# Normal Distribution

A random variable $X$ with probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \qquad -\infty < x < \infty$$

is a normal random variable with parameters $\mu$, where $-\infty < \mu < \infty$, and $\sigma > 0$. Also,

$$E(X) = \mu \quad \text{and} \quad V(X) = \sigma^2$$

and the notation $N(\mu, \sigma^2)$ is used to denote the distribution.

# Empirical Rule

For any normal random variable,

$P(\mu - \sigma < X < \mu + \sigma) = 0.6827$

$P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.9545$

$P(\mu - 3\sigma < X < \mu + 3\sigma) = 0.9973$

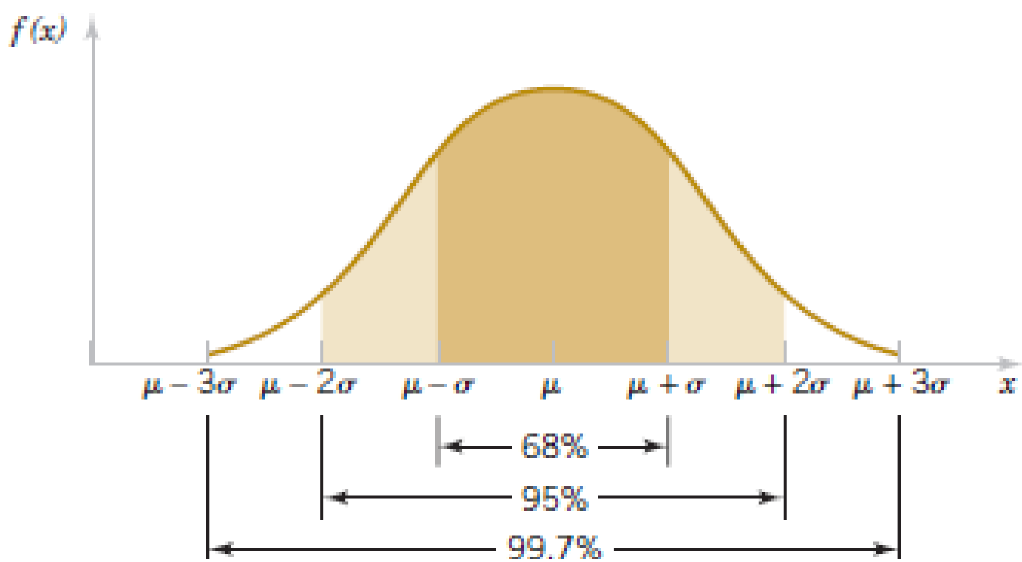



Figure 4-12 Probabilities associated with a normal distribution

# Standardizing a Normal Random Variable

# Standard Normal Random Variable

A normal random variable with

$$\mu = 0 \text{ and } \sigma^2 = 1$$

is called a standard normal random variable and is denoted as $Z$. The cumulative distribution function of a standard normal random variable is denoted as:

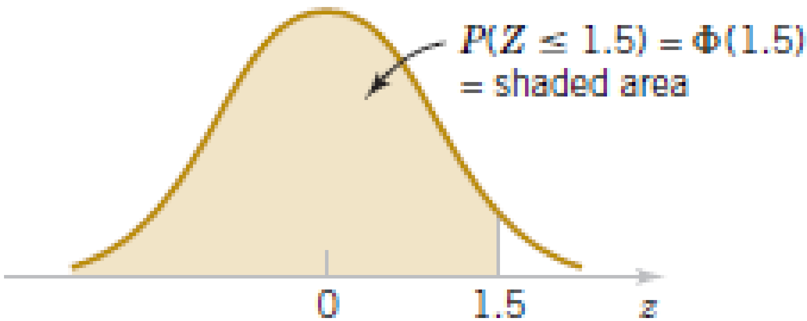
$$\Phi(z) = P(Z \leq z)$$

Values are found in Appendix Table III and by using Excel and Minitab.

# Standard Normal Distribution

Assume $Z$ is a standard normal random variable.

Find $P(Z \leq 1.50)$. Answer: 0.93319




| $z$ | 0.00 | 0.01 | 0.02 | 0.03 |
|---|---|---|---|---|
| 0 | 0.50000 | 0.50399 | 0.50398 | 0.51197 |
| ⋮ | | ⋮ | | |
| 1.5 | 0.93319 | 0.93448 | 0.93574 | 0.93699 |

Standard normal Probability density function

Find $P(Z \leq 1.53)$. Answer: 0.93699

Find $P(Z \leq 0.02)$. Answer: 0.50398

**NOTE :** The column headings refer to the hundredths digit of the value of z in $P(Z \leq z)$.
For example, $P(Z \leq 1.53)$ is found by reading down the z column to the row 1.5 and then selecting the probability from the column labeled 0.03 to be 0.93699.

## Example 1:

Suppose that the current measurements in a strip of wire are assumed to follow a normal distribution with $\mu = 10$ and $\sigma = 2$ mA, what is the probability that the current measurement is between 9 and 11 mA?

***Solution***:

$$P(a \le x \le b) = \Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})$$

$$P(9 \le x \le 11) = \Phi(\frac{11-10}{2}) - \Phi(\frac{9-10}{2})$$

$$= \Phi(+0.5) - \Phi(-0.5)$$

$$= 0.691462 - 0.308538 = 0.382924$$

## TABLE • III Cumulative Standard Normal Distribution

| z | −0.09 | −0.08 | −0.07 | −0.06 | −0.05 | −0.04 | −0.03 | −0.03 | −0.01 | −0.00 |
|---|---|---|---|---|---|---|---|---|---|---|
| −3.9 | 0.000033 | 0.000034 | 0.000036 | 0.000037 | 0.000039 | 0.000041 | 0.000042 | 0.000044 | 0.000046 | 0.00004 |
| −3.8 | 0.000050 | 0.000052 | 0.000054 | 0.000057 | 0.000059 | 0.000062 | 0.000064 | 0.000067 | 0.000069 | 0.00007 |
| −3.7 | 0.000075 | 0.000078 | 0.000082 | 0.000085 | 0.000088 | 0.000092 | 0.000096 | 0.000100 | 0.000104 | 0.00010 |
| −3.6 | 0.000112 | 0.000117 | 0.000121 | 0.000126 | 0.000131 | 0.000136 | 0.000142 | 0.000147 | 0.000153 | 0.00015 |
| −3.5 | 0.000165 | 0.000172 | 0.000179 | 0.000185 | 0.000193 | 0.000200 | 0.000208 | 0.000216 | 0.000224 | 0.00023 |
| −3.4 | 0.000242 | 0.000251 | 0.000260 | 0.000270 | 0.000280 | 0.000291 | 0.000302 | 0.000313 | 0.000325 | 0.00033 |
| −3.3 | 0.000350 | 0.000362 | 0.000376 | 0.000390 | 0.000404 | 0.000419 | 0.000434 | 0.000450 | 0.000467 | 0.00048 |
| −3.2 | 0.000501 | 0.000519 | 0.000538 | 0.000557 | 0.000577 | 0.000598 | 0.000619 | 0.000641 | 0.000664 | 0.00068 |
| −3.1 | 0.000711 | 0.000736 | 0.000762 | 0.000789 | 0.000816 | 0.000845 | 0.000874 | 0.000904 | 0.000935 | 0.00096 |
| −3.0 | 0.001001 | 0.001035 | 0.001070 | 0.001107 | 0.001144 | 0.001183 | 0.001223 | 0.001264 | 0.001306 | 0.001350 |
| −2.9 | 0.001395 | 0.001441 | 0.001489 | 0.001538 | 0.001589 | 0.001641 | 0.001695 | 0.001750 | 0.001807 | 0.001866 |
| −2.8 | 0.001926 | 0.001988 | 0.002052 | 0.002118 | 0.002186 | 0.002256 | 0.002327 | 0.002401 | 0.002477 | 0.002555 |
| −2.7 | 0.002635 | 0.002718 | 0.002803 | 0.002890 | 0.002980 | 0.003072 | 0.003167 | 0.003264 | 0.003364 | 0.003467 |
| −2.6 | 0.003573 | 0.003681 | 0.003793 | 0.003907 | 0.004025 | 0.004145 | 0.004269 | 0.004396 | 0.004527 | 0.004661 |
| −2.5 | 0.004799 | 0.004940 | 0.005085 | 0.005234 | 0.005386 | 0.005543 | 0.005703 | 0.005868 | 0.006037 | 0.006210 |
| −2.4 | 0.006387 | 0.006569 | 0.006756 | 0.006947 | 0.007143 | 0.007344 | 0.007549 | 0.007760 | 0.007976 | 0.008198 |
| −2.3 | 0.008424 | 0.008656 | 0.008894 | 0.009137 | 0.009387 | 0.009642 | 0.009903 | 0.010170 | 0.010444 | 0.010724 |
| −2.2 | 0.011011 | 0.011304 | 0.011604 | 0.011911 | 0.012224 | 0.012545 | 0.012874 | 0.013209 | 0.013553 | 0.013903 |
| −2.1 | 0.014262 | 0.014629 | 0.015003 | 0.015386 | 0.015778 | 0.016177 | 0.016586 | 0.017003 | 0.017429 | 0.017864 |
| −2.0 | 0.018309 | 0.018763 | 0.019226 | 0.019699 | 0.020182 | 0.020675 | 0.021178 | 0.021692 | 0.022216 | 0.022750 |
| −1.9 | 0.023295 | 0.023852 | 0.024419 | 0.024998 | 0.025588 | 0.026190 | 0.026803 | 0.027429 | 0.028067 | 0.028717 |
| −1.8 | 0.029379 | 0.030054 | 0.030742 | 0.031443 | 0.032157 | 0.032884 | 0.033625 | 0.034379 | 0.035148 | 0.035930 |
| −1.7 | 0.036727 | 0.037538 | 0.038364 | 0.039204 | 0.040059 | 0.040929 | 0.041815 | 0.042716 | 0.043633 | 0.044565 |
| −1.6 | 0.045514 | 0.046479 | 0.047460 | 0.048457 | 0.049471 | 0.050503 | 0.051551 | 0.052616 | 0.053699 | 0.054799 |
| −1.5 | 0.055917 | 0.057053 | 0.058208 | 0.059380 | 0.060571 | 0.061780 | 0.063008 | 0.064256 | 0.065522 | 0.066807 |
| −1.4 | 0.068112 | 0.069437 | 0.070781 | 0.072145 | 0.073529 | 0.074934 | 0.076359 | 0.077804 | 0.079270 | 0.080757 |
| −1.3 | 0.082264 | 0.083793 | 0.085343 | 0.086915 | 0.088508 | 0.090123 | 0.091759 | 0.093418 | 0.095098 | 0.096801 |
| −1.2 | 0.098525 | 0.100273 | 0.102042 | 0.103835 | 0.105650 | 0.107488 | 0.109349 | 0.111233 | 0.113140 | 0.115070 |
| −1.1 | 0.117023 | 0.119000 | 0.121001 | 0.123024 | 0.125072 | 0.127143 | 0.129238 | 0.131357 | 0.133500 | 0.135666 |
| −1.0 | 0.137857 | 0.140071 | 0.142310 | 0.144572 | 0.146859 | 0.149170 | 0.151505 | 0.153864 | 0.156248 | 0.158655 |
| −0.9 | 0.161087 | 0.163543 | 0.166023 | 0.168528 | 0.171056 | 0.173609 | 0.176185 | 0.178786 | 0.181411 | 0.184060 |
| −0.8 | 0.186733 | 0.189430 | 0.192150 | 0.194894 | 0.197662 | 0.200454 | 0.203269 | 0.206108 | 0.208970 | 0.211855 |
| −0.7 | 0.214764 | 0.217695 | 0.220650 | 0.223627 | 0.226627 | 0.229650 | 0.232695 | 0.235762 | 0.238852 | 0.241964 |
| −0.6 | 0.245097 | 0.248252 | 0.251429 | 0.254627 | 0.257846 | 0.261086 | 0.264347 | 0.267629 | 0.270931 | 0.274253 |
| −0.5 | 0.277595 | 0.280957 | 0.284339 | 0.287740 | 0.291160 | 0.294599 | 0.298056 | 0.301532 | 0.305026 | 0.308538 |
| −0.4 | 0.312067 | 0.315614 | 0.319178 | 0.322758 | 0.326355 | 0.329969 | 0.333598 | 0.337243 | 0.340903 | 0.344578 |
| −0.3 | 0.348268 | 0.351973 | 0.355691 | 0.359424 | 0.363169 | 0.366928 | 0.370700 | [illegible] | [illegible] | [illegible] |

## TABLE • III Cumulative Standard Normal Distribution (*Continued*)

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.500000 | 0.503989 | 0.507978 | 0.511967 | 0.515953 | 0.519939 | 0.532922 | 0.527903 | 0.531881 | 0.535856 |
| 0.1 | 0.539828 | 0.543795 | 0.547758 | 0.551717 | 0.555760 | 0.559618 | 0.563559 | 0.567495 | 0.571424 | 0.575345 |
| 0.2 | 0.579260 | 0.583166 | 0.587064 | 0.590954 | 0.594835 | 0.598706 | 0.602568 | 0.606420 | 0.610261 | 0.614092 |
| 0.3 | 0.617911 | 0.621719 | 0.625516 | 0.629300 | 0.633072 | 0.636831 | 0.640576 | 0.644309 | 0.648027 | 0.651732 |
| 0.4 | 0.655422 | 0.659097 | 0.662757 | 0.666402 | 0.670031 | 0.673645 | 0.677242 | 0.680822 | 0.684386 | 0.687933 |
| 0.5 | 0.691462 | 0.694974 | 0.698468 | 0.701944 | 0.705401 | 0.708840 | 0.712260 | 0.715661 | 0.719043 | 0.722405 |
| 0.6 | 0.725747 | 0.729069 | 0.732371 | 0.735653 | 0.738914 | 0.742154 | 0.745373 | 0.748571 | 0.751748 | 0.754903 |
| 0.7 | 0.758036 | 0.761148 | 0.764238 | 0.767305 | 0.770350 | 0.773373 | 0.776373 | 0.779350 | 0.782305 | 0.785236 |
| 0.8 | 0.788145 | 0.791030 | 0.793892 | 0.796731 | 0.799546 | 0.802338 | 0.805106 | 0.807850 | 0.810570 | 0.813267 |
| 0.9 | 0.815940 | 0.818589 | 0.821214 | 0.823815 | 0.826391 | 0.828944 | 0.831472 | 0.833977 | 0.836457 | 0.838913 |
| 1.0 | 0.841345 | 0.843752 | 0.846136 | 0.848495 | 0.850830 | 0.853141 | 0.855428 | 0.857690 | 0.859929 | 0.862143 |
| 1.1 | 0.864334 | 0.866500 | 0.868643 | 0.870762 | 0.872857 | 0.874928 | 0.876976 | 0.878999 | 0.881000 | 0.882977 |
| 1.2 | 0.884930 | 0.886860 | 0.888767 | 0.890651 | 0.892512 | 0.894350 | 0.896165 | 0.897958 | 0.899727 | 0.901475 |
| 1.3 | 0.903199 | 0.904902 | 0.906582 | 0.908241 | 0.909877 | 0.911492 | 0.913085 | 0.914657 | 0.916207 | 0.917736 |
| 1.4 | 0.919243 | 0.920730 | 0.922196 | 0.923641 | 0.925066 | 0.926471 | 0.927855 | 0.929219 | 0.930563 | 0.931888 |
| 1.5 | 0.933193 | 0.934478 | 0.935744 | 0.936992 | 0.938220 | 0.939429 | 0.940620 | 0.941792 | 0.942947 | 0.944083 |
| 1.6 | 0.945201 | 0.946301 | 0.947384 | 0.948449 | 0.949497 | 0.950529 | 0.951543 | 0.952540 | 0.953521 | 0.954486 |
| 1.7 | 0.955435 | 0.956367 | 0.957284 | 0.958185 | 0.959071 | 0.959941 | 0.960796 | 0.961636 | 0.962462 | 0.963273 |
| 1.8 | 0.964070 | 0.964852 | 0.965621 | 0.966375 | 0.967116 | 0.967843 | 0.968557 | 0.969258 | 0.969946 | 0.970621 |
| 1.9 | 0.971283 | 0.971933 | 0.972571 | 0.973197 | 0.973810 | 0.974412 | 0.975002 | 0.975581 | 0.976148 | 0.976705 |
| 2.0 | 0.977250 | 0.977784 | 0.978308 | 0.978822 | 0.979325 | 0.979818 | 0.980301 | 0.980774 | 0.981237 | 0.981691 |
| 2.1 | 0.982136 | 0.982571 | 0.982997 | 0.983414 | [illegible] | | | | | |

**Example 2:**

Determine the value for which the probability that a current measurement is below 0.98.

Solution:

$P(X<x)=P(Z<\frac{x-10}{2}) = 0.98$

$Z=\frac{x-10}{2}$

Z=2.05 is the closest value.

$2.05=\frac{x-10}{2}$

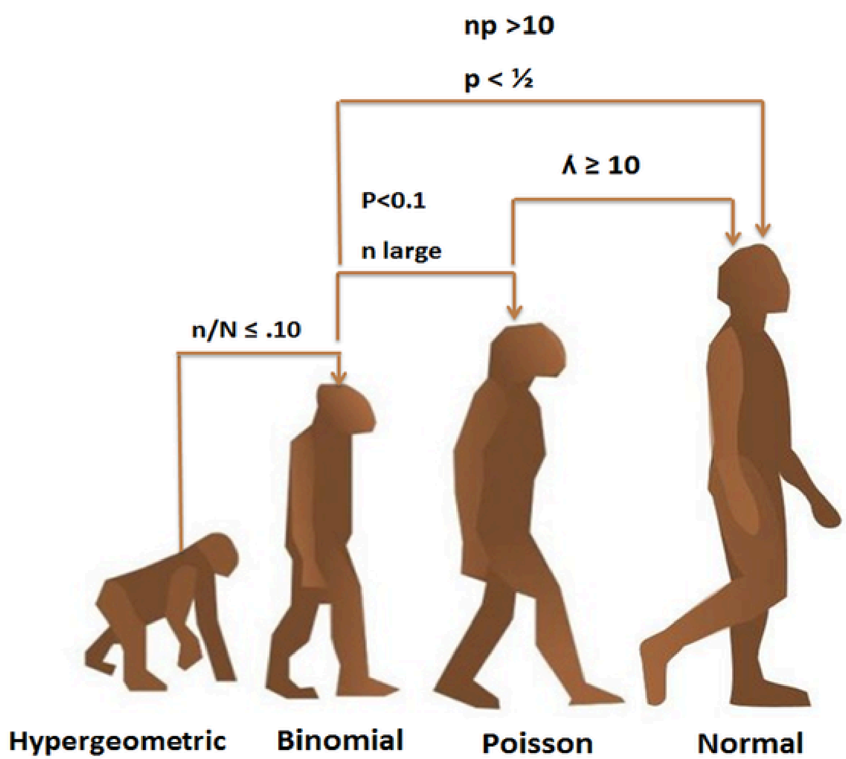
x=14.1 mA

## TABLE • III Cumulative Standard Normal Distribution (*Continued*)

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.500000 | 0.503989 | 0.507978 | 0.511967 | 0.515953 | 0.519939 | 0.532922 | 0.527903 | 0.531881 | 0.535856 |
| 0.1 | 0.539828 | 0.543795 | 0.547758 | 0.551717 | 0.555760 | 0.559618 | 0.563559 | 0.567495 | 0.571424 | 0.575345 |
| 0.2 | 0.579260 | 0.583166 | 0.587064 | 0.590954 | 0.594835 | 0.598706 | 0.602568 | 0.606420 | 0.610261 | 0.614092 |
| 0.3 | 0.617911 | 0.621719 | 0.625516 | 0.629300 | 0.633072 | 0.636831 | 0.640576 | 0.644309 | 0.648027 | 0.651732 |
| 0.4 | 0.655422 | 0.659097 | 0.662757 | 0.666402 | 0.670031 | 0.673645 | 0.677242 | 0.680822 | 0.684386 | 0.687933 |
| 0.5 | 0.691462 | 0.694974 | 0.698468 | 0.701944 | 0.705401 | 0.708840 | 0.712260 | 0.715661 | 0.719043 | 0.722405 |
| 0.6 | 0.725747 | 0.729069 | 0.732371 | 0.735653 | 0.738914 | 0.742154 | 0.745373 | 0.748571 | 0.751748 | 0.754903 |
| 0.7 | 0.758036 | 0.761148 | 0.764238 | 0.767305 | 0.770350 | 0.773373 | 0.776373 | 0.779350 | 0.782305 | 0.785236 |
| 0.8 | 0.788145 | 0.791030 | 0.793892 | 0.796731 | 0.799546 | 0.802338 | 0.805106 | 0.807850 | 0.810570 | 0.813267 |
| 0.9 | 0.815940 | 0.818589 | 0.821214 | 0.823815 | 0.826391 | 0.828944 | 0.831472 | 0.833977 | 0.836457 | 0.838913 |
| 1.0 | 0.841345 | 0.843752 | 0.846136 | 0.848495 | 0.850830 | 0.853141 | 0.855428 | 0.857690 | 0.859929 | 0.862143 |
| 1.1 | 0.864334 | 0.866500 | 0.868643 | 0.870762 | 0.872857 | 0.874928 | 0.876976 | 0.878999 | 0.881000 | 0.882977 |
| 1.2 | 0.884930 | 0.886860 | 0.888767 | 0.890651 | 0.892512 | 0.894350 | 0.896165 | 0.897958 | 0.899727 | 0.901475 |
| 1.3 | 0.903199 | 0.904902 | 0.906582 | 0.908241 | 0.909877 | 0.911492 | 0.913085 | 0.914657 | 0.916207 | 0.917736 |
| 1.4 | 0.919243 | 0.920730 | 0.922196 | 0.923641 | 0.925066 | 0.926471 | 0.927855 | 0.929219 | 0.930563 | 0.931888 |
| 1.5 | 0.933193 | 0.934478 | 0.935744 | 0.936992 | 0.938220 | 0.939429 | 0.940620 | 0.941792 | 0.942947 | 0.944083 |
| 1.6 | 0.945201 | 0.946301 | 0.947384 | 0.948449 | 0.949497 | 0.950529 | 0.951543 | 0.952540 | 0.953521 | 0.954486 |
| 1.7 | 0.955435 | 0.956367 | 0.957284 | 0.958185 | 0.959071 | 0.959941 | 0.960796 | 0.961636 | 0.962462 | 0.963273 |
| 1.8 | 0.964070 | 0.964852 | 0.965621 | 0.966375 | 0.967116 | 0.967843 | 0.968557 | 0.969258 | 0.969946 | 0.970621 |
| 1.9 | 0.971283 | 0.971933 | 0.972571 | 0.973197 | 0.973810 | 0.974412 | 0.975002 | 0.975581 | 0.976148 | 0.976705 |
| 2.0 | 0.977250 | 0.977784 | 0.978308 | 0.978822 | 0.979325 | 0.979818 | 0.980301 | 0.980774 | 0.981237 | 0.981691 |
| 2.1 | 0.982136 | 0.982571 | 0.982997 | 0.983414 | 0.983823 | 0.984222 | 0.984614 | 0.984997 | 0.985371 | 0.985738 |
| 2.2 | 0.986097 | 0.986447 | 0.986791 | 0.987126 | 0.987455 | 0.987776 | 0.988089 | 0.988396 | 0.988696 | 0.988989 |
| 2.3 | 0.989276 | 0.989556 | 0.989830 | 0.990097 | 0.990358 | 0.990613 | 0.990863 | 0.991106 | 0.991344 | 0.991576 |
| 2.4 | 0.991802 | 0.992024 | 0.992240 | 0.992451 | 0.992656 | 0.992857 | 0.993053 | 0.993244 | 0.993431 | 0.993613 |
| 2.5 | 0.993790 | 0.993963 | 0.994132 | 0.994297 | 0.994457 | 0.994614 | 0.994766 | 0.994915 | 0.995060 | 0.995201 |
| 2.6 | 0.995339 | 0.995473 | 0.995604 | 0.995731 | 0.995855 | 0.995975 | 0.996093 | 0.996207 | 0.996319 | 0.996427 |
| 2.7 | 0.996533 | 0.996636 | 0.996736 | 0.996833 | 0.996928 | 0.997020 | 0.997110 | 0.997197 | 0.997282 | 0.997365 |
| 2.8 | 0.997445 | 0.997523 | 0.997599 | 0.997673 | 0.997744 | 0.997814 | 0.997882 | 0.997948 | 0.998012 | 0.998074 |
| 2.9 | 0.998134 | 0.998193 | 0.998250 | 0.998305 | 0.998359 | 0.998411 | 0.998462 | 0.998511 | 0.998559 | 0.998605 |
| 3.0 | 0.998650 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |

# Normal Approximations

- The binomial and Poisson distributions become more bell-shaped and symmetric as their mean value increase.



# Normal Approximation to the Binomial Distribution

If $X$ is a binomial random variable with parameters $n$ and $p$,

$$Z = \frac{X - np}{\sqrt{np(1-p)}}$$

is approximately a standard normal random variable. To approximate a binomial probability with a normal distribution, a continuity correction is applied as follows:

$$P(X = a) = \Phi\left(\frac{a+0.5-np}{\sqrt{np(1-p)}}\right) - \Phi\left(\frac{a-0.5-np}{\sqrt{np(1-p)}}\right) \qquad (1)$$

$$P(a \le X \le b) = \Phi\left(\frac{b+0.5-np}{\sqrt{np(1-p)}}\right) - \Phi\left(\frac{a-0.5-np}{\sqrt{np(1-p)}}\right) \qquad (2)$$

**Example 3:**

**A production process has 25 percent defective rate. A sample size of 50 has been taken from this process. What is the probability that the sample contains between 12 and 14 defective parts?**

**Solution:**

**a) Using Binomial Distribution:**

f(x)= $\binom{n}{x} p^x (1-p)^{(n-x)}$

$P(12 \leq X \leq 14)$=$\sum_{x=12}^{x=14} \binom{50}{x} 0.25^x (1-0.25)^{50-x}$=0.3665

**b)Normal approximation to Binomial Distribution:**

$$P(a \leq X \leq b) = \Phi\left(\frac{b+0.5-np}{\sqrt{np(1-p)}}\right) - \Phi\left(\frac{a-0.5-np}{\sqrt{np(1-p)}}\right)$$

$$P(12 \leq X \leq 14) = \Phi\left(\frac{14+0.5-(50)(0.25)}{\sqrt{50(0.25)(1-0.25)}}\right) - \Phi\left(\frac{12-0.5-(50)(0.25)}{\sqrt{50(0.25)(1-0.25)}}\right)$$

$P(12 \leq X \leq 14) = \Phi(0.65) - \Phi(\text{-}0.33)$=0.72154-0.37077=0.371454

# Normal Approximation to the Poisson

If $X$ is a Poisson random variable with $E(X) = \lambda$ and $V(X) = \lambda$,

$$Z = \frac{X - \lambda}{\sqrt{\lambda}}$$

$$P(X = a) = \Phi\left(\frac{a+0.5-\lambda}{\sqrt{\lambda}}\right) - \Phi\left(\frac{a-0.5-\lambda}{\sqrt{\lambda}}\right) \qquad (1)$$

$$P(a \le X \le b) = \Phi\left(\frac{b+0.5-\lambda}{\sqrt{\lambda}}\right) - \Phi\left(\frac{a-0.5-\lambda}{\sqrt{\lambda}}\right) \qquad (2)$$

## Example:

The Number of hurricanes formed each fall in the Caribbean is Poisson distributed with mean λ= 14. What is the probability that between 10 and 18 hurricanes will be formed next fall?

**a) Using Poisson Distribution:**

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad \text{for} \quad x = 0, 1, 2, 3, \ldots$$

$$P(10 \le X \le 18) = \sum_{x=10}^{x=18} e^{-14}\, 14^x$$

By Using Table A-2 for Poisson distribution:

$P(10 \le X \le 18) =$P$(X \le 18)$- P$(X \le 9) = 0.883 - 0.774 = 0.774$

**b) Approximating Poisson with Normal Distribution**:

Condition:

λ=14>10

$$P(a \le X \le b) = \Phi(\frac{b+0.5-\lambda}{\sqrt{\lambda}}) - \Phi(\frac{a-0.5-\lambda}{\sqrt{\lambda}})$$

$$P(10 \le X \le 18) = \Phi(\frac{18+0.5-14}{\sqrt{14}}) - \Phi(\frac{10-0.5-14}{\sqrt{14}})$$

$P(10 \le X \le 18) = \Phi(1.2) - \Phi(-1.2)$=0.88493-0.115070=0.76986