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1.1

Stator coupling equations:

- $$e_d = -\psi_q - R_s i_d \quad (1)$$
- $$e_q = \psi_d - R_s i_q \quad (2)$$

(a1) Stator dynamics are $(\dot{\psi}_q, \dot{\psi}_d)$ are 'fast' ⇒ average of $\dot{\psi}_q, \dot{\psi}_d$ over time periods relevant to 'slow' variables (like e_d, e_q, i_d, i_q) is negligible. We neglect $\dot{\psi}_q$, $\dot{\psi}_d$.

Rotor Dynamic equations:

- $$e_{fd} = \dot{\psi}_{fd} + R_{fd} i_{fd} \quad (3)$$
- $$0 = \dot{\psi}_{1q} + R_{1q} i_{1q} \quad (4)$$

(a2) ω_r (speed of rotor) is assumed to equal ω_s, which itself is taken as 'synchronous speed'. 1.00 pu. with ω_s as base speed

(a3) System is assumed balanced, all zero sequence component like e_0, i_0, etc. are ~~assumed~~ neglected.

Rotor coupling equations:

- $$\psi_{fd} = x_{ffd} i_{fd} - x_{ad} i_d \quad (5)$$
- $$\psi_{1q} = x_{11q} i_{1q} - x_{aq} i_q \quad (6)$$

(a4) In our system (ee 523), we are NOT modelling damper winding on the d-axis and are modelling one damper winding on the q-axis. This means that $\psi_{1d}, i_{1d}, \psi_{2q}, i_{2q}$ terms are neglected.

$$E_q' = \frac{x_{ad}}{x_{ffd}} \psi_{fd} \quad (7)$$

(From Kundur Ch.5, Pg 180 "Alternative form of machine equations")

↓

Voltage proportional to ψ_{fd}.

Putting (5) in (7):

~~and $E_I = x_{ad} i_{fd}$~~
~~Voltage proportional to i_{fd}.~~

$$E_q' = \frac{x_{ad}}{x_{ffd}} \left\{ x_{ffd} i_{fd} - x_{ad} i_d \right\} \quad (7a)$$

From (5):

$$\boxed{i_{fd} = \frac{\psi_{fd} + x_{ad}\, i_d}{x_{ffd}}} \quad (5a) \qquad \boxed{1.2}$$

~~But from (5):~~

$$\cancel{\psi_{fd} = \frac{\psi_{fd} - x_{ffd}\, i_{fd}}{-x_{ad}}}$$

$$\text{So, } \cancel{E_q' = \frac{x_{ad}}{x_{ffd}}\{ x_{ffd}\, i_{fd} + \psi_{fd}}$$

From (6):

$$\boxed{i_{1q} = \frac{\psi_{1q} + x_{aq}\, i_q}{x_{11q}}} \quad (6a)$$

(8)

$$\boxed{E_d' = \frac{-x_{aq}}{x_{1q}}\, \psi_{1q}}$$

(From Kundur (5.26) Pg 186)

or

$$\boxed{E_d' = \frac{-x_{aq}}{x_{1q}}\{ x_{11}\, i_{1q} - x_q\, i_q \}} \quad (8a)$$

$$\cancel{\psi_q = -e_d - R_s\, i_d} \quad \text{(Kundur (3.164) Pg 97)}$$

$$\cancel{\psi_d = e_q + R_s\, i_q} \quad \text{(Kundur (3.165) Pg 97)}$$

~~Putting (5a) and (6a)~~

From (1) and (2), we have

$$\boxed{\psi_q = -e_d - R_s\, i_d} \quad (1a)$$

$$\boxed{\psi_d = e_q + R_s\, i_q} \quad (2a)$$

~~But~~

$$\cancel{e_d = x_q\, i_q - R_s\, i_d} \quad (9)$$

$$\cancel{e_q = -x_d\, i_d + x_{ad}\, i_{fd} - R_s\, i_q} \quad (10)$$

~~Kundur (3.164) and (3.165) on Pg 97.~~

So, (1a) and (2a) ~~after substitution of (9) and (10) become~~:

$$\cancel{\psi_q = -x_q\, i_q + R_s\, i_d}$$

$$\boxed{\psi_q = -(x_q + x_e)\, i_q + x_q\, i_{1q}} \quad (1b)$$

$$\boxed{\psi_d = -(x_d + x_e)\, i_d + x_d\, i_{fd}} \quad (2b)$$

Kundur (3.127) and (3.128)

$\boxed{1.3}$

Putting 5a and 6a into 1b and 2b;

$$\psi_q = -(x_q + x_l)\, i_q + \cancel{x_{aq}}\, x_q \cdot \frac{1}{x_{11q}} \{ \psi_{1q} + x_{aq} i_q \}$$

$$\psi_d = -(x_d + x_l)\, i_d + x_d \cdot \frac{1}{x_{ffd}} \{ \psi_{fd} + x_{ad} i_d \}$$

$$\dot{\psi}_{fd} = e_{fd} - R_{fd}\, i_{fd} \qquad (9)$$

$$\dot{\psi}_{1q} = -R_{1q}\, i_{1q} \qquad (10)$$

or

$$\psi_d = -\left(x_{ad} + x_l - \frac{x_{ad}^2}{x_{ffd}}\right) i_d + \frac{x_{ad}}{x_{ffd}} \psi_{fd} \qquad (1d)$$

$$\psi_q = -\left(x_{aq} + x_l - \frac{x_{aq}^2}{x_{ffd}}\right) i_q + \frac{x_{aq}}{x_{11q}} \psi_{1q} \qquad (2d)$$

Putting 5a and 6a into 11 and 12:

$$\dot{\psi}_{fd} = e_{fd} - R_{fd} \times \frac{1}{x_{ffd}} \left(\psi_{fd} + x_{ad}\, i_d \right) \qquad (9a)$$

$$\dot{\psi}_{1q} = -R_{1q} \times \frac{1}{x_{11q}} \left(\psi_{1q} + x_{aq}\, i_q \right) \qquad (10a)$$

From (1a), (2a), (1c) and (2c), we get:

$$e_q = -\left(x_{ad} + x_l - \frac{x_{ad}^2}{x_{ffd}}\right) i_d - R_s i_q + \frac{x_{ad}}{x_{ffd}} \psi_{fd} \quad (11)$$

$$e_d = \left(x_{aq} + x_l - \frac{x_{aq}^2}{x_{11q}}\right) i_q - R_s i_d - \frac{x_{aq}}{x_{11q}} \psi_{1q} \quad (12)$$

~~Eff~~ After some substitution, including:

$$x_d' = x_d - \frac{x_{ad}^2}{x_{ffd}} \quad \text{where } x_d = x_{ad} + x_l \quad (s1)$$

$$x_q' = x_q - \frac{x_{aq}^2}{x_{11q}} \quad \text{where } x_q = x_{aq} + x_l \quad (s2)$$

$$E_q' = \frac{x_{ad}}{x_{ffd}} \psi_{fd} \quad (7) \text{ (again)} \quad \& \quad V_q = e_q \quad (s3)$$

$$E_d'' = -\frac{x_{aq}}{x_{11q}} \psi_{1q} \quad (8) \text{ (again)} \qquad V_d = e_d \quad (s4)$$

$\cancel{e_q = E_q' - R_s i_q - x_d i_d}$

$\cancel{e_d = E_d'' - R_s i_d + x_q' i_q}$

$$E_q'' = V_q + R_s i_q + x_d' i_d \quad (13)$$

$$E_d'' = V_d + R_s i_d - x_q' i_q \quad (14)$$

Taking ~~derivative of (13) and (14) wrt time~~:

$\cancel{\dot{E}_q' =}$

Taking the derivative of (7) and (8), we get:

$\boxed{1.5}$

$$\cancel{\frac{x_{ffd}}{x_{ad}}}\ \dot{E}_q' = \left(\frac{x_{ad}}{x_{ffd}}\right) \cdot \left[\left\{ e_{fd} - \frac{R_{fd}}{x_{ffd}}\left(\psi_{fd} + x_{ad}\, i_d\right)\right\}\right]$$

$$\cancel{\text{or} \quad \dot{E}_q' = \left(\frac{x_{ad}}{x_{ffd}}\right)\left[\left\{ e_{fd} - \frac{R_{fd}}{x_{ffd}}\left\{x_{ffd}\, i_{fd}\right\}\right\}\right]}$$

$$\text{or} \quad \cancel{\dot{E}_q' = \frac{x_{ad}}{T_{do}'}\left[e_{fd}\ E_{fd} - \frac{x_{ad}}{x_{ffd}} R_{fd}\, i_d - \frac{R_{fd}}{x_{ffd}}\psi_{fd}\right]}$$

$$\text{or} \quad \dot{E}_q' = \left\{ \frac{\cancel{x_{ad}} \cdot R_{fd}}{x_{ffd}} \left[\frac{x_{ad}\, e_{fd}}{R_{fd}} - \frac{x_{ad}}{x_{ffd}}\left(\psi_{fd} + x_{ad}\, i_d\right)\right]\right.$$

55

$$\text{or} \quad \dot{E}_q' = \frac{1}{\left(\frac{R_{fd}}{x_{ffd}}\right)} \cdot \left[E_{fd} - \left(\frac{x_{ad}^2}{x_{ffd}}\right) i_d - \frac{x_{ad}}{x_{ffd}}\psi_{fd}\right]$$

56 From 51 From 7

$$\text{or} \quad \boxed{\dot{E}_q' = \frac{1}{T_{do}'}\left[E_{fd} - (x_d - x_d')\, i_d - E_q'\right]} \quad \text{III}$$

when

$$\boxed{T_{do}' = \cancel{x_{ffd}}\ \frac{R_{fd}}{x_{ffd}}} \ (56) \qquad \boxed{E_{fd} = e_{fd} \cdot \frac{x_{ad}}{R_{fd}}} \ (55)$$

Similarly,

$$\boxed{\dot{E}_d' = \frac{1}{T_{qo}'}\left[-E_d' + (x_q - x_q')\, i_q\right]} \quad \text{IV}$$

where

$$\boxed{T_{qo}' = \cancel{R_q}\,.\,\frac{R_{1q}}{x_{1q}}} \quad (57)$$

$$\boxed{E_d'' = e_d\,.\,\frac{x_{aq}}{R_{1q}}} \quad (58)$$

Two Axis Model

$$\boxed{\dot{\theta} = (\omega - 1)\,\omega_s} \quad \text{I}$$

$$\boxed{\dot{\omega} = \frac{1}{2H}\left\{P_m - P_e - K_D(\omega - 1)\right\}} \quad \text{II}$$

I to IV form the Two-Axis model.

$$\boxed{\dot{E}_q' = \frac{1}{T_{do}'}\left\{-E_q' - (x_d - x_d')\, i_d + E_{fd}\right\}} \quad \text{III}$$

$$\boxed{\dot{E}_d' = \frac{1}{T_{qo}'}\left\{-E_d' + (x_q - x_q')\, i_q\right\}} \quad \text{IV}$$

2.1

For the steady-state model of the Two-Axis model

$\dot{E}_q' = 0$ in ~~IV~~ III and $\dot{E}_d' = 0$ in IV

or $E_{fd} = E_q' + (x_d - x_d')\, i_d$

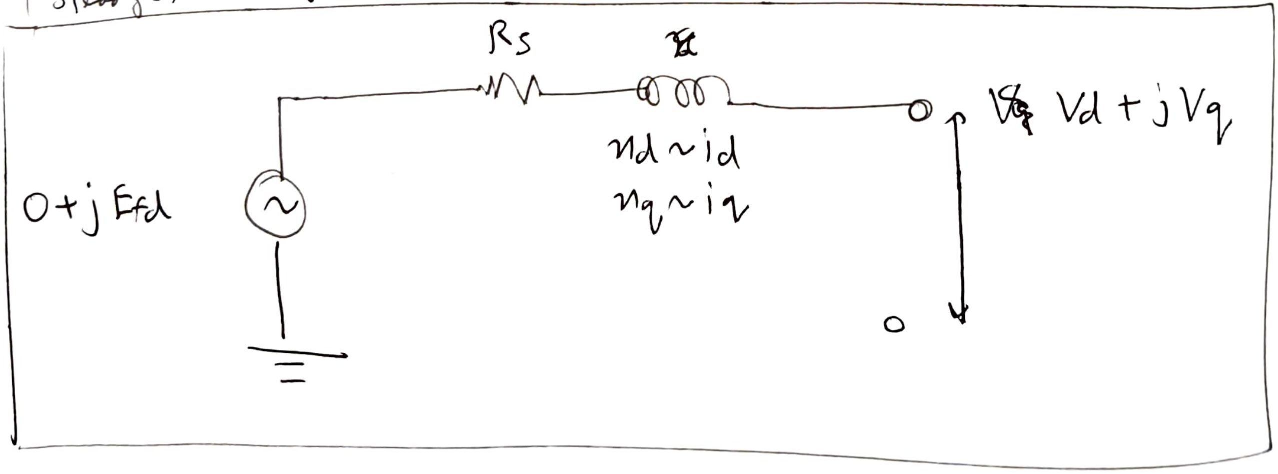
or $E_{fd} = \underbrace{V_q + R_s i_q + x_d' i_d}_{\text{(From (13))}} + (x_d - x_d')\, i_d$.

or $\boxed{E_{fd} = V_q + R_s i_q + x_d i_d}$

or $\boxed{V_q = E_{fd} - R_s i_q - x_d i_d}$ (13 ss)

Illy $\boxed{V_d = -R_s i_d + x_q i_q}$ (14 ss) using (14) and $\dot{E}_d' = 0$

Steady State Diagram



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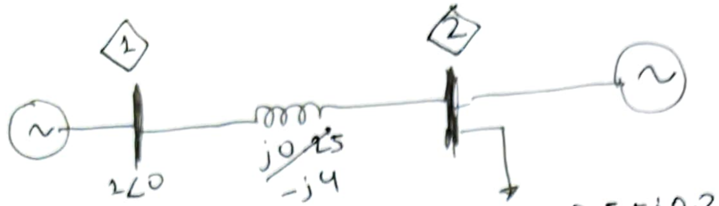
The Anis Model for Two-bus system.



$S_{L2} = 0.5 + j0.2$

$P = 50\%$. $I = 25\%$, $Z = 25\%$

$Q = 50\%$. $Z = 50\%$

$$Y_{bus} = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & -j4 & j4 \\ 2 & j4 & -j4 \end{array} = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & 4\angle -\frac{\pi}{2} & 4\angle \frac{\pi}{2} \\ 2 & 4\angle \frac{\pi}{2} & 4\angle -\frac{\pi}{2} \end{array}$$

$$P_2 = P_{G2} - P_{L2} = |V_2| \sum_{k=1}^{2} |Y_{2k}||V_k| \cos(\gamma_{2k} + \delta_k - \delta_2)$$

$$\text{or } P_2 = P_{G2} - 0.5\{0.5 + 0.25|V_2| + 0.25|V_2|^2\} = 4|V_2| \cos(\tfrac{\pi}{2} + 0 - \delta_2) + 4|V_2|^2 \cos(-\tfrac{\pi}{2})$$

$$\text{or } \boxed{P_2 = P_{G2} - 0.5\{0.5 + 0.25|V_2| + 0.25|V_2|^2\} = 4|V_2| \sin(\delta_2)} \quad g_1(a)$$

$$Q_2 = Q_{G2} - Q_{L2} = -|V_2| \sum_{k=1}^{2} |Y_{2k}||V_k| \sin(\gamma_{2k} + \delta_k - \delta_2)$$

$$\text{or } Q_2 = Q_{G2} - 0.2\{0.5 + 0.5|V_2|^2\} = -4|V_2| \sin(\tfrac{\pi}{2} + 0 - \delta_2) - 4|V_2|^2 \sin(-\tfrac{\pi}{2} + \delta_1 - \delta_1)$$

$$\text{or } \boxed{Q_2 = Q_{G2} - 0.2\{0.5 + 0.5|V_2|^2\} = -4|V_2| \cos(\delta_2) + 4|V_2|^2} \quad g_2(a)$$

$$\boxed{P_{G2} = V_{d2} I_{d2} + V_{q2} I_{q2}}$$

$$\boxed{Q_{G2} = \cancel{V_{d2}}\, V_{q2} I_{d2} - V_{d2} I_{q2}}$$

But $V_{d2} = Re(V_2 \angle (\frac{\pi}{2} - \theta))$

or $V_{d2} = Re(V_2 \angle \delta_2 \angle \frac{\pi}{2} - \theta)$

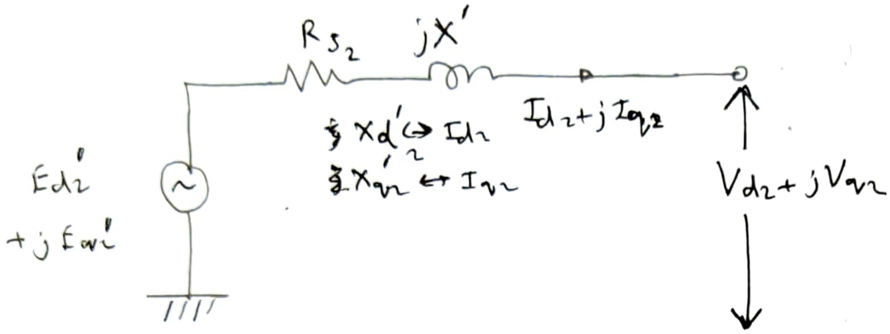
$V_{d2} = V_2 \cos(\delta_2 + \frac{\pi}{2} - \theta_2)$

or $V_{d2} = V_2 \cos(\frac{\pi}{2} - (\theta_2 - \delta_2))$

or $\boxed{V_{d2} = V_2 \sin(\theta_2 - \delta_2)}$ (a)

$\boxed{V_{q2} = V_2 \cos(\theta_2 - \delta_2)}$ (b)

$$\cancel{E'_{q2} = V_{q2} + R_s}$$



$$E'_{d2} - jI_{q2}\cdot(jX'_{q2}) = V_{d2} \qquad jE'_{q2} - jX'_{d2}I_{d2} = jV_{q2}$$

or $\quad E'_{d2} + X'_{q2}I_{q2} = V_{d2} \qquad$ or $\quad E'_{q2} - X'_{d2}I_{d2} = V_{q2}$

or $\quad \boxed{I_{q2} = \dfrac{E'_{d2} - V_{d2}}{-X'_{q2}}} \qquad$ or $\quad \boxed{I_{d2} = \dfrac{E'_{q2} - V_{q2}}{X'_{d2}}}$

$$\boxed{I_{q2} = \frac{E'_{d2} - V_2\sin(\theta_2-\delta_2)}{-X'_{q2}}} \quad (c) \qquad \boxed{I_{d2} = \frac{E'_{q2} - V_2\cos(\theta_2-\delta_2)}{X'_{d2}}} \quad (d)$$

$P_{a2} = V_{d2}I_{d2} + V_{q2}I_{q2} = \cancel{V_{d2}}$.

$$g_1(b) \quad \boxed{P_{a2} = \left(V_2\sin(\theta_2-\delta_2)\right)\cdot\left(\frac{E'_{q2} - V_2\cos(\theta_2-\delta_2)}{X'_{d2}}\right) + \left(V_2\sin(\theta_2-\delta_2)\right)\left(\frac{E'_{d2} - V_2\sin(\theta_2-\delta_2)}{-X'_{q2}}\right)}$$

$$g_2(b) \quad \boxed{Q_{a2} = \left(V_2\cos(\theta_2-\delta_2)\right)\left(\frac{E'_{q2} - V_2\cos(\theta_2-\delta_2)}{X'_{d2}}\right) - \left(V_2\sin(\theta_2-\delta_2)\right)\left(\frac{E'_{d2} - V_2\sin(\theta_2-\delta_2)}{-X'_{q2}}\right)}$$

From $g_1(a)$, $g_1(b)$, we get $\boxed{g_1(x, y) = 0}$

$g_2(a)$, $g_2(b)$, we get $\boxed{g_2(x, y) = 0}$

where $x = \begin{bmatrix} \theta_2 \\ \omega_2 \\ E'_{q2} \\ E'_{d2} \end{bmatrix}$ (not needed here)

$$y = \begin{bmatrix} \delta_2 \\ |V_2| \end{bmatrix}$$

$\dot{x} = f(x, y) =$

$$\dot{\theta}_2 = (\omega_2 - 1)\,\omega_s$$

$$\dot{\omega}_2 = \frac{1}{2H_2}\left\{ P_{m2} - P_{G2} - K_{D2}(\omega_2 - 1) \right\}$$

from $g_1(b)$

$$\dot{E}'_{q2} = \frac{1}{T'_{do2}}\left\{ -E'_{q2} - (x_{d2} - x'_{d2})\, I_{q2} + E_{fd2} \right\}$$

from (c)

$$\dot{E}'_{d2} = \frac{1}{T'_{qo2}}\left\{ -E'_{d2} + (x_{q2} - x'_{q2})\, I_{d2} \right\}$$

from (d)