

# Appendix A

## Network Data

This appendix will provide the data required in order to perform dynamic studies on the test systems used throughout this thesis. In all cases a system base of 100 MVA is used.

### A.1 Two-Area Test Network Data

All original data is adopted from *Power System Stability and Control* by P. Kundur (London: McGraw-Hill, Inc., 1994).

#### A.1.1 Line Impedances

The line impedance data for the network is presented in Table A.1.

#### A.1.2 Load Flow Data

Data required to complete load flow is included in Table A.2, bus 1 is the slack.

#### A.1.3 Generator Dynamic Data

The generator dynamic presented is given in Table A.3 on the machine base.

All generators use the same AVR settings, given below:

$$K_A^{ex} = 200, T_A^{ex} = 0.01, T_B^{TGR} = 10, T_C^{TGR} = 1, E_{fd}^{\min} = -5.5, E_{fd}^{\max} = 5.5.$$

Similarly, PSS settings on all generators are identical, and given below:

$$T_W^{PSS} = 10, T_1^{PSS} = 0.05, T_2^{PSS} = 0.02, T_3^{PSS} = 3, T_4^{PSS} = -5.4, E_{PSS} = 20, E_{PSS}^{\min} = -0.1, E_{PSS}^{\max} = 0.1.$$

**Table A.1** Line data for the Kundur two area test network

From bus	To bus	$R$ (pu)	$X$ (pu)	$B$ (pu)
1	5	0	0.15 / 9	0
2	6	0	0.15 / 9	0
3	11	0	0.15 / 9	0
4	10	0	0.15 / 9	0
5	6	$25 \times 0.0001$	$25 \times 0.001$	$25 \times 0.00175$
10	11	$25 \times 0.0001$	$25 \times 0.001$	$25 \times 0.00175$
6	7	$10 \times 0.0001$	$10 \times 0.001$	$10 \times 0.00175$
9	10	$10 \times 0.0001$	$10 \times 0.001$	$10 \times 0.00175$
7	8	$110 \times 0.0001$	$110 \times 0.001$	$110 \times 0.00175$
7	8	$110 \times 0.0001$	$110 \times 0.001$	$110 \times 0.00175$
8	9	$110 \times 0.0001$	$110 \times 0.001$	$110 \times 0.00175$
8	9	$110 \times 0.0001$	$110 \times 0.001$	$110 \times 0.00175$

**Table A.2** Load flow data for the Kundur two area test network

Bus	$V$ (Pu)	$\theta$ ( $^\circ$ )	$P_G$ (MW)	$P_L$ (MW)	$Q_L$ (MVar)	$Q_C$ (MW)
1	1.03	0	—	—	—	—
2	1.01	—	700	—	—	—
3	1.03	—	719	—	—	—
4	1.01	—	700	—	—	—
7	—	—	—	967	100	200
9	—	—	—	1767	100	350

**Table A.3** Generator dynamic data for the Kundur two area test network

Generator	Rating (MVA)	$X_d$ (pu)	$X'_d$ (pu)	$X''_d$ (pu)	$T'_{d0}$ (s)	$T''_{d0}$ (s)	$X'_q$ (pu)	$X''_q$ (pu)	$T'_{q0}$ (s)	$T''_{q0}$ (s)	$H$ (s)
G1	900	1.8	0.3	0.25	8	0.03	1.7	0.25	0.4	0.05	6.5
G2	900	1.8	0.3	0.25	8	0.03	1.7	0.25	0.4	0.05	6.5
G3	900	1.8	0.3	0.25	8	0.03	1.7	0.25	0.4	0.05	6.175
G4	900	1.8	0.3	0.25	8	0.03	1.7	0.25	0.4	0.05	6.175

## A.2 NETS-NYPS Test Network Data

Full system details, generator and exciter parameters are adopted from *Power System Stability and Control* by P. Kundur (London: McGraw-Hill, Inc., 1994), with PSS settings for G9 sourced from *Power System Oscillations* by G. Rogers (Norwell: Kluwer Academic Publishers, 2000).

### A.2.1 Line Impedances

The line impedance data for the network is presented in Table A.4, including transformer off-nominal turns ratio (*ONR*) where applicable.

### A.2.2 Load Flow Data

Data required to complete load flow is included in Table A.5, G13 connected to bus 65 is the slack.

### A.2.3 Generator Dynamic Data

The generator dynamic presented is given in Tables A.6 and A.7, scaled to the given machine base.

Generators G1–G8 all use type DC1A exciters, with the following parameters:

$$T_R = 0.01, \quad K_A^{ex} = 40, \quad T_A^{ex} = 0.02, \quad E_{ex}^{\min} = -10, \quad E_{ex}^{\max} = 10, \quad T_E^{ex} = 0.785, \quad K_E^{ex} = 1, \\ A_E^{ex} = 0.07, \quad B_E^{ex} = 0.91.$$

Generator G9 uses a type ST1A\_v2 exciter, with the following parameters:

$$T_R = 0.01, \quad K_A^{ex} = 200, \quad E_{fd}^{\min} = -5, \quad E_{fd}^{\max} = 5.$$

Generator G9 is also fitted with a PSS with the following settings:

$$T_W^{PSS} = 10, \quad T_1^{PSS} = 0.05, \quad T_2^{PSS} = 0.01, \quad T_3^{PSS} = 0.05, \quad T_4^{PSS} = 0.02, \quad K_{PSS} = 10, \\ E_{PSS}^{\min} = -0.5, \quad E_{PSS}^{\max} = 0.5.$$

## A.3 HVDC System Details

Details are provided for the HVDC system parameters used for various case studies throughout this thesis.

### A.3.1 LCC-HVDC Line Embedded in Two-Area Network (Sect. 3.1)

LCC-HVDC line parameters (on 400 MW HVDC base with  $V_{dc}^{base} = 200 \text{ kV}$ ):

$$R_{dc} = 0.022, \quad L_{dc} = 2.48 \times 10^{-3}, \quad C_{dc} = 1.08 \times 10^{-3}$$

**Table A.4** Line data for the NETS-NYPS test network

From bus	To bus	$R$ (pu)	$X$ (pu)	$B$ (pu)	$ONR$	From bus	To bus	$R$ (pu)	$X$ (pu)	$B$ (pu)	$ONR$
2	53	0	0.0181	0	1.025	33	34	0.0011	0.0157	0.202	–
6	54	0	0.025	0	1.07	35	34	0.0001	0.0074	0	0.946
10	55	0	0.02	0	1.07	34	36	0.0033	0.0111	1.45	–
19	56	0.0007	0.0142	0	1.07	9	36	0.0022	0.0196	0.34	–
20	57	0.0009	0.018	0	1.009	9	36	0.0022	0.0196	0.34	–
22	58	0	0.0143	0	1.025	16	37	0.0007	0.0089	0.1342	–
23	59	0.0005	0.0272	0	1	31	38	0.0011	0.0147	0.247	–
25	60	0.0006	0.0232	0	1.025	33	38	0.0036	0.0444	0.693	–
29	61	0.0008	0.0156	0	1.025	41	40	0.006	0.084	3.15	–
31	62	0	0.026	0	1.04	48	40	0.002	0.022	1.28	–
32	63	0	0.013	0	1.04	42	41	0.004	0.06	2.25	–
36	64	0	0.0075	0	1.04	18	42	0.004	0.06	2.25	–
17	65	0	0.0033	0	1.04	17	43	0.0005	0.0276	0	–
41	66	0	0.0015	0	1	39	44	0	0.0411	0	–
42	67	0	0.0015	0	1	43	44	0.0001	0.0011	0	–
18	68	0	0.003	0	1	35	45	0.0007	0.0175	1.39	–
36	17	0.0005	0.0045	0.32	–	39	45	0	0.0839	0	–
49	18	0.0076	0.1141	1.16	–	44	45	0.0025	0.073	0	–
16	19	0.0016	0.0195	0.304	–	38	46	0.0022	0.0284	0.43	–
19	20	0.0007	0.0138	0	1.06	1	47	0.0013	0.0188	1.31	–
16	21	0.0008	0.0135	0.2548	–	47	48	0.0025	0.0268	0.4	–
21	22	0.0008	0.014	0.2565	–	47	48	0.0025	0.0268	0.4	–
22	23	0.0006	0.0096	0.1846	–	46	49	0.0018	0.0274	0.27	–
23	24	0.0022	0.035	0.361	–	45	51	0.0004	0.0105	0.72	–
16	24	0.0003	0.0059	0.068	–	50	51	0.0009	0.0221	1.62	–
2	25	0.007	0.0086	0.146	–	37	52	0.0007	0.0082	0.1319	–
25	26	0.0032	0.0323	0.531	–	3	52	0.0011	0.0133	0.2138	–
37	27	0.0013	0.0173	0.3216	–	1	2	0.0035	0.0411	0.6987	–
26	27	0.0014	0.0147	0.2396	–	2	3	0.0013	0.0151	0.2572	–
26	28	0.0043	0.0474	0.7802	–	3	4	0.0013	0.0213	0.2214	–
26	29	0.0057	0.0625	1.029	–	4	5	0.0008	0.0128	0.1342	–
28	29	0.0014	0.0151	0.249	–	5	6	0.0002	0.0026	0.0434	–
1	30	0.0008	0.0074	0.48	–	6	7	0.0006	0.0092	0.113	–
9	30	0.0019	0.0183	0.29	–	5	8	0.0008	0.0112	0.1476	–
9	30	0.0019	0.0183	0.29	–	7	8	0.0004	0.0046	0.078	–
30	31	0.0013	0.0187	0.333	–	8	9	0.0023	0.0363	0.3804	–
1	31	0.0016	0.0163	0.25	–	6	11	0.0007	0.0082	0.1389	–
30	32	0.0024	0.0288	0.488	–	10	11	0.0004	0.0043	0.0729	–
32	33	0.0008	0.0099	0.168	–	12	11	0.0016	0.0435	0	1.06
4	14	0.0008	0.0129	0.1382	–	10	13	0.0004	0.0043	0.0729	–
13	14	0.0009	0.0101	0.1723	–	12	13	0.0016	0.0435	0	1.06
14	15	0.0018	0.0217	0.366	–	1	27	0.032	0.32	0.41	–
15	16	0.0009	0.0094	0.171	–	50	18	0.0012	0.0288	2.06	–

**Table A.5** Load flow data for the NETS-NYPS test network

Bus	$V$ (pu)	$\theta$ (°)	$P_G$ (MW)	$P_L$ (MW)	$Q_L$ (MVar)	Bus	$V$ (pu)	$\theta$ (°)	$P_G$ (MW)	$P_L$ (MW)	$Q_L$ (MVar)
1	—	—	—	252.7	118.56	44	—	—	—	267.55	4.84
3	—	—	—	322	2	45	—	—	—	208	21
4	—	—	—	200	73.6	46	—	—	—	150.7	28.5
7	—	—	—	234	84	47	—	—	—	203.12	32.59
8	—	—	—	208.8	70.8	48	—	—	—	241.2	2.2
9	—	—	—	104	125	49	—	—	—	164	29
12	—	—	—	9	88	50	—	—	—	100	−147
15	—	—	—	320	153	51	—	—	—	337	−122
16	—	—	—	329	32	52	—	—	—	158	30
17	—	—	—	6000	300	53	1.045	—	250	—	—
18	—	—	—	2470	123	54	0.98	—	545	—	—
20	—	—	—	680	103	55	0.983	—	650	—	—
21	—	—	—	274	115	56	0.997	—	632	—	—
23	—	—	—	248	85	57	1.011	—	505	—	—
24	—	—	—	309	−92	58	1.05	—	700	—	—
25	—	—	—	224	47	59	1.063	—	560	—	—
26	—	—	—	139	17	60	1.03	—	540	—	—
27	—	—	—	281	76	61	1.025	—	800	—	—
28	—	—	—	206	28	62	1.01	—	500	—	—
29	—	—	—	284	27	63	1	—	1000	—	—
33	—	—	—	112	0	64	1.0156	—	1350	—	—
36	—	—	—	102	−19.46	65	1.011	0	—	—	—
39	—	—	—	267	12.6	66	1	—	1785	—	—
40	—	—	—	65.63	23.53	67	1	—	1000	—	—
41	—	—	—	1000	250	68	1	—	4000	—	—
42	—	—	—	1150	250						

LCC-HVDC controller parameters:

$$K_P^{Idc,rect} = K_P^{Idc,inv} = 45, \quad K_P^{Vdc,inv} = 65, \quad K_I^{Idc,rect} = K_I^{Idc,inv} = 1000, \quad K_I^{Vdc,inv} = 2000, \\ \gamma^{\min} = 15^\circ.$$

### A.3.2 VSC-HVDC Line Embedded in Two-Area Network (Sect. 3.1)

VSC-HVDC line parameters (on 400 MW HVDC base with  $V_{dc}^{base} = 200$  kV):

$$R_{dc} = 0.044, \quad L_{dc} = 2.8 \times 10^{-4}, \quad C_{dc} = 3.715 \times 10^{-3}.$$

VSC-HVDC controller parameters:

$$K_P^{Vdc} = 3, \quad K_I^{Vdc} = 40, \quad K_I^{Pdc} = K_I^{Qdc} = 20.$$

**Table A.6** Generator dynamic data for the NETS-NYPS test network (1)

Generator	Bus	Rating (MVA)	$X_{lk}$ (pu)	$X_d$ (pu)	$X'_d$ (pu)	$X''_d$ (pu)	$T'_{d0}$ (s)	$T''_{d0}$ (s)
G1	53	100	0.0125	0.1	0.031	0.025	10.2	0.05
G2	54	100	0.035	0.295	0.0697	0.05	6.56	0.05
G3	55	100	0.0304	0.2495	0.0531	0.045	5.7	0.05
G4	56	100	0.0295	0.262	0.0436	0.035	5.69	0.05
G5	57	100	0.027	0.33	0.066	0.05	5.4	0.05
G6	58	100	0.0224	0.254	0.05	0.04	7.3	0.05
G7	59	100	0.0322	0.295	0.049	0.04	5.66	0.05
G8	60	100	0.028	0.29	0.057	0.045	6.7	0.05
G9	61	100	0.0298	0.2106	0.057	0.045	4.79	0.05
G10	62	100	0.0199	0.169	0.0457	0.04	9.37	0.05
G11	63	100	0.0103	0.128	0.018	0.012	4.1	0.05
G12	64	100	0.022	0.101	0.031	0.025	7.4	0.05
G13	65	200	0.003	0.0296	0.0055	0.004	5.9	0.05
G14	66	100	0.0017	0.018	0.00285	0.0023	4.1	0.05
G15	67	100	0.0017	0.018	0.00285	0.0023	4.1	0.05
G16	68	200	0.0041	0.0356	0.0071	0.0055	7.8	0.05

**Table A.7** Generator dynamic data for the NETS-NYPS test network (2)

Generator	Bus	Rating (MVA)	$X_q$ (pu)	$X'_q$ (pu)	$X''_q$ (pu)	$T'_{q0}$ (s)	$T''_{q0}$ (s)	$H$ (s)	$D$
G1	53	100	0.069	0.028	0.025	1.5	0.035	42	4
G2	54	100	0.282	0.06	0.05	1.5	0.035	30.2	9.75
G3	55	100	0.237	0.05	0.045	1.5	0.035	35.8	10
G4	56	100	0.258	0.04	0.035	1.5	0.035	28.6	10
G5	57	100	0.31	0.06	0.05	0.44	0.035	26	3
G6	58	100	0.241	0.045	0.04	0.4	0.035	34.8	10
G7	59	100	0.292	0.045	0.04	1.5	0.035	26.4	8
G8	60	100	0.28	0.05	0.045	0.41	0.035	24.3	9
G9	61	100	0.205	0.05	0.045	1.96	0.035	34.5	14
G10	62	100	0.115	0.045	0.04	1.5	0.035	31	5.56
G11	63	100	0.123	0.015	0.012	1.5	0.035	28.2	13.6
G12	64	100	0.095	0.028	0.025	1.5	0.035	92.3	13.5
G13	65	200	0.0286	0.005	0.004	1.5	0.035	248	33
G14	66	100	0.0173	0.0025	0.0023	1.5	0.035	300	100
G15	67	100	0.0173	0.0025	0.0023	1.5	0.035	300	100
G16	68	200	0.0334	0.006	0.0055	1.5	0.035	225	50

### A.3.3 VSC-HVDC Line Embedded in Two-Area Network (Sect. 3.2)

VSC-HVDC line parameters (on 400 MW HVDC base with  $V_{dc}^{base} = 200$  kV):

$$R_{dc} = 0.044, \quad L_{dc} = 2.8 \times 10^{-4}, \quad C_{dc} = 3.715 \times 10^{-3}.$$

VSC-HVDC controller parameters:

$$K_P^{Vdc} = 3, \quad K_I^{Vdc} = 40, \quad K_I^{Pdc} = K_I^{Qdc} = 20.$$

PSS-based POD controller parameters (on 100 MVA base):

$$T_W^{POD} = 10, \quad T_1^{POD} = T_3^{POD} = 0.5503, \quad T_2^{POD} = T_4^{POD} = 0.1994, \quad K_{POD} = 0.35.$$

Fixed parameters during MLQG LTR tuning:

$$\Gamma = I, \quad W_o = 0.1 \times I, \quad \Theta = 0.001 \times I, \quad \text{and} \quad V_o = 0.001 \times I.$$

### A.3.4 VSC-HVDC Line Embedded in Five-Area Network (Sect. 3.3)

VSC-HVDC line parameters (on 600 MW HVDC base with  $V_{dc}^{base} = 300$  kV):

$$R_{dc} = 0.04, \quad L_{dc} = 2 \times 10^{-4}, \quad C_{dc} = 3.5 \times 10^{-3}.$$

VSC-HVDC controller parameters:

$$K_P^{Vdc} = 20, \quad K_I^{Vdc} = 60, \quad K_I^{Pdc} = K_I^{Qdc} = 20.$$

PSS-based POD controller parameters (on 100 MVA base):

$$T_W^{POD} = 10, \quad T_1^{POD} = T_3^{POD} = 0.9161, \quad T_2^{POD} = T_4^{POD} = 0.1728, \quad K_{POD} = 0.22.$$

Fixed parameters during MLQG LTR tuning:

$$\Gamma = I, \quad W_o = 0.1 \times I, \quad \Theta = 0.001 \times I, \quad \text{and} \quad V_o = 0.001 \times I.$$

### A.3.5 VSC-MTDC Grid Embedded in Five-Area Network (Sect. 3.4)

All data provided is based on a 100 MW HVDC base (with  $V_{DC}^{base} = 500$  kV).

**Table A.8** VSC-MTDC line data

From node	To node	$R_{dc}$ (pu)	$L_{dc}$ (pu)
1	2	0.01	$2.0 \times 10^{-4}$
1	4	0.007	$1.4 \times 10^{-4}$
1	5	0.005	$1.0 \times 10^{-4}$
2	3	0.005	$1.0 \times 10^{-4}$
3	4	0.008	$1.6 \times 10^{-4}$
4	5	0.006	$1.2 \times 10^{-4}$

VSC-MTDC line parameters are given in Table A.8. All converter stations also cause active power flow losses of 1 %.

VSC-MTDC converter capacitance values (at nodes 1–5) are given below in pu:

$$C_{dc} = \{0.275, 0.1875, 0.1625, 0.2625, 0.1375\}.$$

VSC-MTDC controller parameters:

$$K_P^{Vdc} = 20, K_I^{Vdc} = 200, K_I^{Pdc} = 50, K_I^{Qdc} = 20.$$

### A.3.6 Two VSC-HVDC Lines Embedded in Five-Area Network (Sect. 4.2)

VSC-HVDC-1 line parameters (on 600 MW HVDC base with  $V_{dc}^{base} = 300$  kV):

$$R_{dc} = 0.045, L_{dc} = 2.8 \times 10^{-4}, C_{dc} = 3.715 \times 10^{-3}.$$

VSC-HVDC-2 line parameters (on 500 MW HVDC base with  $V_{dc}^{base} = 250$  kV):

$$R_{dc} = 0.045, L_{dc} = 2.8 \times 10^{-4}, C_{dc} = 3.715 \times 10^{-3}.$$

Both VSC-HVDC controller parameters:

$$K_P^{Vdc} = 3, K_I^{Vdc} = 40, K_I^{Pdc} = K_I^{Qdc} = 20.$$

Fixed parameters during MLQG LTR tuning for all supplementary controllers:

$$\mathbf{\Gamma} = \mathbf{I}, \mathbf{W}_o = 0.1 \times \mathbf{I}, \mathbf{\Theta} = 0.001 \times \mathbf{I}, \text{ and } \mathbf{V}_o = 0.001 \times \mathbf{I}.$$



### A.3.7 VSC-HVDC Line Embedded in Two-Area Network (Sect. 5.2)

VSC-HVDC line parameters (on 400 MW HVDC base with  $V_{dc}^{base} = 200$  kV):

$$R_{dc} = 0.045, L_{dc} = 2 \times 10^{-4}, C_{dc} = 3.5 \times 10^{-3}.$$

VSC-HVDC controller parameters:

$$K_P^{Vdc} = 3, K_I^{Vdc} = 40, K_I^{Pdc} = K_I^{Qdc} = 20.$$

#### A.3.7.1 POD Controller Settings (Sect. 5.2.5)

PSS-based POD controller parameters (on 100 MVA base):

$$T_W^{POD} = 10, T_1^{POD} = T_3^{POD} = 0.6726, T_2^{POD} = T_4^{POD} = 0.1088, K_{POD} = 0.12.$$

### A.3.8 VSC-MTDC Grid Embedded in Five-Area Network with Additional Wind Farm (Sect. 6.2)

All data provided is based on a 100 MW HVDC base (with  $V_{DC}^{base} = 500$  kV).

VSC-MTDC line parameters are given in Table A.9. All converter stations also cause active power flow losses of 1 %.

VSC-MTDC converter capacitance values (at nodes 1–6) are given below in pu:

$$C_{dc} = \{0.275, 0.1875, 0.1625, 0.2625, 0.1375, 0.0750\}.$$

VSC-MTDC controller parameters:

$$K_P^{Vdc} = 20, K_I^{Vdc} = 200, K_I^{Pdc} = 50, K_I^{Qdc} = 20.$$

**Table A.9** VSC-MTDC line data

From node	To node	$R_{dc}$ (pu)	$L_{dc}$ (pu)
1	2	0.01	$2.0 \times 10^{-4}$
1	4	0.007	$1.4 \times 10^{-4}$
1	5	0.005	$1.0 \times 10^{-4}$
2	3	0.005	$1.0 \times 10^{-4}$
3	4	0.008	$1.6 \times 10^{-4}$
4	5	0.006	$1.2 \times 10^{-4}$
3	6	0.001	$1.2 \times 10^{-4}$

Fixed parameters during MLQG LTR tuning for all supplementary controllers:

$$\Gamma = \mathbf{I}, \mathbf{W}_o = 0.1 \times \mathbf{I}, \mathbf{\Theta} = 0.001 \times \mathbf{I}, \text{ and } \mathbf{V}_o = 0.001 \times \mathbf{I}.$$

#### A.4 Data for Optimal Power Flow

In Sect. 5.4.5, an optimal power flow solution is incorporated with the five-area test network. The optimisation minimises the total cost of generation for the given loading scenario, where each generator is subject to the standard cost function (A.1).

$$\text{Cost} = c_0 + c_1 P_e + c_2 P_e^2 \text{ \$}/\text{h} \quad (\text{A.1})$$

The coefficient values for each generator are given in Table A.10. For generators G1–9, these are adopted from “Dynamic security-constrained rescheduling of power systems using trajectory sensitivities,” by T. B. Nguyen and M. A. Pai (*IEEE Trans on Power Systems*, vol. 18, pp. 848–854, 2003). For the remaining generators G10–16, these have been derived to achieve nominal generator outputs close to the standard power flow solution. Also included in Table A.10 are the constraints on active and reactive power for each generating unit.  $P^{\max}$  values have been selected as 1.5 times the nominal active power output from the standard power flow solution for G1–10 and 1.25 times for the larger generators G11–16. Also note that all bus voltages are constrained to between 0.9 and 1.1 pu.

**Table A.10** Data for optimal power flow solution with five-area test network

Generator	Bus	$c_0$	$c_1$	$c_2$	$P^{\max}(\text{MW})$	$P^{\min}(\text{MW})$	$Q^{\min}(\text{MVar})$
G1	53	0	6.9	0.0193	375	100	–100
G2	54	0	3.7	0.0111	817.5	100	–100
G3	55	0	2.8	0.0104	975	100	–100
G4	56	0	4.7	0.0088	948	100	–100
G5	57	0	2.8	0.0128	757.5	100	–100
G6	58	0	3.7	0.0094	1050	100	–100
G7	59	0	4.8	0.0099	840	100	–100
G8	60	0	3.6	0.0113	810	100	–100
G9	61	0	3.7	0.0071	1200	100	–100
G10	62	0	3.9	0.0090	750	100	–100
G11	63	0	4.0	0.0050	1250	500	–100
G12	64	0	2.9	0.0040	1687.5	500	–100
G13	65	0	2.5	0.0019	4488.8	2000	–100
G14	66	0	3.3	0.0033	2231.3	500	–100
G15	67	0	3.8	0.0050	1250	500	–100
G16	68	0	3.5	0.0014	5000	3000	–100

# Appendix B

## Eigenvalue Sensitivity Rank Values

This appendix will detail the rank values calculated based on eigenvalue sensitivity.

### B.1 Numerical Example of Rank Calculation

The rank equation (4.1) is used in this example, repeated as (B.1).

$$\text{Rank} = \left| \frac{\partial \lambda_i}{\partial \gamma_j} \right| \left| \frac{\gamma_j}{\lambda_i} \right| \left| \frac{\sigma_{\gamma_j}}{\mu_{\gamma_j}} \right| \quad (\text{B.1})$$

The example is presented to establish the rank value corresponding to the eigenvalue sensitivity of Mode 1 to variations of G16 within the five-area network.

The nominal value for this generator power output  $\mu_{\gamma:G16} = 4000$  MW. The standard deviation of the distribution for G16 for the study presented in Chap. 4 is equal to 25 % of nominal values at  $3\sigma_{\gamma}$ . Therefore  $\sigma_{\gamma:G16} = 333.33$  MW.

As stated in Chap. 4, the rank is calculated using only the real part  $\sigma_i$  substituted for  $\lambda_i$  in (B.1). This is equal to  $-0.1119$  for the nominal operating point. Following a 1 % increase in the power output of G16, this changes to  $-0.1131$ , therefore  $\delta\sigma_{\text{Mode1}} = -0.0012$ .

The rank value can now be calculated as in (B.2), with all calculated values normalised for comparison.

$$\text{Rank} = \left| \frac{-0.0012}{40} \right| \left| \frac{4000}{-0.1119} \right| \left| \frac{333.33}{4000} \right| = 8.936 \times 10^{-3} \quad (\text{B.2})$$

### B.2 Five-Area Test Network (Sect. 4.3)

The normalised rank values for with the centralised controller in place are shown in Table B.1, and with the decentralised controller in Table B.2.

**Table B.1** Normalised rank values with the centralised controller installed

Parameter	Rank value	Parameter	Rank value
G16	1.000	L27	0.066
G14	0.787	G1	0.065
L18	0.507	L47	0.053
G15	0.354	L28	0.052
L42	0.349	G12	0.051
L41	0.337	HVDC2	0.050
G6	0.330	HVDC1	0.049
G4	0.275	L25	0.048
G7	0.248	L4	0.041
G9	0.240	L7	0.039
G5	0.239	L52	0.038
L20	0.219	L1	0.036
G3	0.204	L26	0.033
G8	0.160	L8	0.033
G2	0.158	L51	0.030
L17	0.148	L40	0.028
G11	0.124	L49	0.027
G10	0.088	L50	0.025
L16	0.088	L46	0.022
L15	0.082	L45	0.016
L24	0.082	L39	0.012
L48	0.081	L33	0.011
L21	0.081	L44	0.010
L23	0.077	L12	0.005
L29	0.072	L36	0.005
L3	0.069		

**Table B.2** Normalised rank values with the decentralised controller installed

Parameter	Rank value	Parameter	Rank value
G16	1.000	G11	0.033
L18	0.407	HVDC1	0.033
G14	0.355	L3	0.033
G15	0.203	L27	0.032
L42	0.198	HVDC2	0.031
G6	0.174	G1	0.030
L41	0.167	G10	0.028
G4	0.145	L28	0.025
G5	0.137	L25	0.023
L20	0.133	L4	0.019
G7	0.132	L47	0.018

(continued)

**Table B.2** (continued)

Parameter	Rank value	Parameter	Rank value
G9	0.101	L52	0.018
L17	0.098	L7	0.017
G3	0.097	L26	0.016
G2	0.075	L8	0.014
G8	0.075	L40	0.012
L51	0.059	L49	0.011
G12	0.051	L1	0.009
L16	0.045	L46	0.008
L21	0.044	L45	0.006
L50	0.044	L33	0.005
L23	0.044	L39	0.004
L15	0.043	L44	0.004
L24	0.041	L12	0.003
L29	0.035	L36	0.003
L48	0.035		

**Table B.3** Normalised rank values with and without the PSS-based POD controller

No POD controller		PSS-based POD controller	
Parameter	Rank value	Parameter	Rank value
HVDC	1.000	HVDC	1.000
L2	0.713	L2	0.243
G3	0.670	G3	0.146
G4	0.537	G4	0.141
L1	0.243	L1	0.042
G2	0.089	G2	0.013

### B.3 Two-Area Test Network (Sect. 5.3)

The normalised rank values for the two-area test network are given in Table B.3.

## Appendix C

### PCM Model Functions

The mathematical description of the PCM model function  $g(\Gamma)$  up to fourth order are presented as (C.1–C.4); where  $\mathbf{K}$  is the set of coefficients,  $\gamma_i$  is the  $i$ th uncertain parameter in the set  $\Gamma$ ,  $n_\gamma$  is the number of modelled uncertain parameters, and  $H_o(\gamma_i)$  is the  $o$ th order orthogonal polynomial of  $\gamma_i$ .

$$g_1(\Gamma) = \left[ \sum_{i=1}^{n_\gamma} H_1(\gamma_i) \right] \mathbf{K} \quad (\text{C.1})$$

$$g_2(\Gamma) = \left[ \sum_{i=1}^{n_\gamma} H_1(\gamma_i) + \sum_{i=1}^{n_\gamma} H_2(\gamma_i) + \sum_{i=1}^{n_\gamma-1} \sum_{j=i+1}^{n_\gamma} H_1(\gamma_i) H_1(\gamma_j) \right] \mathbf{K} \quad (\text{C.2})$$

$$g_3(\Gamma) = \left[ \sum_{i=1}^{n_\gamma} H_1(\gamma_i) + \sum_{i=1}^{n_\gamma} H_2(\gamma_i) + \sum_{i=1}^{n_\gamma-1} \sum_{j=i+1}^{n_\gamma} H_1(\gamma_i) H_1(\gamma_j) \right. \\ \left. + \sum_{i=1}^{n_\gamma} H_3(\gamma_i) + \sum_{i=1}^{n_\gamma} \sum_{j=1, j \neq i}^{n_\gamma} H_2(\gamma_i) H_1(\gamma_j) + \sum_{i=1}^{n_\gamma-2} \sum_{j=i+1}^{n_\gamma-1} \sum_{k=j+1}^{n_\gamma} H_1(\gamma_i) H_1(\gamma_j) H_1(\gamma_k) \right] \mathbf{K} \quad (\text{C.3})$$

$$\begin{aligned}
g_4(\Gamma) = & \left[ \sum_{i=1}^{n_\gamma} H_1(\gamma_i) + \sum_{i=1}^{n_\gamma} H_2(\gamma_i) + \sum_{i=1}^{n_\gamma-1} \sum_{j=i+1}^{n_\gamma} H_1(\gamma_i) H_1(\gamma_j) \right. \\
& + \sum_{i=1}^{n_\gamma} H_3(\gamma_i) + \sum_{i=1}^{n_\gamma} \sum_{j=1, j \neq i}^{n_\gamma} H_2(\gamma_i) H_1(\gamma_j) + \sum_{i=1}^{n_\gamma-2} \sum_{j=i+1}^{n_\gamma-1} \sum_{k=j+1}^{n_\gamma} H_1(\gamma_i) H_1(\gamma_j) H_1(\gamma_k) \\
& + \sum_{i=1}^{n_\gamma} H_4(\gamma_i) + \sum_{i=1}^{n_\gamma} \sum_{j=1, j \neq i}^{n_\gamma} H_3(\gamma_i) H_1(\gamma_j) + \sum_{i=1}^{n_\gamma-1} \sum_{j=i+1}^{n_\gamma} H_2(\gamma_i) H_2(\gamma_j) \\
& + \sum_{i=1}^{n_\gamma} \sum_{j=1, j \neq i}^{n_\gamma-1} \sum_{k=j+1}^{n_\gamma} H_2(\gamma_i) H_1(\gamma_j) H_1(\gamma_k) \\
& \left. + \sum_{i=1}^{n_\gamma-3} \sum_{j=i}^{n_\gamma-2} \sum_{k=j+1}^{n_\gamma-1} \sum_{l=k+1}^{n_\gamma} H_1(\gamma_i) H_1(\gamma_j) H_1(\gamma_k) H_1(\gamma_l) \right] \mathbf{K}
\end{aligned} \tag{C.4}$$

# Appendix D

## PCM Model Uncertain Parameter Details

This appendix provides the details for the uncertain system parameters which have been used to produce Probabilistic Collocation Method functions within this thesis.

### D.1 Two-Area Test Network (Sect. 5.2)

#### *D.1.1 Normally Distributed Parameters*

Details for the normally distributed uncertain parameters within the two-area test network are provided in Table D.1.

#### *D.1.2 Uniformly Distributed Parameters*

The VSC-HVDC power flow  $P_{DC}$  follows a uniform distribution. Recursive coefficients have been determined as presented in Table D.2

### D.2 Five-Area Test Network with Standard Power Flow Solution (Sect. 5.4)

#### *D.2.1 Normally Distributed Parameters*

Details for the normally distributed uncertain parameters within the five-area test network when a standard power flow solution is used are provided in Table D.3.



**Table D.1** Normal distribution details for uncertain parameters in the Kundur two area network

	$\gamma_1:G_2$	$\gamma_2:G_3$	$\gamma_3:G_4$	$\gamma_4:L_1$	$\gamma_5:L_2$
$\mu_\gamma$ (MW)	700	719	700	967	1767
$\sigma_\gamma$ (MW)	46.67	47.93	46.67	64.47	117.8

**Table D.2** Recursive coefficients for orthogonal polynomials representing the uniform distribution of the VSC-HVDC line power flow.

Orthogonal polynomial order, $o$	Recursive coefficients	
	a	b
1	200	40,000
2	200	3,333.50
3	200	2,666.80
4	200	2,571.56
5	200	2,539.81

**Table D.3** Normal distribution details for uncertain parameters in the five-area network with standard PF solution

	$\gamma_1:G_4$	$\gamma_2:G_5$	$\gamma_3:G_6$	$\gamma_4:G_7$	$\gamma_5:G_{14}$	$\gamma_6:G_{16}$	$\gamma_7:L_{18}$	$\gamma_8:L_{20}$
$\mu_\gamma$ (MW)	632	505	700	560	1785	4000	2470	680
$\sigma_\gamma$ (MW)	52.67	42.08	58.33	46.67	148.75	333.33	205.83	56.67

### D.3 Five-Area Test Network with Optimal Power Flow Solution (Sect. 5.4.5)

#### D.3.1 Normally Distributed Parameters

Details for the normally distributed uncertain parameters within the five-area test network when an optimal power flow solution is used are provided in Table D.4.

**Table D.4** Normal distribution details for uncertain parameters in the five-area network with OPF solution

	$\gamma_3:L_{28}$	$\gamma_4:L_{29}$	$\gamma_5:L_{41}$	$\gamma_6:L_{42}$	$\gamma_7:L_{47}$	$\gamma_8:L_{48}$
$\mu_\gamma$ (MW)	206	284	1000	1150	203.12	241.2
$\sigma_\gamma$ (MW)	17.17	23.67	83.33	95.83	16.93	20.10

**Table D.5** Recursive coefficients for orthogonal polynomials representing the uniform distributions of  $P_{dc}^{VSC-1}$  and  $P_{dc}^{VSC-2}$ 

Orthogonal polynomial order, $o$	$P_{dc}^{VSC-1}$		$P_{dc}^{VSC-2}$	
	a	b	a	b
1	350	1.003	275	1.004
2	350	7550.00	275	5250.00
3	350	6039.80	275	4199.80
4	350	5823.77	275	4049.49
5	350	5751.43	275	3999.05

### D.3.2 Uniformly Distributed Parameters

The operating capacities of VSC-HVDC-1 and VSC-HVDC-2 ( $P_{dc}^{VSC-1}$  and  $P_{dc}^{VSC-2}$ ) follow uniform distributions. Recursive coefficients have been determined as presented in Table D.5.

These coefficients result in the monic orthogonal polynomials (D.1–D.5) representing  $\gamma_1 : P_{dc}^{VSC-1}$  and (D.6–D.10) representing  $\gamma_2 : P_{dc}^{VSC-2}$ .

$$H_1(\gamma_1) = \gamma_1 - 350 \quad (D.1)$$

$$H_2(\gamma_1) = \gamma_1^2 - 700\gamma_1 + 1.150 \times 10^5 \quad (D.2)$$

$$H_3(\gamma_1) = \gamma_1^3 - 1050\gamma_1^2 + 3.539 \times 10^5\gamma_1 - 3.812 \times 10^7 \quad (D.3)$$

$$H_4(\gamma_1) = \gamma_1^4 - 1400\gamma_1^3 - 7.156 \times 10^5\gamma_1^2 - 1.579 \times 10^8\gamma_1 + 1.267 \times 10^{10} \quad (D.4)$$

$$H_5(\gamma_1) = \gamma_1^5 - 1750\gamma_1^4 - 1.200 \times 10^6\gamma_1^3 - 4.023 \times 10^8\gamma_1^2 + 6.591 \times 10^{10}\gamma_1 - 4.216 \times 10^{12} \quad (D.5)$$

$$H_1(\gamma_2) = \gamma_2 - 275 \quad (D.6)$$

$$H_2(\gamma_2) = \gamma_2^2 - 550\gamma_2 + 7.038 \times 10^4 \quad (D.7)$$

$$H_3(\gamma_2) = \gamma_2^3 - 825\gamma_2^2 + 2.174 \times 10^5\gamma_2 - 3.820 \times 10^7 \quad (D.8)$$

$$H_4(\gamma_2) = \gamma_2^4 - 1100\gamma_2^3 - 4.403 \times 10^5\gamma_2^2 - 7.576 \times 10^7\gamma_2 + 4.720 \times 10^9 \quad (D.9)$$

$$H_5(\gamma_2) = \gamma_2^5 - 1375\gamma_2^4 - 7.388 \times 10^5\gamma_2^3 - 1.935 \times 10^8\gamma_2^2 + 2.469 \times 10^{10}\gamma_2 - 1.225 \times 10^{12} \quad (D.10)$$

## Appendix E

### Modal System Representation

This appendix presents an example of the modal canonical form of a state space system representation. The example presented is of a sixth order generator model with no associated controllers.

The standard system representation as per the form (E.1) and (E.2) is given by (E.3–E.6).

$$\Delta \dot{\mathbf{x}} = \mathbf{A} \Delta \mathbf{x} + \mathbf{B} \Delta \mathbf{u} \quad (\text{E.1})$$

$$\Delta \mathbf{y} = \mathbf{C} \Delta \mathbf{x} + \mathbf{D} \Delta \mathbf{u} \quad (\text{E.2})$$

$$\mathbf{A} = \begin{bmatrix} -0.142 & 0 & 0.044 & 0 & 0 & 0 \\ 0 & 0.777 & 0 & 0.110 & 0 & 0 \\ 20 & 0 & -20 & 0 & 0 & 0 \\ 0 & -28.571 & 0 & -28.571 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 314.159 \\ -0.083 & 0.121 & -0.020 & -0.015 & 0 & 0 \end{bmatrix} \quad (\text{E.3})$$

$$\mathbf{B} = \begin{bmatrix} 0.004 & 0 & 0.098 & 0 & 0 & 0 \\ 0 & 0.020 & 0 & 0 & 0 & 0 \\ 0.496 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.638 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -314.159 \\ 0 & 0 & 0 & 0.010 & -0.010 & 0 \end{bmatrix} \quad (\text{E.4})$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (\text{E.5})$$

$$\mathbf{D} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (\text{E.6})$$

This state-space representation can be transformed to the modal canonical form as per (E.7) and (E.8) using the modal transformation matrix  $\mathbf{M}$ , where  $\mathbf{z} = \mathbf{M}\Delta\mathbf{x}$ .

$$\dot{\mathbf{z}} = \mathbf{\Lambda}\mathbf{z} + \mathbf{B}_M\Delta\mathbf{u} \quad (\text{E.7})$$

$$\Delta\mathbf{y} = \mathbf{C}_M\mathbf{z} + \mathbf{D}\Delta\mathbf{u} \quad (\text{E.8})$$

The new modal state matrices are given as (E.9)–(E.11).

$$\mathbf{\Lambda} = \begin{bmatrix} -0.098 & 0 & 0 & 0 & 0 & 0 \\ 0 & -20.044 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.669 & 0 & 0 & 0 \\ 0 & 0 & 0 & -28.464 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.005 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (\text{E.9})$$

$$\mathbf{B}_M = \begin{bmatrix} 0.044 & 0 & 0.784 & 0 & 0 & 0 \\ -0.245 & 0 & 0.049 & 0 & 0 & 0 \\ 0 & -0.359 & 0 & 0 & 0 & 0 \\ 0 & 0.661 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -9.818 \\ 0 & 0 & 0 & 19.948 & -19.948 & 0 \end{bmatrix} \quad (\text{E.10})$$

$$\mathbf{C}_M = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.062 & -0.004 & 0 & 0 \\ 0.125 & -1.999 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.061 & 0.998 & 0 & 0 \\ 0 & 0 & 0 & 0 & 32 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.001 \end{bmatrix} \quad (\text{E.11})$$