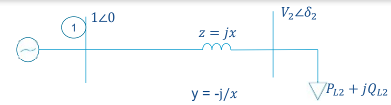
Power-flow Fundamentals

Power-flow Solution Example

1

Simple Power System



$$\overrightarrow{Y_{Bus}} = \begin{bmatrix} \frac{1}{x}\angle -90^0 & \frac{1}{x}\angle 90^0 \\ \frac{1}{x}\angle 90^0 & \frac{1}{x}\angle -90^0 \end{bmatrix}$$

$$P_2 = -P_{L2} = \frac{V_2 \cdot 1}{x}\sin\delta_2$$

$$Q_2 = -Q_{L2} = \frac{{V_2}^2 - V_2 \cdot \cos\delta_2}{x}$$

 2

2

Power-flow Solution

$$-xP_{L2} = V_2 \sin\delta_2$$

$$-{V_2}^2 - xQ_{L2} = -V_2 \cdot \cos\delta_2$$

$${V_2}^2 = (xP_{L2})^2 + {V_2}^4 + 2xQ_{L2}{V_2}^2 + (xQ_{L2})^2$$

$${V_2}^4 + {V_2}^2(2xQ_{L2} - 1) + \left(x^2{P_{L2}}^2 + x^2{Q_{L2}}^2\right) = 0$$

 3

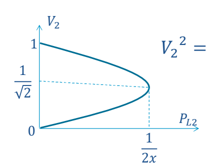
3

Unity Power-factor Load

$${V_2}^4 + {V_2}^2(2xQ_{L2} - 1) + \left(x^2{P_{L2}}^2 + x^2{Q_{L2}}^2\right) = 0$$

$$Q_{L2} = 0 \Longrightarrow Unity\ PF\ Load$$

$${V_2}^4 + {V_2}^2(-1) + \left(x^2{P_{L2}}^2\right) = 0$$

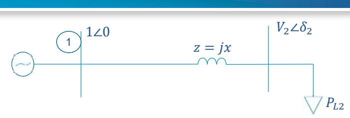


$${V_2}^2 = \frac{1 \pm \sqrt{1 - 4x^2{P_{L2}}^2}}{2}$$

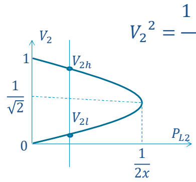
 4

4

Unity Power-factor Load



$${V_2}^2 = \frac{1 \pm \sqrt{1 - 4x^2{P_{L2}}^2}}{2}$$



P_L =0 => V2 = 1 or V2 = 0

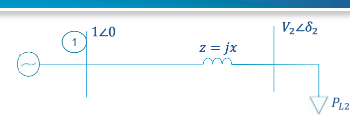
P_L increases => V2 decreases

Large P_L => No solutions

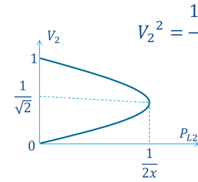
 5

5

Unity Power-factor Load



$${V_2}^2 = \frac{1 \pm \sqrt{1 - 4x^2{P_{L2}}^2}}{2}$$



$P_{limit} = \frac{1}{2x}$ is the static limit.

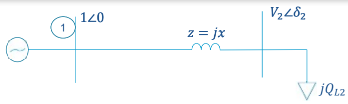
Power-flow equations not solvable when P_L > 1/2x.

Possible system collapse.

 6

6

Purely Reactive Load



$$P_L = 0 \Longrightarrow \textit{Pure Reactive Load}$$

$$V_2^{\,4} + V_2^{\,2}(2xQ_{L2} - 1) + \left(x^2 Q_{L2}^{\,2}\right) = 0$$

$$V_2^{\,2} = \frac{(1 - 2xQ_{L2}) \pm \sqrt{\cancel{4x^2Q_{L2}^{\,2}} + 1 - 4xQ_{L2} \cancel{- 4x^2Q_{L2}^{\,2}}}}{2}$$

7

7

Purely Reactive Load

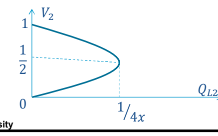
$$P_L = 0 \Longrightarrow \textit{Pure Reactive Load}$$

$$V_2^{\,4} + V_2^{\,2}(2xQ_{L2} - 1) + \left(x^2 Q_{L2}^{\,2}\right) = 0$$

$$V_2^{\,2} = \frac{(1 - 2xQ_{L2}) \pm \sqrt{\cancel{4x^2Q_{L2}^{\,2}} + 1 - 4xQ_{L2} \cancel{- 4x^2Q_{L2}^{\,2}}}}{2}$$

$Q_{L2} > \frac{1}{4x} \Longrightarrow$ No power-flow solution

$\Longrightarrow$ Possible collapse $\Longrightarrow$ Static limit



8

8

Analytical Example Summary

- Bus voltages decline as loads increase
- Multiple power-flow solutions exist
- Solution with nominal voltages the only reasonable solution
- Static limits beyond which power-flow solutions do not exist. Indication of potential system collapse.
- System operation needs to stay well away from static limits.

9

9