E_E 491
Review Session #10

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Economic Dispatch (Ex. 1)

1. Fuel Costs of two generators are given by

$$f_1 = 4P_{G_1} + 0.03P_{G_1}^2, \qquad 0 \leq P_{G_1} \leq 400$$
$$f_2 = 5P_{G_1} + 0.02P_{G_1}^2, \qquad 50 \leq P_{G_2} \leq 400$$

a) Solve ED when the total demand is 200 MW, 400 MW, 600 MW, and 800 MW, respectively.

b) Suppose a third generator is brought online. Recompute ED for total demand of 200 MW, 600 MW, and 1000 MW.

$$f_3 = 8P_{G_3} + 0.01P_{G_3}^2, \qquad 0 \leq P_{G_3} \leq 400$$

$$\mathcal{L} = f_1 + f_2 + \lambda(P_D - P_{G1} - P_{G2})$$

$$\frac{\partial \mathcal{L}}{\partial P_{G1}} = 4 + 0.06 P_{G1} + \lambda(-1) = 0 \Rightarrow P_{G1} = \frac{\lambda - 4}{0.06} \quad \text{— (1)}$$

$$\frac{\partial \mathcal{L}}{\partial P_{G2}} = 5 + 0.04 P_{G2} + \lambda(-1) = 0 \Rightarrow P_{G2} = \frac{\lambda - 5}{0.04} \quad \text{— (2)}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = P_D - P_{G1} - P_{G2} = 0 \Rightarrow P_{G1} + P_{G2} = P_D$$

$$\frac{\lambda - 4}{0.06} + \frac{\lambda - 5}{0.04} = P_D$$

$$41.667\lambda - 191.667 = P_D$$

$$\lambda = \frac{P_D + 191.667}{41.667} \quad \text{— (3)}$$

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Economic Dispatch (Ex. 1) a)

(i) $P_D = 200$ MW

$$\textcircled{3} \Rightarrow \lambda = \frac{200+191.667}{41.667} = 9.4$$

So P_{u1}, P_{u2} can be calculated as:

$$\textcircled{1} \Rightarrow P_{u1} = \frac{9.4-4}{0.06} = 90 \text{ MW} \qquad \textcircled{2} \Rightarrow P_{u2} = \frac{9.4-5}{0.04} = 110 \text{ MW}$$

$$f_1 = 4(90) + 0.03(90)^2 = 603 \text{ \$/h}$$

$$f_2 = 5(110) + 0.02(110)^2 = 792 \text{ \$/h}$$

Hence,

$$\text{Unit cost} = \frac{f_1+f_2}{P_{u1}+P_{u2}} = \frac{603+792}{90+110} = 6.975 \text{ \$/MWh}$$

Economic Dispatch (Ex. 1) a)

(ii) $P_D = 400\,MW$

② $\Rightarrow \lambda = \dfrac{400 + 191.667}{41.667} = 14.20$

So P_{g1}, P_{g2} can be calculated as:

① $\Rightarrow P_{g1} = \dfrac{14.20 - 4}{0.06} = 170\,MW$

② $\Rightarrow P_{g2} = \dfrac{14.20 - 5}{0.04} = 230\,MW$

$f_1 = 4(170) + 0.03(170)^2 = 1547$ \$/h

$f_2 = 5(230) + 0.02(230)^2 = 2208$ \$/h

Hence,

$$\text{Unit cost} = \frac{f_1 + f_2}{P_1 + P_2} = \frac{1547 + 2208}{400} = 9.3875 \text{ \$/MWh}$$

Economic Dispatch (Ex. 1) a)

(iii) $P_D = 600$ MW (5)

$$② \Rightarrow \lambda = \frac{600 + 191.667}{41.667} = 19$$

So P_{41}, P_{42} can be calculated as:

$$① \Rightarrow P_{41} = \frac{19-4}{0.06} = 250 \text{MW}$$

$$② \Rightarrow P_{42} = \frac{19-5}{0.04} = 350 \text{MW}$$

$$f_1 = 4(250) + 0.03(250)^2 = 2875 \ \$/hr$$

$$f_2 = 5(350) + 0.02(350)^2 = 4200 \ \$/h$$

$$\text{Unit cost} = \frac{f_1 + f_2}{P_1 + P_2} = \frac{2875 + 4200}{600} = 11.79 \ \$/MWh$$

Economic Dispatch (Ex. 1) a)

(iv) $P_D = 800$ MW

$P_{G1} = 400$ MW

$P_{G2} = 400$ MW

$$f_1 = 4(400) + 0.03(400)^2 = 6400 \text{ \$/hr}$$

$$f_2 = 5(400) + 0.02(400)^2 = 5200 \text{ \$/hr}$$

$$\text{Unit cost} = \frac{f_1 + f_2}{P_D} = \frac{6400 + 5200}{800} = 14.5 \text{ \$/MWh}$$

Economic Dispatch (Ex. 1) a)

$$\mathcal{L} = f_1 + f_2 + f_3 + \lambda(P_D - P_{G1} - P_{G2} - P_{G3})$$

$$\frac{\partial \mathcal{L}}{\partial P_{G1}} = 4 + 0.06\,P_{G1} + \lambda(-1) = 0 \quad \Rightarrow \quad P_{G1} = \frac{\lambda - 4}{0.06} \quad \text{—} \; (1)$$

$$\frac{\partial \mathcal{L}}{\partial P_{G2}} = 5 + 0.04\,P_{G2} + \lambda(-1) = 0 \quad \Rightarrow \quad P_{G2} = \frac{\lambda - 5}{0.04} \quad \text{—} \; (2)$$

$$\frac{\partial \mathcal{L}}{\partial P_{G3}} = 8 + 0.02\,P_{G3} + \lambda(-1) = 0 \quad \Rightarrow \quad P_{G3} = \frac{\lambda - 8}{0.02} \quad \text{—} \; (3)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = P_D - P_{G1} - P_{G2} - P_{G3} = 0 \quad \Rightarrow \quad P_{G1} + P_{G2} + P_{G3} = P_D$$

$$\frac{\lambda - 4}{0.06} + \frac{\lambda - 5}{0.04} + \frac{\lambda - 8}{0.02} = P_D$$

$$91.667\lambda - 591.667 = P_D$$

$$\lambda = \frac{P_D + 591.667}{91.667} \quad \text{—} \; (4)$$

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Economic Dispatch (Ex. 1) b)

(i) $P_D = 200$ MW

④ $\Rightarrow$ $\lambda = \dfrac{200 + 591.667}{91.667} = 8.636$

① $\Rightarrow$ $P_{G1} = \dfrac{8.636 - 4}{0.06} = 77.3$ MW

② $\Rightarrow$ $P_{G2} = \dfrac{8.636 - 5}{0.04} = 90.9$ MW

③ $\Rightarrow$ $P_{G3} = \dfrac{8.636 - 8}{0.02} = 31.8$ MW

$$\text{Cost} = f_1 + f_2 + f_3$$
$$= 4(77.3) + 0.03(77.3)^2 + 5(90.9) + 0.02(90.9)^2 + 8(31.8) + 0.01(31.8)^2$$
$$= 1372.7 \ \$/\text{hr}$$

$$\text{Unit cost} = \frac{f_1 + f_2 + f_3}{P_{G1} + P_{G2} + P_{G3}} = \frac{1372.7}{200} = 6.86 \ \$/\text{MWh}$$

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Economic Dispatch (Ex. 1) b)

(ii) $P_D = 600$ MW

$$④ \Rightarrow \quad \lambda = \frac{600 + 591.667}{91.667} = 13$$

$$① \Rightarrow \quad P_{41} = \frac{13-4}{0.06} = 150 \text{ MW}$$

$$② \Rightarrow \quad P_{42} = \frac{13-5}{0.04} = 200 \text{ MW}$$

$$③ \Rightarrow \quad P_{43} = \frac{13-8}{0.02} = 250 \text{ MW}$$

$$\text{Cost} = f_1 + f_2 + f_3 = 5700 \ \$/h$$

$$\text{Unit cost} = \frac{f_1+f_2+f_3}{P_1+P_2+P_3} = 9.5 \ \$/MWh$$

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Economic Dispatch (Ex. 1) b)

(iii) $P_D = 1000$ MW i.e(s)

④ $\Rightarrow \lambda = \dfrac{1000 + 591.667}{91.667} = 17.363$

① $\Rightarrow P_{G1} = \dfrac{17.363 - 4}{0.06} = 222.72$ MW

② $\Rightarrow P_{G2} = \dfrac{17.363 - 5}{0.05} = 309.076$ MW

③ $\Rightarrow P_{G3} = \dfrac{17.363 - 8}{0.02} = 468.15$ MW > 400 MW

Hence, fix $P_{G3} = 400$ MW

and $P_{D1} = P_D - P_{G3} = 1000 - 400 = 600$ MW

$P_{G1} + P_{G2} = P_{D1}$

$\dfrac{\lambda - 4}{0.06} + \dfrac{\lambda - 5}{0.04} = 600$

$41.667\,\lambda - 191.667 = 600.$

$\lambda = 19$

$P_{G1} = \dfrac{19 - 4}{0.06} = 250$ MW

$P_{G2} = \dfrac{19 - 5}{0.05} = 350$ MW

$\text{Cost} = 4(250) + 0.03(250)^2 + 5(350) + 0.02(350)^2 + 8(400) + 0.01(400)^2$

$= 11875$

$\text{Unit cost} = \dfrac{11875}{1000} = 11.875$ \$/MWh

Economic Dispatch (Ex. 1) b)

Economic Dispatch (Ex. 2)

2. Consider Example 1 with losses

$$P_{Loss} = 0.00025P_{G_1}^2 + 0.0001P_{G_2}^2$$

Assume $\epsilon = 0.1$ MW. Solve ED when the total demand is 300 MW.

Lossless case

$$\mathcal{L} = f_1 + f_2 + \lambda(P_D + \cancel{P_{loss}}^{0} - P_{G1} - P_{G2})$$

$$\frac{\partial \mathcal{L}}{\partial P_{G1}} = 4 + 0.06 P_{G1} + \lambda(-1) = 0 \quad \Rightarrow \quad P_{G1} = \frac{\lambda - 4}{0.06}$$

$$\frac{\partial \mathcal{L}}{\partial P_{G2}} = 5 + 0.04 P_{G2} + \lambda(-1) = 0 \quad \Rightarrow \quad P_{G2} = \frac{\lambda - 5}{0.04}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = P_D + \cancel{P_{loss}}^{0} - P_{G1} - P_{G2} = 0 \quad \Rightarrow \quad P_{G1} + P_{G2} = P_D$$

$$\frac{\lambda - 4}{0.06} + \frac{\lambda - 5}{0.04} = P_D$$

(a) $P_D = 300\,MW$

$$\frac{\lambda - 4}{0.06} + \frac{\lambda - 5}{0.04} = 300$$

$$\lambda^{(1)} = 11.8$$

and $P_{G1}^{(1)} = \frac{\lambda - 4}{0.06} = 130\,MW$

$P_{G2}^{(1)} = \frac{\lambda - 5}{0.04} = 170\,MW$

Economic Dispatch (Ex. 2)

Lossy case

$$\mathcal{L} = f_1 + f_2 + \lambda (P_D + P_{loss} - P_{g1} - P_{g2})$$

$$\frac{\partial \mathcal{L}}{\partial P_{g1}} = 4 + 0.06 P_{g1} + \lambda (0.0005 P_{g1} - 1) = 0 \quad \Rightarrow \quad P_{g1} = \frac{\lambda - 4}{0.06 + 0.0005\lambda}$$

$$\frac{\partial \mathcal{L}}{\partial P_{g2}} = 5 + 0.04 P_{g2} + \lambda (0.0002 P_{g2} - 1) = 0 \quad \Rightarrow \quad P_{g2} = \frac{\lambda - 5}{0.04 + 0.0002\lambda}$$

Iteration -1

Assume $\lambda^{(-1)} = 12.2$

$$P_{g1}^{(-1)} = \frac{\lambda^{(-1)} - 4}{0.0005\lambda + 0.06} = 124.0545 \text{ MW}$$

$$P_{g2}^{(-1)} = \frac{\lambda^{(-1)} - 5}{0.0002\lambda + 0.04} = 169.6513 \text{ MW}$$

Economic Dispatch (Ex. 2)

Economic Dispatch (Ex. 2)

Iteration 0

$$\lambda^{(0)} = 11.8$$

$$P_{g_1}^{(0)} = 130 \text{ MW}$$

$$P_{g_2}^{(0)} = 170 \text{ MW}$$

$$P_{loss}^{(0)} = 0.00025\,(130)^2 + 0.0001\,(170)^2 = 7.115$$

$$\Delta P^{(0)} = P_D + P_{loss}^{(0)} - P_{g_1}^{(0)} - P_{g_2}^{(0)} = 7.115$$

$$\Delta\lambda^{(0)} = \frac{\lambda^{(0)} - \lambda^{(-1)}}{\sum P_{g_i}^{(0)} - \sum P_{g_i}^{(-1)}}\left[\Delta P^{(0)}\right] = \frac{11.8 - 12.2}{300 - 124.0545 - 169.6513} \times 7.115 = -0.4522$$

Iteration 1

$$\lambda^{(1)} = \lambda^{(0)} + \Delta\lambda^{(0)} = 11.8 - 0.4522 = 11.3478$$

$$P_{g_1}^{(1)} = \frac{\lambda^{(1)} - 4}{0.1005\lambda^{(1)} + 0.06} = 111.8837 \qquad \Sigma P_G^{(1)} = 262.0589$$

$$P_{g_2}^{(1)} = \frac{\lambda^{(1)} - 5}{0.0002\lambda^{(1)} + 0.04} = 150.1752$$

$$P_{loss}^{(1)} = 0.00025(111.8837)^2 + 0.0001(150.1752)^2 = 5.3848$$

$$\Delta P^{(1)} = P_D + P_{loss}^{(1)} - P_{g_1}^{(1)} - P_{g_2}^{(1)} = 43.3258$$

$$\Delta\lambda^{(1)} = \frac{11.3478 - 11.8}{262.0589 - 300} \times 43.3258 = 0.5163$$

Economic Dispatch (Ex. 2)

Iteration 2

$$\lambda^{(2)} = \lambda^{(1)} + \Delta\lambda^{(1)} = 11.8642$$

$$P_{g_1}^{(2)} = \frac{\lambda^{(2)} - 4}{0.0005\lambda^{(2)} + 0.06} = 119.2769$$

$$P_{g_2}^{(2)} = \frac{\lambda^{(2)} - 5}{0.0002\lambda^{(2)} + 0.04} = 161.9946$$

$$\Sigma P_{g_i}^{(2)} = 281.2715$$

$$P_{Loss}^{(2)} = 0.00025(119.2769)^2 + 0.0001(161.9946)^2 = 6.1810$$

$$\Delta P^{(2)} = P_D + P_{Loss}^{(2)} - \Sigma P_{g_i}^{(2)} = 24.9095$$

$$\Delta\lambda^{(2)} = \frac{11.8642 - 11.3478}{281.2715 - 262.0589} \times 24.9095 = 0.6694$$

Economic Dispatch (Ex. 2)

Iteration 3

$$\lambda^{(3)} = \lambda^{(2)} + \Delta\lambda^{(2)} = 12.5336$$

$$P_{g_1}^{(3)} = \frac{\lambda^{(3)} - 4}{0.0005\lambda^{(3)} + 0.06} = 128.7765$$

$$P_{g_2}^{(3)} = \frac{\lambda^{(3)} - 5}{0.0002\lambda^{(3)} + 0.04} = 177.2333$$

$$\Sigma P_{g_i}^{(3)} = 306.0097$$

$$P_{loss}^{(3)} = 0.00025(128.7765)^2 + 0.0001(177.2333)^2 = 7.287$$

$$\Delta P^{(3)} = P_D + P_{loss}^{(3)} - \Sigma P_{g_i}^{(3)} = 1.2773$$

$$\Delta\lambda^{(3)} = \frac{12.5336 - 11.8642}{306.0097 - 281.2715} \times 1.2773 = 0.0346$$

Economic Dispatch (Ex. 2)

Iteration 4

$$\lambda^{(4)} = \lambda^{(3)} + \Delta\lambda^{(3)} = 12.5336 + 0.0346 = 12.5682$$

$$P_{g_1}^{(4)} = \frac{\lambda^{(4)} - 4}{0.0005\lambda^{(4)} + 0.06} = 129.2643$$

$$P_{g_2}^{(4)} = \frac{\lambda^{(4)} - 5}{0.0002\lambda^{(4)} + 0.04} = 178.0175$$

$$\Sigma P_{ui}^{(4)} = 307.2818$$

$$P_{loss}^{(4)} = 0.00025(129.2643)^2 + 0.0001(178.0175)^2 = 7.3463$$

$$\Delta P^{(4)} = P_D + P_{loss}^{(4)} - P_{g_1}^{(4)} - P_{g_2}^{(4)} = 0.0645 < 0.1 \text{ MW}$$

Stop

$$\boxed{\begin{aligned} P_{g_1} &= 129.2643\text{MW} \\ P_{g_2} &= 178.0175\text{MW} \\ P_{loss} &= 7.3463\text{MW} \\ \lambda &= 12.5682 \end{aligned}}$$

Economic Dispatch (Ex. 2)

Questions?