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EE523 Final Exam
5/6/2009 9.30 am to 11.30 pm
(Closed book and closed notes)

Answer as much as you can. Best 4 scores out of the 5 problems will be counted.

1) Consider the power system below.

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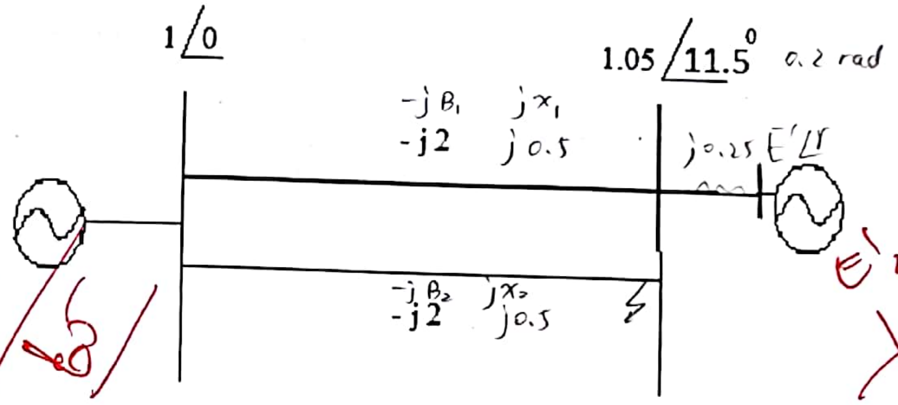

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Generator parameters are: H = 5 sec., $x_d' = 0.25$, $K_D = 5$ pu, and $R_a = 0$. We want to study a fault that occurs close to the generator terminal on the lower transmission line.

a) Assuming classical models, write the dynamic equations for the pre-fault, fault-on and post-fault systems. (15 points)

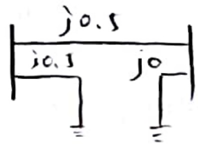
b) Find the pre-fault equilibrium point and solve for the eigenvalues. (Show that the state derivatives are zero at your equilibrium solution.) (10 points)

a)

$$P_G^{pre} = \frac{E_2'\cdot 1}{x_d' + x_1 \| x_2}\sin\theta = \frac{1.05\times 1\sin\theta}{0.25+0.25} = 2.1\sin\theta$$

$$P_G^{post} = \frac{E_2'\cdot 1}{x_d'+x_1}\sin\theta = \frac{1.05\times 1\sin\theta}{0.25+0.5} = 1.4\sin\theta$$

Since the fault is close to bus 2. assume:



$$X_{th} = x_1 \| x_2 = 0.25,\quad V_{th} = \frac{jx_2}{jx_2+jx_1}1\angle 0^\circ = \frac{0.5\cdot 1\angle 0^\circ}{0.5+0.5} = 0.5\angle 0^\circ$$

$$P_G^{fault} = \frac{V_{th}E_2'}{x_d'+x_{th}}\sin\theta = \frac{0.5\times 1.05\sin\theta}{0.25+0.25} = 1.05\sin\theta$$

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Many mistakes.

10

Type 3 model. states $x = \begin{bmatrix} \theta_2 \\ w_2 \end{bmatrix}$

Dynamic equations: $\begin{bmatrix} \dot{\theta}_2 \\ \dot{w}_2 \end{bmatrix} = \begin{bmatrix} (w_2 - 1) w_s \\ \frac{1}{2H}(P_{m2} - P_{e2} - K_D(w_2 - 1)) \end{bmatrix} = \begin{bmatrix} (w_2 - 1)\cdot 377 \\ 0.1(P_{m2} - P_{e2} - 5(w_2 - 1)) \end{bmatrix}$

$P_{m2} = P_{G2}^{pf} = \dfrac{E_2' E_1' \sin\theta_2}{x_d' + x_{21}} = \dfrac{1.05 \times \sin(0.2)}{0.23 + 0.25} = 0.417$

$P_{e2} = \begin{cases} P_{G2}^{pre} = 2.1 \sin\theta_2 & \text{for pre-fault} \\ P_{G2}^{fault} = 1.05 \sin\theta_2 & \text{for fault-on} \\ P_{G2}^{post} = 1.4 \sin\theta_2 & \text{for post-fault} \end{cases}$

Find equilibrium point from power flow

$\begin{cases} \theta_2 = r_2 = 0.2 \\ w_2 = 1 \end{cases}$

plug in to $\begin{cases} \dot{\theta}_2 = (w_2 - 1)\cdot 377 = 0 \\ \dot{w}_2 = 0.1(0.417 - 2.1\cdot\sin 0.2 - 5(1-1)) = -2\times 10^{-5} \approx 0. \end{cases}$

So states derivatives are zero at eq point

$J = \begin{bmatrix} 0 & w_s \\ -0.1\times 2.1\cos\theta_2 & -0.5 \end{bmatrix}\Bigg|_{\substack{\theta_2 = 0.2 \\ w_2 = 1}} = \begin{bmatrix} 0 & 377 \\ -0.21 & -0.5 \end{bmatrix}$

$\lambda I - J = \begin{bmatrix} \lambda & -377 \\ 0.21 & \lambda + 0.5 \end{bmatrix} = 0$, $\lambda(\lambda + 0.5) - (-377)(0.21) = 0$, $\lambda^2 + 0.5\lambda + 79.17 = 0$, $\lambda_{1,2} = -0.25 \pm j8.89$

The system is small-signal stable for pre-fault eq point

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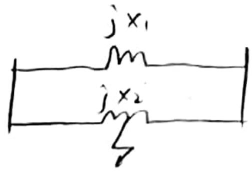


2) Consider the power system in Problem 1. Suppose we want to study a fault in the middle of one of the two transmission lines.

$t_c = 0$

a) Assuming instantaneous clearing, apply equal area criterion and determine whether the system is transient stable or unstable for this contingency. (10 points)

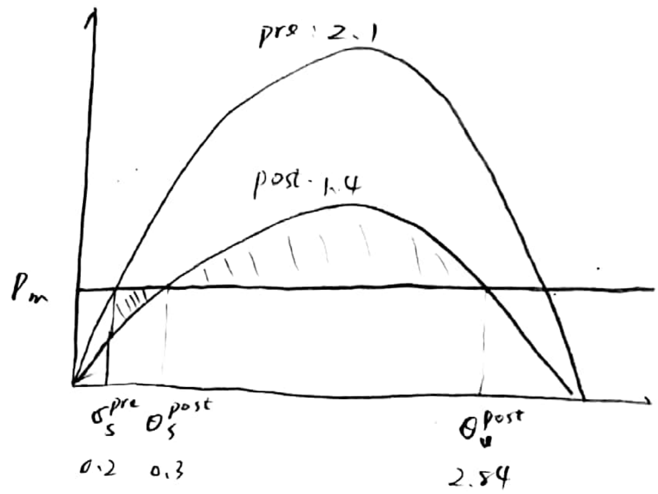
b) Suppose the clearing time is 3 cycles. Again check transient stability using the equal area criterion. (15 points)

1 $t_c = 0$. no fault-on

$P_G^{pre} = 2.1 \sin\theta$

$P_G^{post} = 1.4 \sin\theta$

$P_m = 0.417$



$2.1\sin\theta = 0.417$, $\theta_s^{pre} = 0.20$

$1.4\sin\theta = 0.417$, $\theta_s^{post} = 0.30$

$\theta_u^{post} = \pi - 0.3 = 2.84$

$$Area_{ac} = \int_{\theta_s^{pre}}^{\theta_s^{post}} (P_m - P_G^{post})\, d\theta = \int_{0.2}^{0.3} (0.417 - 1.4\sin\theta)\, d\theta = \left[0.417\,\theta + 1.4\cos\theta\right]\Big|_{\theta=0.2}^{0.3}$$

$$= 0.417 \times 0.1 + 1.4(\cos 0.3 - \cos 0.2) = 0.007$$

$$Area_{dec}^{max} = \int_{\theta_s^{post}}^{\theta_u^{post}} (P_G^{post} - P_m)\, d\theta = \int_{0.3}^{2.54} (1.4\sin\theta - 0.417)\, d\theta = \left[-1.4\cos\theta - 0.417\,\theta\right]\Big|_{\theta=0.3}^{2.54}$$

$$= -1.4(\cos 2.54 - \cos 0.3) - 0.417(2.84 - 0.3) = 1.615$$

$Area_{ac} < Area_{dec}^{max}$. So the system is transient stable.

Correct procedure.
All wrong values

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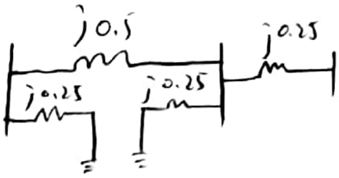
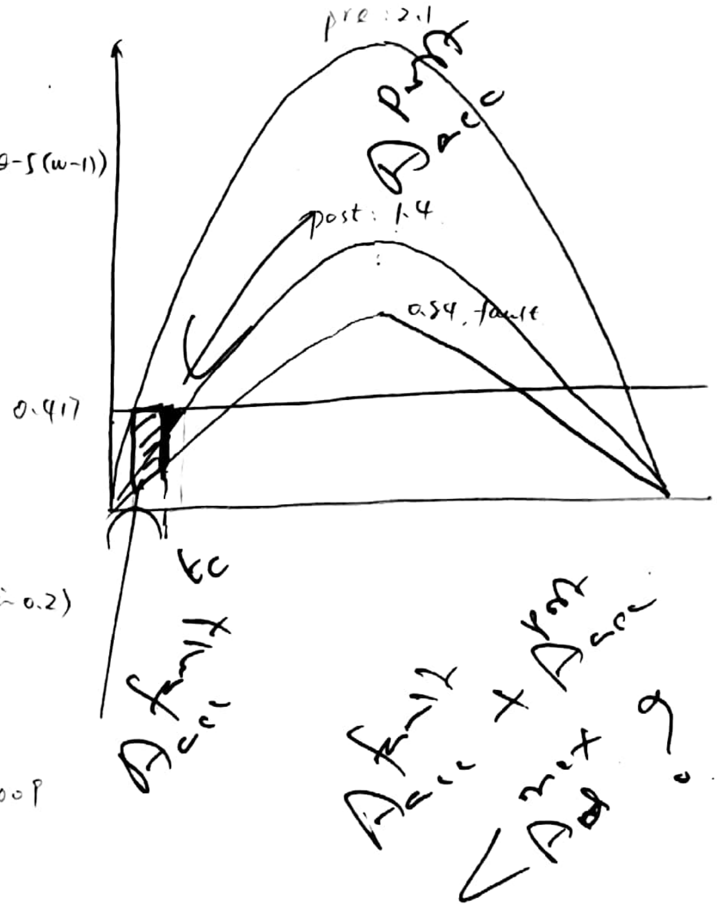
b) $t_c = 3 \text{ cycles} = \frac{3}{60} = 0.05 \text{ sec}$



$$x_{th} = x_1 \parallel \frac{x_2}{2} = \frac{0.5 \times 0.25}{0.5 + 0.25} = \frac{1}{6} = 0.167$$

$$V_{th} = \frac{\frac{x_2}{2}}{\frac{x_2}{2} + x_1} \cdot 1\angle 0^\circ = \frac{0.25\angle 0^\circ}{0.25 + 0.5} = \frac{1}{3}\angle 0^\circ = 0.333\angle 0^\circ$$

$$P_G^{fault} = \frac{V_{th} E_s'}{x_d' + x_{th}} \sin\theta = \frac{0.333 \times 1.05}{0.25 + \frac{1}{6}} \sin\theta = 0.84 \sin\theta$$



$\theta_s^{pre} = 0.2$

$\theta_s^{post} = 0.3$

$\theta_u^{post} = 2.84$

$$\begin{cases} \dot{\theta}_s = (w-1)\cdot 377 \\ \dot{w}_s = 0.1\,(0.417 - 0.84\sin\theta - 5(w-1)) \end{cases}$$

use stepsize $h = 0.01$ sec

$\theta(0.01) = \theta(0) + h\dot{\theta}(0)$

$= 0.2 + h \cdot 0 = 0.2$

$w(0.01) = w(0) + h \cdot \dot{w}(0)$

$= 1 + 0.01 \times 0.1\,(0.417 - 0.84 \cdot \sin 0.2)$

$= 1.00025$

$\theta(0.02) = \theta(0.01) + h\dot{\theta}(0.01)$

$= 0.2 + 0.01 \times 0.00025 \times 377 = 0.2009$

$w(0.02) = w(0.01) + h\,\dot{w}(0.01)$

$= 1.00025 + 0.01 \times 0.1\,(0.417 - 0.84\sin(0.2) - 5 \times 0.00025) = 1.0005$

$\theta(0.03) = \theta(0.02) + h\,\dot{\theta}(0.02)$

$= 0.2009 + 0.01 \times 0.0005 \times 377 = 0.2028$

$w(0.03) = w(0.02) + h\,\dot{w}(0.02)$

$= 1.0005 + 0.01 \times 0.1\,(0.417 - 0.84\sin(0.2009) - 5 \times 0.0005)) = 1.0007$

$\theta(0.04) = \theta(0.03) + h \cdot \dot{\theta}(0.03)$

$= 0.2028 + 0.01 \times 0.0007 \times 377 = 0.2054$

$w(0.04) = w(0.03) + h\,\dot{w}(0.03)$

$= 1.0007 + 0.01 \times 0.1\,(0.417 - 0.84\sin(0.2028) - 5 \times 0.0007)) = 1.0009$

$\theta(0.05) = \theta(0.04) + h \cdot \dot{\theta}(0.04)$

$= 0.2054 + 0.01 \times 0.0008 \times 377 = 0.2088$

$w(0.05) = w(0.04) + h \cdot \dot{\theta}(0.04)$

$= 1.0008 + 0.01 \times a\,l(0.417 - 0.84\sin(0.2054) - 5 \times 0.0008) = 1.0011$

$$Area_{ac} = \int_{\theta_{pre}}^{\theta_{clear}} (P_m - P_G^{fault})\,d\theta$$ 1 α

$$= \int_{0.2}^{0.2088} (0.417 - 0.84\sin\theta)\,d\theta = \left[0.417\theta + 0.84\cos\theta\right]_{\theta=0.2}^{0.2088}$$

$$= 0.417 \times (0.2088 - 0.2) + 0.84 \times (\cos 0.2088 - \cos 0.2) = 0.002$$

$$Area_{dec}^{max} = \int_{\theta_{clear}}^{\theta_u^{post}} (P_G^{post} - P_m)\,d\theta = \int_{0.2088}^{2.84} (1.4\sin\theta - 0.417)\,d\theta$$

$$= \left[-1.4\cos\theta - 0.417\theta\right]\Big|_{\theta=0.2088}^{2.84} = -1.4(\cos 2.84 - \cos 0.2088) - 0.417(2.84 - 0.2088)$$

$$= 1.608$$

$Area_{ac} < Area_{dec}^{max}$, So the system is tranrient stable

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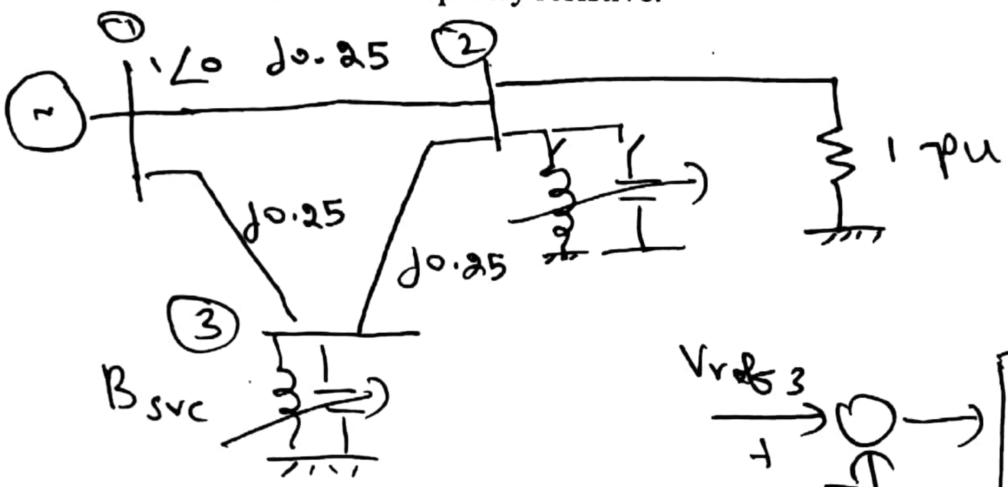
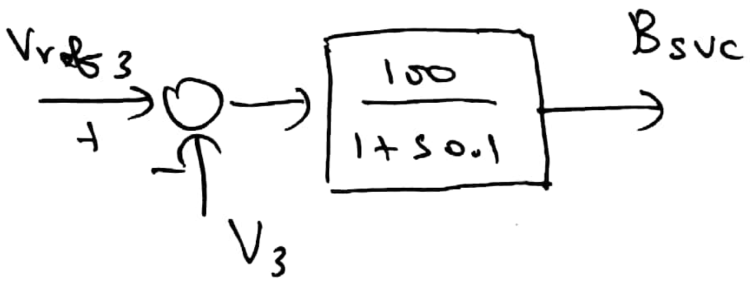
3) Consider the three-bus power system below. For the power-flow problem, bus 2 voltage is kept at 1 pu by manual switching of shunt capacitors and reactors at the bus. And, bus 3 voltage is maintained at 1.05 pu by a Static VAR Compensator (SVC). The dynamic model for SVC is shown below, and the load at bus 2 is modeled to be purely resistive.





a) What shunt compensation is required at buses 2 and 3 as per the power-flow solution? (10 points)

b) Derive the dynamic model for the system and simplify as much as you can. (10 points)

c) Solve the initialization problem (i.e. find the equilibrium solution) (5 points)

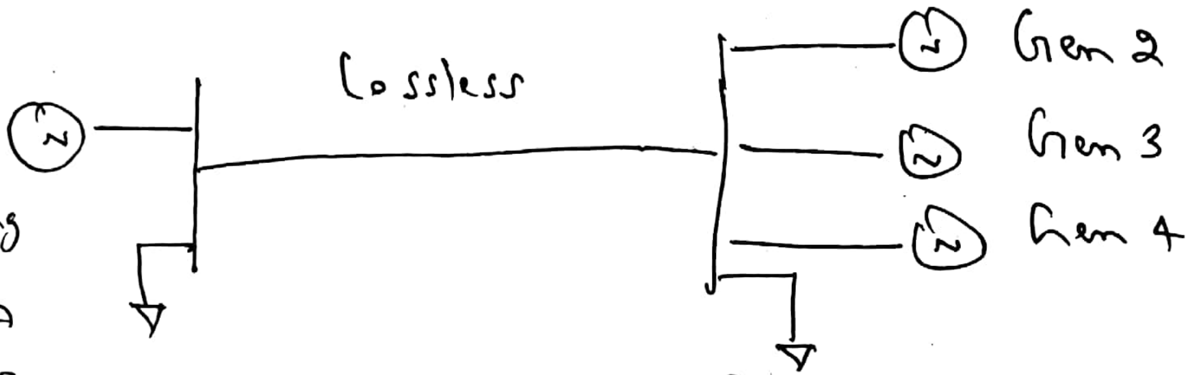
4) Consider the two area system below.

a) Suppose there is a sudden loss of generator 3 in Area 2. Compute the governor responses before and after AGC actions.

(10 points)

b) Suppose the load in Area 1 suddenly increases by 100 MW (instead of gen drop in part a). Compute the governor responses before and after AGC actions. Assume generator 2 to be the slack bus for Area 2.

(15 points)



Rating =1000 MVA

$P_{G1} = 500\,MW$

$P_{L1} = 700\,MW$

$R_1 = 5\%$

$P_L = 200\,MW$

Gen 2 : Rating = 500 MVA $R_2 = 5\%$

$P_{G2} = 200\,MW$

Gen 3 : Rating = 100 MVA, $R_3 = 10\%$

$P_{G3} = 100\,MW$

Gen 4 : Rating = 200 MVA, $R_4 = 5\%$

$P_{G4} = 100\,MW$

a) Gen 3 : $R_3 = 5\%$, Rating 3 = 50 MVA

Governor responses before AGC:

$$\Delta P_L = \frac{100}{1000+500+200} = \frac{1}{17} = 0.0588 \text{ p.u}$$

$\Delta w = R \cdot \Delta P_L = -0.05 \times 0.0588 = -0.0029$ p.u.

$\Delta P_{G1} = 0.0588 \times 1000 = 58.8$ MW

$\Delta P_{G2} = 0.0588 \times 500 = 29.4$ MW

$\Delta P_{G4} = 0.0588 \times 200 = 11.8$ MW

Governor responses after AGC actions.

$\Delta P_{Net1} = P_{Net1}^{actual} - P_{Net1}^{contract} = (-141.2) - (-200) = 58.8$ MW $= 0.0588$ p.u.

$ACE_1 = \Delta P_{Net1} + B_1 \Delta f_1 = 0.0588 + 20 \times (-0.0029) = 0$

$\Delta P_{G1} = -ACE_1 = 0$ MW

$\Delta P_{Net2} = P_{Net2}^{actual} - P_{Net2}^{contract} = (141.2) - (200) = -58.8$ MW $= -0.084$

$ACE_2 = \Delta P_{Net2} + B \Delta f_2 = -0.084 + 20(-0.0029) = -0.1428$ p.u $= -100$ MW

For slack bus Gen 2 : $\Delta P_{G2} = -ACE_2 = 100$ MW

10

$P_{L2} = 200 MW$

b) Rating 1 = 1000 MVA

$P_{G1} = 500 MW$

$P_{L1} = 700 MW$

↓

increase 100 MW

Rating 2 = 500 MVA

$P_{G2} = 200 MW$

Rating 3 = 50 MVA

$P_{G3} = 100 MW$

Rating 4 = 200 MVA

$P_{G4} = 100 MW$

Governor responses before AGC:

$$\Delta P_L = \frac{100}{1000+500+50+1200} = \frac{2}{35} = 0.057 \text{ p.u.}$$

$$\Delta \omega = R\Delta P_L = -0.05 \times \frac{2}{35} = \frac{-1}{350} = -0.0028 \text{ p.u.}$$

$\Delta P_{G1} = 0.057 \times 1000 = 57 MW$

$\Delta P_{G2} = 0.057 \times 500 = 28.6 MW$

$\Delta P_{G3} = 0.057 \times 50 = 2.8 MW$

$\Delta P_{G4} = 0.057 \times 200 = 11.4 MW$

Governor responses after AGC:

$\Delta P_{Net1} = (-243) - (-200) = -43 MW = -0.043 \text{ p.u.}$

$ACE_1 = \Delta P_{Net1} + B_1 \Delta f_1 = -0.043 + 20 \times (-0.0028) = -0.1 \text{ p.u.} = -100 MW$

$\Delta P_{G1} = -ACE_1 = 100 MW$

$\Delta P_{Net2} = (243) - (200) = 43 MW = 0.003 \text{ p.u}$

$ACE_2 = \Delta P_{Net2} + B_2 \Delta f_2 = 0.003 + 20 \times (-0.0028) = 0.$

$\Delta P_{G2} = -ACE_2 = 0$

2

3 marked

Gen 3 , Drop from 8% to ...

-7

8

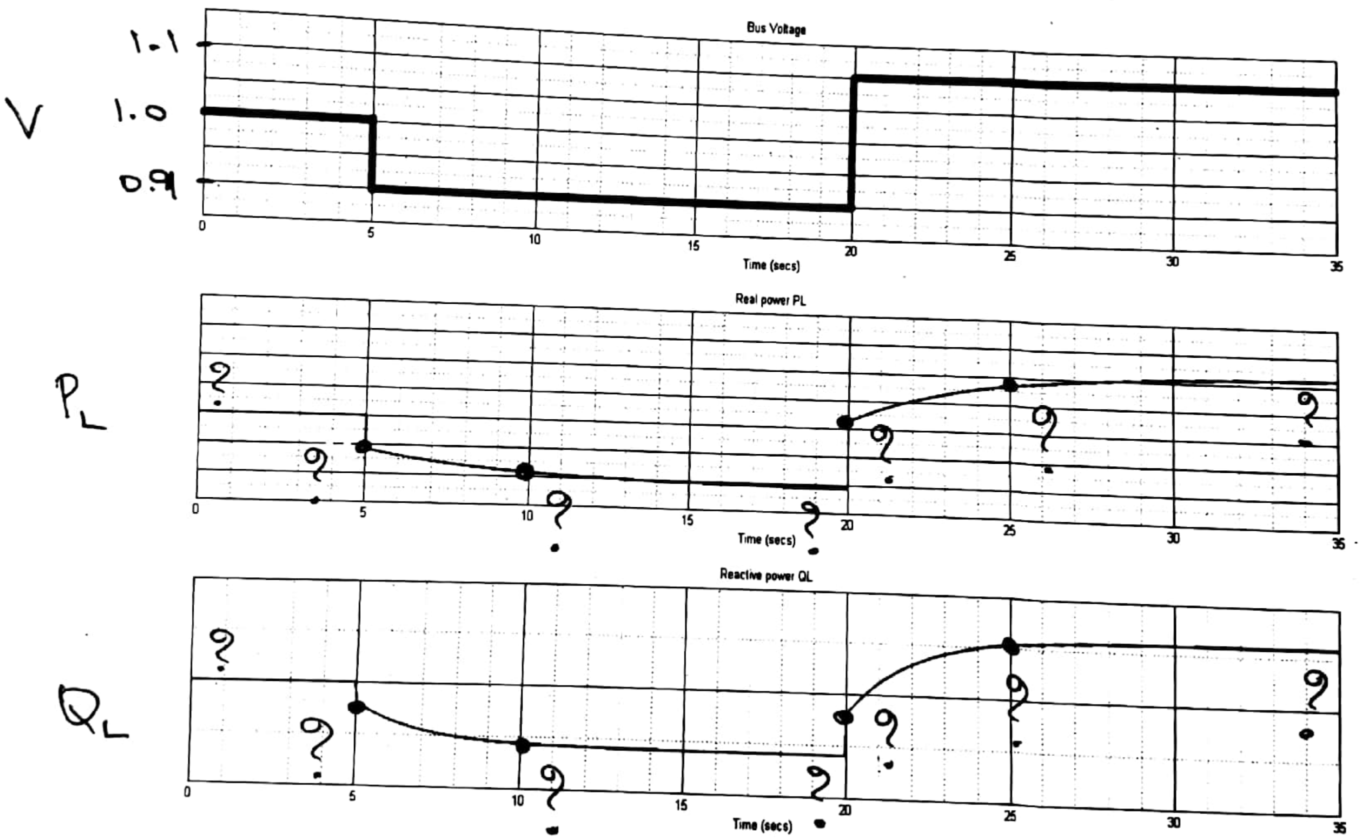
5) Suppose the static and dynamic loads are represented by the following models. Find all the values marked by question marks in the load test responses below.

(25 points)

$$P_L = P_L^S + P_L^D \qquad P_L^S = 0.2V + 0.2V^2$$

$$Q_L = Q_L^S + Q_L^D \qquad Q_L^S = 0.1V$$

$$V \rightarrow \boxed{\frac{0.125}{s+0.25}} \rightarrow P_L^D \qquad V \rightarrow \boxed{\frac{0.1}{s+0.5}} \rightarrow Q_L^D$$



$P_L(0) = ?$ $P_L(5+) = ?$ $P_L(10) = ?$ $P_L(20-) = ?$

$P_L(20+) = ?$ $P_L(25) = ?$ $P_L(35) = ?$

$Q_L(0) = ?$ $Q_L(5+) = ?$ $Q_L(10) = ?$ $Q_L(20-) = ?$

$Q_L(20+) = ?$ $Q_L(25) = ?$ $Q_L(35) = ?$

f) $P_L = P_{LS} + P_{LP} = 0.2V + 0.2V^2 + \frac{0.125}{s+0.25}V$, $\frac{0.125V}{s+0.25} = \frac{0.5V}{1+4s}$, $T_p = 4$

$Q_L = Q_{LS} + Q_{LP} = 0.1V + \frac{0.1}{s+0.5}V$, $\frac{0.1V}{s+0.5} = \frac{0.2V}{1+2s}$, $T_q = 2$

steady state. $s = 0$.

$t = 5_-$, $V = 1$, $P_L = 0.2 \times 1 + 0.2 + \frac{0.125}{0.25} \times 1 = 0.9$

$Q_L = 0.1 + \frac{0.1}{0.5} = 0.3$

$t = 5_+$, $V = 0.9$, $P_L = 0.2 \times 0.9 + 0.2 \times 0.9^2 + \frac{0.125}{0.25} \times 1 = 0.842$

$Q_L = 0.1 \times 0.9 + \frac{0.1}{0.5} \times 1 = 0.29$

$t = 20_-$, $V = 0.9$, $P_L = 0.2 \times 0.9 + 0.2 \times 0.9^2 + \frac{0.125}{0.25} \times 0.9 = 0.792$

$Q_L = 0.1 \times 0.9 + \frac{0.1}{0.5} \times 0.9 = 0.27$

$t = 20_+$, $V = 1.1$, $P_L = 0.2 \times 1.1 + 0.2 \times 1.1^2 + \frac{0.125}{0.25} \times 0.9 = 0.912$

$Q_L = 0.1 \times 1.1 + \frac{0.1}{0.5} \times 0.9 = 0.29$

$t = 35$, $V = 1.1$, $P_L = 0.2 \times 1.1 + 0.2 \times 1.1^2 + \frac{0.125}{0.25} \times 1.1 = 1.012$

$Q_L = 0.1 \times 1.1 + \frac{0.1}{0.5} \times 1.1 = 0.33$

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$$\mathcal{L}^{-1}\left[\frac{0.125}{s+0.25}\right] = 0.125 \times e^{-0.25t}$$

$t=10, \quad \Delta t = 10-5=5 \quad \Delta P_{LD} = 0.125 \times e^{-0.25\times 5} = 0.036 \quad , P_L(10) = 0.842 - 0.036 = 0.806$

$t=25, \quad \Delta t = 25-20=5, \quad \Delta P_{LD} = 0.125 \times e^{-0.25\times 5} = 0.036$

$P_L(25) = 0.912 + 0.036 = 0.948$

0.032

2

$$\mathcal{L}^{-1}\left[\frac{0.1}{s+0.5}\right] = 0.1\, e^{-0.5t}$$

$t=10, \quad \Delta t = 10-5, \quad \Delta \theta_{LD} = 0.1 \times e^{-0.5\times 5} = 0.008 \quad , \theta_L(10) = 0.29 - 0.008 = 0.282$

$t=25, \quad \Delta t = 25-20=5, \quad \Delta \theta_{LD} = 0.1 \times e^{-0.5\times 5} = 0.008 \quad , \theta_L(25) = 0.29 + 0.008 = 0.298$

-2

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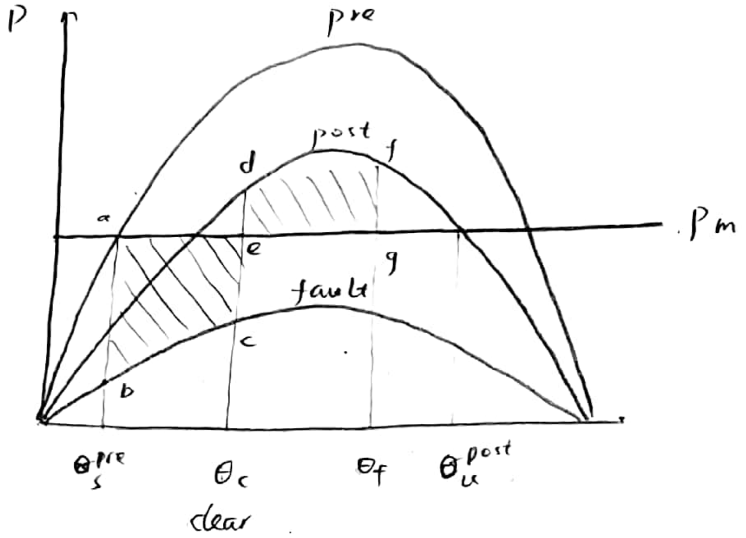
(Bonus) Prove equal area criterion. (5 points)



The swing equation $\frac{T_J}{w_0} \cdot \frac{d\theta^2}{dt^2} = P_m - P_e$ ①

$$\frac{d^2\theta}{dt^2} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d\dot{\theta}}{dt} = \frac{d\theta}{dt}\frac{d\dot{\theta}}{d\theta} = \dot{\theta}\frac{d\dot{\theta}}{d\theta}$$ plug in ①

$$\frac{T_J}{w_0}\dot{\theta}\frac{d\dot{\theta}}{d\theta} = P_e - P_m, \quad \frac{T_J}{w_0}\dot{\theta}\, d\dot{\theta} = (P_e - P_m)\, d\theta$$

$$\int_{\dot{\theta}_s^{pre}}^{\dot{\theta}_c} \frac{T_J}{w_0}\dot{\theta}\, d\dot{\theta} = \int_{\theta_s^{pre}}^{\theta_c} (P_m - P_e)\, d\theta$$

$$\frac{1}{2}\frac{T_J}{w_0}(\dot{\theta}_c^2 - \dot{\theta}_s^{pre\,2}) = \frac{1}{2}\frac{T_J}{w_0}\dot{\theta}_c^2 = \int_{\theta_s^{pre}}^{\theta_c} (P_m - P_e^{fault})\, d\theta$$

Left side = increase of kinetic energy

Right side = the work excess torque does w.r.t. θ, = area abce

Similarly, we have $\frac{1}{2}\frac{T_J}{w_0}(\dot{\theta}_f^2 - \dot{\theta}_c^2) = \frac{1}{2}\frac{T_J}{w_0}\dot{\theta}_c^2 = \int_{\theta_c}^{\theta_f} (P_e^{post} - P_m)\, d\theta$

Decrease of kinetic energy = brake torque = area of defg.

So if θ go beyond the limit θ_u^{post}, $P_m > P_e$, $w\uparrow \Rightarrow w > 1$, $\theta\uparrow$

θ will keep increasing. and the system will be transient unstable

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