State Estimation

Linear State Estimation

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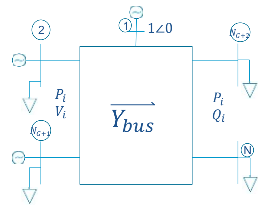
State Estimation

Real-time tool for estimating the voltages, phase angles of a power system



$$\overrightarrow{V_{Bus}} = \begin{bmatrix} V_1\angle\delta_1 \\ \vdots \\ V_N\angle\delta_N \end{bmatrix}$$

$$x = \begin{bmatrix} \delta \\ V \end{bmatrix} \quad \textit{unknown voltages and angles}$$

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State Estimation

- Reality → measure voltages, currents, P_{ij}, Q_{ij} (SCADA system)
  - ➢ Supervisory Control and Data Acquisition System
    - Remote Terminal Units
    - Communication Network
    - Phase angles cannot be measured. Lack of common phase angle reference.

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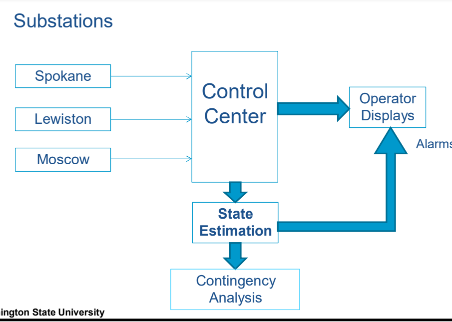
State Estimation



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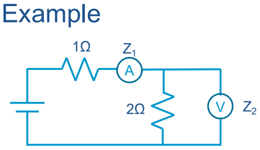
State Estimation

Measurements
- Model errors
- RTU errors

Redundant measurements help get best fit

- Example



Ammeter "twice" as accurate as voltmeter

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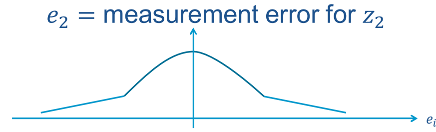
Measurements

$$x_1 = I = state, unknown$$
$$z_1 = I = x_1 + e_1$$

e_1 = measurement error for z_1 = Random variable Gaussian (Normal)

$$z_2 = 2I + e_2 = 2x_1 + e_2$$

e_2 = measurement error for z_2



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Formulation

$$\underline{z} = \underline{H}\,\underline{x} + \underline{e}$$
$$\underline{x} = States\ (known)$$
$$\underline{z} = Measurements$$
$$\underline{H} = Model\ representation$$
$$\underline{e} = Measurement\ errors$$
$$Estimate:\ \hat{x} = ?$$
$$\underline{z} \Longrightarrow Estimation \Longrightarrow \hat{x} = ?$$

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Least Square formulation

Minimize x

$$\sum_{i=1}^{N} \omega_i e_i^2 \Longleftarrow weighted\ total\ error$$
$$= \underline{e}^T \underline{W}\,\underline{e}$$
$$\underline{e} = \underline{z} - \underline{H}\,\underline{x}$$

Minimize w.r.to x

$$(\underline{z} - \underline{H}\,\underline{x})^T \underline{W} (\underline{z} - \underline{H}\,\underline{x}) = g(\underline{x})$$

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Least Square Estimate

$$\frac{\partial g}{\partial \underline{x}} = -2\underline{H}^T \underline{W} (\underline{z} - \underline{H}\,\underline{x}) = 0$$
$$\underline{H}^T \underline{W}\,\underline{z} = \underline{H}^T \underline{W}\,\underline{H}\,\underline{x}$$
$$\boxed{\hat{x} = \underline{G}^{-1} \underline{H}^T \underline{W}\,\underline{z}} \Longleftarrow Least\ Square\ Estimate$$
$$\underline{G} = \underline{H}^T \underline{W}\,\underline{H} = Gain\ Metrix$$

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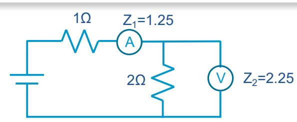
Example



$$\begin{aligned} z_1 &= x_1 + e_1 \\ z_2 &= 2x_1 + e_2 \end{aligned} \Longrightarrow \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} x_1 + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$
$$H = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad W = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\underline{G} = \underline{H}^T \underline{W}\,\underline{H} = [1 \quad 2] \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 6$$

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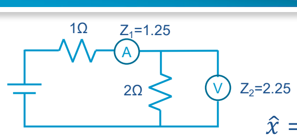
Estimate



$$\hat{x} = \underline{G}^{-1} \underline{H}^T \underline{W}\,\underline{z}$$
$$= \frac{1}{6} [1 \quad 2] \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1.25 \\ 2.25 \end{bmatrix}$$
$$= \frac{1}{6} [2 \quad 2] \begin{bmatrix} 1.25 \\ 2.25 \end{bmatrix}$$
$$= \frac{7}{6} = 1.167$$

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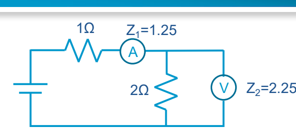
Second case



$$\hat{x} = 1.167$$

Suppose z_1 10 times as accurate as z_2

$$W = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\Longrightarrow \underline{G} = \underline{H}^T \underline{W}\,\underline{H} = [1 \quad 2] \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 14$$

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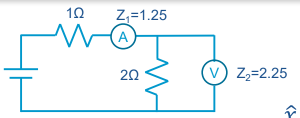
New Estimate



$$\hat{x} = \underline{G}^{-1}\underline{H}^T\underline{W}\,\underline{z}$$
$$= \frac{1}{14}\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}10 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}1.25\\ 2.25\end{bmatrix}$$
$$= \frac{1}{14}\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}12.5\\ 2.25\end{bmatrix}$$
$$= \frac{17}{14} = 1.214$$

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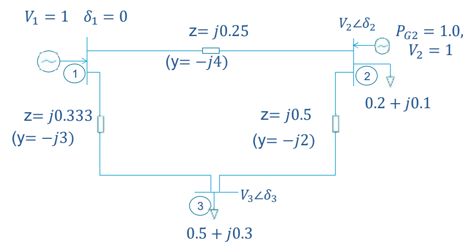
Power System State Estimation

Power System:



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DC Power-flow solution

$$\begin{bmatrix}\delta_2\\ \delta_3\end{bmatrix} = \begin{bmatrix}6 & -2\\ -2 & 5\end{bmatrix}^{-1}\cdot\begin{bmatrix}0.8\\ -0.5\end{bmatrix}$$
$$= \begin{bmatrix}0.1154\\ -0.0538\end{bmatrix}$$

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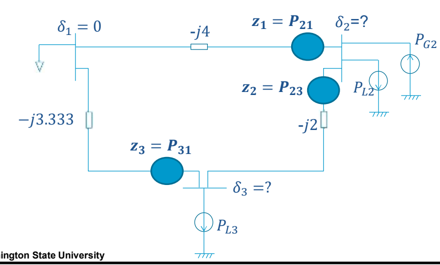
State Estimation

DC powerflow:



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DC Power-flow formulation

$$\delta_2 =? \quad \delta_3 =?$$
$$\underline{x} = \begin{bmatrix}\delta_2\\ \delta_3\end{bmatrix}$$
$$\underline{z_1 = P_{21}} = 4\delta_2 + e_1$$
$$z_2 = P_{23} = 2\delta_2 - 2\delta_3 + e_2$$
$$z_3 = P_{31} = 3\delta_3 + e_3$$
$$\underline{z} = \begin{bmatrix}4 & 0\\ 2 & -2\\ 0 & 3\end{bmatrix}\begin{bmatrix}\delta_2\\ \delta_3\end{bmatrix} + \begin{bmatrix}e_1\\ e_2\\ e_3\end{bmatrix}$$

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DC Power-flow formulation

$$\underline{z} = \begin{bmatrix}4 & 0\\ 2 & -2\\ 0 & 3\end{bmatrix}\begin{bmatrix}\delta_2\\ \delta_3\end{bmatrix} + \begin{bmatrix}e_1\\ e_2\\ e_3\end{bmatrix}$$
$$\begin{bmatrix}\delta_2\\ \delta_3\end{bmatrix} = \begin{bmatrix}0.1154\\ -0.0538\end{bmatrix}$$
$$\underline{z} = \begin{bmatrix}0.4616\\ 0.3384\\ -0.1614\end{bmatrix} + \begin{bmatrix}e_1\\ e_2\\ e_3\end{bmatrix}, \qquad z = \begin{bmatrix}0.45\\ 0.35\\ -0.12\end{bmatrix}$$

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Measurements

$$P_{21} = 0.45$$
$$P_{23} = 0.35$$
$$P_{31} = -0.12$$
$$\underline{W} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$\underline{z} = \begin{bmatrix} 0.45 \\ 0.35 \\ -0.12 \end{bmatrix}$$

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State Estimate

$$\underline{G} = \underline{H}^T \underline{W}\ \underline{H} = \begin{bmatrix} 4 & 2 & 0 \\ 0 & -2 & 3 \end{bmatrix} \underline{W} \begin{bmatrix} 4 & 0 \\ 2 & -2 \\ 0 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 20 & -4 \\ -4 & 13 \end{bmatrix}$$

$$\hat{x} = \underline{G}^{-1} \underline{H}^T \underline{W}\ \underline{z} = \begin{bmatrix} 0.1158 \\ -0.0459 \end{bmatrix} = \begin{bmatrix} \widehat{\delta_2} \\ \widehat{\delta_3} \end{bmatrix}$$

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State Estimation

- LSE Basic Theory Introduced.
- Bad Data Detection – Chi Square tests
- Nonlinear Models in State Estimation
- Topology or Model errors

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