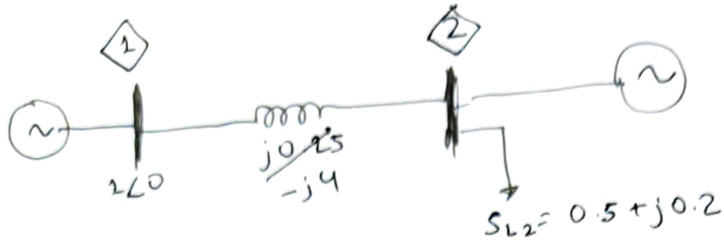
The Anis Model for Two-bus system.



$S_{L2} = 0.5 + j0.2$

$P = 50\%$. $I = 25\%$, $Z = 25\%$.

$Q = 50\%$. $Z = 50\%$.

$$Y_{bus} = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & -j4 & j4 \\ 2 & j4 & -j4 \end{array} = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & 4\angle -\frac{\pi}{2} & 4\angle \frac{\pi}{2} \\ 2 & 4\angle \frac{\pi}{2} & 4\angle -\frac{\pi}{2} \end{array}$$

$$P_2 = P_{G2} - P_{L2} = |V_2| \sum_{k=1}^{2} |Y_{2k}||V_k| \cos(\gamma_{2k} + \delta_k - \delta_2)$$

$$\text{or } P_2 = P_{G2} - 0.5\{0.5 + 0.25|V_2| + 0.25|V_2|^2\} = 4|V_2| \cos(\tfrac{\pi}{2} + 0 - \delta_2) + 4|V_2|^2 \cos(-\tfrac{\pi}{2})$$

$$\text{or } \boxed{P_2 = P_{G2} - 0.5\{0.5 + 0.25|V_2| + 0.25|V_2|^2\} = 4|V_2| \sin(\delta_2)}$$

$g_1(a)$

$$Q_2 = Q_{G2} - Q_{L2} = -|V_2| \sum_{k=1}^{2} |Y_{2k}||V_k| \sin(\gamma_{2k} + \delta_k - \delta_2)$$

$$\text{or } Q_2 = Q_{G2} - 0.2\{0.5 + 0.5|V_2|^2\} = -4|V_2| \sin(\tfrac{\pi}{2} + 0 - \delta_2) - 4|V_2|^2 \sin(-\tfrac{\pi}{2} + \delta_1 - \delta_1)$$

$$\text{or } \boxed{Q_2 = Q_{G2} - 0.2\{0.5 + 0.5|V_2|^2\} = -4|V_2| \cos(\delta_2) + 4|V_2|^2}$$

$g_2(a)$

$$\boxed{P_{G2} = V_{d2} I_{d2} + V_{q2} I_{q2}}$$

$$\boxed{Q_{G2} = \cancel{V_{d2}}\ V_{q2} I_{d2} - V_{d2} I_{q2}}$$

But $V_{d2} = \cancel{V_2 \sin}\ Re(V_2 \angle (\tfrac{\pi}{2} - \theta))$

or $V_{d2} = Re(V_2 \angle \delta_2 \angle \tfrac{\pi}{2} - \theta)$

$V_{d2} = V_2 \cos(\delta_2 + \tfrac{\pi}{2} - \theta_2)$

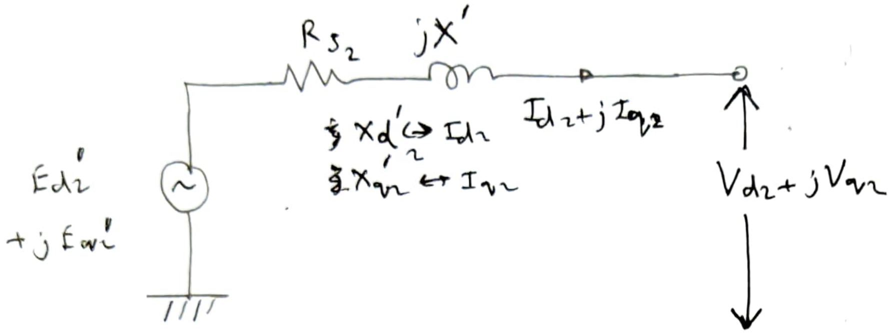
or $V_{d2} = V_2 \cos(\tfrac{\pi}{2} - (\theta_2 - \delta_2))$

or $\cancel{V_{d2} = V_2 \sin(\theta_2 - \delta_2)}$ (a)

or $\boxed{V_{d2} = V_2 \sin(\theta_2 - \delta_2)}$ (a)

$\boxed{V_{q2} = V_2 \cos(\theta_2 - \delta_2)}$ (b)

$\cancel{E'_{q2} = V_{q2} + R_s}$



$$E'_{d2} - jI_{q2} \cdot (jX'_{q2}) = V_{d2} \qquad jE'_{q2} - jX'_{d2} I_{d2} = jV_{q2}$$

$$\text{or} \quad E'_{d2} + X'_{q2} I_{q2} = V_{d2} \qquad \text{or} \quad E'_{q2} - X'_{d2} I_{d2} = V_{q2}$$

$$\text{or} \quad \boxed{I_{q2} = \frac{E'_{d2} - V_{d2}}{-X'_{q2}}} \qquad \text{or} \quad \boxed{I_{d2} = \frac{E'_{q2} - V_{q2}}{X'_{d2}}}$$

$$\text{or} \quad \boxed{I_{q2} = \frac{E'_{d2} - V_2 \sin(\theta_2 - \delta_2)}{-X'_{q2}}} \quad (c) \qquad \boxed{I_{d2} = \frac{E'_{q2} - V_2 \cos(\theta_2 - \delta_2)}{X'_{d2}}} \quad (d)$$

$$P_{a2} = V_{d2} I_{d2} + V_{q2} I_{q2}$$

$$g_1(b) \quad \text{or} \quad \boxed{P_{a2} = \left(V_2 \sin(\theta_2 - \delta_2)\right) \cdot \left(\frac{E'_{q2} - V_2 \cos(\theta_2 - \delta_2)}{X'_{d2}}\right) + \left(V_2 \sin(\theta_2 - \delta_2)\right)\left(\frac{E'_{d2} - V_2 \sin(\theta_2 - \delta_2)}{-X'_{q2}}\right)}$$

$$g_2(b) \quad \boxed{Q_{a2} = \left(V_2 \cos(\theta_2 - \delta_2)\right)\left(\frac{E'_{q2} - V_2 \cos(\theta_2 - \delta_2)}{X'_{d2}}\right) - \left(V_2 \sin(\theta_2 - \delta_2)\right)\left(\frac{E'_{d2} - V_2 \sin(\theta_2 - \delta_2)}{-X'_{q2}}\right)}$$

From $g_1(a)$, $g_1(b)$, we get $g_1(x, y) = 0$

$g_2(a)$, $g_2(b)$, we get $g_2(x, y) = 0$

where $x = \begin{bmatrix} \theta_2 \\ \omega_2 \\ E_{q2}' \\ E_{d2}' \end{bmatrix}$ (not needed here)

$$y = \begin{bmatrix} \delta_2 \\ |V_2| \end{bmatrix}$$

$\dot{x} = f(x, y) =$

$$\dot{\theta}_2 = (\omega_2 - 1)\,\omega_s$$

$$\dot{\omega}_2 = \frac{1}{2H_2}\left\{ P_{m2} - P_{G2} - K_{D2}(\omega_2 - 1) \right\}$$

from $g_1(b)$

$$\dot{E}_{q2}' = \frac{1}{T_{do2}'}\left\{ E_{q2}' - (x_{d2} - x_{d2}')\, I_{q2} + E_{fd2} \right\}$$

from (c)

$$\dot{E}_{d2}' = \frac{1}{T_{qo2}'}\left\{ -E_{d2}' + (x_{q2} - x_{q2}')\, I_{d2} \right\}$$

from (d)
