

# EE 521/ECE 582 – Analysis of Power systems

Class #15 – October 18, 2022

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## Reminders

- Office Hours on Zoom or in Pullman
  - Tuesday (EME 35 or Zoom) – 5-5:30 pm
  - Friday (EME 35 or Zoom) – 1:30-2:30 pm
- Check in Meetings
  - Tuesday 4:30-5 pm
  - Wednesday 4-5 pm (during Office Hours)
- Discussion Set #2

# Discussion Set #2

Assignment #2	Column A: Summarize (150 to 200 word summary of article)	Column B: List of three questions for class to answer?	Column C: Answer the Questions or Add to Discussions (Each response should be between 25 and 100 words)
See Canvas for Papers	Due: 10/18	Due: 10/21	Due: 10/28
Paper #1	Liadi Akande	Noah Allison	All students in class should respond to three of these papers.
Paper #2	Jacob Hastings	Sajjad Uddin Mahmud	All students in class should respond to three of these papers.
Paper #3	Asif Iftekhar Omi	Md Samiul Islam Sagar	All students in class should respond to three of these papers.
Paper #4	Sumanjali Pannala	Saeed Salimi Amiri	All students in class should respond to three of these papers.
Paper #5	Md Mehedi Hasan Tanim	Leonardo Stingini	All students in class should respond to three of these papers.
Paper #6	Hassan Yazdani	Ke Wang	All students in class should respond to three of these papers.

[discussion 2 paper1 GPU-Based Sparse Power Flow Studies With Modified Newtons Method.pdf](#)

[discussion 2 paper2 The continuation power flow a tool for steady state voltage stability analysis.pdf](#)

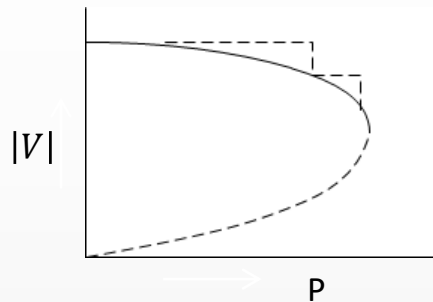
[discussion 2 paper3 Continuation three phase power flow a tool for voltage stability analysis of unbalanced three-phase power systems.pdf](#)

[discussion 2 paper 4 Grid Influences From Reactive Power Flow of Photovoltaic Inverters With a Power Factor Specification of One.pdf](#)

[discussion 2 paper 5 Intergrid A Future Electronic Energy Network.pdf](#)

[Discussion 2 paper 6 Successive-Intersection-Approximation-Based Power Flow Method for Integrated Transmission and Distribution Networks.pdf](#)

## Continuation Power Flow



PV curve  
Nose curve

$$\underline{K} = \underline{f}(\underline{\delta}, \underline{V})$$

$$\text{Let } \lambda \underline{K} - \underline{f}(\underline{\delta}, \underline{V}) = \underline{0}$$

where  $\underline{K}$  is loading profile  $\left| \frac{P}{Q} \right|$

$\lambda$  is loading parameter

$$F(\lambda, \underline{\delta}, \underline{V}) = \underline{0}$$

$$\frac{\partial F}{\partial \delta} d\delta + \frac{\partial F}{\partial V} dV + \frac{\partial F}{\partial \lambda} d\lambda = 0$$

$$\lambda \text{ is an extra unknown, add one more equ } \underbrace{\begin{vmatrix} 0 & \dots & \pm 1 & \dots & 0 \end{vmatrix}}_{e_k} \begin{vmatrix} \frac{d\delta}{dV} \\ \frac{dV}{d\lambda} \end{vmatrix} = 1$$

When  $d\lambda + \text{ve}$ , load increasing

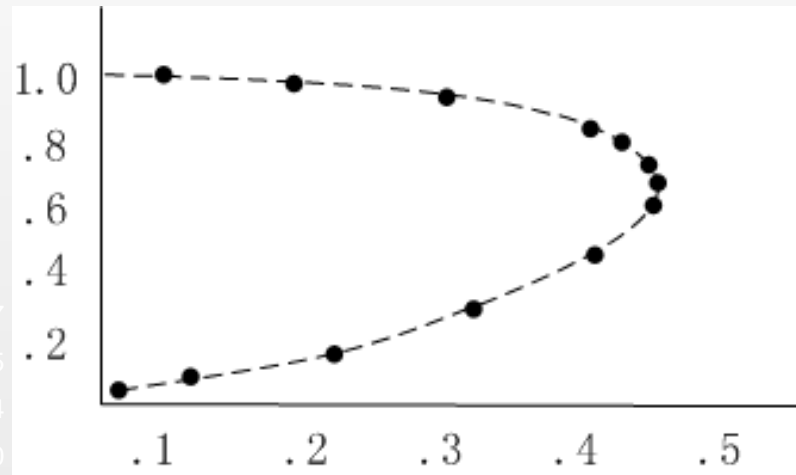
$d\lambda - \text{ve}$ , load decreasing

$dV - \text{ve}$ , voltage decreasing

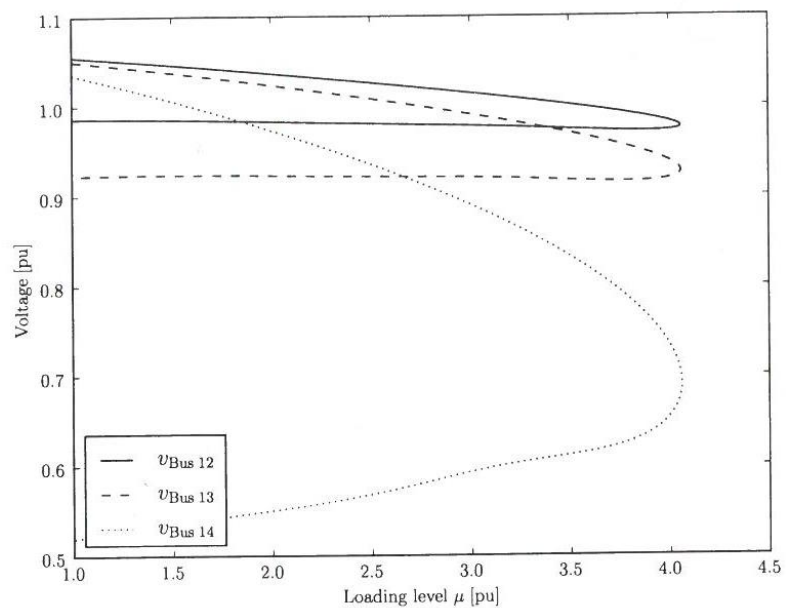
## 2-Bus Example



$\sigma$	$\delta$
.1	-.1017
.1	-.2105
.1	-.3354
.1	-.5050

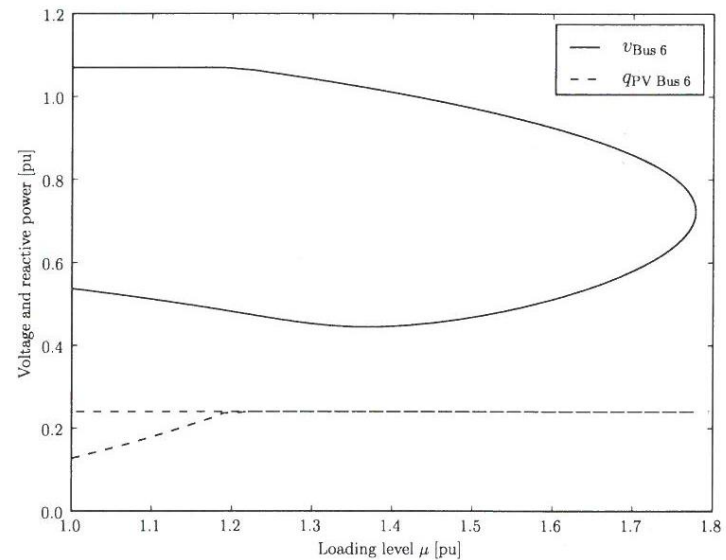
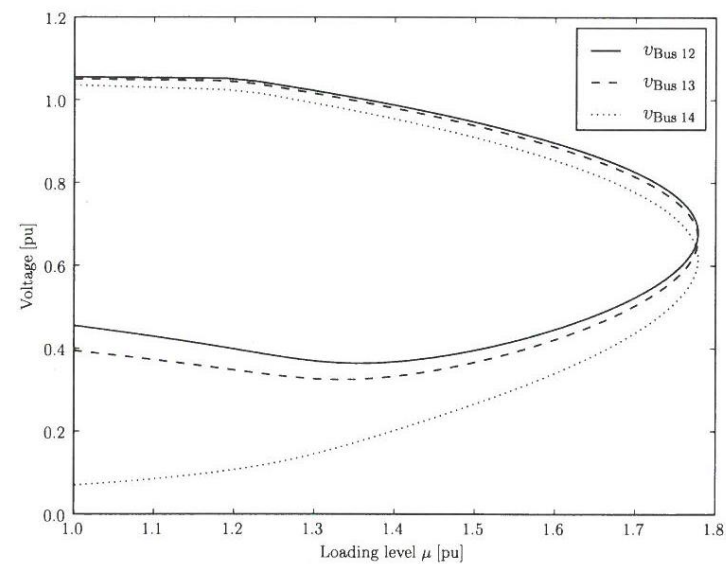


# **What is the solution for the Continuation Power Flow?**



**Fig. 5.9** Nose curve for the IEEE 14-bus system without PV reactive power limits

Federico Milano, Power System Modelling and Scripting, Springer, 2010



**Fig. 5.10** Nose curve for the IEEE 14-bus system enforcing PV generator reactive power limits

## Program #2 – Sparse Matrices Program

- **Full Newton Raphson** -- Take the Jacobian Matrix from Program #1 (with taps) and solve the problem using Sparse Matrix Techniques
- **Fast Decoupled Power Flow** – Use Sparse Matrix Techniques for  $B'$  and  $B''$  and solve. Use Scheme 0 to solve Fast Decoupled Power Flow
- **Extra Credit** -- Fast Decoupled Power Flow – Use Sparse Matrix Techniques for  $B'$  and  $B''$  and solve. Use Scheme 0 to solve Fast Decoupled Power Flow

\*\*\*\* Do not use the SEARCH function as that is not an acceptable method \*\*\*\*



## Program #3 – Continuation Power Flow

- **Develop the Continuation Power Flow** algorithm to determine the P-V curves for all the required buses in the IEEE 14 bus test case with taps. Do not worry about Q limits for this problem.

# Chapter 6

## Optimization

What is optimization? Why optimize?

## Objective

The basic objective of any optimization method is to find the values of the system state variables and/or parameters that minimize some cost function of the system.

Some examples:

Minimize:

- the error between a set of measured and calculated data,
- active power losses,
- the weight of a set of components that comprise the system,
- particulate output (emissions),
- system energy, or
- the distance between actual and desired operating points

## Formulation

Minimize  $f(x, u)$

$$x \in R^n$$

$$u \in R^m$$

Subject to

$$g(x, u) = 0 \text{ (equality constraints)}$$

Power flow equations

$$h(x, u) \leq 0 \text{ (inequality constraints)}$$

Transmission line limits,  $Q_{\text{gen}}$  limits, etc

where  $x$  is the vector of system states and  $u$  is the vector of system parameters.

PQ bus voltage magnitudes and phase

PV bus voltage magnitudes,  $P_{\text{gen}}$

What is state estimation? Why do we care about state estimation?

# What types of measurements do we have on the power grid?

- How good are measurements?
- How do they help?
- How can they hurt?

## Least Squares Estimation (LSE)

**State Estimation** is the process of estimating unknown states from measured quantities.

In many systems, more measurements are made than are necessary to uniquely determine the operating point.

Conversely, not all of the states may be available for measurement.

State estimation gives the “best estimate” of the state of the system in spite of uncertain, redundant, and/or conflicting measurements.



Let  $z$  be the set of measured quantities


The measured values of  $z$  may not equal the true values of  $z^{true}$  due to

- measurement errors,
- missing data, or
- intentional misdirection (hacking)

We'd like to

- Identify bad data
- Minimize the error between the true and measured values

$$e = z - z^{true} = z - Ax$$



States of the system

“squared error”

Minimize  $\|e^2\| = e^T \cdot e = \sum_{i=1}^m \left[ z_i - \sum_{j=1}^m a_{ij} x_j \right]^2$

Minimizing the **square of the error** is desired to negate any effects of sign differences between the measured and true values.

➡ Minimize

$$\begin{aligned}
 e^T \cdot e &= (z - Ax)^T (z - Ax) \\
 &= (z^T - x^T A^T)(z - Ax) \\
 &= z^T z - z^T Ax - x^T A^T z + x^T A^T Ax \\
 &= z^T z - 2x^T A^T z + x^T A^T Ax
 \end{aligned}$$

Minimize  $z^T z - 2x^T A^T z + x^T A^T A x$

Take derivative and set equal to zero:

$$\frac{\partial}{\partial x} (z^T z - 2x^T A^T z + x^T A^T A x) = -2A^T z + 2A^T A x = 0$$

Solving for  $x$ :

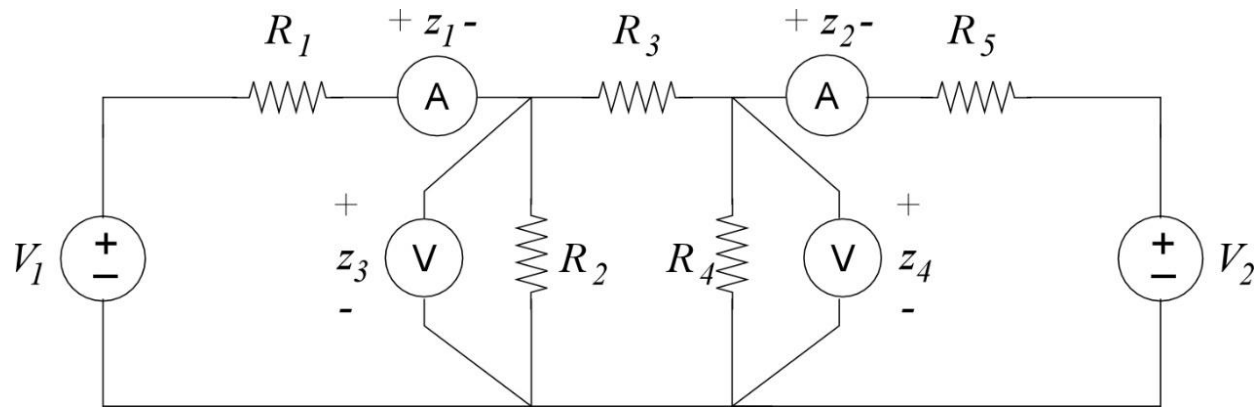
$$A^T A x = A^T z$$

Solve using LU factorization (or other linear solver)

$$\hat{A}x = \hat{b}$$

$$x = (A^T A)^{-1} A^T z$$

Analytic solution



$$R_1 = R_3 = R_5 = 1.5\Omega$$

$$R_2 = R_4 = 1.0\Omega$$

Find the node voltages  
 $V_1$  and  $V_2$

Ammeter 1	$z_1$	4.27 A
Ammeter 2	$z_2$	-1.71 A
Voltmeter 1	$z_3$	3.47 V
Voltmeter 2	$z_4$	2.50 V

Circuit equations

$$-V_1 + R_1 z_1 + z_3 = 0$$

$$-V_2 - R_5 z_2 + z_4 = 0$$

$$z_3 / R_2 - z_1 + (z_3 - z_4) / R_3 = 0$$

$$z_4 / R_4 + z_2 + (z_4 - z_3) / R_3 = 0$$

Yielding:

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} 0.4593 & -0.0593 \\ 0.0593 & -0.4593 \\ 0.3111 & 0.0889 \\ 0.0889 & 0.3111 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Least squares estimation:  $x = (A^T A)^{-1} A^T z$  ← Analytic solution

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \left( \begin{bmatrix} 0.4593 & -0.0593 \\ 0.0593 & -0.4593 \\ 0.3111 & 0.0889 \\ 0.0889 & 0.3111 \end{bmatrix}^T \begin{bmatrix} 0.4593 & -0.0593 \\ 0.0593 & -0.4593 \\ 0.3111 & 0.0889 \\ 0.0889 & 0.3111 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0.4593 & -0.0593 \\ 0.0593 & -0.4593 \\ 0.3111 & 0.0889 \\ 0.0889 & 0.3111 \end{bmatrix}^T \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}$$

Solving for voltages:  $V_1 = 9.8929$   
 $V_2 = 5.0446$

Finding the error:  $e = z - Ax$

$$= \begin{bmatrix} 4.27 \\ -1.71 \\ 3.47 \\ 2.50 \end{bmatrix} - \begin{bmatrix} 0.4593 & -0.0593 \\ 0.0593 & -0.4593 \\ 0.3111 & 0.0889 \\ 0.0889 & 0.3111 \end{bmatrix} \begin{bmatrix} 9.8929 \\ 5.0446 \end{bmatrix}$$

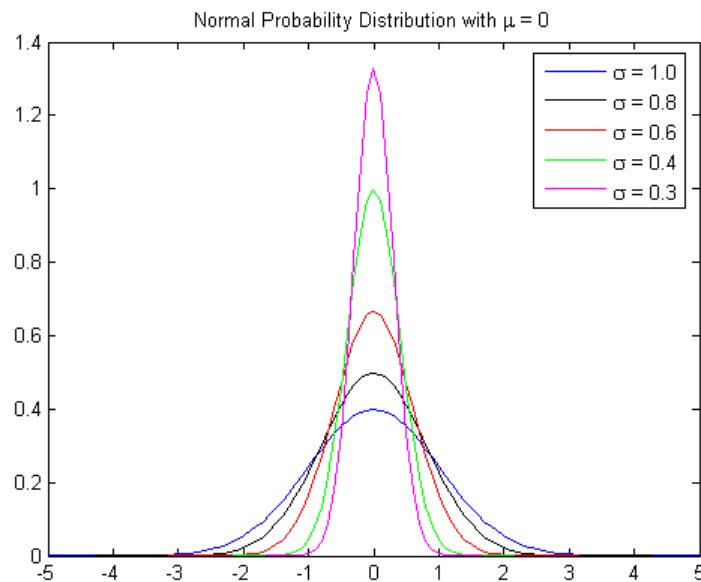
$$= \begin{bmatrix} 0.0255 \\ 0.0205 \\ -0.0562 \\ -0.0512 \end{bmatrix}$$

How do we know if this is a reasonable error?

## Weighted Least Squares Estimation (WLSE)

Suppose we have more confidence in certain types of measurements:

- Certain sensors have been more recently calibrated
- Some types of sensors tend to be more accurate (smaller standard deviation)



We'd like to apply more **weight** to measurements in which there is greater confidence

Introduce a weighting factor:

$$\text{minimize } \|e^2\| = e^T \cdot e = \sum_{i=1}^m w_i \left[ z_i - \sum_{j=1}^m a_{ij} x_j \right]^2$$

In matrix form:

$$A^T W A x = A^T W z \quad \Rightarrow \quad x = (A^T W A)^{-1} A^T W z$$



Let

$$W = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 50 \end{bmatrix}$$

The currents are weighted more heavily than the voltages

Resulting in  $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 9.9153 \\ 5.0263 \end{bmatrix}$

with  $e = \begin{bmatrix} 0.0141 \\ 0.0108 \\ -0.0616 \\ 0.0549 \end{bmatrix}$

original  $V_1 = 9.8929$   
 $V_2 = 5.0446$

Decreased error in current

$$\begin{bmatrix} 0.0255 \\ 0.0205 \\ -0.0562 \\ -0.0512 \end{bmatrix}$$

A plausible weighting matrix that reflects the level of confidence in each measurement set is the inverse of the covariance matrix  $W = R^{-1}$ .

$$W = R^{-1} = \begin{bmatrix} \frac{1}{\sigma_1^2} & 0 & \dots & 0 \\ 0 & \frac{1}{\sigma_2^2} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \frac{1}{\sigma_m^2} \end{bmatrix}$$

Measurements that come from instruments with good consistency (small variance) will carry greater weight than measurements that come from less accurate instruments (high variance).

# Announcements

- Reading Chapter 6
- Remember to start on Discussion Set #2
- Check-in meetings this week
- Start working on Program #3 when done with #1

## Next week

- I'll teach from Vancouver classroom on Tuesday
- Office Hours from Vancouver and via zoom on Tuesday & Wednesday
- Class on next Thursday (10/27) via Zoom or Vancouver classroom (no class in Pullman classroom)