# ISYE435/535 Exp Design For Engineering
# Assignment 1 Key

Prof.: Reinaldo Moraga – T.A.: Ibrahim Oyeyinka

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## Problem 1(a)

The sample mean is

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\bar{x} = \frac{249.6 + 250.4 + 249.9 + 250.1 + 249.7 + 250.3 + 249.8 + 250.2 + 250.0 + 249.5 + 250.6 + 249.9}{12}$$

$$= \frac{3000}{12} = 250$$

**Sample mean:** $\bar{x} = 250$ grams.

## Problem 1(b)

Sample variance:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$s^2 = \frac{1}{11}(1.22) = 0.110909$$

$$s = \sqrt{0.110909} = 0.33303$$

**Sample Variance:** $s^2 = 0.1109$ grams$^2$

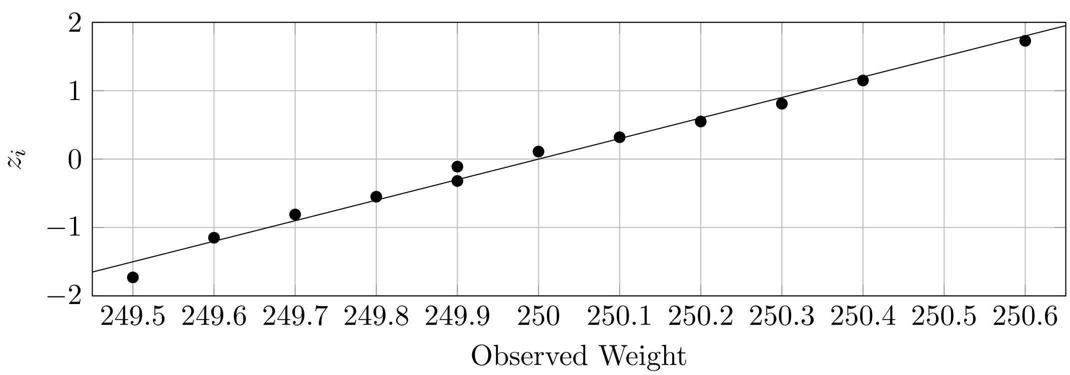
**Sample Standard Deviation:** $s = 0.3330$ grams

## Problem 1(c)

Normal Probability Plot: Ordered data with plotting positions

$$p_i = \frac{i - 0.5}{12}, \qquad z_i = \Phi^{-1}(p_i)$$

| $i$ | $x_{(i)}$ | $p_i$ | $z_i$ |
|---|---|---|---|
| 1 | 249.5 | 0.0417 | -1.73 |
| 2 | 249.6 | 0.1250 | -1.15 |
| 3 | 249.7 | 0.2083 | -0.81 |
| 4 | 249.8 | 0.2917 | -0.55 |
| 5 | 249.9 | 0.3750 | -0.32 |
| 6 | 249.9 | 0.4583 | -0.11 |
| 7 | 250.0 | 0.5417 | 0.11 |
| 8 | 250.1 | 0.6250 | 0.32 |
| 9 | 250.2 | 0.7083 | 0.55 |
| 10 | 250.3 | 0.7917 | 0.81 |
| 11 | 250.4 | 0.8750 | 1.15 |
| 12 | 250.6 | 0.9583 | 1.73 |




The plotted points follow an approximately linear pattern with no clear systematic curvature or extreme outliers. **Thus, the normality assumption appears reasonable.**

## Problem 1(d)

### Model Assumptions

- The sample is randomly selected.
- The observations are independent.
- The population distribution is approximately normal.
- The population variance $\sigma^2$ is unknown.

Since $\sigma$ is unknown and $n = 12$ is small, we use the $t$-distribution with

$$df = n - 1 = 11.$$

**95% Confidence Interval**

The confidence interval for the population mean is

$$\bar{x} \pm t_{\alpha/2,\,n-1}\frac{s}{\sqrt{n}}$$

Given:

$$\bar{x} = 250, \quad s = 0.3330, \quad n = 12, \quad t_{0.025,11} = 2.201$$

$$\frac{s}{\sqrt{n}} = \frac{0.3330}{\sqrt{12}} = 0.0961$$

$$\text{Margin of error} = 2.201(0.0961) = 0.2115$$

$$250 \pm 0.2115$$

$$249.7885 \le \mu \le 250.2115$$

Since 250 lies within the interval, we fail to reject $H_0$ at $\alpha = 0.05$.

## Problem 2(a)

**(Paired $t$-Test)**

**Assumptions**

- The 12 inspectors constitute a random sample (or representative sample) of measurement conditions.
- Each inspector measures the *same* ball bearing using both calipers (paired).
- The paired differences $d_i = x_{1i} - x_{2i}$ are independent across inspectors.
- The population of differences is approximately normal: $d_i \sim N(\mu_d, \sigma_d^2)$.

**Data (inches)**

Define $x_{1i}$ = Caliper 1 reading, $x_{2i}$ = Caliper 2 reading, and

$$d_i = x_{1i} - x_{2i}.$$

| Inspector | Caliper 1 | Caliper 2 | $d_i$ | $d_i^2$ |
|---|---|---|---|---|
| 1 | 0.265 | 0.264 | 0.001 | 0.000001 |
| 2 | 0.265 | 0.265 | 0 | 0 |
| 3 | 0.266 | 0.264 | 0.002 | 0.000004 |
| 4 | 0.267 | 0.266 | 0.001 | 0.000001 |
| 5 | 0.267 | 0.267 | 0 | 0 |
| 6 | 0.265 | 0.268 | −0.003 | 0.000009 |
| 7 | 0.267 | 0.264 | 0.003 | 0.000009 |
| 8 | 0.267 | 0.265 | 0.002 | 0.000004 |
| 9 | 0.265 | 0.265 | 0 | 0 |
| 10 | 0.268 | 0.267 | 0.001 | 0.000001 |
| 11 | 0.268 | 0.268 | 0 | 0 |
| 12 | 0.265 | 0.269 | −0.004 | 0.000016 |
| | | Totals: | $\sum d_i = 0.003$ | $\sum d_i^2 = 0.000045$ |

Let $n = 12$.

**Hypotheses**

A difference in mean measurement corresponds to testing the mean of the paired differences:

$$H_0: \ \mu_1 = \mu_2 \quad \iff \quad H_0 : \mu_d = 0$$

$$H_A: \ \mu_1 \neq \mu_2 \quad \iff \quad H_A : \mu_d \neq 0$$

**Compute $\bar{d}$ and $s_d$**

Sample mean of differences:

$$\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i = \frac{0.003}{12} = 0.00025$$

Sample variance of differences (computational form):

$$s_d^2 = \frac{\sum d_i^2 - \frac{(\sum d_i)^2}{n}}{n-1}$$

$$s_d^2 = \frac{0.000045 - \frac{(0.003)^2}{12}}{11} = \frac{0.000045 - 0.00000075}{11} = \frac{0.00004425}{11} = 0.0000040227$$

$$s_d = \sqrt{0.0000040227} = 0.002006$$

Standard error:

$$SE(\bar{d}) = \frac{s_d}{\sqrt{n}} = \frac{0.002006}{\sqrt{12}} = 0.000579$$

## Problem 2(b)

### Given

$$\sum d_i = 0.003, \quad \sum d_i^2 = 0.000045, \quad n = 12$$

**Assumptions**

- Random sample of inspectors
- Differences are independent
- Differences are normally distributed

**Hypotheses**

$$H_0 : \mu_d = 0, \qquad H_A : \mu_d \neq 0$$

**Test Statistic**

$$\bar{d} = \frac{0.003}{12} = 0.00025$$

$$s_d = 0.002006$$

$$t_0 = \frac{0.00025}{0.002006/\sqrt{12}} = 0.43$$

$$df = 11$$

**P-value (two-sided)**

$$P = 2P(T_{11} > 0.43) = 0.674$$

**Decision Rule**
Reject $H_0$ if $P \leq 0.05$.
Since

$$0.674 > 0.05,$$

we fail to reject $H_0$.
**Conclusion**
There is no significant difference between the two calipers at the 5% level.

## Problem 2(c)

### 95% Confidence Interval for $\mu_d$

We use the paired $t$ confidence interval:

$$\bar{d} - t_{\alpha/2,\,n-1}\frac{s_d}{\sqrt{n}} \leq \mu_d \leq \bar{d} + t_{\alpha/2,\,n-1}\frac{s_d}{\sqrt{n}}$$

$$\bar{d} = 0.00025, \quad s_d = 0.002006, \quad n = 12, \quad df = 11$$

For $\alpha = 0.05$,

$$t_{0.025,11} = 2.201$$

$$0.00025 - 2.201\left(\frac{0.002006}{\sqrt{12}}\right) \le \mu_d \le 0.00025 + 2.201\left(\frac{0.002006}{\sqrt{12}}\right)$$

$$\frac{0.002006}{\sqrt{12}} = 0.000579$$

$$0.00025 - 2.201(0.000579) \le \mu_d \le 0.00025 + 2.201(0.000579)$$

$$2.201(0.000579) = 0.00127$$

$$-0.00102 \le \mu_d \le 0.00152$$

**Conclusion**

Since 0 lies within the interval, there is no significant difference between the mean measurements of the two calipers at the 5% level.

## Problem 3(a)

### Assumptions

- A random sample is taken from each machine.
- The two samples are independent.
- The fill volumes from each machine are normally distributed.
- The population standard deviations are known: $\sigma_1 = 0.015$, $\sigma_2 = 0.018$.

**Compute Sample Means**

Machine 1 data:

$$16.03,\ 16.01,\ 16.04,\ 15.96,\ 16.05,\ 15.98,\ 16.05,\ 16.02,\ 16.02,\ 15.99$$

$$\bar{y}_1 = \frac{16.03 + 16.01 + 16.04 + 15.96 + 16.05 + 15.98 + 16.05 + 16.02 + 16.02 + 15.99}{10} = 16.015$$

Machine 2 data:

$$16.02,\ 16.03,\ 15.97,\ 16.04,\ 15.96,\ 16.02,\ 16.01,\ 16.01,\ 15.99,\ 16.00$$

$$\bar{y}_2 = \frac{16.02 + 16.03 + 15.97 + 16.04 + 15.96 + 16.02 + 16.01 + 16.01 + 15.99 + 16.00}{10} = 16.005$$

Thus,

$$\bar{y}_1 = 16.015, \quad \bar{y}_2 = 16.005$$

$$n_1 = n_2 = 10$$

**Hypotheses**

$$H_0 : \mu_1 = \mu_2$$

$$H_A : \mu_1 \neq \mu_2$$

## Problem 3(b)

**Test Statistic**

Since population standard deviations are known, we use a two-sample $z$-test:

$$z_0 = \frac{\bar{y}_1 - \bar{y}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$z_0 = \frac{16.015 - 16.005}{\sqrt{\frac{0.015^2}{10} + \frac{0.018^2}{10}}} = 1.35$$

At $\alpha = 0.05$ (two-sided test),

$$z_{0.025} = 1.96$$

Decision rule:

$$\text{Reject } H_0 \text{ if } |z_0| > 1.96$$

Since

$$|1.35| < 1.96$$

we **fail to reject** $H_0$.

## Problem 3(c)

**P-value**

For a two-sided test,

$$P = 2P(Z > |z_0|).$$

Since $z_0 = 1.35$,

$$P = 2P(Z > 1.35).$$

From the standard normal table,

$$P(Z < 1.35) = 0.9115,$$

so

$$P(Z > 1.35) = 1 - 0.9115 = 0.0885.$$

$$P = 2(0.0885) = 0.1770.$$

**Decision Rule:**

Reject $H_0$ if $P \leq \alpha = 0.05$.

Since

$$0.1770 > 0.05,$$

we fail to reject $H_0$ at the 5% significance level.

## Problem 3(d)

**(95% Confidence Interval for $\mu_1 - \mu_2$)**

The confidence interval is

$$(\bar{y}_1 - \bar{y}_2) \pm z_{0.025}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(16.015 - 16.005) \pm 1.96\sqrt{\frac{0.015^2}{10} + \frac{0.018^2}{10}}$$

$$0.01 \pm 0.0145$$

$$-0.0045 \leq \mu_1 - \mu_2 \leq 0.0245$$

Since 0 lies in the interval, this confirms that we do not have sufficient evidence to conclude a difference between the two machines.

**Final Conclusion**

At the 5% significance level, there is insufficient evidence to conclude that the two machines fill to different mean volumes.

## Point Breakdown (Assignment 1)

- **Problem 1: Snack Packages (40 points total)**
  1(a) Sample mean (15) 1(b) Sample variance & standard deviation (15) 1(c) Normal probability plot & normality (5) 1(d) 95% CI for $\mu$ & test $H_0$ (5)
- **Problem 2: Calipers & Inspectors (15 points total)**
  2(a) Test for difference in means (5) 2(b) P-value (5) 2(c) 95% CI for difference in means (5)
- **Problem 3: Filling Machines (45 points total)**
  3(a) Hypotheses (15) 3(b) Test at $\alpha = 0.05$ & conclusion (15) 3(c) P-value (5) 3(d) 95% CI for difference in means (10)