① $\bar{x}_i = \frac{x_1 + \cdots + x_n}{n}$ $R_i = |\max(x_{ij}) - \min(x_{ij})|$

$\bar{\bar{x}} = 0.50000 + \left(\frac{\bar{x}_i}{n}\right) \times 0.0001$

6.4 (a)

$0.50345 + 0.50342 + 0.50316 + 0.50315 + 0.50350 + 0.50341$
$+ 0.50326 + 0.50338 + 0.50348 + 0.50336 + 0.50319 + 0.50386$
$+ 0.50354 + 0.50340 + 0.50371 + 0.50349 + 0.50335 + 0.50317$
$+ 0.50340 + 0.50351 + 0.50337 + 0.50328 + 0.50335 + 0.50342$

Row 1 : 3.02009

Row 2 : 3.02053 $\Rightarrow \frac{3.02009 + 3.02053 + 3.02066 + 3.02033}{24}$

Row 3 : 3.02066

Row 4 : 3.02033 $\hat{\mu} = \frac{1}{m}\sum_{i=1}^{m} \bar{X}_i = \frac{12.08161}{24} = 0.503400417$
≈ 0.50340

$\bar{\bar{X}} = 0.50340$

$\bar{R} = 3+4+4+4+4+5+6+4+3+7+8+3+9$
$8+6+8+7+4+3+8+4+2+1+3+2 \Rightarrow \frac{1}{m}\sum_{i=1}^{m} R_i/m =$

Row 1 : 60

Row 2 : 53 $\Rightarrow \frac{60+53}{24} = \frac{113}{24} = 4.70833$

$\bar{R} = \left(\frac{R_i}{n}\right) \times 0.0001 = 0.00047$

$\bar{X}$ - Chart

$UCL = \bar{\bar{X}} + A_2\bar{R} = 0.50340 + 0.577(0.00047) = 0.50367119 \approx 0.50367$

n=5 $CL = \bar{\bar{X}} = 0.50340$

$A_2 = 0.577$ $LCL = \bar{\bar{X}} - A_2\bar{R} = 0.50340 - 0.577(0.00047) = 0.50312881 \approx 0.50313$

R Chart

n=5 $ULC = D_4\bar{R} = 2.114(0.00047) = 0.00099$

$D_4 = 2.114$ $CL = \bar{R} = 0.00047$

$D_3 = 0$ $LCL = D_3\bar{R} = 0(0.00047) = 0$

After removing data point 12 & 15.

Row 1 : 3.02009

Row 2 : 2.51667 $\Rightarrow \dfrac{11.07404}{22} = 0.503365455 \approx 0.50337$

Row 3 : 2.51695 $\quad \bar{\bar{x}} = 0.50337$

Row 4 : 3.02033

$\bar{R} = \dfrac{51+48}{22} = \dfrac{99}{22} = 4.5 \times 0.0001 = 0.00045$

$\bar{x}$ Chart

$UCL = 0.50337 + 0.577(0.00045) = 0.50362965 \approx 0.50363$

$CL = 0.50337$

$LCL = 0.50337 - 0.577(0.00045) = 0.50311035 \approx 0.50311$

R Chart

$UCL = 2.114(0.00045) = 0.0009513 \approx 0.00095$

$CL = 0.00045$

$LCL = 0(0.00045) = 0$

b) Specification : 0.5030 ± 0.0010 (Assuming it's normally distributed)

$UCL = 0.504$

$CL = 0.503$

$LCL = 0.502$

Process Control

$\bar{\bar{x}} = 0.50337$

$\hat{\sigma} = \bar{R}/d_2 = \dfrac{0.00045}{2.326} = 0.000193465 \approx 0.00019$

$P = P(X \geq 0.504) + P(X \leq 0.502)$

$P = 1 - P(X \leq 0.504) + P(X \leq 0.502)$

using normal standard $Z = \dfrac{x-\mu}{\sigma}$

$P = \Phi\left(\dfrac{0.502 - 0.50337}{0.00019}\right) + 1 - \Phi\left(\dfrac{0.504 - 0.50337}{0.00019}\right)$

②

$P = \Phi(-7.210526316) + 1 - \Phi(3.315789474)$

$P = \Phi(\cancel{-7.21}) + 1 - \Phi(3.32)$
$\quad\quad 0$

$P = 1 - 0.999550$

$P = 0.00045 \approx 0.045\%$ (Process is highly capable).

$\Rightarrow$ Checking $PCR = \frac{USL - LSL}{6\sigma} = \frac{0.504 - 0.502}{6(0.00019)} = 1.754385965$

$\approx 1.75 > 1$

$\therefore$ It's capable $C_p > 1$.

6.15 $m = 30$ $\quad \sum_{i=1}^{30} \bar{x}_i = 6{,}000$ $\quad \sum_{i=1}^{30} R_i = 150$

$n = 6$ $\quad \bar{\bar{x}} = \frac{6{,}000}{30} = 200$ $\quad \bar{R} = \frac{150}{30} = 5$

(a) $\bar{x}$ chart

$n=6$ $\quad UCL = \bar{\bar{x}} + A_2\bar{R} = 200 + 0.483(5) = 202.415$

$A_2 = 0.483$ $\quad CL = \bar{\bar{x}} = 200$

$LCL = \bar{\bar{x}} - A_2\bar{R} = 200 - 0.483(5) = 197.585$

R chart

$n=6$ $\quad UCL = D_4\bar{R} = 2.004(5) = 10.02$

$D_4 = 2.004$ $\quad CL = \bar{R} = 5$

$D_3 = 0$ $\quad LCL = D_3\bar{R} = 0(5) = 0$

(b) Specification: 200 ± 5 (Assuming it's normally distributed)

$UCL = 205$

$CL = 200$

$LCL = 195$

Process Control

$\bar{\bar{X}} = 200$

$\hat{\sigma} = \bar{R}/d_2 = \frac{5}{2.534} = 1.973164957 \approx 1.97316$

$\Rightarrow$ Checking PCR $= \frac{UCL - LCL}{6\sigma} = \frac{205-195}{6(1.97316)} = 0.844668788$
≈ 0.84467

$\therefore$ The process isn't capable $C_p < 1$.

C23 $n=4$ $\sum_{i=1}^{30} \bar{x}_i = 12870$ $\sum_{i=1}^{30} s_i = 410$

(a) $m = 30$ $\bar{\bar{x}} = \frac{12870}{30} = 429$ $\bar{s} = \frac{410}{30} = 13.66666667$
≈ 13.66667

$\bar{x}$-chart

$n=4$

$A_3 = 1.628$ $UCL = \bar{\bar{x}} + A_3\bar{s} = 429 + 1.628(13.66667) = 451.2493388 \approx 451.24934$

$CL = \bar{\bar{x}} = 429$

$LCL = \bar{\bar{x}} - A_3\bar{s} = 429 - 1.628(13.66667) = 406.7506612 \approx 406.75066$

s-chart

$n=4$ $UCL = B_4\bar{s} = 2.266(13.66667) = 30.96867422 \approx 30.96867$

$B_4 = 2.266$ $CL = \bar{s} = 13.66667$

$B_3 = 0$ $LCL = B_3\bar{s} = 0(13.66667) = 0$

(b) $\hat{\mu} = \bar{\bar{x}} = 429$

$n=4$ $\hat{\sigma} = \bar{s}/c_4 = \frac{13.66667}{0.9213} = 14.83411484 \approx 14.83411$

$c_4 = 0.9213$

(3)

2-47

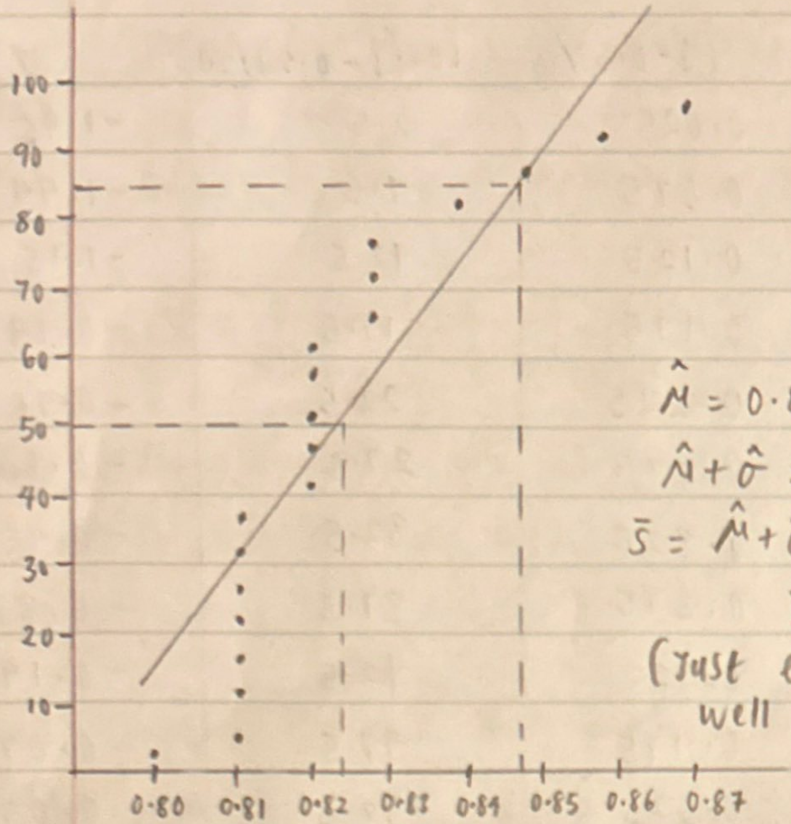
(a)

j	$x_{(j)}$	$(j-0.5)/20$	$100(j-0.5)/20$	Z_j
1	0.80	0.025	2.5	-1.96
2	0.81	0.075	7.5	-1.44
3	0.81	0.125	12.5	-1.15
4	0.81	0.175	17.5	-0.94
5	0.81	0.225	22.5	-0.76
6	0.81	0.275	27.5	-0.60
7	0.81	0.325	32.5	-0.46
8	0.81	0.375	37.5	-0.33
9	0.82	0.425	42.5	-0.19
10	0.82	0.475	47.5	-0.07
11	0.82	0.525	52.5	0.07
12	0.82	0.575	57.5	0.19
13	0.82	0.625	62.5	0.33
14	0.83	0.675	67.5	0.46
15	0.83	0.725	72.5	0.60
16	0.83	0.775	77.5	0.76
17	0.84	0.825	82.5	0.94
18	0.85	0.875	87.5	1.15
19	0.86	0.925	92.5	1.44
20	0.87	0.975	97.5	1.96

$$\bar{x} = \frac{16.48}{20} = 0.824$$

$$s^2 = \frac{\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2/n}{n-1} = \frac{13.586 - (16.48)^2/20}{19}$$

$$s^2 = 0.000341053 \Rightarrow s = \sqrt{s^2} \Rightarrow s = 0.01846761 \simeq 0.01847$$



$\hat{\mu} = 0.824$

$\hat{\mu} + \hat{\sigma} = 0.848$

$\bar{s} = \hat{\mu} + \hat{\sigma} - \hat{\sigma} = 0.848 - 0.824$

$\bar{s} = 0.024$

(Just estimation since the plot isn't well numbered).

∴ It's not normally distributed.

(b) $MR_i = 0.01 + 0.01 + 0.01 + 0.00 + 0.01 + 0.02 + 0.01 + 0.01 + 0.01 + 0.01 + 0.02 + 0.02 + 0.01 + 0.01 + 0.04 + 0.02 + 0.04 + 0.01 + 0.02$

$$\overline{MR} = \sum_{i=1}^{19} MR_i \Big/ n-1 = \frac{0.29}{19} = 0.015263158 \approx 0.01526$$

Control Chart for Individual observation

$n=2$ $\quad UCL = \bar{x} + 3\overline{MR}/d_2 = 0.824 + 3(0.01526)/1.128 \approx 0.86459$

$d_2 = 1.128$ $\quad CL = \bar{x} = 0.824$

$LCL = \bar{x} - 3\overline{MR}/d_2 = 0.824 - 3(0.01526)/1.128 \approx 0.78341$

④

Moving Range Chart

$n=2$ $UCL = D_4 \overline{MR} = 3.267(0.01526) \approx 0.04985$

$D_4 = 3.267$ $CL = \overline{MR} = 0.01526$

$D_3 = 0$ $LCL = D_3 \overline{MR} = 0(0.01526) \approx 0$

(c) $\hat{\mu} = \bar{x} = 0.824$

$\hat{\sigma} = \overline{MR}/d_2 = 0.01526/1.128 = 0.01352836 9 \approx 0.01353$

Note: The process isn't statistical control because batch 18, is above the UCL.

6.19 $n=5$ $\sum_{i=1}^{30} x_i = 607.8$ $\sum_{i=1}^{30} R_i = 144$

(a) $m=30$ $\bar{\bar{x}} = \frac{607.8}{30} = 20.26$ $\bar{R} = \frac{144}{30} = 4.8$

$n=5$ $\bar{x}$-Chart

$A_2 = 0.577$ $UCL = \bar{\bar{x}} + A_2\bar{R} = 20.26 + (0.577)4.8 = 23.0296$

$CL = \bar{\bar{x}} = 20.26$

$LCL = \bar{\bar{x}} - A_2\bar{R} = 20.26 - (0.577)4.8 = 17.4904$

$n=5$ R-Chart

$D_4 = 2.114$ $UCL = D_4\bar{R} = 2.114(4.8) = 10.1472$

$D_3 = 0$ $CL = \bar{R} = 4.8$

$LCL = D_3\bar{R} = 0(4.8) = 0$

(b) $\hat{\mu} = 20.26$, $\hat{\sigma} = \bar{R}/d_2 = 4.8/2.326 \approx 2.06362$

$n=5$ $P = P(X<16) = \Phi\left(\frac{16-20.26}{2.06362}\right) \approx \Phi(-2.06)$

$d_2 = 2.326$ $P = 0.019699$

using normal standard

$Z = \frac{x-\mu}{\sigma}$

$\therefore$ 1.96% will fail lower specification (Assuming normally dist.)