alpha \in [0,1],\ \beta \geq \alpha}$$

Area of the shaded Δ. * $f_{X,Y}(x,y)$ in the region.

For (α,β) in D

i.e. $\alpha \geq 1$, $\beta \in [0,1)$

$F_{X,Y}(x=\alpha, y=\beta) = \iint_{\text{shaded region}} 2\,dx\,dy \qquad (\alpha,\beta) \in D$



or $F_{X,Y}(\alpha,\beta) = \int_{y=0}^{y=\beta} \int_{x=y}^{x=1} 2\,dx\,dy \qquad (\alpha,\beta) \in D$

or $F_{X,Y}(\alpha,\beta) = \int_{y=0}^{y=\beta} 2(1-y)\,dy \qquad (\alpha,\beta) \in D$

or $F_{X,Y}(\alpha,\beta) = 2y - y^2 \Big|_0^\beta \qquad (\alpha,\beta) \in D$

or $$\boxed{F_{X,Y}(\alpha,\beta) = 2\beta - \beta^2 \qquad (\alpha,\beta) \in D \quad \text{or} \quad \alpha \geq 1,\ \beta \in [0,1)}$$

$$f_X(x) = \int_{\forall y} f_{X,Y}(x,y)\,dy$$

or
$$f_X(x) = \begin{cases} 0 & x < 0 \\ \int_{y=0}^{y=x} 2\,dy & x \in [0,1] \\ 0 & x > 1 \end{cases}$$

6(c)

or
$$\boxed{f_X(x) = \begin{cases} 0 & x < 0 \\ 2x & x \in [0,1] \\ 0 & x > 1 \end{cases}}$$ Ans

$$f_Y(y) = \int_{\forall x} f_{X,Y}(x,y)\,dx$$

or
$$f_Y(y) = \begin{cases} 0 & y < 0 \\ \int_{x=y}^{x=1} 2\,dx & y \in [0,1] \\ 0 & y > 1 \end{cases}$$

6(d)

or
$$\boxed{f_Y(y) = \begin{cases} 0 & y < 0 \\ 2(1-y) & y \in [0,1] \\ 0 & y > 1 \end{cases}}$$ Ans

$$f_{X|Y}(x|Y=y) = \frac{f_{X,Y}(x,y=Y)}{f_Y(y)}$$

6(e)

or $f_{X|Y}(x|Y=y) = \begin{cases} \text{not defined} & y<0 \\ \frac{1}{1-y} & y\in(0,1) \\ \text{not defined.} & y\geq 1 \end{cases}$ Ans