Problem 1

A probabilistic experiment with five outcomes is defined below. The probability of each outcome (i.e., of each event consisting of a single outcome) is also shown. For this experiment, a random variable X is defined. Also, three events Q, R, and S are defined. answer the following questions:

Outcomes	Probability	X(W)	Events
A	P({A}) = 0.4	X(A) = 7	Q = {A,B}
B	P({B}) = 0.3	X(B) = 7	R = {B,C,D}
C	P({C}) = 0.1	X(C) = 2	S = {D,□}
D	P({D}) = 0.1	X(D) = 4	T = {D}
□	P({□}) = 0.1	X(□) = 1	

(a.) Are the events R and S independent?

(b.) find the conditional CDF, PMF and PDF for X, given the event Z =Q+RS

(c.) find E[X] and E[X—Z]

(d.) Are the events R and S independent, given that X $\leq$ 5?

(e.) Let's say that you did not know the probabilities of the individual outcomes, but only knew the probabilities of the events Q and T. Would you still be able to find the conditional PM for X given Z?

Problem 2

We say that the event A is almost independent of the event B, if P(A) - 0.01 $\leq P(A|B) \leq$ P(A) + 0.01. answer the following questions:

a. If the event A is almost independent of the event B, is it necessarily true that the event $\bar{A}$ is almost independent of the event B? prove or give a counterexample.

b. If the evnet A is almost independent of the event B, is it true that the event B is almost independent of the event A. prove or give a counterexample.

c. Say that P(A) = 0.6 and P(B)=0.4. Also assume that A is almost independent of B. Is it possible that P(A+B) = 0.75?

Problem 3

The amount of time T required to complete a task is a random variable with PDF $f_T(t) = \frac{3}{t^2}, t \geq 1$. Please answer the following:

(a) find the mean, variance, and standard deviation of T.

(b) find the CDF of T

(c) determine a time threshold t^* such that $P(T \geq t^*) = 0.25$

For the rest of the problem, please consider the following extension. The completed task may either be a success S or a failure. The probability of success depends on how long the task took. In particular, the probability of success for a task of duration $T = \frac{1}{T}$.

(d) find the overall (unconditioned) probability of success P(S).

(e) find the probability density function for the time T, given (hat the task was a success.

(f) find the time threshold t^*, such that $P(S|T \leq t^*) = 0.9$

Problem 4

A random variable X has a pdf $f_X(x) = e^{-x}, x \geq 0$. The random variable Z is the following function of X: $Z = \frac{X^2}{4}$ for $X \leq 2$, while $Z = 1$ for$X \geq 2$. Please find the PDF of Z.

Problem 5

A random variable X has a PDF $f_X(x) = xe^{-x}, x \geq 0$ find the moment-generating function for X

Problem 6

You flip a fair coin. If the coin shows heads, then $Z \sim binom(10, 0.2)$. If the coin shows tails, then $Z \sim geom(0.3)$. Please find the PMF of Z

Problem 7

A random variable X has PDF $f_X(x) = 2e^{-2x}, x \geq 0$. A random variable Z is defined as follows: $Z = \sqrt{X}$ for $0 \leq X < 1$ and $Z = 1$ for $X \geq 1$.

a Find the PDF of Z.

b Find the conditional PDF of X given Z.

Problem 8

Consider a probabilistic experiment where a dart is thrown at a circular dartboard of radius 1, hitting each point with equal probability. We define a random process $X(t)$ as $X(t) = Qt$, where Q is the distance of the dart from the center of the board, and $t \in R^+$. Answer the following questions:

a. Find the first-order PDF for X(t).

b. Find the mean of X(t).

Problem 9

You can play the following game at a Casino. You and your friend each toss a fair six-sided die (the sides are labeled '1',.... ,6', and the tosses are assumed independent). If the number showing on your die is (strictly) larger than the number on the friend's die, your winnings W are the difference between the two numbers. (So, for instance, if you throw a '5' and the instructor throws a '2', your winnings are W= 5 - 2 = 3 dollars). If the number on your die is less than or equal to the number on the the instructor's die, you do not win anything (W = 0),

a. Please sketch the PMF and CDF for W.

b. What is E[W]?

Problem 10

For each of the following statements, please either prove the statement or give a counterexample showing that the statement is not true.

a. Consider three events A, B, and C in a probabilistic experiment. If each pair of events are independent (i.e., A and B are independent, B and C are independent, and A and C are independent),it follows that P(ABC) = P(A)P(B)P(C).

b. For any two events A and B in an probabilistic experiment, it is true that $P(A|B) + P(A|\bar{B}) \leq 1$

c. For any three events A, B, and C in a probabilistic experiment, it is true that $P(A + B + C) = P(A) + P(\bar{A}B) + P(\bar{A}\bar{B}C)$

Problem 11

A casino game works as follows. You first flip a coin which has probability 0.6 of showing tails, and probability 0.4 of showing heads. If the coin shows tails, you toss a fair dice and record the number showing (1, 2,.., 6). If the coin shows heads, you also toss a fair dice but record twice the number showing (2, 4,...,12). Answer the following questions.

a. What are the elementary outcomes of the experiment?

b. How many events can be defined for this experiment?

c. What is the probability that the coin showed Tails' and the number you recorded was '4'?

d. What is the probability that the number recorded is even?

e. Given that the number recorded is even, what's the probability that the coin toss showed heads.

f. Given that the number recorded is even, what's the probability that either the coin toss showed heads or the number recorded was '2'.

Problem 12

Your friend throws a dart at a round dartboard with radius 1, as shown, and is equally likely to hit any point on the dartboard. We define two events for this experiment: A is the event that the top right quadrant is hit, while B is the event that the dart falls within a circle of radius 0.5. We also define a a random variable X, which equals the angle of the dart's location relative to the horizontal axis (in degrees), as shown. Answer the following questions:

a. Please show that the events A and B are independent.

b. Please find the cumulative distribution function and probability density function of X.

c. What is the probability density function of X, given that either A or B occurred.

d. Your friend repeats the dart-throwing experiment 10 times. (Each trial is independent). What is the probability that either A or B occurs on at least 8 trials?

Problem 13

The probability density function for a random variable Y is shown below. Please find the cumulative distribution function for Y. Also, please find $Pr(0.5 < Y < 1)$.

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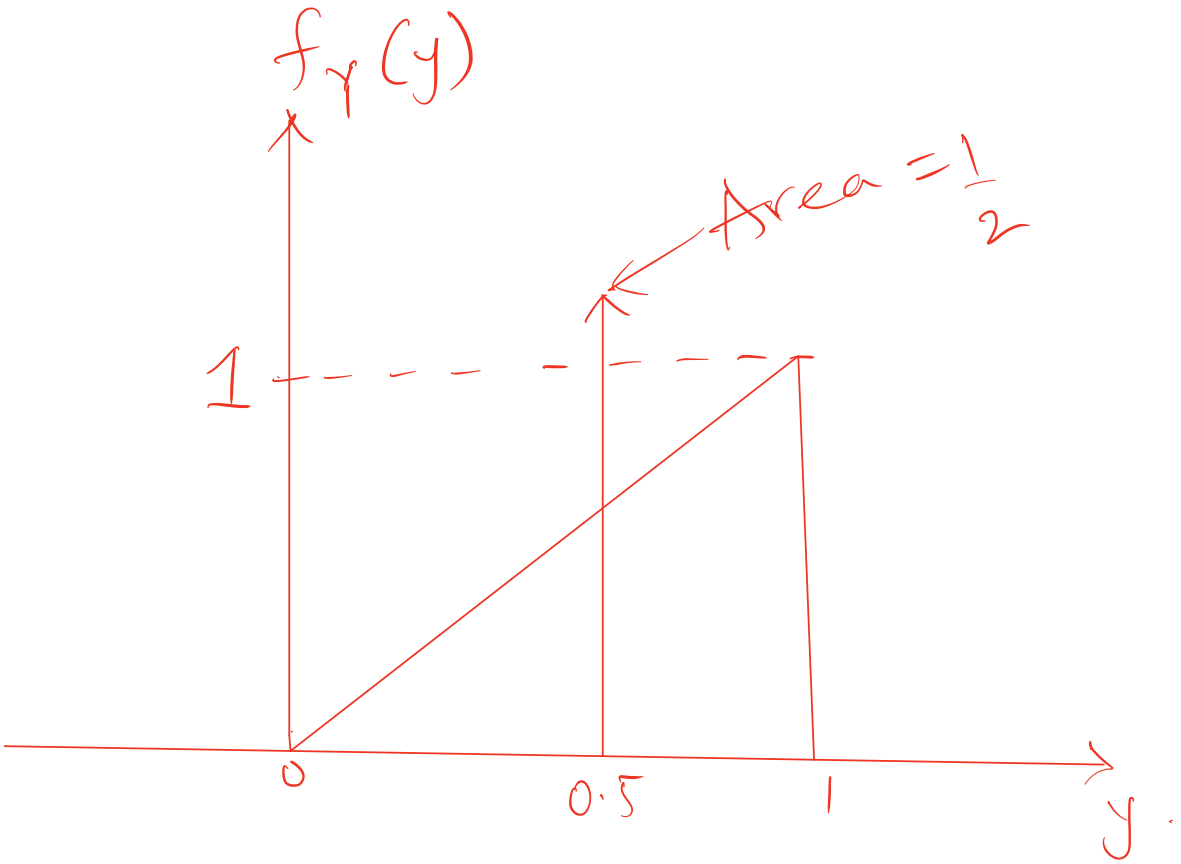
$f_Y(y)$
Area $= \frac{1}{2}$
1
0
0.5
1
y