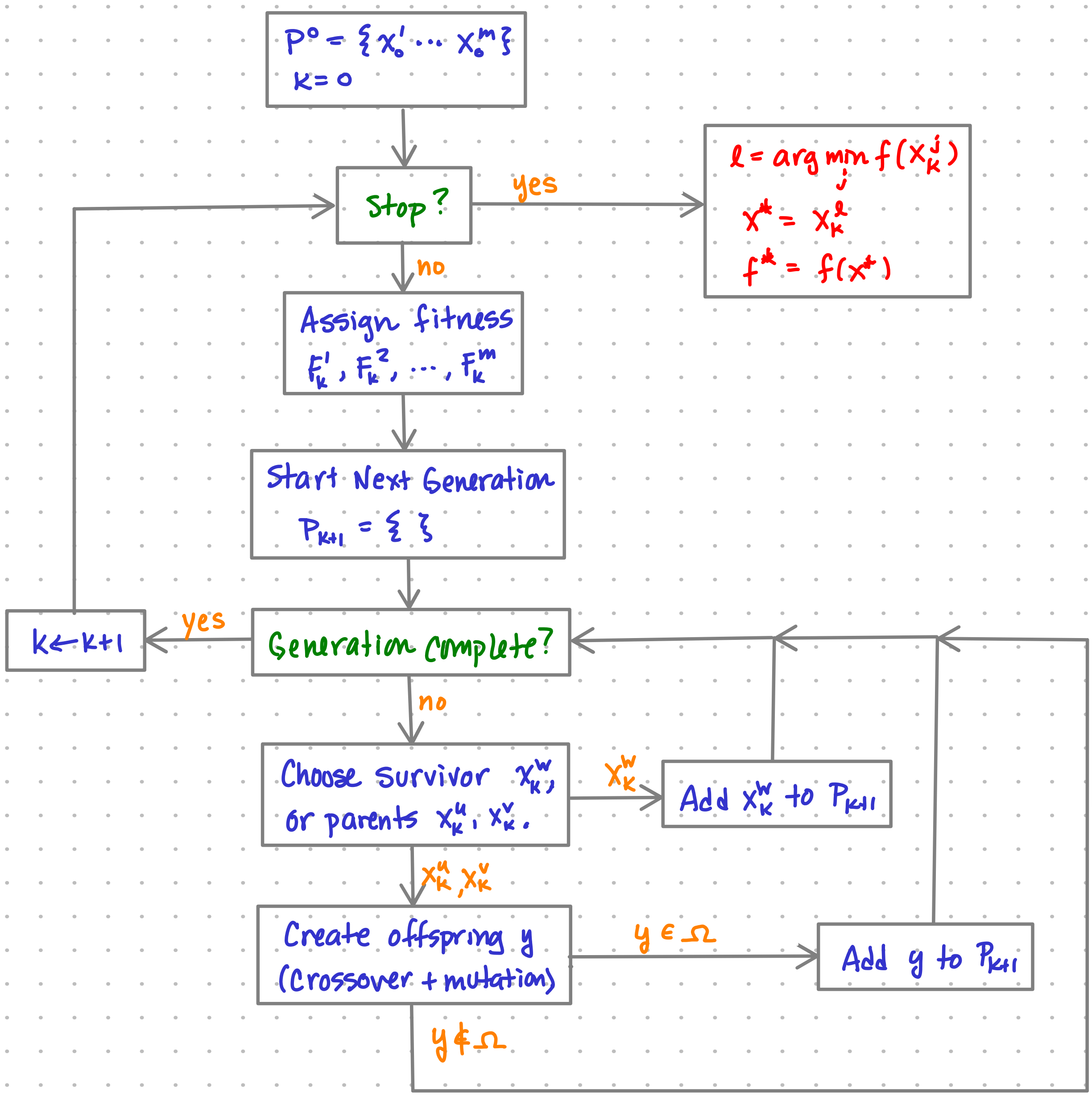
Genetic Algorithms

A GA is a derivative free optimization method whose strength is working to find a global optimum on a bounded domain. The concept is modeled after evolutionary ideas of survival by fitness. Instead of tracking a sequence of best iterates (x_k), GA tracks a sequence of populations $P_k = \{x_k^{(1)}, x_k^{(2)}, \dots, x_k^{(m)}\}$ in which the sequence of best 'persons' is a improving sequence. GA's have the ability to sample, in infinite time, all of the feasible domain through biologically inspired actions of survival, crossover, and mutation. We will consider a basic implementation appropriate for minimization of functions $f: \Omega \to \mathbb{R}$, $\Omega \subset \mathbb{R}^n$ bounded.



This general algorithm does not define several decisions / tasks

(1) How is fitness determined?
(2) How do we choose whether to select a survivor or create an offspring?
(3) How is an offspring defined / created?
(4) How is mutation applied?
(5) When is a population complete?
(6) When do we terminate the algorithm?

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(1) How is fitness determined?

For later use, and in keeping with the biological connection, we consider fitness values which are strictly positive and ordered so that if $f(x_k^u) < f(x_k^v)$ then $F^u > F^v$.

A simple method which determines fitness relative to the current population:

$$f_{max} := \max_i f(x_k^i)$$

$$F_k^i := f_{max} - f(x_k^i) + 1$$

example: $f_k^1 = -3.2,\ f_k^2 = 0.2,\ f_k^3 = 2.6,\ f_k^4 = -1.7,\ f_k^5 = -1.4$

$f_{max} = 2.6$

$F_k = \{6.8\ ; 3.4\ ; 1.0\ ; 5.3\ , 5.0\}$

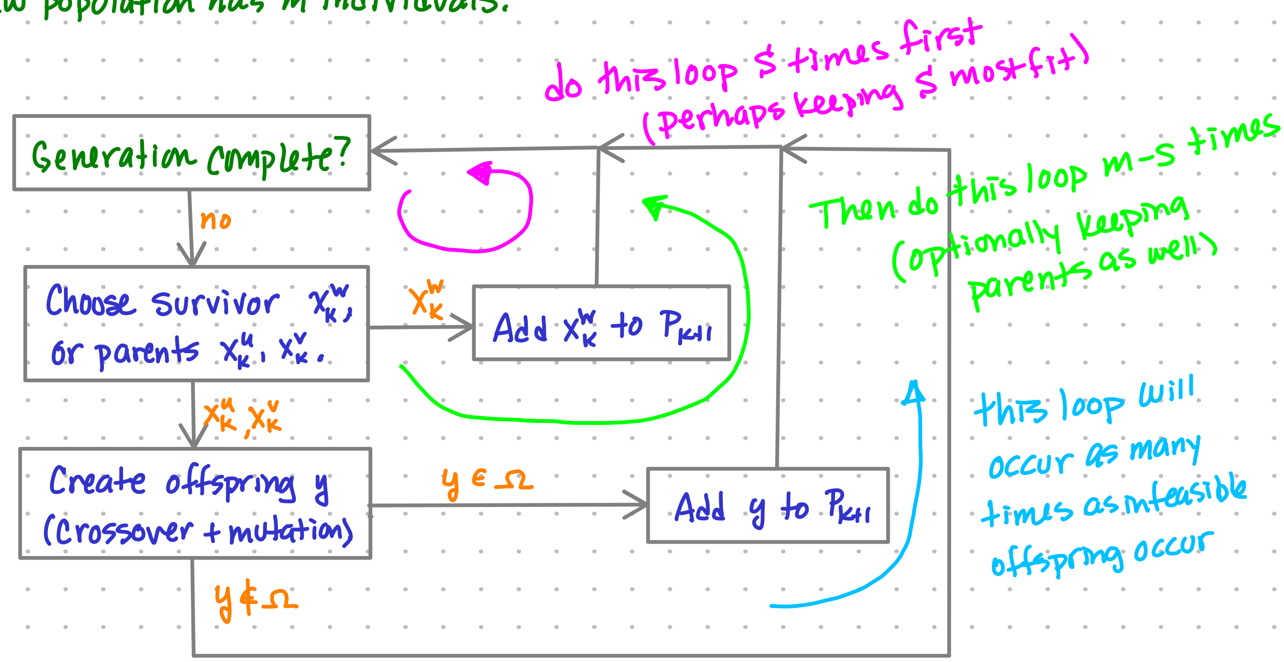
x_k^1 is the most fit (and has best objective value).

② Selecting Survivors and Offspring

Building the next generation population is a combination of survivors and offspring. The only necessary rules are

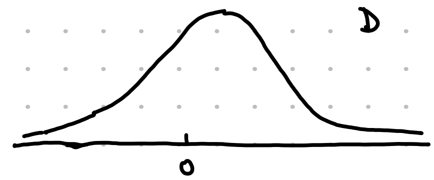
1. The most fit from one generation survives to the next.
2. Every offspring must be feasible.

A simple and popular strategy is to build the next generation starting with $s < m$ survivors (one of which is the most fit) then continue to populate the next generation with offspring of two parents of the current generation, if it survives. Continue until the new population has m individuals.



③ Creating an Offspring
④
⑤ Offspring are created by mixing the properties of parents and allowing for some mutations.

1. Select two parents X_k^u and X_k^v from P_k with probability $f^u / \sum f^i$
2. Set $\theta = F^u / (F^u + F^v)$
3. $y := \theta X_k^u + (1-\theta) X_k^v$. — crossover
4. With probability δ, $y_i \leftarrow y_i + \varepsilon$ — mutation
   where ε is drawn from distribution D.
5. If $y \in \Omega$ it joins population P_{k+1} — survival



⑥ Algorithm Termination

Notice that population fitness is not necessarily monotonic, but the sequence of current best objective value is monotonic. This suggests two reasonable and simple stopping criteria.

1. Stop when a maximum population iteration is reached $k = k_{max}$
2. Stop when the lowest objective value is the same over the previous h iterations.

Hyperparameter list

m	population minimum size
δ	mutation probability
D	mutation probability distribution
k_{max}	maximum allowed number of iterations
h	stagnation limit

other

Dynamic θ?

Let parents survive?

how many mutations per offspring?