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Power-flow Methods

Newton-Raphson Algorithm
for Power-Flow Problem

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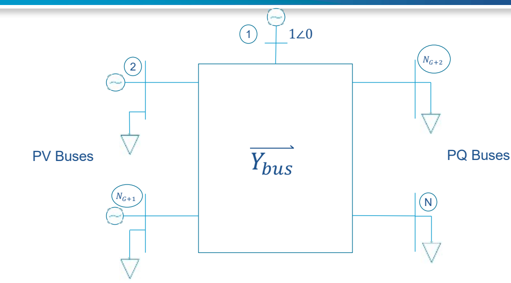
Power-Flow Problem



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NR Features

⇒Fast convergence for good initial conditions

⇒Fast divergence for poor initial conditions

⇒Jacobian computation computationally costly

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Newton-Raphson Algorithm

$$f(x) = 0, x \in \mathbb{R}^n$$
$$x^0$$
$$\Downarrow k = 0$$
$$Evaluate\ \ f(x^k)$$
$$\Downarrow$$
$$|f(x^k)| < \varepsilon \overset{Yes}{\Longrightarrow} Stop$$
$$\Downarrow No$$
$$Evaluate\ J^k = \frac{\partial f}{\partial x}|_{x^k}$$
$$\Downarrow$$
$$\Delta x^k = -(J^k)^{-1} f(x^k) \Longrightarrow x^{k+1} = x^k + \Delta x^k$$

$k = k + 1$

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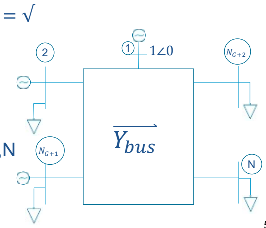
Newton-Raphson Algorithm

Power system:
PV bus: $\delta_i = ?, Q_i = ?, P_i = \surd, V_i = \surd$
PQ bus: $\delta_i = ?, V_i = ?, P_i = \surd, Q_i = \surd$

First step:
Find all voltages $V_i \angle \delta_i$ for i=1,…,N



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Newton-Raphson Algorithm

Then

$$Q_i = \sum_{j=1}^{N} Y_{ij}\, V_i\, V_j \sin(\delta_i - \delta_j - \theta_{ij})$$

can be computed for all PV buses

Newton-Raphson Algorithm is used to compute unknown
V_i and δ_i

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Newton-Raphson Algorithm

$$x = \begin{pmatrix} \delta_2 \\ \vdots \\ \delta_N \\ V_{N_G+2} \\ \vdots \\ V_N \end{pmatrix}$$

all angles

PQ voltages

PV voltages known
V_i is specified for i=1,…,N_G+1

$$f(x) = h(x) - b$$

$$= \begin{pmatrix} p_2(x) \\ \vdots \\ p_N(x) \\ q_{N_G+2}(x) \\ \vdots \\ q_N(x) \end{pmatrix} - \begin{pmatrix} P_2 \\ \vdots \\ P_N \\ Q_{N_G+2} \\ \vdots \\ Q_N \end{pmatrix} = 0$$

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Algorithm

$$x^0$$
$$\Downarrow k = 0$$
$$Evaluate\ f(x^k) = h(x^k) - b$$
$$\Downarrow$$

Largest power mismatch $\quad |f(x^k)| < \varepsilon \overset{Yes}{\Longrightarrow} stop$

$$\Downarrow No$$
$$Evaluate\ J^k = \frac{\partial f}{\partial x}|_{x^k} = \frac{\partial h}{\partial x}|_{x^k}$$
$$\Downarrow$$
$$\Delta x^k = (J^k)^{-1}(-f(x^k)) \Rightarrow x^{k+1} = x^k + \Delta x^k$$

$k = k + 1$

x^0 can be flat initial conditions ($V_i{}^0 = 1$,$\delta_i{}^0 = 0$) or the DC power-flow solution or previous known solution

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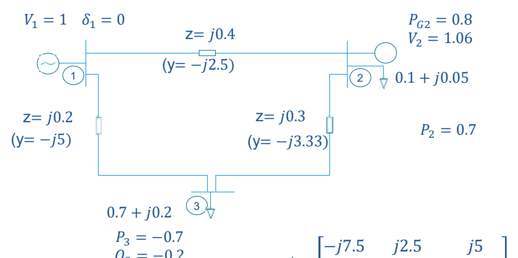
Example 1



$$\overrightarrow{Y_{Bus}} = \begin{bmatrix} -j7.5 & j2.5 & j5 \\ j2.5 & -j5.83 & j3.33 \\ j5 & j3.33 & -j8.33 \end{bmatrix}$$

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Example 1

$$x = \begin{pmatrix} \delta_2 \\ \delta_3 \\ V_3 \end{pmatrix} \quad f(x) = \begin{pmatrix} p_2(x) - P_2 \\ p_3(x) - P_3 \\ q_3(x) - Q_3 \end{pmatrix} \quad \mathrm{h}(x) = \begin{pmatrix} p_2(x) \\ p_3(x) \\ q_3(x) \end{pmatrix}$$

$$\mathrm{b} = \begin{pmatrix} P_2 \\ P_3 \\ Q_3 \end{pmatrix} = \begin{pmatrix} 0.7 \\ -0.7 \\ -0.2 \end{pmatrix}, \qquad \varepsilon = 0.001$$

$$p_i(x) = \sum_{j=1}^{3} Y_{ij}\, V_i\, V_j \cos(\delta_i - \delta_j - \theta_{ij})$$

$$q_i(x) = \sum_{j=1}^{3} Y_{ij}\, V_i\, V_j \sin(\delta_i - \delta_j - \theta_{ij})$$

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Example 1

$$\overrightarrow{Y_{Bus}} = \begin{bmatrix} -j7.5 & j2.5 & j5 \\ j2.5 & -j5.83 & j3.33 \\ j5 & j3.33 & -j8.33 \end{bmatrix}$$

$$p_2(x) = 2.5V_2\, V_1 \cos(\delta_2 - \delta_1 - 90°) + 3.33V_2\, V_3 \cos(\delta_2 - \delta_3 - 90°)$$
$$p_3(x) = 5V_3\, V_1 \cos(\delta_3 - \delta_1 - 90°) + 3.33V_3\, V_2 \cos(\delta_3 - \delta_2 - 90°)$$
$$q_3(x)$$
$$= 5V_3\, V_1 \sin(\delta_3 - \delta_1 - 90°) + 3.33V_3\, V_2 \sin(\delta_3 - \delta_2 - 90°)$$
$$+ 8.33{V_3}^2 \sin(90°)$$

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Jacobian Matrix

$$x = \begin{pmatrix} \delta_2 \\ \delta_3 \\ V_3 \end{pmatrix}, \qquad \mathrm{h}(x) = \begin{pmatrix} p_2(x) \\ p_3(x) \\ q_3(x) \end{pmatrix}$$

$$\mathrm{J} = \left[\frac{\partial h}{\partial x}\right] = \begin{bmatrix} \frac{\partial p_2}{\partial \delta_2} & \frac{\partial p_2}{\partial \delta_3} & \frac{\partial p_2}{\partial V_3} \\ \frac{\partial p_3}{\partial \delta_2} & \frac{\partial p_3}{\partial \delta_3} & \frac{\partial p_3}{\partial V_3} \\ \frac{\partial q_3}{\partial \delta_2} & \frac{\partial q_3}{\partial \delta_3} & \frac{\partial q_3}{\partial V_3} \end{bmatrix}$$

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Jacobian Entries

$$p_2(x) = 2.5V_2\, V_1 \cos(\delta_2 - \delta_1 - 90°) + 3.33V_2\, V_3 \cos(\delta_2 - \delta_3 - 90°)$$

$$\frac{\partial p_2}{\partial \delta_2} = -2.5V_2\, V_1 \sin(\delta_2 - \delta_1 - 90°) - 3.33V_2\, V_3 \sin(\delta_2 - \delta_3 - 90°)$$

$$\frac{\partial p_2}{\partial \delta_3} = 3.33V_2\, V_3 \sin(\delta_2 - \delta_3 - 90°)$$

$$\frac{\partial p_2}{\partial V_3} = 3.33V_2\, \cos(\delta_2 - \delta_3 - 90°)$$

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Iteration 1

$$p_2(x) = 2.5V_2\, V_1 \cos(\delta_2 - \delta_1 - 90°) + 3.33V_2\, V_3 \cos(\delta_2 - \delta_3 - 90°)$$

$$x^0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathrm{h}(x^0) = \begin{bmatrix} p_2(x^0) \\ p_3(x^0) \\ q_3(x^0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -0.1998 \end{bmatrix},$$

$$b - h(x^0) = \begin{bmatrix} 0.7000 \\ -0.7000 \\ -0.0002 \end{bmatrix}$$

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Iteration 1

$$x^0 = \begin{bmatrix} {\delta_2}^0 \\ {\delta_3}^0 \\ {V_3}^0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\frac{\partial p_2}{\partial \delta_2} = -2.5V_2\, V_1 \sin(\delta_2 - \delta_1 - 90°) - 3.33V_2\, V_3 \sin(\delta_2 - \delta_3 - 90°)$$

$$\left.\frac{\partial p_2}{\partial \delta_2}\right|_{x^0} = -2.5 \times 1.06 \times 1 \times \sin(0 - 0 - 90°) - 3.33 \times 1.06 \times 1 \times \sin(0 - 0 - 90°) = 6.18$$

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Jacobian

$$J^0 = \left.\frac{\partial h}{\partial x}\right|_{x^0} = \left.\begin{bmatrix} \frac{\partial p_2}{\partial \delta_2} & \frac{\partial p_2}{\partial \delta_3} & \frac{\partial p_2}{\partial V_3} \\ \frac{\partial p_3}{\partial \delta_2} & \frac{\partial p_3}{\partial \delta_3} & \frac{\partial p_3}{\partial V_3} \\ \frac{\partial q_3}{\partial \delta_2} & \frac{\partial q_3}{\partial \delta_3} & \frac{\partial q_3}{\partial V_3} \end{bmatrix}\right|_{x^0}$$

$$= \begin{bmatrix} 6.18 & -3.53 & 0 \\ -3.53 & 8.53 & 0 \\ 0 & 0 & 8.13 \end{bmatrix}$$

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End of Iteration 1

$$\Longrightarrow \Delta x^0 = (J^0)^{-1}\left(b - h(x^0)\right) = \begin{bmatrix} 0.087 \\ -0.046 \\ 0 \end{bmatrix}$$

$$\Longrightarrow x^1 = x^0 + \Delta x^0 = \begin{bmatrix} 0.087 \\ -0.046 \\ 1 \end{bmatrix}$$

$$\Downarrow k = 1$$

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Iteration 2

$$b - h(x^1) = \begin{bmatrix} 0.0017 \\ -0.0015 \\ -0.0365 \end{bmatrix}$$

$$J^1 = \left.\frac{\partial h}{\partial x}\right|_{x^1} = \begin{bmatrix} 6.13 & -3.50 & 0.4682 \\ -3.50 & 8.50 & -0.6986 \\ 0.4682 & -0.6986 & 8.17 \end{bmatrix}$$

$$\Longrightarrow \Delta x^1 = (J^1)^{-1}\left(b - h(x^1)\right) = \begin{bmatrix} 0.0004 \\ -0.0004 \\ -0.0045 \end{bmatrix}$$

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End of Iteration 2

$$\Longrightarrow x^2 = x^1 + \Delta x^1 = \begin{bmatrix} 0.0874 \\ -0.0465 \\ 0.9955 \end{bmatrix}$$

$$\Downarrow k = 2$$

$$\text{b}-h(x^2) = \begin{bmatrix} 0.000013 \\ -0.000021 \\ -0.00017 \end{bmatrix}$$

$|b - h(x^2)| = 0.00017 < 0.001 \Longrightarrow$ Stop

NR converged in two iterations!

NR Summary

- Good initial condition helps
- Very fast convergence
- Poor initial conditions ⇒ Fast divergence
- Each iteration is time-consuming

$$J^k \Delta x^k = b - h\left(x^k\right)$$