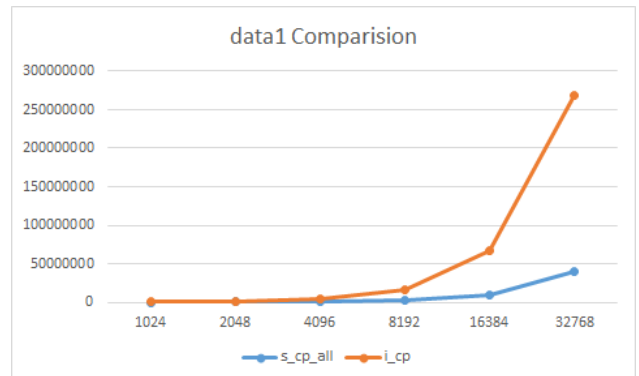
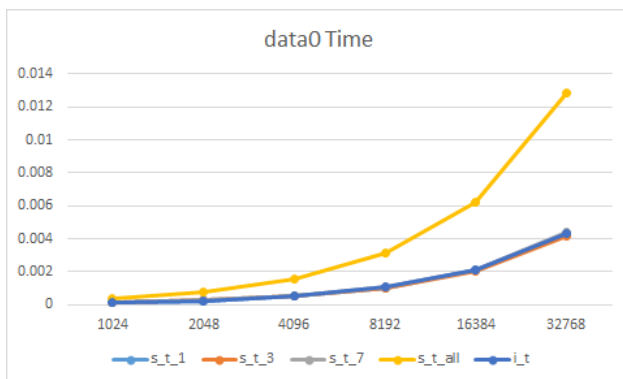
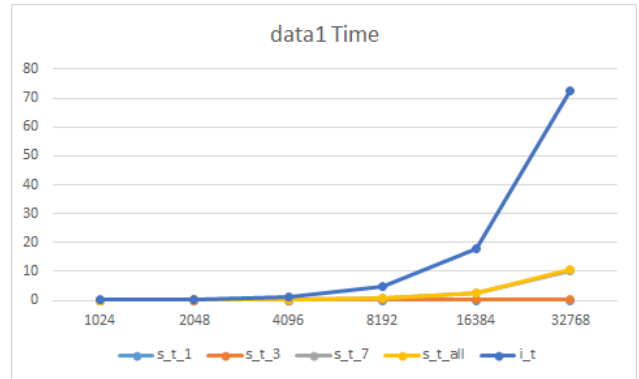


HOMEWOEK 2

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Question 1

As we known, shellsort would become insertion sort when h reduce to one. Meanwhile, the insertion sort will be more efficient when the sequence is almost sorted. The shellsort is more efficient because it take a little time to sort some shorter sequences divided from the original sequence, so that the sequence becomes an almost sorted sequence. By doing so, the insertion sort phase can save much more time.



As we can see in the graph, we deal with a sorted sequence. Time consumption and comparison amount of shellsort are both three times of the insertion sort, since insertion sort just do a traversal but shell do three times.

But as we can see, when we are dealing with a chaotic sequence, insertion sort takes much more time and comparison. Because every data is far from where it should be, it would takes more comparison to find its position.

Question 2

The result and time consumption are as follow:

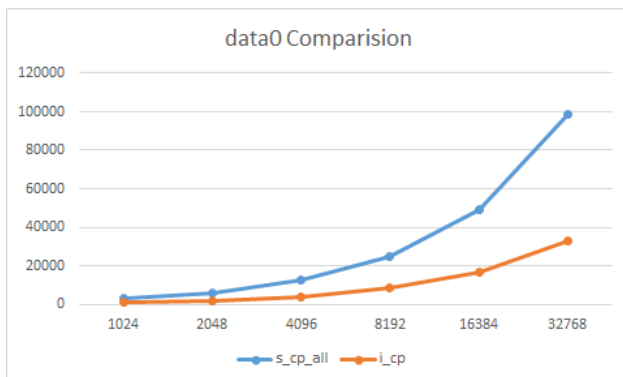
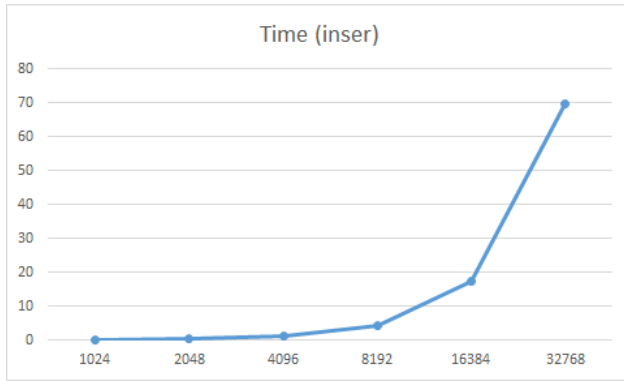
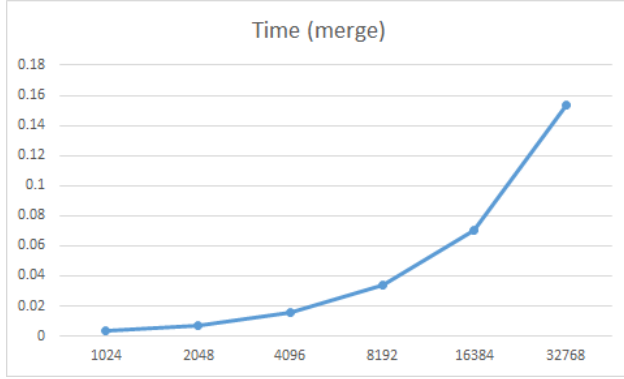


TABLE I
KENDALL TAU DISTANCE

1024	264541
2048	1027236
4096	4183804
8192	16928767
16384	66641183
32768	267933908



My basic logic of deciding KTD is to set one sequence as reference sequence, which means we will take whatever the first value in the reference sequence is the smallest and whatever the last value in the reference sequence is the largest value. Then we will resort the other sequence based on the index of the reference sequence. Then we find the inverse pairs in the 2nd sequence, we find the KTD.

The way to decide the amount of insert pairs would be to find the exchange time of a stable sort algorithm. Cause every exchange means the latter value is smaller than the former value, which is a pair of inverse values.

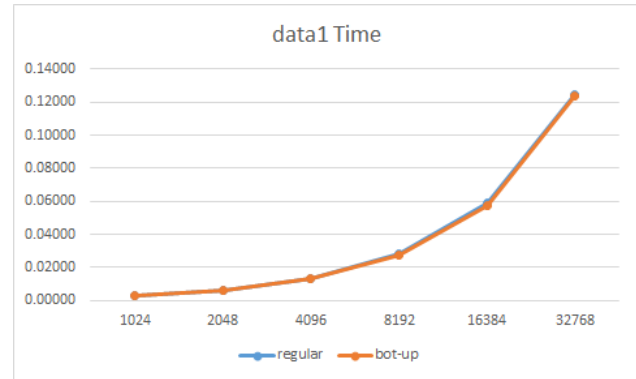
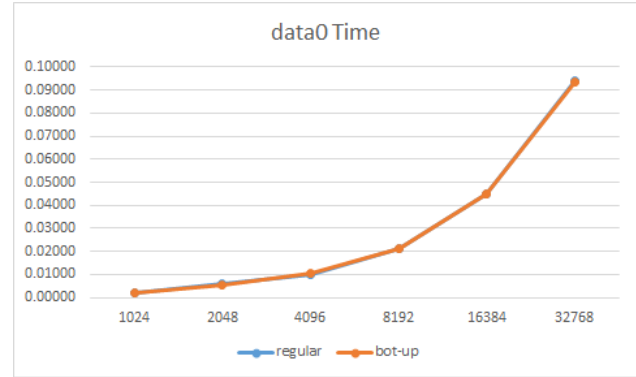
In my implementation, I use both merge sort and insertion sort to find KTD. As expected, the time consumption refers to $O(N \log N)$ and $O(N^2)$.

Question 3

Comparison and array access are the two operations should be considered in merge sort. For the comparisons, every time we finish a merge to the whole array takes N comparisons.

And we will take $\log N$ whole merge, since every time will take half of the array to merge together.

And for array access operation, we can decide from the amount of merge happens. Every merge is going to take a comparison, which means two reads of arrays. And after deciding the smaller value, we will write it to the auxiliary array, which means one read from the original array and one write to the auxiliary array. Then we will write the value from auxiliary array back to original array, which means one read from auxiliary array and one write to the original array. So we total have 6 array access in each merge, 2 writes and 4 reads. For whole process we will have $6N \log N$ array access.



From the time consumption we can see that the two algorithms as expected have a pretty similar time consumption.

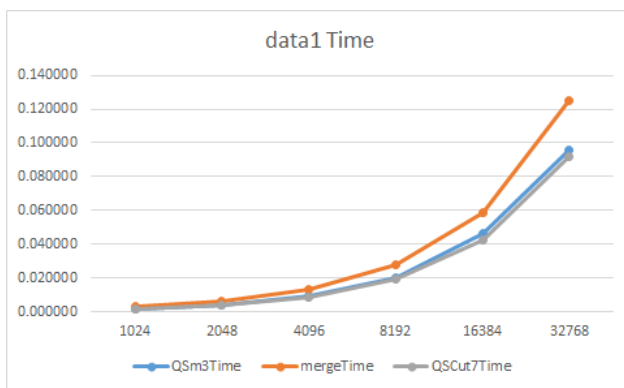
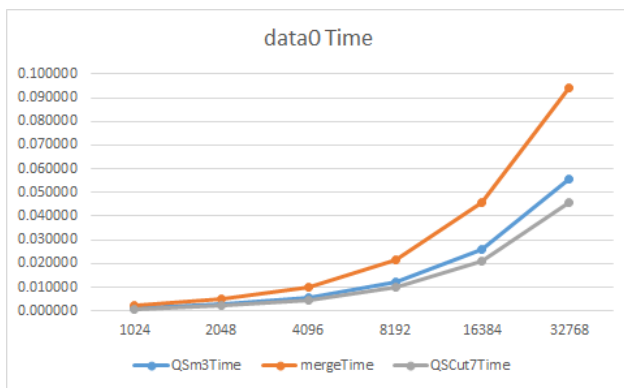
Question 4

For this problem, I assume we know that the sequence we are going to deal with has 1024 repeats of 1, 2048 repeats of 11, 4096 repeats of 111 and 1024 repeats of 1111. So I set four pointers in an auxiliary array points to the positions where the 4 entities are supposed to start. So when one traversal, I can set the entity into where it should be by moving the pointers. Then we write back this auxiliary array to sort it. The process takes N comparisons and $2N$ writes. Which I think should be the smallest time consumption.

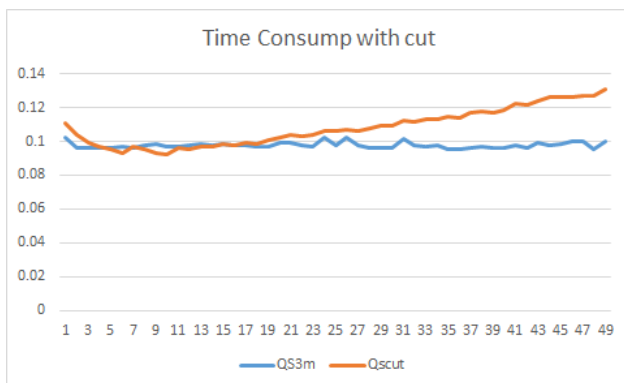
Question 5

As we can see on the follow graph, Quicksort always shows a better performance compared to mergesort. Also, when we

use $N = 7$ as cut-off to insertion, it can save optimize the Quicksort's performance.



Here is the graph shows the time consumption change with cut-off boundary. As we can see when cut-off value falls in range from 7 to 15, cut-off to insertion can help optimizing the Quicksort performance. But when it grows greater than 19, it can not optimize Quicksort any more and time consumptoin shows a increasing trend as teh value keep growing.



Question 6

We mark the sequence from left to right as 1 to 8 except the input and the result. 1

- 1) Mergesort (bottom-up). Every 4 item are sorted.

- 2) Quicksort (standard, no shuffle). Every entities before navy is smaller than it and every entities after navy is greater. And palm is not been exchanged.
- 3) Knuth shuffle. All the entities after silk are in place and plum, pink are not in place, which means this is not insertion sort.
- 4) Mergesort (top-down). The first 12 entities are sorted and the 13 to 18, 19 to 24 are sorted.
- 5) Insertion sort. Every thing beform silk is sorted and every thing after silk is the same as origin.
- 6) Heapsort. If we fill this sequence into a tree from 0 to 23, we can see that all child nodes are greater than their parents.
- 7) Selection sort. Every entity on the right of silk is larger and every entity on the left are sorted.
- 8) Quicksort (3-way, no shuffle). Every entities before navy is smaller than it and every entities after navy is greater. And palm is been exchanged, because it is the median of plum, lime and palm.