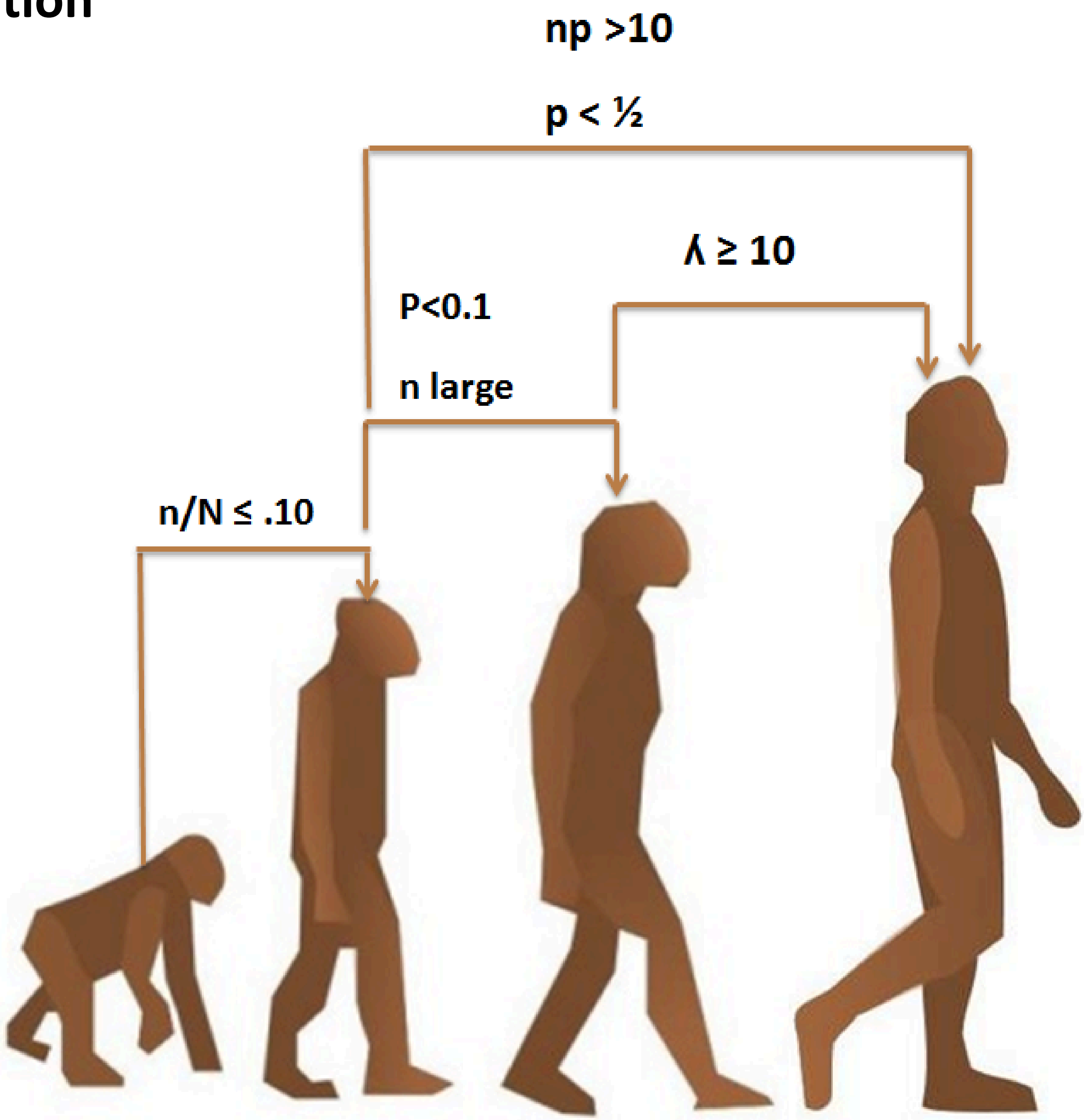
Approximation



Binomial Approximation to Hypergeometric

Hypergeometric Distribution

$$f(x)= \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{K}}$$

Binomial Distribution

$$f(x)= \binom{n}{x}p^x(1-p)^{(n-x)}$$

$$\frac{\boldsymbol{n}}{\boldsymbol{N}} \le 0.10$$

Example:

Suppose that a production lot of 100 Industrial Lamps contains 5 defective parts.

Find the probability that a random sample of 10 lamps from the lot contains no more than 1 defective part.

Using Hypergeometric Distribution:

$$P(X \le 1) = P(X = 0) + P(X = 1) = \frac{\binom{5}{0}\binom{100-5}{10-0}}{\binom{100}{10}} + \frac{\binom{5}{1}\binom{100-5}{10-1}}{\binom{100}{10}}=0.923$$

Binomial Approximation:

$$\frac{\boldsymbol{n}}{\boldsymbol{N}} = \frac{10}{100} = 0.10 \qquad \boldsymbol{p} = \frac{\boldsymbol{K}}{\boldsymbol{N}} = \frac{5}{100} = 0.05$$

f(x)= $\binom{n}{x}p^{x}(1-p)^{(n-x)}$

$P(X \le 1) = P(X=0) + P(X=1) = \sum_{x=0}^{x=1}\binom{10}{x}0.05^{x}\,(1-0.05)^{(10-x)}$=0.914

TABLE A.3 Cumulative Binomial Distribution

n	X	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50
		p = Probability of Occurrence									
2	0	.903	.810	.772	.640	.563	.490	.423	.360	.303	.250
	1	.998	.990	.978	.960	.938	.910	.878	.840	.798	.750
3	0	.857	.729	.614	.512	.422	.343	.275	.216	.166	.125
	1	.993	.972	.939	.896	.844	.784	.718	.648	.575	.500
	2	1.000	.999	.997	.992	.984	.973	.957	.936	.909	.875
4	0	.815	.656	.522	.410	.316	.240	.179	.130	.092	.063
	1	.986	.948	.890	.819	.738	.652	.563	.475	.391	.313
	2	1.000	.996	.988	.973	.949	.916	.874	.821	.759	.687
	3		1.000	.999	.998	.996	.992	.985	.974	.959	.938
5	0	.774	.590	.444	.328	.237	.168	.116	.078	.050	.031
	1	.977	.919	.835	.737	.633	.528	.428	.337	.256	.188
	2	.999	.991	.973	.942	.896	.837	.765	.683	.593	.500
	3	1.000	1.000	.998	.993	.984	.969	.946	.913	.869	.813
	4			1.000	1.000	.999	.998	.995	.990	.982	.969
6	0	.735	.531	.377	.262	.178	.118	.075	.047	.028	.016
	1	.967	.886	.776	.655	.534	.420	.319	.233	.164	.109
	2	.998	.984	.953	.901	.831	.744	.647	.544	.442	.344
	3	1.000	.999	.994	.983	.962	.930	.883	.821	.745	.656
	4		1.000	1.000	.998	.995	.989	.978	.959	.931	.891
	5				1.000	1.000	.999	.998	.996	.992	.984
7	0	.698	.478	.321	.210	.133	.082	.049	.028	.015	.008
	1	.956	.850	.717	.577	.445	.329	.234	.159	.102	.063
	2	.996	.974	.926	.852	.756	.647	.532	.420	.316	.227
	3	1.000	.997	.988	.967	.929	.874	.800	.710	.608	.500
	4		1.000	.999	.995	.987	.971	.944	.904	.847	.773
	5			1.000	1.000	.999	.996	.991	.981	.964	.938
	6					1.000	1.000	.999	.998	.996	.992
8	0	.663	.430	.272	.168	.100	.058	.032	.017	.008	.004
	1	.943	.813	.657	.503	.367	.255	.169	.106	.063	.035
	2	.994	.962	.895	.797	.679	.552	.428	.315	.220	.145
	3	1.000	.995	.979	.944	.886	.806	.706	.594	.477	.363
	4		1.000	.997	.990	.973	.942	.894	.826	.740	.637
	5			1.000	.999	.996	.989	.975	.950	.912	.855
	6				1.000	1.000	.999	.996	.991	.982	.965
	7						1.000	1.000	.999	.998	.996
9	0	.630	.387	.232	.134	.075	.040	.021	.010	.005	.002
	1	.929	.775	.599	.436	.300	.196	.121	.071	.039	.020
	2	.992	.947	.859	.738	.601	.463	.337	.232	.150	.090
	3	.999	.992	.966	.914	.834	.730	.609	.483	.361	.254
	4	1.000	.999	.994	.980	.951	.901	.828	.733	.621	.500
	5		1.000	.999	.997	.990	.975	.946	.901	.834	.746
	6			1.000	1.000	.999	.996	.989	.975	.950	.910
	7					1.000	1.000	.999	.996	.991	.980
	8							1.000	1.000	.999	.998
10	0	.599	.349	.197	.107	.056	.028	.013	.006	.003	.001
	1	.914	.736	.544	.376	.244	.149	.086	.046	.023	.011
	2	.988	.930	.820	.678	.526	.383	.262	.167	.100	.055
	3	.999	.987	.950	.879	.776	.650	.514	.382	.266	.172
	4	1.000	.998	.990	.967	.922	.850	.751	.633	.504	.377

Binomial Distribution

f(x)= $\binom{n}{x}p^x(1-p)^{(n-x)}$

Poisson distribution:

F(x) = $\frac{\lambda^x e^{-\lambda}}{x!}$

p≤0.10 and n Large

Example:

The proportion of industrial lamps that are defective is 0.04. Find the probability that 3 or fewer parts are defective in a sample size of 100.

By using Binomial Distribution:

$P(X \leq 3) = \sum_{x=0}^{x=3}\binom{100}{x}0.04^x\,(1-0.04)^{(100-x)}$=0.429

Poisson Approximation:

P=0.04≤0.10 and n=100 is Large

$$\mu = np = 100(0.04) = 4$$

$$(X \leq 3) = \sum_{x=0}^{x=3} \frac{4^x e^{-4}}{x!} = 0.433$$

TABLE A.2 Cumulative Poisson Distribution

	λ = Mean										
X	*.01*	*.05*	*.1*	*.2*	*.3*	*.4*	*.5*	*.6*	*.7*	*.8*	*.9*
0	.990	.951	.905	.819	.741	.670	.607	.549	.497	.449	.407
1	1.000	.999	.995	.982	.963	.938	.910	.878	.844	.809	.772
2		1.000	1.000	.999	.996	.992	.986	.977	.966	.953	.937
3				1.000	1.000	.999	.998	.997	.994	.991	.987
4						1.000	1.000	1.000	.999	.999	.998
5									1.000	1.000	1.000

X	*1.0*	*1.1*	*1.2*	*1.3*	*1.4*	*1.5*	*1.6*	*1.7*	*1.8*	*1.9*	*2.0*
0	.368	.333	.301	.273	.247	.223	.202	.183	.165	.150	.135
1	.736	.699	.663	.627	.592	.558	.525	.493	.463	.434	.406
2	.920	.900	.879	.857	.833	.809	.783	.757	.731	.704	.677
3	.981	.974	.966	.957	.946	.934	.921	.907	.891	.875	.857
4	.996	.995	.992	.989	.986	.981	.976	.970	.964	.956	.947
5	.999	.999	.998	.998	.997	.996	.994	.992	.990	.987	.983
6	1.000	1.000	1.000	1.000	.999	.999	.999	.998	.997	.997	.995
7					1.000	1.000	1.000	1.000	.999	.999	.999
8									1.000	1.000	1.000

X	*2.2*	*2.4*	*2.6*	*2.8*	*3.0*	*3.5*	*4.0*	*4.5*	*5.0*	*5.5*	*6.0*
0	.111	.091	.074	.061	.050	.030	.018	.011	.007	.004	.002
1	.355	.308	.267	.231	.199	.136	.092	.061	.040	.027	.017
2	.623	.570	.518	.469	.423	.321	.238	.174	.125	.088	.062
3	.819	.779	.736	.692	.647	.537	.433	.342	.265	.202	.151
4	.928	.904	.877	.848	.815	.725	.629	.532	.440	.358	.285
5	.975	.964	.951	.935	.916	.858	.785	.703	.616	.529	.446
6	.993	.988	.983	.976	.966	.935	.889	.831	.762	.686	.606
7	.998	.997	.995	.992	.988	.973	.949	.913	.867	.809	.744
8	1.000	.999	.999	.998	.996	.990	.979	.960	.932	.894	.847
9		1.000	1.000	.999	.999	.997	.992	.983	.968	.946	.916
10				1.000	1.000	.999	.997	.993	.986	.975	.957
11						1.000	999	.998	.995	.989	.980
12							1.000	.999	.998	.996	.991
13								1.000	.999	.998	.996
14									1.000	.999	.999
15										1.000	.999
16											1.000