ISYE435-535 Experimental Design for Engineers
Test #1 Key

Date: Wednesday, February 18, 2026
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NOTE: Students are required to explain and show formulas and rational.

Problem 1 (10 points):

A two-sample *t*-test has been conducted and the sample sizes are $n_1 = n_2 = 12$. The computed value of the test statistic is $t_0 = 2.27$. If the null hypothesis is two-sided, an upper bound on the *P*-value is

(a) 0.10
(b) 0.05 The significance level, α=0.05, becomes the upper bound on the p-value when evaluation the null hypothesis.
(c) 0.025
(d) 0.01
(e) None of the above.

Problem 2 (90 points):

A regional opera company has tried three approaches to solicit donations from 24 potential sponsors. The 24 potential sponsors were randomly divided into three groups of eight, and one approach was used for each group. The dollar amounts of the resulting contributions are shown in the following table.

Approach	Contributions (in $)							
1	1000	1500	1200	1800	1600	1100	1000	1250
2	1500	1800	2000	1200	2000	1700	1800	1900
3	900	1000	1200	1500	1200	1550	1000	1100

(a) Explain clearly and deeply the statistical model to use for this experiment.

Given that this experiment involves one quantitative response (dependent variable) and one factor with three levels, the appropriate statistical model is the one-way fixed-effects ANOVA model, also called the effects model (textbook, pg 58-62):

$$y_{ij} = \mu + \tau_i + \varepsilon_{ij}$$
$$i = 1,2,\dots,a$$
$$j = 1,2,\dots,n$$

where:

- y_{ij} = the jth observation taken at the ith treatment level
- μ = the overall mean (common to all treatments)
- τ_i = the effect of the ith treatment level
- ε_{ij} = the random error term

This model decomposes each observation into three components:

1. Overall mean (μ). Represents the grand average response across all treatments.

2. Treatment effect (τ_i). Represents how much the mean response at level *i* deviates from the overall mean. In other words,

$$\tau_i = \mu_i - \mu$$

   where μ_i is the true mean response at treatment level *i*.

3. Random error (ε_{ij}). Captures all unexplained variability, including:
   - Measurement error
   - Unit-to-unit variability
   - Environmental noise
   - Uncontrolled factors

For the ANOVA test to be valid, the error terms are assumed to satisfy:
1. $\varepsilon_{ij} \sim \text{Normal}(0, \sigma^2)$
2. Errors are independent
3. Constant variance across treatments (homoscedasticity)

Thus,

$$E(y_{ij}) = \mu + \tau_i$$

and

$$\text{Var}(y_{ij}) = \sigma^2$$

The ANOVA tests whether the factor has a significant effect, sing the following hypothesis:

$$H_0: \tau_1 = \tau_2 = \tau_3 = 0$$

which is equivalent to test whether the means at each level are equal:

$$H_0: \mu_1 = \mu_2 = \mu_3$$

versus

$$H_a\text{:At least one mean differs}$$

In order to establish which effects are different, a multiple comparison test is run using Fisher's LSD statistics.

(b) Do the data indicate that there is a difference in results obtained from the three different approaches? Use $\alpha = 0.05$.

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Aproach	2	1362708	681354	9.41	0.001
Error	21	1520625	72411		
Total	23	2883333			

There is a difference between the approaches. This is because the p-value is less than 0.05 and H_0 is rejected. The Fisher's LSD test or the Tukey test will indicate which approaches are different (students can use either one).

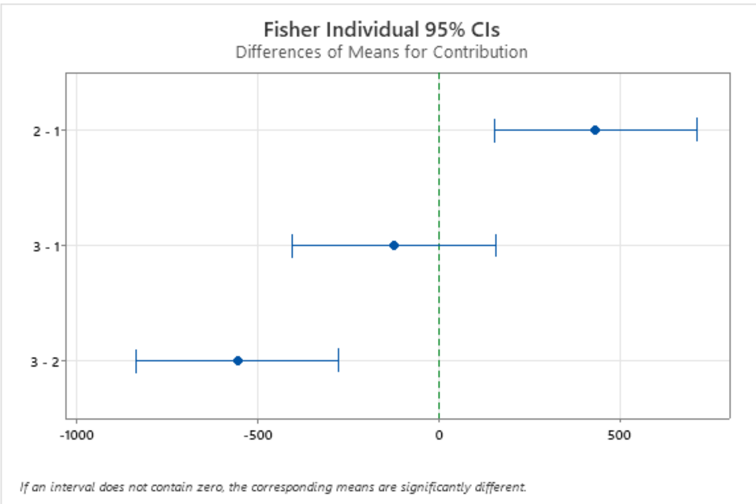
Approach 2 is different than approach 1 and approach 3 as shown in the following output.

Grouping Information Using the Fisher LSD Method and 95% Confidence

Aproach	N	Mean	Grouping
2	8	1737.5	A
1	8	1306	B
3	8	1181.3	B

Means that do not share a letter are significantly different.

The following plot helps to visualize that approach 2 is different from the other two, using a confidence level of 95% simultaneously.



The previous plot is based on the calculations of the following confidence intervals:

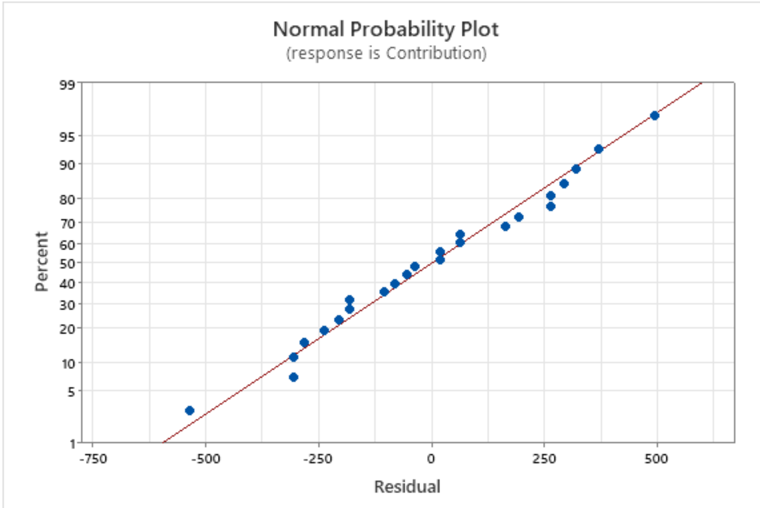
Fisher Individual Tests for Differences of Means

Difference of Levels	Difference of Means	SE of Difference	95% CI	T-Value	Adjusted P-Value
2 - 1	431	135	(151, 711)	3.21	0.004
3 - 1	-125	135	(-405, 155)	-0.93	0.363
3 - 2	-556	135	(-836, -276)	-4.13	0.000

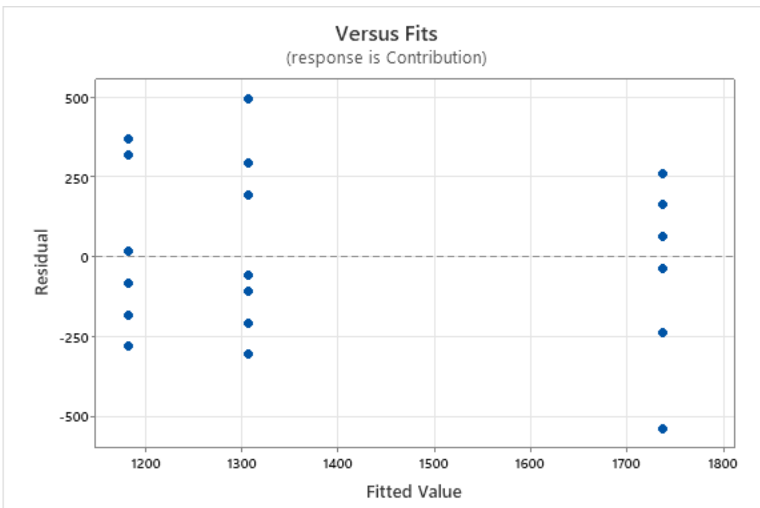
Simultaneous confidence level = 88.16%

It is seen clearly that approach 2 is significantly different from the other approaches at the significance level of 0.05.

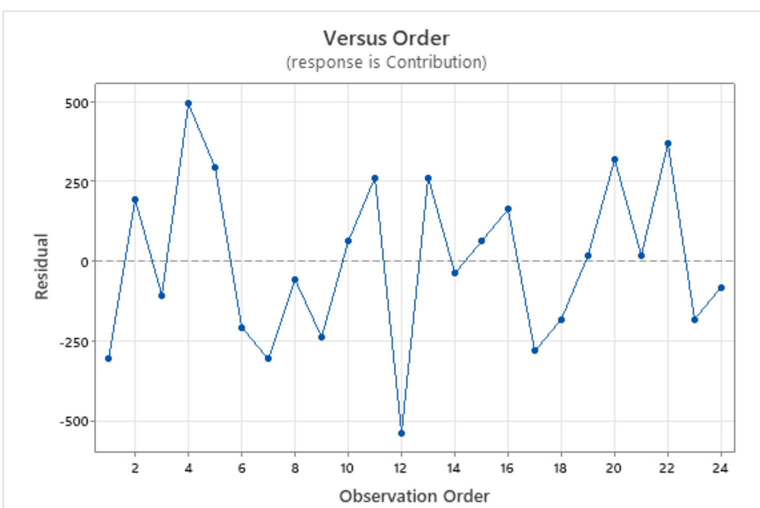
(c) Analyze the residuals from this experiment and comment on the model adequacy.



Students should state the hypothesis being tested with the plot.



Students should state the hypothesis being tested with the plot.



Students should state the hypothesis being tested with the plot.

Conclusion:

There is nothing unusual about the residuals showing that the assumptions of normality in residuals, equal variances and independence are true (students should provide more insights).