Power-flow Methods

DC Power-Flow Solution

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Background

- Nonlinear power-flow approximated to linear equations for ***Lightly loaded*** conditions
- Direct solution. Easy. Fast. Approximate.
- Gives quick estimate of bus voltage phase angles and power-flows
- Used for real-time calculations such as in power markets
- Initial guess for nonlinear iterative methods

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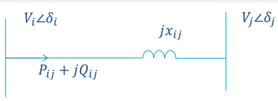
Lossless Transmission Line



$$\vec{I_{ij}} = \frac{V_i\angle\delta_i - V_j\angle\delta_j}{jx_{ij}}$$

$$P_{ij}+jQ_{ij} = V_i\angle\delta_i\left(\frac{V_i}{x_{ij}}\angle(\delta_i - 90°) - \frac{V_j}{x_{ij}}\angle(\delta_j - 90°)\right)^*$$

$$= \frac{{V_i}^2}{x_{ij}}\angle 90° - \frac{V_iV_j}{x_{ij}}\angle(\delta_i-\delta_j+90°)$$

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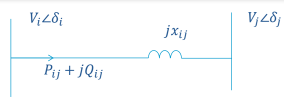
Transmission Line Flows



$$P_{ij}+jQ_{ij} = \frac{{V_i}^2}{x_{ij}}\angle 90° - \frac{V_iV_j}{x_{ij}}\angle(\delta_i-\delta_j+90°)$$

$$P_{ij} = \frac{V_iV_j}{x_{ij}}\sin(\delta_i-\delta_j)$$

$$Q_{ij} = \frac{{V_i}^2}{x_{ij}} - \frac{V_iV_j}{x_{ij}}\cos(\delta_i-\delta_j) = \frac{{V_i}^2 - V_iV_j\cos(\delta_i-\delta_j)}{x_{ij}}$$

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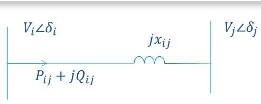
DC Equivalent System

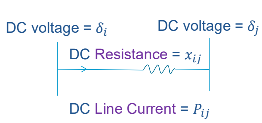
$$P_{ij} = \frac{V_iV_j}{x_{ij}}\sin(\delta_i-\delta_j) \approx \frac{(\delta_i-\delta_j)}{x_{ij}}$$

Q_{ij} neglected



- $V_i \approx 1$
- $\sin(\delta_i-\delta_j) \approx (\delta_i-\delta_j)$

$$P_{ij} = \frac{(\delta_i-\delta_j)}{x_{ij}}$$



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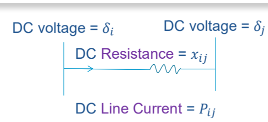
AC Power System to Equivalent DC System

$$P_{ij} = \frac{(\delta_i-\delta_j)}{x_{ij}}$$



AC Power Flow	DC Power Flow
Bus voltage phase angles	Bus voltages
Line MW power-flow	DC line current
Line reactance	Line resistance

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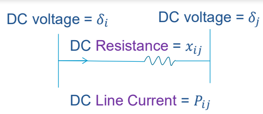
AC Power System to Equivalent DC System

$$P_{ij} = \frac{(\delta_i - \delta_j)}{x_{ij}}$$



AC Power Flow	DC Power Flow
Bus voltage phase angles	Bus voltages
Line MW power-flow	DC line current
Line reactance	Line resistance
Generator	**Current Source**
Slack Bus	**DC Ground**

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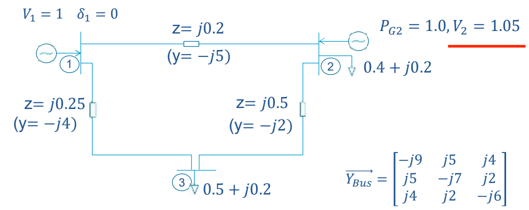
Example 1



$$\overrightarrow{Y_{Bus}} = \begin{bmatrix} -j9 & j5 & j4 \\ j5 & -j7 & j2 \\ j4 & j2 & -j6 \end{bmatrix}$$

$$B_{DC} = -Imag[Y_{Bus}]_{2:3 \atop 2:3} = \begin{bmatrix} 7 & -2 \\ -2 & 6 \end{bmatrix}$$

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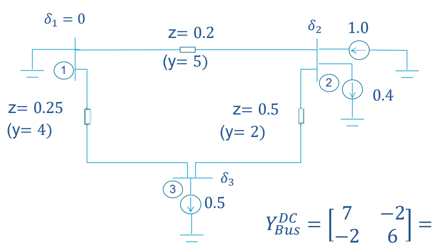
DC Equivalent System



$$Y_{Bus}^{DC} = \begin{bmatrix} 7 & -2 \\ -2 & 6 \end{bmatrix} = B_{DC}$$

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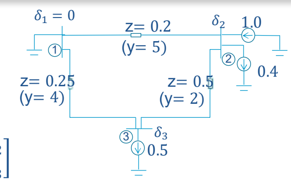
Example 1

$$\begin{bmatrix} P_2 \\ P_3 \end{bmatrix} = B_{DC} \begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix}$$

DC power-flow solution:



$$\begin{bmatrix} 1 - 0.4 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 7 & -2 \\ -2 & 6 \end{bmatrix} \begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix}$$

$$\begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix} = \begin{bmatrix} 7 & -2 \\ -2 & 6 \end{bmatrix}^{-1} \begin{bmatrix} 0.6 \\ -0.5 \end{bmatrix} \Rightarrow \begin{bmatrix} \vec{V_1} \\ \vec{V_2} \\ \vec{V_3} \end{bmatrix} = \begin{bmatrix} 1\angle 0 \\ 1\angle 0.0684 \\ 1\angle -0.0605 \end{bmatrix}$$

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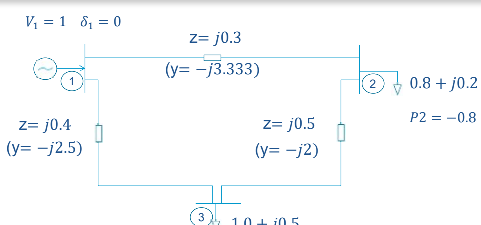
Example 2



$P3 = -1.0$

$$\overrightarrow{Y_{Bus}} = \begin{bmatrix} -j5.833 & j3.333 & j2.5 \\ j3.333 & -j5.333 & j2 \\ j2.5 & j2 & -j4.5 \end{bmatrix}$$

$$B_{DC} = \begin{bmatrix} 5.333 & -2 \\ -2 & 4.5 \end{bmatrix}$$

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Example 2

$$B_{DC} = \begin{bmatrix} 5.333 & -2 \\ -2 & 4.5 \end{bmatrix}$$

$$\begin{bmatrix} P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} -0.8 \\ -1.0 \end{bmatrix} = B_{DC} \begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix}$$

$$\begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix} = \begin{bmatrix} -0.28 \\ -0.3467 \end{bmatrix}$$

DC Power-flow Solution is

$$\begin{bmatrix} \vec{V_1} \\ \vec{V_2} \\ \vec{V_3} \end{bmatrix} = \begin{bmatrix} 1\angle 0 \\ 1\angle -0.28 \\ 1\angle -0.3467 \end{bmatrix}$$

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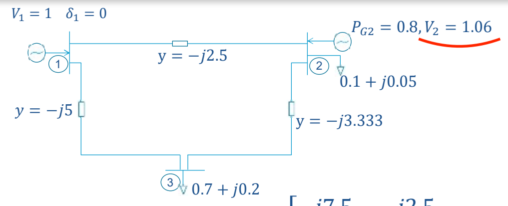
Example 3



$$\overrightarrow{Y_{Bus}} = \begin{bmatrix} -j7.5 & j2.5 & j5 \\ j2.5 & -j5.833 & j3.333 \\ j5 & j3.333 & -j8.333 \end{bmatrix}$$

$$B_{DC} = \begin{bmatrix} 5.833 & -3.333 \\ -3.333 & 8.333 \end{bmatrix}$$

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DC Solution

$$\begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix} = B_{DC}{}^{-1} \begin{bmatrix} P_2 \\ P_3 \end{bmatrix}$$

$$= \begin{bmatrix} 5.833 & -3.333 \\ -3.333 & 8.333 \end{bmatrix}^{-1} \begin{bmatrix} 0.7 \\ -0.7 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0933 \\ -0.0467 \end{bmatrix} rad$$

$$\overrightarrow{V_1} = 1\angle 0, \overrightarrow{V_2} = 1.06\angle 0.0933, \overrightarrow{V_3} = 1\angle -0.0467$$

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Summary

- Nonlinear power-flow approximated to linear equations for ***Lightly loaded*** conditions
- Direct solution. Easy. Fast. Approximate.
- Gives quick estimate of bus voltage phase angles and power-flows
- Approximations invalid for heavily loaded systems
- Does not detect static limits

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