Discrete & Continuous Random Variables

- A discrete random variable is a random variable with a finite or countably infinite range. Its values are obtained by counting.
- A continuous random variable is a random variable with an interval (either finite or infinite) of real numbers for its range. Its values are obtained by measuring.

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Examples of Discrete & Continuous Random Variables

- Discrete random variables:
  - Number of scratches on a surface.
  - Proportion of defective parts among 100 tested.
  - Number of transmitted bits received in error.
  - Number of common stock shares traded per day.
- Continuous random variables:
  - Electrical current and voltage.
  - Physical measurements, e.g., length, weight, time, temperature, pressure.

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Discrete Random Variables

Physical systems can be modeled by the same or similar random experiments and random variables. The distribution of the random variable involved in each of these common systems can be analyzed. The results can be used in different applications and examples.

We often omit a discussion of the underlying sample space of the random experiment and directly describe the distribution of a particular random variable.

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Probability Mass Function

For a discrete random variable X with possible values $x_1, x_2, \ldots, x_n$, a probability mass function(pmf) is a function such that:

$$(1) \quad f(x_i) \geq 0$$

$$(2) \quad \sum_{i=1}^{n} f(x_i) = 1$$

$$(3) \quad f(x_i) = P(X = x_i)$$

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Example1:

A Sample of 100 boards used in heads up display was examined and the number of wave soldering defects per board was counted, with the following results:

Defects	0	1	2	3	4	5
Number of Such	30	35	15	10	6	4

Discrete Probability Distribution:

x_i	0	1	2	3	4	5
$f(x_i)$	0.30	0.35	0.15	0.10	0.06	0.04

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Cumulative Distribution Function and Properties

The cumulative distribution function(cdf), is the probability that a random variable *X* with a given probability distribution will be found at a value less than or equal to *x*.

Symbolically,

$$F(x) = P(X \leq x) = \sum_{x_i \leq x} f(x_i)$$

For a discrete random variable *X*, *F*(*x*) satisfies the following properties:

$$(1)\quad F(x) = P(X \leq x) = \sum_{x_i \leq x} f(x_i)$$

$$(2)\quad 0 \leq F(x) \leq 1$$

$$(3)\quad \text{If } x \leq y, \text{ then } F(x) \leq F(y)$$

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Cumulative Distribution Function For Example 1

x_i	0	1	2	3	4	5
$f(x_i)$	0.30	0.35	0.15	0.10	0.06	0.04
F(x)	0.30	0.65	0.8	0.9	0.96	1

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Expected Value of a Discrete Random Variable

$\mu = E(x) = \sum x f(x)$ for all x

Expected Value or Mean for Example 1:

$\mu = E(x) = (0)(0.30)+(1)(0.35)+(2)(0.15)+3(0.10)+4(0.06)+(5)(0.04)$ =1.39 defects

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Variance Formula Derivations

$$V(X)=\sum_x (x-\mu)^2 f(x)$$ is the definitional formula

$$=\sum_x \left(x^2-2\mu x+\mu^2\right)f(x)$$

$$=\sum_x x^2 f(x)-2\mu\sum xf(x)+\mu^2\sum_x f(x)$$

$$=\sum_x x^2 f(x)-2\mu^2+\mu^2$$

$$=\sum_x x^2 f(x)-\mu^2$$ is the computational formula

The computational formula is easier to calculate manually.

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Variance for Example 1

σ^2=Var(X)=V(X)=$(0^2)(0.30)+(1^2)(0.35)+(2^2)(0.15)+(3^2)(0.10)+(4^2)(0.06)$
$+(5^2)(0.04)-1.39^2=1.8778\ defects^2$

Standard Deviation for Example 1

$\sigma=\sqrt{1.8778}=1.37$ defects

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