Home-work 1

2.1 16.05, 16.03, 16.02, 16.04, 16.05, 16.01, 16.02,
16.02, 16.03, 16.01, 16.00, 16.07

$n = 12$

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\bar{x} = \frac{16.05 + 16.03 + 16.02 + 16.04 + 16.05 + 16.01 + 16.02 + 16.02 + 16.03 + 16.01 + 16.00 + 16.07}{12}$$

$\bar{x} \approx 16.0292$

(b)

i	1	2	3	4	5	6	7	8	9	10	11	12	Sum
x_i	16.05	16.03	16.02	16.04	16.05	16.01	16.02	16.02	16.03	16.01	16.00	16.07	192.35
x_i^2	257.60	256.96	256.64	257.28	257.60	256.32	256.64	256.64	256.96	256.32	256	258.24	3083.2147

$$s^2 = \frac{\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2 / n}{n-1} = \frac{3083.2147 - (192.35)^2/12}{12-1}$$

$$s^2 = \frac{3083.2147 - 3083.210208}{11} = \frac{0.004492}{11} = 0.000408364$$

$s^2 = 0.000408364$

$s = \sqrt{s^2} = 0.02208009$

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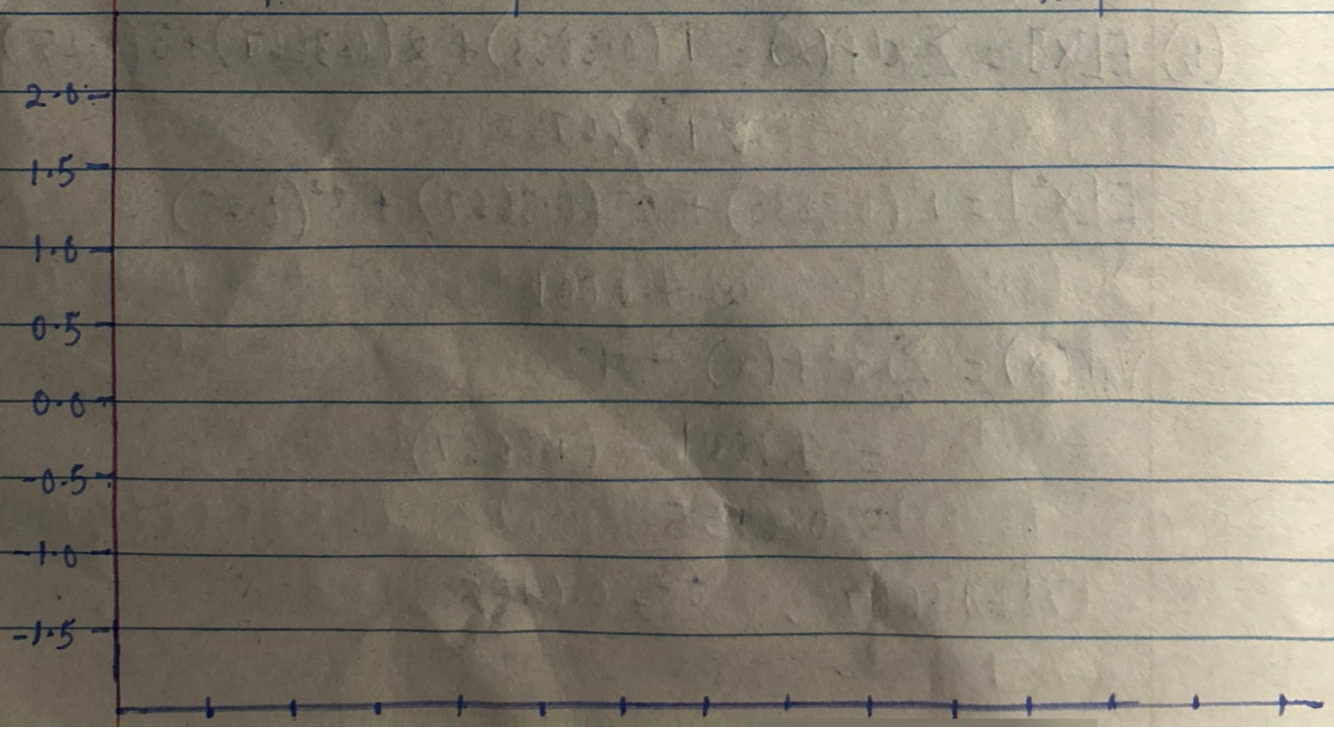
(c) Sorted data: 16.00, 16.01, 16.01, 16.02, 16.02, 16.02, 16.03, 16.03, 16.04, 16.05, 16.05, 16.07

i	$x_{(i)}$	$P_i = \frac{i-0.5}{12}$	$100 \cdot P_i$	z_i
1	16.00	0.04167	4.2%	−1.73
2	16.01	0.12500	12.5%	−1.15
3	16.01	0.20833	20.8%	−0.81
4	16.02	0.29167	29.2%	−0.54
5	16.02	0.37500	37.5%	−0.31
6	16.02	0.45833	45.8%	−0.10
7	16.03	0.54167	54.1%	0.10
8	16.03	0.62500	62.5%	0.31
9	16.04	0.70833	70.8%	0.54
10	16.05	0.79167	79.2%	0.81
11	16.05	0.87500	87.5%	1.15
12	16.07	0.95833	95.8%	1.73



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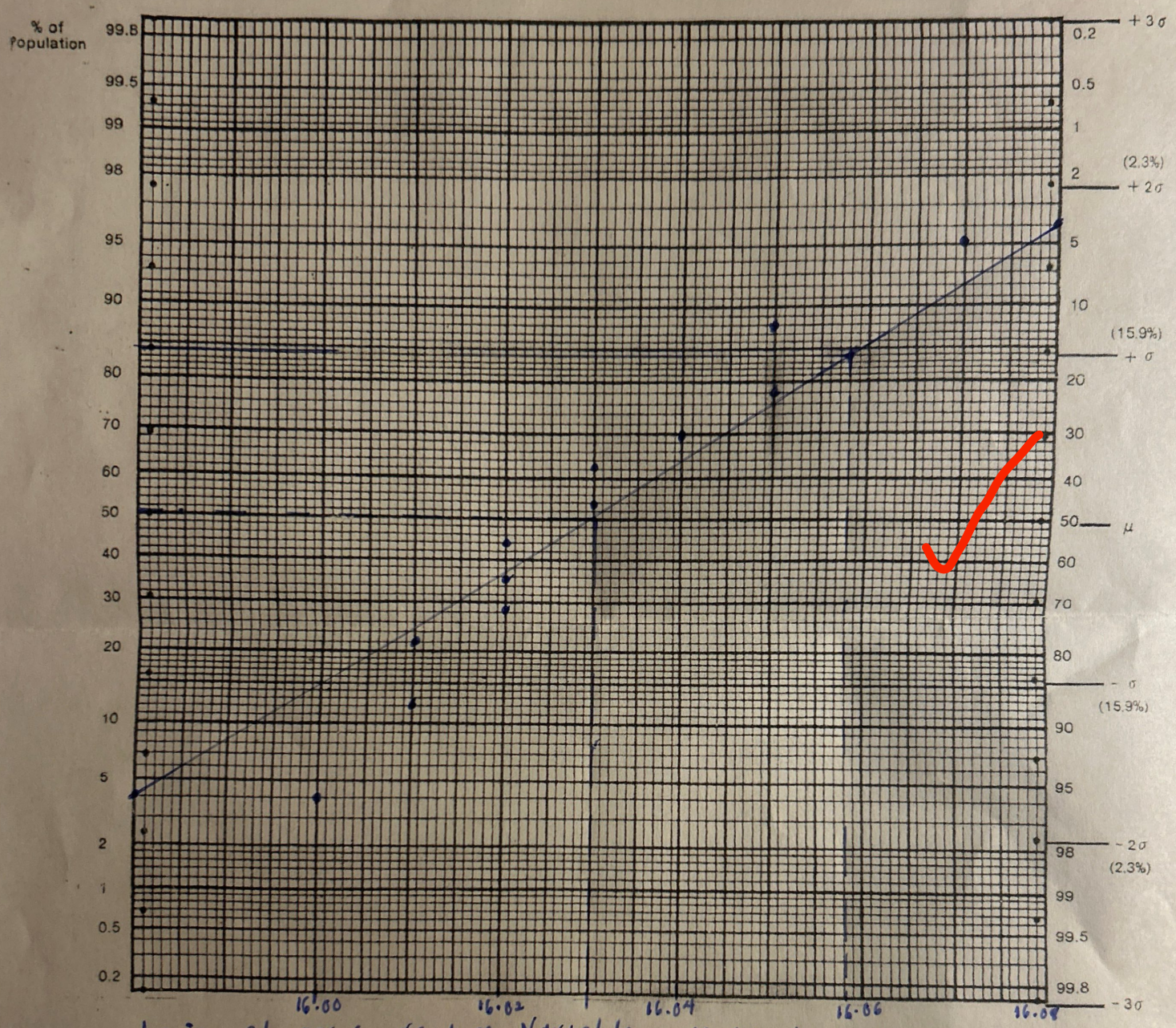

horizonal axis random variable, vertical axis is probability.

mean → $\hat{\mu} = 16.03$

var → $\hat{\mu} + \hat{\sigma} = 16.058$

Std → $\hat{\mu} + \hat{\sigma} - \hat{\mu} = 16.058 - 16.03$

$\sigma = 0.028$

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3.2 Find K ; probabilities must sum to 1

$$\frac{1+3K}{3} + \frac{1+2K}{3} + \frac{0.5+5K}{3} = 1$$

$$= (1+3K) + (1+2K) + (0.5+5K) = 3$$

$$= 2.5 + 10K = 3$$

$$\Rightarrow \frac{10K}{10} = \frac{0.5}{10} \Rightarrow K = 0.05$$

Probabilities

$$P(1) = \frac{1+3(0.05)}{3} = \frac{1.15}{3} \approx 0.3833$$

$$P(2) = \frac{1+2(0.05)}{3} = \frac{1.1}{3} \approx 0.3667$$

$$P(3) = \frac{0.5+5(0.05)}{3} = \frac{0.75}{3} \approx 0.25$$

(b) $E[X] = \sum x\, P(x) = 1(0.3833) + 2(0.3667) + 3(0.25)$

$$\approx 1.8667$$

$$E[X^2] = 1^2(0.3833) + 2^2(0.3667) + 3^2(0.25)$$

$$\approx 4.1001$$

$$Var(X) = \sum_x x^2 f(x) - \mu^2$$

$$= 4.1001 - (1.8667)^2$$

$$\approx 0.6155$$

$\mu = 1.8667$, $\sigma^2 = 0.6155$

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(c) $F(x) = P(X \leq x)$

$F(x) = 0$ for $x < 1$

$F(1) = 0.3833$

$F(2) = 0.3833 + 0.3667 = 0.75$

$F(3) = 0.3833 + 0.3667 + 0.25 = 1$

$$F(x) = \begin{cases} 0 & x < 1 \\ 0.3833 & 1 \leq x < 2 \\ 0.75 & 2 \leq x < 3 \\ 1 & x \geq 3 \end{cases}$$

3.93 Sample $n = 50$, nonconforming $p = 0.02$

X = Sample of 50

$X \sim Bin(n = 50, p = 0.02)$

$\hat{p} = \frac{X}{50}$

$\hat{p} \leq 0.04 \Rightarrow \frac{X}{50} \leq 0.04 \Rightarrow X \leq 2$

$P(\hat{p} \leq 0.04) = P(X \leq 2)$

$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$P(X=x) = \binom{50}{x}(0.02)^x(0.98)^{50-x}$

(i) $P(X=0) = \binom{50}{0}(0.02)^0(0.98)^{50} = (0.98)^{50} \approx 0.36416968$

$P(X=1) = \binom{50}{1}(0.02)^1(0.98)^{49} = 50(0.02)(0.98)^{48} \approx 0.37160171$

$P(X=2) = \binom{50}{2}(0.02)^2(0.98)^{48} = 1225(0.0004)(0.98)^{48}$

≈ 0.18580086

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$P(X \leq 2) \approx 0.36416968 + 0.37160171 + 0.18580086$

$= 0.92157225$

$P(\hat{P} \leq 0.04) = P(X \leq 2) \approx 0.9216$

2.5 $p = 0.10, n = 4$, $X \sim Bin(4, 0.10)$

(a) $P(x=1) = \binom{4}{1}(0.10)^1(0.90)^3$

$^nC_x = \frac{n!}{x!(n-x)!} \Rightarrow \frac{4!}{1!(4-1)!}(0.10)^1(0.90)^3$

$= 4 \times 0.10 \times 0.729 = 0.2916$

(b) $P(x=1)$

$P(X>1) = 1 - [P(X=0) + P(X=1)]$

$= 1 - [0.651 + 0.2916]$

$P(X>1) = 0.0523$

→ Addition Problem

(a) $E[X] = \sum_x x f(x)$

$\mu = 0(0.10) + 1(0.25) + 2(0.30) + 3(0.15) + 4(0.10) + 5(0.06) + 6(0.04)$

$= 2.24$

$E[X^2] = 0^2(0.10) + 1^2(0.25) + 2^2(0.30) + 3^2(0.15) + 4^2(0.10) + 5^2(0.06) + 6^2(0.04)$

$= 7.34$

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$\sigma^2 = Var(x) = 7.34 - (2.24)^2$

$= 7.34 - 5.0176$

$Var(x) = 2.3224$

$\sigma = \sqrt{\sigma^2} = \sqrt{2.3224}$

$\sigma \approx 1.524$

(b) $F(x) = P(X \leq x)$

$F(0) = 0.10$, $F(1) = 0.35$, $F(2) = 0.65$, $F(3) = 0.80$,

$F(4) = 0.90$, $F(5) = 0.96$, $F(6) = 1.00$

$$f(x) = \begin{cases} 0 & x < 0 \\ 0.10 & 0 \leq x < 1 \\ 0.35 & 1 \leq x < 2 \\ 0.65 & 2 \leq x < 3 \\ 0.80 & 3 \leq x < 4 \\ 0.90 & 4 \leq x < 5 \\ 0.96 & 5 \leq x < 6 \\ 1.00 & x \geq 6 \end{cases}$$

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3.13 $p = 0.005$, $n = 100$, $X \sim Bin(100, 0.0005)$

Poisson approximation (Since n Large, p = Small)

$$\lambda = np = 100(0.0005) = 0.05$$

$$X \approx Pois(0.05)$$

(a) If the Stopping rule is Stop when $X \geq 2$

$$P(X \geq 2) = 1 - P(X=0) - P(X=1)$$

$$P(X=0) = e^{-0.05} = 0.95122942$$

$$P(X=1) = e^{-0.05}(0.05) = 0.04756147$$

$$P(X \geq 2) = 1 - (0.95122942 + 0.4756147)$$

$$\approx 0.00121$$

→ Probability of at least one Stop in 40 Samples

$$P(\text{no Stop in one Sample}) = 1 - 0.00121$$

$$= 0.99879089$$

$$P(\text{no Stop in } 40) = (0.99879089)^{40} \approx 0.95276$$

$$P(\text{at least one Stop}) = 1 - 0.95276 = 0.04724$$

$$P(\text{at least one Stop in } 40) \approx 0.04712 \approx 4.72\%$$

(b) If the Stopping rule is Stop when $X \geq 3$

$$P(X \geq 3) = 1 - [P(0) + P(1) + P(2)]$$

$$\lambda = 0.05$$

$$P(2) = \frac{0.05^2}{2} e^{-0.05} = 0.00118904$$

$$\Rightarrow P(0) + P(1) + P(2) = 0.99997993$$

$$P(X \geq 3) = 1 - 0.99997993 = 0.00002007$$

$$P(\text{Stop in one Sample}) \approx 2.01 \times 10^{-5}$$

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