1/31/2026

Home-Work 2

3.21 (a) Lot Size : $N = 25$

Sample Size ; $n = 5$

Acceptance rule ; Accept lot if 0 nonconforming components in Sample.

→ Since we're sampling without replacing from a finite lot, we use the hypergeometric distribution.

$N = 25$

$n = 5$

$K = 2$

$x = 0$

$$P(X=x) = f(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

$$\Rightarrow P(X=0) = \frac{\binom{2}{0}\binom{25-2}{5-0}}{\binom{25}{5}} = \frac{\binom{2}{0}\binom{23}{5}}{\binom{25}{5}}$$

$$= \frac{\left(\frac{2!}{0!(2-0)!}\right)\left(\frac{23!}{5!(23-5)!}\right)}{\left(\frac{25!}{5!(25-5)!}\right)} \quad \frac{33,649}{53130} \approx 0.63$$

$P(X=0) = 63\%$

(b) Approximating hypergeometric to binomial

$P = K/N = 2/25 = 0.08$

$n = 5$

$x = 0$

$$f(x) = \binom{n}{x} p^x (1-p)^{(n-x)}$$

$$\Rightarrow P(X=0) = \binom{5}{0} 0.08^0 (1-0.08)^{(5-0)}$$

$$= \left(\frac{5!}{0!(5-0)!}\right) 0.08^0 (0.92)^{(5)}$$

$$= \left(5!/5!\right) 0.08^0 (0.659081523) = (1)(1)(0.659081523)$$

$$\approx 0.66$$

$P(X=0) = 66\%$

hypergeometric = 0.6333 , Binomial = 0.6591

Absolute error = $|0.6591 - 0.6333| = 0.0258$

Relative absolute error = $0.0258/0.6333 \approx 0.041 = 4\%$

→ No because the $n/N = 0.20 > n/N \leq 0.10$ condition for approximating binomial to hypergeometric distribution.

(c) Suppose $N = 150$, $n = 5$

$n/N = 5/150 \approx 0.033 \leq 0.1$ satisfy the condition for approximating binomial to hypergeometric distribution

Checking : hypergeometric :

$$P(X=0) = \frac{\binom{2}{0}\binom{148}{5}}{\binom{150}{5}} = \frac{\binom{148}{5}}{\binom{150}{5}} = \frac{\left(\frac{148!}{5!(148-5)!}\right)}{\left(\frac{150!}{5!(150-5)!}\right)}$$

$P(X=0) = 0.934228188$

Binomial approximation :

$P = 2/150 = 0.013$

$$P(X=0) = \binom{5}{0} 0.013^0 (1-0.013)^{(5-0)} = \left(\frac{5!}{0!(5-0)!}\right) 0.013^0 (0.9867)^{(5)}$$

$$= \left(\frac{5!}{5!}\right) 0.013^0 (0.935087565) = (1)(1)(0.93508756) = 0.93508756$$

$P(X=0) = 0.93508756$

Absolute error = $|0.93508756 - 0.934228188| = 0.000859375$

Relative absolute error = $0.000859375/0.934228188 \approx 0.00092 = 0.09\%$

∴ $n/N \leq 0.1$ is satisfied and the relative error is significant small (0.09%) so it's satisfactory when $N = 150$

(d) $P(X=0) \leq 0.05$ using hypergeometric pmf, assuming $K=5$ nonconforming items and $N=25$

$P(X=0) \leq 0.05$ or $P(X=0) = 1-P(X=0) \geq 0.95$

$$P(X=0) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

Checking: when $n=10$,

$$P(X=0) = \frac{\binom{5}{0}\binom{25-5}{10-0}}{\binom{25}{10}} = \frac{\binom{5}{0}\binom{20}{10}}{\binom{25}{10}} = \frac{\left(\frac{5!}{0!(5-0)!}\right)\left(\frac{20!}{10!(20-10)!}\right)}{\left(\frac{25!}{10!(25-10)!}\right)}$$

$$= \frac{\left(\frac{20!}{10!\,10!}\right)}{\left(\frac{25!}{10!\,15!}\right)} = \frac{184756}{3268760} = 0.056521739 \approx 0.06$$

> 0.05

when $n=11$,

$$P(X=0) = \frac{\binom{5}{0}\binom{25-5}{11-0}}{\binom{25}{11}} = \frac{\binom{5}{0}\binom{20}{11}}{\binom{25}{11}} = \frac{\left(\frac{5!}{0!(5-0)!}\right)\left(\frac{20!}{11!(20-11)!}\right)}{\left(\frac{25!}{11!(25-11)!}\right)}$$

$$= \frac{\left(\frac{20!}{11!\,19!}\right)}{\left(\frac{25!}{11!\,14!}\right)} = \frac{167960}{4457400} = 0.037681159 \approx 0.04$$

< 0.05

$1 - 0.037681159 \approx 0.96$

Since $0.0376 \leq 0.05$, the minimum sample size is $n=11$

3.23 $P(a \le X \le b) = \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)$, $Z = \frac{X-\mu}{\sigma}$

$LCL = 4.95 \to a$, $UCL = 5.05 \to b$, $\mu = 5$, $\sigma = 0.02$

$P(4.95 \le X \le 5.05) = \Phi\left(\frac{5.05-5}{0.02}\right) - \Phi\left(\frac{4.95-5}{0.02}\right)$

$= \Phi(2.5) - \Phi(-2.5)$ or $2\Phi(2.5) - 1$

$= 0.993790 - 0.0062101$

$= 0.$[illegible]$799 \approx 0.99$

3.27 $P(X < x) = P\left(Z < \frac{x-\mu}{\sigma}\right)$, Let lower limit $= x$

$\mu = 5000$, $\sigma = 50$, $0.5\% \to 0.005$

$P\left(Z \le \frac{x-5000}{50}\right) = 0.005$

$Z_{0.005} = \frac{x-5000}{50} \Rightarrow -2.58 = \frac{x-5000}{50}$

$\Rightarrow -129 = x - 5000 \Rightarrow x = 5000 - 129$

$\Rightarrow x = 4871$, $\therefore$ Lower Limit $= 4871$ end Foot Candles

3.44 Original process $(\lambda = 0.01)$

$\lambda = 0.01$, $K = 1$, Pmf : $f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

$P(X=1) = \frac{e^{-0.01}(0.01)^1}{1!} = 0.009900498$

Improved process $(\lambda = 0.01/2 = 0.005)$

$P'(X=1) = \frac{e^{-0.005}(0.005)^1}{1!} = 0.004975062$

$\frac{P'}{P} = \frac{0.004975062}{0.009900498} = 0.502506231 \approx 0.50 = 50\%$

Effect: Comparing 0.004975 to 0.00990, the probability of finding exctly one defect is cut approximately in half (reduced by $\approx 50\%$)

3.29 Since $np = 0.01 \times 100 = 1$, $p = 0.01$. we can't approximate to normal distribution but $p < 0.10$ and $n = 100$ (Large) we can use the Poisson distribution.

$\because p < 0.10$, $n = 100$; $\hat{p} = \frac{x}{n}$, where x is the number of nonconforming units.

$$\lambda = np = 100(0.01) = 1$$

PMF: $P(X = x) = \frac{1^x e^{-1}}{x!} = \frac{e^{-1}}{x!}$

$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.01(0.99)}{100}} = \sqrt{0.000099} \approx 0.00995$

We want to find the probability that $\hat{p}$ exceeds p by k standard deviations: $P(\hat{p} + k\sigma_{\hat{p}})$

$P(X > np + nk\sigma_{\hat{p}})$

$P(X > 1 + 100k(0.00995))$

$P(X > 1 + 0.995k)$

for $k=1$
$\hat{p} = 0.01 + 1 \times 0.00995$
$= 0.01995$

For $k = 1$, Threshold: $X > 1 + 0.995(1) = 1.995$, Since X must be an Integer, we are looking for $P(X \geq 2)$.

$P(X \geq 2) = 1 - [P(X=0) + P(X=1)]$

For $k=2$
$\hat{p} = 0.01 + 2 \times 0.00995$
$= 0.0299$

$\left[\frac{e^{-1}}{0!} + \frac{e^{-1}}{1!}\right] = 0.36788 + 0.36788$

$= 1 - (0.36788 + 0.36788) = 1 - 0.73576$

$P \approx 0.2642$

For $k = 2$, Threshold: $X > 1 + 0.995(2) = 1 + 1.99 = 2.99$

$P(X \geq 3) = 1 - [P(X=0) + P(X=1) + P(X=2)]$ -2

For $k=3$; $\hat{p} = 0.01 + 3 \times 0.00995 = 0.03985$

For $k=1$, $Z = \frac{0.01995-0.01}{0.00995} = 1$ $\quad P(Z \geq 1) = 0.1587$

$k=2$, $Z = \frac{0.0299-0.01}{0.00995} = 2$ $\quad P(Z \geq 2) = 0.0228$

$P(X=2) = \frac{e^{-1}}{2} = \frac{0.36788}{2} \approx 0.18394$

$K=3$, $Z = \frac{0.03985-0.01}{0.00995} = 3$, $\quad P(Z \geq 3) = 0.00135$

$P(X \leq 2) = 0.73576 + 0.18394 = 0.91970$

$P(X \geq 3) = 1 - 0.91970$

$P \approx 0.0803$

For $K=3$, threshold: $X \geq 1 + 0.995(3) = 1 + 2.985 = 3.985$

$P(X \geq 4) = 1 - P(X \leq 3)$

$P(X=3) = \frac{e^{-1}}{3!} = \frac{0.36788}{6} \approx 0.06131$

$P(X \leq 3) = 0.91970 + 0.06131 = 0.98101$

$P(X \geq 4) = 1 - 0.98101$

$P \approx 0.0190$

For $K=1$: 0.2642

For $K=2$: 0.0803

For $K=3$: 0.0190