EE507 Exam 2
November 17th, 2022

You have three hours to complete the exam. You may use any inanimate reference that you want for the exam; you can also access lecture videos online. Please do not communicate with anyone else, via any medium, about the exam. The exam will be scored out of 85 points.

Problem 1 (10 points)
A random variable X has pdf $f_X(x) = 0.5e^{-|x|}$, where $x \in R$. Please find the moment generating function of X, and use the moment generating function to find the second moment of X.

Problem 2 (10 points)
A discrete random variable X has pmf $p_X(k) = \frac{1}{3}(\frac{2}{3})^k$ for $k = 0,1,2,3, \ldots$ The discrete random variable Y is independent of X, and has the following pmf: $p_Y(k) = \frac{1}{3}$ for k=0,1,2 (and $p_Y(k) = 0$ otherwise). Please find $P(X + Y \leq 2)$.

Problem 3 (15 points)
Two random variables X and Y have joint pdf $f_{X,Y}(x, y)$ which is uniform on the region defined by $1 \leq x \leq 2$ and $0 \leq y \leq x$.

a. Please find the pdf of Y, and the conditional pdf of X given Y=y. Are X and Y independent?
b. Find the MMSE estimate for X given Y=y.

Problem 4 (15 points)
Two random variables X and Y are jointly Gaussian with the following parameters: $m_X = m_Y = 0, \sigma_X^2 = 4, \sigma_Y^2 = 9, \rho = -\frac{1}{2}$. Answer the following questions:

a) Write down the joint pdf for X and Y. (Please write down the actual probability density function, not a notation for the density function.)
b) We define Z=X+Y and W= bX, where b is non-zero constant. Please find the joint pdf of Z and W, leaving your answer in terms of b. (You can use the shorthand notation for the density for this part). For what values of b is cov(Z,W)>0?
c) Please find the MMSE estimate for Y given X=x.

Problem 5 (10 points)
Three random variables X, Y, and Z are defined as follows. The random variable X is uniform on [0,1]. Given X=x, the random variable Y is uniform on [0,x]. Given X=x and Y=y, the random variable Z is uniform on [0,y]. Please find $E[X^2Y^2Z^2]$

Problem 6 (10 points)

Give a definition for a random process, and explain why the concept of random processes is important in engineering.

Problem 7 (15 points)

A time signal is defined as $X(t) = \frac{1}{t+1+C}$, where $t \in R^+$, and C is an exponential random variable with distribution $C \sim \exp(1)$.

a. Please argue that $X(t)$ is a random process.
b. Find the first-order pdf for $X(t)$.