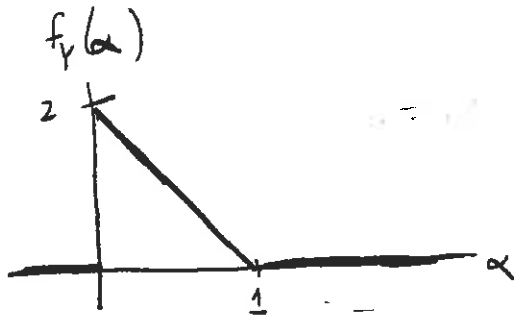


## EE 507, Homework $2\pi + 1$

due November 9

### Problem 8

A random variable  $Y$  has the PDF shown below:



- Please find the maximum probability estimate for  $Y$ .
- Please find the MMSE estimate for  $Y$ .
- Please find the minimum mean absolute error estimate for  $Y$ .

### Problem 9

A random variable  $X$  is uniform on  $[0, 1]$ . Given  $X = x$ , the random variable  $Y$  is exponential with mean  $x^2$ .

- Please find the MMSE estimate for  $Y$  given  $X = x$ . What is the expected error of the estimate, given  $X = x$ ? What is the (overall) average estimation error?
- Please find the MMSE estimate for  $Y^2$  given  $X = x$ . Conceptually, why isn't this estimate just the square of the MMSE estimate of  $Y$  given  $X = x$ ?
- Please find the MMSE estimate of  $X$  given  $Y = y$ . What is the (overall) average estimation error?
- What is the LMMSE estimate for  $Y$  given  $X = x$ ?

## EE 507, Homework 9

due December 4, 2015

### Problem 10

- What is a random process?
- Why are random processes of interest to scientists and engineers?
- Find a signal in the world around you that can be viewed as a random process. Actually measure a sample trajectory of this process.
- When we defined random processes, we associated entire signals with outcomes of an experiment rather than using a different experiments to define the process values at particular times. What is the advantage of this approach?

### Problem 11

An uncertain experiment has two equally likely outcomes, A and B. A random process  $X(\omega, t)$ ,  $t \in R$ , is defined for this experiment as follows:  $X(A, t) = \cos(t)$  and  $X(B, t) = \sin(t)$ .

- Please find the first-order PDF for  $X(t)$ .
- Please find the second-order PDF for  $X(t)$ .
- Please find the  $n$ th-order PDF for  $X(t)$ . (Assume WLOG that the times in the joint PDF are increasing, i.e.  $t_1 < t_2 < \dots < t_n$ ).
- Please find  $E[X(t)]$  and  $R_{XX}(t_1, t_2)$ .

### Problem 12

Consider a discrete-time random process  $X[k]$ ,  $k = 0, 1, 2, \dots$ , where each  $X[k]$  is an independent random variable that equals 0 with probability 0.6 and equals 1 with probability 0.4.

- Please find the  $n$ th-order PMF for  $X[k]$ . (Assume without loss of generality that the times in the joint PMF are increasing.)

b. Please find  $E[X(t)]$ ,  $R_{XX}(t, \tau)$ , and  $C_{XX}(t, \tau)$ .

Now consider the process  $Y[k] = \sum_{i=0}^{k-1} X[i]$ ,  $k = 1, 2, 3, \dots$

c. Please find the first-order PMF for  $Y[k]$ .

d. Please find the  $n$ th-order PMF for  $Y[k]$ . (Assume without loss of generality that the times in the joint PMF are increasing.)

e. Please find  $E[Y(t)]$ ,  $R_{YY}(t, \tau)$ , and  $C_{YY}(t, \tau)$ .

f. Please find  $R_{XY}(t, \tau)$ .

## Problem 13

Let  $T$  be an exponential random variable with mean 1. We define a random process  $X(t)$ ,  $t \in R^+$ , as follows:  $X(t) = 1$  for  $t \leq T$  and  $X(t) = 0$  for  $t \geq T$ . Please find the second-order joint PDF of  $X(t)$ .