ASSIGNMENT 04 of EE 523, SPRING 2023

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1. Type 1 Model

$$E = \text{Adjacency list} = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \end{array} \begin{bmatrix} 5 \\ 6 \\ 11 \\ 10 \\ 1,\ 6 \\ 2,\ 5,\ 7 \\ 6,\ 8 \\ 7,\ 9 \\ 8,\ 10 \\ 4,\ 9,\ 11 \\ 3,\ 10 \end{bmatrix}$$

$$\underset{12\times 1}{x} = \begin{bmatrix} \theta_2 \\ \omega_2 \\ E'_{q2} \\ E'_{d2} \\ \theta_3 \\ \omega_3 \\ E'_{q3} \\ E'_{d3} \\ \theta_4 \\ \omega_4 \\ E'_{q4} \\ E'_{d4} \end{bmatrix}_{12\times 1}$$

$$\underset{20\times 1}{y} = \begin{bmatrix} \delta_2 \\ |V_2| \\ \delta_3 \\ |V_3| \\ \delta_4 \\ |V_4| \\ \delta_5 \\ |V_5| \\ \vdots \\ \delta_{11} \\ |V_{11}| \end{bmatrix}_{20\times 1}$$

1.2

{Note: ✓ implies that the variables are ~~either~~ given (P_{m_i}, X'_{d_i}, X_{d_i}, X'_{q_i}, X_{q_i}, Y_{ik}, γ_{ik}, ~~α_{ik}~~ K_{D_i}, ω_s, E_{fd_i}, M_i ~~or have been initialized, say using power flow~~ ~~(V_k, δ_k for non-gen buses)~~}

$$P_{G_2} - P_{D_2} - P_2 = 0$$

$$\text{or} \quad V_{d_2} I_{d_2} + V_{q_2} I_{q_2} - P_{D_2} - |V_2| \sum_{(2,k)\in E} |Y_{2k} V_k| \cos(\gamma_{2k} + \delta_k - \delta_2)$$

$$\boxed{\begin{aligned} \text{or} \quad & |V_2| \sin(\theta_2 - \delta_2) \cdot \left\{ \frac{E'_{q_2} - V_2 \cos(\theta_2 - \delta_2)}{X'_{d_2}} \right\} \\ & + |V_2| \cos(\theta_2 - \delta_2) \cdot \left\{ \frac{E'_{d_2} - V_2 \sin(\theta_2 - \delta_2)}{-X'_{q_2}} \right\} \\ & - P_{D_2} \\ & - \left[G_{22} |V_2|^2 + |Y_{25} V_5 V_2| \cos(\gamma_{25} + \delta_5 - \delta_2) \right] \\ & = 0 \end{aligned}} \quad g_1$$

$$Q_{G_2} - Q_{D_2} - Q_2 = 0$$

$$\boxed{\begin{aligned} \text{or} \quad & |V_2| \cos(\theta_2 - \delta_2) \left\{ \frac{E'_{q_2} - V_2 \cos(\theta_2 - \delta_2)}{X'_{d_2}} \right\} \\ & \cancel{+} - |V_2| \sin(\theta_2 - \delta_2) \left\{ \frac{E'_{d_2} - V_2 \sin(\theta_2 - \delta_2)}{-X'_{q_2}} \right\} \\ & - Q_{D_2} \\ & - \left[-B_{22} |V_2|^2 - |Y_{25} V_5 V_2| \sin(\gamma_{25} + \delta_5 - \delta_2) \right] \\ & = 0 \end{aligned}} \quad g_2$$

~~Similarly, for busses~~

$$
\begin{aligned}
&|V_3| \sin(\theta_3 - \delta_3) \cdot \left\{ \frac{E'_{q3} - V_3 \cos(\theta_3 - \delta_3)}{x_{d3}} \right\} \\
&+ |V_3| \cos(\theta_3 - \delta_3) \cdot \left\{ \frac{\cancel{E_q}\; E'_{d3} - V_3 \sin(\theta_3 - \delta_3)}{-x'_{q3}} \right\} \\
&- P_{D3} \\
&- \left[G_{33} |V_3|^2 + |Y_{3,11} V_{11} V_3| \cos(\gamma_{3,11} + \delta_{11} - \delta_3) \right] \\
&= 0
\end{aligned}
\qquad (g_3)
$$

$$
\begin{aligned}
&|V_3| \cos(\theta_3 - \delta_3) \cdot \left\{ \frac{E'_{q3} - V_3 \cos(\theta_3 - \delta_3)}{x'_{d3}} \right\} \\
&- |V_3| \sin(\theta_3 - \delta_3) \cdot \left\{ \frac{E'_{d3} - V_3 \sin(\theta_3 - \delta_3)}{-x'_{q3}} \right\} \\
&- Q_{D3} \\
&- \left[- B_{33} |V_3|^2 - |Y_{3,11} V_{11} V_3| \sin(\gamma_{3,11} + \delta_{11} - \delta_3) \right] \\
&= 0
\end{aligned}
\qquad (g_4)
$$

$$|V_4| \sin(\theta_4 - \delta_4) \cdot \left\{ \frac{E'_{q4} - V_4 \cos(\theta_4 - \delta_4)}{x''_{d4}} \right\}$$

$$+ |V_4| \cos(\theta_4 - \delta_4) \cdot \left\{ \frac{E''_{d4} - V_4 \sin(\theta_4 - \delta_4)}{-x''_{q4}} \right\}$$

$$- P_{D4}$$

$$- \left[G_{44} |V_4|^2 - |Y_{4,10} V_{10} V_4| \cos(\gamma_{4,10} + \delta_{10} - \delta_4) \right]$$

$$= 0 \qquad (g5)$$

$$|V_4| \cos(\theta_4 - \delta_4) \cdot \left\{ \frac{E''_{q4} - V_4 \cos(\theta_4 - \delta_4)}{x''_{d4}} \right\}$$

$$- |V_4| \sin(\theta_4 - \delta_4) \cdot \left\{ \frac{E''_{d4} - V_4 \sin(\theta_4 - \delta_4)}{-x''_{q4}} \right\}$$

$$- Q_{D4}$$

$$- \left[-B_{44} |V_4|^2 - |Y_{4,10} V_{10} V_4| \sin(\gamma_{4,10} + \delta_{10} - \delta_4) \right]$$

$$= 0 \qquad (g6)$$

$\boxed{1.5}$

$$\cancel{P_{G5}}^{\,0} - P_{D5} - P_5 = 0$$

or

$$\boxed{-P_{D5} - \left[G_{55}|V_5|^2 + |Y_{51} V_1 V_5| \cos(\gamma_{51} + \delta_1 - \delta_5) + |Y_{56} V_6 V_5| \cos(\gamma_{56} + \delta_6 - \delta_5) \right] = 0}$$

g 7

$$Q_{G5} - Q_{D5} - Q_5 = 0$$

or

$$\boxed{-Q_{D5} - \left[-B_{55}|V_5|^2 - |Y_{51} V_1 V_5| \sin(\gamma_{51} + \delta_1 - \delta_5) - |Y_{56} V_6 V_5| \sin(\gamma_{56} + \delta_6 - \delta_5) \right] = 0}$$

g 8

$$\boxed{-P_{D6} - \left[G_{66}|V_6|^2 + |Y_{62} V_2 V_6| \cos(\gamma_{62} + \delta_2 - \delta_6) + |Y_{65} V_5 V_6| \cos(\gamma_{65} + \delta_5 - \delta_6) + |Y_{67} V_7 V_6| \cos(\gamma_{67} + \delta_7 - \delta_6) \right] = 0}$$

g 9

$$\boxed{-Q_{D6} - \left[-B_{66}|V_6|^2 - |Y_{62} V_2 V_6| \sin(\gamma_{62} + \delta_2 - \delta_6) - |Y_{65} V_5 V_6| \sin(\gamma_{65} + \delta_5 - \delta_6) - |Y_{67} V_7 V_6| \sin(\gamma_{67} + \delta_7 - \delta_6) \right] = 0}$$

g 10

1-6

$$-P_{D_7} - \left[G_{77}|V_7|^2 + |Y_{76} V_6 V_7| \cos(\gamma_{76} + \delta_6 - \delta_7) + |Y_{78} V_8 V_7| \cos(\gamma_{78} + \delta_8 - \delta_7) \right] = 0$$

g11

$$-Q_{D_7} - \left[-B_{77}|V_7|^2 - |Y_{76} V_6 V_7| \sin(\gamma_{76} + \delta_6 - \delta_7) - |Y_{78} V_8 V_7| \sin(\gamma_{78} + \delta_8 - \delta_7) \right] = 0$$

g12

$$-P_{D_8} - \left[G_{88}|V_8|^2 + |Y_{87} V_7 V_8| \cos(\gamma_{87} + \delta_7 - \delta_8) + |Y_{89} V_9 V_8| \cos(\gamma_{89} + \delta_9 - \delta_8) \right] = 0$$

g13

$$-Q_{D_8} - \left[-B_{88}|V_8|^2 - |Y_{87} V_7 V_8| \sin(\gamma_{87} + \delta_7 - \delta_8) - |Y_{89} V_9 V_8| \sin(\gamma_{89} + \delta_9 - \delta_8) \right] = 0$$

g14

$$-P_{D_9} - \left[G_{99}|V_9|^2 + |Y_{98} V_8 V_9| \cos(\gamma_{98} + \delta_8 - \delta_9) + |Y_{9,10} V_{10} V_9| \cos(\gamma_{9,10} + \delta_{10} - \delta_9) \right] = 0$$

g15

$$-Q_{D_9} - \left[\cancel{G_9} - B_{99}|V_9|^2 - |Y_{98} V_8 V_9| \sin(\gamma_{98} + \delta_8 - \delta_9) - |Y_{9,10} V_{10} V_9| \sin(\gamma_{9,10} + \delta_{10} - \delta_9) \right] = 0$$

g16

$$-P_{D_{10}} - \Big[G_{10,10}|V_{10}|^2 + |Y_{10,4} V_4 V_{10}| \cos(\gamma_{10,4} + \delta_4 - \delta_{10}) + |Y_{10,9} V_9 V_{10}| \cos(\gamma_{10,9} + \delta_9 - \delta_{10}) + |Y_{10,11} V_{11} V_{10}| \cos(\gamma_{10,11} + \delta_{11} - \delta_{10}) \Big] = 0 \qquad g17$$

$$-Q_{D_{10}} - \Big[-B_{10,10}|V_{10}|^2 - |Y_{10,4} V_4 V_{10}| \sin(\gamma_{10,4} + \delta_4 - \delta_{10}) - |Y_{10,9} V_9 V_{10}| \sin(\gamma_{10,9} + \delta_9 - \delta_{10}) - |Y_{10,11} V_{11} V_{10}| \sin(\delta_{10,11} + \delta_{11} - \delta_{10}) \Big] = 0 \qquad g18$$

$$-P_{D_{11}} - \Big[G_{11,11}|V_{11}|^2 + |Y_{11,3} V_3 V_{11}| \cos(\gamma_{11,3} + \delta_3 - \delta_{11}) + |Y_{11,10} \cdot V_{10} V_{11}| \cos(\gamma_{11,10} + \delta_{10} - \delta_{11}) \Big] = 0 \qquad g19$$

$$-Q_{D_{11}} - \Big[-B_{11,11}|V_{11}|^2 - |Y_{11,3} V_3 V_{11}| \sin(\gamma_{11,3} + \delta_3 - \delta_{11}) - |Y_{11,10} V_{10} V_{11}| \sin(\gamma_{11,10} + \delta_{10} - \delta_{11}) \Big] = 0 \qquad g20$$

$\forall\ i = \cancel{2}, 3$ and 4

1.8

$$\dot{\theta}_i = (\omega_i - 1)\,\omega_s$$

$$\dot{\omega}_i = \frac{1}{2H_i}\Big[P_{m_i} - \Big[V_i \sin(\theta_i - \delta_i)\cdot\Big\{ \frac{E'_{q_i} - V_i\cos(\theta_i-\delta_i)}{X'_{d_i}} \Big\} + V_i\cos(\theta_i - \delta_i)\cdot\Big\{ \frac{E'_{d_i} - V_i\sin(\theta_i-\delta_i)}{-X'_{q_i}} \Big\}\Big] - K_{D_i}(\omega_i - 1) \Big]$$

$$\dot{E}'_{q_i} = \frac{1}{T'_{do_i}}\Big[-E'_{q_i} - (X_{d_i} - X'_{d_i})\Big\{ \frac{E'_{q_i} - V_i\cos(\theta_i-\delta_i)}{X'_{d_i}} \Big\} + E_{fd_i} \Big]$$

$$\dot{E}'_{d_i} = \frac{1}{T'_{qo_i}}\Big[-E'_{d_i} - (X_{q_i} - X'_{q_i})\Big\{ \frac{E'_{d_i} - V_i\sin(\theta_i-\delta_i)}{-X'_{q_i}} \Big\} \Big]$$

Again, $i = 2, 3, 4$.

$\to$ f_1 to f_{12}

where

f_1 + f_4 are for $i=2$

f_5 + f_8 are for $i=3$

f_9 to f_{12} are for $i=4$

Thus,

$$[g(x,y) = 0]_{20\times 1} \equiv \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \\ g_5 \\ g_6 \\ g_7 \\ \vdots \\ g_{20} \end{bmatrix}_{20\times 1}$$

gen. buss

represent the Type 1 model

load buss

for Kundur's 11 bus system

$$\dot{x} = f(x,y)$$

$$\dot{x} \equiv \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_{12} \end{bmatrix}_{12\times 1}$$

X — X — X — X