Setting Specifications on Discrete Components

- Complex assemblies
- Tolerance stack-up problems
- Linear combinations

8.8.1 Linear Combinations

In many cases, the dimension of an item is a linear combination of the dimensions of the component parts. That is, if the dimensions of the components are $x_1, x_2, \ldots, x_n$, then the dimension of the final assembly is

$$y = a_1x_1 + a_2x_2 + \cdots + a_nx_n \tag{8.38}$$

where the a_c are constants.

If the x_i are normally and independently distributed with mean μ_i and variance σ_i^2, then y is normally distributed with mean $\mu_y = \Sigma_{n=1}^{i} a_i\mu_i$ and variance $\sigma_y^2 = \Sigma_{n=1}^{i} a_i^2 \sigma_i^2$. Therefore, if μ_i and σ_i^2 are known for each component, the fraction of assembled items falling outside the specifications can be determined.

EXAMPLE 8.8 Meeting Customer Specifications

A linkage consists of four components as shown in Figure 8.18. The lengths of x_1, x_2, x_3, and x_4 are known to be $x_1 \sim N(2.0, 0.0004)$, $x_2 \sim N(4.5, 0.0009)$, $x_3 \sim N(3.0, 0.0004)$, and $x_4 \sim N(2.5, 0.0001)$. The lengths of the components can be assumed independent, because they are produced on different machines. All lengths are in inches. Determine the proportion of linkages that meet the customer specification on overall length of 12 ± 0.10.

SOLUTION

To find the fraction of linkages that fall within design specification limits, note that y is normally distributed with mean

$$\mu_y = 2.0 + 4.5 + 3.0 + 2.5 = 12.0$$

and variance

$$\sigma_y^2 = 0.0004 + 0.0009 + 0.0004 + 0.0001 = 0.0018$$

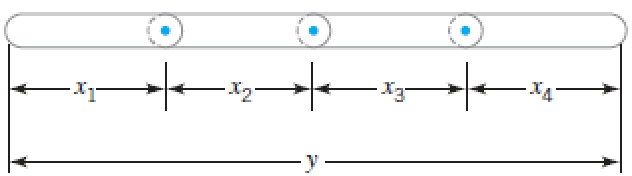

FIGURE 8.18 A linkage assembly with four components.

To find the fraction of linkages that are within specification, we must evaluate

$$\begin{aligned}
P\{11.90 \le y \le 12.10\} &= P\{y \le 12.10\} - P\{y \le 11.90\} \\
&= \Phi\left(\frac{12.10 - 12.00}{\sqrt{0.0018}}\right) - \Phi\left(\frac{11.90 - 12.00}{\sqrt{0.0018}}\right) \\
&= \Phi(2.36) - \Phi(-2.36) \\
&= 0.99086 - 0.00914 \\
&= 0.98172
\end{aligned}$$

Therefore, we conclude that 98.172% of the assembled linkages will fall within the specification limits. This is not a Six Sigma product.

EXAMPLE 8.10: Assembly of a Shaft and a Bearing

A shaft is to be assembled into a bearing. The internal diameter of the bearing is a normal random variable—say, x_1—with mean $\mu_1 = 1.500\text{in.}$ and standard deviation $\sigma_1 = 0.0020\text{in.}$ The external diameter the shaft—say, x_2—is normally distributed with mean $\mu_2 = 1.480\text{in.}$ and standard deviation $\sigma_2 = 0.0040\text{in}$
The assembly is shown in Figure 8.20.

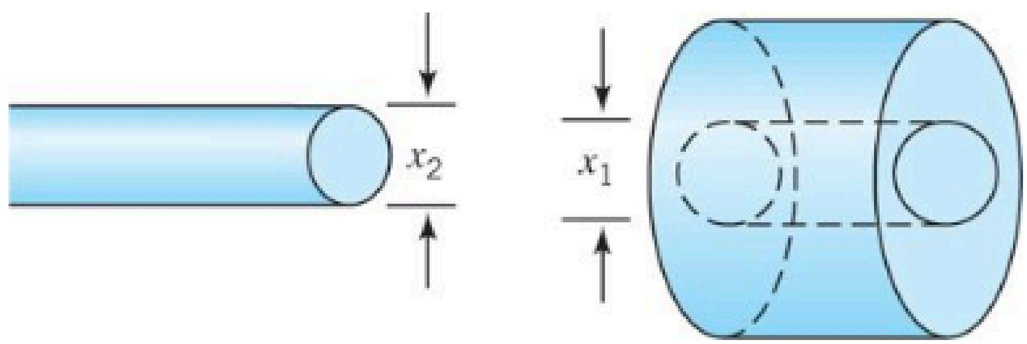

FIGURE 8.20: Assembly of a shaft and a bearing.

When the two parts are assembled, interference will occur if the shaft diameter is larger than the bearing diameter—that is, if

$$y = x_1 - x_2 < 0$$

Note that the distribution of y is normal with mean

$$\mu_y = \mu_1 - \mu_2 = 1.500 - 1.480 = 0.020$$

and variance

$$\sigma_y^2 = \sigma_1^2 + \sigma_2^2 = (0.0020)^2 + (0.0040)^2 = 0.00002$$

Therefore, the probability of interference is

$$\begin{aligned} P\{\text{interference}\} &= P\{y < 0\} = \Phi\left(\frac{0-0.020}{\sqrt{0.00002}}\right) = \\ &\Phi(-4.47) = 0.000004\,(4\,\text{ppm}) \end{aligned}$$

which indicates that very few assemblies will have interference. This is essentially a Six Sigma design.

In problems of this type, we occasionally define a minimum clearance—say, C—such that

$$P\{\text{clearance} < C\} = \alpha$$

Thus, C becomes the natural tolerance for the assembly and can be compared with the design specification. In our example, if we establish $\alpha = 0.0001$ (i.e., only 1 out of 10,000 assemblies or 100 ppm will have clearance less than or equal to C), then we have

$$\frac{C-\mu_y}{\sigma_y} = -Z_{0.0001}$$

or

$$\frac{C-0.020}{\sqrt{0.00002}} = -3.71$$

which implies that $C = 0.020 - (3.71)\sqrt{0.00002} = 0.0034$. That is, only 1 out of 10,000 assemblies will have clearance less than 0.0034 in.