Idaho PERSI: R Modeling Guidebook

August 13, 2020

Section 1: Pension Plan Benefits

I. Retirement Benefit

- Eligibility
  - Service Retirement (Normal Retirement): Age 65 with five years of service including six months of membership service (Section 59-1341).
- Amount of Allowance
  - For each year of credited service, the annual service retirement allowance is 2.0% (2.3%) of the highest 42-month average salary (Section 59-1342).
- Minimum Benefit
  - $60 annual allowance for each year of service. The dollar amounts increase after 1974 according to the rate of cost of living increases in retirement allowances (Section 59-1342).
- Normal Form
  - Straight life retirement allowance plus any death benefit (Section 59-1351).
- Optional Form
  - Actuarial equivalent of the normal form under the options available, according to the mortality and interest basis adopted by the Board (Section 59-1351).

II. Early Retirement*

- Eligibility
  - Age 55 with five years of service, including six months of membership service (contributing members only) (Section 59-1345).
- Amount of Allowance
  - Full accrued service retirement allowance if age plus service equals 90; otherwise, the accrued service retirement allowance, reduced by 3% for each of the first five years by which the early retirement date precedes the date the member would be eligible to receive the full accrued benefit, and by 5.75% for each additional year (Section 59-1342).

III. Vested Retirement Allowance

- Eligibility
  - Former contributing members with five years of membership service are entitled to receive benefits after attaining age 55 (Section 59-1345).
- Benefit
  - Same as early retirement allowance (Section 59-1345).

IV. Disability Retirement Allowance

- Eligibility
  - Five years of membership service (Section 59-1352).
- Amount of Allowance

- Projected service retirement allowance based on accrued service plus service projecrted to age 65 (latter limited to excess of 30 years over accrued service) less any amount payable under workers' compensation law (Section 59-1353).

- Normal Form
  - Temporary annuity to age 65 plus any death benefit. Service retirement allowance becomes payable at age 65 (Section 59-1354).

V. Death Benefit

- After Retirement
  - Under the normal form of the retirement allowance, the excess, if any, of the member's accumulated contributions with interest at retirement over all payments received. Otherwise, payable according to the option elected (Section 59-1361).
- Before Retirement
  - An automatic joint and survivor option applied to the actuarial equivalent of the member's accrued service retirement allowance is paid to the surviving spouse of a member with at least five years of service who dies while: (i) contributing; (ii) not contributing, but eligible for benefits; or (iii) retired for disability... OR
  - If a member with at least five years of service has no spouse, a lump sum payment is made equal to twice the accumulated contributions with interest (Section 59-1361)... OR
  - If a member with at least five years of service has no spouse, a lump sum payment is made equal to twice the accumulated contributions with interest (Section 59-1361).

VI. Employee Contributions

- The financing objective of the Fund is to establish contriubtion rates that will tend to remain level as percentages of payroll. The current total contribution rate is 74.10%:62.53% employer contribution rate and 11.57% employee contribution rate.

Section 2: Actuarial Assumptions

Decrement Assumptions

- Decrement Asssumptions:
  - Active members: death, termination, disability, and retirement.
  - Non-active members: death only
  - Single-decrement environment: probability of decrement (q) = rate of decrement (q')
  - Multiple-decrement environment: q (probability) < q' (rate)
    - 2 decrement environment:
$$\mathbf{q^{(1)} = q'^{(1)}[1 - \frac{1}{2}q'^{(2)}]}$$

```
double_decrement <- function(qr_1, qr_2) {

  qr_1 * (1 - (1/2) * qr_2)

}
```

- 3 decrement environment:

$$\mathbf{q^{(1)} = q'^{(1)}[1 - \frac{1}{2}(q'^{(2)} + q'^{(3)}) + \frac{1}{3}q'^{(2)}q'^{(3)}]}$$

```
three_decrement <- function(qr_1, qr_2, qr_3) {

  qr_1 * (1 - ((1/2) * (qr_2 + qr_3)) + ((1/3) * (qr_2 * qr_3)))

}
```

- Mortality Decrement:
  - Probability of a life age x living n years:

$$\mathbf{P_{x \to n} = \prod_{t=0}^{n-1}(1 - q'^{(m)}_{x+t}) = \prod_{t=0}^{n-1}(p'^{(m)}_{x+t})}$$

  - Note that p and q are complementary probabilties of each other
- Salary Assumption = Merit + Productivity + Inflation
- Interest Assumption (Discount Rate, ARR) = Risk-Free Rate + Risk Premium + Inflation

Idaho Data

Mortality

4. Mortality (Adopted July 1, 2014)

Contributing Members, Service Retirement Members, and Beneficiaries

- **Teachers**

  Males RP-2000 Combined Table for Healthy Individuals for males, set back three years.

  Females RP-2000 Combined Table for Healthy Individuals for females, set back three years.

- **General *Employees and All Beneficiaries***

  Males RP-2000 Combined Table for Healthy Individuals for males, set back one year.

  Females RP-2000 Combined Table for Healthy Individuals for females, set back one year.

  All mortality tables are adjusted with generational mortality adjustments using projection scale AA as shown in Table A-8B of the July 1, 2019 valuation report. The projection scale is applied from the year 2000 to the year in which the mortality assumption is being applied.

- ***Disabled Members***

  For disabled members, the mortality rates used in the valuation are the rates from the RP-2000 table for disabled individuals for respective sexes, with a one-year setback for males and a one-year set forward for females.

  All mortality tables are adjusted with generational mortality adjustments using projection scale AA as shown in Table A-8B of the July 1, 2017 valuation report. The projection scale is applied from the year 2000 to the year in which the mortality assumption is being applied.

The RP-2000 tables are available via this hyperlink https://www.soa.org/globalassets/assets/files/research/exp-study/rp00_mortalitytables.pdf. To get this into a data frame, we read in the pdf and created a data frame in R to export to Excel format.

Male Mortality Table

```r
library(readxl)
library(knitr)

m_mort <- read_excel("male_mortality_tables.xlsx")

kable(m_mort[49:71, ])
```

Age	Employees	Healthy Annuitant	Combined Healthy	Disabled Retiree
49	0.001860	NA	0.001860	0.026404
50	0.001995	NA	0.001995	0.027687
51	0.002138	0.005347	0.002138	0.028975
52	0.002288	0.005528	0.002449	0.030268
53	0.002448	0.005644	0.002667	0.031563
54	0.002621	0.005722	0.002916	0.032859
55	0.002812	0.005797	0.003196	0.034152
56	0.003029	0.005905	0.003624	0.035442
57	0.003306	0.006124	0.004200	0.036732

Age	Employees	Healthy Annuitant	Combined Healthy	Disabled Retiree
58	0.003628	0.006444	0.004693	0.038026
59	0.003997	0.006895	0.005273	0.039334
60	0.004414	0.007485	0.005945	0.040668
61	0.004878	0.008196	0.006747	0.042042
62	0.005382	0.009001	0.007676	0.043474
63	0.005918	0.009915	0.008757	0.044981
64	0.006472	0.010951	0.010012	0.046584
65	0.007028	0.012117	0.011280	0.048307
66	0.007573	0.013419	0.012737	0.050174
67	0.008099	0.014868	0.014409	0.052213
68	0.008598	0.016460	0.016075	0.054450
69	0.009069	0.018200	0.017871	0.056909
70	0.009510	0.020105	0.019802	0.059613
71	0.009922	0.022206	0.022206	0.062583

Female Mortality Table

```
f_mort <- read_excel("female_mortality_tables.xlsx")

kable(f_mort[49:71, ])
```

Age	Employees	Healthy Annuitant	Combined Healthy	Disabled Retiree
49	0.001434	NA	0.001434	0.011535
50	0.001550	NA	0.001550	0.012477
51	0.001676	0.002344	0.001676	0.013456
52	0.001814	0.002459	0.001852	0.014465
53	0.001967	0.002647	0.002018	0.015497
54	0.002135	0.002895	0.002207	0.016544
55	0.002321	0.003190	0.002424	0.017598
56	0.002526	0.003531	0.002717	0.018654
57	0.002756	0.003925	0.003090	0.019710
58	0.003010	0.004385	0.003478	0.020768
59	0.003291	0.004921	0.003923	0.021839
60	0.003599	0.005531	0.004441	0.022936
61	0.003931	0.006200	0.005055	0.024080
62	0.004285	0.006919	0.005814	0.025293
63	0.004656	0.007689	0.006657	0.026600
64	0.005039	0.008509	0.007648	0.028026
65	0.005429	0.009395	0.008619	0.029594
66	0.005821	0.010364	0.009706	0.031325
67	0.006207	0.011413	0.010954	0.033234
68	0.006583	0.012540	0.012163	0.035335
69	0.006945	0.013771	0.013445	0.037635
70	0.007289	0.015153	0.014860	0.040140
71	0.007613	0.016742	0.016742	0.042851

NEED TABLE A-8B

We need Table A-8B to see what the “generational mortality adjustments using projection scale AA”.

However, we can set back one year males and females mortality tables. For disabled members a one-year setback for males a one-year set forward for females.

Modified Male Mortality Table

```
m_mort_mod <- read_excel("male_mortality_tables_mod.xlsx")

kable(m_mort_mod[49:71, ])
```

Age	Employees	Healthy Annuitant	Combined Healthy	Disabled Retiree
49	0.001860	NA	0.001860	0.026404
50	0.001995	NA	0.001995	0.027687
51	0.002138	0.005347	0.002138	0.028975
52	0.002288	0.005528	0.002449	0.030268
53	0.002448	0.005644	0.002667	0.031563
54	0.002621	0.005722	0.002916	0.032859
55	0.002812	0.005797	0.003196	0.034152
56	0.003029	0.005905	0.003624	0.035442
57	0.003306	0.006124	0.004200	0.036732
58	0.003628	0.006444	0.004693	0.038026
59	0.003997	0.006895	0.005273	0.039334
60	0.004414	0.007485	0.005945	0.040668
61	0.004878	0.008196	0.006747	0.042042
62	0.005382	0.009001	0.007676	0.043474
63	0.005918	0.009915	0.008757	0.044981
64	0.006472	0.010951	0.010012	0.046584
65	0.007028	0.012117	0.011280	0.048307
66	0.007573	0.013419	0.012737	0.050174
67	0.008099	0.014868	0.014409	0.052213
68	0.008598	0.016460	0.016075	0.054450
69	0.009069	0.018200	0.017871	0.056909
70	0.009510	0.020105	0.019802	0.059613
71	0.009922	0.022206	0.022206	0.062583

Modified Female Mortality Table

```
f_mort_mod <- read_excel("female_mortality_tables_mod.xlsx")

kable(f_mort_mod[49:71, ])
```

Age	Employees	Healthy Annuitant	Combined Healthy	Disabled Retiree
49	0.001434	NA	0.001434	0.011535
50	0.001550	NA	0.001550	0.012477
51	0.001676	0.002344	0.001676	0.013456
52	0.001814	0.002459	0.001852	0.014465
53	0.001967	0.002647	0.002018	0.015497
54	0.002135	0.002895	0.002207	0.016544
55	0.002321	0.003190	0.002424	0.017598
56	0.002526	0.003531	0.002717	0.018654
57	0.002756	0.003925	0.003090	0.019710
58	0.003010	0.004385	0.003478	0.020768

Age	Employees	Healthy Annuitant	Combined Healthy	Disabled Retiree
59	0.003291	0.004921	0.003923	0.021839
60	0.003599	0.005531	0.004441	0.022936
61	0.003931	0.006200	0.005055	0.024080
62	0.004285	0.006919	0.005814	0.025293
63	0.004656	0.007689	0.006657	0.026600
64	0.005039	0.008509	0.007648	0.028026
65	0.005429	0.009395	0.008619	0.029594
66	0.005821	0.010364	0.009706	0.031325
67	0.006207	0.011413	0.010954	0.033234
68	0.006583	0.012540	0.012163	0.035335
69	0.006945	0.013771	0.013445	0.037635
70	0.007289	0.015153	0.014860	0.040140
71	0.007613	0.016742	0.016742	0.042851

Disability Retirement

8. Disability Retirement (Adopted July 1, 2016)

Annual rates assumed for disability retirement are illustrated in the following table:

	General Employees		Teachers	
Age	Male	Female	Male	Female
25	.01%	.01%	.01%	.05%
35	.03	.01	.02	.04
45	.11	.10	.07	.07
55	.32	.28	.20	.30

We don't need to lag disability retirement data, but the data available in the valuation report does not include intermediate ages. One way to overcome this is through linear interpolation (creating a straight line between the known data points). In base R we can use the approx function.

```r
##### Disability Retirement #####

age_dr <- c(25, 35, 45, 55)
age_dr <- approx(age_dr, n = 31)$y

dr_m <- c(0.0001, 0.0003, 0.0011, 0.0032)
dr_m <- approx(dr_m, n = 31)$y

dr_f <- c(0.0001, 0.0001, 0.0010, 0.0028)
dr_f <- approx(dr_f, n = 31)$y

dis_ret <- cbind(age_dr, dr_m, dr_f)
dis_ret <- as.data.frame(dis_ret)
names(dis_ret) <- c("Age", "General Employees - Male", "General Employees - Female")

kable(head(dis_ret))
```

Age	General Employees - Male	General Employees - Female
25	0.00010	1e-04
26	0.00012	1e-04
27	0.00014	1e-04
28	0.00016	1e-04
29	0.00018	1e-04
30	0.00020	1e-04

```r
#write.xlsx(dis_ret, file = "disability_retirement.xlsx", sheetName = "Disability Retirement")
```

Other Terminations

Exhibit 1
(continued)

7. Other Terminations of Employment (Adopted July 1, 2016)

Assumed annual rates of termination are illustrated below. Rates are based only on years of service.

Years of Service	General Employees		Teachers	
	Male	Female	Male	Female
5	8.8%	10.3%	5.5%	6.0%
10	5.5	6.4	3.1	3.1
15	3.5	4.0	1.9	1.8
20	2.4	2.9	1.3	1.3
25	1.7	2.5	1.2	1.2
30	1.5	2.5	1.2	1.2

```r
yos <- c(5, 10, 15, 20, 25, 30)
yos <- approx(yos, n = 26)$y

ter_m <- c(0.088, 0.055, 0.035, 0.024, 0.017, 0.015)
ter_m <- approx(ter_m, n = 26)$y

ter_f <- c(0.103, 0.064, 0.04, 0.029, 0.025, 0.025)
ter_f <- approx(ter_f, n = 26)$y

oth_ter <- cbind(yos, ter_m, ter_f)
oth_ter <- as.data.frame(oth_ter)
names(oth_ter) <- c("Years of Service", "General Employees - Male", "General Employees - Female")

kable(head(oth_ter))
```

Years of Service	General Employees - Male	General Employees - Female
5	0.0880	0.1030
6	0.0814	0.0952
7	0.0748	0.0874
8	0.0682	0.0796
9	0.0616	0.0718
10	0.0550	0.0640

```
#write.xlsx(oth_ter, file = "other_terminations_retirement.xlsx", sheetName = "Other Terminations")
```

Early Retirement

6. Early Retirement (Adopted July 1, 2016)

Annual rates of retirement assumed to occur among persons eligible for a reduced early retirement benefit are illustrated in the following table:

	General Employees		Teachers	
	Male		Female	
Age	First Year Eligible	Thereafter	First Year Eligible	Thereafter
50	*	*	*	*
55	3%	3%	6%	6%
60	5	6	14	12

* *For all ages younger than the age indicated, withdrawal is assumed to occur (see Section 7).*

REVIEW: Early Retirement w/ 55 as First Year Eligibility

```r
age_e <- c(56, 60)
age_e <- approx(age_e, n = 5)$y

ret_m <- c(0.03, 0.06)
ret_m <- approx(ret_m, n = 5)$y

ret_f <- c(0.06, 0.12)
ret_f <- approx(ret_f, n = 5)$y

early_ret <- cbind(age_e, ret_m, ret_f)
early_ret <- as.data.frame(early_ret)
names(early_ret) <- c("Age", "General Employees - Male", "General Employees - Female")

addon <- c(55, 0.03, 0.06)

early_ret <- rbind(addon, early_ret)

kable(early_ret)
```

Age	General Employees - Male	General Employees - Female
55	0.0300	0.060
56	0.0300	0.060
57	0.0375	0.075
58	0.0450	0.090
59	0.0525	0.105
60	0.0600	0.120

```
#write.xlsx(early_ret, file = "early_retirement.xlsx", sheetName = "Early Retirement")
```

Service Retirement

Exhibit 1
(continued)

5. Service Retirement (Adopted July 1, 2016)

Annual rates of retirement assumed to occur among persons eligible for a service retirement are illustrated in the following table:

	General Employees			
	Male		Female	
Age	First Year Eligible	Thereafter	First Year Eligible	Thereafter
55	22%	10%	26%	18%
60	26	17	26	18
65	33	50	37	52
70	18	20	18	21

	General Employees			
	Male		Female	
Age	First Year Eligible	Thereafter	First Year Eligible	Thereafter
55	19%	5%	10%	10%
60	30	18	26	18
65	36	46	49	49
70	*	*	*	*

* *For all ages older than the age indicated, retirement is assumed to occur immediately.*

NEEDS REVISON: Service Retirement w/ 60 First Year Eligible

```
age_t <- c(55, 60, 65, 70)
age_t <- approx(age_t, n = 16)$y

fye_m <- c(0.22, 0.26, 0.33, 0.18)
fye_m <- approx(fye_m, n = 16)$y

ta_m <- c(0.1, 0.17, 0.5, 0.2)
ta_m <- approx(ta_m, n = 16)$y

fye_f <- c(0.26, 0.26, 0.37, 0.18)
fye_f <- approx(fye_f, n = 16)$y

ta_f <- c(0.18, 0.18, 0.52, 0.21)
ta_f <- approx(ta_f, n = 16)$y

serv_ret <- cbind(age_t, fye_m, ta_m, fye_f, ta_f)
serv_ret <- as.data.frame(serv_ret)

names(serv_ret) <- c("Age", "Male - First Year Eligible", "Male - There after",
                     "Female - First Year Eligible", "Female - There after")

kable(serv_ret)
```

Age	Male - First Year Eligible	Male - There after	Female - First Year Eligible	Female - There after
55	0.220	0.100	0.260	0.180
56	0.228	0.114	0.260	0.180
57	0.236	0.128	0.260	0.180
58	0.244	0.142	0.260	0.180
59	0.252	0.156	0.260	0.180
60	0.260	0.170	0.260	0.180
61	0.274	0.236	0.282	0.248
62	0.288	0.302	0.304	0.316
63	0.302	0.368	0.326	0.384
64	0.316	0.434	0.348	0.452
65	0.330	0.500	0.370	0.520
66	0.300	0.440	0.332	0.458
67	0.270	0.380	0.294	0.396
68	0.240	0.320	0.256	0.334
69	0.210	0.260	0.218	0.272
70	0.180	0.200	0.180	0.210

```r
# Single decrement

library(readxl)
library(knitr)

dec_tables <- read_excel('Idaho_Decrement_071620.xlsx')

dec_tables <- as.data.frame(dec_tables)

kable(head(dec_tables))
```

Age	Mortality - Male	Mortality - Female	DR - Male	DR - Female	SR FYE - Male	SR TA - Male	SR FYE - Female	SR TA - Female	ER - Male	ER - Female
1	0.000637	0.000571	0	0	0	0	0	0	0	0
2	0.000430	0.000372	0	0	0	0	0	0	0	0
3	0.000357	0.000278	0	0	0	0	0	0	0	0
4	0.000278	0.000208	0	0	0	0	0	0	0	0
5	0.000255	0.000188	0	0	0	0	0	0	0	0
6	0.000244	0.000176	0	0	0	0	0	0	0	0

```r
survival_prob <- function(start_age = 20, end_age = 65) {
  i <- start_age
  j <- end_age
  prod(1 - dec_tables[i:j, 2])
}

survival_prob(30, 65)
```

```
## [1] 0.8896971
```

You can create multiple decrements column using the previous equations, then simple call that column

```
library(readxl)
library(tidyverse)
```

```
## -- Attaching packages --------------------------------------------------------- tidyverse 1.3.0 --
```

```
## v ggplot2 3.3.2     v purrr   0.3.4
## v tibble  3.0.3     v dplyr   1.0.0
## v tidyr   1.1.0     v stringr 1.4.0
## v readr   1.3.1     v forcats 0.5.0
```

```
## -- Conflicts ------------------------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

```
library(knitr)


dec_tables <- read_excel('Idaho_Decrement_071620.xlsx')

dec_tables <- as.data.frame(dec_tables)


kable(head(dec_tables))
```

Age	Mortality - Male	Mortality - Female	DR - Male	DR - Female	SR FYE - Male	SR TA - Male	SR FYE - Female	SR TA - Female	ER - Male	ER - Female
1	0.000637	0.000571	0	0	0	0	0	0	0	0
2	0.000430	0.000372	0	0	0	0	0	0	0	0
3	0.000357	0.000278	0	0	0	0	0	0	0	0
4	0.000278	0.000208	0	0	0	0	0	0	0	0
5	0.000255	0.000188	0	0	0	0	0	0	0	0
6	0.000244	0.000176	0	0	0	0	0	0	0	0

```
double_decrement <- function(qr_1, qr_2) {

  qr_1 * (1 - (1/2) * qr_2)

}

three_decrement <- function(qr_1, qr_2, qr_3) {

  qr_1 * (1 - ((1/2) * (qr_2 + qr_3)) + ((1/3) * (qr_2 * qr_3)))

}
```

```r
male_dec_table <- dec_tables %>%
  select(1:2, 4, 6:7, 10)


male_d_dec <- male_dec_table %>%
  mutate(d_dec = double_decrement(`Mortality - Male`, `DR - Male`))


male_t_dec <- male_d_dec %>%
  mutate(t_dec = three_decrement(`Mortality - Male`, `DR - Male`, `SR TA - Male`))



kable(male_t_dec[40:60,])
```

	Age	Mortality - Male	DR - Male	SR FYE - Male	SR TA - Male	ER - Male	d_dec	t_dec
40	40	0.001079	0.00070	0.000	0.000	0.000	0.0010786	0.0010786
41	41	0.001142	0.00078	0.000	0.000	0.000	0.0011416	0.0011416
42	42	0.001215	0.00086	0.000	0.000	0.000	0.0012145	0.0012145
43	43	0.001299	0.00094	0.000	0.000	0.000	0.0012984	0.0012984
44	44	0.001397	0.00102	0.000	0.000	0.000	0.0013963	0.0013963
45	45	0.001508	0.00110	0.000	0.000	0.000	0.0015072	0.0015072
46	46	0.001616	0.00131	0.000	0.000	0.000	0.0016149	0.0016149
47	47	0.001734	0.00152	0.000	0.000	0.000	0.0017327	0.0017327
48	48	0.001860	0.00173	0.000	0.000	0.000	0.0018584	0.0018584
49	49	0.001995	0.00194	0.000	0.000	0.000	0.0019931	0.0019931
50	50	0.002138	0.00215	0.000	0.000	0.000	0.0021357	0.0021357
51	51	0.002449	0.00236	0.000	0.000	0.000	0.0024461	0.0024461
52	52	0.002667	0.00257	0.000	0.000	0.000	0.0026636	0.0026636
53	53	0.002916	0.00278	0.000	0.000	0.000	0.0029119	0.0029119
54	54	0.003196	0.00299	0.000	0.000	0.000	0.0031912	0.0031912
55	55	0.003624	0.00320	0.220	0.100	0.030	0.0036182	0.0034374
56	56	0.004200	0.00000	0.228	0.114	0.034	0.0042000	0.0039606
57	57	0.004693	0.00000	0.236	0.128	0.038	0.0046930	0.0043926
58	58	0.005273	0.00000	0.244	0.142	0.042	0.0052730	0.0048986
59	59	0.005945	0.00000	0.252	0.156	0.046	0.0059450	0.0054813
60	60	0.006747	0.00000	0.260	0.170	0.050	0.0067470	0.0061735