## MUPET Inference details

## March 12, 2025

For all experiments using DiffPIR, we used 1000 iterations and  $\lambda=1$  because this is the value that is most line with classical inverse problem solvers. We also used a batch-size of 32 and performed the tests on Nvidia A40 GPUs or slower. Per inverse problem considered, the tests took between one and two and a half hours to cover the whole data-set.

Tables 1, 2, 3 and 4 are hyper-parameter tests that were run on a single batch of 16 images from LSUN bedroom using the DiffPIR algorithm [1] in VE form as seen in Algorithm 1. Bold denotes the best performance per hyper-parameter pair, while the lowest LPIPS for each score-function is underlined. The hyper-parameters that produced the underlined results were used for the experiments in the main paper.

This means for pixel-wise completion, we used  $\zeta=0.6$  and  $\sigma_T=1$ , for block-wise completion, we used  $\zeta=0.3$  and  $\sigma_T=10$ , while for super-resolution we used  $\zeta=0.6$  and  $\sigma_T=0.1$  for the score-function obtained via MUPET, and  $\zeta=0.3$  and  $\sigma_T=0.01$  for the score-function obtained via DSM. For deblurring, we used  $\zeta=0.6$  and  $\sigma_T=0.1$  for the MUPET score and  $\zeta=0.3$  and  $\sigma_T=1$  for the DSM score.

The hyper-parameter tests suggest that larger selection of  $\sigma_T$  is advised when the inverse problem considered produces observations that are far away from the ground-truth image and neighboring pixels cannot make up for this missing information. Block-wise in-painting appears to be the most affected by this issue.

## References

[1] Yuanzhi Zhu, Kai Zhang, Jingyun Liang, Jiezhang Cao, Bihan Wen, Radu Timofte, and Luc Van Gool. Denoising diffusion models for plug-and-play image restoration. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 1219–1229, 2023.

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Data: Noisy \boldsymbol{x}_T, observation \boldsymbol{y}, noise-level \sigma_n, sequence of noise-levels (\sigma)_i and degradation \boldsymbol{A}, steering parameter \lambda > 0, mixing parameter \zeta \in [0,1];

Result: Image drawn from the posterior \boldsymbol{x}_0;

for i=T,\ldots,1 do
\begin{array}{c} \rho_i \leftarrow \lambda \frac{\sigma_n}{\sigma_i};\\ \boldsymbol{x}_0^{(i)} \leftarrow \boldsymbol{x}_i + \sigma_i^2 s_{\boldsymbol{\theta}}(\boldsymbol{x}_i,\sigma_i);\\ \boldsymbol{x}_0^{(i)} = \operatorname{argmin}_{\boldsymbol{x}} \|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}\|_2^2 + \rho_i \|\boldsymbol{x} - \boldsymbol{x}_0^{(i)}\|_2^2;\\ \hat{\boldsymbol{\epsilon}}_i \leftarrow \boldsymbol{x}_i - \hat{\boldsymbol{x}}_0^{(i)};\\ \boldsymbol{\epsilon}_i \sim \mathcal{N}(0,\boldsymbol{I});\\ \boldsymbol{x}_{i-1} \leftarrow \hat{\boldsymbol{x}}_0^{(i)} + \sigma_{i-1}(\sqrt{1-\zeta}\hat{\boldsymbol{\epsilon}}_i + \sqrt{\zeta}\boldsymbol{\epsilon}_i);\\ \text{and} \end{array}
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endAlgorithm 1: Variance exploding version of DiffPIR used for evaluation

$-\zeta$	$\sigma_T$	Model	PSNR↑	SSIM↑	LPIPS↓
0.0	0.01	DSM	12.639	0.2609	1.0550
		MUPET	12.655	0.2643	1.0517
	0.1	DSM	12.745	0.2333	1.0752
		MUPET	12.803	0.2399	1.0677
0.0	1.0	DSM	13.279	0.1977	1.1031
		MUPET	13.386	0.2015	1.0912
	10.0	DSM	13.280	0.1666	1.0542
	10.0	MUPET	13.264	0.1667	1.0332
	0.01	DSM	12.813	0.2095	0.9659
	0.01	MUPET	15.632	0.3750	0.7737
	0.1	DSM	15.548	0.3917	0.6719
0.3	0.1	MUPET	17.850	0.4590	0.6026
0.3	1.0	DSM	24.216	0.7513	0.1854
	1.0	MUPET	24.090	0.7493	0.1803
	10.0	DSM	23.899	0.7345	0.2060
	10.0	MUPET	23.857	0.7351	0.1986
	0.01	DSM	11.515	0.1535	0.9870
		MUPET	17.939	0.4699	0.6221
	0.1	DSM	19.022	0.5681	0.4005
0.6		MUPET	21.337	0.6378	0.3301
0.6	1.0	DSM	25.059	0.7843	0.1775
		MUPET	24.867	0.7770	0.1650
	10.0	DSM	24.946	0.7782	0.1843
		MUPET	24.861	0.7740	0.1698
	0.01	DSM	9.2260	0.0762	1.0024
		MUPET	20.180	0.5821	0.4838
	0.1	DSM	24.018	0.7670	0.2073
1.0		MUPET	24.413	0.7729	0.1796
1.0	1.0	DSM	25.284	0.7912	0.1885
		MUPET	25.107	0.7836	0.1797
	10.0	DSM	25.318	0.7885	0.1983
		MUPET	25.201	0.7846	0.1836

Table 1: Hyper-parameter grid-search for 90% pixel-wise completion. Bold values indicate the highest MSE, highest SSIM, and lowest LPIPS in each pair of rows.

ζ	$\sigma_T$	Model	PSNR↑	SSIM↑	LPIPS↓
	0.01	DSM	23.759	0.9312	0.0764
		MUPET	23.762	0.9316	0.0764
	0.1	DSM	23.740	0.9293	0.0757
0.0		MUPET	23.740	0.9296	0.0757
0.0	1.0	DSM	23.623	0.9272	0.0719
		MUPET	23.613	0.9269	0.0721
	10.0	DSM	23.177	0.9249	0.0634
	10.0	MUPET	23.125	0.9247	0.0647
	0.01	DSM	23.805	0.9338	0.0725
		MUPET	23.791	0.9341	0.0732
	0.1	DSM	23.856	0.9351	0.0692
0.3		MUPET	23.632	0.9372	0.0711
0.5	1.0	DSM	25.203	0.9425	0.0513
	1.0	MUPET	25.296	0.9455	0.0495
	10.0	DSM	28.016	0.9507	0.0330
	10.0	MUPET	28.006	0.9532	0.0347
	0.01	DSM	23.766	0.9338	0.0728
		MUPET	23.663	0.9364	0.0733
	0.1	DSM	23.970	0.9360	0.0679
0.6		MUPET	23.404	0.9369	0.0720
0.0	1.0	DSM	26.165	0.9505	0.0478
		MUPET	26.304	0.9475	0.0488
	10.0	DSM	28.232	0.9528	0.0375
		MUPET	28.044	0.9546	0.0380
	0.01	DSM	23.654	0.9366	0.0729
		MUPET	23.362	0.9360	0.0737
	0.1	DSM	24.119	0.9400	0.0668
1.0		MUPET	23.549	0.9383	0.0721
1.0	1.0	DSM	27.680	0.9519	0.0457
		MUPET	27.550	0.9549	0.0450
	10.0	DSM	28.586	0.9563	0.0422
		MUPET	28.126	0.9556	0.0436

Table 2: Hyper-parameter search for block-wise in painting with a  $64 \times 64$  box of pixels missing. Bold values indicate the highest MSE, highest SSIM, and lowest LPIPS in each pair of rows.

ζ	$\sigma_T$	Model	PSNR↑	SSIM↑	LPIPS↓
0.0	0.01	DSM	24.440	0.7512	0.3816
	0.01	MUPET	24.444	0.7518	0.3881
	0.1	DSM	24.370	0.7327	0.3465
		MUPET	24.389	0.7348	0.3409
0.0	1.0	DSM	22.003	0.6032	0.3432
		MUPET	21.924	0.5898	0.3448
	10.0	DSM	19.218	0.4220	0.5565
	10.0	MUPET	19.096	0.4000	0.5770
	0.01	DSM	26.005	0.7816	0.2912
	0.01	MUPET	26.056	0.7783	0.2887
	0.1	DSM	26.136	0.7696	0.3173
0.3	0.1	MUPET	26.449	0.7722	0.2625
0.5	1.0	DSM	25.797	0.7437	0.3544
	1.0	MUPET	25.897	0.7470	0.3230
	10.0	DSM	25.239	0.7245	0.3762
	10.0	MUPET	25.264	0.7263	0.3553
	0.01	DSM	25.421	0.7730	0.2915
		MUPET	25.884	0.7750	0.2821
	0.1	DSM	26.405	0.7796	0.2941
0.6	0.1	MUPET	26.529	0.7785	0.2381
0.0	1.0	DSM	26.420	0.7662	0.3254
		MUPET	26.490	0.7705	0.2821
	10.0	DSM	26.173	0.7555	0.3448
		MUPET	26.218	0.7584	0.3089
	0.01	DSM	23.004	0.7316	0.3337
		MUPET	25.871	0.7695	0.3039
	0.1	DSM	26.587	0.7794	0.2927
1.0		MUPET	26.348	0.7772	0.2559
1.0	1.0	DSM	26.535	0.7729	0.3066
		MUPET	26.504	0.7728	0.2825
	10.0	DSM	26.475	0.7696	0.3215
		MUPET	26.534	0.7691	0.2976

Table 3: Comparison of DSM and MUPET models with different  $\zeta$  and  $\sigma_T$  values for 3x super-resolution on LSUN bedroom. Bold values indicate the highest MSE, highest SSIM, and lowest LPIPS in each pair of rows.

ζ	$\sigma_T$	Model	PSNR↑	SSIM↑	LPIPS↓
	0.01	DSM	24.699	0.7304	0.4517
		MUPET	24.710	0.7324	0.4588
	0.1	DSM	24.738	0.7231	0.4148
0.0		MUPET	24.777	0.7256	0.4137
0.0	1.0	DSM	24.303	0.6799	0.2905
		MUPET	24.270	0.6706	0.2846
	10.0	DSM	23.530	0.6193	0.3076
		MUPET	23.372	0.6080	0.3113
	0.01	DSM	26.005	0.7800	0.3190
	0.01	MUPET	26.288	0.7849	0.3002
	0.1	DSM	26.445	0.7818	0.3015
0.3		MUPET	26.765	0.7866	0.2315
0.5	1.0	DSM	26.550	0.7785	0.2757
		MUPET	26.641	0.7800	0.2338
	10.0	DSM	26.529	0.7741	0.2850
	10.0	MUPET	26.556	0.7757	0.2463
	0.01	DSM	25.487	0.7780	0.3150
		MUPET	26.184	0.7860	0.2886
	0.1	DSM	26.646	0.7902	0.2851
0.6	0.1	MUPET	26.864	0.7908	0.2153
0.0	1.0	DSM	26.705	0.7848	0.2823
		MUPET	26.840	0.7889	0.2310
	10.0	DSM	26.696	0.7819	0.2946
		MUPET	26.800	0.7857	0.2500
	0.01	DSM	23.358	0.7591	0.3401
		MUPET	26.026	0.7853	0.3021
	0.1	DSM	26.379	0.7916	0.2885
1.0		MUPET	26.579	0.7877	0.2446
1.0	1.0	DSM	26.870	0.7896	0.2911
		MUPET	26.924	0.7906	0.2587
	10.0	DSM	26.822	0.7866	0.2992
		MUPET	26.895	0.7887	0.2723

Table 4: Comparison of DSM and MUPET models with different  $\zeta$  and  $\sigma_T$  values for deblurring on LSUN bedroom (using a Gaussian blur-kernel with  $\sigma_{\rm blur}=2$ ). Bold values indicate the highest PSNR, highest SSIM, and lowest LPIPS in each pair of rows.