**Problem 1**

A random variable $X$ has probability density function $f_X(x) = 0.2 + 0.2\delta(x)$, for $-2 \leq x \leq 2$. Now consider $Y = g(x)$, where the function $g()$ is defined as follows: $g(x) = x^2$ for $-1 \leq x \leq 1$ and $g(x) = 1$ otherwise. Please find the pdf of $Y$ and also the expected value of $Y$.

**Problem 2**

Let $X$ and $Y$ be jointly Gaussian with $m_x = m_y = 0, \sigma_x^2 = \sigma_y^2 = 4$, and $r = 0.5$.

a. Please write down the joint PDf of $X$ and $Y$.

b. Two other random variable $Z$ and $W$ are computed from $X$ and $Y$ as follows: $\begin{bmatrix} Z \\ W \end{bmatrix} = \begin{bmatrix} a & 0 \\ b & c \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$. Please design the constants $a$, $b$ and $c$ so that $Z$ and $W$ are independent, identically distributed Gaussian random variables with mean 0 and variance 1

**Problem 3**

A random variable $X$ is uniformly distributed on [0, 1]. Unfortunataly, the random variable $X$ cannot be measured directly. Instend, a noisy mensurement $Y$ is taken of the random variable. Specifically, given that $X = x$, the mensurement $Y$ is a uniformly distributed on $[0, x^2]$. Please answer the following questions:

a. Please find the joint probability density function of $X$ and $Y$

b. Please find the marginal probability density function for $Y$.

c. Please find the conditional probability density function for $X$, given $Y = y$.

**Problem 4**

Let X and Y be two independent geometrie random variables, with pmfs $p_X(x) = (0.1)(0.9^x)$ for $x = 0, 1, 2, \ldots..$, and $p_Y(y) = (0.1)(0.9^y)$ for $y = 0, 1, 2, \ldots.$, Let $Z = X + Y$

a. What are the variance of Z and the covariance of $X$ and $Z$?

b. Find the pmf for $Z$.

c. Please find the conditional pmf of $X$ given $Z = z$

**Problem 5**

$X$ and $Y$ are independent identically distributed exponential random variables with parameter $\lambda$ $= 1$. Please find the pdf of $Z = \max(X, Y)$.

**Problem 6**

Consider an experiment where a dart is thrown at a circular dartboard of radius 1, and is equally likely to land anywhere on the dartboard. We define $R$ to be the distance of the dart from the center of the board. Also, we define the random process $X(t), t \in R^+$, as $X(t) = e^{Rt}$, Please find the first-order pdf of $X(t)$.

**Problem 7**

How can a moment-generating function be used to find moments?