ISYE 430/530 — Lab I: Washer Process

Team 5

Micah Calderon — Syed Abrar Zaheer — Jonathan Gomez — Ibrahim Oyeyinka

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Data: Washer Inner-Diameter Data (mm)

Sample	x_{i1}	x_{i2}	x_{i3}	x_{i4}	x_{i5}
1	11.12	11.17	11.20	11.19	11.19
2	11.13	11.15	11.17	11.20	11.21
3	11.22	11.13	11.09	11.10	11.20
4	11.20	11.08	11.14	11.13	11.07
5	11.11	11.17	11.14	11.17	11.27
6	11.19	11.18	11.24	11.13	11.11
7	11.21	11.15	11.24	11.19	11.23
8	11.29	11.39	11.08	11.25	11.28
9	11.23	11.09	11.22	11.18	11.21
10	11.23	11.25	11.27	11.18	11.25
11	11.26	11.16	11.27	11.24	11.26
12	11.27	11.36	11.18	11.21	11.27
13	11.26	11.22	11.19	11.29	11.27
14	11.21	11.23	11.22	11.17	11.20
15	11.32	11.25	11.26	11.24	11.16
16	11.27	11.15	11.20	11.30	11.20
17	11.07	11.20	11.19	11.17	11.35
18	11.24	11.29	11.24	11.27	11.19
19	11.07	11.18	11.28	11.17	11.04
20	11.17	11.34	11.32	11.24	11.29

1. Set up $(\bar{X}, R)$ charts + check normality (assume outliers are assignable causes)

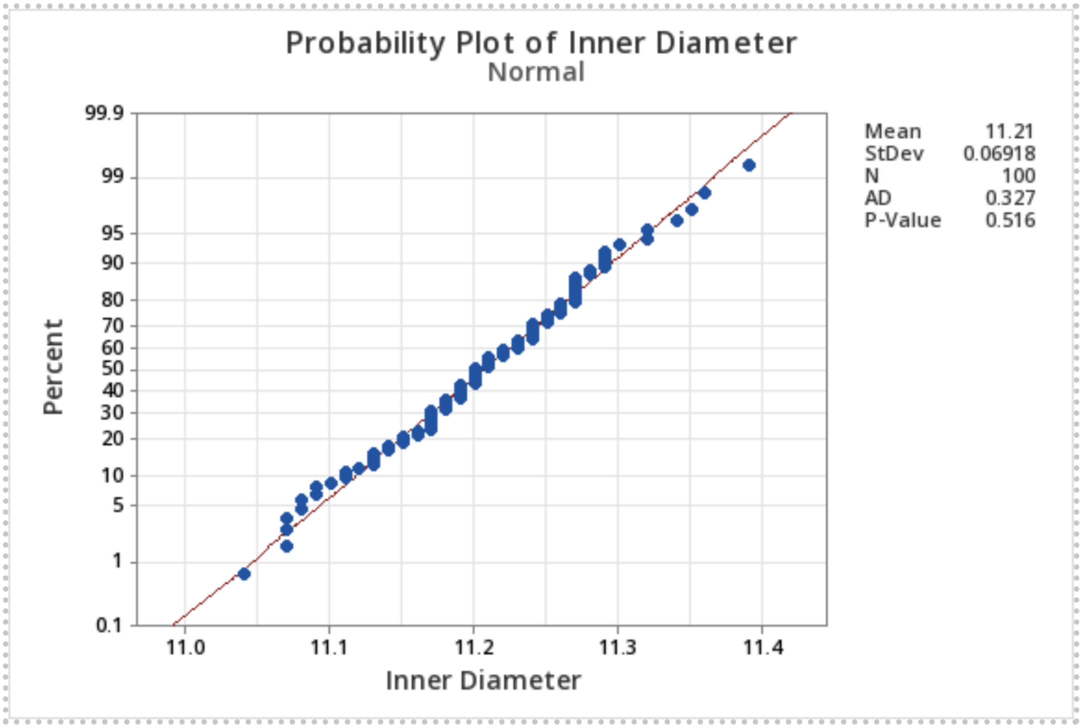
(a) Subgroup statistics

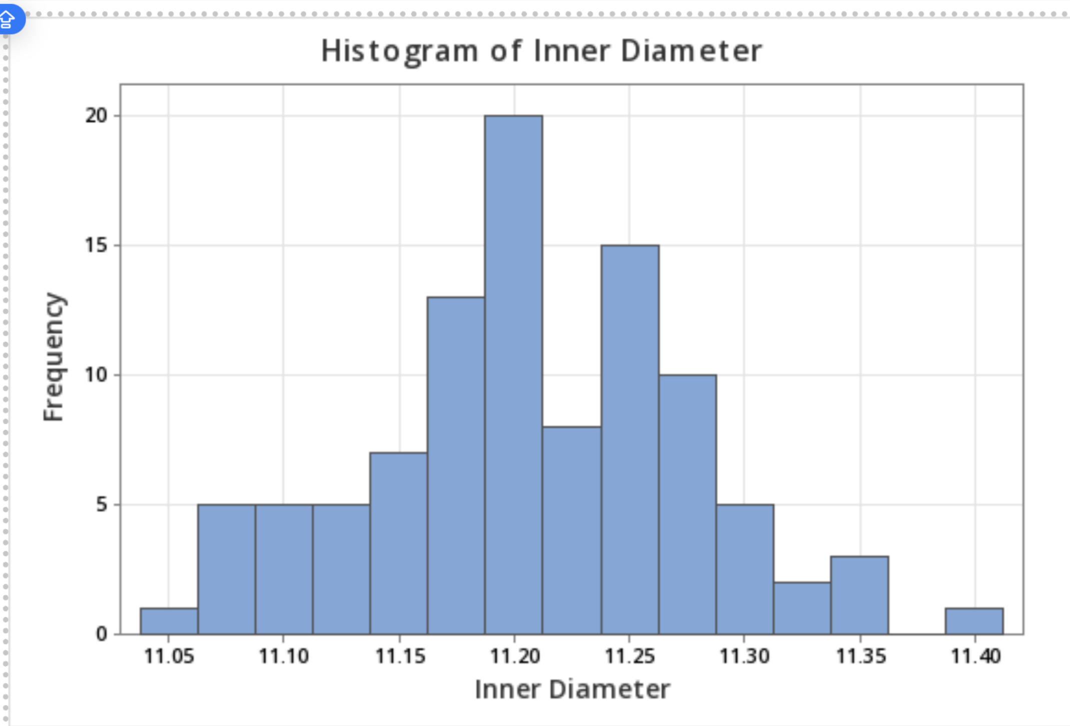
We have $m = 20$ subgroups, each of size $n = 5$. For subgroup i:

$$\bar{x}_i = \frac{1}{n}\sum_{j=1}^{n} x_{ij}, \qquad R_i = \max(x_{ij}) - \min(x_{ij}).$$

Overall:

$$\bar{\bar{X}} = \frac{1}{m}\sum_{i=1}^{m} \bar{x}_i, \qquad \bar{R} = \frac{1}{m}\sum_{i=1}^{m} R_i.$$





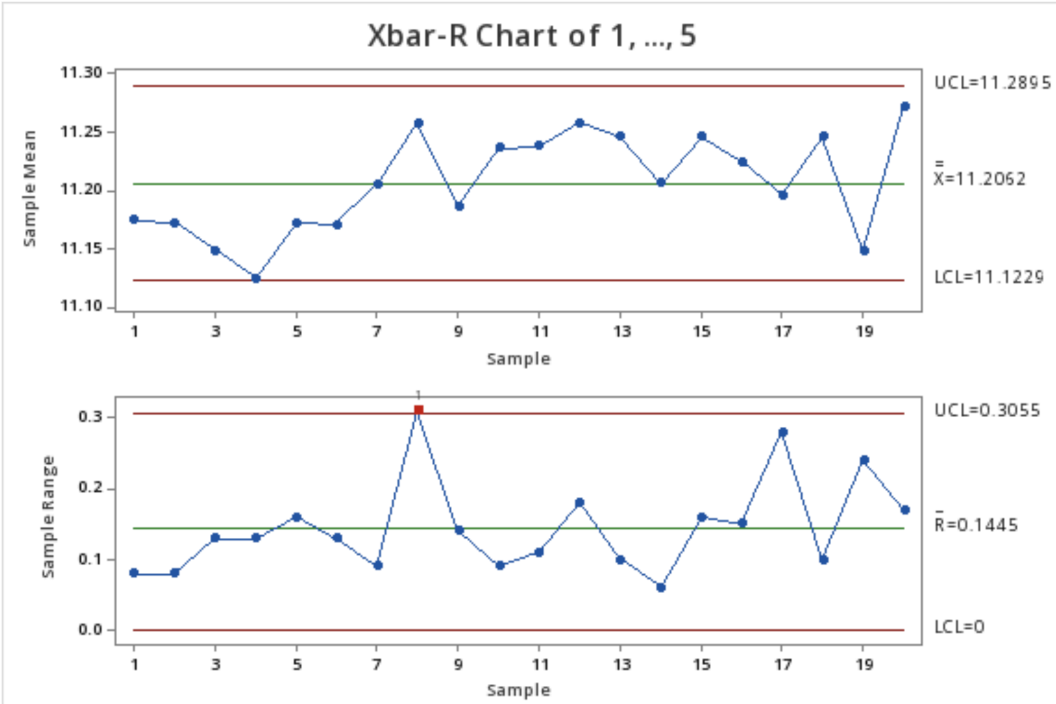
(b) Control limits for $(\bar{X}, R)$

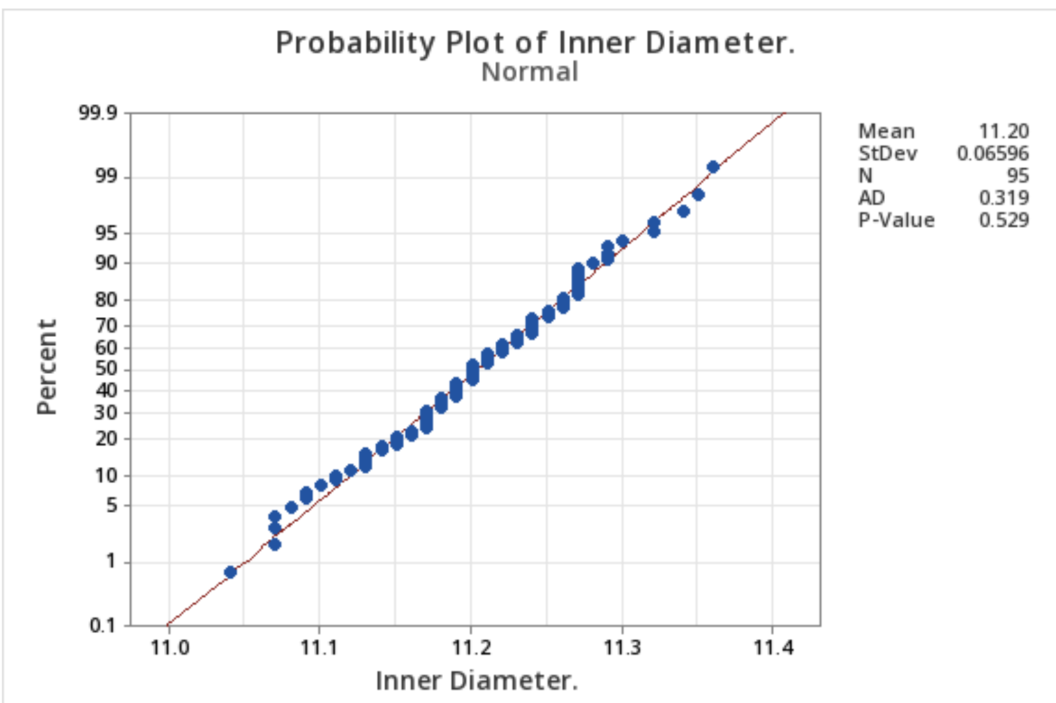
For $n = 5$, constants: $A_2 = 0.577$, $D_3 = 0$, $D_4 = 2.114$.

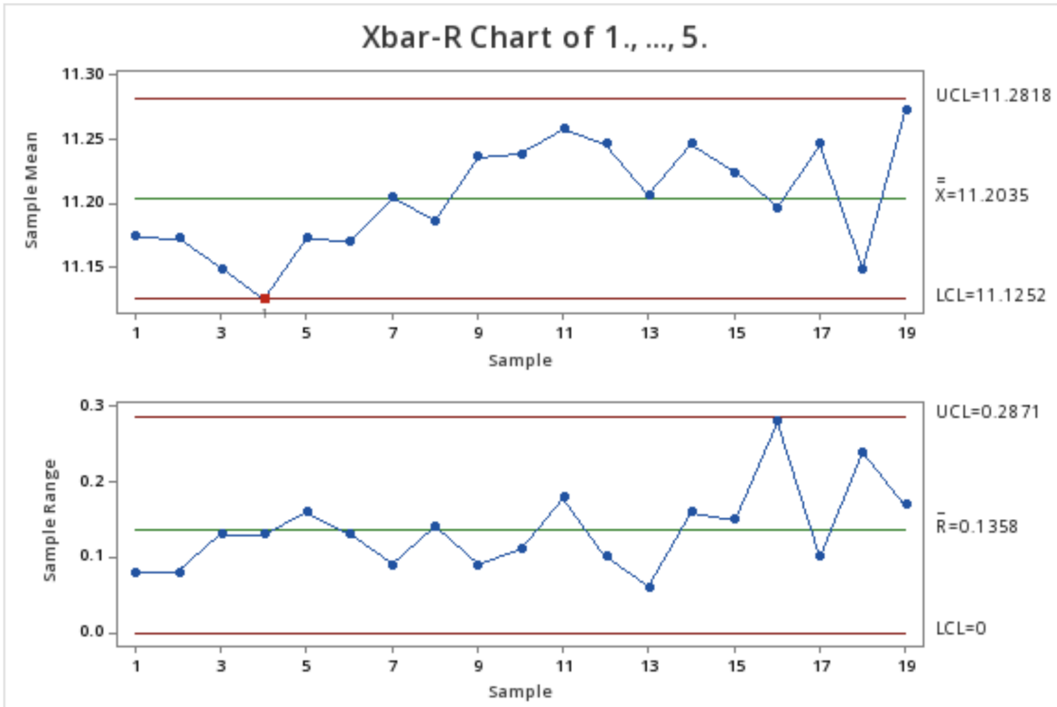
$$\mathrm{UCL}_R = D_4\bar{R}, \quad \mathrm{CL}_R = \bar{R}, \quad \mathrm{LCL}_R = D_3\bar{R}.$$

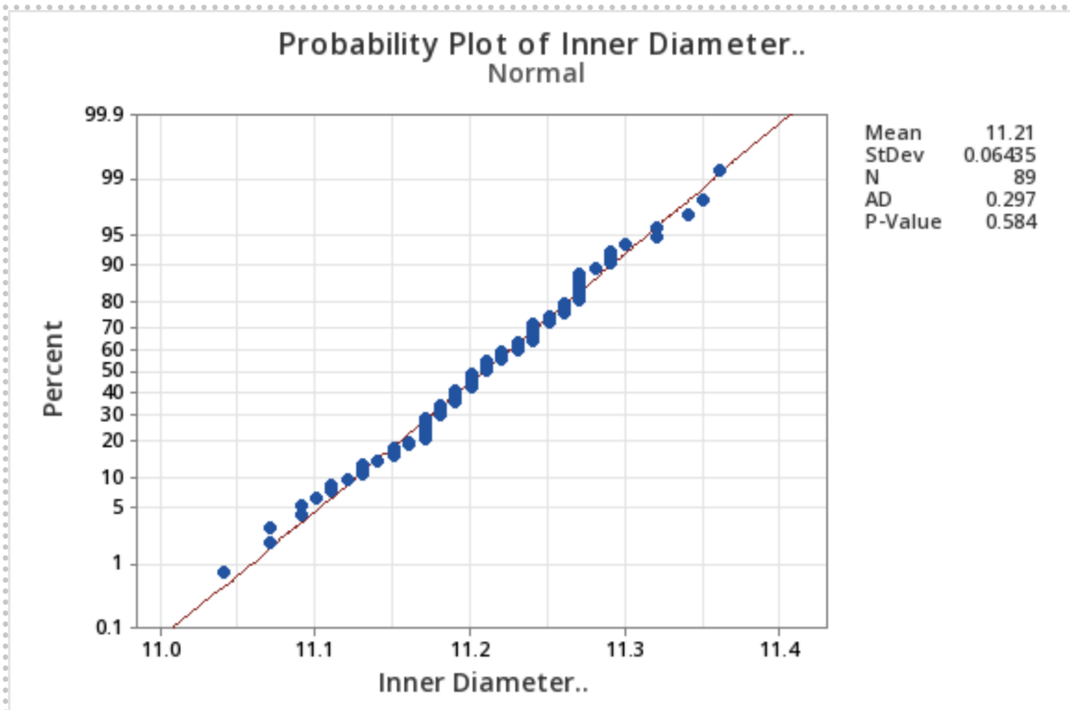
$$\mathrm{UCL}_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}, \quad \mathrm{CL}_{\bar{X}} = \bar{\bar{X}}, \quad \mathrm{LCL}_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}.$$

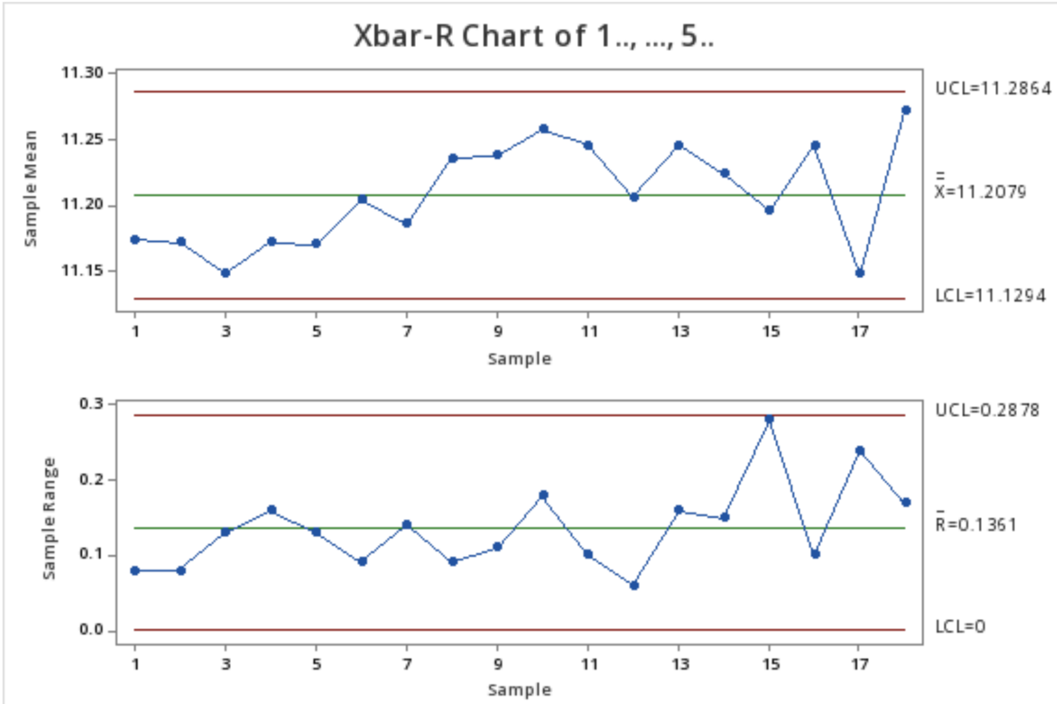
(c) Outliers / assignable causes (from Minitab)






Xbar-R Chart of 1., ..., 5.
Sample Mean
Sample
UCL=11.2818
$\bar{\bar{X}}$=11.2035
LCL=11.1252
Sample Range
Sample
UCL=0.2871
$\bar{R}$=0.1358
LCL=0


Probability Plot of Inner Diameter..
Normal
Percent
Inner Diameter..
Mean 11.21
StDev 0.06435
N 89
AD 0.297
P-Value 0.584



From our Minitab output, the out-of-control samples are treated as assignable causes and removed. For the $(\bar{X}, R)$ analysis, the outliers removed are:

$$\boxed{\text{Sample 8 and Sample 4.}}$$

After removing these, the revised limits are based on the remaining subgroups:

$$\bar{R} = 0.1361, \qquad \bar{\bar{X}} = 11.2079.$$

Thus,

$$\begin{aligned} \mathrm{UCL}_R &= 2.114(0.1361) = 0.2877, \\ \mathrm{CL}_R &= 0.1361, \\ \mathrm{LCL}_R &= 0. \end{aligned}$$

$$\begin{aligned} \mathrm{UCL}_{\bar{X}} &= 11.2079 + 0.577(0.1361) = 11.2864, \\ \mathrm{CL}_{\bar{X}} &= 11.2079, \\ \mathrm{LCL}_{\bar{X}} &= 11.2079 - 0.577(0.1361) = 11.1294. \end{aligned}$$

(d) Normality check (Minitab probability plot)

We checked normality using the normal probability plot / normality test in Minitab. Based on the plot and test output, normality is acceptable for capability and defective-rate calculations (after removing assignable-cause subgroups).

2. Is the process capable? (existing process, USL = 11.40, LSL = 11.00)

For an existing process, we estimate the within-subgroup standard deviation using:

$$\hat{\sigma} = \frac{\bar{R}}{d_2}, \qquad d_2 = 2.326 \text{ for } n = 5.$$

$$\hat{\sigma} = \frac{0.1361}{2.326} = 0.05851.$$

Capability index:

$$C_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{11.40 - 11.00}{6(0.05851)} = \frac{0.40}{0.35106} = 1.139.$$

$$\boxed{C_p \approx 1.14 > 1 \;\Rightarrow\; \text{Process is capable.}}$$

3. Find the defective rate

Assuming normality with $\mu \approx \bar{\bar{X}} = 11.2079$ and $\sigma \approx \hat{\sigma} = 0.05851$:

$$P(\text{defect}) = P(X > \text{USL}) + P(X < \text{LSL}).$$

Standardise:

$$z_U = \frac{11.40 - 11.2079}{0.05851} = 3.2831, \qquad z_L = \frac{11.00 - 11.2079}{0.05851} = -3.5531.$$

So

$$P(\text{defect}) = [1 - \Phi(3.2831)] + \Phi(-3.5531) \approx 0.0007038.$$

$$\boxed{P(\text{defect}) \approx 0.000704 \;=\; 0.0704\% \;\approx\; 704 \text{ ppm.}}$$

4. Repeat Parts 1 and 2 using $(\bar{X}, S)$ charts

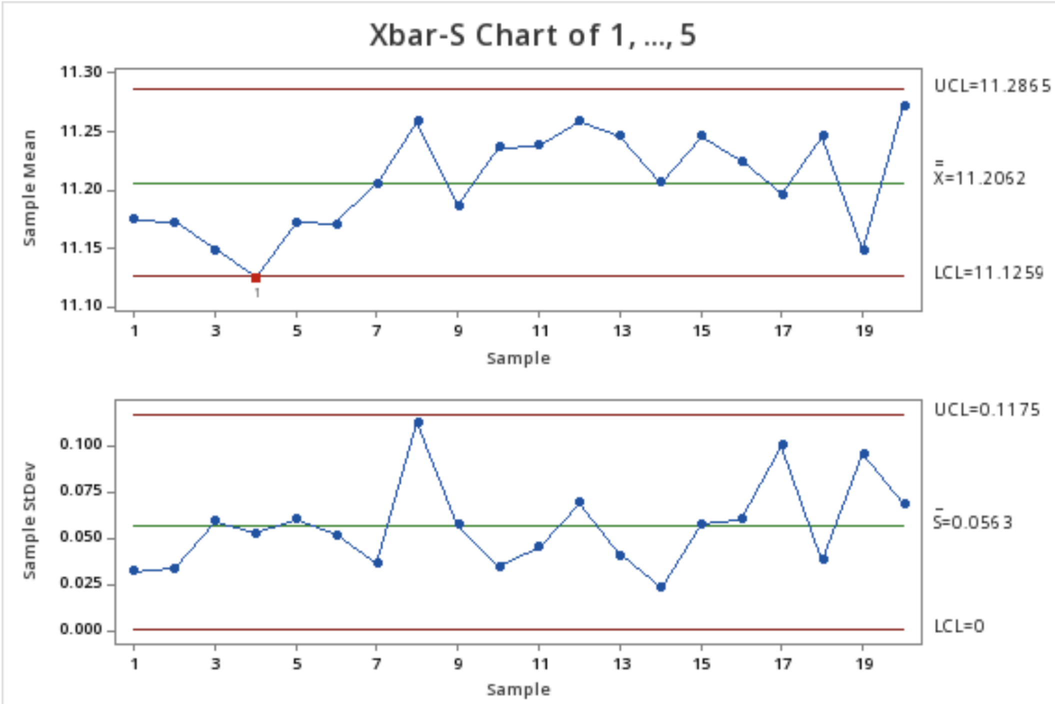
For subgroup i, compute s_i and

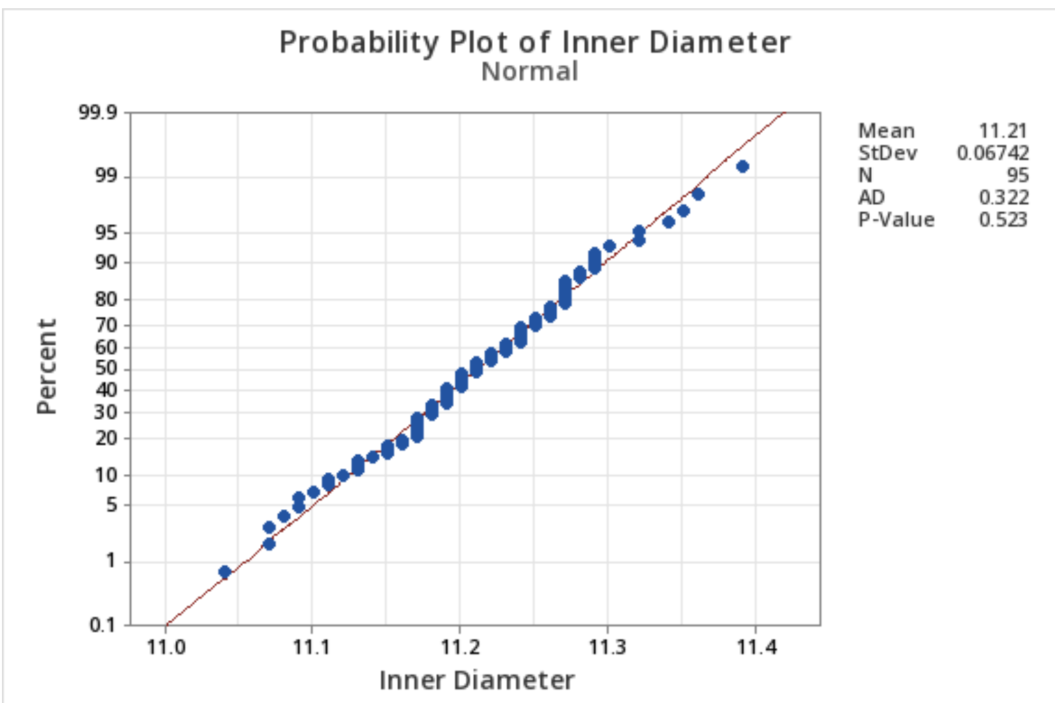
$$\bar{S} = \frac{1}{m}\sum_{i=1}^{m} s_i.$$

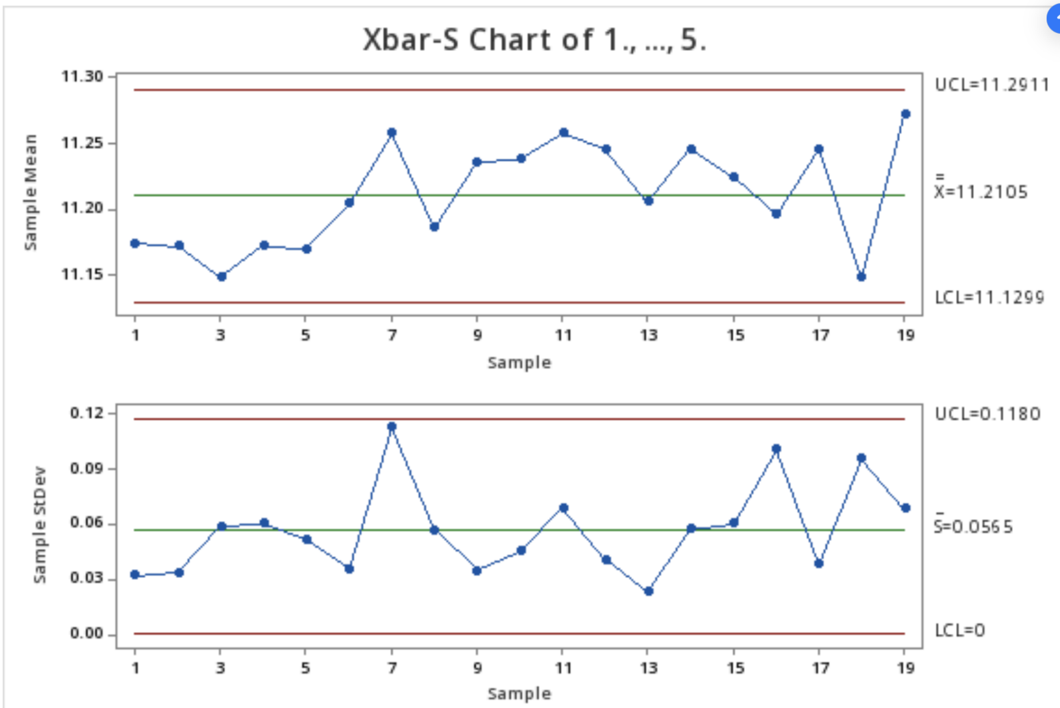
For $n = 5$, constants: $A_3 = 1.427$, $B_3 = 0$, $B_4 = 2.089$.

$$\text{UCL}_S = B_4\bar{S}, \quad \text{CL}_S = \bar{S}, \quad \text{LCL}_S = B_3\bar{S}.$$

$$\text{UCL}_{\bar{X}} = \bar{\bar{X}} + A_3\bar{S}, \quad \text{CL}_{\bar{X}} = \bar{\bar{X}}, \quad \text{LCL}_{\bar{X}} = \bar{\bar{X}} - A_3\bar{S}.$$


Xbar-S Chart of 1, ..., 5
Sample Mean
UCL=11.2865
X=11.2062
LCL=11.1259
Sample
Sample StDev
UCL=0.1175
S=0.0563
LCL=0
Sample


Probability Plot of Inner Diameter
Normal
Percent
Inner Diameter
Mean 11.21
StDev 0.06742
N 95
AD 0.322
P-Value 0.523



From our Minitab output for $(\bar{X}, S)$, the assignable-cause outlier is:

$$\boxed{\text{Sample 4.}}$$

After removal:

$$\bar{S} = 0.0565, \qquad c_4 = 0.9400 \text{ for } n = 5, \qquad \hat{\sigma} = \frac{\bar{S}}{c_4} = \frac{0.0565}{0.9400} = 0.0601.$$

$$C_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{0.40}{6(0.0601)} = 1.11.$$

$$\boxed{C_p \approx 1.11 > 1 \;\Rightarrow\; \text{Process is capable (using } (\bar{X}, S)).}$$