Condition: If m and N is large in relation to n, then hypergeometric distribution can be approximated by Binomial distribution!
Suppose X is a hypergeometric R.V with parameter (n,N,m). Y is a binomial R.V with parameter (n, p=m/N). When N is large, Var(X)=np(1-p) so it will be the same as done with replacement!

$$p(x=i)=\frac{\binom{m}{i}\binom{N-m}{n-i}}{\binom{N}{n}},$$

$$i=0,1,\ldots,n=\binom{n}{i}p^{i}(1-p)^{n-i}=P(Y=i)$$

By letting p=m/N. Since m, N are large, p is constant.

Condition: If n=1, the Binomial distribution is reduced to Bernoulli distribution!
A Bernoulli R.V is just a special case of Binomial R.V with parameter (1,p).

# Cross Functional Flow chart

## Uniform discrete

Let X be a random variable that can take on one of n distinct values,x1,x2,...,xn, with each value having the same probability of occurring. Then X is said to be a Uniform discrete random variable.
The probability mass function (PMF) of X is given by

$$p(X=xi)=\frac{1}{n},$$

$$i=1,2,\ldots,n$$

The set of values X can take is finite or countably infinite, typically {a,a+1,,,,,b}, where a and b are min and max.

$$E[X]=\frac{a+b}{2}$$

$$Var(x)=\frac{(b-a+1)^2-1}{12}$$

## Bernouli

Let X be a random variable that represents the outcome of a Bernoulli trial. Then X is said to be a Bernoulli random variable with parameter $p$, where p is the probability of success (i.e., the probability that $X$ = 1) and 1−p is the probability of failure (i.e., the probability that X=0).
PMF of such Bernoulli R.V is given by

$$\begin{cases} p(0)=1-p \\ p(1)=p \end{cases}$$

E[X]=p
Var(x)=p(1-p)

Bernoulli random variables are often used to model binary outcomes, such as success/ failure, true/false, yes/no situations in experiments.

## Binomial

Suppose now that n independent trials, each of which results a prob p of success and 1-p of failure. Then X is said to be a binomial R.V with parameter (n,p) if X represent the number of success that occurred in n trials.
PMF of a such binomial R.V is given by

$$p(X=i)=\binom{n}{i}p^{i}(1-p)^{n-i}$$

$$,i=0,1,\ldots,n$$

E[X]=np
Var(x)=np(1-p)

5 coins toss, each with probability of heads of ½. Assume outcomes are independent, find PMF of number of Heads?
Let X denotes number of heads got. Then X is a binomial R.V with parameter(5, 1/2).

$$p(0)=\binom{5}{0}(1/2)^{0}(1/2)^{5-0}=1/32$$

$$p(1)=\binom{5}{1}(1/2)^{1}(1/2)^{5-1}=5/32$$

$$p(2)=\binom{5}{2}(1/2)^{2}(1/2)^{5-2}=10/32$$

$$p(3)=\binom{5}{3}(1/2)^{3}(1/2)^{5-3}=10/32$$

$$p(4)=\binom{5}{4}(1/2)^{4}(1/2)^{5-4}=5/32$$

$$p(5)=\binom{5}{5}(1/2)^{5}(1/2)^{5-5}=1/32$$

## Hypergeometric

Let X be a random variable representing the number of successes in a sample of size n drawn without replacement from a population of size N that contains m successes. Then X is said to be a Hypergeometric random variable with parameters N (the population size), m (the number of successes in the population), and n (the sample size).
PMF of a such Hypergeometric R.V is given by

$$p(x=i)=\frac{\binom{m}{i}\binom{N-m}{n-i}}{\binom{N}{n}},$$

$$i=0,1,\ldots,n$$

E[X]=np
Var(x)=np(1-p)(1-(n-1)/(N-1))