EE507 Final Exam
Assigned: 12/14 at 7 AM, due 12/16 at midnight.

Problem 1

Consider a zero mean Gaussian WSS random process $X(t)$ with autocorrelation $R_{XX}(t) = 3e^{-|t|}$.

a. Find the power spectrum of $X(t)$. **(3 points)**
b. Find the joint pdf of $X(7)$ and $X(9)$. **(3 points)**
c. Now consider the random process $Y(t) = 2X(t) + 5$. Please argue that $Y(t)$ is also WSS, and find its mean and autocorrelation. **(3 points)**
d. A linear time invariant system with impulse response $h(t) = e^{-4t}, t \geq 0$, is driven by the random process $X(t)$. The output of the system is denoted by $Z(t)$. Please find the mean, power spectrum, and autocorrelation of $Z(t)$. **(10 points)**
e. Another random process $W(t)$ is defined as $W(t) = (X(t-1))^2$. Please find the mean of $W(t)$. **(3 points)**
f. Bonus: Find the autocorrelation of $W(t)$. **(10 points)** (Note: the bonus problem is rather challenging, so please do it only after you have completed the other parts of the exam.)

Problem 2

A random process $X[k], k = 0,1,2, \ldots$ is defined as follows: each $X[k]$ is an independent exponential random variable with mean 1.

a. Please find the first- and second- order pdfs of $X[k]$. **(4 points)**
b. Find the mean $E(X[k])$ and autocorrelation $R_{XX}[k,j] = E(X[k]X[j])$. **(4 points)**
c. Another random process $Y[k], k = 0,1,2, \ldots$, is defined from $X[k]$ as follows: $Y[k] = 0$ if $X[0], \ldots, X[k]$ are all less than or equal to 2 (i.e. $X[0] \leq 2, X[1] \leq 2, \ldots X[k] \leq 2$); and $Y[k] = 1$ otherwise. Find the first- and second- order pmfs of $Y[k]$. **(7 points)**
d. Please prove that the random process $Y[k]$ converges to 1 in a mean square sense. **(3 points)**

Problem 3

a. A random process $X(t)$ is generated as follows. First, three fair coins are tossed, and the number of heads showing on the three coin tosses is denoted as Y. Then $X(t)$ is defined as $X(t) = Yt$ (where $t \in R$). Please find the mean and first-order pdf of $X(t)$. Also, please determine if $X(0)$ and $X(2)$ are independent. **(6 points)**
b. A second random process $Z(t)$ is generated as follows. First, three fair coins are tossed, and the number of heads showing on the three coin tosses is denoted as Q. Then $Z(t)$ is defined as $Z(t) = Ct$ ($t \in R$), where $C \sim unif([0, Q])$. Please find the mean and first-order pdf of $Z(t)$. **(6 points)**

Problem 4

A randomly chosen bus in Pullman is green (G) with probability 0.9. and is electric (E) with probability 0.8 independently of whether it is green or not. Additionally, a randomly chosen bus is big (B) with probability 0.7, but you don't know whether or not this event is independent of the other defined events. All buses in Pullman have capacity C of at least 11, green electric buses have capacity C of at least 50, and big green electric buses have capacity C of at least 100. Please argue that the expected capacity of a randomly chosen bus in Pullman is at least 60. **(8 points)**

Problem 5

Two random variables X and Y have joint probability density function $f_{X,Y}(x,y)$ that is uniform on the region $-1 \leq x \leq 1, 0 \leq y \leq x^2$.

a. Find $f_{X|Y}(x|Y=y)$ and $var(X \mid Y=y)$. **(6 points)**
b. Find the pdf of $Z = \frac{Y}{X^2}$. **(6 points)**

Problem 6

Two random variables M and N have the following joint probability mass function: $p_{M,N}(m,n) = C0.5^{m+n}$ for $m = 0,1,2,\ldots$ and $n = 0,1,2,\ldots$.

a. Find the constant C. **(3 points)**
b. Find the pmf of $Q = M + N$. **(5 points)**
c. Please find the conditional pmf $p_{M|Q}(m \mid Q = q)$. **(6 points)**

Problem 7

a. True or False: Consider a probabilistic experiment, and a random process $X(t)$ defined from the experiment. Given that a particular event occurs, the trajectory of the random process is uniquely determined. Please explain your answer briefly. **(2 points)**
b. True or False: Consider a probabilistic experiment, and a random process $X(t)$ defined from the experiment. Given the trajectory of the random process (e.g. you know that $X(t) = t$), the outcome of the underlying probabilistic experiment is uniquely determined. Please briefly explain. **(2 points)**
c. True or False: Consider a probabilistic experiment, and a random process $X(t)$ defined from the experiment. The content in $X(t)$ at a particular frequency (i.e., the Fourier transform of $X(t)$ evaluated at that frequency) is a random variable. Please briefly explain. **(2 points)**
d. True or False: An LTI system with impulse response $h(t) = e^{-t} - e^{-4t} + te^{-2t}\cos(t)$ is driven by a zero mean Gaussian wide sense stationary random process $X(t)$ with autocorrelation $R_{xx}(t) = 2e^{-|t|} - e^{-2|t|}$, producing an output $Y(t)$. It follows that $E(Y(17)^3) = 0$. Hint: this question should NOT require any math. **(2 points)**
e. True or False: A wide-sense-stationary random process is always strong sense stationary. **(2 points)**
f. True or False: If two random variables are independent, they are also uncorrelated. **(2 points)**
f. Please write a couple of sentences about an engineering problem which requires understanding random processes **(2 points)**.