Home Work 3

4.5 (a) $H_0 : \mu = 8.25\ cm$

$H_A : \mu \neq 8.25\ cm$

Given: $\sigma = 0.002\ cm$, Sample Size $n = 15$,

Sample mean $\bar{x} = 8.2535\ cm$, Significance level $\alpha = 0.05$

Z-test Statistic: $Z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

$$Z_0 = \frac{8.2535 - 8.25}{0.002/\sqrt{15}} = \frac{0.0035}{0.000516398}$$

$Z_0 = 6.77772$

$Z_0 \approx 6.78$

→ Determine the Critical value and make a decision

For a two sided test at $\alpha = 0.05$, The Critical values are $Z_{\alpha/2} \Rightarrow Z_{0.025} = 1.96$.

Since $|Z_0| = 6.78 > Z_{0.025} = 1.96$, Reject H_0.

* We will reject the null hypothesis and conclude that the mean inside bearing diameter isn't 8.25 cm.

(b) The probability of not detecting a shift when one has occurred is the type II error (β).

Given: $\mu_0 = 8.25$, $\mu_1 = 8.25$, $\sigma = 0.002$

$\mu_1 = \mu_0 + \delta \Rightarrow 8.255 - 8.25 \Rightarrow \delta = 0.005$

Upper bound $= \mu_0 + Z_{\alpha/2}\,\sigma/\sqrt{n} = 8.25 + 1.96\,(0.002/\sqrt{15})$

$= 8.25175$

Lower bound $= \mu_0 - Z_{\alpha/2}\,\sigma/\sqrt{n} = 8.25 - 1.96\,(0.002/\sqrt{5})$

$= 8.24825$

Scanned with CamScanner

$$\beta = \Phi\left(Z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}\right) - \Phi\left(-Z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}\right)$$

$$= \Phi\left(Z_{0.025} - \frac{0.005\sqrt{15}}{0.002}\right) - \Phi\left(Z_{0.025} - \frac{0.005\sqrt{15}}{0.002}\right)$$

$$= \Phi\left(1.96 - \frac{0.0193649 17}{0.002}\right) - \Phi\left(-1.96 - \frac{0.019364917}{0.002}\right)$$

$$= \Phi(-7.7224585) - \Phi(-11.6424585)$$

$\beta \approx 0$

* The probability of not detecting this shift (8.25 to 8.255 cm) is 0. The test will almost surely detect such a large shift.

4.6 (a) $H_0 : \mu = 25h$

$H_A : \mu > 25h$

Sample data: 25.5, 26.1, 26.8, 23.2, 24.2, 25.0, 27.8, 27.3, 25.7

$\bar{x} \approx 26.0h$, $s \approx 1.62h$, $\alpha = 0.05$

t-test statistic:

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

estimate s, since σ is unknown.

$$t_0 = \frac{26.0 - 25}{1.62/\sqrt{10}} \approx 1.95$$

reject H_0: $t_0 > t_{\alpha, n-1} \Rightarrow t_0 > t_{(0.05, 9)}$

$t_0 \approx 1.95 > t_{(0.05,9)} \approx 1.83$, Reject H_0

* We will reject the null hypothesis and conclude with 95% confidence that the mean battery life exceeds 25h.

(b) The manufacturer claims that standard deviation (σ) is less than 1.5 hours (known)

$H_0 : \sigma = 1.5h$

$H_A : \sigma < 1.5h$

Given: $n = 10$, $S \approx 1.62h$, $\alpha = 0.05$

$$\chi_0^2 = \frac{(n-1)S^2}{\sigma_0^2}$$

$$\chi_0^2 = \frac{(10-1)\cdot(1.62)^2}{(1.5)^2} = \frac{9(2.6244)}{2.25} \approx 10.56$$

reject H_0 : $\chi_0^2 < \chi^2_{(1-\alpha, n-1)}$ $\Rightarrow$ $\chi_0^2 \approx 10.56 > \chi^2_{(0.95, 9)} \approx 3.33$

Since $\chi_0^2 > \chi_{(0.95,9)}$, Fail to reject H_0

* At the 0.05 significance level, there is insufficient evidence to support the manufacturer's claim that the standard deviation is less than 1.5 hours.

(The sample standard deviation ≈ 1.62 is slightly higher than the claimed value.

4.12 Given; $n = 500$, nonconforming $X = 65$, sample proportion: $\hat{P} = 65/500$

$\hat{P} = 0.13$, hypothesized proportion: $P_0 = 0.08$, $\alpha = 0.05$

* Estimate process fraction nonconforming: $\hat{P} = 0.13 = 13\%$

95% CL : $\hat{P} \pm Z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}} = 0.13 \pm 1.95\sqrt{\frac{0.13(0.87)}{500}}$

$= 0.13 \pm 1.96(0.0150) \Rightarrow$ 95% CL $= (0.10, 0.16)$

$H_0 : P = 0.08$

$H_A : P \neq 0.08$

Verify the approximation Condition
$nP = 500(0.08) = 40 > 10$
$P = 0.08 < 0.5$
So, we can approximate from binomal to normal.

$$Z_0 = \frac{\hat{P} - P_0}{\sqrt{P_0(1-P_0)/n}} = \frac{0.13 - 0.08}{\sqrt{0.08(0.92)/500}} = \frac{0.05}{0.0121} \approx 4.13$$

$Z_0 \approx 4.13$

→ Determine the Critical value.

$Z_{\alpha/2} \Rightarrow Z_{0.05/2} \Rightarrow Z_{0.025} \approx 1.96$

Since $|Z_0| \approx 4.13 > Z_{0.025} \approx 1.96$, reject H_0

* At the 5% level, there is strong evidence that the true fraction defective isn't 0.08. Based on the sample the process fraction non conforming is estimated as 0.13, which is significantly higher than 0.08.