ASSIGNMENT 04 of EE 523, SPRING 2023

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1. Type 1 Model

E = Adjacency list

$$
\begin{array}{c}
1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11
\end{array}
\begin{bmatrix}
5 \\ 6 \\ 11 \\ 10 \\ 1,\ 6 \\ 2,\ 5,\ 7 \\ 6,\ 8 \\ 7,\ 9 \\ 8,\ 10 \\ 4,\ 9,\ 11 \\ 3,\ 10
\end{bmatrix}
$$

$$
\underset{12\times 1}{x} = \begin{bmatrix} \theta_2 \\ \omega_2 \\ E'_{q2} \\ E'_{d2} \\ \theta_3 \\ \omega_3 \\ E'_{q3} \\ E'_{d3} \\ \theta_4 \\ \omega_4 \\ E'_{q4} \\ E'_{d4} \end{bmatrix}_{12\times 1}
\qquad
\underset{20\times 1}{y} = \begin{bmatrix} \delta_2 \\ |V_2| \\ \delta_3 \\ |V_3| \\ \delta_4 \\ |V_4| \\ \delta_5 \\ |V_5| \\ \vdots \\ \delta_{11} \\ |V_{11}| \end{bmatrix}_{20\times 1}
$$

{Note: ⌀ implies that the variables are ~~either~~ given (P_{mi}, X_{di}', X_{di}, X_{qi}', X_{qi}, Y_{ik}, γ_{ik}, ~~δ_k~~, K_{Di}, ω_s, E_{fdi}, M_i) ~~or have been initialized, say using power flow~~ (~~V_k, δ_k for nongen buses~~)}

1.2

$$P_{G_2} - P_{D_2} - P_2 = 0$$

or $$V_{d_2} I_{d_2} + V_{q_2} I_{q_2} - P_{D_2} - |V_2| \sum_{(2,k) \in E} |Y_{2k} V_k| \cos(\gamma_{2k} + \delta_k - \delta_2)$$

or
$$|V_2| \sin(\theta_2 - \delta_2) \cdot \left\{ \frac{E_{q_2}' - V_2 \cos(\theta_2 - \delta_2)}{X_{d_2}'} \right\}$$
$$+ |V_2| \cos(\theta_2 - \delta_2) \cdot \left\{ \frac{E_{d_2}' - V_2 \sin(\theta_2 - \delta_2)}{-X_{q_2}'} \right\}$$
$$- P_{D_2}$$
$$- \left[G_{22} |V_2|^2 + |Y_{25} V_5 V_2| \cos(\gamma_{25} + \delta_5 - \delta_2) \right]$$
$$= 0$$

g_1

$$Q_{G_2} - Q_{D_2} - Q_2 = 0$$

or
$$|V_2| \cos(\theta_2 - \delta_2) \left\{ \frac{E_{q_2}' - V_2 \cos(\theta_2 - \delta_2)}{X_{d_2}'} \right\}$$
$$- |V_2| \sin(\theta_2 - \delta_2) \left\{ \frac{E_{d_2}' - V_2 \sin(\theta_2 - \delta_2)}{-X_{q_2}'} \right\}$$
$$- Q_{D_2}$$
$$- \left[-B_{22} |V_2|^2 - |Y_{25} V_5 V_2| \sin(\gamma_{25} + \delta_5 - \delta_2) \right]$$
$$= 0$$

g_2

~~Similarly, for buses~~

$$
\begin{aligned}
&|V_3| \sin(\theta_3-\delta_3)\cdot\left\{\frac{E'_{q3} - V_3\cos(\theta_3-\delta_3)}{x_{d3}}\right\} \\
&+ |V_3|\cos(\theta_3-\delta_3)\cdot\left\{\frac{\cancel{E_q}\; E'_{d3} - V_3 \sin(\theta_3-\delta_3)}{-x'_{q3}}\right\} \\
&- P_{D3} \\
&- \left[G_{33}|V_3|^2 + |Y_{3,11}\, V_{11}\, V_3| \cos(\gamma_{3,11}+\delta_{11}-\delta_3)\right] \\
&= 0
\end{aligned}
\qquad (g_3)
$$

$$
\begin{aligned}
&|V_3| \cos(\theta_3-\delta_3)\cdot\left\{\frac{E'_{q3} - V_3\cos(\theta_3-\delta_3)}{x'_{d3}}\right\} \\
&- |V_3|\sin(\theta_3-\delta_3)\cdot\left\{\frac{E'_{d3} - V_3 \sin(\theta_3-\delta_3)}{-x'_{q3}}\right\} \\
&- Q_{D3} \\
&- \left[-B_{33}|V_3|^2 - |Y_{3,11}\, V_{11}\, V_3| \sin(\gamma_{3,11}+\delta_{11}-\delta_3)\right] \\
&= 0
\end{aligned}
\qquad (g_4)
$$

1.4

$$|V_4| \sin(\theta_4 - \delta_4) \cdot \left\{ \frac{E'_{q4} - V_4 \cos(\theta_4 - \delta_4)}{X''_{d4}} \right\}$$

$$+ |V_4| \cos(\theta_4 - \delta_4) \cdot \left\{ \frac{E'_{d4} - V_4 \sin(\theta_4 - \delta_4)}{-X''_{q4}} \right\}$$

$$- P_{D4}$$

$$- \left[G_{44} |V_4|^2 - |Y_{4,10} V_{10} V_4| \cos(\gamma_{4,10} + \delta_{10} - \delta_4) \right]$$

$$= 0 \qquad (g5)$$

$$|V_4| \cos(\theta_4 - \delta_4) \cdot \left\{ \frac{E'_{q4} - V_4 \cos(\theta_4 - \delta_4)}{X''_{d4}} \right\}$$

$$- |V_4| \sin(\theta_4 - \delta_4) \cdot \left\{ \frac{E'_{d4} - V_4 \sin(\theta_4 - \delta_4)}{-X''_{q4}} \right\}$$

$$- Q_{D4}$$

$$- \left[-B_{44} |V_4|^2 - |Y_{4,10} V_{10} V_4| \sin(\gamma_{4,10} + \delta_{10} - \delta_4) \right]$$

$$= 0 \qquad (g6)$$

$\boxed{\cancel{1.6}}$ $\boxed{1.5}$

$$\cancel{P_{G5}}^{\,0} - P_{D5} - P_5 = 0$$

or

$$\boxed{-P_{D5} - \Big[G_{55}|V_5|^2 + |Y_{51} V_1 V_5| \cos(\gamma_{51} + \delta_1 - \delta_5) + |Y_{56} V_6 V_5| \cos(\gamma_{56} + \delta_6 - \delta_5) \Big] = 0}$$

(g 7)

$$Q_{G5} - Q_{D5} - Q_5 = 0$$

or

$$\boxed{-Q_{D5} - \Big[-B_{55}|V_5|^2 - |Y_{51} V_1 V_5| \sin(\gamma_{51} + \delta_1 - \delta_5) - |Y_{56} V_6 V_5| \sin(\gamma_{56} + \delta_6 - \delta_5) \Big] = 0}$$

(g 8)

$$\boxed{\begin{aligned} -P_{D6} - \Big[G_{66}|V_6|^2 &+ |Y_{62} V_2 V_6| \cos(\gamma_{62} + \delta_2 - \delta_6) \\ &+ |Y_{65} V_5 V_6| \cos(\gamma_{65} + \delta_5 - \delta_6) \\ &+ |Y_{67} V_7 V_6| \cos(\gamma_{67} + \delta_7 - \delta_6) \Big] = 0 \end{aligned}}$$

(g 9)

$$\boxed{\begin{aligned} -Q_{D6} - \Big[-B_{66}|V_6|^2 &- |Y_{62} V_2 V_6| \sin(\gamma_{62} + \delta_2 - \delta_6) \\ &- |Y_{65} V_5 V_6| \sin(\gamma_{65} + \delta_5 - \delta_6) \\ &- |Y_{67} V_7 V_6| \sin(\gamma_{67} + \delta_7 - \delta_6) \Big] = 0 \end{aligned}}$$

(g 10)

1-6

$$-P_{D_7} - \left[G_{77}|V_7|^2 + |Y_{76}V_6V_7|\cos(\gamma_{76}+\delta_6-\delta_7) + |Y_{78}V_8V_7|\cos(\gamma_{78}+\delta_8-\delta_7) \right] = 0$$

g11

$$-Q_{D_7} - \left[-B_{77}|V_7|^2 - |Y_{76}V_6V_7|\sin(\gamma_{76}+\delta_6-\delta_7) - |Y_{78}V_8V_7|\sin(\gamma_{78}+\delta_8-\delta_7) \right] = 0$$

g12

$$-P_{D_8} - \left[G_{88}|V_8|^2 + |Y_{87}V_7V_8|\cos(\gamma_{87}+\delta_7-\delta_8) + |Y_{89}V_9V_8|\cos(\gamma_{89}+\delta_9-\delta_8) \right] = 0$$

g13

$$-Q_{D_8} - \left[-B_{88}|V_8|^2 - |Y_{87}V_7V_8|\sin(\gamma_{87}+\delta_7-\delta_8) - |Y_{89}V_9V_8|\sin(\gamma_{89}+\delta_9-\delta_8) \right] = 0$$

g14

$$-P_{D_9} - \left[G_{99}|V_9|^2 + |Y_{98}V_8V_9|\cos(\gamma_{98}+\delta_8-\delta_9) + |Y_{9,10}V_{10}V_9|\cos(\gamma_{9,10}+\delta_{10}-\delta_9) \right] = 0$$

g15

$$-Q_{D_9} - \left[\cancel{G_9} - B_{99}|V_9|^2 - |Y_{98}V_8V_9|\sin(\gamma_{98}+\delta_8-\delta_9) - |Y_{9,10}V_{10}V_9|\sin(\gamma_{9,10}+\delta_{10}-\delta_9) \right] = 0$$

g16

$$-P_{D_{10}} - \Big[G_{10,10}|V_{10}|^2 + |Y_{10,4} V_4 V_{10}| \cos(\gamma_{10,4} + \delta_4 - \delta_{10}) + |Y_{10,9} V_9 V_{10}| \cos(\gamma_{10,9} + \delta_9 - \delta_{10}) + |Y_{10,11} V_{11} V_{10}| \cos(\gamma_{10,11} + \delta_{11} - \delta_{10}) \Big] = 0 \tag{g17}$$

$$-Q_{D_{10}} - \Big[-B_{10,10}|V_{10}|^2 - |Y_{10,4} V_4 V_{10}| \sin(\gamma_{10,4} + \delta_4 - \delta_{10}) - |Y_{10,9} V_9 V_{10}| \sin(\gamma_{10,9} + \delta_9 - \delta_{10}) - |Y_{10,11} V_{11} V_{10}| \sin(\delta_{10,11} + \delta_{11} - \delta_{10}) \Big] = 0 \tag{g18}$$

$$-P_{D_{11}} - \Big[G_{11,11}|V_{11}|^2 + |Y_{11,3} V_3 V_{11}| \cos(\gamma_{11,3} + \delta_3 - \delta_{11}) + |Y_{11,10} \cdot V_{10} V_{11}| \cos(\gamma_{11,10} + \delta_{10} - \delta_{11}) \Big] = 0 \tag{g19}$$

$$-Q_{D_{11}} - \Big[-B_{11,11}|V_{11}|^2 - |Y_{11,3} V_3 V_{11}| \sin(\gamma_{11,3} + \delta_3 - \delta_{11}) - |Y_{11,10} V_{10} V_{11}| \sin(\gamma_{11,10} + \delta_{10} - \delta_{11}) \Big] = 0 \tag{g20}$$

$\forall$ i = 2, 3 and 4

$$\dot{\theta}_i = (\omega_i - 1)\,\omega_s$$

$$\dot{\omega}_i = \frac{1}{2H_i}\Bigg[P_{m_i} - \Big[\cancel{\ } V_i \sin(\theta_i - \delta_i) \cdot \left\{ \frac{E_{q_i}' - V_i \cos(\theta_i - \delta_i)}{X_{d_i}'} \right\} + V_i \cos(\theta_i - \delta_i) \cdot \left\{ \frac{E_{d_i}' - V_i \sin(\theta_i - \delta_i)}{-X_{q_i}'} \right\} \Big] - K_{D_i}(\omega_i - 1) \Bigg]$$

$$\dot{E}_{q_i}' = \frac{1}{T_{do_i}'}\left[-E_{q_i}' - (X_{d_i} - X_{d_i}')\left\{ \frac{E_{q_i}' - V_i \cos(\theta_i - \delta_i)}{X_{d_i}'} \right\} + E_{fd_i} \right]$$

$$\dot{E}_{d_i}' = \frac{1}{T_{qo_i}'}\left[-E_{d_i}' - (X_{q_i} - X_{q_i}')\left\{ \frac{E_{d_i}' - V_i \sin(\theta_i - \delta_i)}{-X_{q_i}'} \right\} \right]$$

Again, i = 2, 3, 4.

$\rightarrow$ $\boxed{f_1 \text{ to } f_{12}}$

where

f_1 + f_4 are for i = 2

f_5 + f_8 are for i = 3

f_9 to f_{12} are for i = 4

Thus,

$$\underbrace{[g(x,y) = 0]}_{20\times 1} \equiv \underbrace{\begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \\ g_5 \\ g_6 \\ g_7 \\ \vdots \\ g_{20} \end{bmatrix}}_{20\times 1}$$

gen. buss (g_1 to g_6) — represent the Type 1 model

load buss (g_7 to g_{20})

for Kundur's 11 bus system

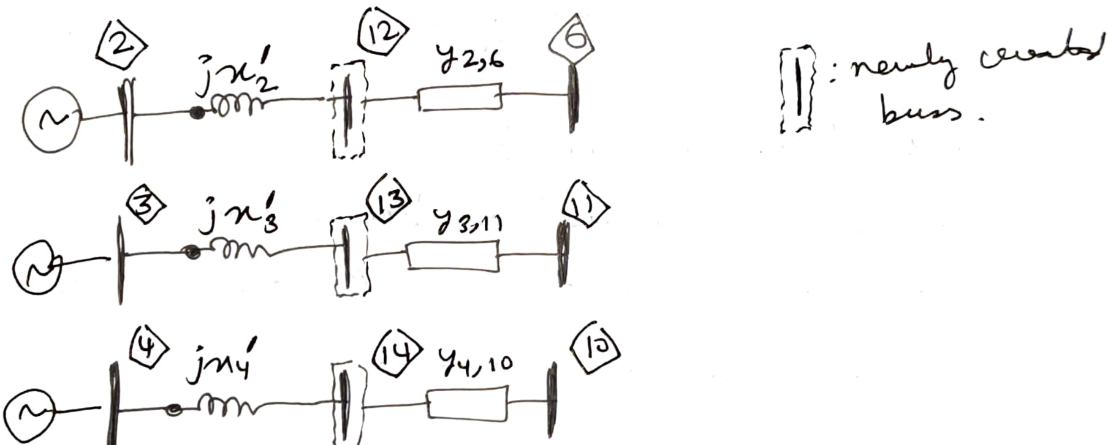
$\dot{x} = f(x,y)$

$$\dot{x} \equiv \underbrace{\begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_{12} \end{bmatrix}}_{12\times 1}$$

Type II Model

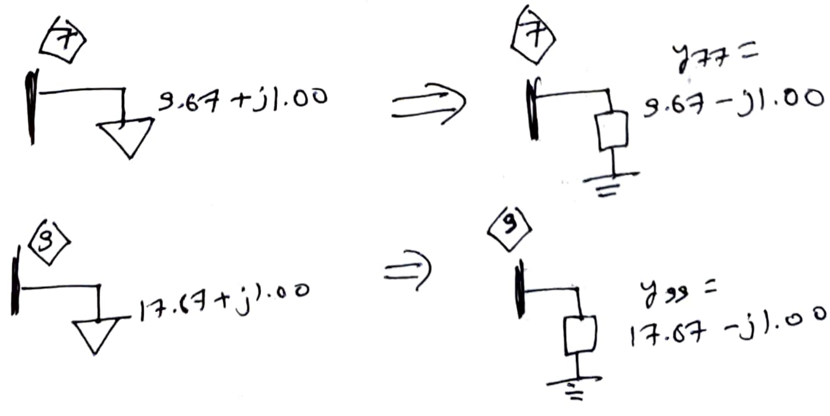
For Type II model we first need to create ~~Ygen~~ Ynet by: ~~adding~~ ~~node~~

1. Adding extra buses at the terminals of every PV bus, and incorporating the generator impedance in the newly created branches.



Note: In Type II model, we ignore saliency, so $x'_{d_i} = x'_{q_i} = x'_i = \dfrac{x'_{d_i\,\text{original}} + x'_{q_i\,\text{original}}}{2}$

2. Converting all loads into ~~an~~ constant impedances.



3. Renumber/ Reorder our buses such that the slack bus is numbered $\boxed{1}$, PV buses are numbered $\boxed{2}$ to $\boxed{N_{pv}+1}$, PQ buses are numbered $\boxed{N_{pv}+2}$ to $\boxed{N}$ and new buses at the terminals of the PV buses are numbered ~~fd~~ $\boxed{2+(N-1)}$ to $\boxed{N_{pv}+1+(N-1)}$

thus making a total of $N + N_{pv}$ buses.

$$\underbrace{N}_{\text{original total count of all buses in the system.}} + \underbrace{N_{pv}}_{\text{number of Non slack PV buses in the system.}}$$

Fortunately, for the Kundur 11 bus system, since the generator buses are already numbered ~~from~~ from $\boxed{1}$ to $\boxed{4}$, we need not do any renumbering.

~~4~~ 4. Finally ~~to~~ form the Ybus, called Ynet from the $N + N_{pv}$ buses:

$$
Y_{net} = \begin{array}{c|cccc:ccccc}
 & 1 & 2 & 3 & 4 & 5 & \cdots & 11 & 12 & 13 \; 14 \\
\hline
1 \\ 2 \\ 3 \\ 4 & & \begin{array}{c} Y_{gg} \\ (N_{pv}+1)\times(N_{pv}+1) \\ 4\times 4 \end{array} & & & & \begin{array}{c} Y_{gn} \\ (N_{pv}+1)\times(N-1) \\ 4\times 10 \end{array} \\
\hdashline
5 \\ \vdots \\ 11 \\ 12 \\ 13 \\ 14 & & \begin{array}{c} Y_{ng} \\ (N-1)\times(N_{pv}+1) \\ 10\times 4 \end{array} & & & & \begin{array}{c} Y_{nn} \\ (N-1)\times(N-1) \\ 10\times 10 \end{array}
\end{array}
$$

$(N_{pv}+N)\times(N_{pv}+N)$

14×14

(5). Identify Y_{gg}, Y_{gr}, Y_{rg} and Y_{rr} from Y_{net}.

$$\underset{4\times4}{Y_{gg}} = Y_{net}\,[1:N_{pv}+1,\ 1:N_{pv}+1]$$

$$\underset{4\times10}{Y_{gr}} = Y_{net}\,[1:N_{pv}+1,\ N_{pv}+2:\text{end}]$$

$$\underset{10\times4}{Y_{rg}} = Y_{net}\,[N_{pv}+2:\text{end},\ 1:N_{pv}+1]$$

$$\underset{10\times10}{Y_{rr}} = Y_{net}\,[N_{pv}+2:\text{end},\ N_{pv}+2:\text{end}].$$

(6) Form Y_{gen}:

$$\underset{4\times4}{Y_{gen}} = \underset{4\times4}{Y_{gg}} - \underset{4\times10}{Y_{gr}}\ \underset{10\times10}{Y_{rr}^{-1}}\ \underset{10\times4}{Y_{rg}}$$

(7) Now we can form the Type II model:

$\forall$ buses $i = 1$ to $N_{pv}+1$

$$\underset{\cancel{4\times1}\ 3\times1}{\dot{\theta}_i} = (\omega_i - 1)\,\omega_s$$

$$\underset{\cancel{4\times1}\ 3\times1}{\dot{\omega}_i} = \frac{1}{2H_i}\left\{P_{m_i} - P_{a_i} - K_{D_i}(\omega_i - 1)\right\}$$

$$\underset{\cancel{4\times1}\ 3\times1}{\dot{E}'_{q_i}} = \frac{1}{T'_{do_i}}\left\{E'_{q_i} - (x_{d_i} - x'_i)\,I_{d_i} + E_{fd_i}\right\}$$

$$\underset{\cancel{4\times1}\ 3\times1}{\dot{E}'_{d_i}} = \frac{1}{T'_{qo_i}}\left\{-E'_{d_i} + (x_{q_i} - x'_i)\,I_{q_i}\right\}$$

where

$$P_{a_i} = \sum_{k=1}^{N_{pv}+1} Y_{gen,ik}\,E'_k E'_i \cos(\gamma_{ik} + \gamma_k - \gamma_i)$$

where $E'_i = \sqrt{E'^2_{q_i} + E'^2_{d_i}}$

$$\gamma_i = \tan^{-1}\left(\frac{E'_{q_i}}{E'_{d_i}}\right)$$

Note: Don't confuse internal voltage angle γ_i / γ_k with the $Y_{gen,ik}$ angle γ_{ik}.

$$I_{d_i} = -\sum_{k=1}^{N_{pv}+1} Y_{gen,ik}\,E'_k \sin(\gamma_k + \gamma_{ik} - \theta_i)$$

$$I_{q_i} = \sum_{k=1}^{N_{pv}+1} Y_{gen,ik}\,E'_k \cos(\gamma_{ik} + \gamma_k - \theta_i)$$

$$x'_i = \frac{x'_{q_i} + x'_{d_i}}{2}$$

(8) We may insert the other known algebraic variables into the differential equations to get the final set of equations: 2.4

$$\boxed{\underset{3q\times 1}{\dot{\theta}_i} = (\omega_i - 1)\,\omega_s}$$

$i = 1$ to $N_{PV}+1$

$$\underset{3q\times 1}{\dot{\omega}_i} = \frac{1}{2H_i}\left[P_{m_i} - \left\{\sum_{k=1}^{N_{PV}+1} Y_{egen\,ik}\left(\sqrt{E_{q_k}'^2 + E_{d_k}'^2}\right)\left(\sqrt{E_{q_i}'^2 + E_{d_i}'^2}\right)\cos\left(\gamma_{ik} + \tan^{-1}\left(\frac{E_{q_k}'}{E_{d_k}'}\right) - \tan^{-1}\left(\frac{E_{q_i}'}{E_{d_i}'}\right)\right)\right\} - K_{D_i}(\omega_i - 1)\right]$$

$$\underset{3q\times 1}{\dot{E}_{q_i}'} = \frac{1}{T_{do_i}'}\left[E_{q_i}' - (x_{d_i} - x_i')\left\{-\sum_{k=1}^{N_{PV}+1} Y_{egen\,ik}\sqrt{E_{q_k}'^2 + E_{d_k}'^2}\,\sin\left(\gamma_{ik} + \tan^{-1}\left(\frac{E_{q_k}'}{E_{d_k}'}\right) - \theta_i\right)\right\} + E_{fd_i}\right]$$

$$\underset{3q\times 1}{\dot{E}_{d_i}'} = \frac{1}{T_{qo_i}'}\left[-E_{d_i}' + (x_{q_i} - x_i')\left\{\sum_{k=1}^{N_{PV}+1} Y_{egen\,ik}\sqrt{E_{q_k}'^2 + E_{d_k}'^2}\,\cos\left(\gamma_{ik} + \tan^{-1}\left(\frac{E_{q_k}'}{E_{d_k}'}\right) - \theta_i\right)\right\}\right]$$

$$x_i' = \frac{\overbrace{x_{q_i}' + x_{d_i}'}^{\text{original values.}}}{2}$$

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