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EE 491: Review of EE361

Transmission Line Models Review

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Homework 1

- Due on Sept 3 2020 @ 9 am

Per Unit Problem:

Common MVA Base should be specified. You can use either **30 MVA Base** or **100 MVA Base**.

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Basics

$v(t) = \sqrt{2}V\cos(\omega t)$ $\qquad \vec{V} = V\angle 0$

$i(t) = \sqrt{2}I\cos(\omega t - \varphi)$ $\qquad \vec{I} = I\angle -\varphi$

$$\begin{aligned}
p(t) &= 2VI\cos(\omega t)\cos(\omega t - \varphi) \\
&= 2VI\cos(\omega t)\,(\cos\omega t\cos\varphi + \sin\omega t\sin\varphi) \\
&= VI\cos\varphi\,[2\cos^2\omega\,t] + VI\sin\varphi\,[2\cos\omega\,t\sin\omega\,t] \\
&= VI\cos\varphi\,[1+\cos 2\,\omega t] + VI\sin\varphi\,[\sin 2\,\omega t] \\
&= P\,[1+\cos 2\,\omega t] + Q\,[\sin 2\,\omega t] \\
&= \quad p_1(t) \quad + \quad p_2(t)
\end{aligned}$$

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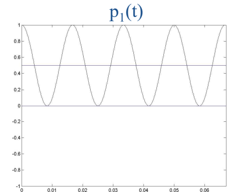
Basics

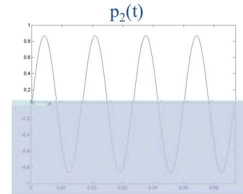
Assume VI=1, φ=60⁰. Then, P=0.5, Q=0.877

$p_1(t)$



Average = 0.5.
Real power.

$p_2(t)$



Average = 0.
Swinging power.

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Basics

$\vec{V} = V\angle 0 \qquad \vec{I} = I\angle -\varphi$

$\vec{S} = \vec{V}\,\vec{I}^{\,*} = VI\cos\varphi + j\,VI\sin\varphi = P + j\,Q$

- P = VI cos φ and Q = VI sin φ
- S = P + j Q
- Real power well-understood.
- Generators are sources. Loads and Losses are sinks.
- Reactive power?

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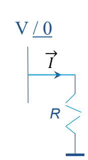
Basic components



$\vec{I} = \frac{V\angle 0}{R} = \frac{V}{R}\angle 0$

$S = \vec{V}\,\vec{I}^{*} = V\angle 0\left(\frac{V}{R}\angle 0\right)^{*} = \frac{V^2}{R} + j\,0$

$P = \frac{V^2}{R} \qquad Q = 0$

Resistors are sinks of real power

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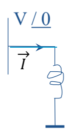
Reactive power



$$\vec{I} = \frac{V\angle 0}{jx} = \frac{V}{x}\angle -90^0$$

$$S = \vec{V}\,\vec{I}^* = V\angle 0\left(\frac{V}{x}\angle -90^0\right)^* = 0 + j\frac{V^2}{x}$$

$$P = 0 \qquad Q = \frac{V^2}{x}$$

Reactors are sinks of reactive power

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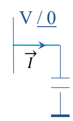
Reactive power



$$\vec{I} = jy * V\angle 0 = Vy\angle 90^0$$

$$S = \vec{V}\,\vec{I}^* = V\angle 0\,(Vy\angle 90^o)^* = 0 - jV^2y$$

$$P = 0 \qquad Q = -V^2y$$

Capacitors are sources of reactive power

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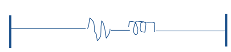
Real power management

- Generators are sources.
- Loads and MW losses are sinks.
- MW losses small.
- Can transfer MW power over long distances.
- Thermal limits.



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Reactive power management

- Generators and capacitors are sources.
- Reactors and loads are sinks.
- Abundance of reactors in the power system.
- Line reactances typically ten times the line resistances. Same for transformers,…

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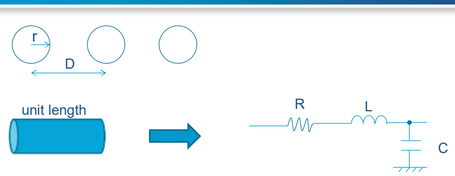
Transmission Line Model



$$L \approx C_1 \ln C_2 \frac{D}{r}$$

$$C \approx \frac{1}{C_3 \ln C_4 \frac{D}{r}}$$

$$R \approx \frac{1}{Area}$$

$$\omega L \approx 0.5\ \Omega/mile$$

$$\omega C \approx 10^{-5}\ S/mile$$

$$R \approx 0.05\ \Omega/mile$$

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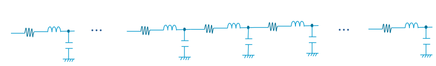
Transmission Line Models

Distributed Parameter Circuit



Line Model

Short
ℓ < 50 miles

Medium
50 miles < ℓ < 150 miles

Long
ℓ > 150 miles

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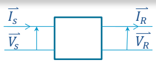
Transmission Line Model



$$\begin{bmatrix}\vec{V_S}\\ \vec{I_S}\end{bmatrix} = \begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}\vec{V_R}\\ \vec{I_R}\end{bmatrix}$$

$$\vec{I_a} = I\angle(-\varphi)$$
$$\vec{I_b} = I\angle(-\varphi - 120^o)$$
$$\vec{I_c} = I\angle(-\varphi + 120^o)$$

$$\vec{S_a} = \vec{V_a}\vec{I_a}^* = VI\angle(\varphi) = P_a + jQ_a$$

$$\vec{S} = \vec{S_a} + \vec{S_b} + \vec{S_c} = 3VI\angle(\varphi) = P + jQ$$ (normally used for Rating)

$$P = 3VI\cos\varphi = \sqrt{3}V_L I_L \cos\varphi$$

$$Q = 3VI\sin\varphi = \sqrt{3}V_L I_L \sin\varphi$$

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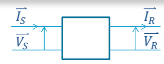
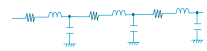
Short Line Model



Short line: ℓ<50 miles

$$\begin{bmatrix}\vec{V_S}\\ \vec{I_S}\end{bmatrix} = \begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}\vec{V_R}\\ \vec{I_R}\end{bmatrix}$$

ABCD parameter representation



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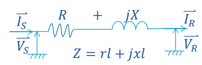
Ignore all capacitances



$$\vec{V_S} = \vec{V_R} + Z\vec{I_R}$$

$$\begin{bmatrix}\vec{V_S}\\ \vec{I_S}\end{bmatrix} = \begin{bmatrix}1 & Z\\ 0 & 1\end{bmatrix}\begin{bmatrix}\vec{V_R}\\ \vec{I_R}\end{bmatrix}$$

$$\vec{V_S} = \vec{V_R} + R\vec{I_R} + jX\vec{I_R}$$

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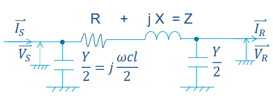
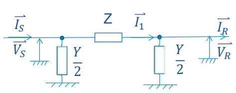
Medium Line Model

Medium line: 50 miles < ℓ < 150 miles

Lump all capacitances ⟹ Put one half on each side



$$R = rl$$
$$X = xl = \omega Ll$$
$$Y = yl = \omega Cl$$



$$\vec{I_1} = \vec{I_R} + \vec{V_R}\frac{Y}{2}$$

$$\vec{V_S} = \vec{V_R} + Z\vec{I_1}$$
$$= \vec{V_R} + Z\vec{I_R} + \frac{ZY}{2}\vec{V_R}$$
$$= \left(1 + \frac{ZY}{2}\right)\vec{V_R} + Z\vec{I_R}$$

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Medium Line Model

$$\vec{I_S} = \vec{I_1} + \vec{V_S}\frac{Y}{2} = \vec{I_R} + \vec{V_R}\frac{Y}{2} + \frac{Y}{2}\left(1 + \frac{ZY}{2}\right)\vec{V_R} + \frac{Y}{2}Z\vec{I_R}$$
$$= \vec{V_R}\left(\frac{Y}{2} + \frac{Y}{2} + \frac{Y^2Z}{4}\right) + \vec{I_R}\left(1 + \frac{YZ}{2}\right)$$
$$= \vec{V_R}\left(Y + \frac{Y^2Z}{4}\right) + \vec{I_R}\left(1 + \frac{YZ}{2}\right)$$

$$\begin{bmatrix}\vec{V_S}\\ \vec{I_S}\end{bmatrix} = \begin{bmatrix}1 + \frac{YZ}{2} & Z\\ Y\left(1 + \frac{YZ}{4}\right) & 1 + \frac{YZ}{2}\end{bmatrix}\begin{bmatrix}\vec{V_R}\\ \vec{I_R}\end{bmatrix}$$

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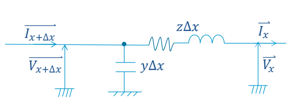
Long Line Model

Long line: ℓ > 150 miles ⟹ Distributed Parameter Circuit



$$\vec{I_{x+\Delta x}} = \vec{I_x} + y\Delta x\vec{V_x} \Longrightarrow \frac{\Delta\vec{I_x}}{\Delta x} = y\vec{V_x}$$

$$\vec{V_{x+\Delta x}} = \vec{V_x} + z\Delta x\vec{I_x} \Longrightarrow \frac{\Delta\vec{V_x}}{\Delta x} = z\vec{I_x}$$

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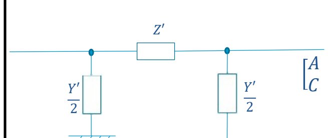
Long Line Model

$$\frac{d\vec{I_x}}{dx} = y\vec{V_x} \quad \Rightarrow \quad \frac{d^2\vec{V_x}}{dx^2} = yz\vec{V_x}$$

$$\frac{d\vec{V_x}}{dx} = z\vec{I_x} \quad \frac{d^2\vec{I_x}}{dx^2} = yz\vec{I_x}$$

$$\Rightarrow \begin{bmatrix}A & B\\ C & D\end{bmatrix} = \begin{bmatrix}\cosh\gamma l & Z_c\sinh\gamma l\\ \frac{1}{Z_c}\sinh\gamma l & \cosh\gamma l\end{bmatrix}$$

$$Z_c = \sqrt{\frac{z}{y}} \;\&\; \gamma = \sqrt{zy}$$



$$\begin{bmatrix}A & B\\ C & D\end{bmatrix} = \begin{bmatrix}1 + \frac{Z'Y'}{2} & Z'\\ Y'\left(1 + \frac{Z'Y'}{4}\right) & 1 + \frac{Z'Y'}{2}\end{bmatrix}$$

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Long Line Model

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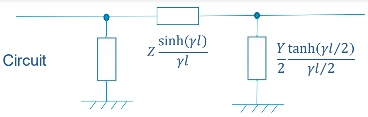
Long line Equivalent π Circuit



Choose $Z' = Z_c \sinh(\gamma l) = Z\frac{\sinh(\gamma l)}{\gamma l}$

$$1 + \frac{Y'Z'}{2} = \cosh(\gamma l)$$

$$\frac{Y'Z'}{2} = \cosh(\gamma l) - 1$$

$$\frac{Y'}{2} = \frac{1}{Z_c}\left(\frac{\cosh(\gamma l) - 1}{\sinh(\gamma l)}\right) = \frac{1}{Z_c}\tanh\left(\frac{\gamma l}{2}\right) = \frac{Y}{2}\frac{\tanh(\gamma l/2)}{\gamma l/2}$$

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Transmission Line Tables Review

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Line parameters can be calculated from Book tables A.3, A.4 and A.5.
Per unit length reactance from Tables A.3 (x_a) and A.4 (x_d)
(Read Section 4.9 in the textbook and Examples 4.3 and 5.1.)

$$x_L = \omega L = C_1 ln\left(C_2\frac{D}{r}\right) = C_1\ ln\left(C_2\frac{1}{r}\right) + C_1 \ln(C_2 D) = x_a + x_d$$

x_a is a fixed parameter for the line; x_d depends on the line spacing D_{eq}.

Per unit length capacitive reactance from Tables A.3 (x'_a) and A.5 (x'_d)

$$x_c = C_3 ln\left(C_4\frac{D}{r}\right) = C_3 \ln\left(C_4\frac{1}{r}\right) + C_3 \ln(C_4 D) = x'_a + x'_d$$

x'_a is a fixed parameter for the line; x'_d depends on the line spacing D_{eq}.

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Book Problem 6.5

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Example: Book (P234 Problem 6.5)

Ostrich

$$D_{eq} = \sqrt[3]{15 \times 15 \times 30} = 18.9\ ft$$
$$l = 70\ mi$$
$$x = x_a + x_d$$

Reactance: *From Table A.3:* $x_a = 0.458$

0.9ft, can also calculated directly using equation

From Table A.4: $x_d(18.9ft) = 0.3507 + \frac{0.0066}{10} \times 9 = 0.3566$

$$\Rightarrow x = 0.8416\ \Omega/mile$$

Resistance: *From Table A.3:* $R(temp = 50^o) = 0.3372\ \Omega/mile$

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TABLE A.3
Electrical characteristics of bare aluminum conductors steel-reinforced (ACSR)†

Code word	Aluminum area, cmil	Stranding Al/St	Layers of aluminum	Outside diameter, in	Resistance: Dc, 20°C, Ω/1,000 ft	Resistance: Ac, 60 Hz: 20°C, Ω/mi	Resistance: Ac, 60 Hz: 50°C, Ω/mi	GMR D_s, ft	Reactance per conductor 1-ft spacing, 60 Hz: Inductive X_a, Ω/mi	Reactance per conductor 1-ft spacing, 60 Hz: Capacitive X'_a, MΩ·mi
Waxwing	266,800	18/1	2	0.609	0.0646	0.3488	0.3831	0.0198	0.476	0.1090
Partridge	266,800	26/7	2	0.642	0.0640	0.3452	0.3792	0.0217	0.465	0.1074
Ostrich	300,000	26/7	2	0.680	0.0569	0.3070	0.3372	0.0229	0.458	0.1057
Merlin	336,400	18/1	2	0.684	0.0512	0.2767	0.3037	0.0222	0.462	0.1055
Linnet	336,400	26/7	2	0.721	0.0507	0.2737	0.3006	0.0243	0.451	0.1040
Oriole	336,400	30/7	2	0.741	0.0504	0.2719	0.2987	0.0255	0.445	0.1032
Chickadee	397,500	18/1	2	0.743	0.0433	0.2342	0.2572	0.0241	0.452	0.1031
Ibis	397,500	26/7	2	0.783	0.0430	0.2323	0.2551	0.0264	0.441	0.1015
Pelican	477,000	18/1	2	0.814	0.0361	0.1957	0.2148	0.0264	0.441	0.1004
Flicker	477,000	24/7	2	0.846	0.0359	0.1943	0.2134	0.0284	0.432	0.0992
Hawk	477,000	26/7	2	0.858	0.0357	0.1931	0.2120	0.0289	0.430	0.0988
Hen	477,000	30/7	2	0.883	0.0355	0.1919	0.2107	0.0304	0.424	0.0980
Osprey	556,500	18/1	2	0.879	0.0309	0.1679	0.1843	0.0284	0.432	0.0981
Parakeet	556,500	24/7	2	0.914	0.0308	0.1669	0.1832	0.0306	0.423	0.0969
Dove	556,500	26/7	2	0.927	0.0307	0.1663	0.1826	0.0314	0.420	0.0965
Rook	636,000	24/7	2	0.977	0.0269	0.1461	0.1603	0.0327	0.415	0.0950
Grosbeak	636,000	26/7	2	0.990	0.0268	0.1454	0.1596	0.0335	0.412	0.0946
Drake	795,000	26/7	2	1.108	0.0215	0.1172	0.1284	0.0373	0.399	0.0912
Tern	795,000	45/7	3	1.063	0.0217	0.1188	0.1302	0.0352	0.406	0.0925
Rail	954,000	45/7	3	1.165	0.0181	0.0997	0.1092	0.0386	0.395	0.0897
Cardinal	954,000	54/7	3	1.196	0.0180	0.0988	0.1082	0.0402	0.390	0.0890
Ortolan	1,033,500	45/7	3	1.213	0.0167	0.0924	0.1011	0.0402	0.390	0.0885
Bluejay	1,113,000	45/7	3	1.259	0.0155	0.0861	0.0941	0.0415	0.386	0.0874
Finch	1,113,000	54/19	3	1.293	0.0155	0.0856	0.0937	0.0436	0.380	0.0866
Bittern	1,272,000	45/7	3	1.345	0.0136	0.0762	0.0832	0.0444	0.378	0.0855
Pheasant	1,272,000	54/19	3	1.382	0.0135	0.0751	0.0821	0.0466	0.372	0.0847
Bobolink	1,431,000	45/7	3	1.427	0.0121	0.0684	0.0746	0.0470	0.371	0.0837
Plover	1,431,000	54/19	3	1.465	0.0120	0.0673	0.0735	0.0494	0.365	0.0829
Lapwing	1,590,000	45/7	3	1.502	0.0109	0.0623	0.0678	0.0498	0.364	0.0822
Falcon	1,590,000	54/19	3	1.545	0.0108	0.0612	0.0667	0.0523	0.358	0.0814
Bluebird	2,156,000	84/19	4	1.762	0.0080	0.0476	0.0515	0.0586	0.344	0.0776

†Most used multilayer sizes.
‡Data, by permission, from Aluminum Association, *Aluminum Electrical Conductor Handbook*, 2nd ed., Washington, D.C., 1982.

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TABLE A.4 Inductive reactance spacing factor X_d at 60 Hz† (ohms per mile per conductor)

Feet	Separation, Inches: 0	1	2	3	4	5	6	7	8	9	10	11
0	......	−0.3015	−0.2174	−0.1682	−0.1333	−0.1062	−0.0841	−0.0654	−0.0492	−0.0349	−0.0221	−0.0106
1	0	0.0097	0.0187	0.0271	0.0349	0.0423	0.0492	0.0558	0.0620	0.0679	0.0735	0.0789
2	0.0841	0.0891	0.0938	0.0984	0.1028	0.1071	0.1112	0.1152	0.1190	0.1227	0.1264	0.1299
3	0.1333	0.1366	0.1399	0.1430	0.1461	0.1491	0.1520	0.1549	0.1577	0.1604	0.1631	0.1657
4	0.1682	0.1707	0.1732	0.1756	0.1779	0.1802	0.1825	0.1847	0.1869	0.1891	0.1912	0.1933
5	0.1953	0.1973	0.1993	0.2012	0.2031	0.2050	0.2069	0.2087	0.2105	0.2123	0.2140	0.2157
6	0.2174	0.2191	0.2207	0.2224	0.2240	0.2256	0.2271	0.2287	0.2302	0.2317	0.2332	0.2347
7	0.2361	0.2376	0.2390	0.2404	0.2418	0.2431	0.2445	0.2458	0.2472	0.2485	0.2498	0.2511
8	0.2523											
9	0.2666											
10	0.2794											
11	0.2910											
12	0.3015											
13	0.3112											
14	0.3202											
15	0.3286											
16	0.3364											
17	0.3438											
18	0.3507											
19	0.3573											
20	0.3635											
21	0.3694											
22	0.3751											
23	0.3805											
24	0.3856											
25	0.3906											
26	0.3953											
27	0.3999											
28	0.4043											
29	0.4086											
30	0.4127											
31	0.4167											
32	0.4205											
33	0.4243											
34	0.4279											
35	0.4314											
36	0.4348											
37	0.4382											
38	0.4414											
39	0.4445											
40	0.4476											
41	0.4506											
42	0.4535											
43	0.4564											
44	0.4592											
45	0.4619											
46	0.4646											
47	0.4672											
48	0.4697											
49	0.4722											

At 60 Hz, in Ω/mi per conductor
$X_d = 0.2794 \log d$
d = separation, ft
For three-phase lines
$d = D_{eq}$

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†From *Electrical Transmission and Distribution Reference Book* by permission of the ABB Power T & D Company, Inc.

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TABLE A.5 Shunt capacitance-reactance spacing factor X_d at 10 Hz (megaohm-miles per conductor)

Feet	Separation, Inches: 0	1	2	3	4	5	6	7	8	9	10	11
0	......	−0.0737	−0.0532	−0.0411	−0.0326	−0.0260	−0.0206	−0.0160	−0.0120	−0.0085	−0.0054	−0.0026
1	0	0.0024	0.0046	0.0066	0.0085	0.0103	0.0120	0.0136	0.0152	0.0166	0.0180	0.0193
2	0.0206	0.0218	0.0229	0.0241	0.0251	0.0262	0.0272	0.0282	0.0291	0.0300	0.0309	0.0318
3	0.0326	0.0334	0.0342	0.0350	0.0357	0.0365	0.0372	0.0379	0.0385	0.0392	0.0399	0.0405
4	0.0411	0.0417	0.0423	0.0429	0.0435	0.0441	0.0446	0.0452	0.0457	0.0462	0.0467	0.0473
5	0.0478	0.0482	0.0487	0.0492	0.0497	0.0501	0.0506	0.0510	0.0515	0.0519	0.0523	0.0527
6	0.0532	0.0536	0.0540	0.0544	0.0548	0.0552	0.0555	0.0559	0.0563	0.0567	0.0570	0.0574
7	0.0577	0.0581	0.0584	0.0588	0.0591	0.0594	0.0598	0.0601	0.0604	0.0608	0.0611	0.0614
8	0.0617											
9	0.0652											
10	0.0683											
11	0.0711											
12	0.0737											
13	0.0761											
14	0.0783											
15	0.0803											
16	0.0823											
17	0.0841											
18	0.0858											
19	0.0874											
20	0.0889											
21	0.0903											
22	0.0917											
23	0.0930											
24	0.0943											
25	0.0955											
26	0.0967											
27	0.0978											
28	0.0989											
29	0.0999											
30	0.1009											
31	0.1019											
32	0.1028											
33	0.1037											
34	0.1046											
35	0.1055											
36	0.1063											
37	0.1071											
38	0.1079											
39	0.1087											
40	0.1094											
41	0.1102											
42	0.1109											
43	0.1116											
44	0.1123											
45	0.1129											
46	0.1136											
47	0.1142											
48	0.1149											
49	0.1155											

At 60 Hz, in MΩ·mi per conductor
$X_d' = 0.06831 \log d$
d = separation, ft
For three-phase lines
$d = D_{eq}$

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Book Problem 6.5

Capacitive Reactance: $x_c = x_a' + x_d'$

$$From\ Table\ A.3:\ x_a' = 0.1057\ \ M\Omega/mile$$

$$From\ Table\ A.5: x_d' = 0.0858 + \frac{0.0016}{10} \times 9 = 0.0872\ \ M\Omega/mile$$

$$\Rightarrow x_c = 0.1929\ \ M\Omega/mile$$

$$\Rightarrow y_c = 0.5184 \times 10^{-5}\ s/mile$$

$$2)\ Z = (0.3372 + j0.8146)\ \ \Omega/mile\ \ \times 70\ mile$$

$$Y = 70 \frac{1}{0.1929 \times 10^6}\ s = 3.6288 \times 10^{-4} S$$

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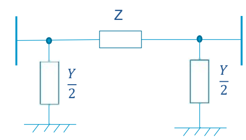
Book Problem 6.5



$$Z = (0.3372 + j0.8146)\ \ \Omega/mile\ \ \times 70\ mile$$

$$Y = 70 \frac{1}{0.1929 \times 10^6}\ s = 3.6288 \times 10^{-4} s$$

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