HWS, die 11/3/2022

Problem 1

- a. Consider three random variables X, Y, and Z. Let's say that $f_{X,Y,Z}(x,y,z) = f_X(x)f_Y(y)f_Z(z)$,
- i.e. that their joint PDF is the product of their individual PDFs. Is it necessarily true that X and Y are independent?
- b. Is your answer to part a different from the answer to the analogous problem for three events? If so, conceptually, why?

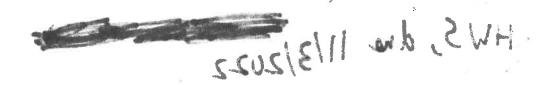
Problem 2

Consider a pair of jointly Gaussian random variables X and Y with parameters $m_x=m_y=0$, $\sigma_x^2=1,\ sigma_y^2=4,\ {\rm and}\ r=0.5.$

- a. Please sketch lines of equal probability for the joint PDF of X and Y.
- b. What are the marginal PDFs of X and Y?
- c. Please find $f_{Y|X}(y \mid X = x)$.
- d. For what value of x is E[Y | X = x] = -2.
- e. What is the PDF of Z = X + Y 1?
- f. Let Z = 2X + 3Y, and let W = X Y. Please find E[Z], var(Z), E[W], var(W), and cov(Z, W). Also, please find the joint PDF of Z and W.
- g. Consider a random variable R = aX + bY. For what constants a and b are R and Y independent?
- h. Let Q = aX, where a equals 1 with probability 0.5 and equals 2 with probability 0.5. Please find the PDF of Q.

Problem 3

- a. Consider two random variables X and Y that are independent, and consider Z = X + Y. Show that the PDF of Z is given by $f_Z(z) = \int_{-\infty}^{\infty} f_Y(z-x) f_X(x) dx$.
- b. For independent X and Y, show that Z = X + Y has PDF $f_Z(z) = \int_{-\infty}^{\infty} f_Y(y) f_X(z-y) dy$.
- c. Say that X and Y are independently and identically distributed exponential random variables with mean 0.5. What is the PDF of Z = X + Y?



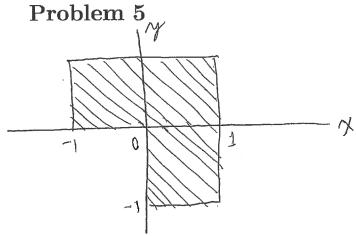
d. Say that X and Y are independent random variables that are both uniform on [0,1]. Please find the PDF of Z = X + Y.

e. Consider the sum of 7 independent random variables that are each uniform on [0,1]. Numerically find the PDF of this sum. What common density does this PDF remind you of?

Problem 4

Say that X and Y are independent random variables that are each uniform on [0,1]. Let $Z = \frac{Y}{Y}$.

- a. Please find the PDF of Z.
- b. Please find $E[X^2 + Y^2]$.
- c. Please find the joint PDF of X and Z, where $Z = \frac{Y}{X}$.
- d. Please find E[XZ], cov(X, Z), and $\rho_{X,Z}$.
- e. Please find the PDF of X given Z, and $E[X \mid Z]$.
- f. Verify that, in this example. $E[E[X \mid Z]] = E[X]$.
- g. Please find $E[XZ \mid Z = z]$.



Consider a pair of random variables X and Y uniformly distributed in the region shown above. Please find E[X], E[Y], E[XY], var(X), var(Y), cov(X,Y), and $\rho_{X,Y}$.

Problem 6 Two random variables X and Y are unformly distributed in the region shown below: Please find: fxy(x,y), Fxy(x,y), fx(x),fy(y), and fxy(x)Y=y).

Problem 7

Is it possible for two random variables X and Y to be each uniterm on (0,1), uncorrelated, and not independent?