EE507 Homework 3

Problem 1

A random variable X has a probability density function $f_X(x) = Cx^{-3}, x \geq 1$.

a) Please find the constant C.
b) Find the mean and variance of X.
c) Find the CDF of X.
d) Please find the PDF of X given that $X \geq 2$. Also, please find the mean value of X given that that $X \geq 2$.

Problem 2

a. A Bernoulli random variable $X \sim B(p)$ is a discrete random variable, which equals 1 with probability *p* and equals *0* with probability *1-p.* Please find the pmf, CDF, and first five moments of a Bernoulli random variable X. (Please leave your answer in terms of *p*).
b. A bi-directional Bernoulli random variable $X \sim BB(p)$ is a discrete random variable, which equals 1 with probability p and equals -1 with probability 1-p. Please find the pmf, CDF, and first five moments of a bi-directional Bernoulli random variable.

Problem 3

Consider a Gaussian random variable $Y \sim N(m = 1, \sigma^2 = 4)$
a. Please find the mean, standard deviation, and second moment of Y.
b. In terms of the standard Gaussian distribution, what is the probability that $2 \leq Y \leq 3$.
c. Please design a zero-mean Gaussian random variable Z such that $P(Z \geq 1) = 0.3$.

Problem 4

A probabilistic experiment has three outcomes, A, B, and C which have probabilities of 0.2, 0.3, and 0.5 respectively. A random variable X is defined as follows: if the experiment has outcome A, then X is exponential with parameter $\lambda = 2$. If the experiment has outcome B, then X is exponential with parameter $\lambda = 1$. If the experiment has outcome C, then X=0.

Please find $P(A \mid X = x)$, $P(B \mid X = x)$, and $P(C \mid X = x)$ as a function of x.

Problem 5

A random variable X is uniformly distributed on the interval [0,2]. Given X=x, the event A occurs with probability 1-x/2, and the event B occurs independently with probability x/2. Please answer the following questions:

a. Please find P(A) and P(B).
b. Find the PDF of X given A.
c. Find P(A|B). Are A and B independent?

Problem 6

Consider a geometric random variable Q with parameter p=0.7.

a. Please find the pmf and mean of Q, given that $Q \leq 4$.
b. Let $R = Q^2$. Please find the pmf of *R.*

Problem 7

Consider throwing a dart at a circular dartboard with unit radius; assume that every point on the dartboard is equally likely. Let X be the distance of the dart from the center of the dartboard. Let $Z = X^a$ where a>0. Please find the pdf of Z (leaving your answer in terms of a). Is Z a uniform random variable for some parameter value a?

Problem 8

The random variable X is uniform on [0,4]. A random variable Y is defined as follows: Y=0 of X<1, and Y=X-1 if $X \geq 1$. Please find the pdf of Y.