

Binomial Distribution

The random variable X that equals the number of trials that result in a success is a binomial random variable with parameters $0 < p < 1$ and $n = 1, 2, \dots$

The probability mass function is:

For constants a and b , the binomial expansion is



Exercise 3-18: Organic Pollution-1

Each sample of water has a 10% chance of containing a particular organic pollutant. Assume that the samples are independent with regard to the presence of the pollutant. Find the probability that, in the next 18 samples, exactly 2 contain the pollutant.

Answer:

Let X denote the number of samples that contain the pollutant in the next 18 samples analyzed. Then X is a binomial random variable with $p = 0.1$ and $n = 18$

$$P(X=2) = \binom{18}{2} (0.1)^2 (0.9)^{16} = 153 (0.1)^2 (0.9)^{16} = 0.2835$$

Using Table:

$$P(X=2) = P(X \leq 2) - P(X \leq 1) = 0.734 - 0.450 = 0.284$$



TABLE A.3 Cumulative Binomial Distribution *continued*

<i>n</i>	<i>X</i>	<i>p</i> = Probability of Occurrence									
		.05	.10	.15	.20	.25	.30	.35	.40	.45	.50
18	0	.397	.150	.054	.018	.006	.002	.000	.000	.000	.000
	1	.774	.450	.224	.099	.039	.014	.005	.001	.000	.000
	2	.942	.734	.480	.271	.135	.060	.024	.008	.003	.001
	3	.989	.902	.720	.501	.306	.165	.078	.033	.012	.004
	4	.998	.972	.879	.716	.519	.333	.189	.094	.041	.015
	5	1.000	.994	.958	.867	.717	.534	.355	.209	.108	.048
	6		.999	.988	.949	.861	.722	.549	.374	.226	.119
	7		1.000	.997	.984	.943	.859	.728	.563	.391	.240
	8			.999	.996	.981	.940	.861	.737	.578	.407
	9			1.000	.999	.995	.979	.940	.865	.747	.593
	10				1.000	.999	.994	.979	.912	.873	.760



Exercise 3-18: Organic Pollution-2

Determine the probability that at least 4 samples contain the pollutant.

Answer:

$$\begin{aligned} P(X \geq 4) &= 1 - P(X < 4) \\ &= 1 - \sum_{x=0}^3 \binom{18}{x} (0.1)^x (0.9)^{18-x} \\ &= 1 - [0.150 + 0.300 + 0.284 + 0.168] \\ &= 0.098 \end{aligned}$$

Using Table:

$$P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0.902 = 0.098$$



Exercise 3-18: Organic Pollution-3

Now determine the probability that $3 \leq X < 7$.

Answer:

$$\begin{aligned} P(3 \leq X < 7) &= \sum_{x=3}^6 \binom{18}{x} (0.1)^x (0.9)^{18-x} \\ &= 0.168 + 0.070 + 0.022 + 0.005 \\ &= 0.265 \end{aligned}$$

Using Table:

$$P(3 \leq X < 7) = P(X \leq 6) - P(X \leq 2) = 0.999 - 0.734 = 0.265$$



Binomial Mean and Variance

If X is a binomial random variable with parameters p and n ,

$$\mu = E(X) = np$$

and

$$\sigma^2 = V(X) = np(1-p)$$



Example 3-19:

For the number of transmitted bit received in error in Example 3-16, $n = 4$ and $p = 0.1$. Find the mean and variance of the binomial random variable.

Answer:

$$\mu = E(X) = np = 4 * 0.1 = 0.4$$

$$\sigma^2 = V(X) = np(1-p) = 4 * 0.1 * 0.9 = 0.36$$

$$\sigma = SD(X) = 0.6$$

