

Hypergeometric Distribution

- A set of N objects contains:
 - K objects classified as success
 - $N - K$ objects classified as failures
- A sample of size n objects is selected without replacement from the N objects randomly, where $K \leq N$ and $n \leq N$.
- Let the random variable X denote the number of successes in the sample. Then X is a hypergeometric random variable with probability mass function



Example 3-27: Parts from Suppliers-1

A batch of parts contains 100 parts from supplier A and 200 parts from Supplier B. If 4 parts are selected randomly, without replacement, what is the probability that they are all from Supplier A?

Answer:

Let X equal the number of parts in the sample from Supplier A.

$$P(X = 4) = \frac{\binom{100}{4} \binom{200}{0}}{\binom{300}{4}} = 0.0119$$



Example 3-27: Parts from Suppliers-2

What is the probability that two or more parts are from supplier A?

Answer:

$$P(X \geq 2) = P(X = 2) + P(X = 3) + P(X = 4)$$

$$= \frac{\binom{100}{2} \binom{200}{2}}{\binom{300}{4}} + \frac{\binom{100}{3} \binom{200}{1}}{\binom{300}{4}} + \frac{\binom{100}{4} \binom{200}{0}}{\binom{300}{4}}$$

$$= 0.298 + 0.098 + 0.0119$$

$$= 0.408$$



Example 3-27: Parts from Suppliers-3

What is the probability that at least one part in the sample is from Supplier A?

Answer:

$$P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{\binom{100}{0} \binom{200}{4}}{\binom{300}{4}} = 0.804$$



Hypergeometric Mean & Variance

If X is a hypergeometric random variable with parameters N , K , and n , then

σ^2 approaches the binomial variance as n / N becomes small.



Poisson Distribution

The random variable X that equals the number of events in a Poisson process is a Poisson random variable with parameter $\lambda > 0$, and the probability mass function is:



Example 3-31: Calculations for Wire Flaws-1

For the case of the thin copper wire, suppose that the number of flaws follows a Poisson distribution With a mean of 2.3 flaws per mm. Find the probability of exactly 2 flaws in 1 mm of wire.

Answer:

Let X denote the number of flaws in 1 mm of wire

$$P(X = 2) = \frac{e^{-2.3} 2.3^2}{2!} = 0.265$$



Example 3-31: Calculations for Wire Flaws-2

Determine the probability of 10 flaws in 5 mm of wire.

Answer :

Let X denote the number of flaws in 5 mm of wire.

$$E(X) = \lambda = 5 \text{ mm} \cdot 2.3 \text{ flaws/mm} = 11.5 \text{ flaws}$$

$$P(X = 10) = e^{-11.5} \frac{11.5^{10}}{10!} = 0.113$$



Example 3-31: Calculations for Wire Flaws-3

Determine the probability of at least 1 flaw in 2 mm of wire.

Answer :

Let X denote the number of flaws in 2 mm of wire.

Note that $P(X \geq 1)$ requires ∞ terms. ☺

$$E(X) = \lambda = 2 \text{ mm} \cdot 2.3 \text{ flaws/mm} = 4.6 \text{ flaws}$$

$$P(X \geq 1) = 1 - P(X = 0) = 1 - e^{-4.6} \frac{4.6^0}{0!} = 0.9899$$



Poisson Mean & Variance

If X is a Poisson random variable with parameter λ , then

$$\mu = E(X) = \lambda \quad \text{and} \quad \sigma^2 = V(X) = \lambda$$

The mean and variance of the Poisson model are the same.

