Consider the following scenario

(a) $f : \mathbb{R}^n \to \mathbb{R}$, $x_0 \in \mathbb{R}^n$.

(b) A Global minimizer exists in $B(x_0, r)$.

(c) For some $\delta > 0$, define $G(x_0; \delta, r) = \{ y = x_0 + a\delta \mid a \in \mathbb{Z}^n, y \in B(x_0, r) \}$.

Some $\bar{x} \in G$ is sufficiently optimal.

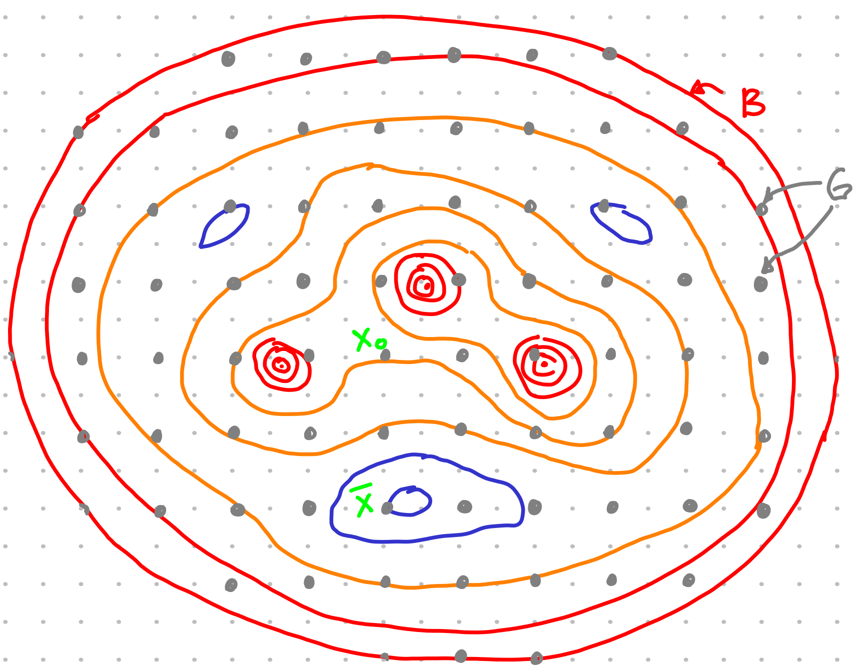
Example:

$$\min f(x) = \frac{1}{2}\|x - p\|^2 - \frac{\mu}{2m}\sum_{k=1}^{m} \ln \|x - p_k\|^2 \quad , \quad p = \frac{1}{m}\sum_{k=1}^{m} p_k .$$

desire to choose $x \in \mathbb{Z}^n$. We can shift coordinates so that $p = \vec{0}$.

Then we have, for $x_0 = \vec{0}$, a global minimizer exists in $B(x_0, r)$ for some large enough r.

$$G(0; 1, r) = \{ y \in \mathbb{Z}^n \mid \|y\| < r \}$$



$\approx \left(\frac{2r}{\delta}\right)^n$ points in G

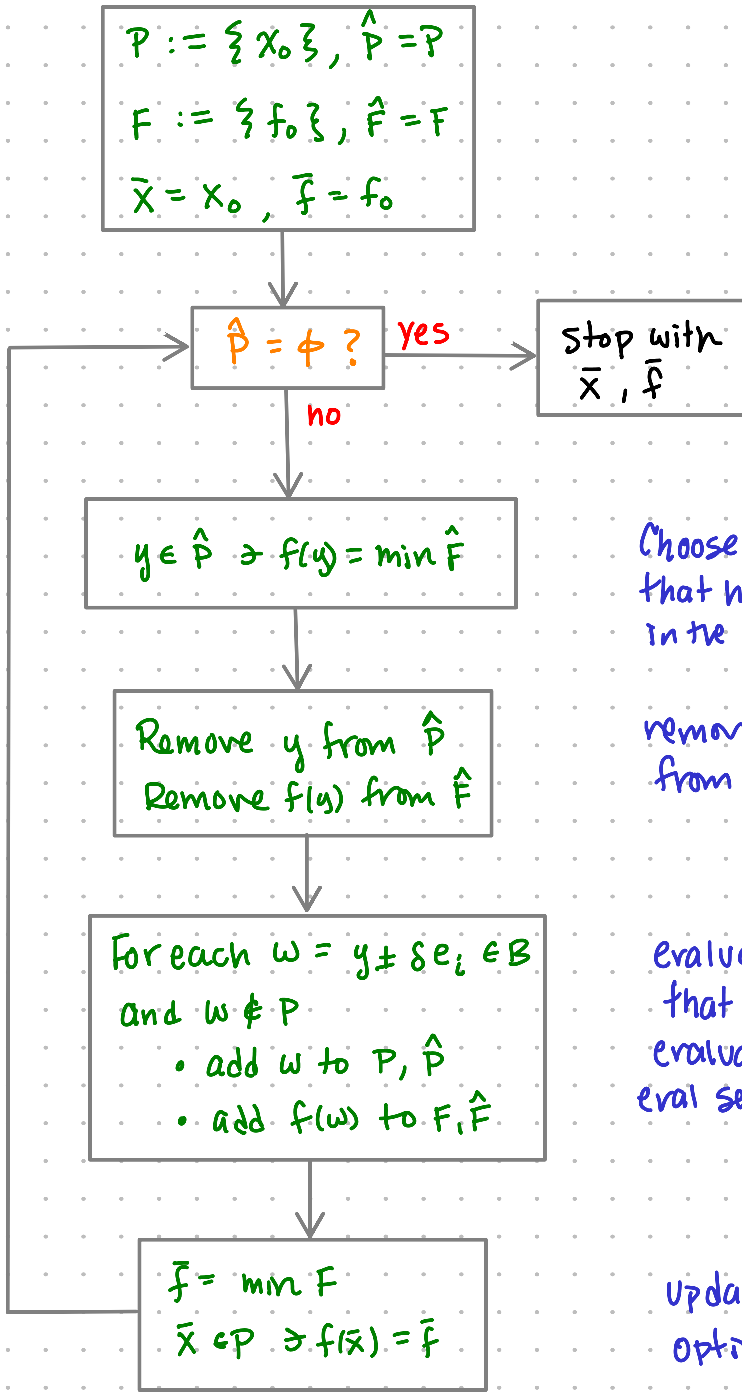
An exhaustive sampling of G yields $\bar{x}$.

How can we efficiently search G so that at any given time we have a good estimate of $\bar{x}$?

Given : $f: \mathbb{R}^n \to \mathbb{R}$, $x_0 \in \mathbb{R}^n$

r such that $\| x^* - x_0 \| < r$

$r > \delta > 0$.



Choose any point in leaf set that has the minimum objective in the leaf set.

remove the chosen point from the leaf set

evaluate all neighbor points that have not already been evaluated and add to the eval set and leaf set.

Update the current optimal point.