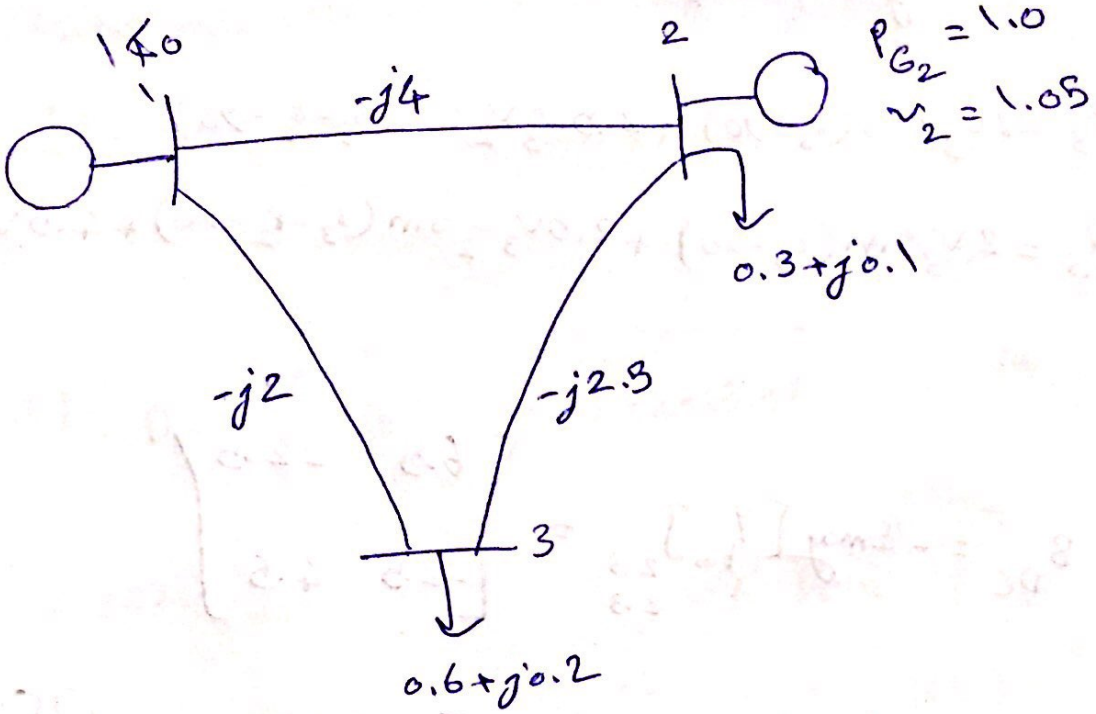
EE 491 - Midterm #1

Fall 2020

1)



a)

$$\vec{Y}_{bus} = \begin{bmatrix} -j6 & j4 & j2 \\ j4 & -j6.5 & j2.5 \\ j2 & j2.5 & -j4.5 \end{bmatrix}$$

b)

	type	known	unknown
1	slack bus	δ, v	P, Q
2	Generator bus	v, P	δ, Q
3	Load bus	P, Q	δ, v

2

c)
$$\begin{cases} P_i = \sum_j v_i v_j y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \\ Q_i = \sum_j v_i v_j y_{ij} \sin(\delta_i - \delta_j - \theta_{ij}) \end{cases}$$

$$\begin{cases} P_3 = 2v_3 \cos(\delta_3 - 90) + 2.5 v_3 v_2 \cos(\delta_3 - \delta_2 - 90) \\ Q_3 = 2v_3 \sin(\delta_3 - 90) + 2.5 v_3 v_2 \sin(\delta_3 - \delta_2 - 90) + 4.5 v_3^2 \end{cases}$$

d)
$$B_{DC} = -\text{Img}[Y_{bus}]_{\substack{2:3 \\ 2:3}} = \begin{bmatrix} 6.5 & -2.5 \\ -2.5 & 4.5 \end{bmatrix}$$

$$\begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix} = B_{DC}^{-1} \begin{bmatrix} P_{G_2} - P_{L_2} \\ -P_{L_3} \end{bmatrix} = \begin{bmatrix} 0.19565 & 0.1087 \\ 0.1087 & 0.28261 \end{bmatrix} \begin{bmatrix} 0.7 \\ -0.6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix} = \begin{bmatrix} 0.071739 \\ -0.093478 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 \\ \vec{v}_3 \end{bmatrix} = \begin{bmatrix} 1 \angle 0 \\ 1.05 \angle 0.071739 \\ 1 \angle -0.093478 \end{bmatrix}$$

3

e) → Line R is negligible compared to line X ($R_L \ll X_L$)

→ The voltage profile is flat ($|V_N| \simeq 1\ pu$)

→ Voltage angle differences between neighboring nodes are small.

($\sin(\delta_i - \delta_j) \approx \delta_i - \delta_j$)

2)

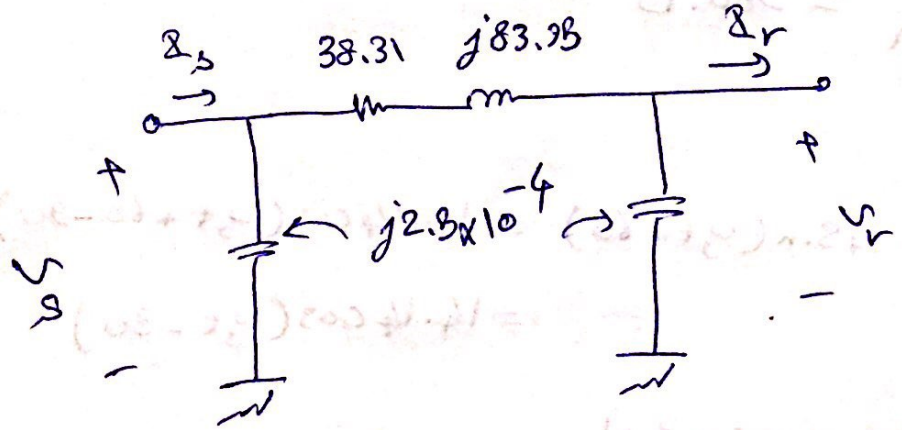
a) $D_{eq} = \sqrt[3]{15.87 \times 15.87 \times 2 \times 15.87} = 20\ ft$

$r = 0.3831\ \Omega/mi \Rightarrow R = 38.31\ \Omega$

$X_L = 0.476 + 0.3633 = 0.8393\ \Omega/mi \Rightarrow X_L = 83.93\ \Omega$

$X_C = 0.1090 + 0.0889 = 0.1979\ M\Omega/mi \Rightarrow Y_C = 5.0531 \times 10^{-4}\ S$

b)



4

c) $A = D = 1 + \frac{YZ}{2} = 0.97884 \angle 0.37^\circ$

$B = Z = 38.31 + j83.95 = 92.278 \angle 65.47^\circ \ \Omega$

$C = Y\left(1 + \frac{YZ}{4}\right) = 5 \times 10^{-4} \angle 90.28^\circ \ S$

d) $\bar{V}_{R,LN} = \frac{\bar{V}_{R,LL}}{\sqrt{3}} = \frac{346.4 \ kV}{\sqrt{3}} = 200 \ kV \angle 0$

$\bar{I}_R = \frac{P}{\sqrt{3} V_{R,LL} \cos\varphi} = \frac{90 \ MW}{\sqrt{3} \times 346.4 \ kV \times 0.9} = 166.67 \ A \angle -\cos^{-1}(0.9)$

$\bar{V}_{S,LN} = A \bar{V}_{R,LN} + B \bar{I}_R = 207.97 \ kV \angle 3.24^\circ$

$\Rightarrow V_{S,LL} = 360.18 \ kV$

3) $v(t) = 14.14 \sin(\omega_s t + 60) = 14.14 \cos(\omega_s t + 60 - 90)$

$= 14.14 \cos(\omega_s t - 30)$

$i(t) = 10 \cos(\omega_s t + 30)$

5

a) $\underline{V} = \frac{14.14}{\sqrt{2}} \angle -30^\circ = 10 \angle -30^\circ$

$\underline{I} = \frac{10}{\sqrt{2}} \angle 30^\circ = 7.07 \angle 30^\circ$

b) $S = \underline{V}\underline{I}^* = (10 \angle -30)\left(\frac{10}{\sqrt{2}} \angle -30\right)$

$= \frac{100}{\sqrt{2}} \angle -60 = 35.355 - j61.237$

c) $P = 35.355$ W $\Rightarrow$ absorbs (sink)

$Q = -61.237$ VAr $\Rightarrow$ delivers (source)