Power-flow Methods

Newton-Raphson Algorithm

-Theory-

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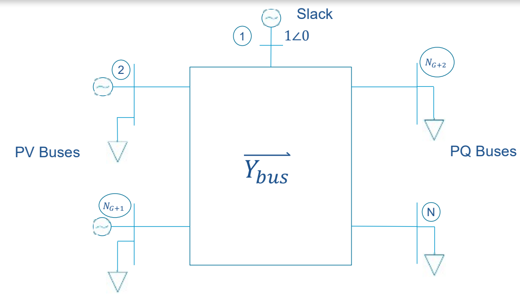
Power-Flow Problem



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DC Method Summary

$$V_i \approx 1$$

$$B_{DC} = -Imag(\overrightarrow{Y_{bus}})_{\substack{2:N\\2:N}}$$

$$\delta_{DC} = \begin{bmatrix} \delta_2 \\ \vdots \\ \delta_N \end{bmatrix} = {B_{DC}}^{-1} P$$

$$P = \begin{bmatrix} P_2 \\ \vdots \\ P_N \end{bmatrix} = \begin{bmatrix} P_{G2} - P_{L2} \\ \vdots \\ P_{GN} - P_{LN} \end{bmatrix}$$

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DC Method

⇒Easy to solve / Fast solution

⇒Good approximation for lightly loaded

⇒Poor approximation under mid/heavy loaded

⇒Cannot detect static limits ⇒ Misleading

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Newton-Raphson Method

$$g(x) = 0$$

$$x \in \mathbb{R}^N$$

$$g: \mathbb{R}^N \to \mathbb{R}^N$$

N nonlinear equations in N unknowns.

$$x^o \Rightarrow Initial\ guess$$

$$\Downarrow k = 0$$

$$g(x) = g(x^k) + \frac{\partial g}{\partial x}|_{x^k}(x - x^k) + \cancel{\boxed{h.o.t}}$$

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NR Derivation

$$0 \approx g(x^k) + J^k(x - x^k)$$

$$J^k = \frac{\partial g}{\partial x}|_{x^k}$$

$$\Delta x^k = (x - x^k)$$

$$x - x^k = -(J^k)^{-1} g(x^k)$$

$$x^{k+1} = x^k - (J^k)^{-1} g(x^k)$$

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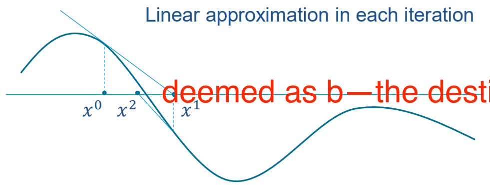
Convergence

Graphical Illustration:

Linear approximation in each iteration



Converges very quickly starting from good initial conditions

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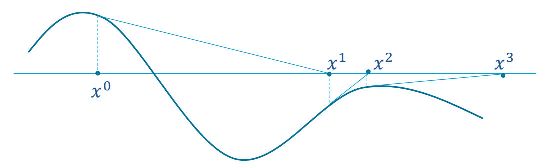
Divergence



Diverges very quickly for poor initial conditions

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NR Algorithm Flow-Chart

$$g(x) = 0$$

$$x^0 \Downarrow k = 0$$

$$Evaluate\ \ g(x^k)$$

$$\Downarrow$$

$$|g(x^k)| < \varepsilon \overset{Ye}{\Longrightarrow} Stop$$

$$\Downarrow No$$

$k = k + 1$

$$Evaluate\ J^k = \frac{\partial g}{\partial x}|_{x^k}$$

$$\Downarrow$$

$$\Delta x^k = -(J^k)^{-1} g(x^k)$$

$$x^{k+1} = x^k + \Delta x^k$$

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Example 1

$$g(x) = \sin x = 0$$

$$\frac{dg}{dx} = \cos x$$

$x^0 = 0.1$ (good initial condition)

$$\Longrightarrow g(x^0) = \sin 0.1 = 0.0998$$

$$\frac{dg}{dx}|_{x^0} = J^0 = 0.995$$

$$\Longrightarrow x^1 - x^0 = -(J^0)^{-1} g(x^0) = -0.1003$$

$$x^1 = 0.1 - 0.1003 = -0.0003 = 3.3 \times 10^{-4}$$

Converges very quickly

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Divergence Example

$x^0 = 1.5$ (poor initial condition)

$$\Longrightarrow g(x^0) = \sin 1.5 = 0.9975$$

$$\frac{dg}{dx}|_{x^0} = J^0 = 0.0707$$

$$\Longrightarrow x^1 - x^0 = -(J^0)^{-1} g(x^0) = -\frac{0.9975}{0.0707}$$

$$x^1 = 1.5 - 14.1089 = -12.6089$$

Diverges very quickly

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NR Algorithm Flow-Chart

$$g(x) = 0$$

$$x^0 \Downarrow k = 0$$

$$Evaluate\ \ g(x^k)$$

$$\Downarrow$$

$$|g(x^k)| < \varepsilon \overset{Yes}{\Longrightarrow} Stop$$

$$\Downarrow No$$

$k = k + 1$

$$Evaluate\ J^k = \frac{\partial g}{\partial x}|_{x^k}$$

$$\Downarrow$$

$$\Delta x^k = -(J^k)^{-1} g(x^k)$$

$$x^{k+1} = x^k + \Delta x^k$$

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Example 2

$$g_1(x_1, x_2) = {x_1}^2 + x_1 x_2 - 1 = 0$$
$$g_2(x_1, x_2) = x_1 x_2 + {x_2}^2 - 2 = 0$$

$$\frac{\partial g}{\partial x} = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} \\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 2x_1 + x_2 & x_1 \\ x_2 & 2x_2 + x_1 \end{bmatrix}$$

$$\begin{bmatrix} {x_1}^0 \\ {x_2}^0 \end{bmatrix} = \begin{bmatrix} 1.1 \\ 0.9 \end{bmatrix}, \varepsilon = 0.01$$

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First Iteration

$$k = 0 \quad \Longrightarrow g(x^0) = \begin{bmatrix} 1.2 \\ -0.2 \end{bmatrix}$$
$$|g(x^0)| = 1.2 > \varepsilon = 0.01$$
$$J^0 = J(x^0) = \begin{bmatrix} 2x_1 + x_2 & x_1 \\ x_2 & 2x_2 + x_1 \end{bmatrix}\Bigg|_{\begin{bmatrix} 1.1 \\ 0.9 \end{bmatrix}}$$
$$= \begin{bmatrix} 3.1 & 1.1 \\ 0.9 & 2.9 \end{bmatrix}$$
$$\Delta x^0 = -(J^0)^{-1} g(x^0) = \begin{bmatrix} -0.4625 \\ 0.2125 \end{bmatrix}$$
$$x^1 = x^0 + \Delta x^0 = \begin{bmatrix} 0.6375 \\ 1.1125 \end{bmatrix}$$

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Second Iteration

$$\Longrightarrow g(x^1) = \begin{bmatrix} 0.1156 \\ -0.0531 \end{bmatrix}$$
$$x^2 = \begin{bmatrix} 0.5779 \\ 1.1542 \end{bmatrix}$$
$$\Longrightarrow g(x^2) = \begin{bmatrix} 0.0011 \\ -0.0007 \end{bmatrix}$$
$$|g(x^2)| = 0.0011 < \varepsilon = 0.01 \Longrightarrow Stop$$
$$x^2 = \begin{bmatrix} 0.5779 \\ 1.1542 \end{bmatrix} \approx \begin{bmatrix} 1/\sqrt{3} \\ 2/\sqrt{3} \end{bmatrix}$$

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NR Algorithm Flow-Chart

$$g(x) = 0$$

$$x^0 \Downarrow k = 0$$
$$Evaluate \;\; g(x^k)$$
$$\Downarrow$$
$$|g(x^k)| < \varepsilon \overset{Yes}{\Longrightarrow} Stop$$
$$\Downarrow No$$
$$Evaluate \; J^k = \frac{\partial g}{\partial x}\Big|_{x^k}$$
$$\Downarrow$$
$$\Delta x^k = -\left(J^k\right)^{-1} g(x^k)$$
$$x^{k+1} = x^k + \Delta x^k$$

$k = k + 1$

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