Foreword from the First Edition

This book is the culmination of ten years of intensive inquiry by Howard Winklevoss into the intricacies of pension costs and actuarial liabilities. It is a welcome and much needed addition to pension literature.

When I began my pension research activities in the early 1950s, I sought in vain for explanations of pension cost behavior of the type that abound in this book. I suspect that many of these relationships were known, perhaps intuitively, by actuaries actively engaged in pension consulting or the pension operations of life insurance companies. If so, there insights and perceptions were apparently treated as proprietary information, to be confined to internal communications or the inner recesses of their minds. With some notable exceptions, the ideas did not find their way into published pension literature.

To a considerable degree, knowledge of pension cost behavior was constrained by the technology of the day. Pension cost calculations were laborious and many shortcuts and approximations were used. With the development of sophisticated, high-speed computers, it has become feasible to refine pension actuarial techniques and assumptions. In particular, it has become feasible through the construction of computer models to simulate the experience of a pension plan over many years, with a changing population mix and a variety of actuarial assumptions, especially those of an economic nature. The fruit of this improved technology and refined actuarial approaches is a keener perception of pension cost behavior under varied circumstances.

To a large extent this book simply quantifies and provides new or improved actuarial notation for long recognized pension cost concepts and procedures. In certain areas, however, new insights and techniques have been developed. Both types of contributions are useful and innovative. The book should serve the needs of pension actuaries of all persuasions, pension consultants, management and labor pension specialists, governmental officials with pension responsibilities, students, and others interested in the dynamics of pensions. Dr. Winklevoss is to be commended for having completed this prodigious and scholarly task.

On a more personal note, it has been an intellectually rewarding experience to work with Dr. Winklevoss on this project and other academic undertakings.

Philadelphia, PA **Dan M. McGill**
December 1976

Preface

Although this book is entitled *Pension Mathematics with Numerical Illustrations,* Second Edition, it is more like the first edition of a new book. With the exception of the first few chapters, the text is a virtual rewrite of the original book. Two topics have been trimmed back from the first edition, namely, the analysis of ancillary benefits and early retirement. This edition covers all of the relevant material on these topics, but the presentation is more concise. The major additions to the book are chapters on (1) statutory funding requirements, (2) pension accounting, (3) funding policy analysis, (4) asset allocation, and (5) retiree health benefits.

The pension industry has changed considerably since the first edition was published in 1977. At that time, ERISA had just been enacted and was being implemented throughout the pension industry. Just prior to its passage, I was involved in several government-sponsored research assignments on the expected cost of alternative vesting provisions, and the Business Roundtable asked me to share my findings at one of their meetings. After my talk, one of the members cornered me and asked, "Why are you so enthusiastic about the pending pension legislation? Don't you know that once they get the first law passed they'll keep on passing them and eventually do more harm than good?" Little did I know how accurate this statement was.

Many of us had such great expectations for ERISA and, while many of its provisions are clearly beneficial, the avalanche of regulations and new pension legislation since its passage has been terrifying. As an example of its complexity, the reader need only turn to page 156 and glance at the series of equations needed to determine the minimum required contributions to a qualified pension plan. Unless Congress simplifies the various statutory and regulatory requirements for defined benefit pension plans, and does it quickly, only historians will be interested in this book, for it will provide the mathematics of a subject no longer relevant to corporate America.

Another major change since the first edition was the promulgation of SFAS 87, which sets forth a comprehensive set of procedures to follow in accounting for pension plans. Corporate America vigorously fought against the FASB requiring such a rigid set of accounting rules but has hardly said a word since its implementation in 1987. One of the reasons for this silence may be that many companies have experienced negative pension expense (i.e., pension income) since the implementation of SFAS 87, which no doubt came as a pleasant surprise. This resulted from strong capital markets in the 1980's and the FASB's requirement that the discount rate used in calculating pension expense be tied to the spot rate on long-term corporate bonds. These rates, in recent years, have been uncharacteristically high relative to inflation, a result that no doubt occurred because of the poor performance of fixed-income investments when inflation was high in the early 1980's. In any event, it will be interesting to see the reaction among plan sponsors if long-term rates decrease at a time when the market value of plan assets drop. Complacency with the accounting standard may turn, once again, to vigorous complaints.

The third area of change in the pension field since the mid-1970's is among practitioners. I recall speaking at an actuarial meeting in the late 1970's where I was asked to defend the use of explicit best estimate assumptions. It did not seem like much of an issue because using such assumptions appeared logical. Conservatism, I argued, could be achieved by contributing more to the plan than the amount determined using best estimate assumptions. To my surprise, only a few individuals at the meeting seemed to embrace the use of explicit best estimate assumptions. This has all changed now, with best estimate assumptions being required for both statutory and accounting calculations. In fact, if anything, the assumptions may have become a little too "best estimate." For example, the first edition of this book used a 7 percent interest rate in all of the illustrations, a rate that was on the high side at the time. This edition uses an 8 percent rate, which, ironically, is on the low side. Has the actuarial community, egged on by SFAS 87 as well as the IRS, become a little too aggressive, with 9 and 10 percent rates being common? Plan sponsors have no doubt enjoyed the effects of using higher interest rates which, in many cases, caused their contributions to be zero and accounting expense to be negative. On the other hand, if future interest rates must be lower than the rates currently used because of sustained low levels of inflation, the adjustment to increased costs may be painful for many corporations.

The author would like to thank all of the individuals who assisted in this edition. I owe special thanks to Dan M. McGill, Emeritus Professor, Wharton School. Nearly 25 years ago Dan hired me to teach at the Wharton School, and my association with him over the ensuing 12 years while I was teaching there was a richly rewarding experience. He gave me considerable encouragement and assistance in completing the first edition as well as the second edition of this book.

Steven R. Strake, senior actuary at Winklevoss Consultants, performed all of the numerical calculations in the book. As readers will soon discover, this represents a huge amount of work for which I am very grateful. This work was performed by Glenn D. Allison in the first edition and the author would like to recognize that contribution in this edition as well. Jing Cheng Liu, Winklevoss Consultants, provided considerable assistance in editing the book, especially with respect to the statutory and regulatory aspects of pension funding.

Two reviewers deserve special thanks for their very thorough and tedious reviews of each and every sentence and equation in the book. The first is both a former student and employee: Debbie L. Benner, William M. Mercer. It has been a pleasure to work with her over the years. David R. Kass, David R. Kass & Company, provided a similarly exhaustive review, for which I am very grateful.

The University Press sought reviews from two academicians with actuarial backgrounds: Frank G. Bensics, University of Hartford, and James C. Hickman, University of Wisconsin. Both individuals provided insightful comments which were incorporated into the book.

Three Pension Research Council members were asked to review the text. In each case, the comments were extensive and beneficial. These individuals, along with two others who assisted in their review, are Donald S. Grubbs, Jr. and Joseph D. Marsden, Grubbs and Company; Marc M. Twinney and Norman

J. Campeau, Ford Motor Company; and Howard Young, University of Michigan.

Three individuals provided very helpful assistance on Chapter 16, Funding and Accounting for Retiree Health Benefits: Harold S. Cooper, Chicago Consulting Actuaries; Charles C. Morgan, The Prudential Asset Management Company; and Dale H. Yamamoto, Hewitt Associates.

Any remaining errors, omissions, or other shortcomings in the book are the sole responsibility of the author. Readers are invited to assist with the next edition by advising the author of any errors and offering suggestions.

Greenwich, CT **Howard E. Winklevoss**
December 1992

Chapter 1

Pension Plan Benefits

The primary function of a pension plan is to provide income to employees in their retirement. Pension plans are not limited to providing retirement income, however, and all plans provide at least some of the following types of benefits: (1) vested termination benefits, (2) disability benefits, and (3) death benefits, the latter consisting of either a lump sum benefit or an annuity to a surviving spouse or other dependents. The eligibility requirements and benefit formulas typically associated with each of these benefits are discussed in this chapter. In addition, the benefits associated with a model pension plan used to illustrate pension costs throughout this book are given.[1]

There are two basic types of pension plans: *defined contribution* (DC) and *defined benefit* (DB). A defined contribution plan, as the name implies, has a defined amount of employer and/or employee contributions set aside each year, often as a specified percentage of salary.[2] The employee's retirement benefit is determined by the size of the accumulation at retirement. For example, if the accumulation totals $100,000 and is applied to provide an annuity payable for life, the annual benefit might fall in the range of $10,000 to $15,000 per year, depending on the em-

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[1]For comprehensive treatments of pension plan design see: Everett T. Allen, Jr., Joseph J. Melone, Jerry S. Rosenbloom and Jack L. VanDerhei, *Pension Planning*, 7th ed. (Homewood, Ill.: Richard D. Irwin, 1992); Jerry S. Rosenbloom, *The Handbook of Employee Benefits: Design, Funding and Administration*, 3rd ed. (Homewood, Ill.: Richard D. Irwin, 1992); and Dan M. McGill and Donald S. Grubbs, Jr., *Fundamentals of Private Pensions*, 6th ed. (Homewood, Ill.: Richard D. Irwin, 1989).

[2]Profit sharing plans, which are a type of "defined contribution" plan, often define the method of allocating contributions among employees rather than defining contributions themselves.

ployee's age at retirement, the interest rate assumed during retirement, the form of the benefit payment (e.g., whether or not payments continue to a surviving spouse after the death of the annuitant), and so forth. The important point is that for a DC plan, yearly contributions to an accumulation fund are *defined*, whereas the benefit ultimately paid to the employee is not known for certain until retirement.

Table 1–1 illustrates the benefits payable, as a percentage of retirement-age salary, for an 8-percent-of-salary DC plan under various assumptions as to employment periods and interest returns on the accumulating funds. The other assumptions used in the illustration are (1) salary increases of 6 percent per year, (2) retirement at age 65, (3) 8 percent interest return in retirement, (4) mortality based on 1971 Group Annuity Mortality (GAM) Table, and (5) benefits payable during the life of the retiree only (i.e., a so-called straight life annuity). The results illustrate a substantial difference in benefits for employees with differing periods of service and/or interest returns on the accumulating funds. The benefit percentages are directly proportional to the contribution rate; hence, a 4 percent contribution formula would produce benefits equal to 50 percent of those shown in Table 1–1.

TABLE 1–1

Retirement Benefits as Percent of Salary at Retirement for 8-Percent-of-Salary DC Plan

Interest	*Entry Ages*		
Return	25	35	45
6%	39%	29%	20%
8%	60	40	24
10%	92	55	30

A defined benefit plan is one under which the retirement benefit (as opposed to the employer's contribution) is the defined quantity, generally expressed in terms of the employee's salary and length of service. For example, the plan might provide a benefit of 1.5 percent of the employee's average salary during the final 5 years of employment for each year of service rendered. An employee with 20 years of service would receive a benefit equal to 30 percent of final average salary. Since this example is based on the employee's salary, the precise benefit will not be known until retirement, as was the case under a defined contribution plan; nevertheless, the *benefit formula*, as opposed to the *contribution formula*, is defined, hence, the term defined benefit plan.

Table 1–2 illustrates the benefits payable, as a percentage of retirement-age salary, for alternative DB formulas that encompass the range typically found among large corporate pension plans. The benefit formulas are based on the employee's final 5-year average salary. The other assumptions used in the illustration are (1) salary increases of 6 percent per year, (2) retirement at age 65, and (3) benefits payable during the life of the retiree only. The 1.5 percent DB formula is roughly equivalent to the 8 percent DC formula assuming the fund earns 8 percent per year. This might lead one to conclude that the costs associated with a 1.5 percent DB plan would equal 8 percent of salary. While this would be true if neither plan offered vesting prior to retirement, it is not the case if such plans provide the minimum vesting standards required by law. The DB plan is generally less costly because the cost of providing vested accrued benefits to terminating employees is typically less than the value of employer contributions taken by employees terminating under the DC plan prior to retirement.

TABLE 1–2

Retirement Benefits as Percent of Salary at Retirement for Alternative DB Plan Formulas

Benefit	*Entry Ages*		
Percent	25	35	45
1.0%	35%	26%	17%
1.5%	52	39	26
2.0%	69	52	35

This book presents the mathematics applicable to defined benefit pension plans. In particular, the various methods of determining employer contributions required to fund such benefits are given. The last chapter of this book considers a related topic; namely, the mathematics associated with retiree health benefit plans.

RETIREMENT BENEFITS

The eligibility requirements and benefit formulas typically associated with DB pension plans are discussed in this section, along with the provisions assumed for the model pension plan.[3] The discussion also points out the minimum plan design requirements and other pertinent rules established under the Employee Retirement Income Security Act (ERISA), as well as subsequent legislation.

Eligibility Requirements

There are two categories of eligibility requirements related to retirement benefits, one setting the requirements for plan membership and the other specifying the requirements for retirement under the plan.

The Employee Retirement Income Security Act of 1974 (ERISA), as amended, places a limit on the age and service requirements for plan membership. An employee must be eligible to join the plan after reaching age 21 and rendering one year of service.[4] Naturally, a plan can provide more liberal eligibility requirements than those specified under ERISA, and some plans have no waiting period whatsoever. Unless the plan is contributory, in which case the employee may elect not to participate, benefits frequently begin to accrue at the time these membership eligibility requirements are met. Alternatively, benefit accruals might be granted retroactively to the employee's date of hire, but the law does not require that pre-eligibility service be included for benefit accrual purposes. The pension cost data presented in this book are based on the assumption that plan membership commences at the date of hire.

At the other end of the employment cycle are the eligibility requirements for retirement. The normal retirement age of the plan is stated in the plan document and frequently specified as age 65. The traditional definition of the normal retirement age is the first age at which retirement can occur without any reduction in the benefits calculated according to the plan's benefit formula. This definition does not apply to many large corporate pension

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[3]Table 1–3 on page 11 summarizes all of the model pension plan benefits.

[4]If the plan provides full and immediate vesting, the eligibility requirements for plan participants can be extended to two years of service.

plans, however, since non-reduced benefits are often provided at ages below the plan's normal retirement, sometimes as early as age 60 for a plan having 65 as the normal retirement age. The relevance of the normal retirement age for such plans is that it is frequently used as the commencement date for deferred pension benefits to vested terminated employees and, in some cases, to disabled employees.

The symbol r is used to denote the normal retirement age in the pension mathematics presented in later chapters, with r' denoting the first age at which an employee becomes eligible for early retirement, with either full benefit accruals or reduced benefits, and r'' denoting the age by which all employees are assumed to be retired. Unless otherwise stated, r is age 65 in the numerical illustrations and r' is assumed to be based on the employee's age and service.[5] The early retirement eligibility provision assumed for the model pension plan is one permitting retirement, on an actuarially reduced basis, upon the attainment of age 55 and the completion of 10 years of credited service. Thus, for most employees, r' will be age 55.

Benefit Amount

The most common type of benefit formula used in pension plans is the so-called *unit benefit formula*, which provides a unit of benefit for each year of credited service. There are three such formulas associated with defined benefit plans: (1) flat dollar, (2) career average, and (3) final average. The flat dollar benefit formula is the simplest of the three, providing a dollar amount, such as \$20, per month for each year of service rendered by the employee. The flat dollar amount is generally increased at periodic intervals by plan amendment, either to keep pace with the inflationary trends in the economy and/or in response to union negotiations. Since it is not permissible under existing IRS regulations to anticipate future benefit increases for funding purposes, the intermittent jumps in the unit benefit can cause erratic contribution patterns and may prevent such plans from reaching full funding.[6]

[5] The oldest retirement age, r'', is assumed to be equal to r unless otherwise indicated.

[6] The funding characteristics associated with this type of benefit formula are illustrated in Chapter 13.

The career average benefit formula provides a benefit defined in terms of some stipulated percentage of the employee's career average salary. For example, a plan might define an employee's benefit accrual as 2 percent of each year's salary.[7] It is permissible under current law to use a larger percentage for salary in excess of the average Social Security wage base than the percentage applied to salary under this level, an integration formula known as the *step rate method.* The difference in the percentages, which is generally limited to a value of .75 percent (or, if lower, the base percentage itself), is allowed so that an employer's plan can compensate for the fact that Social Security benefits are not based on total salary.[8] Thus, if a plan were to use the maximum difference under the step rate benefit formula, in theory at least, total benefits (pension plus the portion of the Social Security benefit attributable to employer contributions) would constitute roughly the same percentage of salary for all employees. A plan using such a formula is said to be *integrated* with Social Security, and is deemed under current law not to be discriminatory in favor of highly paid employees. Although the employer's plan itself does indeed discriminate in favor of higher paid employees, such discrimination is designed to offset the reverse discrimination inherent in Social Security benefits which favor lower paid employees.

The final average benefit formula is one providing a given percentage of the employee's final average (or highest average) salary per year of service. Since the benefits derived from this type of formula are based on the employee's salary near retirement, the percentage need not be as high as the career average formula percentage in order to provide equivalent benefits.

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[7]Career average benefit formulas are often updated by basing past service benefits on the employee's current salary. Thus, the employee's retirement benefit may approximate the benefits of a final average formula. As under a flat dollar benefit formula, the updating process can cause erratic contribution patterns and may prevent such plans from reaching full funding, as illustrated in Chapter 13.

[8]The .75 percent factor must be reduced if the plan permits normal retirement prior to the employee's normal retirement age under Social Security (which is now dependent on the employee's birth date) and/or if the benefit reduction for early retirement is lower than the reduction specified under law. As a practical matter, the factor is .65 for plans that want to use the same benefit formula for all participants, since age 67 is the applicable retirement age under Social Security for younger participants and most plans have age 65 as the normal retirement age.

This formula can also be integrated with Social Security according to the step rate procedure. For example, a plan could provide 1 percent of an employee's final 5-year average salary up to the applicable Social Security wage base and 1.75 percent on the excess salary (a smaller differential may be required, depending on the plan's normal and early retirement provisions).

Another procedure for integrating benefits with Social Security is known as the *offset method*. Under this procedure, which is often used with final average benefit formulas, the retirement benefit of the plan is determined without regard to the Social Security wage base, but the benefit so determined is offset (reduced) by up to .75 percent of the retiring employee's final 3-year average compensation (up to the covered compensation under Social Security) per year of service. In no event can the reduced benefit be less than 50 percent of the unreduced benefit.[9]

The formula used for the model pension plan is 1.5 percent of the participant's final 5-year average salary for each year of credited service. Social Security integration is not used in order to simplify the various analyses; however, many pension plans are in fact integrated with Social Security. The normal annuity form is a straight life annuity, with early retirement benefits actuarially reduced (i.e., a reduction that offsets the otherwise higher cost of benefits beginning sooner and expected to be paid longer to participants retiring early).

VESTED BENEFITS

An employee has a vested benefit if its payment at retirement is no longer contingent upon remaining in the service of the employer. When an employee terminates employment with a vested benefit, the benefit amount generally becomes payable at the plan's normal retirement age; however, some plans permit pay-

[9]Another offset methodology, which was widely used prior to the Tax Reform Act of 1986, was to reduce the final average formula benefit by a percentage of the employee's Primary Insurance Amount (PIA) payable under Social Security, with the offset generally limited to 50 percent, but possibly as high as 83 1/3 percent. While this method is still used, the benefit payable (when combined with Social Security benefits) cannot represent a higher percentage of salary for higher paid employees than for lower paid employees.

ments to begin at an early retirement age, usually with an actuarial reduction for early payment.[10]

Eligibility Requirements

The Tax Reform Act of 1986 (TRA '86) requires that pension benefits vest according to one of two schedules: (1) full vesting upon the completion of 5 years of service or (2) graded vesting, with 20 percent of the accrued benefit vested after completing 3 years of service and an additional 20 percent per year thereafter, reaching full vesting after 7 years of service.[11] Vesting after 5 years of service is used for the model pension plan.

Benefit Amount

In most cases, the benefit accruals used to determine vested benefits are those defined by application of the retirement benefit formula. The cost of providing benefits to vested terminating employees could be minimized by adopting a formula with disproportionate benefits provided at older ages and/or longer periods of service; however, the law has established guidelines to prevent such *backloading*.

In all cases, benefits attributable to employee contributions are fully and immediately vested. At the time of termination, employees may be entitled to a return of their contributions, usually with interest, instead of leaving them in the plan and receiving a deferred retirement benefit. Prior to the passage of ERISA, the employee's election to take back contributions often had the effect of forfeiting all of the benefits associated with employer contributions.

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[10] Most plans pay small vested benefits as lump sums to terminated employees and many plans allow lump sum payments for larger amounts as well.

[11] ERISA initially mandated three alternative vesting schedules: (1) full vesting after 10 years, (2) graded vesting, with 25 percent vesting after 5 years, increasing by 5 percent per year for the next 5 years and then 10 percent per year for the next five years, and (3) Rule-of-45 vesting, with 50 percent vesting when the participant's age and years of service total 45, and an additional 10 percent for each of the 5 subsequent years.

DISABILITY BENEFITS

Two types of disability benefits are found in pension plans, one providing a deferred pension to disabled employees beginning at the plan's normal retirement age and continuing for life, and the second providing benefits that commence after a specified waiting period, such as 6 or 9 months, and continuing for life. The former type is generally found in combination with a long-term disability (LTD) benefit program operating outside the pension plan, with the LTD plan providing the desired level of benefits from the time of disability to the plan's normal retirement age.

Eligibility Requirements

The eligibility provisions for disability benefits vary widely among plans, but a minimum age or service requirement, or both, usually exists. The disability entitlement might coincide with eligibility for early retirement, for example, age 55 and 10 years of service, or with eligibility for vesting. The model pension plan provides employees with their accrued benefits, payable immediately, if disability occurs after age 40 with 10 years of credited service.

Benefit Amount

The most common method used to define disability benefits is simply to apply the retirement benefit formula to the employee's salary and years of service at the time of disability. Some plans use the total potential service of the employee up to the plan's normal retirement instead of actual service at the date of disability. This is almost invariably the case if the disability pension is the deferred type used in conjunction with an LTD program. In some instances, the amount of the disability benefit is a flat amount per month, irrespective of the employee's service or accrued benefit. The benefit assumed for the model pension plan is the unreduced accrued benefit commencing at the time of disability and payable for life.

DEATH BENEFITS

Death benefits may consist of a lump sum distribution, such as a flat dollar amount or, for active employees, some multiple of salary. Alternatively, the death benefit may take the form of an annuity payable to a surviving spouse.

Eligibility Requirements

Death benefit eligibility provisions are often related to the plan's eligibility for vesting. In fact, ERISA requires that a surviving spouse option be made available to plan members at the time they become vested. The benefit must be payable by the age the employee would have been eligible for early retirement, or it can be made available earlier. In either case, an actuarial reduction can be applied if the benefit commences prior to the employee's normal retirement age.

The model pension plan provides a surviving spouse benefit upon satisfying the requirements for vesting (i.e., 5 years of service).

Benefit Amount

If a surviving spouse benefit is payable, the amount is generally some percentage of the deceased participant's accrued benefit. A common formula is 50 percent of the participant's attained age accruals, possibly reduced for early retirement or for the actuarial cost of the surviving spouse benefit itself. The benefit payable under the model pension plan is 50 percent of the employee's attained age accrued benefit without reduction.

A summary of the benefits provided under the model pension plan is given in Table 1–3. While some plans require employee contributions, the model plan, as indicated in Table 1–3, does not require such contributions.

TABLE 1-3

Summary of Model Plan Benefits

I. Retirement Benefit	
A. Eligibility	
1. Normal Retirement	Age 65
2. Early Retirement	Age 55 and 10 years of service
B. Benefit	1.5 percent of final 5-year average salary per year of service, payable for life, actuarially reduced for early retirement
II. Vested Benefit	
A. Eligibility	Full vesting after 5 years of service
B. Benefit	Accrued benefit (based on the retirement benefit formula applied to final average salary and service at termination), payable at age 65 for life
III. Disability Benefit	
A. Eligibility	Age 40 and 10 years of service
B. Benefit	Accrued unreduced benefit, payable immediately for life
IV. Death Benefit	
A. Eligibility	5 years of service
B. Benefit	50 percent of accrued benefit, payable for life of surviving spouse, commencing when employee would have been eligible for early retirement
V. Employee Contributions	None

Chapter 2

Actuarial Assumptions

This chapter discusses the actuarial assumptions used to calculate pension costs and liabilities. These include various rates of decrement applicable to plan members, future salary estimates for plans with benefits linked to salary, and future interest returns on plan assets. In addition to a general discussion of actuarial assumptions, the specific assumptions used with the model pension plan are given.

DECREMENT ASSUMPTIONS

Active plan participants are exposed to the contingencies of death, termination, disability, and retirement, whereas nonactive members are exposed to death.[1] These contingencies are dealt with in pension mathematics by various *rates of decrement*. A rate of decrement refers to the proportion of participants leaving a particular status due to a given cause, under the assumption that there are no other decrements applicable. If such a rate is used in a single-decrement environment (i.e., where there are in fact no other decrements applicable), it is also equal to the *probability of decrement*. For example, since retired employees exist in a single-decrement environment, being exposed only to mortality, the applicable rate of mortality at a given age is identical to the probability of dying at that age.

[1]Technically, some nonactive participants are "exposed" to the contingency of re-entry into active status, or what could be called an incremental assumption. This contingency is not considered in the mathematics developed in this book; however, increments to the entire active membership via new hires are considered.

The rate of decrement in a multiple-decrement environment (i.e., where more than one decrement is operating), is *not* equal to the probability of decrement. Active employees exist in a multiple-decrement environment, being exposed to mortality, termination, disability, and retirement; hence, the rate of decrement is *not* equal to the probability of decrement because the other decrements prevent participants from being exposed to the contingency throughout the year.

A typical assumption for transforming a *rate* into a *probability* for a multiple-decrement environment is that all decrements occur on a uniform basis throughout the year, referred to as the uniform distribution of death (UDD) assumption. With $q'^{(k)}$ denoted as the rate of decrement for cause k and $q^{(k)}$ (without the prime symbol) as the probability of decrement, the transformation of a rate into a probability in a double-decrement environment ($k = 1, 2$) under the UDD assumption is given by

$$q^{(1)} = q'^{(1)}\left[1 - \tfrac{1}{2}q'^{(2)}\right]. \tag{2.1a}$$

If both rates were 10 percent, the corresponding probabilities would be 9.5 percent.

The value of $q^{(1)}$ for a three-decrement environment becomes

$$q^{(1)} = q'^{(1)}\left[1 - \tfrac{1}{2}(q'^{(2)} + q'^{(3)}) + \tfrac{1}{3}q'^{(2)}q'^{(3)}\right], \tag{2.1b}$$

and for a four-decrement environment, we have

$$\begin{aligned} q^{(1)} = q'^{(1)}\Big[1 &- \tfrac{1}{2}(q'^{(2)} + q'^{(3)} + q'^{(4)}) \\ &+ \tfrac{1}{3}(q'^{(2)}q'^{(3)} + q'^{(2)}q'^{(4)} + q'^{(3)}q'^{(4)}) \\ &- \tfrac{1}{4}(q'^{(2)}q'^{(3)}q'^{(4)})\Big]. \end{aligned} \tag{2.1c}$$

Again, if each rate were 10 percent, the corresponding probabilities would be 9.0333 and 8.5975 percent, respectively, for the three- and four-decrement cases.

The mathematics and numerical analysis presented in this book are based on an approximation to the UDD assumption in the case of three or more decrements. The value of $q^{(1)}$ for $k = 3$ is approximated by

$$\begin{aligned} q^{(1)} &\approx q'^{(1)}\left[1 - \tfrac{1}{2}q'^{(2)}\right]\left[1 - \tfrac{1}{2}q'^{(3)}\right] \\ &\approx q'^{(1)}\left[1 - \tfrac{1}{2}(q'^{(2)} + q'^{(3)}) + \tfrac{1}{4}q'^{(2)}q'^{(3)}\right], \end{aligned} \tag{2.2a}$$

and, for $k = 4$, approximated by

$$\begin{aligned} q^{(1)} &\approx q'^{(1)}\left[1 - \tfrac{1}{2}q'^{(2)}\right]\left[1 - \tfrac{1}{2}q'^{(3)}\right]\left[1 - \tfrac{1}{2}q'^{(4)}\right] \\ &\approx q'^{(1)}\Big[1 - \tfrac{1}{2}(q'^{(2)} + q'^{(3)} + q'^{(4)}) \\ &\qquad + \tfrac{1}{4}(q'^{(2)}q'^{(3)} + q'^{(2)}q'^{(4)} + q'^{(3)}q'^{(4)}) \\ &\qquad - \tfrac{1}{8}(q'^{(2)}q'^{(3)}q'^{(4)})\Big]. \end{aligned} \tag{2.2b}$$

The error in this approximation is quite small, causing $q^{(1)}$ to be understated by $\frac{1}{12}q'^{(2)}q'^{(3)}$ for the three-decrement case and by

$$\tfrac{1}{12}(q'^{(2)}q'^{(3)} + q'^{(2)}q'^{(4)} + q'^{(3)}q'^{(4)}) - \tfrac{1}{8}(q'^{(2)}q'^{(3)}q'^{(4)})$$

for the four-decrement case. For example, if all rates were 10 percent, the probabilities would be understated by .000833 and .002375, respectively, for the three- and four-decrement cases. Since a 10 percent rate is greater than most decrement rates, the error in the above approximations is of no practical significance.

The above shows, and general reasoning would suggest, that the probability of decrement is *smaller* than the rate of decrement in a multiple-decrement environment. The degree of reduction is dependent on the number and magnitude of the competing decrements. In a pension plan environment, the reduction can be substantial for active employees, owing both to the number of decrements and the relative size of some decrements, such as termination and retirement rates.

The remaining portion of this chapter is devoted to a discussion of various pension plan decrement rates, or the equivalent of probabilities in a single-decrement environment. Chapter 3 considers these rates in a multiple-decrement environment, the proper context for pension plans.

As noted previously, the prime symbol on $q'^{(k)}$ indicates a rate of decrement in a multiple-decrement environment, while $q^{(k)}$ denotes the corresponding probability of decrement. The following four rates will be discussed:

$q'^{(m)}$ = mortality rate	$q'^{(t)}$ = termination rate
$q'^{(d)}$ = disability rate	$q'^{(r)}$ = retirement rate

Mortality Decrement

Mortality among active employees, of course, eliminates the retirement benefit obligation, while mortality among pensioners terminates the ongoing obligation. On the other hand, mortality may create another form of benefit obligation. Mortality prior to retirement may trigger a lump sum benefit based on a flat-dollar amount or some multiple of salary, or the commencement of annual payments to a surviving spouse, either for a specified period of time or for life. Similarly, death in retirement may result in the continuation of all or some portion of the deceased's pension, either to the estate, to a surviving spouse, or to some other beneficiary.

Age is the most obvious factor related to mortality. Annual mortality rates become progressively higher as age increases, beginning at approximately 0.05 percent at age 20, reaching 2 percent by age 65, and increasing to 100 percent at the end of the human life span, generally assumed to be age 100 or 110. A second factor related to mortality is gender. Females tend to have lower mortality rates than males at every age. Empirical studies have shown that for ages near retirement, a 5-year age setback in the male table allows one to achieve a reasonably good approximation to the female mortality, with a somewhat larger setback at younger ages and a smaller setback at older ages. There are other factors, such as occupation, that tend to be related to mortality, but these usually are not taken into account unless the circumstances call for a more refined evaluation.

If the pension plan population is large, mortality rates based on its past experience may be developed. The development of such rates normally involves some combination of the most recent mortality experience of the group, past trends in its mortality, and a subjective element reflecting anticipated future changes in mortality. A more sophisticated approach is to develop a series of mortality rate schedules, one for each future calendar year. The theory underlying this procedure is that the mortality rate for an employee currently age x, for example, will differ from the mortality rate for an employee reaching this age 10 or 20 years from the present time.

The mortality rates used to illustrate pension costs are based on the 1971 Group Annuity Mortality (GAM) Table for males, as shown in Table 2–1.[2]

TABLE 2–1

1971 Group Annuity Mortality Rates for Males

x	$q_x^{'(m)}$	x	$q_x^{'(m)}$	x	$q_x^{'(m)}$	x	$q_x^{'(m)}$	x	$q_x^{'(m)}$
20	0.00050	40	0.00163	60	0.01312	80	0.08743	100	0.32983
21	0.00052	41	0.00179	61	0.01444	81	0.09545	101	0.35246
22	0.00054	42	0.00200	62	0.01586	82	0.10369	102	0.37722
23	0.00057	43	0.00226	63	0.01741	83	0.11230	103	0.40621
24	0.00059	44	0.00257	64	0.01919	84	0.12112	104	0.44150
25	0.00062	45	0.00292	65	0.02126	85	0.13010	105	0.48518
26	0.00065	46	0.00332	66	0.02364	86	0.13932	106	0.53934
27	0.00068	47	0.00375	67	0.02632	87	0.14871	107	0.60607
28	0.00072	48	0.00423	68	0.02919	88	0.15849	108	0.68744
29	0.00076	49	0.00474	69	0.03244	89	0.16871	109	0.78556
30	0.00081	50	0.00529	70	0.03611	90	0.17945	110	1.00000
31	0.00086	51	0.00587	71	0.04001	91	0.19049		
32	0.00092	52	0.00648	72	0.04383	92	0.20168		
33	0.00098	53	0.00713	73	0.04749	93	0.21299		
34	0.00105	54	0.00781	74	0.05122	94	0.22654		
35	0.00112	55	0.00852	75	0.05529	95	0.24116		
36	0.00120	56	0.00926	76	0.06007	96	0.25620		
37	0.00130	57	0.01004	77	0.06592	97	0.27248		
38	0.00140	58	0.01089	78	0.07260	98	0.29016		
39	0.00151	59	0.01192	79	0.07969	99	0.30913		

The probability of an individual surviving n years is an important calculation for pension plans. If the rate of mortality at age x is denoted by $q_x^{'(m)}$, the probability of a life age x living to age $x + 1$ is given by the complement of the mortality rate and denoted by $p_x^{(m)}$. For the general case, the probability of a life age x living n years is denoted by ${}_np_x^{(m)}$ and may be expressed in terms of the specific rates of mortality at age x through age $x + n - 1$ as follows:

$$ {}_np_x^{(m)} = \prod_{t=0}^{n-1} (1 - q_{x+t}^{'(m)}) = \prod_{t=0}^{n-1} p_{x+t}^{(m)}. \tag{2.3} $$

[2]As shown in Chapter 12, switching from the GAM 71 to the GAM 83 table increases pension liabilities and long-run costs by about 7 percent.

With r denoted as the plan's normal retirement age, the probability of surviving to this age, ${}_{r-x}p_x^{(m)}$ for $x \leq r$, and the probability of surviving beyond this age, ${}_{x-r}p_r^{(m)}$ for $x \geq r$, are given in Table 2–2 for various values of x and for r equal to 65.

TABLE 2–2

Mortality-Based Survival Probabilities

x	${}_{65-x}p_x^{(m)}$	x	${}_{x-65}p_{65}^{(m)}$
20	0.8099	65	1.0000
25	0.8121	70	0.8740
30	0.8149	75	0.6988
35	0.8187	80	0.4947
40	0.8241	85	0.2856
45	0.8326	90	0.1273
50	0.8485	95	0.0411
55	0.8767	100	0.0083
60	0.9225	105	0.0007
65	1.0000	110	0.0000

These probabilities illustrate that retirement-related pension costs are reduced by 10 to 20 percent by pre-retirement mortality. For example, if the average age of a group of active employees were 40, pension costs would be reduced by 15 to 20 percent due to the mortality assumption, since ${}_{25}p_{40}^{(m)}$ is equal to 0.8241. This cost reduction, however, would be partially or even fully offset by pre-retirement death benefits. Pension costs after retirement, of course, are affected significantly by the post-retirement survival probability, ${}_{x-r}p_r^{(m)}$, which approaches zero at an increasing rate as x increases. As indicated in Table 2–2, half of the individuals at age 65 are expected to reach age 80, but then only half of these survivors will reach age 85, and only half of those will reach age 90.

Termination Decrement

The termination (or withdrawal) decrement, like the mortality decrement, prevents some employees from reaching the plan's normal or early retirement ages. If the employee is vested, the accrued benefit is payable at some future age, generally the normal retirement age of the plan, or it may be payable at an earlier age with an appropriate actuarial reduction. Whereas the termination decrement, in the absence of vesting, reduces pension costs, this reduction is partially offset by the corresponding cost of vesting.

A multitude of factors enter into the determination of employee termination rates, but two factors consistently found to be important are age and length of service. The older the employee and/or the longer the period of service, the less likely it is that termination will occur. Consequently, termination rates frequently have both an age and a service dimension, known as *select and ultimate* rates. The term "select" denotes rates applicable for a specified period beyond the employee's entry age, and the term "ultimate" denotes rates applicable to ages beyond that point. Most schedules have a three to five year select period, although it is still common to find schedules based on age alone because of computational simplicity. In some cases, additional dimensions, such as gender, occupational or compensation level, and vesting status, are used in determining termination rates. Finally, just as mortality might be assumed to decrease in future years, termination rates may also be assumed to change. In this case a series of termination rate schedules would be used, reflecting historical trends and subjective estimates of expected future changes.

Table 2-3 displays the select and ultimate termination rates used for illustrating pension costs, given by quinquennial entry ages from 20 through 60. The select period is 5 years and the nearest select schedule is applied to employees with intermediate entry ages. Since the model pension plan to which these rates are applicable permits early retirement at age 55 with 10 years of service, the termination rates are defined to be zero at and beyond each entrant's qualification for early retirement.

The rate of termination at age x is denoted by $q_x'^{(t)}$.[3] The probability that the employee will remain in service for one year, excluding consideration of other decrements, is equal to the complement of this rate, that is, $p_x'^{(t)} = 1 - q_x'^{(t)}$. The probability of surviving n years may be found by taking the product of n such complements from age x to age $x + n - 1$, denoted by ${}_np_x'^{(t)}$. Table

[3]Since this rate is based on the employee's entry age y, the symbol $q_{x,y}'^{(t)}$ could be used to make this functional relationship explicit. The subscript y is not used in this text for notational simplicity.

TABLE 2–3
Select and Ultimate Termination Rates

	Entry Ages, y								
x	20	25	30	35	40	45	50	55	60
20	0.2431								
21	0.2245								
22	0.2071								
23	0.1908								
24	0.1757								
25	0.1616	0.2119							
26	0.1486	0.1749							
27	0.1365	0.1506							
28	0.1254	0.1340							
29	0.1152	0.1207							
30	0.1059	0.1059	0.1682						
31	0.0974	0.0974	0.1397						
32	0.0896	0.0896	0.1160						
33	0.0827	0.0827	0.0966						
34	0.0764	0.0764	0.0814						
35	0.0708	0.0708	0.0708	0.1281					
36	0.0658	0.0658	0.0658	0.1013					
37	0.0614	0.0614	0.0614	0.0820					
38	0.0575	0.0575	0.0575	0.0684					
39	0.0541	0.0541	0.0541	0.0586					
40	0.0512	0.0512	0.0512	0.0512	0.0942				
41	0.0487	0.0487	0.0487	0.0487	0.0751				
42	0.0466	0.0466	0.0466	0.0466	0.0616				
43	0.0448	0.0448	0.0448	0.0448	0.0526				
44	0.0433	0.0433	0.0433	0.0433	0.0466				
45	0.0421	0.0421	0.0421	0.0421	0.0421	0.0686			
46	0.0410	0.0410	0.0410	0.0410	0.0410	0.0547			
47	0.0402	0.0402	0.0402	0.0402	0.0402	0.0463			
48	0.0394	0.0394	0.0394	0.0394	0.0394	0.0420			
49	0.0388	0.0388	0.0388	0.0388	0.0388	0.0399			
50	0.0382	0.0382	0.0382	0.0382	0.0382	0.0382	0.0538		
51	0.0376	0.0376	0.0376	0.0376	0.0376	0.0376	0.0462		
52	0.0370	0.0370	0.0370	0.0370	0.0370	0.0370	0.0417		
53	0.0362	0.0362	0.0362	0.0362	0.0362	0.0362	0.0391		
54	0.0354	0.0354	0.0354	0.0354	0.0354	0.0354	0.0371		
55	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0345	0.0522	
56	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0333	0.0419	
57	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0319	0.0359	
58	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0302	0.0324	
59	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0281	0.0297	
60	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0258	0.0500
61	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0230	0.0343
62	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0197	0.0258
63	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0160	0.0199
64	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0118	0.0127

2–4 gives the probability of remaining in service until age 65 from each entry age shown, as well as the probability of remaining in service five years (i.e., until full vesting is attained), based on the rates in Table 2–3.[4]

TABLE 2–4

Termination-Based Survival Probabilities for Various Entry Ages

y	${}_5p_y^{(t)}$	${}_{65-y}p_y^{(t)}$
20	0.3104	0.0355
25	0.4206	0.1009
30	0.5250	0.2023
35	0.6309	0.3347
40	0.7101	0.4791
45	0.7723	0.6400
50	0.8002	0.6815
55	0.8220	0.7457
60	0.8648	0.8648

It is clear that retirement-related costs, especially for younger employees, are significantly reduced by the termination assumption.[5] Moreover, since vested benefits are much smaller than the employee's retirement benefit, the added cost associated with vesting does not fully offset the cost reduction due to terminations.

Disability Decrement

Disability among active employees, like mortality and termination, prevents qualification for retirement benefits and, in turn, lowers retirement-based costs; however, disability-based costs may be greater or less than this reduction, depending on the plan's disability provision. As noted in Chapter 1, a typical disability benefit might provide an annual pension, beginning after a relatively short waiting period, based on the employee's benefits

[4]Note the use of the symbol y in Table 2-4 to denote the employee's entry age, a convention used throughout this book.

[5]Pension liabilities for the entire plan are not as sensitive to termination rates as suggested by the probabilities given in Table 2-4. This is the case since older employees, for whom the termination decrements are small, account for a disproportionately large amount of the total liability. These relationships are analyzed in Chapter 12.

accrued to date, or based on a service-projected normal retirement benefit. When disability benefits are provided outside the pension plan, it is common to continue crediting the disabled employee with service until normal retirement, at which time the auxiliary plan's benefits cease and the employee begins receiving a retirement pension. The mortality rate assumption under either type of benefit should be based on disabled-life mortality instead of the "healthy life" mortality rates applicable to other plan members.

The disabled-life mortality rates used to illustrate pension costs are given in Table 2–5, and the survival probabilities based on these rates are given in Table 2–6, where ${}^{d}q_x^{\prime(m)}$ and ${}^{d}p_x^{(m)}$ denote disabled-life mortality rates and survival probabilities, respectively. Comparing these rates and probabilities with those in Tables 2–1 and 2–2 reveals that disabled-life mortality is significantly greater than the mortality of non-disabled lives.

Several factors are associated with disability among active employees, the most notable ones being age, gender, and occupation. In the interest of simplicity, and since disability benefits generally represent a relatively small portion of the plan's total financial obligations, the disability assumption used to illustrate costs is related to age alone. These rates are given in Table 2–7 and the corresponding symbol is $q_x^{\prime(d)}$. If one considers *only* the disability decrement, the probability of surviving n years, ${}_np_x^{(d)}$, is the product of n factors, $1 - q_x^{\prime(d)}$ or $p_x^{(d)}$, for ages x through age $x + n - 1$.

Table 2–8 illustrates disability-related survival probabilities by showing the probability of surviving to age 65 for quinquennial ages from 20 through 65. These rates suggest that retirement-related costs might be reduced 10 to 15 percent as a result of the disability decrement, an effect quite similar to that caused by mortality.

Figure 2–1 shows a graph of the mortality, termination, and disability survival probabilities from age x to age 65 for an age-30 entrant ($30 \leq x \leq 65$) under the illustrative assumptions. The termination-based survival probability is considerably smaller than the other two survival functions below age 50. The survival probabilities based on mortality and disability are quite similar, as noted earlier.

TABLE 2-5

Disabled-Life Mortality Rates

x	${}^{d}q_x^{\prime(m)}$	x	${}^{d}q_x^{\prime(m)}$	x	${}^{d}q_x^{\prime(m)}$	x	${}^{d}q_x^{\prime(m)}$	x	${}^{d}q_x^{\prime(m)}$
20	0.00840	40	0.01454	60	0.03488	80	0.09654	100	0.35919
21	0.00853	41	0.01511	61	0.03663	81	0.10171	101	0.40694
22	0.00872	42	0.01570	62	0.03847	82	0.10715	102	0.46409
23	0.00891	43	0.01633	63	0.04042	83	0.11287	103	0.53204
24	0.00910	44	0.01699	64	0.04248	84	0.11890	104	0.61229
25	0.00930	45	0.01770	65	0.04465	85	0.12524	105	0.70640
26	0.00951	46	0.01845	66	0.04695	86	0.13191	106	0.81619
27	0.00973	47	0.01924	67	0.04938	87	0.13893	107	0.94334
28	0.00996	48	0.02009	68	0.05195	88	0.14630	108	1.00000
29	0.01021	49	0.02097	69	0.05466	89	0.15404		
30	0.01048	50	0.02191	70	0.05754	90	0.16219		
31	0.01077	51	0.02290	71	0.06056	91	0.17094		
32	0.01108	52	0.02395	72	0.06375	92	0.18059		
33	0.01141	53	0.02506	73	0.06713	93	0.19154		
34	0.01177	54	0.02624	74	0.07069	94	0.20429		
35	0.01216	55	0.02749	75	0.07444	95	0.21944		
36	0.01258	56	0.02881	76	0.07841	96	0.23769		
37	0.01303	57	0.03020	77	0.08259	97	0.25984		
38	0.01351	58	0.03167	78	0.08700	98	0.28679		
39	0.01401	59	0.03323	79	0.09165	99	0.31954		

TABLE 2-6

Disabled-Life Survival Probabilities

x	${}_{65-x}^{\ d}p_x^{(m)}$	x	${}_{x-65}^{\ d}p_{65}^{(m)}$
20	0.4219	65	1.0000
25	0.4408	70	0.7757
30	0.4629	75	0.5575
35	0.4895	80	0.3618
40	0.5227	85	0.2049
45	0.5659	90	0.0968
50	0.6238	95	0.0354
55	0.7044	100	0.0076
60	0.8214	105	0.0003
65	1.0000	110	0.0000

TABLE 2-7

Disability Rates

x	$q_x^{\prime(d)}$	x	$q_x^{\prime(d)}$	x	$q_x^{\prime(d)}$
20	0.0003	35	0.0004	50	0.0031
21	0.0003	36	0.0005	51	0.0034
22	0.0003	37	0.0006	52	0.0038
23	0.0003	38	0.0007	53	0.0042
24	0.0003	39	0.0008	54	0.0046
25	0.0003	40	0.0009	55	0.0050
26	0.0003	41	0.0010	56	0.0054
27	0.0003	42	0.0012	57	0.0060
28	0.0003	43	0.0014	58	0.0068
29	0.0003	44	0.0016	59	0.0080
30	0.0004	45	0.0018	60	0.0098
31	0.0004	46	0.0020	61	0.0124
32	0.0004	47	0.0022	62	0.0160
33	0.0004	48	0.0025	63	0.0208
34	0.0004	49	0.0028	64	0.0270

TABLE 2-8

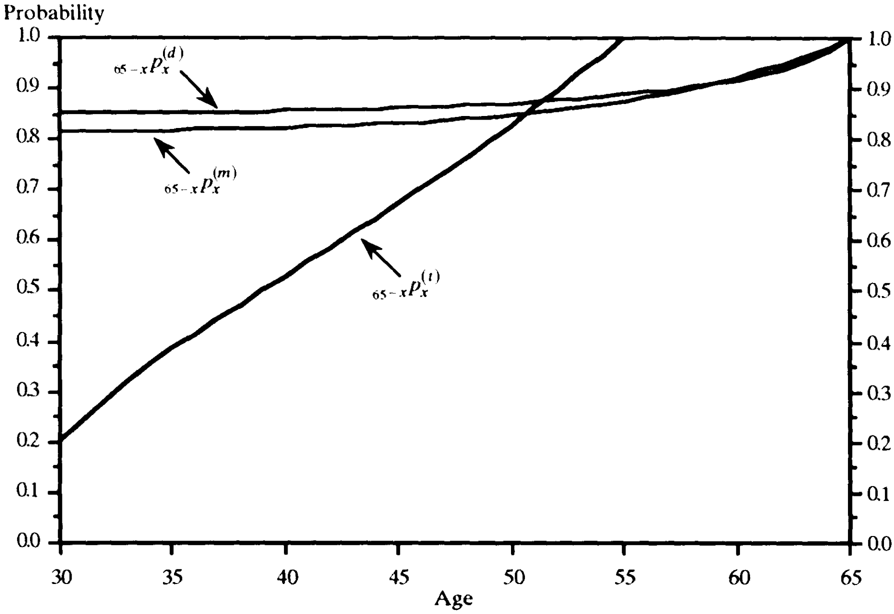
Disability-Based Survival Probabilities

x	${}_{65-x}p_x^{(d)}$	x	${}_{65-x}p_x^{(d)}$
20	0.8498	45	0.8619
25	0.8511	50	0.8717
30	0.8524	55	0.8886
35	0.8541	60	0.9168
40	0.8567	65	1.0000

Retirement Decrement

Unlike the other decrement factors that preclude the receipt of a pension benefit, the retirement decrement initiates such payments. Retirement prior to the plan's normal retirement age, as noted in the previous chapter, is called early retirement. The benefits paid to employees retiring early are generally less than the benefit accruals earned to the date of early retirement. This reduction may be a flat percentage per year prior to normal retirement, such as 6 percent, or it may be an *actuarial equivalent* reduction. The latter reflects as precisely as possible the loss of interest, the lower benefit-of-survivorship, and the longer life ex-

FIGURE 2–1
Single-Decrement Survival Probabilities from Age x to Age 65



pectancy associated with early retirement.[6] The subject of actuarial equivalence and the financial implications of providing a greater-than-actuarially-equivalent early retirement benefit are discussed in Chapter 9.

The degree to which employees elect to retire early, given the fact that the plan permits early retirement, is a function of numerous economic and sociological factors. Length of service, health status, level of pension benefits, occupational status, gender, and the ages at which Social Security benefits are payable tend to affect the incidence of early retirement. Other factors, no doubt, could be detected in a given plan, and, since the basis of early retirement varies widely among plans, retirement rates are generally constructed from the experience of the particular plan under consideration. Although it has been customary to use an average retirement age as a surrogate for the more precise early (and late) retirement distribution, age-specific rates over eligible

[6]The benefit of survivorship is considered later in this book. Essentially, it refers to the concept that a portion of the cost attributable to benefits for those employees who survive in service is provided by contribution forfeitures on behalf of those who do not survive.

retirement ages, for example, from age 55 through age 70, are frequently used, especially in connection with large corporate plans. Since there has been a distinct trend toward increased early retirements in recent years, selecting future early retirement rates may be more subjective than the other actuarial assumptions discussed up to this point.

The retirement rate of decrement at age x is denoted by $q_x'^{(r)}$. All of the mathematics and numerical analyses presented through Chapter 8 are based on a single retirement age of 65. Chapter 9 analyzes early retirement, and all subsequent material through the remainder of the book is based on the retirement rates presented in Table 2–9. The average retirement age for this schedule is 61.4. Since Social Security benefits are first available at age 62, many plans experience a relatively high incidence of retirements at this age. The rates in Table 2–9 reflect this phenomenon.

TABLE 2–9

Early Retirement Rates

Age	*Retirement Rates*
55	.05
56	.05
57	.05
58	.05
59	.05
60	.20
61	.30
62	.40
63	.30
64	.30
65	1.00

SALARY ASSUMPTION

If the plan's benefits are a function of salary, estimates of the employee's future salaries are required. These estimates involve consideration of three factors: (1) salary increases due to merit, (2) increases due to labor's share of productivity gains, and (3) increases due to inflation.[7]

[7] Another factor that is sometimes relevant is a "catch up" or "slow down" allowance, but this does not constitute a theoretical component of the long-run salary increase assumption.

Merit Component

Merit increases are those achieved by employees as they progress through their career, reflecting their increased contribution to the organization. The rate of increase from this source typically diminishes as the employee becomes older. Since merit increases are exclusive of inflation and overall productivity gains achieved by the employment group as a whole, they have little effect on the aggregate payroll of all employees over time, provided the distribution of employees by age and service does not change significantly. As existing employees grow older and earn higher salaries due to merit increases, a continual flow of new employees with relatively lower salaries enter the population. The net result is a stable year-to-year distribution of merit-related salaries for the entire group of active employees.

The merit scale for a group of employees can be estimated by comparing the differences in salaries among employees at various ages and with various periods of service in a given year. A cross-sectional analysis of this type eliminates the effect of inflation and productivity increases. In many cases, a constant rate of increase at each age is used to approximate the age-specific rates of a typical merit scale.

The merit scale used to illustrate pension costs is given in Table 2–10. This scale is unity at age 20, hence, the salary of an employee entering at this age is expected to increase 2.8 times by retirement due to merit increases alone. Similarly, the retirement-date salary of an age-30 entrant is expected to be about 1.9 greater than the entry-age salary, determined by dividing the age-65 factor by the age-30 factor (2.769 ÷ 1.487). The merit scale shows a continually decreasing rate of salary progression, beginning at 4.5 percent at age 20 and declining to nearly zero percent by age 64.

Productivity Component

The second factor that affects the salaries of the entire group of employees is labor's share of productivity gains. This factor, which is difficult to estimate, may have diminished in importance over the years, and it varies among industries. A productivity

TABLE 2–10

Merit Salary Scale

x	*Scale*	*x*	*Scale*	*x*	*Scale*
20	1.000	35	1.749	50	2.460
21	1.045	36	1.802	51	2.497
22	1.091	37	1.854	52	2.532
23	1.138	38	1.906	53	2.565
24	1.186	39	1.958	54	2.596
25	1.234	40	2.008	55	2.624
26	1.284	41	2.059	56	2.651
27	1.334	42	2.108	57	2.674
28	1.384	43	2.157	58	2.696
29	1.436	44	2.204	59	2.715
30	1.487	45	2.250	60	2.731
31	1.539	46	2.295	61	2.745
32	1.592	47	2.339	62	2.756
33	1.644	48	2.381	63	2.764
34	1.697	49	2.422	64	2.769

component of 1 percent per annum is assumed in the numerical illustrations.[8]

Inflation Component

The third and most significant factor affecting an employee's future salary is inflation. This factor is more likely to be represented by a constant compound rate, unlike the merit component which generally increases salary at a decreasing rate with age. This need not be the case, however, and empirical trends and/or subjective beliefs may suggest a series of rates that increase or decrease for a period of time to an ultimate level.

The inflation component used to project salaries for the model plan is assumed to be 4 percent per year. Although this rate may appear to be low for various periods throughout history, it is consistent with long-term average inflation rates. Since the inflation rate for pension plans is used to project salaries over relatively long periods of time (up to 45 years), the 4 percent rate

[8]There is some question whether any productivity gains will occur in the future for some groups of employees. Nevertheless, it can be argued that the salaries of these groups must necessarily keep pace with the salaries of other groups; hence, the productivity factor is still relevant. In any event, the productivity factor for a given plan would be set with careful consideration being given to historical data and subjective views regarding the future.

may indeed represent a prudent assumption. The cost impact of other rates of inflation is examined in Chapter 12.

Total Salary Increase

Table 2–11 shows the age-64 salary multiple and the corresponding compound rate of increase for several entry ages, using the previously discussed merit scale, a 1 percent productivity component, and a 4 percent inflation component.[9] For entrants at ages 25 and 30, the salary projection factors to age 64 are about 15 and 10, respectively. This is approximately equal to a 7 percent compound rate of increase.

TABLE 2–11

Salary Projections Inclusive of Merit, Productivity, and Inflation

Entry Age y	*Age-64 Salary as Multiple of Entry-Age Salary*	*Equivalent Compound Rate of Increase*
20	23.70	0.075
25	15.04	0.072
30	9.78	0.069
35	6.52	0.067
40	4.45	0.064
45	3.11	0.062
50	2.23	0.059
55	1.64	0.056
60	1.23	0.054

INTEREST ASSUMPTION

The interest assumption has a powerful effect on pension costs, since it is used to find the present value of financial obligations due 20, 40, and even 60 years from the valuation date. Although it is common to find this assumption set at a constant compound rate, this is a special case of the more general assumption that would allow the rate of interest to vary over time. As with most actuarial assumptions, an element of subjectivity is in-

[9]Inflation and productivity will increase total payroll, while the merit component will have little effect, as noted previously. Thus, the model plan assumptions can be thought of as increasing total payroll by about 5 percent per year, while increasing a given participant's salary by 5.5 to 7.5 percent per year, depending on age.

volved in establishing the interest rate to be used in the valuation of pension costs and liabilities.

The interest assumption is generally set at a level representing the return expected to be achieved on the plan's assets in future years, although it is not uncommon to find rates being used that are ostensibly lower or higher than such expectations. In any event, the interest assumption, like the salary assumption, can be viewed as consisting of three components: (1) a risk-free rate of return, (2) a premium for investment risk, and (3) a premium for inflation.

Risk Free Rate

The risk free rate is one that would prevail for an investment completely secure as to principal and yield in an environment with no current or anticipated inflation. An estimate of this theoretical component would be the difference between short-term treasury bills and inflation, a difference that varies widely from year to year, and is nearly zero over very long periods of time. Although empirical estimates are inconclusive, it is generally agreed that the long-term equilibrium risk free rate falls in the range of 1 to 2 percent. The illustrative rate used is 1 percent.

Investment Risk

The second interest rate component is the investment risk inherent in the current and future portfolio of plan assets. A different investment risk, and hence risk premium, may be associated with each investment, although it is generally practicable to break investments down only into several broad classes for assignment of the risk premium. Table 2–12 shows the development of the risk premium used for the illustrations in this book. Two asset classes are used along with a typical asset mix, having 50 percent fixed-income securities and 50 percent equities. The expected risk premium for the portfolio is 3 percent. Table 2–12 also indicates the typical range in risk premiums for these two asset classes.

TABLE 2-12

Investment Risk Estimates

	Risk Premium		*Asset*	*Expected*
Asset Class	*Range*	*Assumption*	*Mix*	*Premium*
Bonds	1 to 3%	2%	50%	1%
Stocks	3 to 5%	4%	50%	2%
				3%

Inflation Component

A premium for the current and anticipated rate of inflation is the third interest rate component. This factor, it will be remembered, was present in the salary assumption also, and in this sense the salary and interest assumptions have a common link. As noted earlier, the assumed rate of future inflation is likely to have a higher subjective element than most actuarial assumptions, since near term rates may not be good indicators of long-term inflation rates. In some cases, it may be appropriate to use a graded inflation rate, beginning with the current year's rate and grading downward or upward to some ultimate level. As noted previously, a constant inflation rate of 4 percent has been selected for illustrative purposes.

Total Interest Rate

The individual components of the interest rate assumption used in this book total 8 percent, consisting of a 1 percent risk free rate, a 3 percent risk premium, and a 4 percent inflation rate. Although it is not claimed that these values are necessarily appropriate for a given plan, they have been selected with an eye toward their general appropriateness.

Chapter 3

Basic Actuarial Functions

The purpose of this chapter is to introduce several actuarial functions used in the development of pension mathematics throughout the remainder of the book. The discussion begins with the composite survival function and interest function, perhaps the two most basic concepts in pension mathematics. Pension plan benefit functions are then presented, followed by a discussions of annuities, the latter representing a combination of interest and survival functions.

COMPOSITE SURVIVAL FUNCTION

The composite survival function represents the probability that an active plan participant survives in service for a given period, based on all of the decrement rates to which the employee is exposed. Whereas the probability of surviving one year in a single-decrement environment is equal to the complement of the rate of decrement, the probability of surviving one year in a multiple-decrement environment is equal to the product of such complements for each applicable rate of decrement. The probability of an active participant aged x surviving one year is

$$p_x^{(T)} = (1 - q_x'^{(m)})(1 - q_x'^{(t)})(1 - q_x'^{(d)})(1 - q_x'^{(r)}), \quad (3.1a)$$

or equivalently,

$$p_x^{(T)} = p_x'^{(m)} p_x'^{(t)} p_x'^{(d)} p_x'^{(r)}. \quad (3.1b)$$

This same probability can be expressed in terms of multiple-decrement probabilities:

$$p_x^{(T)} = 1 - (q_x^{(m)} + q_x^{(t)} + q_x^{(d)} + q_x^{(r)}). \quad (3.2a)$$

A common approximation for multiple-decrement probabilities is illustrated for the mortality probability as follows:

$$q_x^{(m)} \approx q_x^{\prime(m)}\,(1 - \tfrac{1}{2}q_x^{\prime(t)})\,(1 - \tfrac{1}{2}q_x^{\prime(d)})\,(1 - \tfrac{1}{2}q_x^{\prime(r)}). \tag{3.2b}$$

The probability of surviving in active service for n years is equal to the product of successive one-year composite survival probabilities:

$$_np_x^{(T)} = \prod_{t=0}^{n-1} p_{x+t}^{(T)}. \tag{3.2c}$$

Table 3–1 shows the probability of surviving to age 65 from each attained age, based on nine different entry ages, under the various rates of decrement set out in Chapter 2. Since the termination decrement includes a 5-year select period, probabilities are given for each of the first five years subsequent to the nine entry ages. The chance of reaching retirement, at least under the illustrative assumptions, is quite low even up through attained age 50, where the probability is 0.62. Since retirement-related pension cost estimates are in direct proportion to the survival probability (i.e., the lower the probability the lower are such estimates), it is clear that discounting for the various decrements reduces pension costs significantly. Obviously, the cost reduction is mitigated to the extent that a benefit is provided with each type of decrement (e.g., a vested termination benefit, a death benefit, or a disability benefit).

The calculation of survival probabilities in accordance with the above definitions can be quite arduous. The probabilities, however, are readily obtainable from a *service table*. This table shows the hypothetical number of employees, from an arbitrary initial number, who survive to each future age. The initial number, or radix, is generally taken to be some large value, such as 100,000 or 1,000,000, and $l_x^{(T)}$ is the notation for the survivors at age x.[1] The total number of employees leaving active service during the year is denoted by $d_x^{(T)}$ and defined as

$$d_x^{(T)} = l_x^{(T)} q_x^{(T)}. \tag{3.3a}$$

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[1] Although not indicated by the symbol $l_x^{(T)}$, it is understood to represent the number of survivors at age x who entered the plan at age y. At a later point the more general symbol $l_{x,y}^{(T)}$ will be used, which makes the entry age variable explicit.

TABLE 3–1
Probability of Surviving in Service to Age 65, ${}_{65-x}p_x^{(T)}$

Entry Age	Select Ages y	$y+1$	$y+2$	$y+3$	$y+4$	x	Attained Age
20	0.02	0.03	0.04	0.05	0.07	0.08	25
						0.10	26
						0.11	27
						0.13	28
						0.15	29
25	0.07	0.09	0.11	0.13	0.15	0.17	30
						0.19	31
						0.21	32
						0.23	33
						0.25	34
30	0.14	0.17	0.20	0.23	0.25	0.28	35
						0.30	36
						0.32	37
						0.34	38
						0.36	39
35	0.24	0.28	0.31	0.33	0.36	0.38	40
						0.40	41
						0.43	42
						0.45	43
						0.47	44
40	0.35	0.38	0.41	0.44	0.47	0.49	45
						0.52	46
						0.54	47
						0.57	48
						0.60	49
45	0.47	0.51	0.54	0.57	0.60	0.62	50
						0.65	51
						0.69	52
						0.72	53
						0.75	54
50	0.51	0.55	0.58	0.61	0.64	0.79	55
						0.80	56
						0.81	57
						0.82	58
						0.84	59
55	0.59	0.63	0.67	0.70	0.74	0.85	60
						0.87	61
						0.89	62
						0.92	63
						0.96	64
60	0.74	0.79	0.84	0.89	0.94	1.00	65

Total decrements from the active population equal the sum of each separate decrement:

$$d_x^{(T)} = d_x^{(m)} + d_x^{(t)} + d_x^{(d)} + d_x^{(r)} \tag{3.3b}$$

$$= l_x^{(T)} \left(q_x^{(m)} + q_x^{(t)} + q_x^{(d)} + q_x^{(r)} \right) . \tag{3.3c}$$

Table 3–2 illustrates the concept of a service table for 1,000,000 entrants at age 20, based on the decrement assumptions specified in Chapter 2.[2] The probability of an age-20 entrant surviving in active service to age 65 is easily found from this table:

$${}_{65-20}p_{20}^{(T)} = \frac{l_{65}^{(T)}}{l_{20}^{(T)}} = \frac{24{,}448}{1{,}000{,}000} = 0.0244 . \tag{3.4a}$$

Similarly, the probability of an employee age 40 surviving to age 65 would be

$${}_{25}p_{40}^{(T)} = \frac{l_{65}^{(T)}}{l_{40}^{(T)}} = \frac{24{,}448}{65{,}276} = 0.3745 . \tag{3.4b}$$

Although service tables are an important source of computational efficiency, even when working with high speed computers, they are used only occasionally in presenting the theory of pension mathematics in this book.

INTEREST FUNCTION

The interest function is used to discount a future payment to the present time. It plays a crucial role in determining pension costs and, like the survival function of the previous section, it reduces such values. If i_t is the interest rate assumed for the tth year, the present value of one dollar due in n years is given by

$$\frac{1}{(1 + i_1)(1 + i_2) \cdots (1 + i_n)} , \tag{3.5a}$$

and, if $i_1 = i_2 = \cdots = i_n$, we have

$$\frac{1}{(1 + i)^n} . \tag{3.5b}$$

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[2]To the extent that any rates are entry-age dependent, such as termination rates, a separate service table is applicable to each entry age.

TABLE 3–2

Service Table

x	$l_x^{(T)}$	$d_x^{(m)}$	$d_x^{(t)}$	$d_x^{(d)}$	$d_x^{(r)}$	$d_x^{(T)}$
20	1,000,000	442	243,002	263	0	243,708
21	756,292	350	169,718	201	0	170,270
22	586,023	286	121,314	158	0	121,757
23	464,265	238	88,543	126	0	88,907
24	375,358	202	65,921	103	0	66,226
25	309,132	176	49,933	85	0	50,194
26	258,938	156	38,460	72	0	38,688
27	220,251	140	30,049	62	0	30,251
28	189,999	129	23,814	53	0	23,996
29	166,004	119	19,113	47	0	19,280
30	146,724	112	15,529	56	0	15,697
31	131,027	107	12,754	50	0	12,911
32	118,116	103	10,576	45	0	10,725
33	107,392	101	8,875	41	0	9,017
34	98,375	99	7,510	38	0	7,647
35	90,727	98	6,419	35	0	6,552
36	84,176	98	5,534	41	0	5,673
37	78,503	99	4,816	46	0	4,960
38	73,543	100	4,224	50	0	4,374
39	69,169	102	3,738	54	0	3,893
40	65,276	104	3,338	57	0	3,499
41	61,777	108	3,004	60	0	3,172
42	58,605	114	2,727	69	0	2,910
43	55,695	123	2,491	76	0	2,690
44	53,006	133	2,290	83	0	2,506
45	50,499	144	2,121	89	0	2,354
46	48,145	156	1,969	94	0	2,219
47	45,926	169	1,841	99	0	2,108
48	43,818	181	1,721	107	0	2,009
49	41,808	194	1,616	115	0	1,925
50	39,884	206	1,517	121	0	1,845
51	38,039	219	1,424	127	0	1,769
52	36,270	230	1,335	135	0	1,700
53	34,570	241	1,244	142	0	1,628
54	32,942	252	1,159	148	0	1,559
55	31,383	267	0	156	0	423
56	30,960	286	0	166	0	452
57	30,508	305	0	182	0	487
58	30,020	326	0	203	0	529
59	29,491	350	0	235	0	585
60	28,907	377	0	281	0	659
61	28,248	405	0	348	0	753
62	27,495	433	0	436	0	869
63	26,626	459	0	549	0	1,008
64	25,618	485	0	685	0	1,170
65	24,448	0	0	0	24,448	24,448

The following simplifying definition is used in connection with the present value function:

$$v = \frac{1}{(1+i)}. \tag{3.6}$$

Thus, v^n represents the present value of one dollar due in n years at an annual compound rate of interest equal to i.

The interest function, v^t, begins at a value of unity for $t = 0$ and approaches zero as t approaches infinity, provided $i > 0$. Also, v^t is inversely related to i, taking on the value of unity for $i = 0$ and approaching zero as i approaches infinity. Table 3–3 illustrates the significance of the interest factor. In addition to showing v^t for the illustrative interest assumption of 8 percent, the function is also evaluated for interest rates of 6 and 10 percent. Since the interest factor is associated with each entrant's potential future age, a period of about 70 years for entrants age 30 or younger, the interest discount function is tabulated at 5-year intervals for 70 years.

The significance of the interest rate function is readily apparent. For an age-30 entrant, v^{65-30} is 0.07 at an 8 percent rate of interest, 0.13 at 6 percent, and 0.04 at 10 percent. Retirement-related cost estimates are directly related to this factor and, since the interest discount extends beyond retirement, the total effect of the interest assumption is even greater than the effect of v^{r-y} alone.

Figure 3–1 shows the survival function from age x to age 65 and the interest function over this same age interval for an age-30 entrant under the model assumptions. This graph indicates that the interest function, based on an 8 percent rate, is smaller than the survival function at all ages, although this relationship may not always hold. The product of the interest and survival functions is frequently encountered in pension mathematics. Since both functions are less than unity, their product is quite small over most attained ages.

SALARY FUNCTION

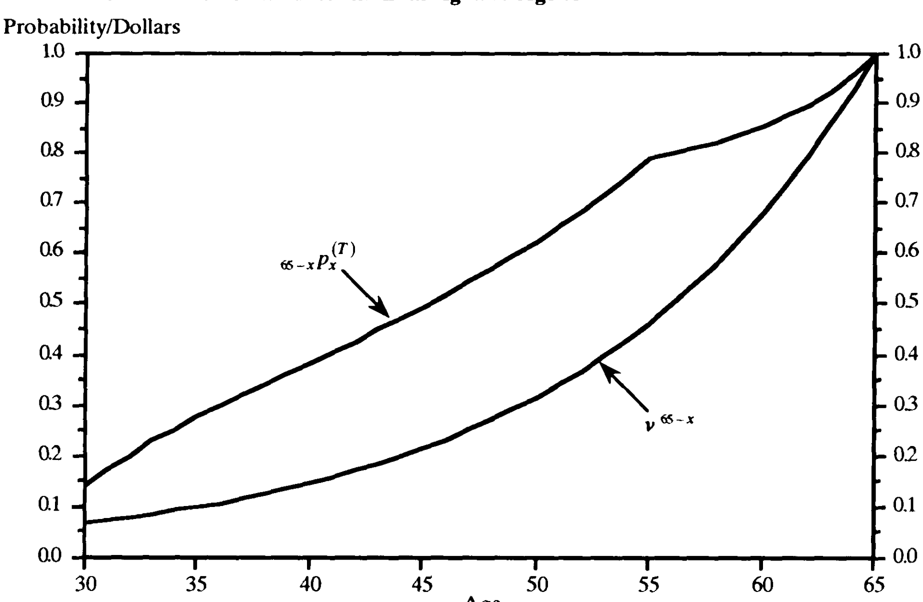
If a pension plan has benefits expressed in terms of salary, it is necessary to develop salary-related notation and procedures for estimating future salary. The current dollar salary for a par-

TABLE 3-3

Compound Interest Function, v^t

	Interest Rate		
t	6%	8%	10%
0	1.0000	1.0000	1.0000
5	0.7473	0.6806	0.6209
10	0.5584	0.4632	0.3855
15	0.4173	0.3152	0.2394
20	0.3118	0.2145	0.1486
25	0.2330	0.1460	0.0923
30	0.1741	0.0994	0.0573
35	0.1301	0.0676	0.0356
40	0.0972	0.0460	0.0221
45	0.0727	0.0313	0.0137
50	0.0543	0.0213	0.0085
55	0.0406	0.0145	0.0053
60	0.0303	0.0099	0.0033
65	0.0227	0.0067	0.0020
70	0.0169	0.0046	0.0013

FIGURE 3-1
Survival and Interest Functions from Age x to Age 65



ticipant age x is denoted by s_x, and S_x represents the *cumulative* salary from entry age y up to, but not including, age x.[3] Thus, for $x > y$ we have

$$S_x = \sum_{t=y}^{x-1} s_t. \tag{3.7}$$

In order to estimate the dollar salary at age x, based on the employee's age-y salary, the following formula is used:

$$s_x = s_y \frac{(SS)_x}{(SS)_y}[(1 + I)(1 + P)]^{(x-y)}, \tag{3.8a}$$

where

s_y = entry-age dollar salary

$(SS)_x$ = merit salary scale at age x

I = rate of inflation

P = rate of productivity reflected in the salary increases.

An age-y entrant's salary at age x can also be defined in terms of the age-z salary ($y < z < x$):

$$s_x = s_z \frac{(SS)_x}{(SS)_z}[(1 + I)(1 + P)]^{(x-z)}. \tag{3.8b}$$

If all of the salary increase assumptions were met from age y to age z, the employee's salary at age z would be equal to

$$s_z = s_y \frac{(SS)_z}{(SS)_y}[(1 + I)(1 + P)]^{(z-y)}. \tag{3.8c}$$

Substituting (3.8c) for s_z in (3.8b) reduces the latter to (3.8a), showing that s_x is identical, under the salary increase assumptions, whether derived from the entry age salary or the attained age salary.

Table 3–4 shows the salary function per dollar of entry-age salary based on the previously discussed merit scale, 1 percent

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[3]It is to be understood throughout the remainder of this book that s_x and S_x are dependent on each participant's entry age y, despite the fact that y does not appear in the symbol. The symbols $s_{x,y}$ and $S_{x,y}$ could be used to denote these two functions, which have the virtue of making the entry age explicit. These symbols are used to make the entry age variable explicit at a later point where the equations require a summation over all combinations of x and y.

TABLE 3-4

Salary Function per Dollar of Entry Age Salary, $s_x \div s_y$

	Entry Age, y				
x	20	30	40	50	60
20	1 000				
21	1.097				
22	1.203				
23	1.317				
24	1.442				
25	1.575				
26	1.721				
27	1.877				
28	2.045				
29	2.228				
30	2.422	1 000			
31	2.632	1.087			
32	2.859	1.180			
33	3.100	1.280			
34	3.360	1.387			
35	3.636	1.501			
36	3.934	1.624			
37	4.249	1.754			
38	4.587	1.894			
39	4.948	2.043			
40	5.328	2.200	1 000		
41	5.736	2.368	1.077		
42	6.166	2.546	1.157		
43	6.625	2.735	1.244		
44	7.108	2.935	1.334		
45	7.619	3.146	1.430		
46	8.160	3.369	1.532		
47	8.733	3.605	1.639		
48	9.334	3.854	1.752		
49	9.969	4.116	1.871		
50	10.632	4.389	1.996	1 000	
51	11.331	4.678	2.127	1.066	
52	12.065	4.981	2.264	1.135	
53	12.833	5.298	2.409	1.207	
54	13.638	5.630	2.560	1.283	
55	14.474	5.976	2.717	1.361	
56	15.354	6.339	2.882	1.444	
57	16.262	6.714	3.052	1.530	
58	17.215	7.107	3.231	1.619	
59	18.203	7.515	3.417	1.712	
60	19.226	7.938	3.609	1.808	1 000
61	20.291	8.377	3.808	1.908	1.055
62	21.391	8.831	4.015	2.012	1.113
63	22.526	9.300	4.228	2.119	1.172
64	23.695	9.782	4.447	2.229	1.232

productivity factor, and 4 percent inflation rate, illustrated for decennial entry ages from 20 through 60. The age-64 salary is 24 times greater than the age-20 starting salary. At the other extreme, an age-60 entrant's initial salary is expected to increase by 23 percent by age 64. Retirement-related cost estimates are directly proportional to an employee's final 5-year average salary under the model benefit formula used to illustrate pension costs in this book; consequently, it is clear from Table 3–4 that the growth in a participant's future salary can increase pension cost estimates substantially. This is in contrast to the interest rate and decrement probabilities, both of which have a decreasing effect on pension cost estimates.

BENEFIT FUNCTION

The benefit function is used to determine the amount of benefits paid at retirement, vested termination, disablement, and death. This function, the interest function, and the survival function provide the basic components required to formulate pension costs, as shown in subsequent chapters. In this section consideration is given to the three most common types of benefit formulas used with defined benefit pension plans.

The symbol b_x denotes the annual benefit accrual during age x to age $x + 1$ for an age-y entrant, and is referred to as the *benefit accrual function*. The benefit accrual function can equal the formula accruals or, as discussed in this section, some other definition of accruals, such as a portion of the participant's projected retirement-age benefit. The accrued benefit, denoted by B_x, is equal to the sum of each attained age accrual up to, but not including, age x. This function is called the *accrued benefit function* and is defined for $x > y$ by

$$B_x = \sum_{t=y}^{x-1} b_t. \tag{3.9}$$

The convention of using lower and upper case letters, with the upper case denoting a summation of lower case functional values, was also used for the salary scale.[4]

[4]The benefit functions, b_x and B_x, are dependent on the employee's entry age y, suggesting the notation $b_{x,y}$ and $B_{x,y}$, which is not used here for simplicity.

Flat Dollar Unit Benefit

Under a flat dollar unit benefit formula, b_x is equal to the annual benefit payable per year of service. These values are found to range from about \$150 to \$300 per year of service. Although the attained age subscript is shown here, it is unnecessary since the accrual is independent of age. The accrued benefit is a years-of-service multiple of the benefit accrual, that is,

$$B_x = (x - y)\, b_x . \tag{3.10}$$

Career Average

The career average benefit formula has the following definitions for the benefit accrual and the accrued benefit functions at age x:

$$b_x = k\, s_x , \tag{3.11a}$$

$$B_x = k\, S_x , \tag{3.11b}$$

where k denotes the proportion of attained age salary provided as an annual benefit accrual. The benefit functions under the career average formula follow precisely the pattern of the attained age salary and the cumulative salary discussed and illustrated previously.

Final Average

The final average benefit formula is somewhat more complicated. Let n denote the number of years over which the participant's salary prior to retirement is to be averaged, and let k equal the proportion of the average salary provided per year of service. The projected retirement benefit, assuming retirement occurs at the beginning of age r, is defined as

$$B_r = k\,(r - y)\frac{1}{n} \sum_{t=r-n}^{r-1} s_t , \tag{3.12a}$$

or more simply

$$B_r = k\,(r - y)\frac{1}{n}(S_r - S_{r-n}) . \tag{3.12b}$$

The attained age benefit accrual and accrued benefit functions can be defined in several ways under this benefit formula.

One approach is to defined B_x according to the benefit formula based on the participant's current salary average:

$$B_x = k\,(x-y)\tfrac{1}{n}\,(S_x - S_{x-n}), \tag{3.13}$$

where n is the smaller of the years specified in the benefit formula or $x - y$. The corresponding benefit accrual at age x can be determined by the following basic relationship:

$$b_x = B_{x+1} - B_x. \tag{3.14a}$$

Substituting the definition of B_x given in (3.13) and simplifying, we have:

$$\begin{aligned} b_x &= k\,\tfrac{1}{n}\,(S_{x+1} - S_{x+1-n}) \\ &\quad + k\,\tfrac{1}{n}(x-y)\big[(S_{x+1} - S_{x+1-n}) - (S_x - S_{x-n})\big] \qquad (3.14b) \\ &= k\,\tfrac{1}{n}\,(S_{x+1} - S_{x+1-n}) + k\,\tfrac{1}{n}\,(x-y)\big[s_x - s_{x-n}\big]. \qquad (3.14c) \end{aligned}$$

The first term on the right side of (3.14c) is the portion of the benefit accrual earned in the current year based on the participant's current n-year salary average, while the second term represents the portion earned as a result of the increase in the n-year salary average base. The increase in the salary base, which is represented by $\frac{1}{n}\,(s_x - s_{x-n})$ in (3.14c), is multiplied by years-of-service to date. Therefore, the benefit accrual during age x includes an implicit updating of the previous accruals which can cause b_x to be a steeply increasing function of x.

As will be shown later, steeply increasing benefit accrual and accrued benefit functions produce even steeper pension cost functions under some methods of determining pension costs and liabilities, characteristics that may be undesirable. Consequently, two modifications to benefit functions that are based on the plan's formula have been developed for funding purposes in order to mitigate this effect.

The *constant dollar* (*CD*) modification defines the benefit accrual function as a pro rata share of the participant's retirement-age projected benefit:

$$^{CD}b_x = \frac{B_r}{(r-y)}, \qquad (y \le x < r) \tag{3.15a}$$

a constant for all x. The accrued benefit function is a years-of-service multiple of this constant:

$$^{CD}B_x = \frac{B_r}{(r-y)}(x - y). \qquad (y \leq x \leq r) \qquad (3.15b)$$

While the constant dollar modification is applicable to a career average benefit formula, it has no effect on a flat dollar unit benefit formula.

The *constant percent* (*CP*) modification defines b_x as a constant percent of salary. The appropriate percentage of attained age salary is found by dividing the projected benefit by the cumulative projected salary of the age-y entrant:

$$^{CP}b_x = \frac{B_r}{S_r}s_x, \qquad (y \leq x < r) \qquad (3.16a)$$

$$^{CP}B_x = \frac{B_r}{S_r}S_x. \qquad (y \leq x \leq r) \qquad (3.16b)$$

The constant percent modification has no effect on a career average benefit formula, just as the constant dollar version has no effect on a flat dollar benefit formula.

Table 3–5 shows the benefit accrual and accrued benefit functions under the final average benefit formula used in this book (i.e., 1.5 percent of final 5-year average salary per year of service) for an age-30 entrant. The constant dollar and constant percent versions are illustrated along with the unmodified version of this benefit formula. In each case, the benefit functions are expressed as a percentage of the employee's projected retirement benefit, with retirement assumed to occur at the beginning of age 65. The constant percent version has only a minor effect as compared to the unmodified benefit functions; however, the constant dollar version produces significant differences. This modification allocates a constant 2.86 percent, or 1/35, of the projected benefit per year of service. The constant dollar version develops an accrued benefit equal to 50 percent of the projected benefit when one-half of the expected career has been completed, while the allocation under the constant percent is only 25 percent at this point, and the unmodified version is 18 percent. Thus, even though all of the accrued benefit functions begin at zero and attain B_r by age r, the modifications, and particularly the constant dollar version, cause the intermediate values to be different. Figures 3–2 and 3–3 show these relationships graphically.

TABLE 3-5

Benefit Accrual and Accrued Benefit Functions Expressed as a Percent of the Projected Benefit

	Unmodified		Constant Percent		Constant Dollar	
Age	b_x	B_x	${}^{CD}b_x$	${}^{CD}B_x$	${}^{CP}b_x$	${}^{CP}B_x$
30	0.32	0.00	0.67	0.00	2.86	0.00
31	0.35	0.32	0.73	0.67	2.86	2.86
32	0.38	0.67	0.79	1.41	2.86	5.71
33	0.41	1.06	0.86	2.20	2.86	8.57
34	0.45	1.47	0.93	3.06	2.86	11.43
35	0.58	1.92	1.01	4.00	2.86	14.29
36	0.66	2.49	1.09	5.01	2.86	17.14
37	0.75	3.15	1.18	6.10	2.86	20.00
38	0.84	3.90	1.28	7.28	2.86	22.86
39	0.95	4.74	1.38	8.56	2.86	25.71
40	1.07	5.70	1.48	9.93	2.86	28.57
41	1.19	6.76	1.59	11.42	2.86	31.43
42	1.33	7.95	1.71	13.01	2.86	34.29
43	1.47	9.28	1.84	14.73	2.86	37.14
44	1.63	10.75	1.98	16.57	2.86	40.00
45	1.80	12.39	2.12	18.54	2.86	42.86
46	1.99	14.19	2.27	20.66	2.86	45.71
47	2.18	16.18	2.43	22.93	2.86	48.57
48	2.39	18.36	2.60	25.36	2.86	51.43
49	2.62	20.75	2.77	27.95	2.86	54.29
50	2.86	23.37	2.96	30.73	2.86	57.14
51	3.11	26.23	3.15	33.68	2.86	60.00
52	3.38	29.34	3.35	36.83	2.86	62.86
53	3.66	32.71	3.57	40.19	2.86	65.71
54	3.96	36.38	3.79	43.76	2.86	68.57
55	4.28	40.34	4.02	47.55	2.86	71.43
56	4.61	44.62	4.27	51.57	2.86	74.29
57	4.96	49.23	4.52	55.84	2.86	77.14
58	5.32	54.19	4.79	60.36	2.86	80.00
59	5.71	59.51	5.06	65.15	2.86	82.86
60	6.10	65.22	5.35	70.21	2.86	85.71
61	6.51	71.32	5.64	75.56	2.86	88.57
62	6.95	77.83	5.95	81.20	2.86	91.43
63	7.38	84.78	6.26	87.15	2.86	94.29
64	7.84	92.16	6.59	93.41	2.86	97.14
65		100.00		100.00		100.00

FIGURE 3–2
Attained Age Benefit Accrual Functions as a Percent of the Projected Retirement Benefit

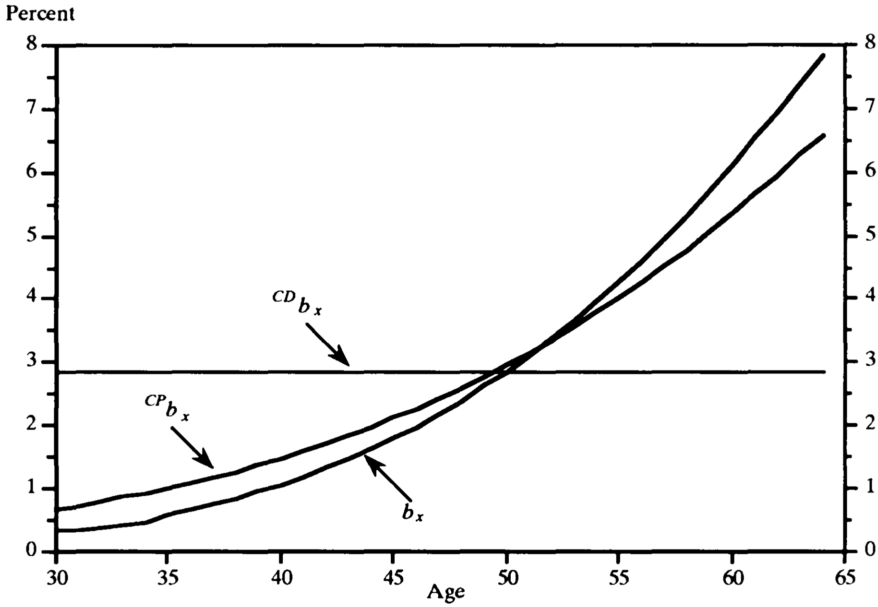
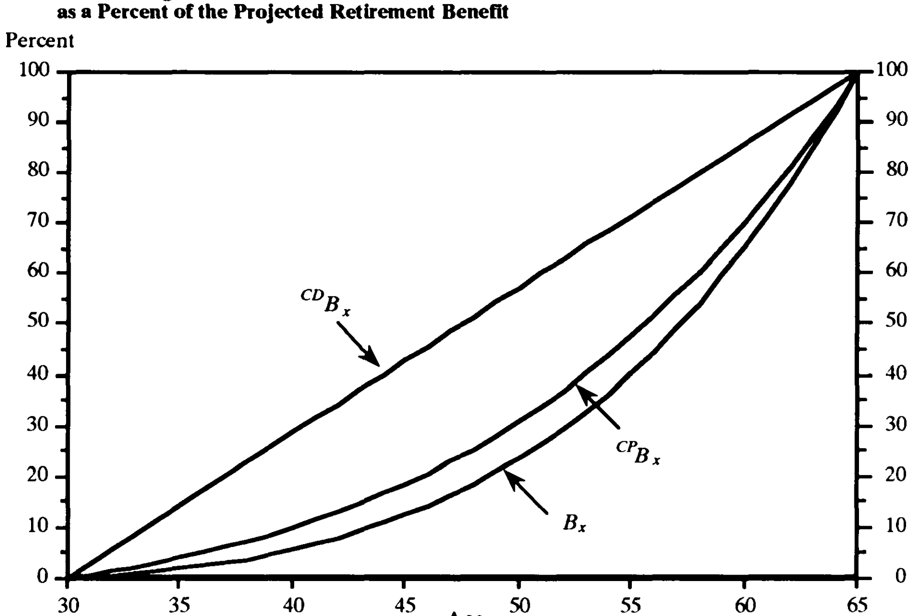


FIGURE 3–3
Attained Age Accrued Benefit Functions as a Percent of the Projected Retirement Benefit



ANNUITY FUNCTIONS

Annuities represent a combination of the survival and interest functions. Most annuities are based on the mortality-only survival function, and the material below reflects this emphasis. However, this section also defines a temporary employment-based annuity which uses the composite survival function, since this annuity is required for some funding methods.

Straight Life Annuity

If retirement benefits cease upon death, the annuity is called a *straight life annuity* and its present value, assuming an annual benefit of one dollar payable at the beginning of age x, is given by

$$\ddot{a}_x = \sum_{t=0}^{\infty} {}_tp_x^{(m)} v^t . \tag{3.17a}$$

The infinity sign is used as the upper limit of the summation for simplicity, since ${}_tp_x^{(m)}$ becomes zero beyond some advanced age.[5]

A special case of the life annuity is when the interest rate is zero, in which case we have, simply, one plus the curtate life expectancy (i.e., based on whole years only) at age x. This is expressed notationally as

$$e_x = \left[\sum_{t=0}^{\infty} {}_tp_x^{(m)} \right] - 1 . \tag{31.7b}$$

Period Certain Life Annuity

It is not uncommon to find an n-year *period certain life annuity* used as the basis for distributing pension benefits. During the certain period, benefits are payable whether or not the annuitant is alive. This type of annuity is a combination of an n-year *period certain annuity* plus an n-year *deferred life annuity*:

$$\ddot{a}_{\overline{x:\overline{n|}}} = \left[\sum_{t=0}^{n-1} v^t \right] + {}_np_x^{(m)} v^n \ddot{a}_{x+n}$$

---

[5]An approximation for the present value of an annuity payable m times a year, with payments at the beginning of each period, is found by subtracting $(m - 1)/2m$ from the annuity payable annually. Since retirement benefits are paid monthly, 11/24 should be subtracted from the above annuity to approximate a monthly payment of 1/12 of a dollar. The annual-pay annuity is assumed throughout this book for simplicity.

$$= \ddot{a}_{\overline{n}|} + {}_{n|}\ddot{a}_x . \tag{3.18}$$

These equations introduce notations deserving comment. The bar over the subscript $x{:}\overline{n}|$ signifies that the annuity is paid until the last status fails, where the two statuses represent (1) the life of the plan member and (2) the n-year period. Thus, the period certain life annuity is technically a *last survivor annuity*. An annuity discussed subsequently is $\ddot{a}_{x:\overline{n}|}$ without the bar. This annuity is known as an n-year *temporary life annuity* which pays until the *first* of the two statuses fails and is technically a *joint annuity*. Finally, (3.18) uses two other symbols: $\ddot{a}_{\overline{n}|}$ indicating an n-year *period certain annuity* and ${}_{n|}\ddot{a}_x$ denoting the present value of an n-year *deferred life annuity.*

Joint and Survivor Annuity

Another type of annuity is known as the *joint and survivor annuity*. The term "joint" suggests that the payment amount is based on more than one status, and the term "survivor" suggests that it pays as least some amount until the last status fails. For example, a 50 percent joint and survivor annuity pays one dollar annually while both statuses are alive (usually husband and wife, but not necessarily restricted to couples), and reduces to 50 cents after the first death. Let x denote the age of the plan member, z the joint annuitant's age, and k the portion of the annual benefit paid to the survivor after the first death, regardless of who dies first. The $100k$ percent joint and survivor annuity may be represented as

$$
\begin{aligned}
{}^{k}\ddot{a}_{xz} = \sum_{t=0}^{\infty} v^{t}\big[& {}_tp_x^{(m)}\,{}_tp_z^{(m)} + k\,{}_tp_x^{(m)}\,(1 - {}_tp_z^{(m)}) \\
& + k\,{}_tp_z^{(m)}\,(1 - {}_tp_x^{(m)})\big].
\end{aligned} \tag{3.19a}
$$

The first term inside the brackets represents a payment of \$1 if both x and z are alive at time t, the second term represents a payment of \$$k$ if only x is alive, while the third term represents a payment of \$$k$ if only z is alive.

A widely used variation of this annuity is known as a *contingent* joint and survivor annuity. Under this form, the annuity benefit is reduced only if the plan member is the first to die. The survivor's benefit might be any portion, with one-half and two-thirds representing choices usually available. A $100k$ percent contingent joint and survivor annuity may be expressed as

$$\overset{k}{\ddot{a}}_{\underset{1}{x}z} = \sum_{t=0}^{\infty} v^t \big[{}_tp_x^{(m)}\, {}_tp_z^{(m)} + {}_tp_x^{(m)}\,(1 - {}_tp_z^{(m)}) + k\, {}_tp_z^{(m)}\,(1 - {}_tp_x^{(m)}) \big], \qquad (3.19b)$$

where the 1 under the x subscript stipulates that, if x is the first to die, only k dollars are continued to z. The bracketed expression represents a payment of \$1 if both x and z are alive at time t, a payment of \$1 if x is alive and z is not alive, and a payment of \$$k$ if z is alive and x is not alive. This expression reduces to

$$\overset{k}{\ddot{a}}_{\underset{1}{x}z} = \sum_{t=0}^{\infty} v^t \big[{}_tp_x^{(m)} + k\, {}_tp_z^{(m)} - k\, {}_tp_x^{(m)}\, {}_tp_z^{(m)} \big]. \qquad (3.19c)$$

In this form, the bracketed term represents a payment of \$1 to x regardless of whether or not z is alive, a payment of \$$k$ to z regardless of whether or not x is alive, and since this would result in a total payment of \$$(1 + k)$ in the event both are alive in year t, \$$k$ is subtracted if both are alive at time t.

Refund Annuities

Pension plans that require employee contributions frequently provide death benefits in retirement equal to the excess, if any, of the employee's accumulated contributions at retirement date over the cumulative benefits received up to the time of death. If the difference is paid in a lump sum, the annuity is termed a *modified cash refund annuity*, whereas if the difference is paid by continuing the benefit payments to a named beneficiary, the annuity is termed a *modified installment refund annuity*.[6]

Let C_r denote the employee's accumulated contributions at retirement and n' denote such contributions per dollar of retirement benefit (i.e., $n' = C_r \div B_r$). The present value at age r of the modified cash refund annuity may be written as

[6] Cash refund and installment annuities guarantee a return of the annuity's purchase cost. The term *modified* is used in connection with pension plans to indicate that the guarantee involves a return of accumulated employee contributions.

$$^{MCR}\ddot{a}_r = \sum_{t=0}^{\infty} v^t \, {}_tp_r^{(m)}\left[1 + v\, q_{r+t}^{(m)} \max\{(n' - t - 1), 0\}\right]. \quad (3.20)$$

This formulation assumes that the lump sum payment, if any, will be made at the end of the year of death.

The modified installment refund annuity is equal to the sum of an n'-year annuity certain plus an n'-year deferred life annuity (i.e., a period certain life annuity, where the certain period is determined by the ratio of employee contributions at retirement to annual benefit payments):

$$^{MIR}\ddot{a}_r = \ddot{a}_{\overline{n'}|} + {}_{n'|}\ddot{a}_x . \quad (3.21)$$

Table 3–6 shows numerical values for the annuities discussed up to this point under alternative interest rate, mortality rate, and attained-age assumptions. Life expectancies at age 55, 65, and 70 are shown in Section I. A 25 percent change in the underlying mortality rate is seen to affect this statistic by less than 25 percent. Section II of Table 3–6 shows annuity certain values, with the last row displaying an annuity certain for a period equal to an age-65 individual's life expectancy. Note that this value is greater than an age-65 life annuity shown in Section V. Figure 3–4 is useful in reasoning through this relationship. The area for both annuity representations is, of course, equal. Since their respective annuity values involve the product of each payment (or expected payment in the case of the life annuity) times v^t, for t ranging from zero to 45 (110 – 65), the life annuity payment stream involves much smaller values of v^t beyond age 65 + e_{65} than the values of v^t used with the annuity certain.

Section III of Table 3–6 shows the very substantial difference between a last survivor annuity and a temporary life annuity, both involving 10-year periods. Section IV illustrates the relatively small impact of providing a 100 percent contingent benefit versus a 50 or 75 percent contingency. Likewise, a 10-year difference in the age of the contingent annuitant has little impact on the annuity value.

Section V shows life annuity values for ages 55, 65, and 70. A 25 percent change in the underlying mortality rate has only slightly less impact than a 2 percentage point change in the interest rate. Finally, the modified cash refund and modified installment refund annuities are shown in Section VI of the table. Since employee contributions at retirement are assumed to be equal to 5 times annual benefits, the installment refund annuity is equal to a 5-year and age-65 last survivor annuity.

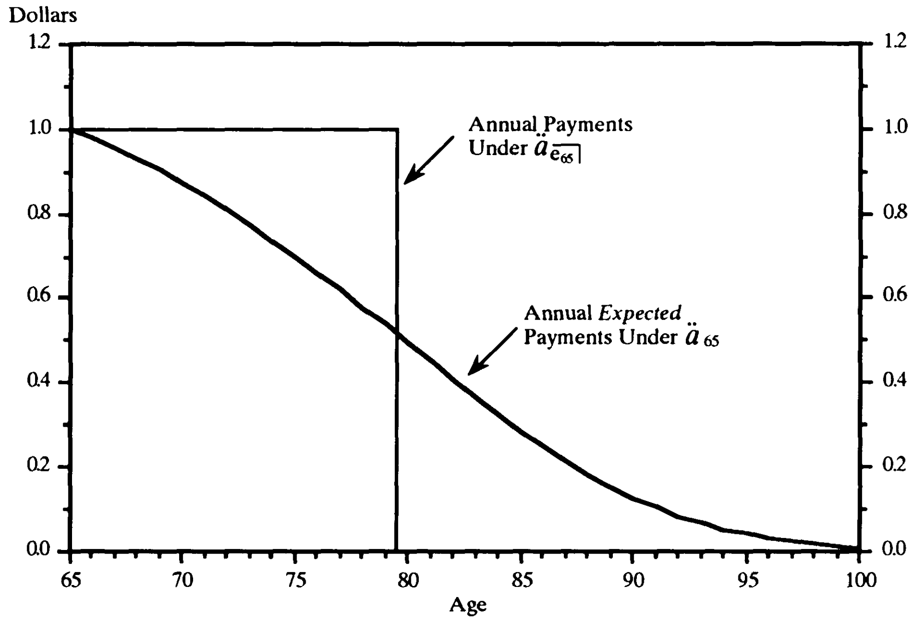
TABLE 3–6

Annuity Values Under Alternative Interest and Mortality Assumptions

Type of Annuity	Interest Rates:	6%	8%			10%
	Mortality as % of GAM-1971:	100%	75%	100%	125%	100%
I. Life Expectancy	e_{55}		24.95	22.21	20.21	
	e_{65}		17.00	14.61	12.91	
	e_{70}		13.57	11.41	9.89	
II. Annuity Certain	$\ddot{a}_{\overline{5}\rceil}$	4.47		4.31		4.17
	$\ddot{a}_{\overline{10}\rceil}$	7.80		7.25		6.76
	$\ddot{a}_{\overline{15}\rceil}$	10.29		9.24		8.37
	$\ddot{a}_{\overline{e_{65}}\rceil}$	10.13	9.85	9.12	8.50	8.27
III. Last Survivor and Temporary Life Annuities	$\ddot{a}_{\overline{65:\overline{10}\rceil}}$	10.55	9.80	9.34	8.98	8.36
	$\ddot{a}_{65:\overline{10}\rceil}$	6.98	6.69	6.51	6.35	6.10
IV. Contingent Joint and Survivor Annuity	${}^{50}\ddot{a}_{65:60}$	11.09	10.22	9.65	9.17	8.53
	${}^{75}\ddot{a}_{65:60}$	11.78	10.71	10.18	9.72	8.94
	${}^{100}\ddot{a}_{65:60}$	12.46	11.21	10.70	10.26	9.35
	${}^{50}\ddot{a}_{65:55}$	11.46	10.43	9.89	9.44	8.70
	${}^{50}\ddot{a}_{65:65}$	10.75	10.02	9.41	8.90	8.36
V. Life Annuity	$\ddot{a}_{55}$	12.24	10.90	10.45	10.06	9.10
	$\ddot{a}_{65}$	9.73	9.24	8.60	8.08	7.71
	$\ddot{a}_{70}$	8.35	8.23	7.52	6.95	6.84
VI. Modified Cash and Installment Refund Annuities	${}^{MCR}\ddot{a}_{65}$	9.93	9.39	8.80	8.32	7.90
	${}^{MIR}\ddot{a}_{65}$	9.92	9.38	8.78	8.31	7.88

$n' = C_r \div B_r = 5$ for modified cash and installment refund annuities.

FIGURE 3–4
Life Annuity vs. Annuity Certain for Life Expectancy



Temporary Annuities

Temporary annuities, as will be shown in later chapters, are required for some actuarial cost methods. In this case, consideration is given to an employment-based annuity, subject to multiple decrements, rather than the typical retirement-related annuity involving only mortality. As noted earlier, the temporary annuity has $x{:}\overline{n}|$ as its subscript, indicating that the payments cease at the end of n years or, if sooner, at the time the status x fails. In addition, a superscript T is added to the annuity symbol to signify a multiple-decrement environment. Equation (3.22) sets forth the basic definition of this annuity:

$$\ddot{a}^{T}_{x:\overline{n}|} = \sum_{t=0}^{n-1} {}_tp_x^{(T)} v^t . \tag{3.22}$$

The value of n is often set at $r - x$ in the context of pension mathematics, resulting in an annuity running from the participant's attained age x up to, but not including, retirement age.

The withdrawal decrement may have a unique effect on this annuity, as shown in Table 3–7. Generally one would anticipate the present value of an annuity running to age 65 to become smaller at older ages. However, because of the withdrawal decrement, the value of the annuity for most entry ages reaches a maximum at some age in between the employee's entry age and retirement age. This maximum for an age-20 entrant is 8.18 at ages 41 and 42, with younger and older ages having smaller values. For example, at age 20, the 45-year annuity takes on a value of only 4.00.

An important variation of $\ddot{a}^T_{x:\overline{r-x}|}$ is represented by ${}^s\ddot{a}^T_{x:\overline{r-x}|}$, the superscript s denoting that the annuity is salary-based, as defined by

$$ {}^s\ddot{a}^T_{x:\overline{r-x}|} = \sum_{t=x}^{r-1} \frac{S_t}{S_x} {}_{t-x}p_x^{(T)} v^{t-x}. \tag{3.23} $$

This formula represents the present value of an employee's future salary from age x to age r, per unit of salary at age x.

Table 3–8 replicates Table 3–7 based on the annuity defined in (3.23). This annuity, as one would expect, develops larger attained age values than the unit-based annuity given in Table 3–7. Whereas the unit-based annuity for an age-20 entrant reaches a maximum value of 8.18 at age 41, the salary-based annuity reaches a maximum of 14.35 at age 36. By age 64, however, both annuity values are equal to unity. As was the case for the annuity values shown in Table 3–7, the values given in Table 3–8 for the same attained age, but for different entry ages, are affected to some extent by the select period of the termination assumption and the elimination of termination rates after the first early retirement qualification age.

Figure 3–5 provides a graphical comparison of the annuities specified by equations (3.22) and (3.23) for the age-30 entrant. This graph shows that these annuities can take on unique shapes during an employee's career.

TABLE 3–7

Present Value of a Temporary Employment-Based Life Annuity from Age x to Age 65, $\ddot{a}^{T}_{x:\overline{65-x}|}$

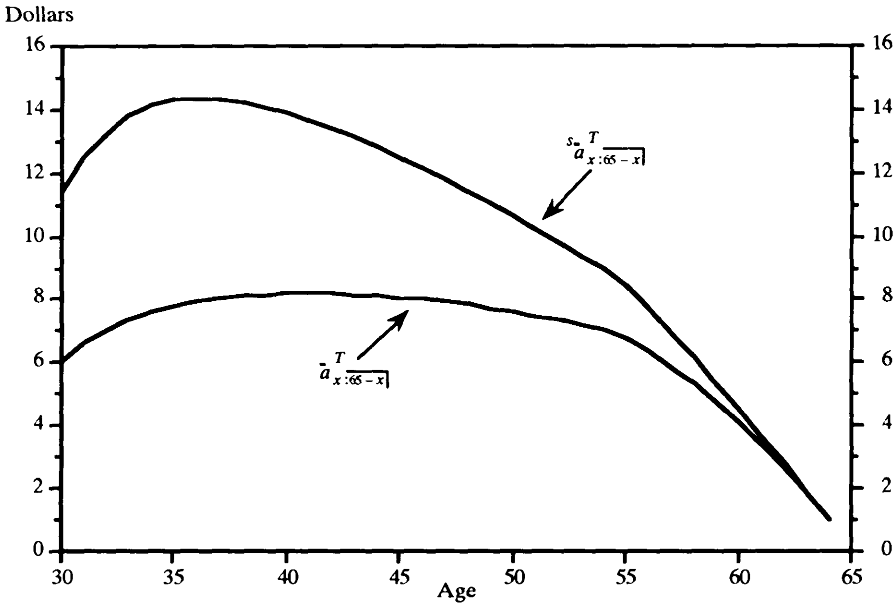
	Entry Age, y				
x	20	30	40	50	60
20	4.00				
21	4.29				
22	4.58				
23	4.88				
24	5.19				
25	5.49				
26	5.79				
27	6.08				
28	6.36				
29	6.63				
30	6.88	6.07			
31	7.11	6.59			
32	7.32	7.03			
33	7.51	7.38			
34	7.67	7.63			
35	7.81	7.81			
36	7.93	7.93			
37	8.02	8.02			
38	8.09	8.09			
39	8.14	8.14			
40	8.17	8.17	7.55		
41	8.18	8.18	7.83		
42	8.18	8.18	8.00		
43	8.16	8.16	8.08		
44	8.12	8.12	8.09		
45	8.07	8.07	8.07		
46	8.00	8.00	8.00		
47	7.93	7.93	7.93		
48	7.84	7.84	7.84		
49	7.73	7.73	7.73		
50	7.62	7.62	7.62	7.09	
51	7.49	7.49	7.49	7.01	
52	7.35	7.35	7.35	6.86	
53	7.19	7.19	7.19	6.67	
54	7.01	7.01	7.01	6.44	
55	6.80	6.80	6.80	6.17	
56	6.34	6.34	6.34	5.86	
57	5.85	5.85	5.85	5.50	
58	5.32	5.32	5.32	5.09	
59	4.74	4.74	4.74	4.64	
60	4.12	4.12	4.12	4.12	3.84
61	3.44	3.44	3.44	3.44	3.30
62	2.70	2.70	2.70	2.70	2.64
63	1.89	1.89	1.89	1.89	1.87
64	1.00	1.00	1.00	1.00	1.00

TABLE 3–8

Present Value of a Temporary Employment-Based and Salary-Based Life Annuity from Age x to Age 65, ${}^{s}\ddot{a}^{T}_{x:\overline{65-x}|}$

	Entry Age, y				
x	20	30	40	50	60
20	6.68				
21	7.39				
22	8.13				
23	8.87				
24	9.61				
25	10.33				
26	11.01				
27	11.65				
28	12.24				
29	12.75				
30	13.20	11.42			
31	13.58	12.47			
32	13.88	13.27			
33	14.11	13.84			
34	14.25	14.18			
35	14.34	14.34			
36	14.35	14.35			
37	14.31	14.31			
38	14.22	14.22			
39	14.07	14.07			
40	13.89	13.89	12.72		
41	13.66	13.66	13.01		
42	13.40	13.40	13.08		
43	13.12	13.12	12.98		
44	12.81	12.81	12.77		
45	12.49	12.49	12.49		
46	12.15	12.15	12.15		
47	11.79	11.79	11.79		
48	11.42	11.42	11.42		
49	11.04	11.04	11.04		
50	10.65	10.65	10.65	9.75	
51	10.25	10.25	10.25	9.44	
52	9.83	9.83	9.83	9.06	
53	9.40	9.40	9.40	8.62	
54	8.95	8.95	8.95	8.14	
55	8.49	8.49	8.49	7.63	
56	7.72	7.72	7.72	7.08	
57	6.95	6.95	6.95	6.50	
58	6.16	6.16	6.16	5.89	
59	5.35	5.35	5.35	5.23	
60	4.54	4.54	4.54	4.54	4.22
61	3.70	3.70	3.70	3.70	3.55
62	2.83	2.83	2.83	2.83	2.77
63	1.94	1.94	1.94	1.94	1.92
64	1.00	1.00	1.00	1.00	1.00

FIGURE 3–5
Unit-Based and Salary-Based Temporary Annuity Values from Age x to Age 65



Chapter 4

Pension Plan Population Theory

BASIC CONCEPTS

This chapter deals with the population of pension plan members, which consists of several subpopulations. Active employees, of course, make up the primary group within the plan's membership, and the following discussion focuses on them. Retired employees represent another subpopulation, and for present purposes, this group is assumed to consist only of members who retired directly from the service of the employer. This is in contrast to defining a retired member as any one of several types of benefit recipients. Vested terminated employees make up a third group, which can be further divided into those in the benefit deferral period and those receiving benefits. Disabled employees make up the fourth subpopulation for plans providing disability benefits, and beneficiaries, generally surviving spouses, make up a fifth subpopulation.

Stationary Population

The discussion of pension plan populations begins at the most elementary level, namely, with the concept of a stationary population. A population is considered to be stationary when its size and age distribution remain constant year after year. If the decrement rates associated with the population are constant, and if a constant number of new entrants flows into the population each year, a stationary condition will exist after *n* years, where *n* equals the oldest age in the population less the youngest age.[1]

Understanding the concept of a stationary population can be facilitated by considering a simplified example. Assume that a population has four ages (x, x+1, x+2, and x+3), that the rates of decrement for each age are 1/4, 1/3, 1/2, and 1, respectively, and that 100 new employees are hired each year, all at age x. The first few years experience for a population exposed to these conditions is given in Table 4–1.

TABLE 4–1
Development of a Stationary Population

Year	Ages: / Decrement Rates:	x 1/4	$x+1$ 1/3	$x+2$ 1/2	$x+3$ 1	$x+4$	Total Size
1	New Entrants →	100					100
2	New Entrants →	100	75				175
3	New Entrants →	100	75	50			225
4	New Entrants →	100	75	50	25		250
5	New Entrants →	100	75	50	25	0	250
6	New Entrants →	100	75	50	25	0	250
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
	Stationary Age Distribution:	40%	30%	20%	10%	0%	100%

After the first year, the original group of 100 employees hired at age x are age x+1 and total 75 in number. Since 100 new employees are hired at age x each year, the total population at the beginning of year 2 is 175 members. Continuing this process for three years (i.e., until the beginning of year 4), produces a population with a constant size of 250 members and a stationary distribution, as shown in percentage form at the bottom of Table 4–1. Thus, the population becomes stationary after *n* years,

[1] If the population is assumed to be continuous over each age interval, rather than discrete as is assumed in this chapter for simplicity, then *n* would be equal to the first age at which no survivors exist less the youngest age. In other words, one year would be added to the value of *n* for the continuous case.

where n in this case is equal to three (i.e., the oldest age in the population, $x+3$, less the youngest age, x).

Since pension benefits are tied to service, it is also relevant to point out that the service distribution of a stationary population also becomes constant after n years.[2] Table 4–1 shows that 40 percent of the stationary population has zero years of service, 30 percent has one year of service, and so forth.

A pension plan population, unlike the example shown in Table 4–1, has multiple entry ages, and it is logical to inquire whether or not the concept of a stationary population applies in this case. To show that it does apply, one need only conceptualize a multiple entry age population as a series of single entry age populations, with each subpopulation representing a given entry age. Consequently, a multiple entry age population of active employees will become stationary after m years, where m equals the largest retirement-age/entry-age spread among the various subpopulations.

Mature Population

The concept of a mature population is only slightly different from, and somewhat more general than, a stationary population. In fact, a stationary population is a special case of a mature population. Both concepts involve a constant year-to-year age and service distribution, but whereas the stationary population maintains a constant size, this need not be the case for a mature population. If the increments to the population (newly hired employees) increase at a *constant rate*, the population will attain a constant percentage age and service distribution in precisely the same length of time as required for a population to become stationary. Moreover, the size of the mature population will grow at precisely the same *rate* as the growth in new entrants.

These characteristics are illustrated in Table 4–2 where the number of new entrants is doubled each year (i.e., a 100 percent growth rate). The decrement assumptions are the same as those used in Table 4–1. The age distribution, which is shown at the bottom of the table, is constant year after year, but considerably

[2]This assumes, of course, that the entry age distribution of new entrants is fixed.

different from the age distribution developed in Table 4–1. This is the case for the service distribution also.

TABLE 4–2
Development of a Mature Population

Years	Ages: / Decrement Rates:	x / 1/4	x + 1 / 1/3	x + 2 / 1/2	x + 3 / 1	x + 4	Total Size
1		100					100
2		200	75				275
3		400	150	50			600
4		800	300	100	25		1,225
5		1,600	600	200	50	0	2,450
6		3,200	1,200	400	100	0	4,900
7		6,400	2,400	800	200	0	9,800
8		12,800	4,800	1,600	400	0	19,600
.		.	.	.	.	.	.
.		.	.	.	.	.	.
.		.	.	.	.	.	.
Mature Age Distribution:		65%	25%	8%	2%	0%	100%

Throughout the remainder of this book, the term mature population will be used, even in those cases where a stationary population applies, since it is the more general of the two concepts.

Undermature and Overmature Populations

A population is considered to be undermature if its age and service distribution has a larger proportion of younger, short-service employees than that of a mature population that faces the same decremental factors, is of the same size, and experiences the same entry age distribution. An overmature population is one that has a disproportionately large number of employees at older ages and with longer periods of service than that of a mature population based on the same decrement and entry age assumptions. Generally, growing industries are characterized by firms having undermature populations, while declining industries have firms with overmature populations.

An example of an undermature population is given in Table 4–3, where the number of new entrants increases by 100 employees each year, representing a continually decreasing rate of growth. The membership distribution in this example, parenthetically expressed in percentage form, asymptotically approaches the same distribution as that for the stationary population discussed previously in Table 4–1. After 100 years the population's age and service distribution is nearly identical to that of the corresponding stationary population.

TABLE 4–3
Development of an Undermature Population

Years	Ages: / Decrement Rates:	*x* 1/4	*x* + 1 1/3	*x* + 2 1/2	*x* + 3 1	*x* + 4	Total Size
1		100 (100%)					100
2		200 (73%)	75 (27%)				275
3		300 (60%)	150 (30%)	50 (10%)			500
4		400 (53%)	225 (30%)	100 (13%)	25 (3%)		750
5		500 (50%)	300 (30%)	150 (15%)	50 (5%)	0	1,000
6		600 (48%)	375 (30%)	200 (16%)	75 (6%)	0	1,250
7		700 (47%)	450 (30%)	250 (17%)	100 (7%)	0	1,500
9		800 (46%)	525 (30%)	300 (17%)	125 (7%)	0	1,750
.		.	.	.	.	.	.
.		.	.	.	.	.	.
100		10,000 (40.4%)	7,425 (30.0%)	4,900 (19.8%)	2,425 (9.8%)	0	24,750
.		.	.	.	.	.	.
.		.	.	.	.	.	.
.		.	.	.	.	.	.
	Asymptotic Age Distribution:	40%	30%	20%	10%	0%	100%

Finally, Table 4–4 illustrates the development of an overmature population, created by hiring 1,000 employees in the first year and 100 *fewer* employees each year thereafter. The degree to which the population is overmature is determinable by comparing its age and service distribution to that of the stationary population based on the same decrement rates (given in the lower part of Table 4–4 for convenience).

TABLE 4–4
Development of an Overmature Population

Years	*Ages:*	x	$x+1$	$x+2$	$x+3$	$x+4$	*Total Size*
	Decrement Rates:	1/4	1/3	1/2	1		
1		1,000 (100%)					1,000
2		900 (55%)	750 (45%)				1,650
3		800 (41%)	675 (34%)	500 (25%)			1,975
4		700 (35%)	600 (30%)	450 (23%)	250 (13%)		2,000
5		600 (34%)	525 (30%)	400 (23%)	225 (13%)	0	1,750
6		500 (33%)	450 (30%)	350 (23%)	200 (13%)	0	1,500
7		400 (32%)	375 (30%)	300 (24%)	175 (14%)	0	1,250
9		300 (30%)	300 (30%)	250 (25%)	150 (15%)	0	1,000
Stationary Age Distribution:		40%	30%	20%	10%	0%	100%

Size-Constrained Population

For purposes of illustration, the populations discussed thus far have had their membership size as a dependent variable and the number of newly hired employees as the independent variable. This is not an appropriate assumption in dealing with pension plan populations, since the sponsoring firm determines the size of its employee group on business considerations. Switching the dependent and independent variables has significant implications for the resulting age and service distributions.

The data in Table 4–5 show the results of a time-dependent plan population based on the same assumptions as before except for membership size, which is assumed to be constant at 1,000 employees from the inception of the population onward. Although the data are presented in terms of numbers of employees, one need only shift the decimal point one position to the left to obtain the percentage distribution.

TABLE 4–5
Development of a Size-Constrained Population

Years	Ages: / Decrement Rates:	*x* 1/4	*x* + 1 1/3	*x* + 2 1/2	*x* + 3 1	*x* + 4	Total Size
1		1,000 *					1,000
2		250	750 *				1,000
3		313	188	500 *			1,000
4		391	234	125	250 *		1,000
5		488 *	293	156	63	0	1,000
6		360	366 *	195	78	0	1,000
7		388	270	244 *	98	0	1,000
8		407	291	180	122 *	0	1,000
9		411 *	305	194	90	0	1,000
10		391	308 *	203	97	0	1,000
11		399	294	205 *	102	0	1,000
12		402	299	196	103 *	0	1,000
13		401 *	302	200	98	0	1,000
14		398	301 *	201	100	0	1,000
15		400	299	200 *	101	0	1,000
16		400	300	199	100 *	0	1,000
.		.	.	.	.	.	.
.		.	.	.	.	.	.
.		.	.	.	.	.	.
	Ultimate:	400	300	200	100	0	1,000

The age and service distribution of the constant-size population is quite erratic at first, but becomes less so as time progresses. Although the age distribution in this example converges to that of the stationary population discussed in Table 4–1, the convergence is not smooth. In fact, a continually smaller ripple is seen to flow through the population during each successive year. This ripple, which begins with the initial group of 1,000 employees hired, is noted by asterisks in Table 4–5.

A size-constrained population will *generally* converge to its stationary counterpart created without a size constraint. The length of time required for the convergence is a function of the number of ages in the population and the rates of decrements at each of its ages. Naturally, the more attained ages, other things being equal, the longer it will take for the size-constrained population to come within a predetermined tolerance level of its stationary counterpart. Moreover, the lower the age-specific rates of decrement, the longer it generally takes for a predetermined tolerance level to be reached. The latter generalization can be appreciated by considering the following extreme example.

Suppose the age-specific decrements are zero for each age up to the last age, at which point the decrement rate is 100 percent. If a size constraint is imposed, the population will never converge to the uniform distribution that would result in the absence of the size constraint. In this case the population's age distribution will cycle indefinitely, with each n-year cycle consisting of n different distributions of 100 percent of the membership at each possible attained age, where n is the number of ages in the population. At the other extreme is the case where the rate of decrement is 100 percent at each age. The population under this assumption will be mature from its inception, since no employees will survive beyond the first age, and the entire labor force will be rehired at this age each year.

Up to this point the discussion of a size-constrained population has dealt with a single entry age population. This is unrealistic for pension plans, since they conform to the more general multiple entry age, size-constrained population. When the multiple entry age situation was discussed in the context of a stationary population without a size constraint, it was suggested that one conceptualize the population as consisting of a collection of single entry age populations. The situation is not nearly as simple when a size constraint is imposed. For example, it does not follow that each entry age subsector of the overall population will receive the same number of new entrants as the number of employees decrementing from that subsector each year. This is the case even if the hiring age distribution is held constant over time, unless the population is mature. The ultimate effect is that the population tends to approach a mature status sooner than it takes the longest entry age subpopulation to become mature under a size constraint. A numerical illustration of the convergence pattern of a multiple entry age population is given in the following section.

MODEL PLAN POPULATION

Pension costs are analyzed both for individual participants and for various model pension plan populations in this book. When dealing with the plan as a whole, it is necessary to assume an age and service distribution of plan members, a specific salary structure of active employees, and a benefit structure of nonactive plan members. In the interest of generality, numerous plan populations, in varying states of maturity, are assumed for the numerical illustrations. Rather than select the model population arbitrarily, the experience of a single plan population is simulated over a period of 50 years. In order to simulate the various maturity statuses, the initial plan population is undermature and is then forced through a mature state to an overmature status. This is accomplished by first having the size of the population increase at a decreasing rate and eventually decrease at an increasing rate.

The hiring age distribution and salary scale for new entrants during the 50-year simulation is given in Table 4–6. The average hiring age is 28, and the salary scale reflects one half of the previously discussed merit scale from age 20 to each of the specific entry ages. The total salary of active employees throughout the 50-year simulation increases according to the productivity and inflation rates specified earlier, that is, 1 percent and 4 percent,

TABLE 4–6

Hiring Age Distribution and Salary Scale

Entry Age	*Hiring Distribution*	*Salary Scale*
20	0.277	1.0000
25	0.290	1.1171
30	0.152	1.2437
35	0.101	1.3747
40	0.086	1.5042
45	0.049	1.6252
50	0.016	1.7301
55	0.015	1.8122
60	0.014	1.8655

respectively. The annual rate of increase in salary for the total plan population, however, will be somewhat more than 5 percent because of the maturation that takes place in the population of active employees.

Table 4–7 shows various statistics for the simulated plan population. The number of active employees, expressed as a percentage of the initial group, doubles during the first 25 years and then decreases to its original size over the succeeding 25 years. Thus, the population experiences (1) rapid growth, (2) gradual growth, (3) gradual decline, and (4) rapid decline during the 50-year period. Since the initial population of active employees is undermature, the simulated population experiences various undermature, approximately mature, and overmature statuses. Several statistics associated with the corresponding *mature* population for the underlying decrement assumptions are given at the bottom of Table 4–7 for comparison.

The average age of active employees begins at age 35, increases to 40.7 after 25 years, and to age 47.3 by the end of the 50-year period. The average for the mature population is age 41.3, a value reached by the simulated population during its 29th year. The average service period of employees begins at 5 years, increases to 9.7 years after 25 years, and to 16.6 years after 50 years. The average period of service for the mature population is 10.6 years, a value reached by the simulated population in its 30th year. These statistics show that even though the active population in its 30th year is not perfectly mature, its average age and service at that point are very nearly equal to that of the corresponding mature population. The speed with which the simulated population attains an approximately mature status is due, in part, to the assumption of multiple entry ages.

Table 4–7 also shows the total and average salaries of active employees, expressed as a percent of their respective values at the outset of the 50-year period. The ratio for total salaries increases to exactly double the ratio for average salaries after 25 years, a relationship consistent with the increase in the number of active employees. After 50 years, however, the total salary percentage is identical to the average salary percentage, since the population has returned to its initial size by this time. The rate of growth in average salary is approximately 5.9 percent during the first 25 years, and approximately 5.6 percent during the last 25 years.

TABLE 4-7

Population Statistics

Year	Number as Percent of Initial Size	Average Age	Average Service	Total Salary as Percentage of Initial	Average Salary as Percentage of Initial	Number as a Percent of Actives: Retired	Vested Terminated	Disabled	Surviving Spouses
0	100.0	35.0	5.0	100.0	100.0	0.0	0.0	0.0	0.0
1	107.8	34.8	4.9	114.6	106.3	1.7	2.4	0.0	0.1
2	115.4	35.0	5.0	130.9	113.5	2.0	5.0	0.1	0.2
3	122.6	35.3	5.2	148.1	120.8	2.3	7.8	0.1	0.3
4	129.4	35.6	5.3	166.2	128.5	2.7	10.9	0.1	0.4
5	136.0	35.9	5.5	185.5	136.4	3.1	14.2	0.2	0.5
6	142.2	36.2	5.7	205.8	144.7	3.5	17.8	0.2	0.7
7	148.2	36.5	5.9	227.3	153.4	4.0	21.6	0.3	0.8
8	153.8	36.8	6.1	250.0	162.5	4.5	25.7	0.3	1.0
9	159.0	37.0	6.3	273.7	172.1	5.0	30.0	0.4	1.2
10	164.0	37.3	6.6	298.9	182.2	5.6	34.4	0.4	1.4
11	168.6	37.6	6.8	325.1	192.8	6.2	39.1	0.5	1.6
12	173.0	37.8	7.0	352.9	204.0	6.9	43.8	0.6	1.8
13	177.0	38.1	7.2	381.8	215.7	7.6	48.7	0.7	2.1
14	180.6	38.3	7.4	412.0	228.1	8.3	53.7	0.8	2.3
15	184.0	38.6	7.6	443.6	241.1	9.1	58.9	0.9	2.6
16	187.0	38.8	7.9	476.5	254.8	9.9	64.1	1.0	2.9
17	189.8	39.0	8.1	510.9	269.2	10.8	69.3	1.1	3.2
18	192.2	39.3	8.3	546.5	284.4	11.7	74.6	1.3	3.5
19	194.2	39.5	8.5	583.3	300.3	12.7	79.9	1.4	3.8
20	196.0	39.7	8.7	621.5	317.1	13.7	85.3	1.6	4.1
21	197.4	39.9	8.9	660.9	334.8	14.7	90.6	1.7	4.5
22	198.6	40.1	9.1	701.6	353.3	15.9	95.9	1.9	4.8
23	199.4	40.3	9.3	743.3	372.8	17.1	101.2	2.1	5.2
24	199.8	40.5	9.5	785.9	393.3	18.3	106.5	2.3	5.6
25	200.0	40.7	9.7	829.8	414.9	19.5	111.7	2.5	5.9
26	199.8	40.9	9.9	874.5	437.7	20.8	116.8	2.7	6.3
27	199.4	41.0	10.1	920.1	461.4	22.2	121.8	2.9	6.7
28	198.6	41.2	10.2	966.3	486.6	23.6	126.8	3.1	7.1
29	197.4	41.3	10.4	1,012.8	513.1	24.9	131.6	3.2	7.4
30	196.0	41.5	10.6	1,060.3	541.0	26.3	136.3	3.4	7.8
31	194.2	41.7	10.8	1,107.7	570.4	27.6	140.9	3.6	8.2
32	192.2	41.8	10.9	1,155.5	601.2	29.1	145.3	3.8	8.5
33	189.8	42.0	11.1	1,202.9	633.8	30.5	149.5	4.0	8.9
34	187.0	42.1	11.3	1,249.7	668.3	31.8	153.6	4.2	9.2
35	184.0	42.3	11.5	1,296.5	704.6	33.1	157.5	4.4	9.6
36	180.6	42.5	11.7	1,342.1	743.1	34.4	161.1	4.5	9.9
37	177.0	42.7	11.8	1,386.8	783.5	35.6	164.6	4.7	10.2
38	173.0	42.9	12.0	1,429.6	826.4	36.8	167.8	4.8	10.5
39	168.6	43.1	12.2	1,470.1	871.9	38.0	170.8	5.0	10.8
40	164.0	43.3	12.5	1,509.0	920.1	39.0	173.6	5.1	11.0
41	159.0	43.5	12.7	1,544.6	971.5	40.0	176.1	5.2	11.3
42	153.8	43.7	13.0	1,577.2	1,025.5	41.0	178.3	5.3	11.5
43	148.2	44.0	13.2	1,605.4	1,083.3	41.8	180.2	5.4	11.7
44	142.2	44.3	13.5	1,628.6	1,145.3	42.6	181.8	5.5	11.9
45	136.0	44.7	13.9	1,647.6	1,211.4	43.3	183.1	5.6	12.0
46	129.4	45.0	14.3	1,659.8	1,282.7	43.9	184.1	5.6	12.2
47	122.6	45.5	14.7	1,666.0	1,358.9	44.4	184.7	5.7	12.3
48	115.4	46.0	15.2	1,663.8	1,441.8	44.8	185.0	5.7	12.4
49	107.8	46.6	15.8	1,652.2	1,532.6	45.1	185.0	5.8	12.5
50	100.0	47.3	16.6	1,632.1	1,632.1	45.3	184.6	5.8	12.5
Mature Population:		41.3	10.6			23.3	103.0	2.9	6.7

These values exceed 5 percent (i.e., 1 percent productivity and 4 percent inflation) because of the maturation of the population and its interaction with the merit salary scale.

The last four columns of Table 4–7 show the number of non-active plan members as a percentage of active employees. There are no nonactives at the outset of the simulation, by definition, but by the end of 25 years, retired employees total 19.5 percent, vested termination employees (in both a pre-and post-retirement status) total 111.7 percent, disabled employees total 2.5 percent, and surviving spouses total 5.9 percent. After 50 years, retired employees total 45.3 percent of active employees, a considerably overmature state in comparison to the 23.3 percent for the mature population (see the last row of Table 4–7). The corresponding number of vested terminated employees after 50 years is 184.6 percent, as compared to 103.0 percent for the mature population. The largest number of disabled employees and surviving spouses total only 5.8 and 12.5 percent, respectively, even for the extremely overmature population at the end of the 50-year period.

Chapter 5

Pension Liability Measures

A variety of liability measures are associated with pension plans, each one having a specified purpose. Some liabilities represent the financial obligations of the plan, either on a plan termination or ongoing basis, while others simply represent mathematical byproducts of various actuarial cost methods used for funding pension plans. Although the latter are not liabilities in the true sense of the word, they are referred to as *actuarial liabilities* to distinguish them from the term liability as used in the fields of finance and accounting. The accounting profession has promulgated several specific pension liability measures, and another set of liabilities is defined by federal statutes in determining minimum required and maximum tax deductible contributions. Finally, since none of these liabilities may represent what management believes to be the "true" long-term financial obligation of the plan, *economic liabilities* are sometimes used to evaluate a plan's funded status.

The purpose of this chapter is to present the fundamental mathematics of alternative liability measures. The specific liabilities defined by various federal statutes are presented in Chapter 10, at which point the rules associated with pension funding are discussed. Similarly, the liabilities used in pension accounting are presented in Chapter 11, at which point the entire subject of pension accounting is presented. Economic liabilities are discussed in Chapter 14. Although the liabilities presented in this chapter encompass the general definitions of the various federal statutory, accounting, and economic liabilities, there are other aspects to these liabilities that are discussed in their respective chapters.

At this point only the liability associated with retirement is considered, and then only for a single age r. The corresponding liabilities for vested termination benefits, disability benefits, death benefits, and early retirement benefits are defined in Chapters 8 and 9.

PLAN TERMINATION LIABILITY

The plan termination liability (PTL), sometimes referred to as the plan's *legal liability*, is equal to the present value of all accrued benefits, both for active and retired employees. Assuming that the benefit is in the form of an annuity payable for the lifetime of the retiree, equation (5.1a) defines this liability for a participant age x prior to retirement, while (5.1b) is applicable after retirement:

$$(PTL)_x = B_x \; {}_{r-x}p_x^{(m)} \; v^{r-x} \; \ddot{a}_r, \qquad \text{for } x \leq r \qquad (5.1a)$$

where

B_x = accrued benefit as defined by the plan

${}_{r-x}p_x^{(m)}$ = probability of living from age x to age r

v^{r-x} = interest discount from age x to age r[1]

$\ddot{a}_r$ = present value, at age r, of a life annuity;

$$(PTL)_x = B_r \; \ddot{a}_x, \qquad \text{for } x \geq r \qquad (5.1b)$$

where

B_r = retirement benefit payable for life

$\ddot{a}_x$ = present value, at age x, of a life annuity.

The $(PTL)_x$ function increases sharply with age prior to retirement, since the first three factors in (5.1a) increase with age, while the fourth term is constant. After retirement, the $(PTL)_x$ function decreases according to the annuity function, since the benefit function is constant.

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[1]In practice, the interest rate used to evaluate the liability associated with a plan termination may be based on non-uniform rates instead of a constant rate, as assumed in this presentation.

The $(PTL)_x$ function for active employees utilizes the mortality survival function, whereas all of the other liability measures presented in this chapter involve the composite survival function, which includes decrements for termination and disability. The mortality survival function is appropriate for the $(PTL)_x$ function since only death would prevent the participant from receiving the accrued benefit at retirement if the plan were terminated.[2] The participant's future employment status or disability status would have no bearing on the receipt of the accrued retirement benefit.

The mathematical definition of $(PTL)_x$ for retired participants is the same for all liability measures. Hence, the remainder of this chapter is devoted to alternative liability measures for active participants (i.e., all x's $\leq r$).

PLAN CONTINUATION LIABILITY

The plan continuation liability for accrued benefits, sometimes referred to as the *ongoing liability* for accrued benefits, measures the financial obligation under the assumption that the plan will continue to exist. In this case, future employment and disability statuses are relevant. Equation (5.2a) defines this liability measure for an active participant at age x:

$$^{AB\,r}(PCL)_x = B_x \; {}_{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r. \tag{5.2a}$$

The AB prescript to the plan continuation liability symbol indicates that the liability is based on the accrued benefit as defined by the plan. The first edition of this book expressed this liability in terms of a service prorated projected benefit, defined by (3.15b). Other experts believe it should be expressed in terms of a salary prorated benefit, as defined by (3.16b), or the "accrued benefit" implicitly defined by a given actuarial cost method. Any of these definitions can represent a meaningful plan continuation liability, depending on one's philosophy. The plan continuation liability also includes ancillary benefits; hence, the prescript r is used to denote that only retirement benefits are being evaluated at this point, a convention used hereafter until the liability associated with ancillary benefits is defined.

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[2]Under federal law, even non-vested employees become fully vested in their accrued benefits if a single-employer plan is terminated with sufficient assets.

The plan continuation liability and the plan termination liability can be expressed in terms of each other:

$$^{AB\,r}(PCL)_x = \frac{_{r-x}p_x^{(T)}}{_{r-x}p_x^{(m)}}\; ^r(PTL)_x \tag{5.2b}$$

$$= \frac{_{r-x}p_x^{(m)}}{_{r-x}p_x^{(T)}}\; ^r(PCL)_x . \tag{5.2c}$$

Under identical actuarial assumptions, the $^{AB\,r}(PCL)_x$ function will be lower in value than the $(PTL)_x$ function until retirement, at which point they become equal. If the liability associated with termination, disability, and death benefits were to be included, however, the $^{AB\,r}(PCL)_x$ may equal or exceed the $(PTL)_x$. If the plan had no disability or death benefits and the employee were fully vested, then the two liability values would be equal if the vested termination liability were included in the $^{AB\,r}(PCL)_x$ function. In other words, in both cases only death prior to retirement would prevent the participant from receiving the accrued benefit.

As a practical matter, the actuarial assumptions used in evaluating these two liabilities are likely to be different. The interest rate used with the $(PTL)_x$ function, for example, might logically approximate the rate at which the plan sponsor could "sell" the liability to an insurance carrier, whereas the interest rate used for $^{AB\,r}(PCL)_x$ is likely to represent the plan sponsor's expected long-run return on plan assets. Thus, it is difficult to predict the relative values of the plan termination and plan continuation liabilities.

Table 5–1 shows these two liability values during the career of an age-30 entrant, expressed as a percentage of the age-65 value. Post-65 values are also provided through age 100. The model assumptions are used in the illustration, but without ancillary benefits included in $^{AB\,r}(PCL)_x$. Figure 5–1 plots the values in Table 5–1, graphically illustrating that the bulk of an employee's liability is associated with ages near retirement.

ACTUARIAL LIABILITIES

Several actuarial cost methods are used with pension plans, and each method has an associated actuarial liability. In general

TABLE 5-1

Plan Termination and Plan Continuation Liabilities as a Percentage of the Age-65 Value

x	$(PTL)_x$	${}^r(PCL)_x$	x	$B_r\,\ddot{a}_x$
30	0.00	0.00	65	100.00
32	0.04	0.01	66	97.38
34	0.11	0.03	68	92.06
36	0.22	0.08	70	86.72
38	0.41	0.16	72	81.48
40	0.69	0.32	74	76.28
42	1.13	0.57	76	70.93
44	1.79	1.00	78	65.57
46	2.77	1.69	80	60.47
48	4.21	2.79	82	55.76
50	6.30	4.55	84	51.42
52	9.32	7.32	86	47.42
54	13.65	11.64	88	43.69
56	19.83	17.71	90	40.17
58	28.61	25.84	92	36.84
60	41.05	37.64	94	33.57
62	58.69	55.03	96	30.38
64	83.74	81.48	98	27.29
65	100.00	100.00	100	24.26

terms, a cost method's actuarial liability is equal to the present value of benefits allocated to date, which can be expressed as follows:

$$ {}^r(AL)_x = B'_x \; {}_{r-x}p_x^{(T)} \, v^{r-x} \, \ddot{a}_r, \tag{5.3} $$

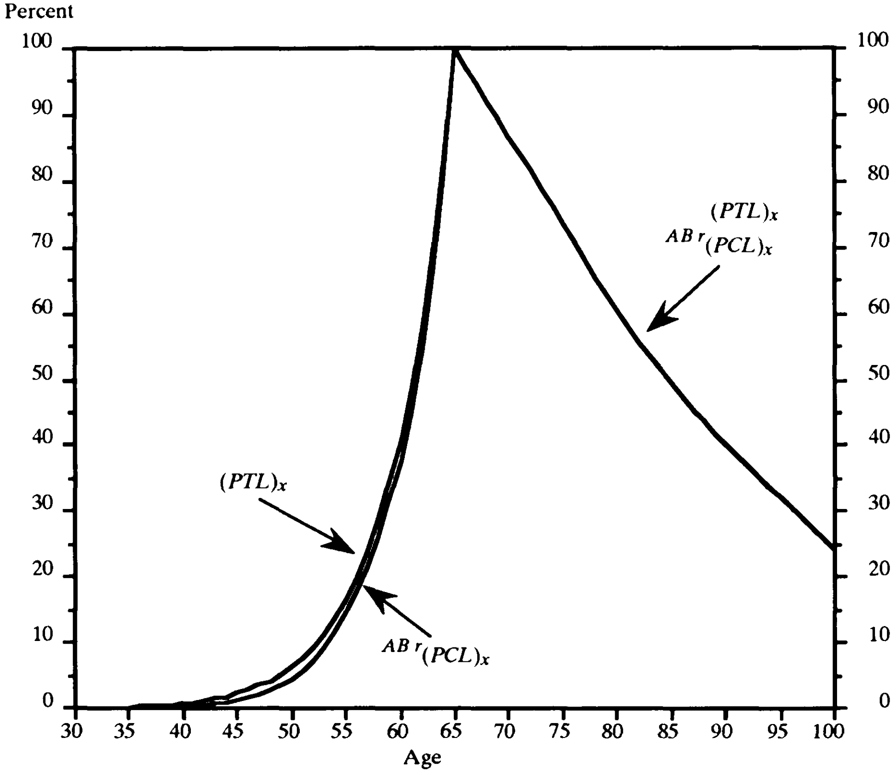
where B'_x represents the benefits allocated under a given actuarial cost method, as discussed at a later point in this chapter. Observe that, if the benefit function is equal to the accrued benefit as defined by the plan, the expression is identical to the plan continuation liability given by (5.2a).

The actuarial liability of a given cost method may also be viewed as the portion of the participant's *present value of future benefits* (PVFB) allocated under the method. The ${}^r(PVFB)_x$ function equals the present value of the participant's *total projected* retirement benefit:

$$ {}^r(PVFB)_x = B_r \; {}_{r-x}p_x^{(T)} \, v^{r-x} \, \ddot{a}_r. \tag{5.4a} $$

The ${}^r(PVFB)_x$ function is the same as the ${}^r(AL)_x$ function

FIGURE 5–1
Plan Termination and Plan Continuation Liabilities as a Percentage of the Age-65 Value



evaluated with B_r instead of B_x'. Since the actuarial liability represents the proportion of $^r(PVFB)_x$ allocated by the actuarial cost method being used, a generalized actuarial liability definition can be expressed in the following manner:

$$^r(AL)_x = k\ ^r(PVFB)_x, \tag{5.4b}$$

where k is a fraction dependent on each cost method and defined in subsequent sections of this chapter.

Accrued Benefit Method

The actuarial liability under the accrued benefit (AB) method, sometimes referred to as the *unit credit method*, is equal to the present value of accrued benefits (i.e., equation (5.3) evaluated with the accrued benefit as defined by the plan):

$$^{AB\,r}(AL)_x = B_x \; {}_{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r \tag{5.5a}$$

$$= \frac{B_x}{B_r} \; {}^{r}(PVFB)_x . \tag{5.5b}$$

In this case, the value of k in (5.4b) is a ratio of the accrued benefit to the age-r benefit. Note that the accrued benefit actuarial liability is defined mathematically to be the same as the $^{AB\,r}(PCL)_x$ function; however, these values may not be equal for a given plan because different actuarial assumptions might logically be used for each liability measure.

Benefit Prorate Methods

There are two benefit prorate methods, generally referred to as *projected unit credit methods*. The actuarial liability under the first version uses the accrued benefit function defined by (3.15b), i.e., a service proration of the participant's projected retirement benefit yielding a constant dollar benefit allocated to each attained age. The actuarial liability is defined by equations (5.6a) and (5.6b), where the BD prescript denotes the "**b**enefit prorate, constant **d**ollar" version:

$$^{BD\,r}(AL)_x = \frac{x-y}{r-y} B_r \; {}_{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r \tag{5.6a}$$

$$= \frac{x-y}{r-y} \; {}^{r}(PVFB)_x . \tag{5.6b}$$

In this case, the value of k in (5.4b) is the ratio of current service to projected service at retirement.

The actuarial liability under the second version uses the accrued benefit defined by (3.15b), i.e., a salary proration of the participant's projected retirement benefit yielding a benefit allocated to each attained age equal to a constant percent of salary:

$$^{BP\,r}(AL)_x = \frac{S_x}{S_r} B_r \; {}_{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r \tag{5.7a}$$

$$= \frac{S_x}{S_r} \; {}^{r}(PVFB)_x . \tag{5.7b}$$

The BP prescript denotes the "**b**enefit prorate, constant **p**ercent" version. The value of k in (5.4b) is the ratio of the participant's cumulative salary at age x to the projected cumulative salary at age r.

These two liability measures will be somewhat greater than the accrued benefit actuarial liability discussed above, assuming the same actuarial assumptions.

Cost Prorate Methods

There are two cost prorate methods, sometimes referred to as *projected benefit cost methods* or *entry age cost methods.* Again the liabilities can be defined in terms of prorated retirement benefits, but in this case the proration is based on temporary employment-based annuities. As we will see in Chapter 6, the pension cost under one version is equal to a constant dollar amount throughout the employee's career, whereas the other version has costs equal to a constant percent of the employee's salary. The prescripts *CD* (cost prorate, constant **d**ollar) and *CP* (cost prorate, constant **p**ercent) are used to designate each type.

The actuarial liability under the constant dollar version is given by (5.8a) and (5.8b), while the constant percent version is given by (5.9a) and (5.9b):

$$^{CD\,r}(AL)_x = \frac{\ddot{a}^T_{y:\overline{x-y}|}}{\ddot{a}^T_{y:\overline{r-y}|}} B_r \; {}_{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r \tag{5.8a}$$

$$= \frac{\ddot{a}^T_{y:\overline{x-y}|}}{\ddot{a}^T_{y:\overline{r-y}|}} \; {}^r(PVFB)_x\,; \tag{5.8b}$$

$$^{CP\,r}(AL)_x = \frac{{}^s\ddot{a}^T_{y:\overline{x-y}|}}{{}^s\ddot{a}^T_{y:\overline{r-y}|}} B_r \; {}_{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r \tag{5.9a}$$

$$= \frac{{}^s\ddot{a}^T_{y:\overline{x-y}|}}{{}^s\ddot{a}^T_{y:\overline{r-y}|}} \; {}^r(PVFB)_x\,. \tag{5.9b}$$

In each case, the projected benefit is prorated by a ratio of temporary employment-based annuities, with the ratio increasing to unity by retirement.[3] The annuities were defined previously by (3.22) and (3.23), and the ratios represent the value of k in (5.4b) for each method.

If salary is a non-decreasing function of x, the following inequalities hold, indicating the relative size of the actuarial liabilities under each of the five cost methods described above:

$$0 \le \frac{B_x}{B_r} \le \frac{S_x}{S_r} \le \frac{x-y}{r-y} \le \frac{{}^s\ddot{a}^T_{y:\overline{x-y}|}}{{}^s\ddot{a}^T_{y:\overline{r-y}|}} \le \frac{\ddot{a}^T_{y:\overline{x-y}|}}{\ddot{a}^T_{y:\overline{r-y}|}} \le 1\,. \tag{5.10}$$

In theory there need not be any restriction on the value of k in (5.4b), provided that it is zero at age y and attains unity at age r. Consequently, an infinite number of actuarial liabilities exist, only a few of which are formally recognized.

Table 5–2 shows the various actuarial liabilities for an age-30 entrant under the model actuarial assumptions, all expressed as a percent of the age-65 values (values that is identical among the methods). Note that the actuarial liabilities are relatively small throughout most of the employee's career as compared to the age-65 value. These values indicate that a substantial portion of a plan's actuarial liability may be associated with individuals in or near retirement, even though the number of such individuals may be relatively small. Figure 5–2 shows a graph of the attained age pattern of the actuarial liabilities during an employee's career.

Figure 5–3 shows the various actuarial liabilities for the model pension plan, using the population in the 25th year of the simulation shown in Table 4–7. The values are all expressed as a percent of the present value of future benefits. These liabilities are determined for the entire plan membership, and they include the liabilities associated with ancillary benefits, a subject yet to be discussed. The liability associated with retired employees is constant among the various methods; however, the liability for active employees differs substantially. Since cost methods are designed to have plan assets accumulate to their respective actuarial liabilities, it is clear that different cost methods can have a significant effect on the ultimate level of plan assets. This, in turn, suggests that plan contributions during the early stages of

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[3]At this point it is premature to provide the logic as to why these two actuarial liabilities can be defined by prorating the projected benefit by the annuity ratios, a subject taken up in Chapter 6.

funding will be higher for those methods with larger actuarial liabilities; however, such contributions eventually will be lower because of the investment returns associated with a larger asset base. These relationships will be explored more fully in later chapters.

TABLE 5–2

Actuarial Liability and $^{r}(PVFB)_x$ Functions as a Percentage of the Age-65 Value

	Accrued	Benefit Prorate Methods		Cost Prorate Methods		
	Benefit	Constant	Constant	Constant	Constant	
Age	Method	Percent	Dollar	Percent	Dollar	$^{r}(PVFB)_x$
30	0.00	0.00	0.00	0.00	0.00	0.95
31	0.00	0.01	0.04	0.11	0.20	1.24
32	0.01	0.02	0.09	0.25	0.45	1.55
33	0.02	0.04	0.16	0.43	0.75	1.90
34	0.03	0.07	0.26	0.64	1.08	2.27
35	0.05	0.11	0.38	0.89	1.46	2.68
36	0.08	0.16	0.53	1.18	1.88	3.12
37	0.12	0.22	0.72	1.52	2.36	3.61
38	0.16	0.30	0.95	1.92	2.90	4.16
39	0.23	0.41	1.23	2.39	3.51	4.78
40	0.32	0.54	1.56	2.93	4.19	5.47
41	0.43	0.71	1.96	3.55	4.96	6.24
42	0.57	0.92	2.44	4.27	5.83	7.10
43	0.76	1.19	3.00	5.10	6.80	8.07
44	1.00	1.52	3.67	6.04	7.90	9.16
45	1.30	1.93	4.45	7.13	9.13	10.39
46	1.69	2.43	5.38	8.37	10.52	11.77
47	2.17	3.05	6.47	9.80	12.09	13.32
48	2.79	3.82	7.76	11.43	13.86	15.08
49	3.57	4.77	9.27	13.30	15.86	17.07
50	4.55	5.94	11.04	15.45	18.14	19.32
51	5.78	7.37	13.13	17.90	20.71	21.88
52	7.32	9.13	15.58	20.72	23.64	24.78
53	9.24	11.29	18.46	23.95	26.96	28.08
54	11.64	13.93	21.83	27.65	30.74	31.83
55	14.63	17.16	25.77	31.87	35.02	36.08
56	17.71	20.37	29.34	35.44	38.51	39.50
57	21.40	24.18	33.40	39.42	42.38	43.30
58	25.84	28.68	38.01	43.88	46.69	47.52
59	31.18	34.03	43.28	48.90	51.50	52.24
60	37.64	40.41	49.34	54.57	56.92	57.56
61	45.46	48.07	56.35	61.04	63.08	63.62
62	55.03	57.32	64.54	68.50	70.16	70.59
63	66.81	68.60	74.22	77.22	78.43	78.72
64	81.48	82.54	85.84	87.55	88.21	88.36
65	100.00	100.00	100.00	100.00	100.00	100.00

FIGURE 5–2
Actuarial Liabilities as a Percentage of the Age-65 Values

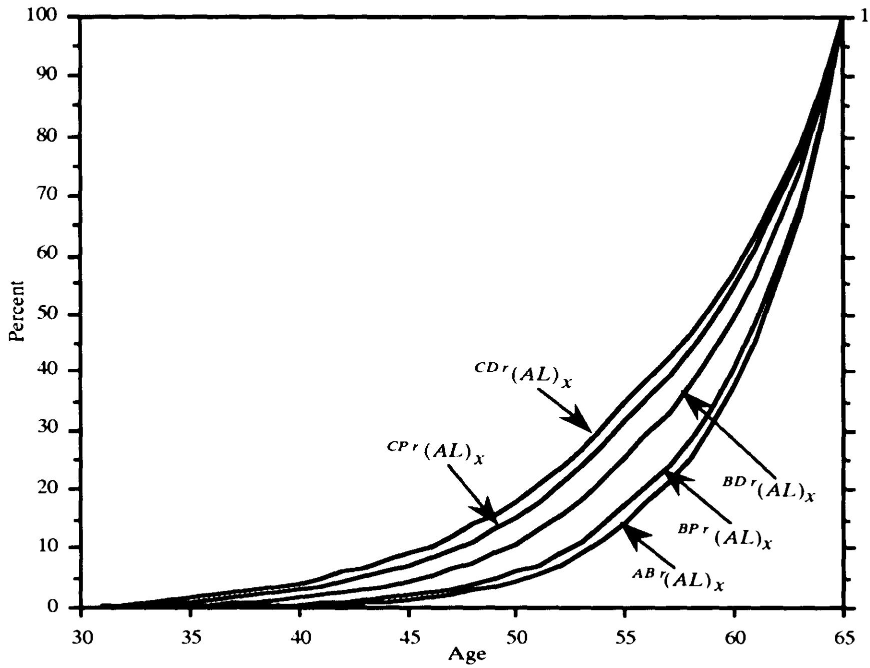
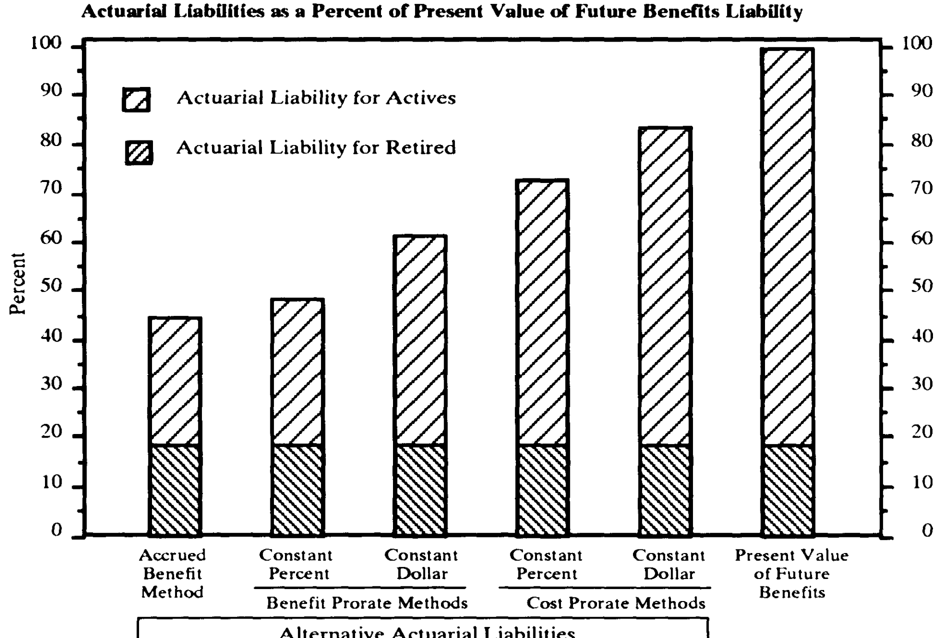


FIGURE 5–3
Actuarial Liabilities as a Percent of Present Value of Future Benefits Liability



Chapter 6

Normal Costs

Pension costs can be categorized into two fundamental types: *normal costs* and *supplemental costs.* Normal costs represent the annual cost attributed to the current year of service rendered by active participants, with such costs being defined by one of several actuarial cost methods. In theory, the actuarial accumulation of normal costs from entry age to retirement age will be equal to the liability for the employee's pension benefit at retirement (the retirement-date actuarial liability). The experience of the plan, however, will not precisely match the underlying actuarial assumptions. Moreover, the plan may have granted benefit credits to years prior to its formation (i.e., for periods when normal costs were not calculated), and/or benefit changes or actuarial assumption changes may occur from time to time. Hence, actual normal costs will not accumulate to the retirement-date liability. Supplemental costs are designed to resolve the difference between the theoretical and actual accumulation of normal costs, again according to a specified methodology.

This chapter defines and discusses normal costs, while Chapter 7 deals with supplemental costs. Once both of these cost functions are defined, the minimum and maximum tax deductible contribution limits imposed by federal statutes and the accounting expense required by FASB can be set forth.

Normal costs can be determined on a participant by participant basis, with the plan's overall costs equal to the sum of each individual's costs, or they can be determined by a nearly equivalent calculation involving an aggregation of plan participants. These two methodologies suggest another type of classification for actuarial cost methods, namely, *individual* versus *aggregate.* The term "aggregate" will be reserved herein to refer to this type of aggregation, as opposed to the aggregation of normal and supplemental costs, which is sometimes used in the pension literature and in practice. Moreover, unless the term "aggregate" is used, it is to be understood that the discussion pertains to an "individual" normal cost method.

The discussion begins with a generalized normal cost function, followed by specific definitions. At this point only the normal cost associated with retirement benefits (based on retirement at age r) is considered. The normal cost associated with ancillary benefits is given in Chapter 8.

GENERALIZED NORMAL COST FUNCTION

The retirement-benefit normal cost (NC) for an employee aged x can be represented by the following generalized function:

$$^{r}(NC)_x = b_x' \; {}_{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r . \qquad (y \leq x < r) \qquad (6.1)$$

The prescript on the $^{r}(NC)_x$ notation indicates that only retirement benefits are considered. Any normal cost can be specified by the appropriate definition of b_x', as described more fully in this chapter.[1] Before defining the normal cost under various actuarial cost methods, we will examine some fundamental normal cost relationships.

In general, normal costs are designed to amortize $^{r}(PVFB)_y$ over the employee's working lifetime, the pattern of amortization payments being governed by the particular actuarial cost method. Thus, the present value of a participant's future normal costs at age y is equal to $^{r}(PVFB)_y$. Notationally, this relationship may be expressed as follows, assuming that normal costs are made at the beginning of each age from the employee's entry age y to one year prior to retirement age r:

$$^{r}(PVFB)_y = {}^{r}(PVFNC)_y = \sum_{t=y}^{r-1} {}^{r}(NC)_t \; {}_{t-y}p_y^{(T)} \; v^{t-y} . \qquad (6.2a)$$

This equation of equality can be demonstrated by writing the left side of (6.2a) in its basic form and substituting (6.1) for $^{r}(NC)_t$:

[1] Normal costs are defined for ages $y \leq x < r$. This age range, however, will not be repeated for each normal cost equation presented.

$$B_r \; {}_{r-y}p_y^{(T)} \; v^{r-y} \; \ddot{a}_r = \sum_{t=y}^{r-1} \left[b_x' \; {}_{r-t}p_t^{(T)} \; v^{r-t} \; \ddot{a}_r\right] {}_{t-y}p_y^{(T)} \; v^{t-y}. \quad (6.2b)$$

The product of ${}_{r-t}p_t^{(T)}$ and ${}_{t-y}p_y^{(T)}$ is ${}_{r-y}p_y^{(T)}$ and the product of v^{r-t} and v^{t-y} is v^{r-y}. Thus, (6.2b) reduces to the defined relationship:

$$B_r = \sum_{t=y}^{r-1} b_x'. \quad (6.2c)$$

This relationship is applicable for the normal costs under all actuarial cost methods, and illustrates that normal costs do indeed amortize ${}^r(PVFB)_y$ over the period from age y to age r.

Continuing with the amortization concept, it also follows that the actuarial liability at age x is equal to the present value of future benefits (PVFB) at that age less the present value of future normal costs (PVFNC) yet to be made (i.e., the portion of ${}^r(PVFB)_x$ not yet amortized):

$${}^r(AL)_x = {}^r(PVFB)_x - {}^r(PVFNC)_x. \quad (6.3a)$$

Demonstrating that this relationship holds, we write ${}^r(PVFB)_x$ and ${}^r(PVFNC)_x$ in their basic form, as follows:

$$\begin{aligned} {}^r(AL)_x &= B_r \; {}_{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r \\ &\quad - \sum_{t=x}^{r-1} \left[b_x' \; {}_{r-t}p_t^{(T)} \; v^{r-t} \; \ddot{a}_r\right] {}_{t-x}p_x^{(T)} \; v^{t-x}. \end{aligned} \quad (6.3b)$$

The bracketed term in equation (6.3b) represents the normal cost function. The right side of (6.3b) can be written as

$$= B_r \; {}_{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r - \left(\sum_{t=x}^{r-1} b_x'\right) {}_{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r, \quad (6.3c)$$

and since

$$B_r - \sum_{t=x}^{r-1} b_x' = B_x', \quad (6.3d)$$

equation (6.3b) simplifies to

$${}^r(AL)_x = B_x' \; {}_{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r. \quad (6.3e)$$

Equation (6.3e) is the actuarial liability definition given by (5.3); hence, the relationship defined by (6.3a) is valid.

Another definition of the actuarial liability in terms of normal costs is the so-called retrospective approach, in contrast to the prospective approach studied above. Under this definition the actuarial liability is equal to the accumulated value of past normal costs (AVPNC):[2]

$$^{r}(AL)_x = {}^{r}(AVPNC)_x. \tag{6.4a}$$

The right side of this equation can be expressed as

$$^{r}(AL)_x = \sum_{t=y}^{x-1} {}^{r}(NC)_t \; (1 + i)^{x-t} \frac{1}{_{x-t}p_t^{(T)}} \tag{6.4b}$$

and, substituting the normal cost definition given in (6.1), we have

$$^{r}(AL)_x = \sum_{t=y}^{x-1} \left[b_t' \; _{r-t}p_t^{(T)} \; v^{r-t} \; \ddot{a}_r\right] (1 + i)^{x-t} \frac{1}{_{x-t}p_t^{(T)}} \tag{6.4c}$$

which reduces to

$$^{r}(AL)_x = \left(\sum_{t=y}^{x-1} b_t'\right) {}_{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r \tag{6.4d}$$

$$= B_x' \; _{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r. \tag{6.4e}$$

Equation (6.4e) is equal to the prospective actuarial liability definition given by (5.3); hence, the actuarial accumulation of past normal costs at age x is equal to the actuarial liability at that age.

The prior normal costs in equation (6.4b) increase to the current age by two factors: the benefit of interest and what is referred to as the "benefit of survivorship." The benefit of interest is a straightforward concept, but the benefit of survivorship may not be at first glance. In order to explore the latter, equation

[2]In actual fact, past normal costs may not accumulate to the value of $^{r}(PVFB)_x$ because of the granting of past service credits for years prior to the plan establishment, plan changes, actuarial assumption changes, or experience deviation from actuarial assumptions.

(6.4b) is written with ${}_{r-x}p_x^{(T)}$ replaced by its corresponding service table components:[3]

$$ {}^{r}(AL)_x = \sum_{t=y}^{x-1} {}^{r}(NC)_t \, (1 + i)^{x-t} \, \frac{l_t^{(T)}}{l_x^{(T)}}. \tag{6.5} $$

In this form we see that a normal cost is generated on behalf of all of the hypothetical employees at each age t, yet the total accumulation at age x is allocated to the lesser number of those who survive in service to this age, hence, the term benefit of survivorship in service. Although this effect is called the "benefit of survivorship," the normal cost is determined with full recognition of this gain from non-survivors.

In theory, normal costs can take on any positive or negative value during an employee's working lifetime. The only theoretical restriction on age-specific normal cost values is that their present value (or accumulated value) satisfy the above relationships. Since an infinite number of normal cost patterns could be determined such that these conditions hold, there exists an infinite number of possible actuarial cost methods, only a few of which are formally recognized and discussed in this book.

A more general retrospective definition of the actuarial liability, appropriate both before and after retirement, is given by

$$ {}^{r}(AL)_x = \sum_{t=y}^{x-1} [{}^{r}(NC)_t - B_t] \, (1 + i)^{x-t} \, \frac{1}{{}_{x-t}p_t^{(T)}}. \tag{6.6} $$

The annual pension benefit, B_t, in this formulation is zero prior to retirement and equal to B_r after retirement. Conversely, ${}^{r}(NC)_x$ would be positive prior to retirement and zero after retirement.

NORMAL COST UNDER ACTUARIAL COST METHODS

Each actuarial cost method discussed in Chapter 5 has a corresponding normal cost function. In effect, the normal cost represents the growth in the actuarial liability from one year to the next, reflecting a larger accrued benefit along with interest and

[3] $l_x^{(T)}$ is the number of survivors out of an arbitrary number of employees beginning at some entry age y and who are exposed to the various decrements from y to x. See Chapter 3, Table 3–2, for an example of the $l_x^{(T)}$ function.

survivorship adjustments. As indicated by the general normal cost function given in (6.1), the normal cost represents the present value of the current year's benefit accrual. Normal costs differ among actuarial cost methods by the benefit deemed to accrue at each attained age.

Accrued Benefit Method

The *individual* normal cost under the accrued benefit method is defined by equation (6.1) with b_x determined by the natural accruals that result from applying the plan's benefit formula to the employee's current service and, if applicable, current salary:

$$^{AB\,r}(NC)_x = b_x \ {}_{r-x}p_x^{(T)} \ v^{r-x} \ \ddot{a}_r. \tag{6.7a}$$

The normal cost for the entire plan is equal to the sum of the normal costs for each participant.

The *aggregate* accrued benefit (AAB) normal cost for the entire active plan membership is defined as:

$$^{AAB\,r}(NC) = \left(\sum l_{x,y} \ b_{x,y}\right) \left[\frac{\sum l_{x,y} \ ^{r}(PVFB)_{x,y}}{\sum l_{x,y} \ B_{r,y}}\right] \tag{6.7b}$$

where

$\sum$ = summation over all entry age, attained age combinations ($y < r$; $y \leq x < r$)

$l_{x,y}$ = number of age-y entrants currently age x

$b_{x,y}$ = benefit accrual at age x for an age-y entrant

$B_{r,y}$ = accrued benefit at age r for an age-y entrant

$^{r}(PVFB)_{x,y}$ = present value of future benefits at age x for an age-y entrant.

If there is only one active employee, then (6.7b) simplifies to the normal cost under the individual accrued benefit method given by (6.7a).

The individual normal cost version is used in practice and recognized by federal statutes as a valid method for use with benefit formulas other than the final average salary type. The aggregate version is of only theoretical interest, yet the formulation establishes a methodology for aggregate methods in general, namely, that the numerator and denominator of such methods are weighted by the number of participants at each x,y combination, with the result then multiplied by the total number of plan participants (or compensation). The numerical results of the individual and aggregate methodologies, while not identical, are typically quite similar.

Benefit Prorate Methods

Specifying b_x in (6.1) as previously defined in either (3.15a) or (3.16a) defines the normal cost under two benefit prorate methods. The first definition represents a service proration of B_r which produces a constant dollar benefit accrual, while the second represents a salary proration which produces a benefit accrual equal to a constant percent of salary. The equation for each is given below, with the prescripts indicating the type of cost method and type of proration, respectively:

$$^{BD\,r}(NC)_x = \frac{B_r}{r-y}\ {}_{r-x}p_x^{(T)}\ v^{r-x}\ \ddot{a}_r; \tag{6.8}$$

$$^{BP\,r}(NC)_x = \frac{B_r}{S_r}\ s_x\ {}_{r-x}p_x^{(T)}\ v^{r-x}\ \ddot{a}_r. \tag{6.9}$$

The actuarial liability associated with these normal costs are given by equations (5.6a) and (5.7a), respectively.

Both of these individual methods have aggregate counterparts which, again, are only of theoretical interest inasmuch as they are seldom, if ever, used in practice. First, equations (6.8) and (6.9) are rewritten in an equivalent form using the $^r(PVFB)_x$ function:

$$^{BD\,r}(NC)_x = \frac{^r(PVFB)_x}{r-y}; \tag{6.10}$$

$$^{BP\,r}(NC)_x = s_x\ \frac{^r(PVFB)_x}{S_r}. \tag{6.11}$$

The aggregate versions, which involve determining the numerator and denominator for the entire group of active employees, is given by the following two equations:

$$^{ABD\,r}(NC) = \left(\sum l_{x,y}\right)\left[\frac{\sum l_{x,y}\ {}^r(PVFB)_{x,y}}{\sum l_{x,y}\ (r-y)}\right]; \tag{6.12}$$

$$^{ABP}\,{}^{r}(NC) = \left(\sum l_{x,y}\; s_{x,y}\right)\left[\frac{\sum l_{x,y}\;{}^{r}(PVFB)_{x,y}}{\sum l_{x,y}\; S_{r,y}}\right] \tag{6.13}$$

where

$s_{x,y}$ = salary at age x for an age-y entrant

$S_{r,y}$ = cumulative salary from age y to r.

Cost Prorate Methods

The traditional terminology for this family of normal costs is the *projected benefit cost method* or *entry age cost method.* As we will see, the term "cost prorate" is more precise in describing the methodology used with these methods.

The previously described normal cost methods make the *benefit accrual* associated with the employee the independent variable and the corresponding normal cost the dependent variable. In contrast, the cost prorate methods designate the *normal cost* as the independent variable and the corresponding benefit accrual, which can be derived from the normal cost function, becomes the dependent variable. The switching of the independent and dependent variables has significant financial implications.

There are two forms of this method: one with normal costs equal to a constant dollar amount throughout the employee's working lifetime, and the other with costs equal to a constant percentage of the employee's salary, hence, the term *cost prorate method.* The first form is typically used with plans where benefits are not based on salary, while the second is used with career average and final average benefit formulas.

The "cost prorate, constant dollar" normal cost is defined by first writing the fundamental identity that future normal costs at age y must equal the present value of future benefits at that age, and then solving for the constant dollar normal costs.

$$^{CD}\,{}^{r}(NC)_y\; \ddot{a}^{T}_{y:\overline{r-y}|} = {}^{r}(PVFB)_y\,; \tag{6.14a}$$

$$^{CD}\,{}^{r}(NC)_y = \frac{{}^{r}(PVFB)_y}{\ddot{a}^{T}_{y:\overline{r-y}|}}. \tag{6.14b}$$

The temporary annuity was defined previously by (3.22). The normal cost at age y, by definition, is applicable to all attained ages under this method. Equation (6.14b) also illustrates, once again, that the present value of future benefits is amortized by normal costs.

The "cost prorate, constant percent" normal cost can be determined first by equating the present value of a portion, K, of the participant's future salary to the present value of future benefits:

$$K\ s_y\ {}^s\ddot{a}^T_{y:\overline{r-y}|} = {}^r(PVFB)_y\,; \tag{6.15a}$$

$$K = \frac{{}^r(PVFB)_y}{s_y\ {}^s\ddot{a}^T_{y:\overline{r-y}|}}. \tag{6.15b}$$

Then, the normal cost at age x is simply this factor times attained age salary:

$$^{CP\,r}(NC)_x = K\ s_x\,. \tag{6.15c}$$

If salary is an increasing function of age, the normal cost under this version represent an ever-increasing dollar amount.

The actuarial liabilities associated with these two methods were given previously by equations (5.8a) and (5.9a), respectively. The more conventional expressions for these actuarial liabilities, however, are the prospective definitions:

$$^{CD\,r}(AL)_x = {}^r(PVFB)_x - {}^{CD\,r}(NC)_x\ \ddot{a}^T_{x:\overline{r-x}|}\,; \tag{6.16}$$

$$^{CP\,r}(AL)_x = {}^r(PVFB)_x - {}^{CP\,r}(NC)_x\ {}^s\ddot{a}^T_{x:\overline{r-x}|}\,. \tag{6.17}$$

The equality of these expressions with those of (5.8b) and (5.9b) is as follows, using (6.16) to illustrate the identity. First, the normal cost symbol in (6.16) is replaced by (6.14b):

$$^{CD\,r}(AL)_x = {}^r(PVFB)_x - \frac{{}^r(PVFB)_y}{\ddot{a}^T_{y:\overline{r-y}|}}\ \ddot{a}^T_{x:\overline{r-x}|}\,. \tag{6.18a}$$

Replacing ${}^r(PVFB)_y$ by ${}_{x-y}p_y^{(T)}\ v^{x-y}\ {}^r(PVFB)_x$ and factoring out ${}^r(PVFB)_x$, one obtains

$$^{CD\,r}(AL)_x = {}^r(PVFB)_x \left[1 - \frac{{}_{x-y}p_y^{(T)}\ v^{x-y}\ \ddot{a}^T_{x:\overline{r-x}|}}{\ddot{a}^T_{y:\overline{r-y}|}}\right]. \tag{6.18b}$$

With a common denominator, (6.18b) becomes

$$^{CD\,r}(AL)_x = {}^r(PVFB)_x \left[\frac{\ddot{a}^T_{y:\overline{r-y}|} - {}_{x-y}p_y^{(T)}\, v^{x-y}\, \ddot{a}^T_{x:\overline{r-x}|}}{\ddot{a}^T_{y:\overline{r-y}|}} \right]. \tag{6.18c}$$

The numerator represents a temporary employment-based annuity running from age y to age x, i.e., a temporary annuity from y to r minus a deferred temporary annuity payable from x to r. Thus, equation (6.18c) simplifies to (5.8b). The same result can be obtained for the constant-percent-of-salary version.

The above formulations of the cost prorate methods do not produce a benefit accrual factor, b_x, that can be used in the generalized normal cost equation (6.1). Such a factor can be derived, however, by replacing the normal cost notation in (6.1) with the cost prorate normal cost and solving for b_x. This is illustrated using the cost prorate, constant dollar normal cost:

$$\frac{^r(PVFB)_y}{\ddot{a}^T_{y:\overline{r-y}|}} = b_x\, {}_{r-x}p_x^{(T)}\, v^{r-x}\, \ddot{a}_r; \tag{6.19a}$$

$$b_x = \frac{B_r\, {}_{x-y}p_y^{(T)}\, v^{x-y}}{\ddot{a}^T_{y:\overline{r-y}|}}. \tag{6.19b}$$

The second and third factors of the numerator decrease significantly with age, implying that the portion of B_r allocated to each attained age is a sharply decreasing function. This is in contrast to the portion of B_r allocated under the other methods discussed previously, where B_r is increasing or constant. The corresponding benefit accrual for the cost prorate, constant percent method is

$$b_x = \frac{s_x\, B_r\, {}_{x-y}p_y^{(T)}\, v^{x-y}}{s_y\, {}^s\ddot{a}^T_{y:\overline{r-y}|}}. \tag{6.19c}$$

The *aggregate* versions of these two normal cost methods can be written in the following manner:

$$^{ACD\,r}(NC) = \left(\sum l_{x,y}\right) \left[\frac{\sum l_{x,y}\, {}^r(PVFB)_y}{\sum l_{x,y}\, \ddot{a}^T_{y:\overline{r-y}|}} \right]; \tag{6.20a}$$

$$^{ACP\,r}(NC) = \left(\sum l_{x,y}\, s_{x,y}\right) \left[\frac{\sum l_{x,y}\, {}^r(PVFB)_y}{\sum l_{x,y}\, s_y\, {}^s\ddot{a}^T_{y:\overline{r-y}|}} \right]. \tag{6.20b}$$

Summary of Normal Costs Under Actuarial Cost Methods

The *individual* normal costs defined thus far can all be expressed as a fraction of the ${}^{r}(PVFB)_x$ function, much the same way their corresponding actuarial liabilities were expressed:

$$ {}^{r}(NC)_x = k \; {}^{r}(PVFB)_x \tag{6.21} $$

where k is defined as follows:

k	*Actuarial Cost Method*
$\dfrac{b_x}{B_r}$	Accrued Benefit Method
$\dfrac{s_x}{S_r}$	Benefit Prorate Constant Percent Method
$\dfrac{1}{r-y}$	Benefit Prorate Constant Dollar Method
$\dfrac{s_x \; {}_{x-y}p_y^{(T)} \; v^{x-y}}{s_y \; {}^{s}\ddot{a}^{T}_{y:\overline{r-y}\vert}}$	Cost Prorate Constant Percent Method
$\dfrac{{}_{x-y}p_y^{(T)} \; v^{x-y}}{\ddot{a}^{T}_{y:\overline{r-y}\vert}}$	Cost Prorate Constant Dollar Method

In theory there need not be any restriction on the value of k in (6.21), provided that the sum of such fractions, from entry age y to one year prior to retirement r, equal unity. Thus, an infinite number of actuarial cost methods exist, since this condition can be met by an infinite number of attained age patterns of k.

It is interesting to consider two extreme cases for the value of k, where k is equal to zero at all ages except one, at which age it takes on the value of unity. If the single age is the employee's entry-age y, the entire projected benefit is accounted for at that point. The normal cost, of course, would equal ${}^{r}(PVFB)_y$ and the actuarial liability at each age thereafter would equal ${}^{r}(PVFB)_x$. This method is known as *initial funding* in the context of pension plan funding.

If the single age for which k is equal to unity is the participant's retirement age, then the normal cost is equal to ${}^{r}(PVFB)_r$ for this single age. The actuarial liability is zero up to this age, and equals ${}^r(PVFB)_x$ thereafter (for $x > r$). In the context of pension plan funding, this method is known as *terminal funding*.

Table 6–1 shows the various normal costs for an age-30 entrant under the model actuarial assumptions, all expressed as a percent of attained age salary. These values are plotted in Figure 6–1. The methods produce normal cost values that are relatively dispersed near the employee's entry age, reasonably close midway through the employee's career, and substantially different as the employee approaches retirement. Clearly, those methods with the lowest initial costs have the highest costs near retirement, and vice versa.

The cost patterns under the accrued benefit cost method and the benefit prorate (constant percent) method might appear to be undesirable inasmuch as they increase sharply throughout the employee's career. However, a large plan census with a relatively stable age and service distribution will produce a reasonably constant normal cost percentage for the entire plan under all actuarial cost methods. In this case, the normal cost percentages will differ among the methods depending on the average age and service of plan participants. The benefit prorate methods will produce the lowest costs for a relatively undermature active employee population and vice versa for a relatively overmature population. These dynamics will be displayed at a later point in this book.

The percentage of the age-30 entrant's projected retirement benefit allocated to each age under each actuarial cost method and the model assumptions is given in Table 6–2 and graphed in Figure 6–2. Table 6–3 shows the cumulative percentages given in Table 6–2, with these values being graphed in Figure 6–3. The cost prorate methods have cumulative benefit allocations that are far greater than those of the benefit prorate methods. For example, one-half of the projected benefit is allocated by age 35 under the constant dollar version and by age 40 under the constant percent version. This is in sharp contrast to the accrued benefit method which allocates one-half of the projected benefit by age 57.

Figure 6–4 shows the various normal costs for the model pension plan and the mature population, all expressed as a percent of payroll. These normal cost values include the cost associated with ancillary benefits, a subject yet to be discussed. The normal

TABLE 6–1

Normal Cost Functions as a Percent of Attained Age Salary

Age	*Accrued Benefit Method*	*Benefit Prorate Methods*		*Cost Prorate Methods*	
		Constant Percent	*Constant Dollar*	*Constant Percent*	*Constant Dollar*
30	0.12	0.25	1.05	3.24	6.08
32	0.19	0.41	1.46	3.24	5.15
34	0.32	0.59	1.82	3.24	4.38
36	0.50	0.81	2.13	3.24	3.74
38	0.73	1.09	2.44	3.24	3.21
40	1.04	1.43	2.76	3.24	2.76
42	1.45	1.86	3.10	3.24	2.39
44	1.99	2.40	3.46	3.24	2.07
46	2.71	3.08	3.87	3.24	1.81
48	3.65	3.94	4.34	3.24	1.58
50	4.89	5.05	4.88	3.24	1.39
52	6.54	6.48	5.52	3.24	1.22
54	8.70	8.32	6.27	3.24	1.08
56	11.14	10.33	6.91	3.24	0.96
58	13.79	12.42	7.41	3.24	0.86
60	17.12	15.05	8.04	3.24	0.77
62	21.44	18.45	8.86	3.24	0.69
64	27.31	23.10	10.02	3.24	0.62

FIGURE 6–1

Normal Costs as a Percent of Salary Under Various Actuarial Cost Methods

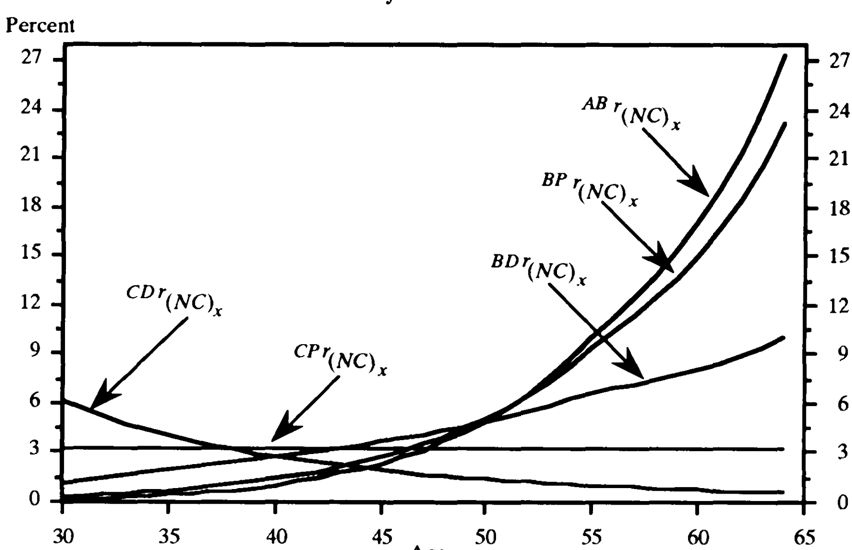


TABLE 6–2

Percentage of Projected Retirement Benefit Allocated to Each Age Under Various Actuarial Cost Methods

Age	Accrued Benefit Method	Benefit Prorate Methods: Constant Percent	Benefit Prorate Methods: Constant Dollar	Cost Prorate Methods: Constant Percent	Cost Prorate Methods: Constant Dollar
30	0.32	0.67	2.86	8.78	16.48
32	0.37	0.79	2.86	6.34	10.09
34	0.50	0.93	2.86	5.09	6.89
36	0.67	1.09	2.86	4.35	5.03
38	0.85	1.28	2.86	3.80	3.76
40	1.08	1.48	2.86	3.36	2.86
42	1.34	1.71	2.86	2.99	2.21
44	1.64	1.98	2.86	2.67	1.71
46	2.00	2.27	2.86	2.39	1.33
48	2.40	2.60	2.86	2.13	1.04
50	2.86	2.96	2.86	1.90	0.81
52	3.38	3.35	2.86	1.68	0.63
54	3.96	3.79	2.86	1.48	0.49
56	4.61	4.27	2.86	1.34	0.40
58	5.31	4.79	2.86	1.25	0.33
60	6.08	5.35	2.86	1.15	0.27
62	6.91	5.95	2.86	1.04	0.22
64	7.79	6.59	2.86	0.92	0.18

FIGURE 6–2

Percentage of Projected Retirement Benefit Allocated to Each Age Under Various Actuarial Cost Methods

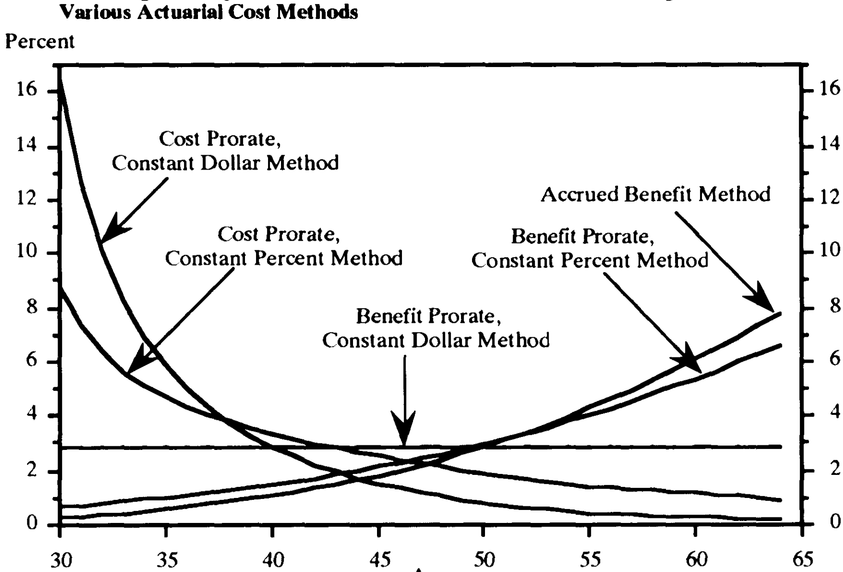


TABLE 6–3

Cumulative Percentage of Projected Retirement Benefit Allocated to Each Age Under Various Actuarial Cost Methods

	Accrued	Benefit Prorate Methods		Cost Prorate Methods	
Age	*Benefit Method*	*Constant Percent*	*Constant Dollar*	*Constant Percent*	*Constant Dollar*
30	0.00	0.00	0.00	0.00	0.00
32	0.65	1.41	5.71	16.12	29.16
34	1.45	3.06	11.43	28.08	47.50
36	2.53	5.01	17.14	37.84	60.24
38	3.95	7.28	22.86	46.24	69.60
40	5.77	9.93	28.57	53.60	76.65
42	8.05	13.01	34.29	60.13	82.02
44	10.87	16.57	40.00	65.94	86.17
46	14.33	20.66	45.71	71.14	89.39
48	18.52	25.36	51.43	75.79	91.90
50	23.55	30.73	57.14	79.93	93.85
52	29.53	36.83	62.86	83.61	95.38
54	36.58	43.76	68.57	86.86	96.57
56	44.82	51.57	74.29	89.72	97.50
58	54.38	60.36	80.00	92.35	98.25
60	65.38	70.21	85.71	94.80	98.88
62	77.96	81.20	91.43	97.05	99.40
64	92.21	93.41	97.14	99.08	99.82
65	100.00	100.00	100.00	100.00	100.00

FIGURE 6–3

Cumulative Percentage of Projected Retirement Benefit Allocated to Each Age Under Various Actuarial Cost Methods

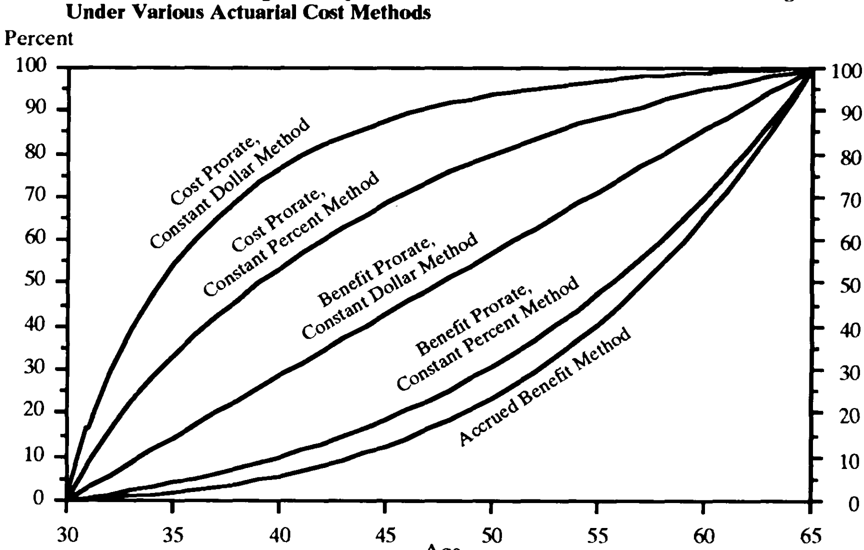
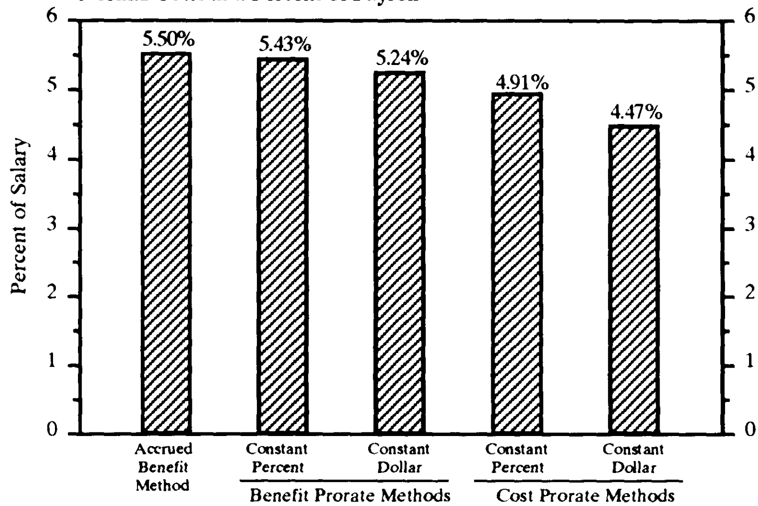


FIGURE 6–4
Normal Costs as a Percent of Payroll



costs for each method are not substantially different; the accrued benefit cost method has a 5.5% normal cost, while at the other extreme, the cost prorate, constant dollar method has a 4.5% normal cost. These differences, however, become more significant when compared for undermature and overmature plans.

PLAN TERMINATION COST METHOD

This chapter has defined the normal cost for each of the actuarial liabilities set forth in Chapter 5. In addition, the point was made that there exists an infinite number of actuarial liabilities and corresponding normal costs. Therefore, it is logical to ask what the so-called "normal cost" would be for the plan continuation and plan termination liabilities presented in Chapter 5. Since the plan continuation liability for accrued benefits is mathematically identical to the actuarial liability under the accrued benefit cost method, it stands to reason that they have identical normal costs.

The normal cost for the plan termination liability can be derived by examining the progression of year-to-year values.[4] The normal cost at age x is equal to the difference between the present value of the liability at age $x + 1$ less the liability at age x:

[4]The normal cost for each of the previously defined actuarial cost methods can also be defined in terms of the annual growth in their actuarial liability.

$$^{PT}(NC)_x = p_x^{(T)}\, v\, (PTL)_{x+1} - (PTL)_x. \tag{6.22a}$$

Upon substituting the components making up the PTL, equation (6.22a) becomes

$$^{PT}(NC)_x = p_x^{(T)}\, v \left[B_{x+1}\ {}_{r-x-1}p_{x+1}^{(m)}\ v^{r-x-1}\, \ddot{a}_r\right] - \left[B_x\ {}_{r-x}p_x^{(m)}\, v^{r-x}\, \ddot{a}_r\right]. \tag{6.22b}$$

Equation (6.22b) reduces to

$$^{PT}(NC)_x = \left[B_{x+1}\ p_x^{(T)}\ {}_{r-x-1}p_{x+1}^{(m)} - B_x\ {}_{r-x}p_x^{(m)}\right] v^{r-x}\, \ddot{a}_r. \tag{6.22c}$$

Since $p_x^{(T)} = p_x^{(m)}\, p_x^{(w)}\, p_x^{(d)}\, p_x^{(r)}$, equation (6.22c) may be written as

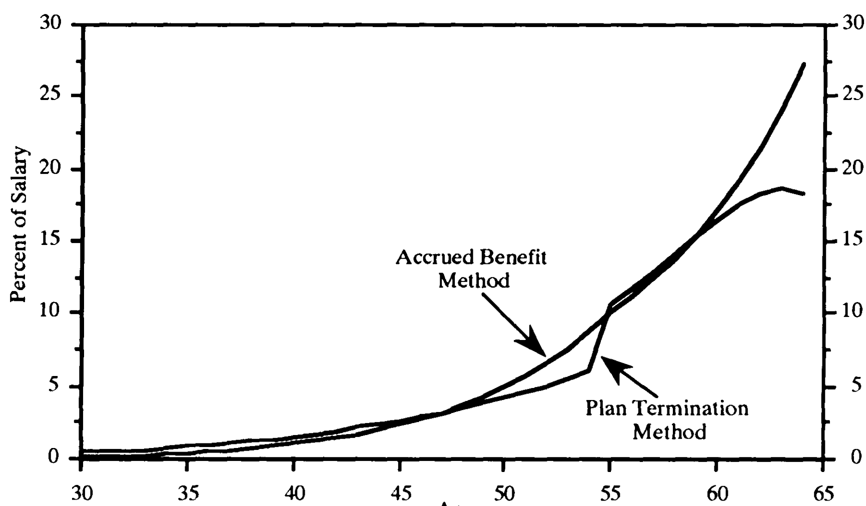
$$^{PT}(NC)_x = \left[B_{x+1}\, p_x^{(w)}\, p_x^{(d)}\, p_x^{(r)} - B_x\right] {}_{r-x}p_x^{(m)}\, v^{r-x}\, \ddot{a}_r. \tag{6.22d}$$

Although B_{x+1} exceeds B_x, it is possible for this excess to be more than offset by the product of the withdrawal, disability, and retirement rates, especially for young employees. If this were to occur, the normal cost as given by (6.22d) could take on negative values at some ages beyond age y.

Figure 6-5 compares the normal cost of the plan termination liability method to that of the accrued benefit cost method for an age-30 entrant.

FIGURE 6-5

Normal Cost Under Plan Termination Method vs. Accrued Benefit Method



Chapter 7

Supplemental Costs

Theoretically, the accumulation of past normal costs will precisely equal the cost method's actuarial liability determined prospectively (i.e., the present value of future benefits less the present value of future normal costs). By the same token, if contributions are made to the trust fund each year in the amount of the normal cost, then theoretically plan assets will also equal the actuarial liability. There are several reasons, however, why these equalities will not obtain.

- *Experience Variations:* The experience of the plan will differ from the underlying actuarial assumptions, referred to as *actuarial gains and losses.*
- *Assumption Changes:* The actuarial assumptions will be changed from time to time.
- *Benefit Changes:* The plan's benefit formula may be increased or decreased periodically by plan amendment, with the change frequently being retroactive.
- *Past Service Accruals:* The plan sponsor may have granted benefit credits to years prior to the establishment of the plan, sometimes referred to as the *past service liability.*

Additionally, plan assets may not equal the plan's actuarial liability for yet a fifth reason:

- *Contribution Variances:* The plan sponsor may contribute more or less than the normal cost under the cost method in use.

The discrepancy that develops between plan assets and the prospectively determined actuarial liability is called the plan's *unfunded liability.* Although the term "unfunded liability" is used, the discrepancy could be either positive or negative. For example, if the plan's assets exceed the actuarial liability, a surplus (i.e., negative unfunded liability) would exist.

Supplemental costs are designed to amortize the plan's unfunded liability. In this sense, they are like the plan's normal costs which are designed to amortize $^r(PVFB)_y$ from age y to r, but which may fail to do so because of one or more of the factors listed above. The failure of normal costs to amortize $^r(PVFB)_y$ creates an unfunded liability which, in turn, generates a supplemental cost.

The supplemental cost can be a one-time cost equal in value to the unfunded liability created during a given year, or it can extend over a period of years. Supplemental costs can, at least in theory, take on any pattern over any time period, or the supplemental costs can be geared to the corresponding normal cost pattern of the actuarial cost method in use.

The supplemental cost associated with experience variations and assumption changes are almost always formulated on a group basis. In other words, the unfunded liability is determined in the aggregate for all plan members and amortized thereafter either without reference to individual participants or with reference only to the current group of active participants.

The unfunded liabilities and supplemental costs associated with benefit changes, particularly past service accruals, are dealt with on an individual or group basis, depending on the actuarial cost method being used and/or the preference of the plan sponsor or actuary. The purpose of this chapter is to describe alternative supplemental cost methods. First, however, unfunded liabilities that give rise to such costs are defined.

UNFUNDED ACTUARIAL LIABILITY

Since the plan's unfunded actuarial liability is generally determined on a group basis rather than on a participant by participant basis, the notation in this section is similarly based on the entire membership. The plan's total unfunded liability (UL) at the beginning of year t, sometimes referred to as the supplemental liability, is equal to the difference between the actuarial liability and plan assets:

$$(UL)_t = (AL)_t - (Assets)_t \tag{7.1}$$

where

$(AL)_t$ = actuarial liability at beginning of year t

$(Assets)_t$ = plan assets at beginning of year t.

The unfunded liability created during year n from all sources is equal to the difference between the *actual* unfunded liability at the beginning of the next year less the *expected* unfunded liability at that point:

$$(\Delta_n UL) = (UL)_{n+1} - E[(UL)_{n+1}] \tag{7.2a}$$

where

$(\Delta_n UL)$ = unfunded liability (positive or negative) developed during year n

$(UL)_{n+1}$ = actual unfunded liability at the beginning of year $n+1$

$E[(UL)_{n+1}]$ = expected unfunded liability at year end, or the beginning of year $n + 1$ prior to any contributions.

The expected unfunded liability at year end is calculated as of the beginning of year n, under the assumption that none of the previously listed contingencies occur (i.e., experience variations, assumption changes and so forth), as discussed below.

First, $E[(UL)_{n+1}]$ in (7.2a) is replaced by its component parts, as defined by (7.1):

$$(\Delta_n UL) = (UL)_{n+1} - \{E[(AL)_{n+1}] - E[(Assets)_{n+1}]\}. \tag{7.2b}$$

The expected actuarial liability at $n + 1$ is given by

$$E[(AL)_{n+1}] = [(AL)_n + (NC)_n - B_n](1+i) \tag{7.3}$$

where $(NC)_n$ is the plan's normal cost during year n and B_n represents the benefits paid, both assumed to occur at the beginning of the year (or, equivalently, adjusted with interest to the beginning of the year). In other words, the expected actuarial liability is equal to the prior year's actuarial liability plus the normal cost less the benefits paid, all increased with interest. A liability-based unfunded liability results when the actuarial liability at $n+1$ differs from $E[(AL)_{n+1}]$.

The expected assets at $n+1$ may be written as

$$E[(Assets)_{n+1}] = [(Assets)_n + (Cont)_n - B_n](1+i) \qquad (7.4)$$

where $(Cont)_n$ represents the contributions to the plan at the beginning of year n. An asset-based unfunded liability results when assets at $n+1$ differ from $E[(Assets)_{n+1}]$.

Substituting (7.3) and (7.4) into (7.2b), then canceling and rearranging terms, we have

$$(\Delta_n UL) = (UL)_{n+1} - [(UL)_n + (NC)_n - (Cont)_n](1+i). \qquad (7.5)$$

In words, (7.5) shows that no unfunded liability will develop during year n if that year's unfunded liability, $(UL)_n$, plus the net change in the unfunded liability, $(NC)_n - (Cont)_n$, all accumulated with interest, are equal to the unfunded liability at the beginning of the next year.

Observe from (7.5) that if $(NC)_n = (Cont)_n$, and if $(\Delta_n UL)$ is zero for the year, then $(UL)_{n+1} = (UL)_n(1+i)$. In this case contributions equal to the normal cost *plus* the present value of the interest on the unfunded liability will keep the unfunded liability constant from year to year, that is,

$$(Cont)_n = (NC)_n + v\,i\,(UL)_n \qquad (7.6a)$$

$$= (NC)_n + d\,(UL)_n \qquad (7.6b)$$

where $d = vi$, the rate of discount. Generally, however, the plan sponsor will want to reduce the unfunded liability through the payment of supplemental costs.

The incremental unfunded liability created during year n can be categorized into the five factors listed previously. Experience variations can be determined by (7.5), if we assume that none of the other four factors occur. Moreover, it is possible to further allocate experience variations according to each actuarial assumption used in the valuation process.[1] The unfunded liability attributed to assumption changes and/or benefit changes can be determined by simply calculating the actuarial liability at time n with the prior set of assumptions and/or benefits and noting the

[1] Allocating experience variations by source is beyond the scope of this book. It should be noted, however, that the variation attributed to one assumption may affect the variation attributed to another. In other words, the allocation process may not be unique.

difference. Past service accruals, likewise, are generally recorded at the time such accruals are granted. Contribution variances, of course, can be determined by a simple comparison. As we will see, it is necessary to amortize each year's change in the unfunded liability in order to satisfy the FASB accounting requirements and, under most actuarial cost methods, to determine the ERISA minimum required and maximum tax deductible contribution limitations.

Unless steps are taken to amortize the unfunded liabilities that are created each year, they will grow with interest. Thus, the nth unfunded liability change, equal to the unamortized portion of the unfunded liability created during some prior year, has the following value from year t to year $t + 1$:

$$(\Delta_n UL)_{t+1} = (\Delta_n UL)_t (1 + i) \,. \tag{7.7}$$

Supplemental costs, designed to amortize the year-to-year unfunded liability changes, can be either explicit or implicit and, for the latter, either individual-based or group-based. All such variations are defined in this section.

EXPLICIT SUPPLEMENTAL COSTS

If a given unfunded liability change is amortized by a method that bears no relationship to the actuarial cost method or the plan participants, then it is known as an *explicit supplemental cost method.* Explicit costs can be used with all actuarial cost methods studied in Chapter 6, and they are probably the most widely used of all supplemental cost methods. As noted above, each year's unfunded liability change grows with interest, much like any other debt. Thus, it is necessary to pay interest on the unfunded liability plus a portion of the outstanding balance if the quantity it to be amortized. Since contributions are assumed herein to be paid in advance, the annual interest is equal to d, the rate of discount defined previously, times the unfunded liability balance remaining after the contribution. In theory, any portion of the unfunded liability could be paid in the current year; however, the typical procedure is to amortize the increment over a finite period according to a specified pattern.

The so-called *straight line method* calls for annual payments equal to the interest on the outstanding balance plus $1/m$ of the original debt (assuming an m-year amortization period). Thus, the beginning-of-year jth supplemental cost payment ($1 \leq j \leq m$) associated with the nth unfunded liability is as follows:

$$(SC_n)_j = d\left[(\Delta_n ULB)_j - \frac{1}{m}(\Delta_n UL)\right] + \frac{1}{m}(\Delta_n UL) \qquad (7.8a)$$

where

$(SC_n)_j$ = jth supplemental cost for the nth unfunded liability increment

$(\Delta_n ULB)_j$ = unfunded liability balance at the beginning of the jth year for the nth unfunded liability change

$(\Delta_n UL)$ = nth unfunded liability developed during a prior year.

If the amortization payments occur at the end of the year, as is the case for pension accounting (see Chapter 11), then the first section of (7.8a) would equal interest times the unfunded liability balance (rather than discount times the unfunded liability balance reduced by the current year's payment).

An alternative and widely used method is to amortize the unfunded liability increment with a series of constant dollar payments, with each payment representing both interest and principal. The supplemental cost at the beginning of the jth year during the m-year amortization period under the *constant dollar amortization method* is

$$(SC_n)_j = \frac{(\Delta_n UL)}{\ddot{a}_{\overline{m}|}} \qquad (7.8b)$$

where

$\ddot{a}_{\overline{m}|}$ = m-year period certain annuity.

Another explicit approach, argued by some to be the most appropriate for salary-based pension benefit formulas, is to have a series of increasing supplemental cost payments, with the annual increase equal to the non-merit portion of the salary assumption. The objective, of course, is to produce a cost pattern approximately equal to a constant percent of payroll. The *constant percent amortization method* can be defined as follows:

$$(SC_n)_j = \frac{(\Delta_n UL)}{{}^s\ddot{a}_{\overline{m}|}}\left[(1 + I)(1 + P)\right]^{j-1} \qquad (7.8c)$$

where

$^{s}\ddot{a}_{\overline{m}|}$ = m-year period certain annuity, with payments increasing by the inflation and productivity components of the salary assumption

I = inflation component of salary assumption

P = productivity component of salary assumption

j = jth supplemental cost payment ($1 \leq j \leq m$).

Figures 7–1a and 7–1b show the annual payments, first in nominal dollars and then in dollars that are "deflated" by the non-merit salary components (i.e., 5 percent), under the three explicit supplemental cost methods. These data assume an amortization period of 15 years, interest of 8 percent (the model interest rate assumption), salary increases of 5 percent (the model inflation plus productivity assumption), and an initial unfunded liability of 100 dollars. Figures 7–2a and 7–2b show the remaining unfunded liability as a percent of the original amount, again in nominal dollars and salary-adjusted dollars.[2]

IMPLICIT SUPPLEMENTAL COSTS

Supplemental costs calculated according to the same principles used in determining normal costs are referred to as *implicit methods*. Each cost method studied in Chapter 6 has a supplemental cost counterpart. Sometimes in practice, or in the literature, a so-called "normal cost" is calculated under these methods, which include the normal cost as previously defined plus the implicit supplemental cost yet to be defined in this section. While it may be unnecessary, and undoubtedly inconvenient, to separate this total cost into its underlying components, it is nevertheless imprecise and confusing not to do so. As stated previously,

[2] If the amortization period under the constant percent method is in the range of 20 to 30 years, its supplemental costs are less than the interest due on the unfunded liability for a period of time; hence, the outstanding balance increases. When expressed as a percent of total annual payroll, however, the outstanding balance represents a continually smaller amount.

FIGURE 7–1a
Supplemental Costs per $100 of Unfunded Liability Under Alternative Methods

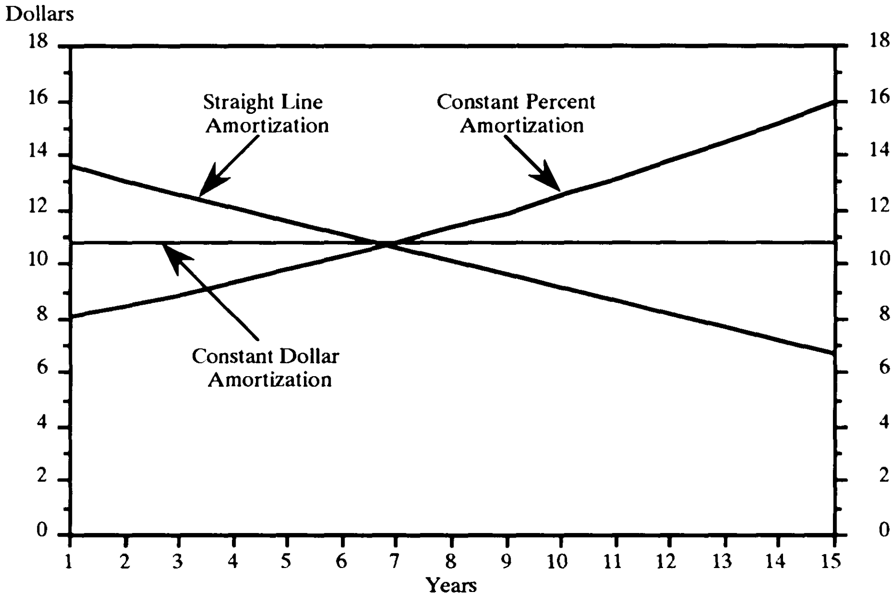


FIGURE 7–1b
Supplemental Costs per $100 of Unfunded Liability Under Alternative Methods
(Dollars "deflated" by 5%, equal to the non-merit salary increase components)

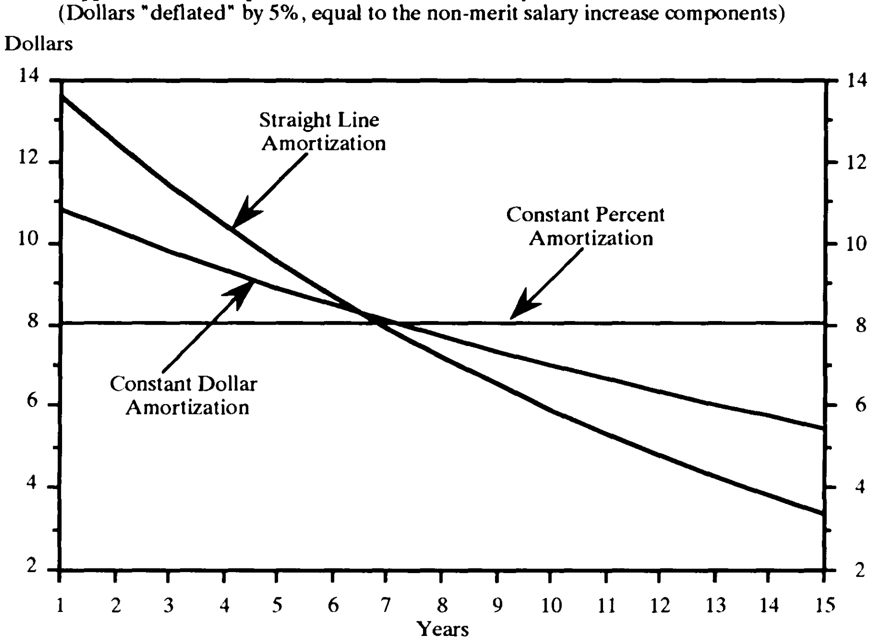


FIGURE 7–2a

Unfunded Liability Balance per $100 of Initial Balance Under Alternative Methods

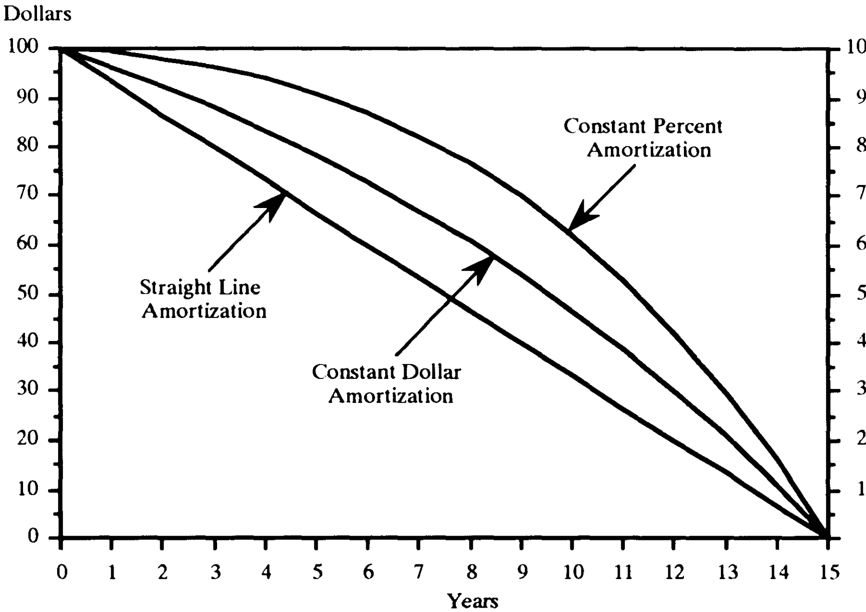
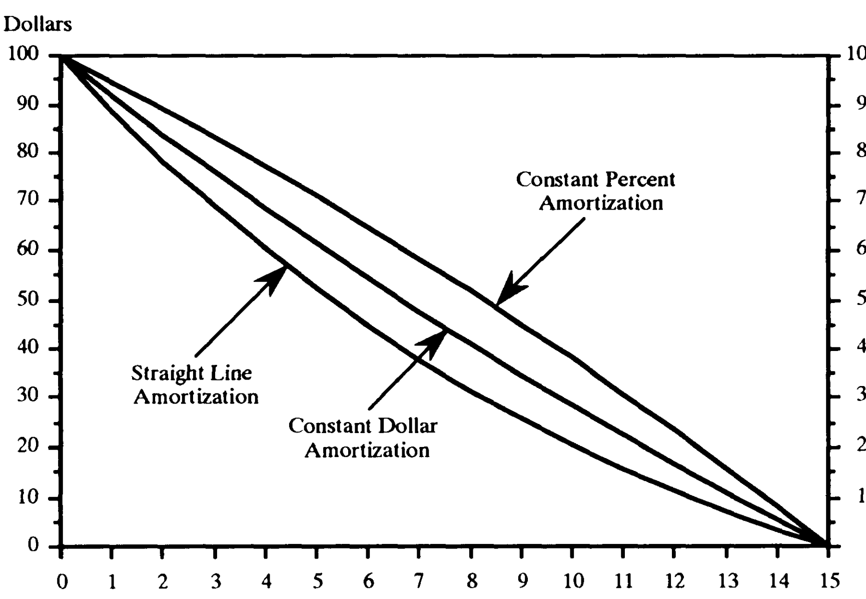


FIGURE 7–2b

Unfunded Liability Balance per $100 of Initial Balance Under Alternative Methods
(Dollars "deflated" by 5%, equal to the non-merit salary increase components)



the term normal cost used in this book never includes any supplemental costs.

Whereas explicit supplemental costs are almost exclusively determined on a group basis, implicit supplemental costs are sometimes determined on a participant-by-participant basis, especially for amortizing past service credits. The other sources of unfunded liability, however, are likewise almost always amortized on a group basis. In fact, a relatively common practice is to use an implicit supplemental cost method for amortizing past service credits and an explicit method to amortize all others sources of unfunded liability. Thus, it is possible to have both types of supplemental cost methods being used on different sources of unfunded liability.

Since implicit supplemental costs are geared to the various actuarial cost methods, the following discussion is organized accordingly.

Accrued Benefit Method

The normal cost under the accrued benefit method, it will be recalled, was found by using the natural benefit accruals for b_x in (6.1). This formulation is repeated here for convenience:

$$^{AB\,r}(NC)_x = b_x \;_{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r. \tag{6.7a}$$

The implicit supplemental cost can be expressed as follows:

$$^{AB}(SC_n)_x = C_n \; b_x \;_{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r \tag{7.9}$$

where

$^{AB}(SC_n)_x$ = accrued benefit supplemental cost at age x for nth unfunded liability change

C_n = coefficient to the benefit accrual for the nth unfunded liability change.

During the age that an unfunded liability is created (the nth unfunded liability change in the above formulation), a coefficient, C_n, is determined such that the present value (from attained age x to retirement) of the future designated supplemental costs is equal to the unfunded liability change. Determining the present value of future normal costs, which is an intermediate step in deriving C_n, can be avoided by using the identity given by (6.3a):

$$^{r}(AL)_x = {}^{r}(PVFB)_x - {}^{r}(PVFNC)_x. \tag{6.3a}$$

In other words, the present value of future normal costs (PVFNC) can be expressed as the present value of future benefits less the actuarial liability. Thus, C_n can be determined as follows:

$$C_n = \frac{(\Delta_n UL)_x}{(PVFB)_x - (AL)_x} \tag{7.10}$$

where

$(\Delta_n UL)_x$ = nth unfunded liability created during age x.

If the implicit supplemental cost under the accrued benefit method is relegated to determining only past service credits (the typical usage of this method), or perhaps benefit changes, then the determination of C_n becomes even more straightforward. To illustrate, assume that the plan is initiated at the employee's age z, yet benefit credits are granted from the employee's entry age. The past service benefit would be equal to B_z. The value of C_n is equal to the following:

$$C_n = \frac{B_z}{B_r - B_z}. \tag{7.11a}$$

In words, C_n is equal to the past service benefit divided by the future service benefit. Applying this ratio to the benefit accrual function in (7.9) is logical, since the sum of such "modified accruals" from age z to retirement is B_z, the past service benefit being funded by the supplemental costs. The plan's total cost, excluding any other supplemental costs that might be appropriate, is found by using the following benefit accrual, where the superscript T indicates the *total* benefit accrual (i.e., the normal cost benefit accrual plus the supplemental cost "benefit accrual" for past service credits):

$$b_x^T = (C_n + 1)\, b_x = \frac{B_r}{B_r - B_z}\, b_x. \tag{7.11b}$$

Therefore, the attained age benefit accrual is increased by the amount that the projected benefit, B_r, exceeds the future service benefit, $B_r - B_z$. This increase is a logical consequence of allocating the past service accrual, B_z, over future ages.

The *aggregate* version of the accrued benefit method has a total cost defined as follows, again assuming that only the supplemental cost for past service accruals are considered:

$$^{AAB\,r}(TC)_t = \left(\sum l_{x,y}\; b_{x,y}\right)\left[\frac{\sum l_{x,y}\; {}^r(PVFB)_{x,y}}{\sum l_{x,y}\; (B_{r,y} - B_{z,y})}\right]. \qquad (7.11c)$$

If only one employee is considered, the "total accrual" in (7.11c) reduces to (7.11b). As new employees enter the plan in future years, their value for $B_{z,y}$ will be zero; hence, eventually equation (7.11c) will be equal to the aggregate accrued benefit method given by equation (6.7b).

Benefit Prorate Methods

The *constant dollar* and *constant percent* versions of the benefit prorate method have normal costs defined by substituting the benefit accrual functions given by (3.15a) and (3.16a) for b_x in (6.1):

$$^{CD}b_x = \frac{B_r}{(r-y)}, \qquad (y \le x < r) \qquad (3.15a)$$

$$^{CP}b_x = \frac{B_r}{S_r} s_x . \qquad (y \le x < r) \qquad (3.16a)$$

These benefit accruals would be modified by a C_n factor, analogous to the modification in (7.9), in determining their respective implicit supplemental costs. In other words, the "modified accruals" would be determined such that their present value, from attained age x to retirement, equaled the unfunded liability increment to be amortized.

Restricting consideration to amortizing past service accruals would give the total accrual at age x as follows for the constant dollar version:

$$b_x^T = \frac{B_r}{r-y} + \frac{\frac{B_r}{r-y}(z-y)}{r-z} = \frac{B_r}{r-z}. \qquad (7.12a)$$

The term to the right of the first equal sign is the retirement benefit accrual under the benefit prorate method, whereas the second term is the supplemental cost "benefit accrual," the latter equal to the past service allocation (the numerator) divided by future service credits.[3] These two accrual factors reduce to the

[3]Note that the past service "allocation" is not defined to be equal to the true (or legally defined) past service accrual, but rather, to the past service years

projected benefit divided by future service credit, an intuitively appealing result. In other words, the constant dollar normal cost plus past-service-credit supplemental cost is found by simply allocating the projected benefit over future service.[4]

The analogous total benefit factor for the constant percent benefit prorate method is given by

$$b_x^T = \frac{B_r}{S_r - S_z} s_x . \tag{7.12b}$$

Again, the projected benefit is prorated by future service salary from age z to age r instead of from entry age to age r.

Finally, the *aggregate* versions of these two methods have total costs (normal cost plus past-service-accruals supplemental cost) defined by the following two equations:

$$^{ABD}\,{}^{r}(TC)_t = \left(\sum l_{x,y}\right) \left[\frac{\sum l_{x,y}\ {}^{r}(PVFB)_{x,y}}{\sum l_{x,y}\ (r - z)}\right]; \tag{7.13a}$$

$$^{ABP}\,{}^{r}(TC)_t = \left(\sum l_{x,y}\ s_{x,y}\right) \left[\frac{\sum l_{x,y}\ {}^{r}(PVFB)_{x,y}}{\sum l_{x,y}\ (S_{r,y} - S_{z,y})}\right]. \tag{7.13b}$$

Cost Prorate Methods

Normal costs under the cost prorate methods are equal to a constant dollar amount or a constant percent of salary during each employee's career. By the same token, implicit supplemental costs amortize the unfunded liability in the same manner from the age at which the unfunded liability is created to retirement. If the nth unfunded liability were created at age x, the corresponding supplemental costs from that age to retirement can be represented as follows:

$$^{CD}(SC_n)_x = \frac{(\Delta_n UL)_x}{\ddot{a}^T_{x:\overline{r-x|}}}; \tag{7.14a}$$

times the constant dollar benefit accruals. This is necessarily the case, since implicit supplemental costs are based on principles identical to those used in determining the cost method's normal costs.

[4]This methodology could also be useful if the first funding age exceeds the employee's plan eligibility age, in which case the methodology used must make up for accruals that occurred before funding commenced.

$$^{CP}(SC_n)_x = \frac{(\Delta_n UL)_x}{s_x\, {}^{s}\ddot{a}^{T}_{x:\overline{r-x|}}}. \qquad (7.14b)$$

The $^{CD}(SC_n)_x$ supplemental cost is a constant dollar amount applicable to each future age. The dollar value of the constant percent supplemental cost is found by multiplying the salary at each future age by the $^{CP}(SC_n)_x$ fraction.

The *individual* implicit supplemental costs under the cost prorate methods are generally used to amortize the liability associated with the plan's prior service accruals, and are sometimes used to amortize the other sources of unfunded liability. Using the constant dollar version for illustrative purposes, the unfunded liability associated with past service accruals at age z (the assumed starting age of the plan) can be expressed as previously set out in equation (5.8b):

$$^{CD\,r}(AL)_z = \frac{\ddot{a}^{T}_{y:\overline{z-y|}}}{\ddot{a}^{T}_{y:\overline{r-y|}}}\, {}^{r}(PVFB)_z. \qquad (7.15a)$$

If this quantity is amortized from age z to age r by $\ddot{a}^{T}_{z:\overline{r-z|}}$, adding it to the normal cost under this method yields the following total cost:

$$^{CD}(TC)_x = \frac{{}^{r}(PVFB)_y}{\ddot{a}^{T}_{y:\overline{r-y|}}} + \frac{\ddot{a}^{T}_{y:\overline{z-y|}}\, {}^{r}(PVFB)_z}{\ddot{a}^{T}_{y:\overline{r-y|}}\, \ddot{a}^{T}_{z:\overline{r-z|}}}. \qquad (7.15b)$$

Establishing a common denominator, we have

$$^{CD}(TC)_x = \frac{{}^{r}(PVFB)_y\, \ddot{a}^{T}_{z:\overline{r-z|}} + \ddot{a}^{T}_{y:\overline{z-y|}}\, {}^{r}(PVFB)_z}{\ddot{a}^{T}_{y:\overline{r-y|}}\, \ddot{a}^{T}_{z:\overline{r-z|}}}. \qquad (7.15c)$$

Equation (7.15c) can be rewritten in the following manner:

$$^{CD}(TC)_x = \frac{{}^{r}(PVFB)_z}{\ddot{a}^{T}_{z:\overline{r-z|}}}\left[\frac{{}_{z-y}p^{(T)}_{y}\, v^{z-y}\, \ddot{a}^{T}_{z:\overline{r-z|}} + \ddot{a}^{T}_{y:\overline{z-y|}}}{\ddot{a}^{T}_{y:\overline{r-y|}}}\right]$$

$$= \frac{{}^{r}(PVFB)_z}{\ddot{a}^{T}_{z:\overline{r-z|}}}. \qquad (7.15d)$$

The bracketed term is equal to unity, since the numerator is the sum of a deferred temporary employment-based annuity from age

z to r, plus a temporary annuity from age y to z. Thus, we see that the total cost for a participant under the constant dollar cost prorate method, when past service accruals are amortized implicitly, is equal to the present value of future benefits at the start of the plan amortized over future service. The same result holds for the constant percent version of this method.

An approach to determining the annual cost at age x under either version of the cost prorate method, assuming all supplemental liability increments and decrements are to be amortized by the implicit methodology, is illustrated for the constant dollar version. First, solving (6.16) for the normal cost, we have

$$^{CD}\,{}^{r}(NC)_x = \frac{{}^{r}(PVFB)_x - {}^{CD}\,{}^{r}(AL)_x}{\ddot{a}^{T}_{x:\overline{r-x}|}}. \tag{7.16a}$$

In words, the normal cost, which is a constant dollar amount for each attained age, is equal to the portion of the PVFB not yet funded by normal costs (i.e., the PVFB less the actuarial liability) amortized from age x to retirement. If the actuarial liability is replaced by the assets allocated to the employee, then any discrepancy between the actuarial liability and assets (or the unfunded liability balance) will be amortized from age x to retirement (i.e., the implicit amortization method). Thus, the total cost can be expressed as

$$^{CD}\,{}^{r}(TC)_x = \frac{{}^{r}(PVFB)_x - (Assets)_x}{\ddot{a}^{T}_{x:\overline{r-x}|}}. \tag{7.16b}$$

If no unfunded liability exists for the employee, (7.16b) simply produces the constant dollar normal cost. If the plan is started at age z, and no other unfunded liability is created, the total cost given by (7.16b) becomes equal to the total cost given by (7.15d), that is, where the past service accruals are amortized over the ages z to r. The important point, however, is that equation (7.16b), and its analogue for the constant percent version, can be used to amortize implicitly the year-to-year (positive or negative) changes in the unfunded liability over the expected future working lifetime of the plan participant.

The total cost under the *aggregate* versions of the cost prorate methods, assuming all unfunded liabilities are amortized implicitly, are given by the following equations:

$$^{ACD\,r}(TC)_t = \left(\sum l_{x,y}\right)\left[\frac{\left\{\sum l_{x,y}\ {}^r(PVFB)_y\right\} - (Assets)_t}{\sum l_{x,y}\ \ddot{a}^{T}_{y:\overline{r-y}|}}\right]; \quad (7.17a)$$

$$^{ACP\,r}(TC)_t = \left(\sum l_{x,y}\ s_{x,y}\right)\left[\frac{\left\{\sum l_{x,y}\ {}^r(PVFB)_y\right\} - (Assets)_t}{\sum l_{x,y}\ s_y\ {}^s\ddot{a}^{T}_{y:\overline{r-y}|}}\right]. \quad (7.17b)$$

These two versions are popular, in part because of computational convenience, and are often referred to as the *aggregate method.*

EXPLICIT AND IMPLICIT SUPPLEMENTAL COST COMBINATIONS

As noted previously, more than one supplemental cost method can be used with a given actuarial cost method. For example, the individual implicit method might be used for past service accruals under either the accrued benefit or benefit prorate methods, while amortizing all other sources of unfunded liability by an *aggregate* explicit method over m years.

A frequently used method combines the *aggregate* version of the cost prorate method with both *aggregate* implicit and explicit supplemental costs. The appropriate modifications to (7.17a) and (7.17b) are given below, where $\sum SC$ equals the sum of all explicit supplemental costs.

$$^{ACD\,r}(TC)_t = \left(\sum l_{x,y}\right)\left[\frac{\left\{\sum l_{x,y}\ {}^r(PVFB)_y\right\} - (Assets)_t - (ULB)_t}{\sum l_{x,y}\ \ddot{a}^{T}_{y:\overline{r-y}|}}\right] + \sum SC; \quad (7.18a)$$

$$^{ACP\,r}(TC)_t = \left(\sum l_{x,y}\ s_{x,y}\right)\left[\frac{\left\{\sum l_{x,y}\ {}^r(PVFB)_y\right\} - (Assets)_t - (ULB)_t}{\sum l_{x,y}\ s_y\ {}^s\ddot{a}^{T}_{y:\overline{r-y}|}}\right] + \sum SC. \quad (7.18b)$$

The unfunded liability balance (ULB) represents the balance, if any, of the unfunded liability amortized by an explicit method. If the ULB is zero, then (7.18a) and (7.18b) equal (7.17a) and (7.17b). If the ULB is equal to the entire unfunded liability, which requires that the ULB be determined annually using the actuarial liability under the individual cost prorate method, the total cost will be nearly equal to the individual prorate normal cost, the only difference being attributable to the numerical variation caused by the averaging process in (7.18a) and (7.18b).

Another common procedure, sometimes referred to as the *frozen initial liability method* or *frozen entry age method*, is to determine the plan's total unfunded liability at a point in time and amortize it by an explicit method. Similarly, any unfunded liability for future plan or assumption changes is added to this so-called "frozen" liability and amortized explicitly. Experience variations, however, are amortized by the *aggregate* implicit methodology.[5] Under this version, the ULB in the above equations would be the current year's balance of the frozen initial unfunded liability (plus any additions thereto).

Still another variation on this theme is to define the unfunded liability at the outset of the plan equal to the present value of accrued benefits (i.e., the actuarial liability as determined under the accrued benefit method). Future benefit and assumption changes are also amortized explicitly, again based on the accrued benefit actuarial liability. Experience variations, however, are amortized by the *aggregate* implicit methodology. This procedure is referred to as the *attained age normal method.*

SUMMARY OF ACTUARIAL COST METHODS

Five normal cost methods were discussed in Chapter 6, as listed in Table 7-1. Three of these methods are benefit-based and two are cost-based. All accomplish the same goal of accounting for or accumulating sufficient funds to meet the benefit obligations under a defined benefit pension plan; however, the incidence of cost during the employee's career is different. This chapter listed five sources of unfunded liabilities and four supplemental cost methods to fund or account for such unfunded liabilities. Again, the goal of each supplemental cost method is the same, namely, to accumulate funds to eliminate the unfunded liability. They differ by the incidence of costs and the period over which the unfunded liability is resolved. It was also noted that more than one supplemental cost method can be used on the various unfunded liability sources. Finally, both the normal cost and supplemental cost methods can be based on either individual

[5] A variant on this methodology, sometimes called the "frozen, frozen" initial liability method, is to amortize all future changes in the unfunded liability by the implicit methodology.

participants or on the group of participants. Consequently, there are a large number of potential normal cost/supplemental cost combinations, only a few of which are used in practice. Chapter 10 includes a discussion of the more commonly used combinations.

TABLE 7-1
Summary of Actuarial Cost Methods

Normal Cost Methods	**Supplemental Cost Methods**
Accrued Benefit Method	Explicit Methods:
Benefit Prorate Methods:	Straight Line
Constant Dollar	Level Dollar
Constant Percent	Level Percent
Cost Prorate Methods:	Implicit Method
Constant Dollar	
Constant Percent	**Sources of Unfunded Liability**
	Experience Deviations
	Assumption Changes
	Benefit Changes
	Past Service Accruals
	Contribution Variances

Chapter 8

Ancillary Benefits

The mathematics associated with various ancillary benefits is presented in this chapter. The first section defines the one year *term cost* of each ancillary benefit and the present value of future benefits, both of which are relevant to defining the cost of ancillary benefits under various actuarial cost methods.

TERM COST CONCEPT

The one year term cost associated with a given ancillary benefit is the liability expected to be created during the upcoming year. For example, if the benefit under consideration is a lump sum death benefit, the term cost is equal to the cost of one year's term insurance in the amount of the death benefit. In the past, term cost methodology was often used because of its computational simplicity, even though the plan's retirement benefits were funded according to a particular actuarial cost method. With the advent of high speed computers, this methodology is seldom used for private plans; however, it is sometimes used with public pension plans. Term cost is used in this chapter as a precursor to determining the cost of ancillary benefits under various cost methods.

VESTED TERMINATION BENEFITS

Term Cost

The term cost (TC) of vested termination benefits for an employee age x is given by

$$ {}^{v}(TC)_x = g_x^{(v)} \, B_x \, q_x^{(t)} \; {}_{r-x-1}p_{x+1}^{(m)} \, v^{r-x} \, \ddot{a}_r \tag{8.1} $$

where

$g_x^{(v)}$ = grading function equal to the proportion of accrued benefit vested at age x

B_x = accrued benefit at age x as defined by the plan benefit formula

$q_x^{(t)}$ = probability of terminating during age x

${}_{r-x-1}p_{x+1}^{(m)}$ = probability of living from age $x + 1$ to retirement.

This formulation shows that the term cost of vesting is the expected liability associated with the contingency that the employee may terminate vested during age x.[1] The term cost of vesting is zero until the first vesting age (denoted by z in this discussion). The grading function in (8.1) takes on the value of zero during this interval, resulting in a zero term cost. Additionally, the term cost is zero at and beyond the first early retirement qualification age, since the termination probability in (8.1) is zero at these ages. Generally, the cost pattern between these ages will be increasing; however, this need not be the case if the decrease in the attained age termination probability more than offsets the product of the survival probability and the interest discount function, both of which increase with age.

Present Value of Future Vested Termination Benefits

The PVFB for vested termination benefits can be expressed by taking the present value of the employee's future term cost of vesting:

$$ {}^{v}(PVFB)_x = \sum_{k=x}^{r'-1} {}_{k-x}p_x^{(T)} \, v^{k-x} \, {}^{v}(TC)_k $$

$$ = \left[\sum_{k=x}^{r'-1} g_k^{(v)} \, B_k \; {}_{k-x}p_x^{(T)} \, q_k^{(t)} \; {}_{r-k-1}p_{k+1}^{(m)} \right] v^{r-x} \, \ddot{a}_r . \tag{8.2} $$

[1]The grading function and accrued benefit function could be evaluated at age $x + 1/2$ to approximate a fractional year's credit. Similarly, the survival function could be evaluated at this fractional age as well. These refinements are not made in the mathematical notation in this chapter in order to simplify the equations.

This function increases until the first vesting age, after which it may or may not increase for a period of time (depending on the underlying actuarial assumptions) and then decreases to zero by r' (the first early retirement eligibility age).

DISABILITY BENEFITS

Term Cost

The term cost of disability for an employee age x can be expressed by

$$^{d}(TC)_x = g_x^{(d)} \, B_x \, q_x^{(d)} \; {}_w^d p_{x+1}^{(m)} \; v^{w+1} \, \ddot{a}_{x+w}^{d} \tag{8.3}$$

where

$g_x^{(d)}$ = grading function equal to the proportion of accrued benefit provided if disability occurs during age x

$q_x^{(d)}$ = probability of becoming disabled during age x

w = waiting period before disability benefits commence

${}_w^d p_{x+1}^{(m)}$ = probability that a *disabled* life age x lives w years

$\ddot{a}_{x+w}^{d}$ = life annuity based on disabled-life mortality.

This formulation assumes that the disability benefit is a function of the plan's accrued benefit. If this is not the case, the benefit function in (8.3) would be changed accordingly. The survival function and the annuity are both based on disabled-life mortality.

The term cost of disability will be zero prior to the first qualification age and will generally increase thereafter, although this need not be the case under some benefit formulas or actuarial assumptions.

Present Value of Future Disability Benefits

The present value of future disability benefits can be expressed by the following equation:

$$
{}^{d}(PVFB)_x = \sum_{k=x}^{r-1} {}_{k-x}p_x^{(T)}\, v^{k-x}\, {}^{d}(TC)_k
$$

$$
= \left[\sum_{k=x}^{r-1} g_k^{(d)}\, B_k\, {}_{k-x}p_x^{(T)}\, q_k^{(d)}\, {}_{w}^{d}p_{k+1}^{(m)} \right] v^{k+w+1-x}\, \ddot{a}_{k+w+1}^{d} . \quad (8.4)
$$

SURVIVING SPOUSE BENEFITS

Term Cost

Equation (8.5) represents the term cost of a surviving spouse benefit, where the benefit paid is a life annuity to the spouse commencing on the employee's death and equal to some fraction of the deceased employee's accrued benefit:

$$
{}^{s}(TC)_x = M\, g_x^{(s)}\, B_x\, q_x^{(m)}\, v\, \ddot{a}_{x+u+1} \quad (8.5)
$$

where

M = probability that the participant has a surviving spouse at death

$g_x^{(s)}$ = grading function equal to the proportion of accrued benefit provided to a surviving spouse if death occurs during age x

$q_x^{(m)}$ = probability of dying during age x

u = number of years (positive or negative) that, when added to the participant's age, yields an assumed age for the surviving spouse

$\ddot{a}_{x+u+1}$ = life annuity based on the spouse's age at the death of the participant.

The expected liability created by the possibility that the employee age x may die during the year has the same general form as the expected vested and disability liabilities. The coefficient M reflects the probability that the participant is married at the time of death, a probability frequently set in the 80 to 85 percent range. The grading function controls the portion of the accrued benefit payable to the surviving spouse at the participant's age x, and the spouse's annuity represents the cost of providing the benefit for the life of the spouse.

Present Value of Future Surviving Spouse Benefits

Taking the present value of future term costs associated with the surviving spouse benefit, we have the following expression for the present value of future benefits function:

$$^{s}(PVFB)_x = \sum_{k=x}^{r-1} {}_{k-x}p_x^{(T)} \, v^{k-x} \, {}^{s}(TC)_k$$

$$= M \left[\sum_{k=x}^{r-1} g_k^{(s)} \, B_k \, {}_{k-x}p_x^{(T)} \, q_k^{(m)} \right] v^{k+1-x} \, \ddot{a}_{k+u+1} \, . \quad (8.6)$$

Notice that (8.6) does not take into account the probability that the spouse survives to each potential future age for which the employee might die. Rather, the M coefficient is used to approximate the probability that the participant has a living spouse at all ages from age x to age $r-1$. It is possible to define M as the probability of having a spouse at age x only, and to use a survival function to account for the spouse's probability of living until the employee's age of death, but this is unduly complex given the relatively small cost of this ancillary benefit.

NUMERICAL ILLUSTRATION

Table 8–1 shows the term cost functions, expressed as a percentage of salary, and the present value of future benefit functions, expressed as a percentage of the retirement-based present value of future benefits function, for an age 30-entrant under the three ancillary benefits of the model plan.

ANCILLARY BENEFITS UNDER ACTUARIAL COST METHODS

The total cost of both ancillary and retirement benefits under various individual actuarial cost methods is considered in this section. The aggregate versions of each cost method are not presented, but the principles are the same.

Accrued Benefit Method

The normal cost under the accrued benefit method, consistent with the underlying theory of this method, is equal to the

TABLE 8–1

Ancillary Benefit Cost Functions:
Term Costs as a Percent of Salary and PVFB Functions as a Percent of $^{r}(PVFB)_x$

	Vested Termination Benefits		Disability Benefits		Surviving Spouse Benefits	
Age	Term Cost	$100 \cdot \frac{^{v}(PVFB)_x}{^{r}(PVFB)_x}$	Term Cost	$100 \cdot \frac{^{d}(PVFB)_x}{^{r}(PVFB)_x}$	Term Cost	$100 \cdot \frac{^{s}(PVFB)_x}{^{r}(PVFB)_x}$
30	0.00	19.76	0.00	16.84	0.00	10.42
31	0.00	19.76	0.00	16.84	0.00	10.42
32	0.00	19.76	0.00	16.84	0.00	10.42
33	0.00	19.76	0.00	16.84	0.00	10.42
34	0.00	19.76	0.00	16.84	0.00	10.42
35	0.33	19.76	0.00	16.84	0.01	10.42
36	0.40	19.28	0.00	16.84	0.01	10.41
37	0.46	18.75	0.00	16.84	0.01	10.40
38	0.53	18.17	0.00	16.84	0.02	10.38
39	0.60	17.55	0.00	16.84	0.02	10.36
40	0.69	16.88	0.12	16.84	0.03	10.33
41	0.77	16.17	0.14	16.72	0.04	10.30
42	0.87	15.41	0.18	16.59	0.05	10.26
43	0.98	14.61	0.23	16.42	0.07	10.21
44	1.10	13.75	0.28	16.22	0.09	10.15
45	1.25	12.84	0.33	15.99	0.12	10.08
46	1.40	11.87	0.38	15.74	0.16	9.99
47	1.58	10.83	0.44	15.46	0.20	9.87
48	1.78	9.73	0.53	15.15	0.26	9.73
49	2.00	8.56	0.62	14.80	0.34	9.55
50	2.26	7.32	0.71	14.41	0.43	9.34
51	2.53	6.00	0.81	14.00	0.54	9.09
52	2.84	4.60	0.94	13.55	0.68	8.79
53	3.16	3.13	1.07	13.07	0.85	8.44
54	3.51	1.60	1.20	12.55	1.05	8.03
55	0.00	0.00	1.37	12.00	1.26	7.55
56	0.00	0.00	1.51	11.41	1.41	7.02
57	0.00	0.00	1.72	10.79	1.57	6.44
58	0.00	0.00	1.99	10.10	1.74	5.81
59	0.00	0.00	2.39	9.33	1.96	5.14
60	0.00	0.00	2.98	8.45	2.20	4.41
61	0.00	0.00	3.83	7.39	2.47	3.63
62	0.00	0.00	5.01	6.09	2.75	2.80
63	0.00	0.00	6.59	4.48	3.07	1.91
64	0.00	0.00	8.65	2.47	3.42	0.97
65	0.00	0.00	0.00	0.00	0.00	0.00

present value of a deferred annuity of b_x (the formula benefit accrual), where the present value is based on the possibility that the employee may either terminate vested and be entitled to a deferred vested benefit, become disabled and receive a disability benefit, die and leave a surviving spouse with a benefit, or retire and receive a retirement benefit. The total normal cost for a participant age x can be represented by

$$^{AB\,T}(NC)_x = \left[b_x \sum_{k=x}^{r-1} {}_{k-x}p_x^{(T)}\, v^{k-x} \left(q_k^{(t)}\, {}^vF_k + q_k^{(d)}\, {}^dF_k + q_k^{(m)}\, {}^sF_k \right) \right] + b_x\, {}^rF_r \quad (8.7)$$

where each F_k function represents the value of the benefit payable at each decrement:

$$\begin{aligned} {}^vF_k &= g_k^{(v)}\ {}_{r-k-1}p_{k+1}^{(m)}\ v^{r-k}\ \ddot{a}_r \\ {}^dF_k &= g_k^{(d)}\ {}_w^dp_{k+1}^{(m)}\ v^{w+1}\ \ddot{a}_{k+w+1}^d \\ {}^sF_k &= M\ g_k^{(s)}\ v^1\ \ddot{a}_{k+u+1} \\ {}^rF_r &= {}_{r-x}p_x^{(T)}\ v^{r-x}\ \ddot{a}_r. \end{aligned}$$

The actuarial liability under the accrued benefit method is found by substituting B_x for b_x in equation (8.7).

Benefit Prorate Methods

The normal cost under the constant dollar version of the benefit prorate method is given by

$$^{BD\,T}(NC)_x = \left[\sum_{k=x}^{r-1} \frac{B_k}{(k-y)}\ {}_{k-x}p_x^{(T)}\, v^{k-x} \left(q_k^{(t)}\, {}^vF_k + q_k^{(d)}\, {}^dF_k + q_k^{(m)}\, {}^sF_k \right) \right] + \frac{B_r}{(r-y)}\, {}^rF_r. \quad (8.8)$$

The benefit accrual allocated to each age at which ancillary benefits are applicable is the projected benefit at that age divided by the years-of-service from entry age to that age. This factor is represented by the first term to the right of the summation sign in (8.8). Otherwise, the equation for the benefit prorate method is the same as that for the accrued benefit method.[2]

The actuarial liability is found by multiplying each of the prorated benefit components in (8.8) by the employee's year-of-

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[2] An alternative methodology is to apply the current year's benefit accrual percentage to the employee's projected salary. If the benefit accrual is a constant percentage of salary for each year of service, this methodology is identical to the methodology presented here; however, if the benefit accrual is non-linear, then the alternative methodology would be the preferred approach.

service to date [i.e., $(x - y)$], which produces the appropriate accrued benefit under the constant dollar version of this method.[3]

The normal cost under the constant percent benefit prorate method is found by making the appropriate modifications to the prorated accrued benefit:

$$^{BP\,T}(NC)_x = \left[\sum_{k=x}^{r-1} \frac{B_k}{S_k} s_k \; {}_{k-x}p_x^{(T)} \; v^{k-x} \left(q_k^{(t)} \; {}^vF_k + q_k^{(d)} \; {}^dF_k + q_k^{(m)} \; {}^sF_k\right)\right] + \frac{B_r}{S_r} s_x \; {}^rF_r . \quad (8.9)$$

The actuarial liability is determined by substituting S_x for s_x in equation (8.9).

Cost Prorate Methods

The normal cost associated with ancillary benefits under either version of the cost prorate method is determined by dividing the age-y present value of future ancillary benefits by the appropriate employment-based life annuity. Thus, the normal cost under the constant dollar method for both retirement and ancillary benefits is as follows:

$$^{CD\,T}(NC)_x = \frac{{}^v(PVFB)_y + {}^d(PVFB)_y + {}^s(PVFB)_y + {}^r(PVFB)_y}{\ddot{a}^T_{y:\overline{r-y}|}}$$

$$= \frac{{}^T(PVFB)_y}{\ddot{a}^T_{y:\overline{r-y}|}} . \quad (8.10a)$$

The actuarial liability, determined prospectively, is given by

$$^{CD\,T}(AL)_x = {}^T(PVFB)_x - {}^T(NC)_x \; \ddot{a}^T_{x:\overline{r-x}|} . \quad (8.10b)$$

The percent of salary, K, used with the constant percent version is found by substituting $s_y \, {}^s\ddot{a}^T_{y:\overline{r-y}|}$ for $\ddot{a}^T_{x:\overline{r-x}|}$ in (8.10a), and the last term in equation (8.10b) would be $K s_x \, {}^s\ddot{a}^T_{x:\overline{r-x}|}$.

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[3]There are other possible definitions of the benefit prorate method. For example, one could derive a constant dollar benefit accrual for each ancillary benefit, or one could derive an aggregate constant dollar benefit accrual for all benefits taken together. These equations were presented in the first edition of this book; however, since that time the above formulation has become the accepted approach.

If the actuarial liability associated with only vested benefits, for example, were found using equation (8.10b), it can be reasoned that its value would be negative at and beyond the first early retirement eligibility age. This is the case, since the present value of future vested benefits would be zero, while the present value of future vesting-related normal costs would continue to have a positive value up to age r. This same phenomenon may also occur with other ancillary benefits at older ages. There are three approaches to eliminating this effect if it is deemed inappropriate, especially for plans consisting of a substantial number of older employees where the negative actuarial liabilities associated with ancillary benefits may be non-trivial. One approach is to use the term cost method for ancillary benefits while maintaining the cost prorate method for retirement benefits and, possibly, one or more ancillary benefits. A second approach is to use the accrued benefit method for the ancillary benefits in question. A final approach is to develop normal costs for each ancillary benefit such that the present value of future benefits is amortized over the period for which such benefits are applicable. In the case of vested benefits, the employment-based annuity used in the denominator of (8.10a) would run from age y to age r'. Although a negative actuarial liability may still occur just prior to age r', it would not be substantial.

ANCILLARY BENEFITS FOR ALTERNATIVE LIABILITY MEASURES

The plan termination liability assumes that the plan is terminated on the date of the valuation; hence, ancillary benefits are not applicable. Ancillary benefits, however, are included in the plan continuation liability. The mathematics are identical to those of the actuarial liability under the accrued benefit cost method (i.e., equation (8.7) with B_x substituted for b_x).

COMPARISON OF ANCILLARY BENEFIT COSTS

Table 8–2 shows the cost of each ancillary benefit as a percent of retirement benefits under each normal cost method at various attained ages for an age-30 entrant. Table 8–3 provides these same data for the actuarial liabilities. Finally, Figure 8–1a shows normal costs with ancillary benefits included for the model pension plan, while Figure 8–1b shows the corresponding actuarial liabilities, each expressed as a percent of payroll.

TABLE 8–2

Ancillary Benefits Under Alternative Normal Costs as a Percent of Retirement Benefit Normal Cost

Age:	30	35	40	45	50	55	60
Vested Benefits							
^{AB}NC	185.40	174.64	96.66	51.76	20.89	0.00	0.00
^{BD}NC	47.41	45.88	32.58	21.32	10.25	0.00	0.00
^{BP}NC	115.25	109.89	67.92	39.44	16.98	0.00	0.00
^{CD}NC	19.76	19.76	19.76	19.76	19.76	19.76	19.76
^{CP}NC	19.76	19.76	19.76	19.76	19.76	19.76	19.76
Disability Benefits							
^{AB}NC	43.17	43.17	42.22	32.69	23.78	16.20	9.33
^{BD}NC	22.73	22.73	22.53	20.10	16.89	13.09	8.39
^{BP}NC	34.61	34.61	34.05	27.97	21.47	15.27	9.08
^{CD}NC	16.84	16.84	16.84	16.84	16.84	16.84	16.84
^{CP}NC	16.84	16.84	16.84	16.84	16.84	16.84	16.84
Survivor Benefits							
^{AB}NC	26.36	26.13	23.73	20.82	16.57	10.82	4.85
^{BD}NC	14.03	14.00	13.58	12.83	11.27	8.36	4.30
^{BP}NC	21.20	21.08	19.77	17.91	14.79	10.08	4.70
^{CD}NC	10.42	10.42	10.42	10.42	10.42	10.42	10.42
^{CP}NC	10.42	10.42	10.42	10.42	10.42	10.42	10.42

TABLE 8–3

Ancillary Benefits Under Alternative Actuarial Liabilities as a Percent of Retirement Benefit Actuarial Liability

Age:	30	35	40	45	50	55	60
Vested Benefits							
${}^{AB}AL$	0.00	185.40	102.30	55.38	23.53	0.00	0.00
${}^{BD}AL$	0.00	47.41	33.76	22.42	11.38	0.00	0.00
${}^{BP}AL$	0.00	115.25	71.24	41.92	19.03	0.00	0.00
${}^{CD}AL$	0.00	19.76	16.00	11.88	6.50	–0.60	–0.22
${}^{CP}AL$	0.00	19.76	14.38	9.67	4.19	–2.61	–1.08
Disability Benefits							
${}^{AB}AL$	0.00	43.17	43.17	33.65	24.62	16.89	10.10
${}^{BD}AL$	0.00	22.73	22.73	20.39	17.24	13.49	9.00
${}^{BP}AL$	0.00	34.61	34.61	28.62	22.12	15.86	9.80
${}^{CD}AL$	0.00	16.84	16.84	15.88	14.25	11.85	8.35
${}^{CP}AL$	0.00	16.84	16.84	15.61	13.80	11.36	7.99
Survivor Benefits							
${}^{AB}AL$	0.00	26.36	23.98	21.17	17.07	11.45	5.42
${}^{BD}AL$	0.00	14.03	13.63	12.93	11.49	8.73	4.75
${}^{BP}AL$	0.00	21.20	19.92	18.15	15.18	10.62	5.24
${}^{CD}AL$	0.00	10.42	10.31	10.03	9.27	7.47	4.35
${}^{CP}AL$	0.00	10.42	10.26	9.92	9.07	7.17	4.08

FIGURE 8–1a

Normal Costs With Ancillary Benefits Under Alternative Actuarial Cost Methods

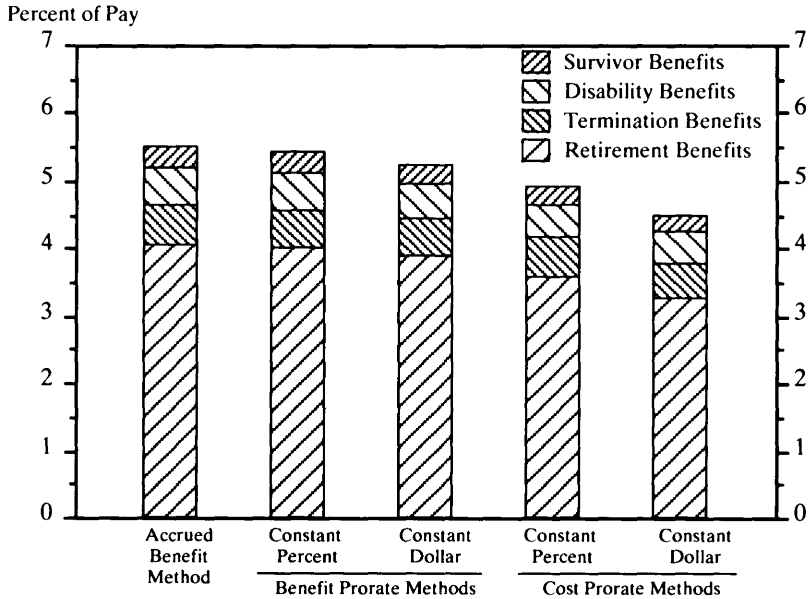
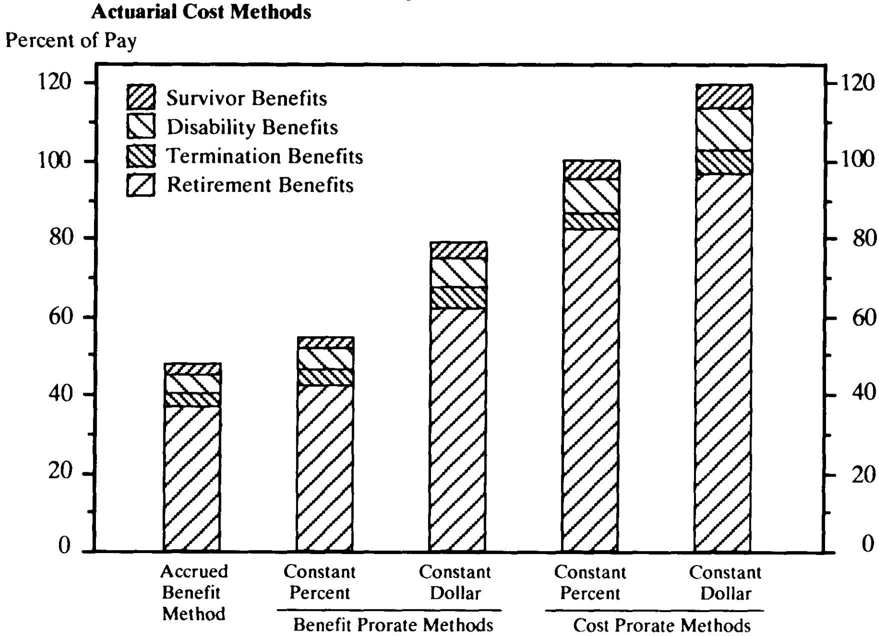


FIGURE 8–1b

Actuarial Liabilities With Ancillary Benefits Under Alternative Actuarial Cost Methods



Chapter 9

Multiple Retirement Ages

The mathematics presented thus far have assumed that retirement occurs at a single retirement age r. Most plans, however, provide for early and late retirement, sometimes on an *actuarially equivalent* basis. If the benefit provided at early or late retirement has the same present value as the benefit payable at normal retirement, then the retirement benefit is deemed to be actuarially equivalent to the normal retirement benefit. One might surmise that such a benefit would be cost neutral to the plan sponsor; however, as we will see, the cost effect is dependent on the actuarial cost method in use.

The first section of this chapter defines the concept of actuarial equivalence, while the second section defines pension costs for alternative actuarial cost methods under the assumption of multiple retirement ages. Although the aggregate cost methods are not considered explicitly, the previously discussed analogies of weighting the numerator and denominator of the individual cost methods to obtain the results under the aggregate methods still hold.

ACTUARIAL EQUIVALENCE

The symbol ${}^*g_k^{(r)}$ represents a retirement grading function which, when multiplied by a participant's natural (or formula) accrued benefit, B_x, produces actuarially equivalent benefits.[1] The principle underlying an actuarially equivalent benefit is one

[1]The asterisk is used to denote the special case when the value of the grading function is equal to the *actuarially equivalent* grading function.

that seeks to provide the participant with an adjusted benefit such that its present value evaluated at the retirement age in question is equal to the present value of the *unadjusted* benefit payable at the normal retirement age of the plan. This relationship for retirement at age k relative to normal retirement at age r is given by

$$ {}^{*}g_k^{(r)}\, B_k\, \ddot{a}_k = B_k\; {}_{r-k}p_k^{(m)}\, v^{r-k}\, \ddot{a}_r . \tag{9.1} $$

This equation assumes that $r > k$. If $r < k$, then the reciprocal of the survival probability is used. The remaining discussion assumes that $r > k$; however, the same principles apply to the opposite case.

The left side of (9.1) is the present value at age k of the reduced early retirement benefit payable immediately, while the right side represents the present value (also evaluated at age k) of the participant's accrued benefit assumed to be payable at age r. An expression for the actuarially equivalent grading function is obtained by solving (9.1):

$$ {}^{*}g_k^{(r)} = \frac{{}_{r-k}p_k^{(m)}\, v^{r-k}\, \ddot{a}_r}{\ddot{a}_k} = \frac{{}_{r-k|}\ddot{a}_x}{\ddot{a}_k} . \tag{9.2a} $$

The actuarially equivalent grading function, consisting of the ratio of an $(r-k)$-year deferred life annuity at age k to a nondeferred life annuity at age k, is clearly less than unity for all $k < r$ (and greater than unity for all $k > r$). The effect of alternative interest and mortality rates can be seen more clearly by rewriting (9.2a) in the following manner:

$$ {}^{*}g_k^{(r)} = \frac{{}_{r-k}p_k^{(m)}\, v^{r-k}\, \ddot{a}_r}{\ddot{a}_{k:\overline{r-k|}} + {}_{r-k}p_k^{(m)}\, v^{r-k}\, \ddot{a}_r} . \tag{9.2b} $$

The denominator of (9.2b) is less sensitive to changes in interest and mortality because of the temporary annuity factor. Thus, the actuarial equivalent grading function is inversely related to interest and mortality: the higher the interest and/or mortality rates the smaller will be the actuarially equivalent early retirement benefit. Males, for example, have higher mortality than females; hence, their actuarially equivalent reduction for early retirement would exceed the reduction for females if such reductions were based on gender as opposed to using a unisex mortality assumption. Similarly, actuarially equivalent reductions would be greater in a high interest rate environment compared to a low interest rate environment. As a practical matter, the plan document must define the actuarially equivalent methodology, which typically does not change with changes in the valuation interest rate or the market rate of interest. Thus, early retirement adjustments may be greater or less than the plan-based or market-based actuarial adjustment.

Table 9–1 gives the value of the actuarial equivalent grading function for ages 55 through 70, with age 65 representing the plan's normal retirement age. The effect of alternative interest and mortality rates are also shown. Interestingly, the actuarially equivalent grading function under the standard assumptions (8 percent interest and GAM-71 mortality assumptions) decreases by nearly 10 percent for each successively younger early retirement age. Thus, the grading function is approximately equal to $.9^{k-r}$ for $k < r$. An employee retiring early at age 61, for example, would receive about two-thirds of the benefit earned to date, while an employee retiring at age 55 would receive about one-third. The data in Table 9–1 show that rather wide variations in

TABLE 9–1

Actuarial Equivalent Grading Function

		Actuarial Equivalent Grading Function Under Alternative Assumptions				
		Interest Rate Sensitivity		*Mortality Rate Sensitivity*		
Age	${}^{*}g_k^{(r)}$	$i = .06$	$i = .10$	$.5\, q_k^{'(m)}$	$1.5\, q_k^{'(m)}$	$\frac{1}{{}^{*}g_k^{(r)}}$
55	0.33	0.39	0.28	0.38	0.29	3.02
56	0.37	0.42	0.32	0.41	0.33	2.73
57	0.41	0.46	0.36	0.46	0.37	2.46
58	0.45	0.50	0.40	0.50	0.41	2.22
59	0.50	0.55	0.46	0.55	0.46	1.99
60	0.56	0.60	0.52	0.61	0.52	1.79
61	0.62	0.66	0.59	0.67	0.59	1.60
62	0.70	0.73	0.67	0.74	0.67	1.43
63	0.79	0.81	0.76	0.81	0.76	1.27
64	0.89	0.90	0.87	0.90	0.87	1.13
65	1.00	1.00	1.00	1.00	1.00	1.00
66	1.13	1.12	1.15	1.11	1.15	0.88
67	1.29	1.25	1.33	1.24	1.34	0.78
68	1.47	1.41	1.54	1.38	1.56	0.68
69	1.68	1.59	1.79	1.54	1.82	0.59
70	1.94	1.80	2.09	1.73	2.15	0.52

the interest and mortality rate assumptions have a relatively modest impact on the actuarially equivalent grading function at most ages. Below age 60, however, the impact becomes more significant.

If the plan were to provide accrued benefits instead of actuarially equivalent benefits at early or late retirement, then the added cost would be represented by the reciprocal of the grading function, shown in the last column of Table 9–1. While this factor represents the true economic cost, the nominal cost depends on the actuarial cost method in use.

PRESENT VALUE OF FUTURE BENEFITS

The present value of future benefits under multiple retirement ages is presented as an introduction to the notation and because of its close relationship to the various actuarial cost methods. Equation (9.3) defines this function for a participant currently age x who is eligible to retire at any age from r' to r''.

$$^{r'}(PVFB)_x = \sum_{k=x}^{r''} g_k^{(r)} \, B_k \; {}_{k-x}^{\;\;r}p_x^{(T)} \, q_k^{(r)} \, v^{k-x} \, \ddot{a}_k \tag{9.3}$$

where

r' = first age for which the employee qualifies for early retirement

r'' = age at which all employees are assumed to have retired

$g_k^{(r)}$ = proportion of the participant's accrued benefit payable if retirement occurs at the beginning of age k

${}_{k-x}^{\;\;r}p_x^{(T)}$ = probability of surviving in employment $(k - x)$ years, where retirement decrements are included with mortality, termination, and disability decrements

$q_k^{(r)}$ = probability of retiring at the *beginning* of age k.

The prescript r' to the $(PVFB)_x$ function is used to signify that costs are determined according to multiple retirement ages, whereas the r prescript has been used to denote this function based on a single retirement age, usually the normal retirement age but not restricted to that age. This formulation assumes, for simplicity, that all retirements occur at the beginning of the age in question, an assumption made throughout this book. The grading function gives the portion of the participant's accrued benefit available for retirement at age k. If full benefits are payable at all retirement ages, this function takes on the value of unity. It is more likely, however, that the benefit payable at some early retirement ages will be less than the attained age accrued benefit, and in some cases will be actuarially reduced. Beyond the normal retirement age, the grading function might exceed unity if the plan provides for actuarial or formula increases.

The present value of future benefits function evaluated with multiple retirement ages, and assuming actuarially equivalent early retirement benefits, is denoted by ${}^{*r'}(PVFB)_x$ and found by substituting (9.2a) for the grading function in (9.3): [2]

$$
{}^{*r'}(PVFB)_x = \sum_{k=x}^{r} \left[\frac{{}_{r-k}p_k^{(m)} \, v^{r-k} \, \ddot{a}_r}{\ddot{a}_k} \right] \cdot B_k \; {}_{k-x}^{\;\;r}p_x^{(T)} \, q_k^{(r)} \, v^{k-x} \, \ddot{a}_k . \tag{9.4a}
$$

By expressing the survival function in terms of individual survival probabilities and noting that there are no termination rates beyond the first early retirement eligibility age, equation (9.4a) can be written as follows:

$$
{}^{*r'}(PVFB)_x = \left[\sum_{k=x}^{r} B_k \; {}_{k-x}p_x^{(d)} \; {}_{k-x}p_x^{(r)} \, q_k^{(r)} \right] \cdot {}_{r-x}p_r^{(m)} \; {}_{r-x}p_r^{(t)} \, v^{r-x} \, \ddot{a}_r . \tag{9.4b}
$$

An intuitively appealing approximation to the ${}^{*r'}(PVFB)_x$ function evaluated for an actuarially equivalent grading function is found by assuming no disability rates beyond r'. This allows the disability survival function to be extracted from the summation as follows:

$$
{}^{*r'}(PVFB)_x \approx \left[\sum_{k=x}^{r} B_k \; {}_{k-x}p_x^{(r)} \, q_k^{(r)} \right] {}_{r-x}p_r^{(T)} \, v^{r-x} \, \ddot{a}_r . \tag{9.4c}
$$

[2] For simplicity, this discussion assumes that the maximum retirement age is the normal retirement age r (i.e., $r'' = r$). The same general conclusions hold for retirements beyond age r, however, the reciprocal of the survival probability in (9.4a) would have to be used for these ages.

Note the use of the composite survival function in place of the individual survival rates for termination, disability, and mortality. The quantity in brackets represents the *expected* early retirement benefit, denoted by $E(B)$. Thus, an approximation to the actuarially equivalent early retirement function is

$$^{*r'}(PVFB)_x \approx E(B) \; _{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r = \frac{E(B)}{B_r} \; ^{r}(PVFB)_x. \quad (9.4d)$$

Equation (9.4c) shows that, if early retirement is permitted on an actuarially equivalent basis, the $^{r}(PVFB)_x$ function evaluated at $E(B)$ instead of B_r provides an approximation to $^{*r'}(PVFB)_x$. The approximation under estimates the true value of $^{*r'}(PVFB)_x$, but the error is relatively small for the typical set of disability rates. If B_x increases with age then $E(B) < B_r$; hence, the present value of future benefits is reduced for plans providing actuarially equivalent early retirement benefits.

ACCRUED BENEFIT METHOD

The normal cost under the accrued benefit method with multiple retirement ages is given by equation (9.5), with the benefit function equal to the natural (or formula) accruals:

$$^{AB \; r'}(NC)_x = b_x \sum_{k=x}^{r''-1} g_k^{(r)} \; _{k-x}^{\;\;r}p_x^{(T)} \; q_k^{(r)} \; v^{k-x} \; \ddot{a}_k. \quad (9.5)$$

The actuarial liability is found by substituting B_x for b_x in (9.5).

Evaluation (9.5) with actuarially equivalent early retirement benefits (again, assuming for simplicity, that r is the last retirement age) produces the following equation:

$$^{AB \; *r'}(NC)_x = b_x \left[\sum_{k=x}^{r-1} \; _{k-x}p_x^{(d)} \; _{k-x}p_x^{(r)} \; q_k^{(r)} \right]$$

$$\cdot \; _{r-x}p_x^{(m)} \; _{r-x}p_x^{(t)} \; v^{r-x} \; \ddot{a}_r. \quad (9.6)$$

If the disability survival probability from r' to r is reasonably constant (i.e., low or nonexistent disability rates), it too can be extracted from the summation sign without introducing a substantial error. The remaining terms inside the bracket sum to unity; hence, an approximation to (9.6) is simply the accrued benefit method evaluated for retirement at age r. The same approximation holds for this method's actuarial liability. Thus, unlike ${}^{r'}(PVFB)_x$, the normal cost and actuarial liability under the accrued benefit method are practically unaffected if early retirement occurs with actuarially reduced benefits. Although this is the result that one would expect, it turns out to be the only actuarial cost method where this it true, as shown subsequently.

BENEFIT PRORATE METHODS

The normal cost for the constant dollar version of the benefit prorate method is given by

$$^{BD}\,{}^{r'}(NC)_x = \sum_{k=x}^{r''-1} \frac{B_k}{(k-y)}\, g_k^{(r)}\; {}_{k-x}^{\;\;r}p_x^{(T)}\; q_k^{(r)}\; v^{k-x}\, \ddot{a}_k. \tag{9.7}$$

The actuarial liability can be expressed as

$$^{BD}\,{}^{r'}(AL)_x = \sum_{k=x}^{r''} \frac{B_k}{(k-y)}\,(x-y)\; g_k^{(r)}\; {}_{k-x}^{\;\;r}p_x^{(T)}\; q_k^{(r)}\; v^{k-x}\, \ddot{a}_k. \tag{9.8}$$

In each case, the benefit is projected to each successive retirement age and prorated from that age back to the employee's entry age. Clearly, if there is only one retirement age, r, then (9.7) simplifies to (6.8) and (9.8) simplifies to (5.6a).

The corresponding equations for the constant percent method are as follows: [3]

$$^{BP}\,{}^{r'}(NC)_x = \sum_{k=x}^{r''-1} \frac{B_k}{S_k}\, s_x\, g_k^{(r)}\; {}_{k-x}^{\;\;r}p_x^{(T)}\; q_k^{(r)}\; v^{k-x}\, \ddot{a}_k; \tag{9.9}$$

$$^{BP}\,{}^{r'}(AL)_x = \sum_{k=x}^{r''} \frac{B_k}{S_k}\, S_x\, g_k^{(r)}\; {}_{k-x}^{\;\;r}p_x^{(T)}\; q_k^{(r)}\; v^{k-x}\, \ddot{a}_k. \tag{9.10}$$

Evaluating the normal cost given in (9.7) with actuarially equivalent early retirement benefits (assuming $k < r$) gives

$$^{BD}\,{}^{*r'}(NC)_x = \left[\sum_{k=x}^{r-1} \frac{B_k}{(k-y)}\; {}_{k-x}p_x^{(d)}\; {}_{k-x}p_x^{(r)}\; q_k^{(r)}\right] \cdot {}_{r-x}p_x^{(m)}\; {}_{r-x}p_x^{(t)}\; v^{r-x}\, \ddot{a}_r. \tag{9.11}$$

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[3]The normal cost under these methods can be defined by deriving a benefit allocation factor that is, indeed, a constant value (as a dollar amount or as a percent of salary) which can then be substituted for the benefit factor in (9.5). This methodology was presented in the first edition of this book. The equations given above, however, have become the accepted approaches.

Again, if the disability survival function is reasonably constant over the various early retirement ages, the bracketed term represents the *average* benefit allocation factor. Since this term will be less than the benefit factor evaluated at normal retirement [i.e., $B_r \div (r-y)$ from equation (6.8)], actuarially equivalent early retirement under the constant dollar benefit prorate method will be less expensive than normal retirement. The corresponding actuarial liability, of course, will be less also.

The same conclusion can be reached for the constant percent benefit prorate method. In this case, the analogous term to the bracketed term in (9.11) involves the salary function.

COST PRORATE METHODS

The normal cost under the constant dollar cost prorate method with multiple retirement ages is equal to the following ratio:

$$^{CD\,r'}(NC)_x = \frac{^{r'}(PVFB)_y}{^{r'}\ddot{a}^T_{y:\overline{r-y}|}} \tag{9.12}$$

where $^{r'}\ddot{a}^T_{y:\overline{r-y}|}$ represents the present value of a temporary employment-based annuity including early retirement decrements from age r' to r''. The corresponding equation for the normal cost under the constant percent version is given by

$$^{CP\,r'}(NC)_x = \frac{^{r'}(PVFB)_y}{s_y \; ^{r's}\ddot{a}^T_{y:\overline{r-y}|}} \, s_x \tag{9.13}$$

where the r' prescript to the temporary annuity symbol again denotes the assumption of early retirement decrements.

The actuarial liability under the constant dollar version, evaluated prospectively, may be expressed as

$$^{CD\,r'}(AL)_x = {}^{r'}(PVFB)_x - \frac{^{r'}(PVFB)_y}{^{r'}\ddot{a}^T_{y:\overline{r-y}|}} \; {}^{r'}\ddot{a}^T_{x:\overline{r-x}|}, \tag{9.14}$$

while that of the constant percent version is found by replacing the annuities in (9.14) with salary-based annuities multiplied by the appropriate salary function. Equations (5.8b) and (5.9b), which express the actuarial liability as a ratio of annuities times the present value of future benefits function, hold for multiple retirement ages but only for $x \le r'$.

The normal cost evaluated with actuarially equivalent early retirement benefits (again, assuming age r is the last retirement age) is found by the appropriate substitution of the grading function into the numerators of (9.12) and (9.13). An approximation, made by assuming the disability survival probability is constant from r' to r, is as follows:

$$^{CD\;*r'}(NC)_x \approx \frac{E(B)\; _{r-y}p_y^{(T)}\; v^{r-y}\; \ddot{a}_r}{^{r'}\ddot{a}^{T}_{y:\overline{r-y}|}} \tag{9.15}$$

where $E(B)$ was defined in conjunction with (9.4c). In words, (9.15) equals the expected early retirement benefit evaluated at normal retirement age, divided by an $(r-y)$-year employment based life annuity, where the latter includes the retirement decrements from r' to r. The decrease in $E(B)$ relative to B_r is generally greater than the decrease in the employment based life annuity for early retirement relative to its value at normal retirement. Thus, it can be concluded that actuarially reduced early retirement generally produces a *lower* cost than the cost for normal retirement. The same conclusion holds for the constant percent version, the analogous equation to (9.15) being as follows:

$$^{CP\;*r'}(NC)_x \approx \frac{E(B)\; _{r-y}p_y^{(T)}\; v^{r-y}\; \ddot{a}_r}{s_y\; ^{r's}\ddot{a}^{T}_{y:\overline{r-y}|}}\; s_x. \tag{9.16}$$

The actuarial liability associated with actuarially equivalent early retirement benefits is given by (19.17a), which can be expressed more compactly for $x \le r'$ by (19.17b):

$$^{CD\;*r'}(AL)_x \approx E(B)\; _{r-x}p_x^{(T)}\; v^{r-x}\; \ddot{a}_r - \frac{E(B)\; _{r-y}p_y^{(T)}\; v^{r-y}\; \ddot{a}_r}{^{r'}\ddot{a}^{T}_{y:\overline{r-y}|}}\; ^{r'}\ddot{a}^{T}_{x:\overline{r-x}|} \tag{9.17a}$$

$$\approx E(B)\; _{r-x}p_x^{(T)}\; v^{r-x}\; \ddot{a}_r\; \frac{^{r'}\ddot{a}^{T}_{y:\overline{x-y}|}}{^{r'}\ddot{a}^{T}_{y:\overline{r-y}|}}. \quad (x \le r') \tag{9.17b}$$

In both cases the disability survival probability is assumed to be constant from r' to r. The corresponding actuarial liability equations for the constant percent version can be obtained by substituting the salary-based temporary annuities for the temporary annuities in (9.17a) and (9.17b). The annuities in (9.17a) must be multiplied by the appropriate salary function s_x in the numerator and s_y in the denominator, whereas the salary functions cancel out in (9.17b), since s_y is required for both the numerator and the denominator.

It turns out that, like the normal cost of the cost prorate method under actuarially equivalent early retirement benefits, the actuarial liability given by the above equations is *less* than the corresponding liability based on normal retirement.

RELATIVE COST OF EARLY RETIREMENT

The normal cost and actuarial liability *early retirement cost ratio* (ERCR), equal to the ratio of the cost of an early retirement benefit to the cost of a normal retirement benefit, is given by (9.18a) when the early retirement grading function is not specified and by (9.18b) when actuarially equivalent benefits are provided, both based on attained age x ($x < k$):

$$_k(ERCR)_x = g_k^{(r)} \frac{B_k}{B_r} \frac{1}{{}_{r-k}p_k^{(T)}} \frac{1}{v^{r-k}} \frac{\ddot{a}_k}{\ddot{a}_r} C_k ; \qquad (9.18a)$$

$$\overset{*}{_k}(ERCR)_x = \frac{B_k}{B_r} \frac{{}_{r-k}p_k^{(m)}}{{}_{r-k}p_k^{(T)}} C_k \qquad (9.18b)$$

where the coefficient C_k has the following values:

$$= \frac{B_r}{B_k} \quad \text{for accrued benefit method}$$

$$= \frac{S_r}{S_k} \quad \text{for benefit prorate, constant percent method}$$

$$= \frac{r-y}{k-y} \quad \text{for benefit prorate, constant dollar method}$$

$$= \frac{{}^s\ddot{a}^T_{y:\overline{r-y|}}}{{}^s\ddot{a}^T_{y:\overline{k-y|}}} \quad \text{for cost prorate, constant percent method}$$

$$= \frac{\ddot{a}^T_{y:\overline{r-y|}}}{\ddot{a}^T_{y:\overline{k-y|}}} \quad \text{for cost prorate, constant dollar method}$$

$$= 1 \quad \text{for the } (PVFB)x \text{ function.}$$

All of the definitions of C_k exceed unity, implying that the relative cost of early retirement under each of the actuarial cost methods is *greater* than the relative cost of early retirement under the $(PVFB)_x$ function. Table 9–2 gives the $_k(ERCR)_x$ based on equation (9.18a) for each of the actuarial cost methods under the model assumptions and for the $(PVFB)_x$ function. The data show that the relative cost of early retirement, based on nonreduced accrued benefits, is significantly higher under the accrued benefit and benefit prorate methods than it is under the cost prorate methods.

TABLE 9–2

Relative Cost of Early Retirement With Full Benefit Accruals for Various Normal Costs

Age	*Accrued Benefit Method*	*Benefit Prorate Methods* Constant Percent	*Benefit Prorate Methods* Constant Dollar	*Cost Prorate Methods* Constant Percent	*Cost Prorate Methods* Constant Dollar	*PVFB*
65	1.00	1.00	1.00	1.00	1.00	1.00
64	1.16	1.15	1.10	1.08	1.07	1.07
63	1.34	1.30	1.20	1.16	1.14	1.13
62	1.52	1.46	1.30	1.22	1.19	1.19
61	1.73	1.63	1.39	1.29	1.25	1.24
60	1.95	1.82	1.49	1.35	1.29	1.28
59	2.19	2.01	1.58	1.40	1.33	1.31
58	2.46	2.21	1.67	1.45	1.36	1.34
57	2.74	2.43	1.76	1.49	1.39	1.36
56	3.06	2.66	1.84	1.53	1.41	1.37
55	3.40	2.90	1.93	1.56	1.42	1.38
54	3.91	3.27	2.09	1.65	1.48	1.43
53	4.50	3.68	2.25	1.74	1.54	1.48
52	5.17	4.14	2.43	1.82	1.60	1.53
51	5.93	4.65	2.61	1.91	1.65	1.57
50	6.79	5.21	2.80	2.00	1.70	1.60

Table 9–3 gives the $_k^*(ERCR)_x$ based on equation (9.18b), i.e., where actuarially equivalent benefits are provided. The relative cost of actuarially reduced benefits under the cost prorate methods is similar to the relative cost of the $(PVFB)_x$ function, all of which are considerably less than unity and decrease as the age is lowered. The accrued benefit method, at the other extreme, exceeds unity and represents an increasing function of lower attained ages.

The analysis up to this point has been in terms of the cost of early retirement at age k relative to the cost at age r. The cost of early retirement at a given age with full benefits versus actuari-

TABLE 9–3

Relative Cost of Early Retirement With Actuarially Reduced Benefit Accruals for Various Normal Costs

Age	Accrued Benefit Method	Benefit Prorate Methods: Constant Percent	Benefit Prorate Methods: Constant Dollar	Cost Prorate Methods: Constant Percent	Cost Prorate Methods: Constant Dollar	PVFB
65	1.00	1.00	1.00	1.00	1.00	1.00
64	1.03	1.01	0.98	0.96	0.95	0.95
63	1.05	1.02	0.94	0.91	0.89	0.89
62	1.07	1.02	0.91	0.86	0.84	0.83
61	1.08	1.02	0.87	0.80	0.78	0.77
60	1.09	1.02	0.83	0.75	0.72	0.71
59	1.10	1.01	0.79	0.70	0.67	0.66
58	1.11	1.00	0.75	0.65	0.61	0.60
57	1.11	0.99	0.71	0.60	0.56	0.55
56	1.12	0.97	0.68	0.56	0.51	0.50
55	1.13	0.96	0.64	0.52	0.47	0.46
54	1.17	0.98	0.63	0.49	0.44	0.43
53	1.22	1.00	0.61	0.47	0.42	0.40
52	1.27	1.02	0.60	0.45	0.39	0.38
51	1.33	1.04	0.58	0.43	0.37	0.35
50	1.38	1.06	0.57	0.41	0.35	0.33

ally reduced benefits is equal to the reciprocal of ${}^{*}g_k^{(r)}$ and is valid for all actuarial cost methods. Table 9–1 provided this statistic, indicating a substantial cost increase for full benefits.

Figure 9–1 shows, for the model pension plan, the cost of normal retirement, early retirement with full benefits, and early retirement with actuarially reduced benefits based on the retirement rates presented in Table 2.9 of Chapter 2.

FIGURE 9–1a

Normal and Early Retirement Under Alternative Normal Costs

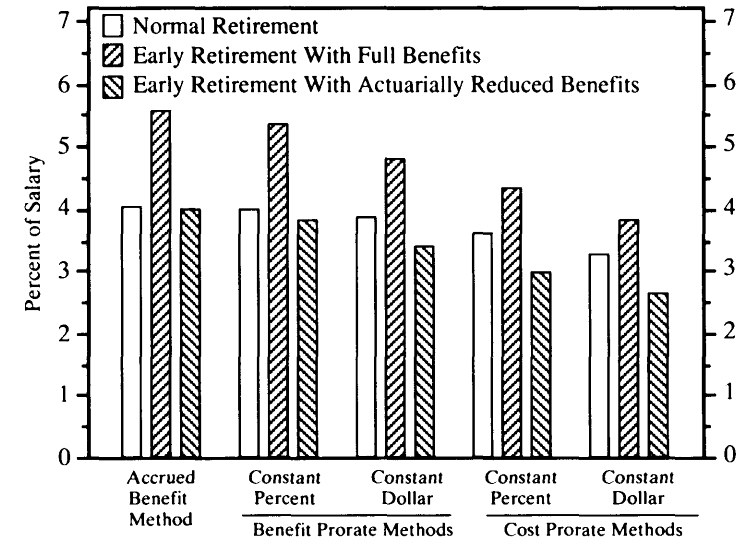
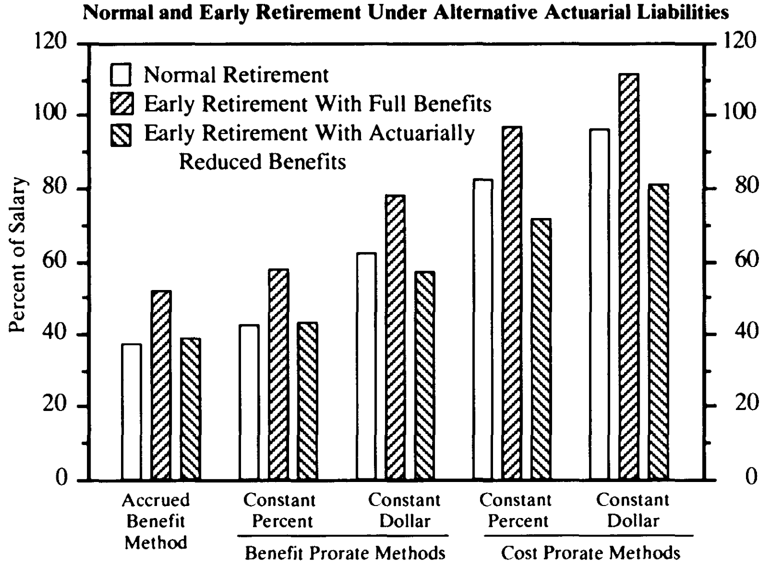


FIGURE 9–1b

Normal and Early Retirement Under Alternative Actuarial Liabilities



Chapter 10

Statutory Funding Requirements

For decades the Internal Revenue Code has placed requirements on pension plans receiving favorable tax treatment and has limited the maximum amount of tax deduction for employer contributions. In 1974 the Employee Retirement Income Security Act (ERISA) was enacted, representing a monumental piece of legislation designed primarily to secure the benefit rights of plan participants by mandating minimum vesting and funding requirements. The law itself, followed by an avalanche of regulations and additional legislation over the ensuing 19 years, has placed a heavy burden on employers who operate qualified defined benefit plans in accordance with its massive and complex rules, with one of the most significant sets of rules being enacted under the Omnibus Budget Reconciliation Act of 1987 (OBRA '87). Some experts believe that, despite laudable intentions, ERISA, expanded by subsequent legislation and stupefying regulations, is self-defeating, as evidenced by the large number of plans that have been terminated since its enactment.

This chapter discusses only a relatively small part of pension legislation, namely, the rules associated with minimum required contributions and maximum tax deductible contributions for single-employer pension plans.[1]

[1]For a comprehensive treatment of pension plan legislation, see Dan M. McGill and Donald S. Grubbs, Jr., *Fundamentals of Private Pensions*, 6th ed. (Homewood, Ill.: Richard D. Irwin, 1989).

MINIMUM REQUIRED CONTRIBUTIONS

The enrolled actuary for the plan is required to keep a running balance of the contributions made to the plan each year as compared to the minimum required contributions. A *funding standard account* (FSA) is used for this purpose and is reported to the IRS in Schedule B of Form 5500. If cumulative contributions exceed the minimum required contributions, the funding standard account will show a *credit balance;* if contributions fall short of the required minimum, the FSA shows a *funding deficiency.* In either case, next year's minimum contribution is affected by the prior year's FSA balance.

The minimum required contributions due at the end of year t (indicated by using $t+1$), can be expressed as follows:

$$^{Min}(EC)_{t+1} = Min \left\{ \begin{array}{c} \left[(NC)_t + (\sum SC)_t - (FSA)_t\right](1+i) + (AFC)_t(1+i') \\ ^{AL}(FFL)_{t+1} - (FSA)_t(1+i) \\ ^{CL}(FFL)_{t+1} - (FSA)_t(1+i) \end{array} \right\} \tag{10.1}$$

where

$^{Min}(EC)_{t+1}$	=	minimum required contribution determined as of the end of year t
$(NC)_t$	=	normal cost under a statutory funding method
$(\sum SC)_t$	=	sum of explicit supplemental costs
$(FSA)_t$	=	funding standard account balance from end of prior year (i.e., beginning of year t), with credit balances positive and funding deficiencies negative
$(AFC)_t$	=	additional funding charge for year t
i	=	valuation interest rate
i'	=	current liability interest rate
$^{AL}(FFL)_{t+1}$	=	year-end value of full funding limit based on actuarial liability of statutory funding method
$^{CL}(FFL)_{t+1}$	=	year-end value of full funding limit based on current liability.

Equation (10.1) indicates that the minimum required contribution includes (1) the *normal cost* under one of the statutory funding methods yet to be discussed, plus (2) the *supplemental costs* (both positive and negative) based on amortization periods set forth by statute, less (3) the *funding standard account balance* from the prior year (credit balances being positive and deficiencies being negative), plus (4) an *additional funding charge* if the plan's funded status falls below a specific target, as discussed later in this chapter. These four components are increased by interest to the end of the year, the first three being increased by the valuation rate of interest and the fourth by the interest rate used in calculating the plan's *current liability* (see later discussion). In no event, however, is the minimum required contribution greater than the full funding limit defined by the actuarial liability under the statutory funding method or the full funding limit defined by the current liability, both of which are reduced by the funding standard account balance from the prior year adjusted with interest. These two full funding limits are defined in this chapter.

If employer contributions are paid prior to year end, the minimum required contribution is reduced for interest at the valuation interest rate. For example, if uniform contributions are made at the end of each quarter, then the minimum quarterly payment would be

$$^{Qrt}(EC)_t = \frac{^{Min}(EC)_{t+1}}{s_{\overline{1}|}^{(4)}} \tag{10.2}$$

where

$^{Qrt}(EC)_t$ = minimum required quarterly contribution

$s_{\overline{1}|}^{(4)} = (1+i)^{3/4} + (1+i)^{1/2} + (1+i)^{1/4} + 1\,.$

As noted previously, if contributions are less than the minimum required amount, an *accumulated funding deficiency* will result. This deficiency must be corrected within 8 1/2 months after the plan year, otherwise the IRS will impose a 10 percent excise tax and will give the plan sponsor a specified time period by which the deficiency and excise tax must be paid. If the deficiency is not paid within this period, a 100 percent excise tax will be imposed.

A waiver of the funding deficiency may be granted by the IRS if a substantial, but temporary, business hardship exists such that contributions equal to the minimum funding requirement are deemed to be adverse to participants. Three waivers are permitted during any 15-year period, with the dollar amount of each waiver being amortized over five years. The interest rate used in amortizing a waiver must be the larger of 150 percent of the federal midterm rate in effect at the beginning of the plan year, the valuation interest rate or the current liability interest rate.

The following sections define and/or elaborate on each of the components making up the minimum required contributions given by (10.1).

Statutory Funding Methods

ERISA and IRS regulations identify the various funding methods that may be used in determining the normal and supplemental cost components of equation (10.1). Table 10–1 identifies each method using the terminology recommended by the Joint Committee on Pension Terminology (JCPT).[2] The statutory term for each method, along with the generic terms developed in this book, are also provided. The characteristics of each method are given in Table 10–2.

The actuarial assumptions used with a statutory funding method must represent the actuary's best estimate of future experience under the plan.[3] In addition, the benefit limits, IRC §415, and the maximum limit on compensation, IRC §401(a)(17), must be included without projection of future increases.

Unit Credit Method (Unprojected). Both the JCPT and ERISA use the term *unit credit* to refer to two different funding methods, one without a benefit projection (described in this section) and one with a benefit projection (described in the next section). The unit credit method (unprojected) is equivalent to the following:

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[2]The Pension Terminology report of the JCPT was published in 1981 and adopted by the American Academy of Actuaries, the Conference of Actuaries in Public Practice, the Society of Actuaries, and the American Society of Pension Actuaries.

[3]ERISA initially allowed the use of actuarial assumptions that, in the aggregate, represented the actuary's best estimate of future experience under the plan. TRA '86 imposed the requirement that each individual assumption represent the actuary's best estimate.

TABLE 10–1

Statutory Funding Method Terminology

JCPT Term	*Statutory Term*	*Generic Term*
Unit Credit Method (unprojected)	Unit Credit Method (unprojected)	Accrued Benefit Method with Explicit Supplemental Costs
Unit Credit Method (projected)	Unit Credit Method (projected)	Benefit Prorate Method with Explicit Supplemental Costs (only Constant Dollar version approved)
Entry Age Method	Entry Age Normal Method	Cost Prorate Method with Explicit Supplemental Costs (Constant Dollar or Constant Percent)
Attained Age Method	Attained Age Normal Method (individual calculations)	Cost Prorate Method (Constant Dollar or Constant Percent) Explicit supplemental costs applied to Unit Credit unfunded actuarial liability at plan formation, plan amendment, and assumption changes; implicit supplemental costs on all other sources
Individual Level Method	Individual Level Premium Method	Cost Prorate Method (Constant Dollar or Constant Percent) Implicit supplemental costs on all sources except non-benefit based gains and losses for which explicit supplemental costs are used
Frozen Entry Age Method	Frozen Initial Liability Method	Cost Prorate Method (Constant Dollar or Constant Percent; group-based calculations) Explicit supplemental costs applied to entry age unfunded actuarial liability at plan formation, plan amendment ,and assumption changes; implicit supplemental costs on all other sources
Frozen Attained Age Method	Attained Age Normal Method (group calculations)	Same as Attained Age Method but with group-based calculations
Aggregate Method	Aggregate Method	Cost Prorate Method with Implicit Supplemental Costs (Constant Dollar or Constant Percent; group-based calculations)
Individual Aggregate Method	Individual Aggregate Method	Same as Aggregate Method but with individual-based calculations

TABLE 10–2

Characteristics of Statutory Funding Methods

			Supplemental Cost Methodology for:			
JCPT Term	*Basic Methodology*	*Census Calculation*	*Initial Unfunded Liability*	*Plan Changes*	*Assumption Changes*	*Actuarial Gains and Losses*
Unit Credit Method (unprojected)	Accrued Benefit	Individual	Explicit	Explicit	Explicit	Explicit
Unit Credit Method (projected)	Benefit Prorate	Individual	Explicit	Explicit	Explicit	Explicit
Entry Age Method	Cost Prorate	Individual	Explicit	Explicit	Explicit	Explicit
Attained Age Method	Cost Prorate	Individual	Explicit (based on unit credit act. lib.)	Explicit (based on unit credit act. lib.)	Explicit (based on unit credit act. lib.)	Implicit
Individual Level Method	Cost Prorate	Individual	Implicit	Implicit	Implicit	Benefit-based: Implicit Other: Explicit
Frozen Entry Age Method	Cost Prorate	Group	Explicit	Explicit	Explicit	Implicit
Frozen Attained Age Method	Cost Prorate	Group	Explicit (based on unit credit act. lib.)	Explicit (based on unit credit act. lib.)	Explicit (based on unit credit act. lib.)	Implicit
Aggregate Method	Cost Prorate	Group	Implicit	Implicit	Implicit	Implicit
Individual Aggregate Method	Cost Prorate	Individual	Implicit	Implicit	Implicit	Implicit

Normal Cost:	Accrued Benefit Method
Supplemental Cost:	Explicit methodology on all unfunded liability sources.

The unit credit method (unprojected) can be used with flat dollar unit benefit and career average benefit formulas; however, it is not acceptable for use with benefit formulas based on final average salary.

Unit Credit Method (Projected). The unit credit method (projected) is equivalent to:

Normal Cost:	Benefit Prorate Method (constant dollar version only)
Supplemental Cost:	Explicit methodology on all unfunded liability sources.

As noted in Chapter 11, pension accounting requires a variation of this method to be used in determining pension expense; hence, many plans have adopted this approach for ERISA purposes even though the amortization methodologies are different and, often, the interest rates will be different.

The constant percent version of the benefit prorate method is not permitted in determining ERISA contribution limits.[4]

Entry Age Method. The entry age normal method has the following components:

Normal Cost:	Cost Prorate Method (constant dollar or percent version)
Supplemental Cost:	Explicit methodology on all unfunded liability sources.

The constant percent version is typically used with salary-based benefit formulas, while the constant dollar version is used with flat dollar unit benefit formulas.

Attained Age Method. The attained age method is equivalent to the following:

Normal Cost:	Cost Prorate Method (constant dollar or percent version)
Supplemental Cost:	Explicit methodology on unfunded liability at plan formation, plan changes and assumption changes based on unit credit actuarial liability. Implicit methodology on gains and losses and difference between unit credit actuarial liability and cost prorate actuarial liability.

[4]The prohibition of a given funding method for determining IRS contribution limits does not preclude the plan sponsor from using that method to establish contributions to the plan, provided, of course, that such contributions fall within the ERISA limits.

As noted, unit credit actuarial liability is used to determine the initial unfunded liability (or when ERISA became applicable, if later), plan changes and assumption changes. Explicit supplemental costs are used for these unfunded liability sources. The excess of the entry age actuarial liability over the unit credit actuarial liability is amortized by the implicit methodology, along with actuarial gains and losses.

Individual Level Method. The individual level method is equivalent to the following:

Normal Cost:	Cost Prorate Method (constant dollar or percent version)
Supplemental Cost:	Implicit methodology on all unfunded liability sources except non-benefit based actuarial gains and losses for the explicit methodology is used.

The implicit amortization period runs from the employee's attained age to retirement, as opposed to the *open group* methodology that amortizes unfunded liabilities over the future lives or compensation of each year's group of active employees.

Frozen Entry Age Method. The frozen entry age method is based on a group (or aggregate) calculation methodology and is equivalent to the following:

Normal Cost:	Cost Prorate Method (constant dollar or percent version)
Supplemental Cost:	Explicit methodology on all unfunded liability sources except actuarial gains and losses.

Explicit supplemental costs are used on the initial unfunded liability and the unfunded liability associated with plan amendments and assumption changes.[5] In determining the initial unfunded liability to be amortized explicitly, the entry age method's actuarial liability is used. The implicit supplemental cost methodology is used on actuarial gains and losses.

Frozen Attained Age Method. The frozen attained age method is identical to the attained age method except that it involves a group calculation rather than an employee-by-employee calculation. It is equivalent to the following:

[5]Some experts believe that assumptions changes can be amortized explicitly or implicitly under the frozen initial liability method, provided they are treated consistently from year to year.

Normal Cost:	Cost Prorate Method (constant dollar or percent version)
Supplemental Cost:	Same as Attained Age Method, but with group calculation methodology.

Aggregate Method. This method is equivalent to the following:

Normal Cost:	Cost Prorate Method (constant dollar or percent version)
Supplemental Cost:	Implicit methodology on all unfunded liability sources.

Unlike the individual methods, where the normal cost and actuarial liability are determined on a participant-by-participant basis and then summed for the entire plan, the aggregate method is based on a group calculation (i.e., the present value of future benefits for all participants is divided by the present value of future payroll or future lifetimes, depending on which version is being used).[6] The total cost so determined is referred to by ERISA as the "normal cost" of this method, even though it equals an underlying normal cost plus an implicit supplemental cost.

Since the aggregate method has only a "normal cost" (which, by definition, includes the implicit supplemental costs), the minimum and maximum contribution limits are virtually identical.[7] Under this method, the implicit amortization period depends on the age composition of active employees. If the active group is overmature, the amortization period could be the equivalent of 10 years or less, whereas if the group is undermature, the period might equate to 15 or even 20 years. In any case, this method almost assuredly amortizes experience variations over a longer period than the explicit amortization requirement of 5 years for minimum contributions, and it amortizes plan amendments over a shorter period than the explicit requirement of 30 years (see subsequent discussion of statutory amortization periods). On the other hand, since this methodology funds each

[6] The term "aggregate" is not clearly defined in either the actuarial literature or ERISA. In most contexts the term refers to aggregating normal costs and supplemental costs, while in others it refers to the use of a group calculation. As noted previously, the term aggregate is used in this book to refer only to the group calculation methodology.

[7] Maximum tax deductible contributions are determined as of the end of the year, whereas minimum required contributions are determined on a basis that assumes quarterly payments throughout the year; hence, the minimum will be somewhat smaller than the maximum due to the interest assumed on contributions made prior to year end.

year's *remaining* unfunded liability over each year's future salaries (or lifetimes) of active employees, the amortization period theoretically extends over an infinite time period, even though the vast majority of the liability is funded during the first 10 to 20 years.

Individual Aggregate Method. This is the same as the Aggregate Method but with individual instead of group calculation methodology. This method requires that assets be allocated to plan participants in a reasonable and consistent manner. This method is equivalent to the following:

Normal Cost:	Cost Prorate Method (constant dollar or percent version)
Supplemental Cost:	Implicit methodology on all unfunded liability sources using individual calculations with assets allocated to participants.

Statutory Amortization Periods

The second component of equation (10.1) is the supplemental costs associated with prior increases or decreases in the unfunded liability that are not yet fully amortized. As noted in the previous section, some statutory funding methods amortize one or more of the unfunded liability sources with *implicit supplemental costs*, whereby the amortization schedule is determined by the same methodology used to determine the method's normal cost.[8] For these methods, the statutory amortization period is, in fact, the implicit period. However, for funding methods that use explicit supplemental costs (i.e., an n-year, level dollar, amortization) for one or more of the unfunded liability sources, the maximum amortization period is set by statute.

ERISA and IRS regulations define the maximum amortization periods to be used in determining explicit supplemental costs, as indicated in Table 10–3. OBRA '87 shortened the amor-

[8]See Chapter 7 for a detailed description of supplemental cost methodologies. The implicit supplemental cost methodology is often referred to as the *spread gain method*, since the amortization of the unfunded liability is spread out over future normal costs. The explicit supplemental cost methodology is often referred to as the *immediate recognition method*, because the unfunded liability is immediately recognized (or determined) and then amortized over n years.

TABLE 10–3

Amortization Periods for Minimum Required Contributions

Unfunded Liability Source	*ERISA*	*OBRA* '87 (*After* 1/1/88)
Initial Unfunded Liability for plans existing as of 1/1/74	40 years	
Initial Unfunded Liability for plans formed after of 1/1/74	30 years	
Plan Amendments	30 years	
Assumption Changes	30 years	10 years
Actuarial Gains and Losses	15 years	5 years
Funding Waivers	15 years	5 years
Change in Funding Method	See Text	
Current Liability Full Funding Limit applicable when Minimum Required Contribution would otherwise apply	n/a	10 years

tization periods for changes in actuarial assumptions, actuarial gains and losses, and funding waivers that occur after January 1, 1988; however, the longer periods initially established under ERISA continue to apply to previously identified unfunded liabilities.

The rules for amortizing the unfunded liability due to a change in funding methods (the penultimate entry in Table 10–3) depend on whether the unfunded liability increases or decreases as a result of the change. The amount of the unfunded liability change is equal to the total unfunded liability under the new method less the sum of the outstanding amortization balances as of the date of the change, less the FSA credit balance or plus the funding deficiency. If the unfunded liability decreases, the amortization period for minimum required contributions is 30 years. If the unfunded liability increases, the period is 40 years (or 30 years if the plan was started after January 1, 1974) less the time interval from the application of ERISA to the current date or, if longer, the lesser of 15 years or the weighted average future working lifetime of active employees.

The last entry in Table 10–3 relates to the case where the current liability full funding limit (see subsequent discussion) reduces the minimum required contribution that would otherwise apply. This reduction is amortized over 10 years.

The supplemental cost at the beginning of year t during an m-year amortization period under the explicit method was given in Chapter 7 by equation (7.8b):

$$(SC_n)_t = \frac{(\Delta_n UL)}{\ddot{a}_{\overline{m|}}} \qquad (1 \leq t \leq m) \qquad (7.8b)$$

where

$(\Delta_n UL)$ = nth unfunded liability (positive or negative) developed during a prior year

$\ddot{a}_{\overline{m|}}$ = m-year period certain annuity.

The sum of such supplemental cost payments, with the amortization periods equal to the statutory maximums set forth in Table 10–3, represents the second component making up the minimum required contribution in (10.1).

If the valuation rate of interest changes, new supplemental cost limits are determined by dividing the beginning-of-year outstanding balance for each amortization by an annuity evaluated at the new interest rate for the period remaining in the amortization schedule.

If the statutory funding method uses explicit supplemental costs for one or more of the unfunded liability sources, a large number of supplemental cost schedules can develop. These schedules can be combined, as follows. First, the positive unfunded liabilities and negative unfunded liabilities are aggregated separately. Then, for the larger of the two, an amortization period is derived such that the corresponding supplemental costs amortize the unfunded liability balance.[9] Finally, the net unfunded liability (i.e., the sum of the positive and negative amounts) is amortized over the derived amortization period in determining the new, net, supplemental cost amount.

As a general rule, it is not beneficial to combine supplemental cost schedules for determining minimum contributions, especially those associated with plan design changes, as these are needed explicitly in determining the additional funding charge, as discussed below.

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[9]The derived amortization period may be fractional or it can be rounded down if the larger of the two amortization bases is positive or rounded up if the larger is negative.

Additional Funding Charge

The fourth component of the minimum required contribution given by (10.1) is designed to speed up the funding of underfunded plans. The formula is as follows:

$$(AFC)_t = Min \begin{Bmatrix} {}^{Old}(SC)_t + {}^{New}(SC)_t + {}^{CE}(SC)_t - {}^{I/PA}(SC)_t \\ (UCL)_t \end{Bmatrix} \qquad (10.3)$$

where

$(AFC)_t$ = additional funding charge

${}^{Old}(SC)_t$ = supplemental cost associated with *old* unfunded current liability established in 1988 and amortized over the succeeding 18 years

${}^{New}(SC)_t$ = supplemental cost associated with the *new* unfunded current liability as of the current year (excluding any remaining *old* unfunded current and any unfunded contingent event liabilities)

${}^{CE}(SC)_t$ = supplemental costs associated with contingent events

${}^{I/PA}(SC)_t$ = sum of all explicit supplemental costs for the initial unfunded actuarial liability and plan amendments that are not yet fully amortized

$(UCL)_t$ = unfunded current liability.

The unfunded current liability, $(UCL)_t$, is used explicitly in equation (10.3) and the current liability is involved in determining the old and new supplemental costs; hence, it is appropriate to define this liability before discussing the other components of (10.3).

Current Liability

The current liability for participants in pay status at age x, having earned a benefit of B_r, is given by

$$(CL)_x = B_r\, \ddot{a}_x . \quad (x \geq r) \qquad (10.4a)$$

If the participant in question terminated employment through a disability, then the annuity factor in (10.4a) should reflect disabled-life mortality. The current liability for non-active participants who terminated at age z with a vested benefit of B_z and who are currently age x with benefits payable at age r is given by

$$(CL)_x = B_z \; {}_{r-x}p_x^{(m)} \; v^{r-x} \; \ddot{a}_r . \quad (z \leq x \leq r) \tag{10.4b}$$

This assumes that the participant has elected out of the mandatory 50 percent survivor benefit required by the Retirement Equity Act (REA) of 1984 or, equivalently, that the survivor benefit is provided on an actuarially equivalent basis. If the survivor benefit is subsidized by the plan sponsor, then a death benefit representing the value of the survivor annuity would be included in (10.4b).

The current liability for active employees is not precisely defined by statute or regulations; however, the calculation must be based on an ongoing rather than a terminating plan scenario. A reasonable interpretation would be to include all ancillary benefits; however, some experts believe that some benefits (e.g., disability and death benefits) need not be included in this calculation. With the broader interpretation, the current liability for active employees can be expressed as follows:

$$(CL)_x = B_x \sum_{k=x}^{r''} {}_{k-x}p_x^{(T)} \; v^{k-x} \cdot \left(q_k^{(t)} \; {}^{v}F_k + q_k^{(d)} \; {}^{d}F_k + q_k^{(m)} \; {}^{s}F_k + q_k^{(r)} \; {}^{r}F_k \right) \tag{10.4c}$$

where each F_k function represents the value of the benefit payable at each decrement. These values for the model pension plan were defined in equation (8.7) of Chapter 8.

Each actuarial assumption, except for the interest rate, must reflect the actuary's best estimate of the plan's future experience solely with respect to that assumption. The interest rate is mandated by law to fall between 90 and 110 percent of the weighted average yield on 30-year Treasury bonds. The weights begin at 40 percent for the most recent year and equal 30, 20, and 10 percent, respectively, for the prior three years.

The unfunded current liability, which is used in determining the *additional funding charge* for underfunded plans, is defined as

$$(UCL)_t = Max \left\{ \begin{matrix} (CL)_t - \left[(AV)_t - {}^{CB}(FSA)_t \right] \\ 0 \end{matrix} \right\} \tag{10.5}$$

where

$(AV)_t$ = actuarial value of assets at beginning of year t[10]

${}^{CB}(FSA)_t$ = credit balance in the funding standard account at beginning of year t (i.e., from the end of prior year).

The *additional funding charge* defined by (10.3) is equal to the sum of three supplemental costs less a fourth, but in no case is it greater than the plan's unfunded current liability as defined by (10.5).

The first three supplemental cost components of the additional funding charge are discussed below. The fourth component represents a subtraction of the supplemental cost for unfunded liabilities associated with the initial unfunded liability and for plan amendments.

Old Supplemental Costs. This component of (10.3) represents the supplemental costs associated with the unfunded current liability, if any, that existed in 1988 (the *old* unfunded current liability). The amortization period for this liability is 18 years, beginning in 1989:

$$ {}^{Old}(SC)_t = \frac{{}^{Old}(UL)_t}{\ddot{a}_{\overline{18-(t-89)}|\,i'_t}} \tag{10.6}$$

where

${}^{Old}(UL)_t$ = balance of unfunded *old* current liability

$\ddot{a}_{\overline{18-(t-89)}|\,i'_t}$ = annuity certain for the remaining years in the 18-year amortization period, based on the interest rate used with the current liability.

If the current liability interest rate never changed, this supplemental cost would not have to be recalculated. However, the current liability interest rate changes almost yearly, therefore, it is necessary to accumulate last year's outstanding balance to the beginning of the current year (i.e., last year's balance less last year's supplemental cost, all accumulated with last year's current liability interest rate) and then divide this balance by an annuity certain for the remainder of the 18 years at the new current liability interest rate.

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[10]The actuarial value of assets is discussed in the final section of this chapter. It may represent the market value of assets or a smoothing of market and/or book values.

New Supplemental Costs. This component of (10.3) represents the supplemental cost associated with the new, or current year's, unfunded current liability, if any:

$$^{New}(SC)_t = \left[(UCL)_t - {}^{CE}(UL)_t - {}^{Old}(UL)_t\right]$$
$$\cdot\left[.30 - .25\, Max\left\{ \begin{matrix}(FR)_t - .35 \\ 0\end{matrix} \right\}\right] \tag{10.7}$$

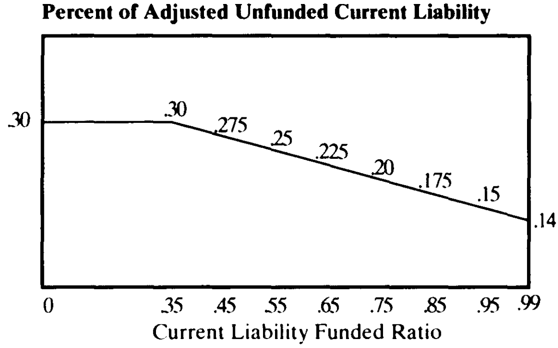
where,

$^{CE}(UL)_t$ = unfunded liability associated with prior contingent events

$(FR)_t$ = funded ratio equal to actuarial value of assets less the FSA credit balance, all divided by the current liability.

This cost represents a percentage of the *adjusted* unfunded current liability. The adjustment involves subtracting the unfunded liability associated with contingent events and the *old* unfunded liability, since these supplemental costs are explicitly included in the additional funding charge in (10.3). The adjusted unfunded current liability is multiplied by a percentage [the second row of (10.7)] that runs from a high of 30 percent for plans that have a funded ratio of 35 percent or less, to a low of 14 percent for plans that have a 99 percent funded ratio. A graph of this percentage for various funded ratios is given in Figure 10–1.

FIGURE 10–1
Percent of Adjusted Unfunded Current Liability



Contingent Event Supplemental Costs. An unpredictable contingent event benefit is any benefit not contingent upon age, service, compensation, death, disability, or other event that is reasonably and reliably predictable. For example, a plant shutdown or massive layoff would constitute an unpredictable contingent event. If the plan experiences such events that are not assumed in the actuarial assumptions, then the additional funding charge must reflect the amortization of the associated liability. The supplemental cost, which represents a component of the additional funding charge in (10.3), is the greater of two values:

$$^{CE}(SC)_t = Max \begin{Bmatrix} ^{CE}B_t\,[1 - (FR)_t]\,(TP)_t \\ ^{CE}_{7\text{-}pay}(SC)_t \end{Bmatrix} \tag{10.8}$$

where

$^{CE}B_t$ = current year's benefit payments associated with all contingent events

$(TP)_t$ = transition percentage, equal to 20 percent for 1993 and increasing by 10 percent per year to 100 percent by 2001

$^{CE}_{7\text{-}pay}(SC)_t$ = supplemental costs associated with amortizing contingent event liabilities over 7 years.

The first value in (10.8) is a percentage (i.e., fraction) of the current year's benefit payments associated with prior contingent events, where the percentage is equal to the complement of the current liability funded ratio times a transition percentage. The transition percentage is 20 percent for 1993, increasing by 10 percentage points per year to 100 percent by year 2001. Alternatively, if the aggregate supplemental cost associated with amortizing each contingent event liability over a 7-year period from the date of the contingent event is greater, then this amount represents the contingent event supplemental cost.

Table 10–4 summarizes the equations discussed thus far for determining the minimum required contribution. The two full funding limit tests are also shown on this summary, and discussed in the following two sections.

Actuarial Liability Full Funding Limit

The actuarial liability under the statutory funding method used by the plan sponsor is used to define one of two *full funding limits* (FFL) associated with minimum required contribu-

TABLE 10–4

Minimum Required Contribution (End of Year)

$$^{Min}(EC)_{t+1} = Min \begin{Bmatrix} \left[(NC)_t + (\sum SC)_t - (FSA)_t\right](1+i) + (AFC)_t(1+i') \\ ^{AL}(FFL)_{t+1} - (FSA)_t(1+i) \\ ^{CL}(FFL)_{t+1} - (FSA)_t(1+i) \end{Bmatrix}$$

where

$$(AFC)_t = Min\left\{\left[^{Old}(SC)_t + {}^{New}(SC)_t + {}^{CE}(SC)_t - {}^{I/PA}(SC)_t\right] \text{ or } (UCL)_t\right\}$$

where

$$^{Old}(SC)_t = {}^{Old}(UL)_t \div \ddot{a}_{\overline{18-(t-89)}|}\, i'_t$$

$$^{New}(SC)_t = \left[(UCL)_t - {}^{CE}(UL)_t - {}^{Old}(UL)_t\right]\left[.30 - .25\, Max\begin{Bmatrix}(FR)_t - .35 \\ 0\end{Bmatrix}\right]$$

$$^{CE}(SC)_t = Max\left\{\left[^{CE}B_t\,(1 - (FR)_t)\,(TP)_t\right] \text{ or } {}^{CE}_{7\text{-}pay}(SC)_t\right\}$$

$$(UCL)_t = Max\begin{Bmatrix}(CL)_t - \left[(AV)_t - {}^{CB}(FSA)_t\right] \\ 0\end{Bmatrix}$$

where

$$^{AL}(FFL)_{t+1} = \left[\left\{(AL)_t + (NC)_t\right\} - Min\begin{Bmatrix}(MV)_t - {}^{CB}(FSA)_t \\ (AV)_t - {}^{CB}(FSA)_t\end{Bmatrix}\right](1+i)\,.$$

$$^{CL}(FFL)_{t+1} = 1.5\left[\left\{(CL)_t + {}^{CL}(NC)_t\right\}(1+i') - E(B)_t\,(1+\tfrac{1}{2}i')\right]$$

$$- \left[Min\begin{Bmatrix}(MV)_t - {}^{CB}(FSA)_t \\ (AV)_t - {}^{CB}(FSA)_t\end{Bmatrix}(1+i) - E(B)_t\,(1+\tfrac{1}{2}i)\right]$$

tions.[11] The FFL based on the plan's actuarial liability will be referenced by ${}^{AL}(FFL)_t$. The second FFL, defined by the current liability yet to be discussed, will be referenced by ${}^{CL}(FFL)_t$.

The ${}^{AL}(FFL)_t$ is equal to the year-end value of the liability *less* the smaller of the market value or actuarial value of assets (excluding any contributions for the current year and less any credit balance in the funding standard account). If the adjusted asset value exceeds the actuarial liability, the minimum required contribution is zero. The FFL based on the statutory funding method's actuarial liability can be represented as follows:

$$
\begin{aligned}
{}^{AL}(FFL)_{t+1} = {} & \left[\{(AL)_t + (NC)_t\}(1+i) - E(B)_t(1+\tfrac{1}{2}i)\right] \\
& - \left[Min \begin{Bmatrix} (MV)_t - {}^{CB}(FSA)_t \\ (AV)_t - {}^{CB}(FSA)_t \end{Bmatrix} (1+i) - E(B)_t(1+\tfrac{1}{2}i) \right] \qquad (10.9a)
\end{aligned}
$$

where

${}^{AL}(FFL)_{t+1}$ = year-end full funding limit based on the statutory funding method's actuarial liability for year t

$E(B)_t$ = expected benefit payments during year t

$(MV)_t$ = market value of assets at beginning of year t

$(AV)_t$ = actuarial value of assets at beginning of year t

${}^{CB}(FSA)_t$ = credit balance in the funding standard account at beginning of year t (i.e., from the end of prior year).

Since both the actuarial liability and plan assets in (10.9a) are adjusted for expected benefit payments, the benefit terms cancel and the full funding limit can be expressed by

$$
{}^{AL}(FFL)_{t+1} = \left[\{(AL)_t + (NC)_t\} - Min \begin{Bmatrix} (MV)_t - {}^{CB}(FSA)_t \\ (AV)_t - {}^{CB}(FSA)_t \end{Bmatrix} \right] (1+i). \qquad (10.9b)
$$

The actuarial value of plan assets can represent any valuation method that takes into account fair market value. Several such methods are discussed in the final section of this chapter.

[11] If the statutory funding method involves implicit supplemental costs (e.g., individual level premium, frozen entry age, attained age, or aggregate methods), the ${}^{AL}(FFL)_t$ is based on the actuarial liability under the entry age normal method, and the normal cost for that method is also used in the calculation.

Current Liability Full Funding Limit

The current liability is used in defining a second full funding limit (FFL):

$$
\begin{aligned}
{}^{CL}(FFL)_{t+1} &= 1.5\left[\left\{(CL)_t + {}^{CL}(NC)_t\right\}(1+i') - E(B)_t(1+\tfrac{1}{2}i')\right] \\
&\quad - \left[Min\begin{Bmatrix}(MV)_t - {}^{CB}(FSA)_t \\ (AV)_t - {}^{CB}(FSA)_t\end{Bmatrix}(1+i) - E(B)_t(1+\tfrac{1}{2}i)\right] \qquad (10.10)
\end{aligned}
$$

where

$^{CL}(FFL)_{t+1}$ = year-end full funding limit based on the current liability for year t

$(CL)_t$ = current liability at beginning of year t

$^{CL}(NC)_t$ = current liability normal cost at beginning of year t, equal to (10.4c) evaluated with b_x instead of B_x

i' = current liability interest rate

i = valuation interest rate.

The interest rate used in projecting the current liability to year end is the rate used in calculating the current liability, while the interest rate used to adjust assets is the valuation rate of interest.

If the full funding limitation based on the actuarial liability is reached, then all supplemental cost schedules in the FSA are eliminated for future years.[12] This is the case even if credits exceed charges, implying that when the plan comes out of full funding, such credits are not available to reduce future contributions. If the full funding limitation based on the current liability is reached, while the actuarial liability FFL is not, then the amortization bases are maintained and an additional basis is established to fund the contribution that would be required without regard to the current liability FFL over 10 years.

---

[12] If contributions are constrained by the full funding limitation, any accumulated funding deficiency resulting from the normal operation of the FSA is eliminated by a *full funding credit* which results in the FSA having a non-negative balance for the year in question.

Funding Standard Account

The FSA, which is reported in Schedule B of Form 5500, shows the charges and credits for the applicable year, as indicated in Table 10–5a for which the plan year is assumed to run from 1/1/xx to 12/31/xx. Table 10–5a can also serve as a worksheet in determining the current year's minimum contribution. By completing the schedule with employer contributions assumed to be zero (line i), any funding deficiency that results represents the minimum required contribution payable at the end of the year. Table 10–5b shows a schedule for developing the additional funding charge (item e) in the FSA.

Additional Interest Charge Due to Late Quarterly Contributions

The plan sponsor must make quarterly pension contributions by the 15th day following the end of each plan-year quarter. The amount of the quarterly contributions can be 25 percent of the prior year's minimum contribution or 22.5 percent (90 percent of 25 percent) of the current year's minimum. The prior year's minimum for this purpose is the year-end value determined without considering the credit balance or funding deficiency. The current year's minimum for this purpose is the minimum as of the beginning of the year without considering the credit balance or funding deficiency, unless the latter is not eliminated by payment within 8 1/2 months after year end. If the plan has unfunded contingent event liabilities, the supplemental costs for these liabilities are not included in the minimum required calculation for quarterly contribution purposes. Instead, the quarterly contributions are increased by the larger of (1) 25 percent of the 7-year supplemental costs associated with the contingent event unfunded liabilities, or (2) the unfunded percentage of the unfunded contingent event benefit payments for 3 months prior to the month of the quarterly payment. The prior year's credit balance, if any, can be used in lieu of quarterly contributions.

As indicated by item f in Table 10–5a, the FSA is charged for any interest penalty paid on late quarterly contributions, with the offsetting credit, of course, being an increase in employer contributions equal to the interest charge. The penalty interest rate is equal to the greater of the current liability rate or 175 percent of

TABLE 10–5a
Funding Standard Account

Charges	
a. Prior year funding deficiency, if any	________
b. Normal cost as of 1/1/xx	________
c. Amortization charges:	
(i) Funding waivers	________
(ii) Other than waivers	________
d. Interest to 12/31/xx on a, b, and c	________
e. Additional funding charge, if applicable (see Table 10-5b)	________
f. Additional interest charge due to late quarterly contributions	________
g. Total charges (a through f)	________
Credits	
h. Prior year credit balance, if any	________
i. Employer contributions during the period from 1/1/xx to 12/31/xx	________
j. Amortization credits	________
k. Interest to 12/31/xx on h, i and j	________
l. Miscellaneous credits:	
(i) FFL credit before reflecting 150% of current liability component	________
(ii) Additional credit due to 150% of current liability component	________
(iii) Waived funding deficiency	________
(iv) Total miscellaneous credits	________
m. Total credits (h through l)	________
n. Credit balance: (if m < g, show difference) or funding deficiency: (if g < m, show difference)	________
Reconciliation	
p. Current year's accumulated reconciliation account:	
(i) Due to additional funding charge as of 1/1/xx	________
(ii) Due to additional interest charges as of 1/1/xx	________
(iii) Due to waived funding deficiency	________
(iv) Total as of 12/31/xx	________

the federal midterm rate as of the first month of the plan year. The additional interest charge is equal to the short fall amount times the penalty rate from the quarterly contribution due date to the actual contribution date, less the valuation rate from the

TABLE 10–5b

Development of Additional Funding Charge

a. Current liability as of 1/1/xx	______
b. Adjusted assets as of 1/1/xx (actuarial value – prior year credit balance)	______
c. Funded current liability percentage (100 b ÷ a)	______%
d. Unfunded current liability as of 1/1/xx (b – a)	______
e. Outstanding balance of unfunded old liability as of 1/1/xx	______
f. Liability attributable to any unpredictable contingent event benefit	______
g. Unfunded new liability (d – e – f)	______
h. Unfunded new liability amount (____% of g)	______
i. Unfunded old liability amount	______
j. Deficit reduction contribution (h + i)	______
k. Net amortization charge for certain bases	______
l. Unpredictable contingent event amount:	______%
(i) Benefits paid during year due to unpredictable contingent event	______
(ii) Unfunded current liability percent (1 – % in c)	______
(iii) Transition percentage	______
(iv) Product of (i), (ii) and (iii)	______
(v) Amortization of all unpredictable contingent event liabilities	______
(vi) Greater of (iv) or (v)	______
m. Additional funding charge as of 1/1/xx [excess of j over k plus l(vi)]	______
n. Assets needed to increase current liability percentage to 100% (line d)	______
o. Lesser of m or n	______
p. Interest to 12/31/xx on item o at current liability interest rate	______
q. Additional funding charge (o + p)	______

quarterly contribution due date to the earlier of the actual contribution date or the end of the plan year.

Miscellaneous Credits

Miscellaneous credits shown in Table 10–5a allow one to keep the funding standard account in balance. The miscellaneous credits involve three parts:

$$(MC)_t = Max\begin{Bmatrix}\left[{}^{FD}(FSA)_{t+1} - {}^{AL}(FFL)_{t+1}\right]\\ 0\end{Bmatrix} +$$

$$Max\begin{Bmatrix}Max\begin{Bmatrix}\left[{}^{FD}(FSA)_{t+1} - {}^{CL}(FFL)_{t+1}\right]\\ 0\end{Bmatrix} - Max\begin{Bmatrix}\left[{}^{FD}(FSA)_{t+1} - {}^{AL}(FFL)_{t+1}\right]\\ 0\end{Bmatrix}\\ 0\end{Bmatrix}$$

$$+ \; {}^{FD}(Waiver)_t \qquad (10.11)$$

where

$(MC)_t$ = miscellaneous credits

${}^{FD}(FSA)_{t+1}$ = year-end funding deficiency, determined without regard to prior year credit balance and current year contribution

${}^{AL}(FFL)_{t+1}$ = year-end full funding limit based on actuarial liability under statutory funding method

${}^{CL}(FFL)_{t+1}$ = year-end full funding limit based on current liability

${}^{FD}(Waiver)_t$ = funding deficiency waiver.

The first credit is the excess of the funding deficiency over the actuarial liability FFL, if any. In other words, if this FFL is applicable, then the funding deficiency (which would otherwise be a required minimum contribution) must be offset by a credit in the FSA. Secondly, if the current liability FFL is lower than the actuarial liability FFL, then this amount likewise must be offset by a credit in the FSA. Finally, the minimum required contribution is reduced by any funding waiver granted by the IRS for the current year.

An equation of equilibrium can be used to check if the FSA has been properly completed; namely, the unfunded actuarial liability must equal the outstanding unamortized liability balance less the FSA balance:

$$(AL)_t - (AV)_t = (ULB)_t - (FSA)_t \qquad (10.12a)$$

where

$(AL)_t$ = actuarial liability under statutory funding method at beginning of year t

$(AV)_t$ = actuarial value of assets at beginning of year t

$(ULB)_t$ = unamortized liability balance at beginning of year t

$(FSA)_t$ = funding standard account balance at beginning of year (i.e., from end of prior year, with credit balances assumed to be positive and funding deficiencies assumed to be negative).

However, this equation will not balance if there has been (1) additional funding charges, (2) additional interest charges on late contributions, or (3) waived funding deficiencies. The reconciliation account in the FSA (see the bottom of Table 10–5a) is used to account for these differences.

The entries for the additional funding charge and additional interest charge are determined by accumulating each prior year's outstanding balance in the reconciliation account with interest at the valuation rate and adding to these balances any prior year charges. The outstanding balance due to waivers equals the prior year's outstanding balance increased at the valuation interest rate less any year-end amortization amount.

Hence, the FSA equation of equilibrium becomes

$$(AL)_t - (AV)_t = (ULB)_t - (FSA)_t - (RA)_t \qquad (10.12b)$$

where

$(RA)_t$ = reconciliation account balance.

Alternative Minimum Funding Standard

As an alternative to the minimum contribution derived from (10.1), or equivalently, the FSA, the plan may elect to contribute according to an *alternative minimum funding standard.* The charges to the alternative account are equal to

- the accrued benefit normal cost (or, if lower, the normal cost under the funding method in use), plus
- the excess, if any, of the accrued benefit actuarial liability over the market value of plan assets.

This alternative is available only if the statutory funding method is the entry age normal method. The regular FSA must be maintained while the alternative is in use, with its accumulated funding deficiency being amortized over 5 years if the plan reverts back to the regular FSA for determining minimum contributions.

If the alternative FSA produces a lower required contribution, then the difference, multiplied by the complement of the employer's marginal tax rate, can be viewed as a loan to the employer, with loan interest rate equal to the valuation rate of interest multiplied by the compliment of the employer's marginal tax rate. This might represent an attractive loan in some cases, depending on the employer's tax rate and cash requirements.

MAXIMUM TAX DEDUCTIBLE CONTRIBUTIONS

There are statutory limitations on the amount of contributions to a qualified pension plan that an employer can deduct, with a 10 percent excise tax being imposed on contributions in excess of such limits.[13] Figure 10–2 sets forth this contribution limit.

FIGURE 10–2

Maximum Tax Deductible Contribution

Maximum of
- 1. Minimum of
  - A. Maximum of
    - 1. Normal Cost + 10-Year Supplemental Cost
    - 2. Minimum Required Contribution
  - B. Actuarial liability FFL
  - C. Current liability FFL
- 2. Unfunded Current Liability

In words, the maximum contribution is equal to the lesser of items (1) and (2), or, if larger, item (3):

(1) the normal cost under the statutory funding method plus maximum limit adjustments based on 10-year amortiza-

[13] Excess contributions may be deducted in future tax years (*carry-over contributions*), but are subject to the 10 percent excise tax for each tax year they remain nondeductible.

tions of the various unfunded liability balances, or if larger, the minimum required contribution

(2) the two full funding limits at year end (without adjustment for any credit balance)

(3) the unfunded current liability at year end, based on the actuarial value of assets.

The two full funding limits applicable to *maximum tax deductible contributions* are nearly identical to equations (10.9b) and (10.10) except that assets are not reduced for the FSA credit balance, but are reduced by any employer contributions that are not yet deductible.

Since experience variations are amortized over 5 years in determining minimum required contributions and over 10 years for determining maximum tax deductible contributions, it is possible that a plan's minimum required contribution could exceed its maximum tax deductible contribution. Under these circumstances, the minimum would be deductible, provided it does not exceed the full funding limits yet to be discussed.

Each year actual employer contributions must be allocated to the outstanding balances for all of the maximum amortizations in order to determine when they becomes zero. The portion of the employer's prior year total contribution applied to the maximum supplemental cost limits can be determined by

$$^{SC}C_{t-1} = \frac{^{Qrt}C_{t-1}\, s_{\overline{1}|}^{(4)}}{(1+i)} - (NC)_{t-1} \tag{10.13a}$$

where

$^{SC}C_{t-1}$ = portion of prior year's total contribution applied to the maximum supplemental cost limits

$^{Qrt}C_{t-1}$ = prior year's quarterly contributions

$s_{\overline{1}|}^{(4)}$ = factor for accumulating quarterly contributions with interest to end of year[14]

$$s_{\overline{1}|}^{(4)} = (1+i)^{3/4} + (1+i)^{1/2} + (1+i)^{1/4} + 1 .$$

[14] If contributions are not both uniform and paid quarterly, then the interest adjustment would be changed accordingly.

$(NC)_{t-1}$ = prior year's normal cost.

The prior year's supplemental cost contributions allocated to each unfunded liability base is prorated to each of the maximum supplemental cost limits as follows:

$$^{SC_n}C_{t-1} = \frac{(SC_n)_{t-1}}{(\sum SC)_{t-1}}\ ^{SC}C_{t-1} \tag{10.13b}$$

where

$^{SC_n}C_{t-1}$ = portion of prior year's contribution applied to nth supplemental cost for determining maximum contributions

$(SC_n)_{t-1}$ = nth supplemental cost from prior year for determining maximum contributions

$(\sum SC)_{t-1}$ = aggregate supplemental costs from prior year.

The outstanding balance for the current year is then determined as

$$(\Delta_n UL)_t = \left[(\Delta_n UL)_{t-1} - {}^{SC_n}C_{t-1}\right](1+i) \tag{10.13c}$$

where

$(\Delta_n UL)_t$ = nth unfunded liability balance in year t.

If the valuation interest rate is changed between the times $t-1$ and t, a revised maximum supplemental cost limit must be determined. The first step is to determine the number of years remaining in the maximum amortization schedule by equating the unfunded liability balance with the present value of future supplemental costs at the prior year's interest rate, and then solving for x in the following: [15]

$$(\Delta_n UL)_t = (SC_n)_t\, \ddot{a}_{\overline{x}|\, i_{t-1}}. \tag{10.13d}$$

Once the value of x has been determined, the revised supplemental cost for determining maximum contributions under the new interest rate is simply

$$(SC_n)_t = \frac{(\Delta_n UL)_t}{\ddot{a}_{\overline{x}|\, i_t}}. \tag{10.13e}$$

[15] The derived amortization period may be fractional or it can be rounded to an integer.

For maximum tax deductible contributions, supplemental costs of unequal length may also be combined. First, all of the unfunded liabilities (negative and positive amounts) are combined to obtain a net unfunded liability. Then a weighted average amortization period is calculated, as illustrated below assuming only two unfunded liability amounts:

$$Yrs = \frac{\left|(\Delta_n UL)_t\right|(Yrs)_n + \left|(\Delta_{n+1} UL)_t\right|(Yrs)_{n+1}}{\left|(\Delta_n UL)_t\right| + \left|(\Delta_{n+1} UL)_t\right|} \tag{10.14}$$

where

Yrs = years for amortizing the combined unfunded liabilities

$\left|(\Delta_n UL)_t\right|$ = absolute value of nth unfunded liability balance

$(Yrs)_n$ = amortization years remaining for the nth supplemental cost.

The net supplemental cost is determined by dividing the net unfunded liability by an annuity certain for the weighted average period found in (10.14).

An alternative to the above procedure when dealing with the supplemental costs for determining the maximum tax deductible contribution, is to use the *fresh start* method. At any time, all amortization bases can be replaced with a single base equal to the plan's unfunded liability amortized over 10 years. The fresh start never produces a larger maximum contribution than was allowed prior to the fresh start.

If employer contributions exceed the maximum contribution limit, the excess may be carried forward and deducted in a future year, but a yearly 10 percent excise tax is applicable on the portion of the excess contribution that remains non-deductible.

NUMERICAL ILLUSTRATION OF STATUTORY FUNDING REQUIREMENTS

An illustrative projection of the minimum required contribution and maximum tax deductible contribution for the model pension plan is given in Table 10–7, based on the valuation and experience assumptions provided in Table 10–6. The return on plan assets is assumed to vary between 5 and 10 percent every 5

TABLE 10–6

Valuation and Experience Assumptions

Valuation Assumptions (same as Model Assumptions):	
Mortality:	71 GAM (Table 2–1), for health lives, Table 2–5 for disabled lives
Termination:	Select and Ultimate (Table 2–3)
Disability:	Table 2–7
Retirement:	Distribution from age 55 to 65 (Table 2–9)
Salary Increase:	5% + Merit Scale (Table 2–10)
Interest Rate:	8% used for all cost and liability calculations
Asset Method:	5-year average of market value
Cost Method:	Constant Dollar Benefit Prorate
FSA:	Zero credit balance
Experience Assumptions:	
Decrements:	Same as Valuation Assumptions
Salary Increase:	Same as Valuation Assumptions
Investment Return:	10% for 5 years, 5% for 5 years, 10% for 5 years, then 5%
Ad hoc COLA:	25% of cumulative inflation every 3 years
Contributions:	ERISA minimum required contribution
Membership:	Population in 25th year from Table 4–7, held constant thereafter

years in order to generate actuarial gains and losses for illustrative purposes. The initial plan membership is equal to the population in the 25th year from Table 4–7; however, the size of the active population is assumed to be constant during the 20 year projection as opposed to the decreasing active membership assumption used beyond year 25 in Table 4–7.

Normal costs remain relatively constant at about 4.4 percent of payroll throughout the 20 year projection period. The initial unfunded liability produces a constant dollar supplemental cost that decreases as a percentage of payroll, beginning at 1.43 percent and reducing to .56 percent after 20 years. *Ad hoc* cost-of-living increases, equal to 25 percent of cumulative inflation, are assumed to be granted every 3 years. These plan amendments have a small impact on costs, amounting to only .5 percent of salary by the end of the 20 year projection period. Actuarial gains and losses, resulting from returns of 5 percent for 5 years, then 10 percent for 5 years, and so forth, have a cumulative impact of just over 2 percent of payroll in the worst year. In most

TABLE 10–7

Projection of ERISA Minimum Required and Maximum Tax Deductible Contribution
(Values Expressed as a Percent of Payroll)

		Supplemental Costs						
Year	*Normal Cost*	*Initial Unfunded Liability*	*Plan Amendment*	*Actuarial Loss (Gain)*	*Minimum Required Contributions*	*Normal Cost (EOY)*	*10-Year Supplemental Costs*	*Maximum Tax Deductible Contribution*
1	4.47	1.43	0.00	0.00	5.89	4.83	2.25	7.08
2	4.39	1.37	0.00	0.40	6.16	4.75	2.42	7.16
3	4.36	1.31	0.00	0.55	6.22	4.71	2.42	7.13
4	4.36	1.24	0.09	0.48	6.17	4.70	2.44	7.15
5	4.36	1.18	0.09	0.22	5.85	4.71	2.17	6.88
6	4.37	1.13	0.08	–0.24	5.34	4.72	1.78	6.50
7	4.38	1.07	0.18	–0.75	4.88	4.73	1.74	6.47
8	4.39	1.02	0.17	–0.79	4.79	4.74	1.70	6.43
9	4.39	0.97	0.17	–0.40	5.13	4.75	1.81	6.56
10	4.40	0.92	0.27	0.35	5.94	4.75	2.28	7.04
11	4.41	0.88	0.26	1.45	7.00	4.76	2.67	7.43
12	4.41	0.83	0.25	2.06	7.55	4.77	2.87	7.63
13	4.42	0.79	0.36	2.13	7.69	4.77	3.09	7.86
14	4.42	0.76	0.34	1.73	7.25	4.77	2.91	7.68
15	4.42	0.72	0.32	0.91	6.37	4.77	2.57	7.34
16	4.42	0.68	0.44	–0.30	5.24	4.78	2.32	7.10
17	4.42	0.65	0.42	–0.97	4.53	4.78	2.03	6.81
18	4.42	0.62	0.40	–1.05	4.40	4.78	1.96	6.74
19	4.42	0.59	0.52	–0.62	4.91	4.78	2.34	7.12
20	4.42	0.56	0.49	0.28	5.76	4.78	2.63	7.41

TABLE 10–8

Projection of ERISA Minimum Required and Maximum Tax Deductible Contribution Under Alternative Funding Methods
(Values Expressed as a Percent of Payroll)

	Unit Credit (Projected)			Entry Age			Frozen Entry Age			Aggregate		
Year	*NC*	*Min*	*Max*	*NC*	*Min*	*Max*	*NC*	*Min*	*Max*	*NC*	*Min*	*Max*
1	4.47	5.89	7.08	4.07	7.45	8.76	3.87	7.25	8.54	6.95	6.95	7.51
2	4.39	6.16	7.16	4.05	7.70	8.82	4.07	7.32	8.59	6.89	6.89	7.45
3	4.36	6.22	7.13	4.04	7.71	8.73	4.16	7.26	8.49	6.75	6.75	7.29
4	4.36	6.17	7.15	4.04	7.59	8.67	4.14	7.19	8.45	6.64	6.64	7.17
5	4.36	5.85	6.88	4.04	7.18	8.30	4.03	6.93	8.14	6.33	6.33	6.84
6	4.37	5.34	6.50	4.04	6.57	7.82	3.84	6.59	7.74	5.96	5.96	6.43
7	4.38	4.88	6.47	4.03	6.02	7.70	3.76	6.49	7.68	5.84	5.84	6.31
8	4.39	4.79	6.43	4.03	5.87	7.59	3.83	6.42	7.57	5.74	5.74	6.20
9	4.39	5.13	6.56	4.02	6.17	7.66	4.01	6.47	7.60	5.76	5.76	6.22
10	4.40	5.94	7.04	4.02	6.98	8.10	4.29	6.74	7.94	6.03	6.03	6.52
11	4.41	7.00	7.43	4.02	6.84	8.47	4.65	5.79	8.18	6.25	6.25	6.75
12	4.41	7.55	7.63	4.02	7.43	8.63	4.86	5.94	8.24	6.32	6.32	6.82
13	4.42	7.69	7.86	4.01	7.56	8.80	4.90	6.05	8.37	6.40	6.40	6.91
14	4.42	7.25	7.68	4.01	7.05	8.21	4.80	5.90	8.12	6.19	6.19	6.68
15	4.42	6.37	7.34	4.01	6.07	6.09	4.59	5.63	7.74	5.87	5.87	6.34
16	4.42	5.24	7.10	4.01	4.79	5.51	4.27	5.39	7.50	5.63	5.63	6.08
17	4.42	4.53	6.81	4.01	3.99	5.22	4.11	5.18	7.20	5.38	5.38	5.81
18	4.42	4.40	6.74	4.01	3.85	5.20	4.13	5.15	6.69	5.29	5.29	5.72
19	4.42	4.91	7.12	4.01	4.41	5.65	4.29	5.40	6.31	5.54	5.54	5.98
20	4.42	5.76	7.41	4.01	5.36	6.04	4.58	5.64	5.84	5.72	5.72	6.17

years, the impact is between negative and positive one percent. Since assets are averaged over five years and actuarial gains and losses are additionally amortized over 5 years, the effects of gains and losses on contribution limits are minimal. As indicated in Table 10–7, minimum required contributions fluctuate in the range of 4.5 to 7.5 percent over the projection period.

Minimum required contributions are determined at the beginning of the year in this projection, while maximum tax deductible contributions are an end-of-year calculation. These values for the illustrative projection are shown in the last portion of Table 10–7. Normal costs are increased to the end of the year by the interest assumption and supplemental costs are based on 10-year schedules. The difference between the minimum and maximum is about 1 percent of payroll in the early years of the projection, increasing to about 2 percent in later years.

Table 10–8 shows the normal costs, minimum required contributions, and maximum tax deductible contributions under three additional funding methods. The entry age (EA) and frozen entry age (FEA) methods have similar minimums and maximums, with FEA generally having a somewhat greater spread between the two limits. The aggregate method has minimums lower than both FEA and EA; however, its maximum is only about .5 percent of payroll greater (a result solely due to beginning-of-year versus end-of-year calculations).

STATUTORY ASSET VALUES

As noted previously, two asset values are used in the various calculations associated with minimum required and maximum tax deductible contributions: *market value* and *actuarial value.* Market value, as the name implies, represents the fair market value of plan assets as of the valuation date. While the plan sponsor may elect to set the actuarial value equal to market value, in many cases an *asset valuation method* is used to establish the plan's actuarial value.

Generally speaking, an asset valuation method is designed to smooth the year-to-year fluctuations in market value, which, in turn, has a smoothing effect on statutory contribution limits. Various asset valuation methods exist for accomplishing this smoothing process, the principal ones of which are discussed in this section. ERISA permits any asset valuation methodology that *reflects* market value; however, if the actuarial value in any given year falls outside a 20 percent corridor of market value, then the actuarial value must be adjusted to comply with this 20 percent limit.[16]

Weighted Average Method

A weighted average of book value and market value is an asset valuation method that can reduce the volatility of the actuarial value of assets, the degree to which depends on the percent weights and the extent of portfolio turnover. Naturally, if turnover is heavy, especially just before the valuation date, then the difference between market and book values may be quite small. As an aside, a properly designed asset valuation method should have minimal influence, if any, on whether or not gains are realized, a characteristic that the weighted average method does not possess.[17] This method can be expressed as follows:

$$(AV)_t = k\ (MV)_t + (1-k)\ (BV)_t \tag{10.15}$$

where

$(AV)_t$ = actuarial value of assets at beginning of year t

$(MV)_t$ = market value of assets at beginning of year t

$(BV)_t$ = book value of assets at beginning of year t

k = portion of market value used in weighting.

Average Ratio Method

The average ratio method adjusts the current book value of assets by an n-year average of market-to-book ratios:

[16] ERISA initially permitted bonds to be evaluated at their amortized book value; however, OBRA '87 eliminated this alternative unless *bond dedication* is being used (i.e., where bond coupon and maturities are matched with expected benefit payments and/or the duration of bonds and liabilities are matched).

[17] For example, if the market value were to increase substantially during the year, there may be a motivation to trade the portfolio near year end only because the book value and, hence, the actuarial value, would reflect the entire increase rather than a weighted average increase, and vice versa, if market assets decreased significantly during the year. Such trading may not be in the best interest of the pension plan; thus, asset valuation methods that have this potential conflict are generally regarded as undesirable.

$$(AV)_t = (BV)_t \frac{1}{n} \left[\frac{(MV)_{t-1}}{(BV)_{t-1}} + \frac{(MV)_{t-2}}{(BV)_{t-2}} + \cdots + \frac{(MV)_{t-n}}{(BV)_{t-n}} \right]. \quad (10.16)$$

This method is sensitive to portfolio turnover, since it utilizes the book value of assets; hence, like the weighted average method, it may not be a desirable technique.

N-Year Moving Average Method

Perhaps the most commonly used method is a 3 to 5 year moving average of market values. The mathematics for this approach is given by

$$(AV)_t = (MV)_t - \frac{n-1}{n} (CG)_{t-1} - \frac{n-2}{n} (CG)_{t-2} - \cdots - \frac{1}{n} (CG)_{t-n+1} \quad (10.17)$$

where

n = averaging period

$(CG)_t$ = capital gains (or losses), both realized and unrealized, during year t.

If the averaging period were 5 years, for example, then 80 percent of the prior year's capital gain (or loss) would be eliminated from the current market value, 60 percent of the gain (or loss) two years prior would be eliminated, and so forth. Conversely, as each year goes by, an additional 20 percent of each year's capital gain (or loss) is recognized in the actuarial value. This approach, while implicitly adjusting for the growth (or decline) in assets over time, places a uniform emphasis on each year's capital gain (or loss) during the averaging period. A variation of this method is to place disproportional weight on more recent experience.

Write-Up Method

Under the write-up method, the prior year's actuarial value of assets, appropriately adjusted for contributions and benefit payments, is increased by a specified yield, typically the interest rate used in the actuarial valuation:

$$(AV)_t = \left[(AV)_{t-1} + C_{t-1} - B_{t-1} \right] (1 + i) \quad (10.18)$$

where

C_t = employer contributions during year t

B_t = benefit payments during year t.

This method, which ignores both market and book values, produces a smooth progression of valuation assets. If the actuarial value strays beyond the ERISA 20 percent corridor, adjusting assets to the corridor would, of course, produce a discontinuity in this progression.

Corridor Method

A variation on the write-up method is to compute a preliminary actuarial value of assets according to (10.18) and compare this value to a predetermined corridor of market value, e.g., 85 to 110 percent of market. If the preliminary value falls outside the corridor, this method adjusts the preliminary value by 100k percent of the difference between the corridor and the preliminary value. Denoting the preliminary value determined by (10.18) with a prime notation, we have

$$(AV)_t = (AV)_t' + k\left[c_1\,(MV)_t - (AV)_t'\right] \tag{10.19a}$$

when $(AV)_t' < c_1\,(MV)_t$, and

$$(AV)_t = (AV)_t' - k\left[(AV)_t' - c_2\,(MV)_t\right] \tag{10.19b}$$

when $(AV)_t' > c_2\,(MV)_t$

where

k = adjustment fraction

c_1 = proportion of market value defining lower corridor limit

c_2 = proportion of market value defining upper corridor limit.

This method has a number of parameters. The value of k is often set in the range of .25 to .33 and the corridor coefficients, which need not be symmetrical, are frequently in the range of .10 to .20.

Table 10–9 shows the mean and standard deviation of the actuarial value of assets, expressed as a percent of these statistics for market value, under the various asset valuation methods discussed in this section. These results are based on a stochastic simulation of a 50-50 stock-bond portfolio, with standard deviations of annual returns equal to 18 percent for stocks and 10 percent for bonds, or an average of 14 percent for the portfolio. The dividend for stocks is assumed to be 4 percent and the coupon rate for bonds 7 percent. Turnover for each asset class is assumed to be 25 percent per year. The results are based on the 10th year of a stochastic simulation, with contributions defined as the minimum required contribution under the constant dollar benefit prorate method.

TABLE 10–9

Effect of Alternative Asset Valuation Methods on the Actuarial Value of Plan Assets

Valuation Method	*Average*	*Std. Dev.*
Market Value	100.0	17.6
Weighted Average ($k = .5$)	97.5	14.3
Average Ratio:		
$n = 3$	100.2	18.8
$n = 5$	99.7	17.9
Moving Average:		
$n = 3$	98.0	15.6
$n = 5$	96.9	13.9
Write-Up ($i = 8\%$)	95.6	11.7
Corridor ($i = 8\%; c_1 = c_2 = .15$):		
$k = .33$	95.7	11.7
$k = .25$	95.6	11.7

The weighted average method, with the weighting factor equal to .5, shows that valuation assets lag market assets by 2.5 percent on average, with the standard deviation being reduced from 17.6 to 14.3 percent. The average ratio method, under the stated assumptions, has little effect on the actuarial value of assets as compared to market value. The n-year moving average method, evaluated at 3 and 5 years, shows the tradeoff between lower volatility and the lag in actuarial assets as compared to market assets. Both the write-up and corridor methods show a substantial decrease in volatility with about a 5 percent lag in asset values. The relationships shown in Table 10–9, of course, would change under different assumptions; nevertheless, they indicate that any asset valuation method that reduces asset volatility produces, on average, a lower value of assets, assuming a positive growth in such assets. The effect of an asset valuation method on pension costs is similar, namely, lower volatility along with somewhat higher average costs.

Chapter 11

Pension Accounting

In 1985 the Financial Accounting Standards Board (FASB) promulgated Statement 87 (SFAS 87), *Employers' Accounting for Pensions*, effective for fiscal years beginning after December 15, 1986. The primary objective of this statement is to achieve consistency, uniformity, and comparability with respect to pension plan accounting among plan sponsors. SFAS 87 specifies the methodology used to determine pension expense, termed the *net periodic pension cost*, the liability, if any, to be reported on the employer's balance sheet, and the various disclosure items to be reported in the sponsor's financial statements.[1]

This chapter describes the essential components of pension accounting as set forth in SFAS 87 using previously defined equations. The discussion begins with two liability values that are an integral part of the SFAS 87.

LIABILITY VALUES

SFAS 87 defines the *Accumulated Benefit Obligation* (ABO) and the *Projected Benefit Obligation* (PBO), both of which must be disclosed in the employer's financial statements. In addition, the ABO is used to determine whether a liability must be reported on the employer's balance sheet, while the PBO plays an integral part in determining annual pension costs.

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[1]SFAS 87 applies both to qualified pension plans (the subject of this book) and non-qualified plans. Because of the effect of the benefit limits (IRC §415) and the maximum limit on compensation [IRC §401(a)(17)] the SFAS 87 costs of supplemental non-qualified plans have become more significant.

Accumulated Benefit Obligation (ABO)

The ABO is equal to the present value of accrued benefits, taking into account all future decrements. The ABO for an active employee age x is given by (11.1a), shown on the following page.[2]

The summation in (11.1a) runs from attained age x to the oldest assumed retirement age r''. Since this period involves ages for which some of the benefits are not applicable, zero values for the grading function and/or decrement rates at these ages eliminate the present value calculation. For example, both the retirement grading function and the retirement decrement are zero prior to age r', while the grading functions associated with the various ancillary benefits are defined to be zero at those ages for which benefits are not applicable. The ABO for non-active members is found by multiplying their accrued benefit by the appropriate annuity factor.

The ABO mathematics are identical to (1) the plan continuation liability, (2) the actuarial liability under the accrued benefit cost method, and (3) the statutory current liability. However, each of these may be based on different actuarial assumptions.[3] Each actuarial assumption used in determining the ABO must represent the *employer's* best estimate of the anticipated experience under the plan. While this still leaves the employer some latitude in the selection process, prior accounting promulgations were silent in this regard. With respect to the discount assumption, which is one of the most significant assumptions, SFAS 87 requires the use of a so-called settlement rate, i.e., the interest rate for which the pension obligation could be settled through the purchase of annuities. Rather than explicitly determining such rates each year, two proxies are suggested: (1) the current

[2]Although each term in (11.1a) has been defined previously, these definitions are provided again for convenience.

[3]The plan continuation liability for retirement benefits at a single retirement age was given by (5.2a), and the actuarial liability under the accrued benefit method by (5.5a). The actuarial liability under the accrued benefit method with ancillary benefits is equal to (8.7), with the benefit accrual being replaced by the accrued benefit. This liability for multiple retirement ages was given by (9.5), again with the benefit accrual being replaced by the accrued benefit (and the summation going to age r''). The statutory current liability is given by (10.4a) through (10.4c).

$$(ABO)_x = B_x \sum_{k=x}^{r''} {}_{k-x}p_x^{(T)}\, v^{k-x} \cdot \left(q_k^{(t)}\, {}^vF_k + q_k^{(d)}\, {}^dF_k + q_k^{(m)}\, {}^sF_k + q_k^{(r)}\, {}^rF_k\right) \tag{11.1a}$$

where

B_x = accrued benefit based on service, salary and the plan's benefit accrual rate determined at age x[4]

${}_{k-x}p_x^{(T)}$ = probability that an employee age x will survive in employment to age k

v^{k-x} = discount, at rate i, from age x to age k

$q_k^{(t)}$ = probability of terminating employment at age k

${}^vF_k$ = value of termination benefit at age k (for model plan, termination grading function times mortality-based life annuity deferred to age r)

$q_k^{(d)}$ = probability of becoming disabled at age k

${}^dF_k$ = value of disability benefit at age k (for model plan, disability grading function times disabled-mortality-based life annuity deferred to end of waiting period)

$q_k^{(m)}$ = probability of dying at age k

${}^sF_k$ = value of death benefit at age k (for model plan, survivor grading function times probability of having a surviving spouse times life annuity, reflecting age of spouse)

$q_k^{(r)}$ = probability of retiring at age k

${}^rF_k$ = value of retirement benefit at age k (for model plan, retirement grading function times life annuity).

[4]Equation (11.1a) shows the accrued benefit as a constant for each future decrement age; however, if the accrued benefit for a given employee at age x is constrained by maximum benefit or maximum salary limits for qualified plans under federal statutes, the indexed value of these limits should be used at each future decrement age. In this case, the accrued benefit would increase until the indexed limit no longer has an effect.

spot yield on high-yield, long-term corporate bonds (e.g., corporate bonds with a maturity of 20 or 30 years), and (2) the most recent interest rates promulgated by the Pension Benefit Guaranty Corporation (PBGC), or some reasonable approximation of such rate. These actuarial assumption requirements apply also to the PBO and the calculation of net periodic pension cost.

The ABO for SFAS 87 purposes must be a fiscal year-end value (or no more than three months prior to the date of the sponsor's financial statements). In many cases, the valuation will have been prepared six to twelve months earlier; hence, the ABO must be projected to the measurement date. There are two approaches that may be used. The first is to perform a detailed projection of the plan census, plus an estimate of new entrants, to the measurement date. This projection must reflect salary and service increases for active members, decrements for all plan members, and, of course, age increases for all members. Any material plan amendments must also be taken into account.

The second approach is to project the valuation results, with appropriate approximations.[5] Equation (11.1b) is appropriate for a full year's projection: [6]

$$(ABO)_{t+1} \approx \left[(ABO)_t + K(ABO)_t\right](1+i) - E(B)_t\left(1+\tfrac{1}{2}i\right) \tag{11.1b}$$

where

$(ABO)_t$ = ABO for all plan members at time t

i = discount rate used with ABO

$E(B)_t$ = expected benefit payments during year t

K = fraction of ABO to account for service and (if appropriate) salary increases.

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[5]While it is permissible to project the ABO (as well as the PBO) to the measurement date, the actuarial assumptions and the benefits valued must be appropriate at the measurement date. If these differ from the original valuation, then a new valuation must be run with appropriate changes before the projection set forth in (11.1b) is used.

[6]The fractional coefficient in the last term is intended to represent a weighted average of benefit payments throughout the year. If benefits are paid at the beginning of the month, and are expected to be uniform during the year, then 13/24 would be the correct coefficient; however, if benefits are expected to increase (or if lump-sum payments are expected on a non-uniform basis), then the coefficient should reflect this non-uniform expectation. For simplicity, the 1/2 coefficient is used in this equation and others presented in this chapter.

The K coefficient to $(ABO)_t$ is determined precisely as

$$K = \left[\sum^{all\,x} \frac{B_{x+1} - B_x}{B_x} (ABO)_x\right], \tag{11.2}$$

where the summation is over all plan members and B_x is the accrued benefit at age x.[7] Various approximations to the fractional growth in the accrued benefit can be made, depending on the type of benefit formula.

Vested Benefit Obligation (VBO)

SFAS 87 requires the disclosure of the vested benefit obligation, again a fiscal year-end value. This obligation is simply equal to the vested percent, if any, of each plan member's ABO. Some experts, however, believe that ancillary benefits, such as disability and pre-retirement death benefits, should not be included in this liability because employees do not become vested in such benefits. In other words, a vested benefit is one to which the employee is entitled after termination, with disability and death benefits not falling in this category.

Since minimum vesting requirements are five years under federal statute, and most non-vested employees are relatively young, the VBO is very close in value to the ABO for many pension plans. In fact, these values are often so close that the employer's financial statements simply indicate that they are not materially different, rather than showing the VBO separately.

Projected Benefit Obligation (PBO)

The PBO is equal to the present value of prorated retirement benefits, where the proration is based on service. The mathematics are the same as the actuarial liability under the constant dollar version of the benefit prorate actuarial cost method. For an active employee age x, the PBO is given by

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[7]Equation (11.2) represents the normal cost under the accrued benefit cost method, i.e., the present value of the increase in the accrued benefit which is equal to the benefit accrual.

$$(PBO)_x = \sum_{k=x}^{r''} {}^{CD}B_k \; {}_{k-x}p_x^{(T)} \; v^{k-x}$$

$$\cdot \left(q_k^{(t)} \; {}^{v}F_k + q_k^{(d)} \; {}^{d}F_k + q_k^{(m)} \; {}^{s}F_k + q_k^{(r)} \; {}^{r}F_k \right) \quad (11.3a)$$

where

$${}^{CD}B_k = \frac{B_k}{(k-y)} (x-y)$$

= accrued benefit projected to age k, prorated by the ratio of current service to projected service at age k.

Each F_k function in (11.3a) represents of the value of the benefit payable at each decrement [see explicit definitions in connection with equation (8.7)].

The prorated accrued benefit in (11.3a) is assumed to be appropriate to each type of decrement, with the grading functions modifying this benefit for any reduction (e.g., the model plan has a survivor benefit equal to 50 percent of the accrued benefit and an early retirement benefit that is actuarially reduced). There are plan designs, however, that require a prorated accrued benefit to be defined separately for one or more decrements. The fundamental principle underlying SFAS 87 is that the allocated accrued benefit (or attributed benefit) follow the plan's formula accruals, and there are certain other requirements that can lead to a unique definition of the attributed benefit for various decrements, as discussed below.

If the retirement benefit, for example, has a years-of-service maximum, then the proration runs over the shorter of actual service or the maximum service. If the maximum for a given plan member occurs at age m, for example, then the proration factor in the above defined attributed benefit would be $(m - y)$ for $k > m$ instead of $(k - y)$. In this case, the attributed benefit increases for ages beyond age m only because of assumed salary increases. If the maximum applies equally to each ancillary benefit, then the attributed benefit can apply to all decrements; otherwise, a different attributed benefit must be defined.

If the benefit formula accrual rate is non-linear (for example, x percent of salary for the first 15 years, and y percent thereafter), then the attributed benefit must be defined as the current year's benefit accrual percentage times the employee's projected salary at age k. This allocation, unlike the one in (11.3a), can produce an ill-behaved benefit allocation, exacerbated if combined with a maximum benefit, for example. SFAS 87 also makes a distinction for non-service related benefits that are subject to vesting, in which case the attribution period is over the non-vested ages, with the entire benefit being allocated at the point of full vesting. On the other hand, non-service related benefits that are not subject to vesting are to be prorated in the manner set forth in (11.3a).[8]

In addition to the best-estimate actuarial assumptions discussed above, if the plan sponsor has recurring plan changes for active and/or retired employees (e.g., systematic increases in a career average benefit formula), then such benefit increases are to be anticipated in the calculation of the PBO. As with the ABO, limitations imposed by federal statute for determining any maximum tax deductible contributions (e.g., non-indexed benefit and salary limits) must reflect expected increases.[9]

For disclosure purposes, the PBO must be projected to the sponsor's fiscal year end (unless a new valuation has been completed within 3 months prior to this date). Again, a valuation can be performed on a projected census or, as was the case for the ABO, the valuation results can be projected. The following approximation can be used (assuming a full year projection):

$$(PBO)_{t+1} \approx \left[(PBO)_t + (SC)_t\right](1+i) - E(B)_t\,(1+\tfrac{1}{2}i) \qquad (11.3b)$$

where $(SC)_t$ is the normal cost under the constant dollar benefit prorate method (termed "service cost" under SFAS 87, as discussed in the following section). Since the assumptions and benefits valued must be appropriate for the measurement date, not the annual valuation date, a new valuation may be required before the results can be projected by (11.3b).

NET PERIODIC PENSION COST

Table 11–1 outlines the components of the net periodic pension cost, each of which is discussed in this section. For this dis

[8]The FASB may not have had enough exposure to the variety of pension plan designs in formulating the attribution methodology for SFAS 87. The attribution methodology given in (11.3a) is preferable, on both theoretical and practical grounds, to the allocation methodology set forth in SFAS 87.

[9]These items must also be included in the plan's service cost, yet to be discussed.

TABLE 11-1

Net Periodic Pension Cost Components

Service Cost:	Normal cost under constant dollar benefit prorate method, adjusted with interest to end of year
+	
Interest Cost:	PBO (adjusted for expected distributions during year) times discount rate
–	
Expected Return on Assets:	Market-related value of assets (adjusted for expected distributions and contributions during year) times expected rate of return on assets
+	
Amortization Costs:	Amortization Methodology:
Transition Obligation (Asset)	Straight line over average future service of plan participants expected to receive benefits or, optionally, 15 years, if greater
+	
Prior Service Cost	Fixed schedule over the future service of plan participants expected to receive benefits
+	
Net Loss (Gain)	Rolling schedule over future service of plan participants expected to receive benefits

cussion, it is assumed that the valuation is on the first day of the plan sponsor's fiscal year.[10]

Service Cost

The service cost (SC) is defined as the normal cost under the constant dollar version of the benefit prorate method, increased by a full year's interest. The benefit accrual under this method is

[10]The service cost, if evaluated prior to this time, can be projected to the beginning of the fiscal year with appropriate approximations and adjustments. The assumptions are those used in the prior fiscal year's measurement date, while the benefits are as of the beginning of the fiscal year, generally the same unless a plan change has been adopted. However, the PBO used in this calculation, and also projected to the beginning of the measurement date, will generally not be the PBO disclosed at the end of the year, since another valuation will undoubtedly have been made by that time.

equal to the accrued benefit at each future age prorated by years of service to that age.[11]

The symbol $(SC)_x$ is used to denote the service cost under SFAS 87 for an employee age x. Equation (11.4) defines the service cost, which is identical to (11.3a) except that the constant dollar accrued benefit is replaced by the constant dollar benefit accrual (equal to the change in the accrued benefit from age x to age $x + 1$), and one year's interest is included in the SFAS 87 service cost: [12]

$$(SC)_x = \left[\sum_{k=x}^{r''} {}^{CD}b_{k}\, {}_{k-x}p_x^{(T)} v^{k-x} \left(q_k^{(t)}\, {}^{v}F_k + q_k^{(d)}\, {}^{d}F_k + q_k^{(m)}\, {}^{s}F_k + q_k^{(r)}\, {}^{r}F_k \right) \right] (1+i) . \tag{11.4}$$

The service cost for the entire plan during year t, $(SC)_t$, is equal to the sum of (11.4) for each active plan member. There are no other choices available under SFAS 87 for determining this component of the net periodic pension cost. Hence, the FASB has eliminated a potential source of cost variation among plans and, for that matter, the same plan over time.

Some would argue that the methodology of prorating benefits by service is an odd conclusion for the FASB to have reached. Two of the fundamental propositions set forth in SFAS 87 are that pension benefits represent deferred wages and that the projected benefit should be earned, and accounted for, proportionately, as opposed to simply accounting for the legally earned benefit accrual. Logic, then, would lead one to prorate the projected benefit by salary. In other words, the normal cost under the constant percent version of the benefit prorate method would be appropriate. The service cost under this method generally would be somewhat higher; however, the amortization costs (yet to be discussed) would be lower since the actuarial liability under the constant percent version is less than that for the constant dollar version of this method.

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[11]The previous discussion regarding the proration of benefits for the PBO, including maximum service limits and non-linear benefit accruals, applies also to the service cost equation.

[12]The interest cost is included since the service cost, while calculated at the beginning of the year, is reported at the end of the year. This interest cost could have been included in the SFAS 87 "interest cost" component of net periodic pension cost.

Whereas the service cost is the normal cost of a specific actuarial cost method (increased for one year's interest), the second and third pension cost components (i.e., SFAS 87 interest cost and amortization cost) are, in effect, what has been termed heretofore as the plan's supplemental cost.

Interest Cost

The second component of the net periodic pension cost is interest on the projected benefit obligation (PBO), with the latter reduced for expected benefit payments throughout the year, $E(B)_t$. Thus, the interest cost (IC) is as follows, where i is the discount rate used in the PBO calculation and the time-weighted expected benefit payments is assumed to be 50 percent of such payments for illustrative purposes:

$$(IC)_t = i\left[(PBO)_t - \tfrac{1}{2}E(B)_t\right]. \tag{11.5a}$$

As noted previously, the discount rate used in (11.5a) must represent a current settlement rate; hence, it will change whenever there is a material change in such rates. Generally, as the discount rate increases, the interest cost component of the net periodic pension cost decreases (and vice versa), since an increase in i generally produces a greater porportionate decrease in $(PBO)_t$.

Expected Return on Assets

The third component is the expected return on the market-related value of assets (MRA), with the latter adjusted for expected benefit payments, $E(B)_t$, and expected contributions, $E(C)_t$, during the year (the time-weighting factor is again assumed to be 50 percent; however, this would not be appropriate if either of these items was not uniform throughout the year):

$$(EROA)_t = i'\left[(MRA)_t - \tfrac{1}{2}E(B)_t + \tfrac{1}{2}E(C)_t\right] \tag{11.5b}$$

where i' is equal to the plan sponsor's best-estimate of the expected long-term return on assets. This interest rate, unlike the rate used with the PBO, is not intended to fluctuate substantially in the short term. Moreover, there is considerably more latitude in selecting this rate than the discount rate used with the PBO. If plan assets are substantial, there is an opportunity for the plan sponsor to exert some management over this element of net periodic pension cost, undoubtedly a result not intended by the FASB.

The market-related value of assets used in (11.5b) may represent the actual market value or, optionally, a smoothed market value (such as 3 or 5 year average), provided that the smoothing process does not extend beyond 5 years.[13]

The interest cost less the expected return on assets represents interest on the unfunded obligation. This can be seen more easily if the discount rate and expected return on assets is assumed to be identical and equal to i. Equation (11.5a) less (11.5b) produces

$$\begin{matrix}\textit{Net}\\ \textit{Interest}\\ \textit{Cost}\end{matrix} = i\left[(PBO)_t - (MRA)_t - \tfrac{1}{2}E(C)_t\right]. \tag{11.5c}$$

In the context of pension funding, (11.5c) would be the annual interest charge on the unfunded liability, with the excess supplemental cost payment being applied toward the amortization of the unfunded liability. For SFAS 87, the interest cost is determined separately from the principal amortization. Again, denoting i as the interest cost and i' as the expected return on assets, (11.5a) less (11.5b) can be written as follows:

$$\begin{aligned}\begin{matrix}\textit{Net}\\ \textit{Interest}\\ \textit{Cost}\end{matrix} &= i\left[(PBO)_t - (MRA)_t - \tfrac{1}{2}E(C)_t\right]\\ &\quad + (i-i')\left[(MRA)_t - \tfrac{1}{2}E(B)_t + \tfrac{1}{2}E(C)_t\right]\end{aligned} \tag{11.5d}$$

In other words, SFAS 87 includes a funding-type interest cost plus the difference in interest rates, $(i - i')$, times the market-related value of assets (adjusted for expected cash flow). The latter can have a substantial affect on the net periodic pension cost, depending on the difference in interest rates and the size of plan assets. Moreover, since it is impossible for the plan's experience to satisfy two interest rates simultaneously, the dual interest assumption under SFAS 87 guarantees that interest-based gains and losses will occur each year, unless $i = i'$.

[13]Chapter 10 included a discussion of alternative asset valuation methods, some of which would qualify for determining the market-related value of assets under SFAS 87.

Amortization Cost

Three different amortization methods are required under SFAS 87, depending on the source of the unrecognized obligation (or asset). Since the interest cost is determined separately, as discussed above, all of the methods amortize only the principal portion of the obligation (asset).

Future Service of Employees Expected to Receive Benefits. The three amortization methods yet to be discussed involve *the future service of employees expected to receive benefits*. The future service (FS) for an employee age x is as follows:

$$(FS)_x = \sum_{k=x}^{r''-1} {}_{k-x}p_x^{(T)}. \tag{11.6a}$$

Equation (11.6a), however, does not necessarily reflect the future service of a plan participant *expected to receive benefits*. This can be seen by transforming (11.6a) into service table notation:

$$(FS)_x = \sum_{k=x}^{r''-1} \frac{l_k^{(T)}}{l_x^{(T)}} \tag{11.6b}$$

$$= \sum_{k=x}^{r''-1} \frac{\sum_{t=k}^{r''-1} \left[d_t^{(t)} + d_t^{(d)} + d_t^{(m)} + d_t^{(r)}\right]}{l_x^{(T)}}. \tag{11.6c}$$

The terms in the numerator of (11.6c) indicate "when and how" the individuals at each future age will leave the plan; however, there is no indication of whether they will be eligible for a benefit. With $E_t^{(v)}$ defined as unity if the individual is eligible for a benefit (for this symbol, a vested benefit) at age t and as zero if not eligible, equation (11.6d) properly accounts for those who are expected to receive a benefit at the time of decrement:

$$(FS)_x = \sum_{k=x}^{r''-1} \frac{\sum_{t=k}^{r''-1} \left[E_t^{(v)}\, d_t^{(t)} + E_t^{(d)}\, d_t^{(d)} + E_t^{(s)}\, d_t^{(m)} + E_t^{(r)}\, d_t^{(r)}\right]}{l_x^{(T)}}. \tag{11.6d}$$

The *average* future service (AFS) is found by summing (11.6d) for all plan members and dividing by the number of participants *expected to receive benefits*. The "expected to receive benefits" (ERB) function for one individual is defined by

$$(ERB)_x = \sum_{k=x}^{r''-1} {}_{k-x}p_x^{(T)}\left[E_k^{(v)}q_k^{(t)} + E_k^{(d)}q_k^{(d)} + E_k^{(s)}q_k^{(m)} + E_k^{(r)}q_k^{(r)}\right]. \tag{11.7}$$

The *average* future service of plan participants expected to receive benefits is given by

$$(AFS)_t = \frac{(FS)_t}{(ERB)_t} \tag{11.8}$$

where the t subscript indicates that these functions are for all plan participants in year t. Average future service, as defined in (11.8) typically will fall in the range of 8 to 15 years.

Prepaid (Accrued) Expense. Another item involved in determining the SFAS 87 amortization cost is the so-called prepaid (accrued) expense. If cumulative contributions to the pension plan from its inception exceed the cumulative accounting expense, the difference is referred to as a *prepaid expense*. In this case, pension funding has "run ahead" of pension accounting, creating an accounting asset that must be accounted for (or amortized) in the future. On the other hand, if the cumulative pension expense exceeds cumulative contributions, the difference is an *accrued expense*. In this case, pension accounting has "run ahead" of pension funding, creating an accounting obligation that must be amortized in the future.

Denoting $(CC)_t$ as cumulative contributions from plan inception to year t and $(CE)_t$ as cumulative expense, a positive difference in (11.9) is a prepaid expense, while a negative difference is an accrued expense (even though it is frequently written without the negative sign):[14]

$$\textit{Prepaid (Accrued) Expense} = \sum (CC)_t - \sum (CE)_t. \tag{11.9}$$

Transition Obligation (Asset). This item, defined as the beginning of the first year to which SFAS 87 applies, is equal to (1) the PBO less (2) the market value of assets (MA) plus any prepaid (or less any accrued) pension expense:

$$\begin{array}{l}\textit{Transition}\\ \textit{Obligation}\\ \textit{(Asset)}\end{array} = (PBO)_t - (MA)_t + \left[\sum (CC)_t - \sum (CE)_t\right]. \tag{11.10}$$

[14]Cumulative contributions and expense are easily determined by adding the current year's values to a running balance.

If the bracketed term in (11.10) is negative, then an accrued expense is deducted from the otherwise determined obligation, whereas a prepaid expense is added if the term is positive. This is logical because an accrued expense implies that a portion of the obligation has already been accounted for (or expensed) and should be excluded from the amortization. Similarly, a prepaid expense implies that a portion of the obligation has not yet been accounted for (or expensed); hence, it should be included in the amortization.

The transition obligation (asset) is amortized on a straight line basis (i.e., equal installments of principal) over the *average future service of plan participants expected to receive benefits*, equation (11.8) determined at the transition date, or, optionally, 15 years, if greater.[15] This schedule is fixed at the date of transition and is not affected by future changes in benefits or assumptions. The annual amortization cost will be positive if the value of (11.10) is positive (indicating a transition liability) and negative if the transition item is an asset.

Prior Service Cost. The prior service cost is defined as the change in the PBO due to a plan amendment. Its value is zero in the first year of SFAS 87 application, and takes on positive (negative) values as plan benefits are increased (decreased) after this time. The prior service cost is amortized over the future service of plan participants expected to receive benefits as of the date of the plan amendment (i.e., a closed group amortization). This is in contrast to the transition obligation which uses the *average* future service of plan participants expected to receive benefits. Each year's minimum amortization is based on the proportion of the originally determined future service that is expected to be worked in the upcoming year. Thus, the amortization cost (AC) during age k for prior service cost (PSC) created at the employee's age x is given by

$$(AC)_k = \frac{{}_{k-x}(FS)_x}{(FS)_x}\,(PSC)_x \quad (k \geq x) \tag{11.11}$$

where ${}_{k-x}(FS)_x$ represents the future service from age k to $k + 1$. The amortization schedule is determined once and for all at the date of plan amendment by recording the subtotals for each value

[15]If all, or almost all, of the plan's participants are inactive, their average life expectancy can be used as the amortization period.

of k in (11.6d). The amortization cost for the entire plan is found by aggregating the numerator and denominator of (11.11) and applying this fraction to the plan's past service cost for the plan amendment in question.

The prior service cost amortization represents a continually decreasing portion of the initial prior service cost, and the amortization schedule is not complete until the last employee at the time of the plan amendment leaves employment. Since this amortization approach is numerically complex, the plan sponsor can elect to amortize the prior service cost more rapidly, for example, over the *average* future service period as defined by (11.8), or any consistently applied more rapid amortization (even immediate recognition). The annual prior service cost amortization payments will be positive if the plan amendment increases the PBO (the usual case) and negative if the PBO decreases as a result of the plan change.

Net Loss (Gain). The cumulative unrecognized net loss (gain) (i.e., the cumulative amount from the date SFAS 87 is applied less any prior amortizations) is determined in the following manner:

$$\begin{matrix}\text{Loss} \\ \text{(Gain)}\end{matrix} = \text{PBO} - \left[\begin{matrix}\text{Market-Related} \\ \text{Value of Assets}\end{matrix} + \begin{matrix}\text{Unrecognized} \\ \text{Transition} \\ \text{Obligation}\end{matrix} + \begin{matrix}\text{Unrecognized} \\ \text{Prior Service} \\ \text{Cost}\end{matrix} - \begin{matrix}\text{Prepaid} \\ \text{(Accrued)} \\ \text{Pension} \\ \text{Expense}\end{matrix}\right]. \tag{11.12}$$

The portion of this loss (gain) that must be amortized is the value produced by (11.13), provided that the value is positive; otherwise there is no minimum amortization:

$$\begin{matrix}\text{Minimum} \\ \text{Amount to be} \\ \text{Amortized}\end{matrix} = \left|\begin{matrix}\text{Loss} \\ \text{(Gain)}\end{matrix}\right| - .10 \bullet \text{Max}\left[\begin{matrix}\text{PBO} \\ \text{Market-Related} \\ \text{Value of Assets}\end{matrix}\right]. \tag{11.13}$$

In words, the cumulative gain (or loss) in excess of 10% of the PBO (or market-related value of assets, if larger) must be amortized.

The amortization is based on a "rolling" schedule, as opposed to the two types of fixed schedules used with the transition obligation and prior service cost. Each year the value of (11.8) is evaluated for the entire group of active participants, with its reciprocal being used to determine the proportion of the minimum loss (gain) amortized in the current year.[16] In the following year a new minimum amount, if any, is determined and amortized according to a newly determined reciprocal of (11.8) and so forth.[17] The amortization payment for the year in question will be positive for a net loss and negative for a net gain.

A corridor smaller than 10%, or even a zero corridor, can be used in (11.13) if done consistently. Since each year's amortization is determined independently of the prior year's, it is possible to have loss (gain) amortization in one year and not the next.

The net loss (gain) reflects both the experience deviations during the year as well as actuarial assumption changes. This is unique to pension accounting, since for pension funding purposes, actuarial assumption changes are treated as a separate item for amortization purposes.

Disclosure of Net Periodic Pension Cost

The components of the net periodic pension cost must be disclosed in the employer's financial statements. SFAS 87 requires a disclosure of the *actual return on assets* along with the other components discussed above. Since these statements are prepared after the close of the firm's fiscal year, the actual return on plan assets is known.

Table 11–2 shows the disclosure items. Note that the asset-based loss (gain) is subtracted from the expected return on assets (line 3 in Table 11–2) and then added into the amortization items (line 4). While these offsetting entries do not affect the value of the net periodic pension cost, the disclosure shows the actual return on assets.

The asset-based loss (gain) can be determined by

$$\begin{matrix}\textit{Asset-Based} \\ \textit{Loss (Gain)}\end{matrix} = (EROA)_t - \left[(MA)_{t+1} + B_t - C_t - (MA)_t\right] \quad (11.14)$$

[16]For plans primarily covering inactive participants, the average life expectancy can be used.

[17]It should be noted that this "rolling" schedule would never amortize a given dollar amount, since amortizing $1/n$ th of each year's remaining balance will never fully extinguish the original balance. This is not necessarily a deficiency in the amortization methodology, however, since actuarial gains and losses should be offsetting over time, especially if the underlying assumptions are best-estimates, as required under SFAS 87.

where

$$(EROA)_t = \text{expected return on assets, equation (11.5b)}$$
$$B_t = \text{actual distribution (benefit payments)}$$
$$C_t = \text{actual contributions}$$
$$(MA)_t = \text{market assets at beginning of year}$$
$$(MA)_{t+1} = \text{market assets at end of year } t.$$

TABLE 11-2

Disclosure of Net Periodic Pension Cost

1. Service Cost	Equation (11.4)
2. Interest Cost	Equation (11.5a)
3. Actual Return on Assets	Expected Return on Assets – Asset-Based Loss (Gain)
4. Net Amortization and Deferrals	Transition Obligation + Prior Service Cost + Loss (Gain) + Asset-Based Loss (Gain)
5. Net Periodic Pension Cost	(1) + (2) – (3) + (4)

Numerical Illustration of Net Periodic Pension Cost

Table 11–3 shows a 20-year projection of the net periodic pension cost and its various components for the model pension plan. The valuation and experience assumptions are identical to those listed in Tables 10–8 and 10–9 of Chapter 10 used to project minimum and maximum statutory contributions. The expected return on assets is an additional assumption required for the SFAS 87 projection; 8 percent is assumed in the projection. The "expected return on assets" and the "asset-based loss (gain)" are shown in Table 11–3 for clarity even though they are not disclosure items *per se*.

TABLE 11-3

Projection of SFAS 87 Net Periodic Cost Disclosure Items
(End-of-Year Values as Percent of Payroll)

Year	(1) *Service Cost*	(2) *Interest Cost*	(3) *Expected Return*	(4) *Asset Loss (Gain)*	(5) *Actual Return* (3 − 4)	(6) *Transition Obligation Amortization*	(7) *Prior Service Cost Amortization*	(8) *Loss (Gain) Amortization*	(9) *Net Amortization* (6 + 7 + 8 − 4)	(10) *Net Periodic Pension Cost*
1	4.83	8.10	7.36	-1.66	9.02	1.01	0.00	0.00	2.67	6.57
2	4.75	8.43	7.65	-2.04	9.69	0.97	0.00	0.00	3.01	6.49
3	4.71	8.68	7.96	-2.37	10.32	0.92	0.00	0.00	3.29	6.36
4	4.70	8.98	8.29	-2.63	10.92	0.88	0.09	0.00	3.60	6.37
5	4.71	9.15	8.64	-2.83	11.47	0.84	0.09	0.00	3.75	6.15
6	4.72	9.31	9.01	3.02	5.99	0.80	0.09	0.00	-2.14	5.91
7	4.73	9.56	9.25	3.32	5.92	0.76	0.19	0.00	-2.38	5.98
8	4.74	9.69	9.38	3.52	5.85	0.72	0.18	0.00	-2.62	5.95
9	4.75	9.82	9.44	3.65	5.79	0.68	0.17	0.00	-2.79	5.99
10	4.75	10.05	9.47	3.71	5.76	0.65	0.28	0.00	-2.78	6.27
11	4.76	10.16	9.50	-2.04	11.54	0.62	0.27	0.00	2.93	6.31
12	4.77	10.26	9.65	-2.51	12.16	0.59	0.26	0.00	3.36	6.23
13	4.77	10.47	9.90	-2.90	12.79	0.56	0.37	0.00	3.83	6.28
14	4.77	10.55	10.20	-3.21	13.41	0.53	0.35	0.00	4.10	6.02
15	4.77	10.63	10.52	-3.43	13.95	0.51	0.32	0.00	4.26	5.72
16	4.78	10.82	10.83	3.64	7.19	0.00	0.41	0.00	-3.24	5.18
17	4.78	10.88	10.98	3.95	7.03	0.00	0.39	0.00	-3.56	5.06
18	4.78	10.93	11.01	4.15	6.86	0.00	0.35	0.00	-3.80	5.05
19	4.78	11.10	10.96	4.25	6.70	0.00	0.44	0.00	-3.82	5.36
20	4.78	11.13	10.86	4.28	6.59	0.00	0.42	0.00	-3.86	5.46

The service cost, like the normal cost projection in Chapter 10, is relatively constant throughout the projection period.[18] The interest cost increases continuously, from about 8 percent of payroll to 11 percent, an increase resulting from the PBO growing faster than payroll due to the aging of the population. The expected return on assets parallels the interest cost, since assets are approximately equal to the PBO (see Table 11–5 yet to be discussed). The asset loss (gain) alternates between a loss and a gain every 5 years as a result of the actual return on assets assumed in the experimental design. Actual returns, of course, are equal to the expected return less the asset loss (or gain).

The amortization of the transition obligation is a continually decreasing percent of payroll over 15 years, starting at 1.01 percent and reducing to .51 percent by the 15th year. The *ad hoc* COLA's, amounting to 25 percent of cumulative inflation every 3 years, have only a minor amortization cost in the early years of the projection, reaching an ultimate level of about .4 percent of payroll by the end of 20 years.

The amortization of the loss (gain) from investment experience alternating from 5 percent higher to 5 percent lower than the 10 percent expected return on assets has no impact on costs. This illustrates that a 10 percent corridor of the PBO (or market-related value of assets, if greater) provides a substantial cushion for smoothing the actual return on assets.

The net periodic pension expense, representing the interaction of these components, fluctuates in the range of 5.5 to 6.5 percent of payroll throughout the projection period.

BALANCE SHEET

The SFAS 87 accounting requirements nearly always cause contributions and pension expense to be different. This contrasts with past accounting rules and practice, under which contributions and expenses were nearly always equal. Thus, there are annual balance sheet entries recording the difference between these

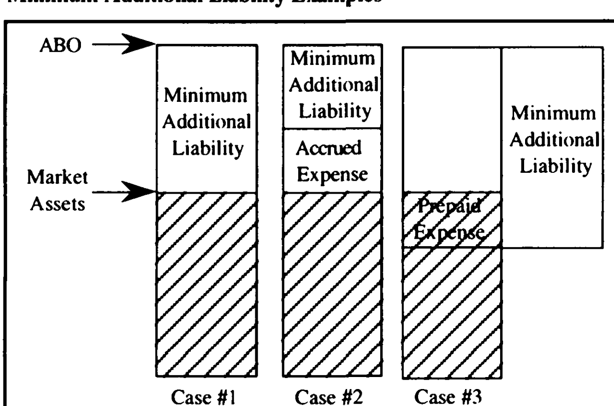
[18]Since the projection of contributions in Chapter 10 is based on the benefit prorate cost method, its normal cost is the same as the service cost for SFAS 87 purposes under the specified assumptions (and assuming there are no applicable maximum benefit and salary limitations). The service cost shown in Table 11–3 is 8 percent larger than the normal cost shown in Table 10–8 because it includes one year's interest.

two quantities, with a prepaid expense (accounting asset) being generated if cumulative contributions exceed cumulative expenses, and an accrued expense (accounting liability) recorded when the reverse it true.

Minimum Additional Liability

SFAS 87 defines a *minimum additional liability* that must be entered on the sponsor's balance sheet. This liability is equal to the difference between the ABO and the market value of assets (reduced for any accrued expense or increased for any prepaid expense). Figure 11–1 illustrates three cases for which a minimum additional liability is required. In case #1 the minimum additional liability is simply the difference between the ABO and market assets, since there is no accrued or prepaid expense. Case #2 has a smaller minimum additional liability because of the existence of an accrued expense (i.e., expensing has "run ahead" of funding). Since an accrued expense is an accounting liability, it is deducted in determining the minimum additional liability recorded on the sponsor's balance sheet. Case #3 has a larger minimum additional liability because of a prepaid expense (i.e., funding has "run ahead" of expensing). Since the prepaid expense is an accounting asset, it must be added in determining the minimum additional liability.

FIGURE 11–1
Minimum Additional Liability Examples



Intangible Asset

If an additional liability is recorded on the balance sheet, an intangible asset can be recorded to avoid a reduction in stockholders' equity. However, the *maximum* intangible asset is limited to the sum of (1) the unrecognized transition obligation and (2) the unrecognized prior service cost. If the minimum additional liability exceeds the maximum intangible asset, stockholders' equity is reduced by the difference.

Reconciliation of Funded Status

The employer's financial statements must include a reconciliation of funded status. The items disclosed in this reconciliation are as given in Table 11–4. It should be noted that the ABO and PBO disclosure items might be based on a valuation different from the valuation used to determine the net periodic pension cost. Figure 11–2 has been constructed to illustrate this situation for a plan having a 1/1 to 12/31 valuation year and a 7/1 to 6/30 fiscal year.

Numerical Illustration of Financial Statement Disclosure Items

A 20-year projection of the SFAS 87 disclosure items are shown in Table 11–5, based on the same assumptions as the projection of the net periodic pension cost given previously. The VBO is within 2 percentage points of the ABO, a result that would be expected given the maturity of the population and a 5-year vesting assumption. The ABO is 78 percent of the PBO at the outset of the projection and climbs steadily to 83 percent by the end of 20 years. Market assets were equal to 85 percent of the PBO in the first year and, because of the initially favorable investment experience, climb to 100 percent after 5 years. This relationship fluctuates between about 90 and 105 percent, as the investment experience fluctuates from 5 percent to 15 percent during the 20-year projection.

The unrecognized transition obligation equals a rapidly declining percentage of the PBO. The unrecognized prior service cost is quite small, never exceeding 2.5 percent of the PBO. The unrecognized loss (gain) from the alternating investment experi-

TABLE 11–4
Reconciliation of Funded Status

1. PBO	Year-end value {or projected year-end value from equation (11.3b) }
2. Market Assets	Year-end value
3. Funded Status	(1) – (2)
4. Unrecognized Transition Obligation	Beginning of year value less current year's amortization amount
5. Unrecognized Prior Service Cost	Beginning of year value less current year's amortization amount
6. Unrecognized Net Loss (Gain)	(3) – (4) – (5) + {Prepaid (Accrued) Expense at B.O.Y. – Net Periodic Pension Cost + Contributions}
7. Prepaid (Accrued) Pension Cost	(3) + (4) + (5) + (6)
8. Additional Liability	Based on year-end value of ABO {or projected year-end value from equation (11.1b) }
9. (Pension Liability) Prepaid Pension Cost	(7) + (8)

FIGURE 11–2
Valuation Year vs. Fiscal Year

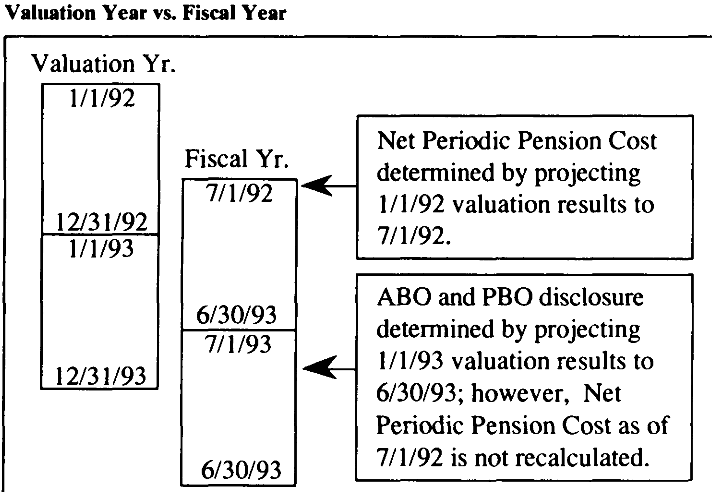


TABLE 11-5

Projection of SFAS 87 Financial Statement Disclosure Items
(Values Expressed as Percent of PBO)

Year	*VBO*	*ABO*	*PBO*	*Market Assets*	*Funded Status*	*Unrecognized Transition Obligation*	*Unrecognized Prior Service Cost*	*Unrecognized Loss (Gain)*	*Additional Liability*	*Prepaid (Accrued) Cost*
1	76.83	77.99	100.00	85.33	14.67	14.67	0.00	0.00	0.00	0.00
2	77.58	78.84	100.00	88.42	11.58	12.58	0.00	-1.60	0.00	-0.60
3	78.03	79.37	100.00	91.76	8.24	10.81	0.00	-3.41	0.00	-0.84
4	78.51	79.92	100.00	94.25	5.75	9.19	0.96	-5.29	0.00	-0.89
5	78.70	80.18	100.00	97.55	2.45	7.86	0.82	-7.21	0.00	-0.99
6	78.84	80.41	100.00	100.64	-0.64	6.68	0.70	-9.17	0.00	-1.16
7	79.31	80.82	100.00	97.63	2.37	5.56	1.61	-6.32	0.00	-1.51
8	79.53	81.01	100.00	95.26	4.74	4.64	1.37	-3.53	0.00	-2.27
9	79.75	81.19	100.00	92.87	7.13	3.81	1.15	-0.83	0.00	-3.01
10	80.14	81.57	100.00	89.78	10.22	3.03	2.01	1.74	0.00	-3.43
11	80.32	81.73	100.00	88.28	11.72	2.38	1.69	4.19	0.00	-3.46
12	80.49	81.89	100.00	91.76	8.24	1.79	1.40	2.29	0.00	-2.77
13	80.85	82.24	100.00	94.72	5.28	1.25	2.23	0.16	0.00	-1.64
14	80.99	82.37	100.00	98.86	1.14	0.79	1.84	-2.05	0.00	-0.55
15	81.12	82.50	100.00	102.82	-2.82	0.37	1.50	-4.36	0.00	0.33
16	81.44	82.81	100.00	105.05	-5.05	0.00	2.33	-6.62	0.00	0.76
17	81.53	82.91	100.00	102.80	-2.80	0.00	1.93	-3.96	0.00	0.77
18	81.61	82.99	100.00	100.07	-0.07	0.00	1.56	-1.26	0.00	0.37
19	81.89	83.27	100.00	96.11	3.89	0.00	2.42	1.38	0.00	-0.09
20	81.94	83.32	100.00	93.65	6.35	0.00	2.01	3.96	0.00	-0.38

ence never exceeds 10 percent of the PBO. Since this is below the minimum amortization corridor, the loss (gain) amortization payments are zero, as previously observed in Table 11–3.

The projection did not require an additional liability to be recorded on the sponsor's balance sheet because assets at all times exceeded the ABO. The last column in Table 11–5 shows that contributions were approximately equal to pension costs, since only a minimum prepaid (accrued) cost occurs during the projection period.

Table 11–6 shows a reconciliation of SFAS 87 values from the end of one year to the end of the next year. This format, with illustrative numbers included, may be useful in better understanding the interaction of the various SFAS 87 components.

CRITIQUE OF SFAS 87

In many respects the SFAS 87 promulgation achieves its objectives of consistency, uniformity, and comparability of pension accounting among plan sponsors. There are several aspects of the promulgation, however, that, if changed, would result in an improved set of accounting procedures. These are briefly discussed in the order of their importance.

Salary vs. Benefit Proration

The service proration methodology used for the service cost and PBO should be changed to a proration based on salary. As noted earlier, the salary proration is consistent with the view that pensions are deferred wages. For example, by the time an employee earns 50 percent of his (or her) career compensation, 50 percent of the career pension benefit should be deemed to have been earned. A byproduct of this recommendation is that the salary assumption becomes less critical, since it is both in the projection of the benefit and in the proration.[19] This would further standardize pension cost accounting among plan sponsors.

[19]The salary assumption in the proration should be the same as in the benefit projection. Thus, a flat benefit formula (e.g., $150 per year of service) would not be affected by the salary proration methodology.

TABLE 11-6

Reconciliation of SFAS 87 Values

		Plan Assets	PBO	Transition Obligation	Loss (Gain)	Prior Service Cost	Periodic Pension Cost	Accrued (Prepaid) Pension Cost
	End of Prior Year	100	(100)	(50)	(10)	40		20
	Service Cost		(10)				10	
	Interest Cost		(15)				15	
	Expected Return on Assets	10					(10)	
Amortization Amounts for:	Transition Obligation			5			(5)	
	Loss (Gain)				2		(2)	
	Past Service Cost					(3)	3	
	Periodic Pension Cost						11	11
	Benefit Payments	(2)	2					
	Contributions	4						(4)
Gain (Loss)	Asset-Based	3			(3)			
	Liability-Based		5		(5)			
	End of Current Year	115	(118)	(45)	(16)	37		(27)

Linear vs. Formula Proration

The projected benefit should be uniformly prorated from entry age to each future decrement age. This is in contrast to the SFAS 87 procedure of allocating benefits according to the plan's benefit formula which, due to various maximums, front or back loading formulas, step-up or step-down formulas, offsets and so forth, can produce anomalous prorations. This recommendation should be implemented regardless of whether the service or salary proration scheme is used.

Interest Cost

Basing the interest cost on a so-called settlement rate, reflective of the cost of annuities, overemphasizes a "wind up" or termination contingency. Annuity rates include insurance company risk, expense, and profit charges which should not be part of the interest rate determination. On the other hand, if the FASB desires the discount rate to reflect market conditions, then the rate should be keyed to the spot rate on investment-grade, long-term corporate bonds as of the measurement date. Changes should be made only when the index causes the prior year's rate to be changed by more than a meaningful amount, such as 50 basis points. In any case, the author favors allowing the long-term return on assets to be selected with an eye towards management's best estimate of such return, taking into account the asset allocation policy of the plan.

Gain (Loss) Amount

It would be useful to show the effect of discount rate changes, as well as other actuarial assumption changes, separate from the effects of experience differing from the underlying assumptions. This is particularly important in light of the continually changing discount rate. This should be done whether the amortization of these two items are treated the same or differently.

Gain (Loss) Amortization

The 10 percent corridor around the larger of the PBO or market-related assets can produce some strange results. For example, (1) a plan that has assets equal to or larger than the PBO has a 10 percent asset-based cushion, (2) a plan that has assets equal to 50 percent of the PBO has a 20 percent asset-based cushion, and (3) a plan that has assets equal to 20 percent of the PBO has a 50 percent asset-based cushion. If anything, one might be inclined to be harsher on lower funded plans rather than more lenient in the degree of asset volatility that enters the accounting expense. If the *lower* of these two values were used, then the benefit to lower funded plans would be removed; however, plans with funded ratios in excess of 100 percent would still be penalized (e.g., a plan with a 200 percent funded plan would have only a 5 percent cushion). This problem could be resolved if one simply based the corridor on assets alone, but then PBO-based gains and losses would be affected differently for different funded ratios.

The shortcoming in the gain (loss) corridor is that it focuses on the wrong item. Since the ultimate objective is to adjust *costs* if gains and losses become too significant, on the one hand, without being overly sensitive, on the other, the corridor might be based on the service cost. For example, the amortization amount associated with the full unrecognized gain (loss) might be determined and, then, the portion of the amortization payment outside a 10 percent corridor of the service cost, for example, might be recognized.

Amortization Periods

The future service of employees, instead of the future service of employees *expected to receive benefits*, should be used. While the SFAS 87 approach is conceptually correct, it is computationally complex and simply not worth the effort, in spite of the availability of high speed computers.

VBO Disclosure

The separate disclosure of the VBO adds little value and should be eliminated as a required disclosure item.

Terminology

Past service cost should be "past service obligation (asset)." Similarly, the transition obligation should be "transition obligation (asset)." Loss (gain) should be "experience loss (gain), and changes in assumptions should be labeled "assumption change obligation (asset)." These cosmetic changes would facilitate understanding, which is one of the SFAS 87 objectives.

Chapter 12

Alternative Actuarial Assumptions

The purpose of this chapter is to investigate the sensitivity of pension costs (1) to changes in valuation assumptions and (2) to changes in the experience of the plan. This analysis will provide insight into the relative importance of the various assumptions used with pension plans.

The valuation sensitivities show the relative impact of each actuarial assumption change on the normal cost and actuarial liability of three actuarial cost methods: the accrued benefit method, the constant dollar benefit prorate method, and the constant percent cost prorate method. The impact on minimum required contributions, of course, would depend on the funded status of the plan.

The experience sensitivities are based on maintaining the valuation assumptions while varying the projected experience of the plan over time. These analyses are based on the constant dollar benefit prorate method, with contributions being made at the minimum required level.

MORTALITY RATES

The impact of mortality is shown in Table 12–1, where the mortality rate multiple indicates the change made to the model assumption. For example, a multiple of .50 indicates that the age-specific rates are reduced to one half the standard rates (except for the rate at the assumed end of the life span, which retains a value of unity), while a multiple of 1.50 indicates a 50 percent increase in the age-specific rates (unless a value greater than unity results, in which case unity is used). As noted in Chapter 2, changes in mortality rates affect the cost of surviving spouse benefits in the opposite direction of the cost of retirement, vesting, and disability benefits.

The impact of variations in the mortality assumption is relatively uniform across funding methods and for both the normal cost and actuarial liability. Variations of up to 25 percent in such rates affect costs and liabilities by 10 to 15 percent, a relatively minor impact given the fact that a 25 percent increase or decrease in mortality rates is a substantial change. The last row of Table 12–1 shows the impact of changing the mortality assumption from the GAM–71 to the GAM–83 table, indicating that long-run costs can be expected to increase by about 7 percent.[1]

TABLE 12–1

Effect of Alternative Mortality Rates

Mortality Rate Multiple	*Accrued Benefit*		*Constant Dollar Benefit Prorate*		*Constant Percent Cost Prorate*	
	NC	*AL*	*NC*	*AL*	*NC*	*AL*
.50	120.3	120.7	121.4	121.1	122.3	121.0
.75	108.9	109.1	109.4	109.2	109.8	109.2
1.00	100.0	100.0	100.0	100.0	100.0	100.0
1.25	86.6	86.4	85.8	86.1	85.2	86.1
1.50	76.7	76.5	75.3	76.0	74.3	75.9
GAM 83	107.2	106.9	107.7	107.2	108.1	107.2

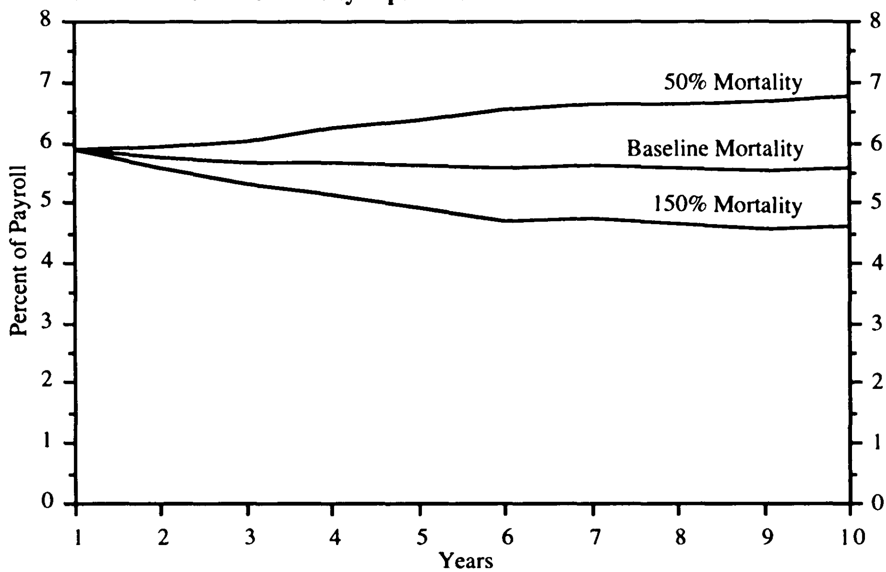
Figure 12–1 shows the financial implications if the valuation assumptions are held constant at the GAM–71 table while the experience of the plan is either 50 percent greater or less than this assumption. By the end of 10 years, costs are affected by about one percent of payroll, or approximately 20 percent of the baseline costs. These results assume that actuarial gains and losses generated from the mortality experience are amortized over 5 years, consistent with the minimum required contribution requirements.

After 10 years of the mortality experience differing from the valuation assumption by 50 percent, the cost impact is about the same as changing the mortality valuation assumption in the initial year, as indicated in Table 12–1. On the other hand, if a plan

[1]Short-term costs could be greater or less than 7 percent, depending on the plan's funded status.

were to experience this mortality and then change the valuation assumption, the net effect on costs would be a combination of the results in Table 12–1 and Figure 12–1, or a cost impact of approximately 40 percent for this example.

FIGURE 12–1
Effect of Alternative Mortality Experience



While valuation mortality rates will occasionally need to be changed, and while experience will fluctuate from the underlying assumption from time to time, variations in the mortality assumption are not likely to have a substantial impact on costs. As Figure 12–1 indicates, even extreme deviations in mortality over an extended period of time have a comparatively minor impact on costs.

TERMINATION RATES

Termination rates for active employees are not only greater in magnitude than mortality rates, they are also subject to considerably more variation, both among plans and for a given plan over time. Although a 50 percent variation in the mortality assumption for a large group of plan members is unlikely, this is not the case for termination rates. Table 12–2 indicates the impact of 25 and 50 percent changes in termination rates for valuation purposes.

Results under the accrued benefit method are hardly affected by these changes, nor is the actuarial liability under the other cost methods. This is the case since the dominant portion of the normal cost under the accrued benefit method, as well as the actuarial liabilities under all methods, are for participants who are at an age for which the withdrawal rates are small or zero. The benefit prorate method shifts the incidence of normal costs to younger ages as compared to the accrued benefit method, and the cost prorate method has an even greater shift; hence, the normal costs under these methods are affected more by a change in termination rates. Since the actuarial liabilities are affected minimally, the plan's supplemental cost will likewise be affected minimally; therefore, the effect on total costs will be less than the effect on normal costs.

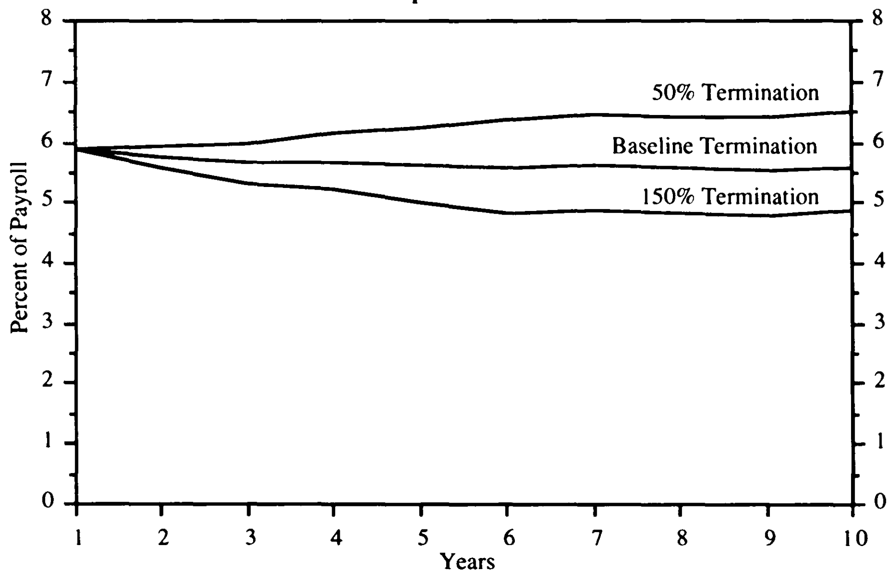
TABLE 12–2

Effect of Alternative Termination Rates

Termination Rate Multiple	*Accrued Benefit*		*Constant Dollar Benefit Prorate*		*Constant Percent Cost Prorate*	
	NC	*AL*	*NC*	*AL*	*NC*	*AL*
.50	102.2	100.4	115.2	104.0	127.4	100.1
.75	101.0	100.2	106.6	101.8	113.3	100.2
1.00	100.0	100.0	100.0	100.0	100.0	100.0
1.25	98.2	99.7	90.4	97.1	77.3	98.9
1.50	96.6	99.4	83.6	94.8	60.2	97.2

Figure 12–2 shows the results of experiencing 10 years of termination rates running 50 percent higher and 50 percent lower than the underlying actuarial assumption. Pension costs are affected by less than 20 percent by the end of the 10-year projection, a result similar to the impact of a 50 percent deviation in mortality rates over this period. As noted previously, however, there is a much greater likelihood of this differing experience occurring with termination rates than with mortality rates. Additionally, as was the case with mortality sensitivities, the results in Table 12–2 would be applicable any time termination rates are changed during the projection in Figure 12–2. Consequently, a plan could experience the combined financial impact of both sensitivity analyses.

FIGURE 12-2
Effect of Alternative Termination Experience



DISABILITY RATES

Table 12-3 displays the financial implications of changing the valuation disability rates by 25 and 50 percent. These changes have virtually no impact on either costs or liabilities. This occurs for two reasons: (1) the disability rates are relatively small and even large proportionate changes in such rates do not have a significant impact, and (2) the change in disability-based costs is largely offset by the change in retirement-based costs. For example, if disability rates are increased, the cost of disability increases but the reduction in retirement-related costs virtually offsets this increase. As indicated in Figure 12-3, there is virtually no perceptible difference in costs over a 10 year period if disability rates deviate by 50 percent from the underlying actuarial assumption.

It will be recalled that the disability provision under the model plan provides the accrued benefit, payable immediately for life, upon disability, provided the employee is age 40 and has 10 years of service. Other disability provisions may be affected

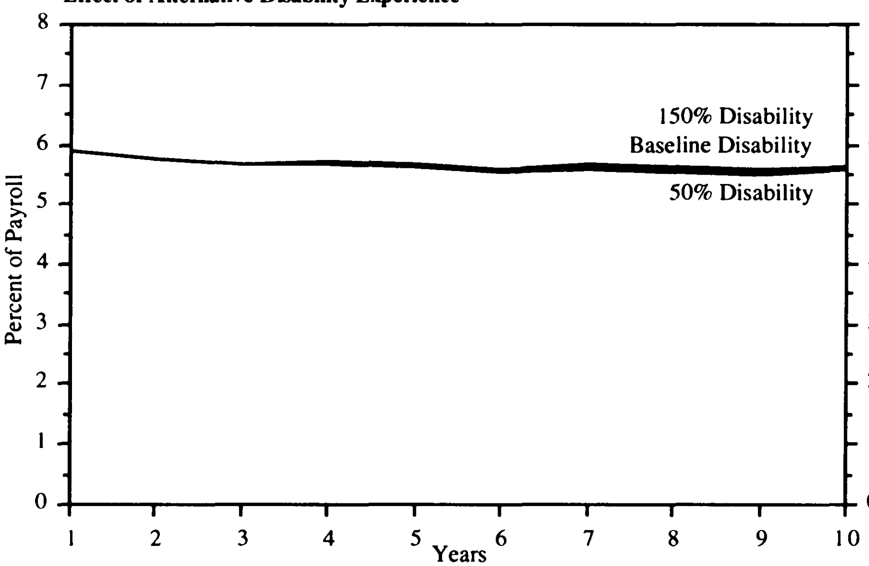
TABLE 12-3

Effect of Alternative Disability Rates

Disability Rate Multiple	Accrued Benefit NC	Accrued Benefit AL	Constant Dollar Benefit Prorate NC	Constant Dollar Benefit Prorate AL	Constant Percent Cost Prorate NC	Constant Percent Cost Prorate AL
.50	98.9	99.4	99.4	99.5	99.7	99.7
.75	99.5	99.7	99.7	99.8	99.8	99.8
1.00	100.0	100.0	100.0	100.0	100.0	100.0
1.25	101.1	100.6	100.6	100.5	100.3	100.3
1.50	102.1	101.2	101.1	100.9	100.6	100.7

FIGURE 12-3

Effect of Alternative Disability Experience



differently than the results presented here; however, it is unlikely that disability rates or disability experience will have a major impact on the financial results of the pension plan.[2]

[2]Public sector pension plans, and particularly police and firefighter plans, are notorious for abusing the disability provisions of the plan; hence, both the disability valuation assumption and experience may be very important to the financial condition of such plans.

RETIREMENT RATES

The costs of providing *actuarially reduced* early retirement benefits for retirements both earlier and later than the standard assumption are shown in Table 12–4. The normal cost and actuarial liability under the accrued benefit method are only minimally affected by changes in the incidence of retirement. The actuarial liabilities of the benefit prorate and cost prorate methods are only moderately affected, whereas the normal costs of these methods are affected up to 25 percent (i.e., early retirements reduce costs while delayed retirements increase costs).[3]

TABLE 12–4

Effect of Alternative Retirement Rates

Average Retirement Age	*Accrued Benefit*		*Constant Dollar Benefit Prorate*		*Constant Percent Cost Prorate*	
	NC	*AL*	*NC*	*AL*	*NC*	*AL*
65.0	106.6	100.9	117.2	109.2	120.7	113.8
63.4	103.8	100.6	109.8	105.2	111.9	107.6
61.4	100.0	100.0	100.0	100.0	100.0	100.0
59.4	97.1	99.4	90.7	95.1	87.5	93.0
57.5	89.6	98.6	78.8	89.6	75.1	85.0

If retirements are assumed for valuation purposes to conform to the distribution of retirements given by Table 2–9, while actual retirements conform either to a distribution of rates averaging 57.5 or to 100 percent of retirements at age 65, then costs will be affected by actuarial gains and losses. As indicated in Figure 12–4, the financial effects of retirement-age deviations have a fairly significant effect on pension costs after a few years, with such costs being increased by about 25 percent after 5 years when retirements are older than expected, and decreased by 25 percent or more after only 3 or 4 years when retirements are earlier than expected. The cost of providing non-reduced accrued benefits at early retirement, as opposed to actuarially reduced benefits, is considered in the following chapter.

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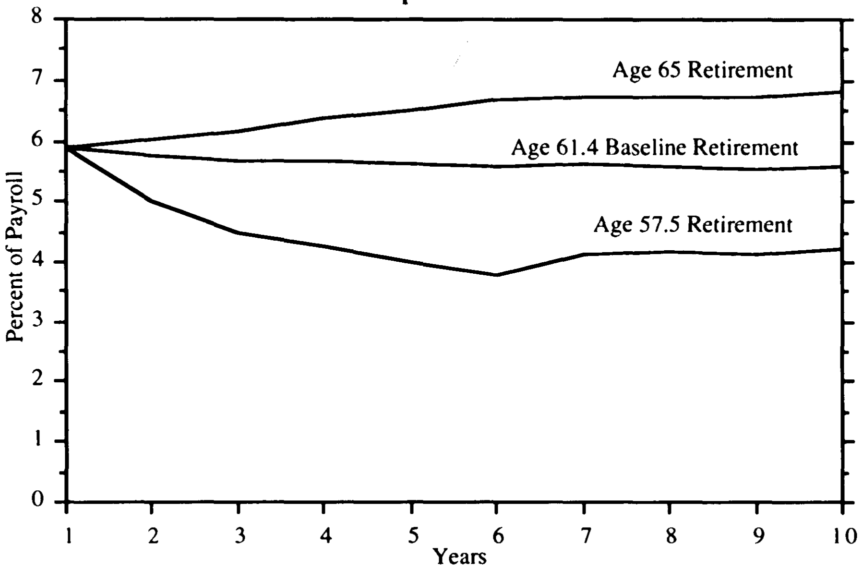
[3]If the plan provides an early retirement reduction that only approximates a true actuarial reduction, such as 6 percent per year below the plan's normal retirement age, the sensitivity of alternative retirement ages may be greater than indicated in Table 12–4.

FIGURE 12–4
Effect of Alternative Retirement Experience



SALARY RATES

Pension costs are directly proportional to the level of benefits provided under the plan. Consequently, for plans with a salary-based benefit formula, the assumed rate of growth in salaries has an important bearing on costs. The underlying theory for future salary increases, as set out in Chapter 2, is that merit, productivity, and inflation represent the components of the increases. The model merit scale, although conforming to a concave function, has about a 2 percent compounding effect for an age-30 entrant, the productivity factor is assumed to be 1 percent, and the inflation factor 4 percent. The first two components, merit and productivity, are relatively stable as compared to the inflation component. The purpose of this section is to analyze the impact of introducing each salary increase component into the salary assumption, and to study the effects of various levels of inflation, both in the valuation assumptions and the experience of the plan.

Table 12–5 shows the results of alternative valuation salary assumptions. The normal cost under the accrued benefit method changes by about 10 percent for each 2 percentage points change in the salary rate. This is in sharp contrast to its actuarial liability, which not only is minimally affected by different salary assumptions, but is affected in the opposite direction. As the salary rate is decreased, the actuarial liability increases and vice versa. The reason for this is that a change in the salary assumption affects each participant's *assumed* accrued benefits under this method. In particular, the flatter the salary assumption, the *larger* the assumed salaries of plan participants for prior years; this in turn increases the assumed accrued benefits. Thus, for a given set of current salaries, the flatter the salary, the larger will be the actuarial liability under the accrued benefit method.

TABLE 12–5

Effect of Alternative Salary Rates

Salary Rate	*Accrued Benefit* NC	*Accrued Benefit* AL	*Constant Dollar Benefit Prorate* NC	*Constant Dollar Benefit Prorate* AL	*Constant Percent Cost Prorate* NC	*Constant Percent Cost Prorate* AL
0	56.0	109.1	64.7	85.1	43.0	84.8
M	65.8	107.2	70.0	87.3	53.1	87.1
M+P	73.3	105.7	74.4	89.2	60.1	89.2
M+P+2%	87.3	102.7	85.3	93.8	77.3	94.2
M+P+4%	100.0	100.0	100.0	100.0	100.0	100.0
M+P+6%	111.6	97.5	120.4	108.2	129.4	106.6
M+P+8%	122.2	95.2	149.0	119.1	166.5	113.8
M+P+10%	131.9	93.1	190.1	133.9	211.8	121.1

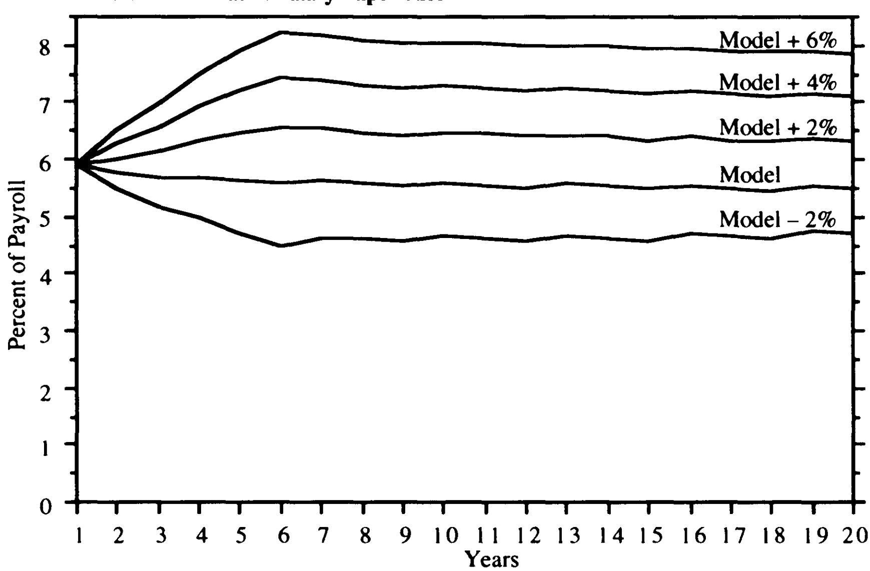
M = Merit Scale; P = Productivity Assumption of 1%

The normal cost under the benefit prorate method is affected by about 10 percent for each 1 percentage point change in the salary assumption, or about double the impact for the accrued benefit method. The actuarial liability is affected by about half of this amount. Finally, the cost prorate method is affected even more, with a 2 percentage point change bringing about a 30 percent change in costs. The actuarial liability, however, is affected to a lesser degree than that of the benefit prorate method.

In theory, at least, it is inappropriate to alter the inflation component of the salary assumption without a commensurate change in the assumed investment return. The effects of simultaneous changes in these two assumptions is studied at a later point in this chapter.

The effects of experiencing salary increases greater or less than the valuation assumptions is provided in Figure 12–5. After 5 years, costs are affected by about 20 percent for each 2 percentage point deviation in the experience from the underlying valuation salary assumption. This impact, however, does not continue to increase, with the relative differential after 20 years being approximately the same as after 5 years.[4] Since salary deviations of 2 to 4 percent for several years can easily occur, this actuarial assumption merits closer scrutiny than the decrement assumptions, for which substantial deviations are less likely to occur.

FIGURE 12-5
Effect of Alternative Salary Experience



INTEREST RATES

The interest rate assumption, like the salary assumption, has associated with it an underlying theory that was discussed in Chapter 2. This theory states that the interest assumption consists of three components: one to account for the risk-free rate of return, one to account for the risk inherent in the portfolio of assets held, and one to account for inflation. These components, it will be remembered, are 1 percent, 3 percent, and 4 percent, respectively, for the model assumptions.

[4]This projection, as well as the following two, are for 20 years instead of 10 years. The time period was extended to illustrate that pension costs reach an ultimate level rather than continuing to increase or decrease for a given experience deviation.

Table 12–6 shows the results of introducing the various interest rate components and varying the inflation rate around the 4 percent model assumption. A brief inspection of this table shows that costs are more sensitive to the interest rate assumption than they are to any parameter thus far studied. For example, assuming zero interest causes costs to increase by 400 to 900 percent, depending on the cost measure. A more meaningful analysis is the impact of a change in the inflation component of the interest assumption. The cost prorate method is the most sensitive of the various measures to such changes. A zero inflation rate component (or a 5 percent interest rate) has the effect of increasing the normal cost nearly threefold, while a 10 percent inflation component (or a 14 percent interest rate) reduces costs to one fourth of the cost under the model assumption. The actuarial liability is only about half as sensitive to changes in the interest rate as the normal cost.

TABLE 12–6

Effect of Alternative Interest Rates

Interest Rate	*Accrued Benefit*		*Constant Dollar Benefit Prorate*		*Constant Percent Cost Prorate*	
	NC	*AL*	*NC*	*AL*	*NC*	*AL*
0	580.5	387.7	895.0	494.4	880.4	365.2
RF	440.4	310.9	638.4	381.8	665.2	304.8
RF+RP	213.9	176.9	260.3	196.2	289.4	182.4
RF+RP+2%	142.6	130.1	156.4	136.4	168.6	133.3
RF+RP+4%	100.0	100.0	100.0	100.0	100.0	100.0
RF+RP+6%	73.1	79.7	67.6	76.7	60.7	77.5
RF+RP+8%	55.4	65.5	47.8	61.0	37.8	61.2
RF+RP+10%	43.2	55.1	35.3	50.0	24.2	49.8

RF = Risk Free Rate of 1%; RP = Risk Premium of 3%

The normal costs under the other two cost methods are less sensitive to interest rate changes than the cost prorate normal cost, although their actuarial liability values are affected by about the same amount.

The rule-of-thumb that pension costs are altered by 6 to 7 percent for each 1/4 of one percent change in the interest rate is well known and used often in connection with pension plans. If we take the midpoint of this range, or 6.5 percent, the rule tells us that a 1 percentage point increase in the interest rate will reduce costs by 22 percent [100 $(1-1.065^{-4})$], while a 1 percentage point reduction will increase costs by 29 percent [100 (1.065^4-1)]. The results of a 2 percentage point increase or decrease reduces costs by 60 percent or increases costs by 160 percent, respectively, while a 4 percentage point change reduces costs by 37 percent or increases costs by 274 percent. The normal cost under the benefit and cost prorate methods conform to this rule reasonably well, but their actuarial liabilities as well as the normal cost under the accrued benefit method appear to follow a 4 percent rule rather than the 6 to 7 percent rule.

The inflation component of the interest rate assumption was altered in this section without a simultaneous change in the inflation component of the salary assumption. While the results are both interesting and important, it is believed that the sensitivity of pension costs to the inflation parameter as analyzed in the next section, is somewhat more meaningful and valuable.

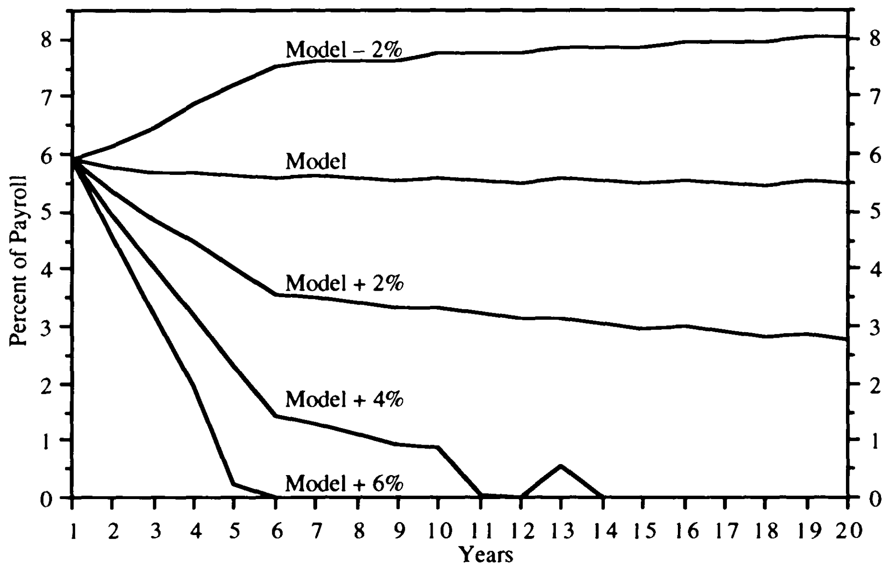
Figure 12–6 shows the implications of experiencing asset returns different from the 8 percent valuation interest rate. The impact on costs continues to grow throughout the 10 year period, with a 2 percentage point deviation causing costs to be affected by over 30 percent (or about 2 percent of payroll) by the end of the 10 year projection.

It is ironic that there is almost a perfect positive correlation between the importance of a given assumption, whether for valuation purposes or for experience purposes, and the assumption's stability and degree to which it can be predicted. The interest rate is by far the most important assumption and by far the most difficult to establish accurately. On the other hand, mortality rates, for which reasonable variations are not particularly crucial to pension costs, are highly predictable.

INFLATION RATES

The effect of changing the assumed rate of inflation, a component of both the salary rate and interest rate assumptions, is analyzed in this section. Some plan sponsors naively believe that equal changes in the interest rate and salary rate (in this case being brought about by a change in the inflation component of each) will tend to cancel out, since these two assumptions have counterbalancing effects on pension costs. This is not the case,

FIGURE 12–6
Effect of Alternative Investment Experience



however, since the salary scale operates up to the participant's retirement age, while the interest discount factor extends to the end of the assumed life span.[5] Thus, a change in the inflation component of the interest assumption will have a *greater* impact on pension costs than its counterpart in the salary assumption.

The results of assuming an inflation component of zero up through 10 percent are given in Table 12–7. Pension costs, as expected, are inversely related to changes in the inflation rate: the higher the rate of inflation the lower the dollar cost of the plan *for the current year.* Although the *dollar* cost of the plan experiencing high inflation will eventually be greater at some future point in time than if lower inflation were to be experienced, the cost as a *percentage of payroll* will be less. The impact among cost methods and between the normal cost and actuarial liability is reasonably constant.

Theoretically, if a sponsoring firm's earnings were to be perfectly insulated from the effects of inflation, then greater rates of inflation might be viewed as a cost reducing factor in a relative

[5]This would not be true if the plan has a cost-of-living clause, in which case the inflation component of the salary assumption would extend beyond retirement.

sense. The more typical case, however, is where inflation impairs the earnings potential of the firm and the corresponding increase in pension dollar costs simply adds to other problems created by inflation. The effects of inflation are even more serious if this component in the salary assumption becomes applicable at ages beyond retirement due to cost-of-living escalators, as will be seen in the following chapter.

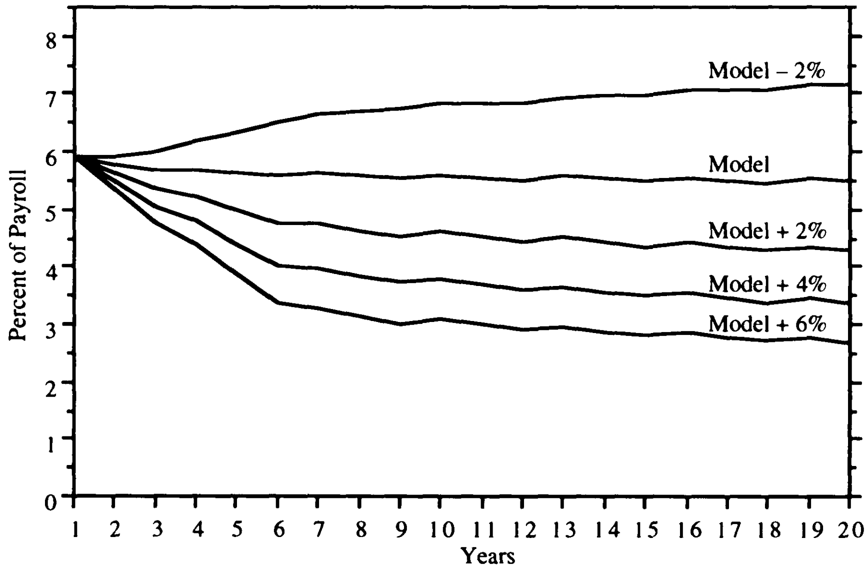
TABLE 12–7

Effect of Alternative Inflation Rates

Inflation Rate	*Accrued Benefit*		*Constant Dollar Benefit Prorate*		*Constant Percent Cost Prorate*	
	NC	*AL*	*NC*	*AL*	*NC*	*AL*
0%	161.9	187.6	175.6	164.0	178.9	161.1
2%	125.2	133.7	130.0	126.0	131.0	125.1
4%	100.0	100.0	100.0	100.0	100.0	100.0
6%	81.9	77.8	79.3	81.6	78.8	81.9
8%	68.4	62.6	64.4	68.1	63.8	68.5
10%	58.2	51.7	53.3	57.9	52.7	58.2

Figure 12–7 shows the financial results if inflation is different from the base case projection. It will be recalled that the pension plan being projected provides an *ad hoc* COLA every 3 years equal to 25 percent of cumulative inflation. The higher the level of inflation, the lower is the cost of the pension plan, even with the *ad hoc* COLAs being given. After 10 years of inflation deviating from the valuation assumption, the ultimate level of costs is obtained, with costs remaining at that level throughout the remainder of the projection.

FIGURE 12–7
Effect of Alternative Inflation Experience



Chapter 13

Alternative Plan Benefits

The purpose of this chapter is to consider the financial implications of several alternative plan designs. The analysis is segregated according to benefit category: retirement, vested, disability, and surviving spouse benefits. The effects of various benefit formulas and eligibility requirements within each category are studied.

RETIREMENT BENEFITS

Three separate analyses are performed on the plan's retirement benefits. First, the retirement formula itself is examined, with comparisons being made among final average, career average, and flat dollar unit benefit formulas. Secondly, the benefits payable at early retirement are considered. The model plan provides actuarially equivalent benefits, and the analysis compares the effect of providing full benefits at early retirement. Finally, the benefits provided after retirement are considered. The model plan provides *ad hoc* COLAs every 3 years equal to 25 percent of cumulative inflation, and the analysis considers alternative inflation-based adjustments to retirement benefits.

Alternative Benefit Formulas

The benefit formula is, of course, the most important factor affecting the costs and liabilities of the plan. The model plan has benefits based on the employee's final 5-year average salary, a relatively common formula among corporate pension plans. Table 13–1 shows the financial implications if the averaging period is reduced to 3 years, increased to the employee's entire career, or if the formula is equal to a flat dollar unit benefit.

TABLE 13–1

Effect of Alternative Benefit Formulas

Salary Average Period	Accrued Benefit		Constant Dollar Benefit Prorate		Constant Percent Cost Prorate	
	NC	AL	NC	AL	NC	AL
FAS (3 Years)	105.5	103.5	105.6	103.7	105.7	103.9
FAS (5 Years)	100.0	100.0	100.0	100.0	100.0	100.0
Career Average	57.8	100.0	71.5	84.2	70.7	79.9
Flat Dollar	56.0	100.0	58.9	79.5	40.0	79.1

FAS = Final Average Salary; Flat Dollar = Flat Dollar Unit Benefit. All formulas are equivalent before salary increase assumption is applied.

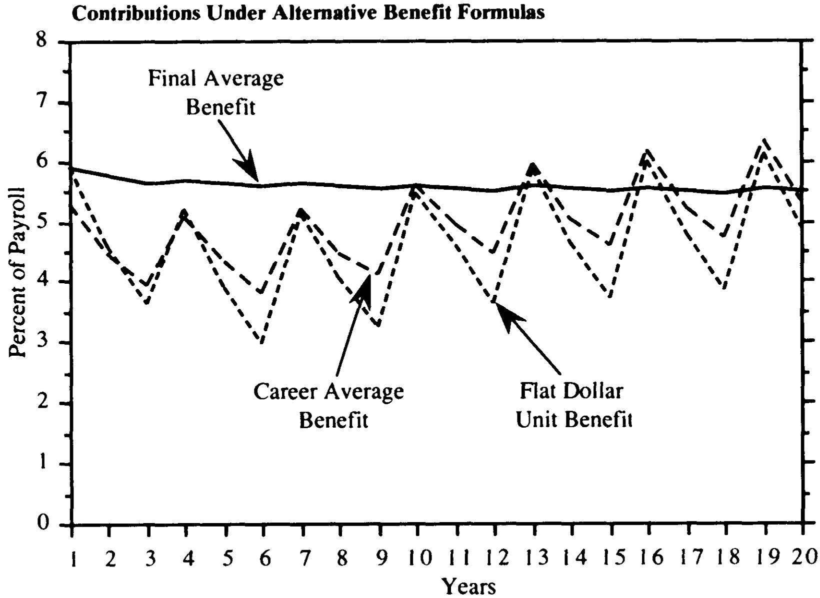
Reducing the final average period from 5 to 3 years increases costs and liabilities by about 5 percent. This result is a function of the salary increase assumption which, for the model pension plan, is 5 percent plus the merit scale, the latter having little impact at ages near retirement. A higher salary assumption would increase the cost impact of a 3-year final average period and vice versa.

Expanding the salary averaging period to the employee's entire career has a dramatic financial impact. Normal costs are reduced by 40 percent under the accrued benefit method and by 30 percent under the benefit and cost prorate methods. The actuarial liability under the accrued benefit method is not affected, while the other two methods have a 15 to 20 percent reduction. The flat dollar unit benefit formula affects the accrued benefit method by a similar amount; however, the normal costs under the benefit and cost prorate methods are reduced by 40 and 60 percent, respectively, as compared to the costs under the final 5 year average formula.

Many career average plans have benefit updates that, in the long run, make them the equivalent of final average formulas. Similarly, flat dollar unit benefit plans, which are typically associated with collectively bargained employees, often have benefits that are negotiated to keep pace with inflation, with such benefit increases being provided retroactively. These updating processes causes such plans to be less funded than their final average counterparts. In the past, actuaries often used lower-than-expected interest rates to implicitly fund some portion of the inevitable future benefit increases in advance. This practice is no longer permissible, and the requirement that explicit, best estimate assumptions be used exacerbates the underfunding problem associated with these plans. The underfunding of flat dollar unit benefit plans was part of the motivation for the *deficit reduction contribution* under OBRA '87. However, it may be more appropriate for Congress to allow (or even require) tax deductible contributions to be based on an assumed increase in the unit benefit of such formulas, perhaps limited to 3 or 4 percent.

Figures 13–1 shows the long-term costs of a final 5-year average formula compared to a career average formula and a flat dollar unit benefit formula, where the latter two are updated every 3 years to the benefits provided under the final average plan.[1] As expected, the cost pattern for the two amended plans is quite erratic due to the updating procedure.

FIGURE 13–1
Contributions Under Alternative Benefit Formulas



[1] Assets as a percent of the actuarial liability under all three plans were set equal at the outset of the projection in order to have a valid comparison of the long-run costs of the different benefit formulas.

The economic liability is the same for all three plans, since each plan delivers essentially the same final average benefit.[2] The economic liability funded ratio under the final average benefit formula is approximately 100 percent during the projection; however, the same ratio under the career average plan is in the 75 percent range and for the flat-dollar unit benefit plan is in the 65 percent range. Since the long-run level of assets is lower, the long-run costs of the amended plans eventually will be higher than the costs under the final average plan.

If it is the intention of the plan sponsor to update a career average plan, for example, to mimic a final average plan, it may be better to adopt a final average plan in the first place. Under a periodically updated career average plan, the funded status will be lower, long-run costs will be higher, and the pattern of costs will be erratic. On the other hand, the advantage of periodic updates is that the employer has greater control over the costs and liabilities of the plan.

Alternative Early Retirement Benefits

The benefits provided at early retirement, like the basic benefit formula itself, can have a significant effect on pension costs. In recent years more participants are retiring early and more plans are providing benefits that exceed full actuarial reductions. The cost effect of providing non-reduced early retirement benefits for various early retirement assumptions relative to the cost of retirement at age 65 is given in Table 13–2. The base case early retirement assumption, shown previously in Table 2–9, has an average retirement age of 61.4. This distribution has an increased normal cost of 20 percent for the accrued benefit method and about 10 percent for the other two cost methods as compared to age-65 retirement.

The cost of providing non-reduced early retirement benefits relative to the cost of providing actuarially reduced benefits for retirements at the same ages is given in Table 13–3. The cost impact is significant. For example, the age 61.4 early retirement distribution shows a 30 percent increase in costs and about a 20

[2]The economic liability, which represents management's best-estimate of the true economic obligation of the plan, is discussed in Chapter 14.

TABLE 13–2

Effect of Non-Reduced Benefits at Alternative Retirement Ages as a Percent of Values for Retirement at Age 65

Average Retirement Age	*Accrued Benefit*		*Constant Dollar Benefit Prorate*		*Constant Percent Cost Prorate*	
	NC	*AL*	*NC*	*AL*	*NC*	*AL*
65.0	100.0	100.0	100.0	100.0	100.0	100.0
63.4	109.0	106.4	105.1	104.3	104.1	103.2
61.4	122.1	115.2	112.0	110.0	109.9	107.1
59.4	137.0	124.4	119.2	115.4	115.9	110.7
57.5	148.7	135.9	123.6	121.5	121.6	114.7

TABLE 13–3

Effect of Non-Reduced Benefits at Alternative Retirement Ages as a Percent of Values of Actuarially Reduced Benefits

Average Retirement Age	*Accrued Benefit*		*Constant Dollar Benefit Prorate*		*Constant Percent Cost Prorate*	
	NC	*AL*	*NC*	*AL*	*NC*	*AL*
65.0	100.0	100.0	100.0	100.0	100.0	100.0
63.4	112.0	106.7	112.2	108.4	112.3	109.1
61.4	130.1	116.2	131.2	120.1	132.6	121.9
59.4	150.4	126.3	154.0	132.5	159.8	135.5
57.5	176.8	139.1	184.0	148.0	195.5	153.5

percent increase in actuarial liabilities. Thus, the dual effect of early retirements and non-reduced benefits can be significant.

Cost-of-Living Adjustments (COLA)

Some plans, mostly or perhaps exclusively in the public sector, allow the benefits of retired employees to increase automatically, usually according to a rate which is tied to a national inflation index. The purpose of this section is to investigate the cost consequences of an automatic benefit escalator under various rates of inflation versus paying these same benefits on an *ad hoc* basis. Table 13–4 shows that costs and liabilities under all three funding methods are increased by about 8 percent for each one percentage point increase in the COLA assumption. For example, providing an automatic 3 percent COLA increases costs and liabilities by about 25 percent. This, of course, is the long-term

TABLE 13–4

Effect of Automatic Cost-of-Living Adjustments

*Automatic COLA Rate**	*Accrued Benefit*		*Constant Dollar Benefit Prorate*		*Constant Percent Cost Prorate*	
	NC	*AL*	*NC*	*AL*	*NC*	*AL*
0%	100.0	100.0	100.0	100.0	100.0	100.0
1%	107.7	107.3	107.7	107.4	107.7	107.4
2%	116.4	115.4	116.4	115.6	116.5	115.7
3%	126.2	124.6	126.2	124.9	126.4	125.1
4%	137.3	134.9	137.4	135.5	137.6	135.7

*The inflation rate in the salary and interest assumptions also adjusted to this level (from 4% base case).

increase in costs. If a plan without an automatic COLA were to adopt a 3 percent COLA, the near-term costs would be additionally increased by the unfunded liability created at the time of adoption.

Figure 13–2 shows the cost implications of providing these same benefits on a *ad hoc* basis by amending the plan every three years. The base case, it will be recalled, provides 25 percent of cumulative inflation (or 1 percent) every 3 years. After 20 years of *ad hoc* benefit increases, costs are about 1 percentage point of payroll higher (or a change in costs of 15 to 20 percent) for each one percentage point of COLA provided on an *ad hoc* basis.

Nearly all employers provide benefits adjustments to retirees on an *ad hoc* basis, partly because they can control future benefit increases and partly because of the high cost of installing an automatic COLA, even one with a cap of 3 or 4 percent. Most public plans, on the other hand, provide benefit adjustments of 2 to 4 percent on an automatic basis.

VESTED TERMINATION BENEFITS

The minimum required vesting period is 5 years of service. Table 13–5 shows the effects on normal costs and actuarial liabilities of alternative service requirements, ranging from zero years to 20 years. These results indicate that alternative vesting requirements have a rather modest impact on total costs based on the termination rates used in the model pension plan. In fact, eliminating the service requirement completely only increases costs by about 1 percentage point.

FIGURE 13–2

Effect of *Ad Hoc* Cost-of-Living Adjustments

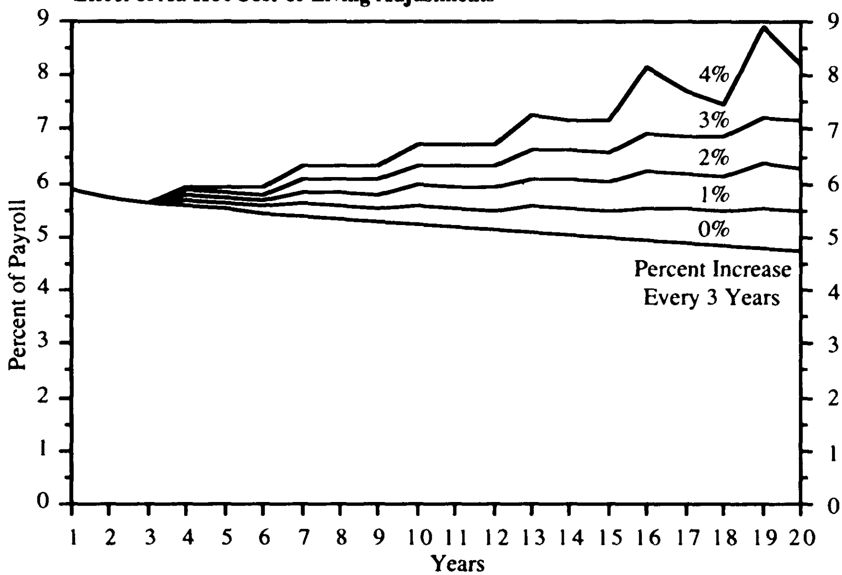


TABLE 13–5

Effect of Alternative Service Requirements for Vesting

Vesting Service Requirement	*Accrued Benefit*		*Constant Dollar Benefit Prorate*		*Constant Percent Cost Prorate*	
	NC	*AL*	*NC*	*AL*	*NC*	*AL*
20 Years	92.8	97.7	92.2	97.6	90.1	99.2
15 Years	95.3	98.7	94.8	98.7	93.2	100.0
10 Years	97.8	99.5	97.5	99.6	96.6	100.2
5 Years	100.0	100.0	100.0	100.0	100.0	100.0
0 Years	101.1	100.1	101.3	100.1	101.9	99.6

DISABILITY BENEFITS

The disability provision used in the model pension plan requires 10 years of service and age 40. Table 13–6 shows the financial implications of alternative age and service requirements. As noted, there is little impact on costs of such changes, based on the disability rates and benefits for the model pension plan.

TABLE 13–6

Effect of Alternative Age and Service Requirements for Disability Benefits

Disability Requirements:		Accrued Benefit		Constant Dollar Benefit Prorate		Constant Percent Cost Prorate	
Age	*Service*	*NC*	*AL*	*NC*	*AL*	*NC*	*AL*
55	10 Yrs	97.6	99.0	97.5	98.8	97.3	98.9
40	10 Yrs	100.0	100.0	100.0	100.0	100.0	100.0
None	10 Yrs	100.1	100.0	100.1	100.0	100.1	100.0
40	0 Yrs	100.6	100.1	100.7	100.1	101.0	100.0
None	0 Yrs	100.9	100.2	101.0	100.2	101.3	100.0

SURVIVING SPOUSE BENEFITS

The surviving spouse benefit for the model pension plan provides 50 percent of the employee's accrued benefit commencing when the employee would have been eligible for early retirement, generally age 55 for most employees. Table 13–7 shows the financial impact of several alternatives with respect to the amount of the benefit and the commencement date.

TABLE 13–7

Effect of Alternative Surviving Spouse Benefits

Survivor Benefit	*Accrued Benefit NC*	*Accrued Benefit AL*	*Constant Dollar Benefit Prorate NC*	*Constant Dollar Benefit Prorate AL*	*Constant Percent Cost Prorate NC*	*Constant Percent Cost Prorate AL*
50% Deferred	100.0	100.0	100.0	100.0	100.0	100.0
100% Deferred	104.3	102.1	104.5	102.6	104.8	102.6
50% Immediate	100.8	100.6	100.9	100.5	101.0	100.4
100% Immediate	105.9	103.0	106.2	103.5	106.8	103.2

Chapter 14

Funding Policy

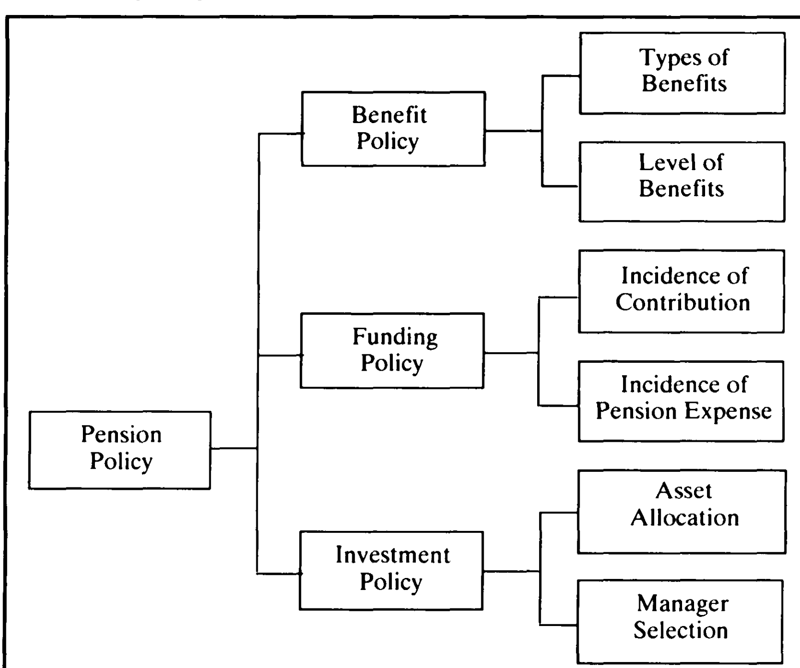
The statutory rules for determining minimum required and maximum tax deductible contributions were set forth in Chapter 10, and the accounting rules for determining annual pension costs were discussed in Chapter 11. Neither of these sets of rules, however, necessarily provides guidance to plan sponsors regarding the level of contributions that should be made to the pension plan each year. Statutory requirements, of course, place boundaries on such contributions; however, these boundaries themselves can be influenced by the choice of funding method, actuarial assumptions, and asset valuation method. Accounting rules, while not providing as much flexibility as statutory rules, nevertheless involve the selection of several factors that can affect both near-term and long-term accounting costs. Finally, there exists an interaction between contributions and accounting costs. While accounting costs have no impact on contributions or their statutory limits, the reverse is not true since contributions affect plan assets which, in turn, affect accounting costs.

The plan sponsor's *funding policy* should set forth objectives regarding future contribution patterns, funded ratios, and accounting costs. The implementation of this policy involves selecting various methods and assumptions that, while conforming to the various statutory and accounting rules, are designed to meet management's objectives. Prior to the passage of ERISA and subsequent legislation, and before the promulgation of SFAS 87, it was relatively straightforward to implement an articulated funding policy. This is no longer the case, and plan sponsors must now accept the reality that it may not be feasible to have contributions or expense conform to a rational funding policy.

As illustrated in Figure 14–1, there are two additional policies to be considered in conjunction with the plan's funding policy, namely, the *benefit policy* and *investment policy*. The benefit policy sets forth the sponsor's objectives with regard to the type and level of benefits to be provided under the plan, and the method for delivering such benefits. The investment policy sets forth the risk and return objectives of the sponsor, resulting in an asset allocation strategy (i.e., the percentage of assets to be invested in various asset classes) and guidelines for selecting and monitoring investment managers. For the analyses in this chapter, it is assumed that the benefit policy and investment policy are already established; however, all three policies should be developed jointly in formulating the sponsor's overall *pension policy*.

The purpose of this chapter is to illustrate the development and implementation of a funding policy for the model pension plan. While the policy selected for the analysis may not be appropriate for any given plan because of the sponsor's unique fi-

FIGURE 14–1
Pension Policy Components



nancial and tax circumstances, the analysis sets forth a general methodology that may be useful in approaching this important task.

ECONOMIC LIABILITY

One of the first decisions a plan sponsor should make in formulating a funding policy is the ultimate level of assets that the plan should accumulate. In other words, what funding target should be adopted as part of the funding policy? Should assets systematically accumulate to the plan continuation liability, the plan termination liability, the ABO, the PBO, the current liability, or the actuarial liability of the actuarial cost method used for funding?

In addition to the liability (or funding) definition, there are benefit and assumption issues. For example, if the plan has a career average formula with periodic increases being given to mimic a final average formula, should the benefits used in setting the funding target be based on the benefit formula specified in the plan document or should it reflect the impact of anticipated periodic increases? As another example of the benefit issue, should fixed benefits after retirement be used in the funding target or should expected *ad hoc* COLA benefits be considered? Finally, what actuarial assumptions should be used? Should the interest rate be the SFAS 87 spot rate, the SFAS 87 expected return on assets, the PBGC interest rates, the interest rate used for determining the ERISA funding limits, the current liability interest rate, or some other rate based on management's views?

The concept of an economic liability has evolved to assist plan sponsors in defining an appropriate funding target. This liability represents management's best estimates with regard to future benefits and future plan experience, and represents what management believes is a rational allocation of benefits over the working career of an active employee. Since pensions can be viewed as deferred wages, correlating the benefit allocation with salary has a strong intellectual appeal. Consequently, the economic liability is frequently selected as the actuarial liability under the constant percent benefit prorate method.

The economic liability, if defined with best-estimate benefits, best-estimate assumptions, and the constant percent benefit prorate method, may be very different from the various liabilities noted above, especially if the latter are evaluated according to the plan's legal benefits with the actuarial assumptions currently being used for such liabilities. On the other hand, it may well be that the sponsor is satisfied with a given liability value, e.g., the PBO under SFAS 87, in which case this measure is defined as the economic liability for determining the plan's funding policy.

For illustrative purposes, the economic liability defined in Table 14–1 is used in the subsequent analysis of the model pension plan. The economic liability for the model pension plan is given in Figure 14–2 along with five additional liability values for comparison purposes.

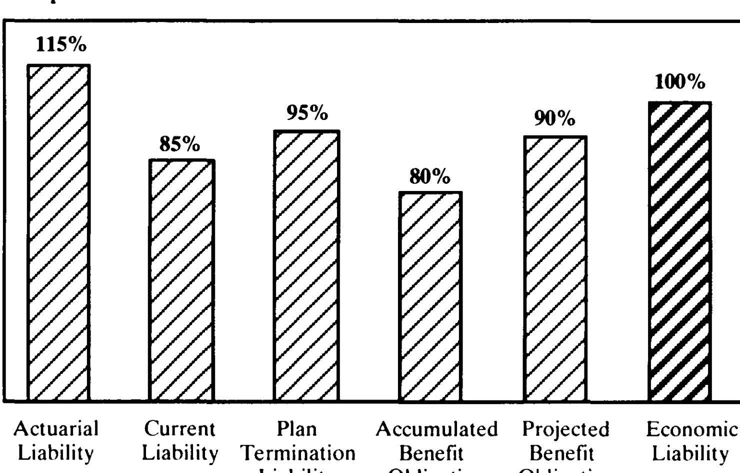
TABLE 14–1

Economic Liability Definition

Benefits: Based on the final average benefit formula, but with benefits assumed to increase after retirement at 25 percent of the inflation rate (i.e., 1 percent based on the 4 percent inflation assumption)
Benefit Allocation: Prorated by salary
Assumptions: Same as used with the model plan

FIGURE 14–2

Comparison of Alternative Liabilities



FUNDING POLICY OBJECTIVES

In addition to the funding target, the plan sponsor must select a funding period over which assets are to reach the funding target, the pattern of contributions and accounting expense, and the degree of conservatism associated with the funding policy. For illustrative purposes, it is assumed that the plan sponsor has developed the funding objectives set forth in Table 14–2. In effect, this policy states that assets are to accumulate to the economic liability of the plan over a period of 10 years, with a 70 percent probability that this funding objective is achieved, and with contributions (and accounting costs) equal to a level percentage of payroll.

TABLE 14–2

Illustrative Funding Policy

- ***Funding Target:*** The market value of assets should systematically approach the plan's economic liability.
- ***Funding Period:*** The funded ratio, based on the economic liability, should equal or exceed 100 percent at the end of 10 years.
- ***Contribution Pattern:*** Contributions should be approximately equal to a level percentage of payroll, having a relatively high degree of year-to-year stability, yet maintaining maximum flexibility to deviate from this pattern if financial circumstances so dictate.
- ***Accounting Cost Pattern:*** Same as contribution objectives.
- ***Confidence Level:*** The funded target should be met at the end of 10 years with a 70 percent confidence factor.

FINANCIAL MODELING

The implementation of a funding policy requires financial modeling, including current year valuations, deterministic projections, and stochastic projections. Actuarial valuations involve the determination of the plan's liabilities and costs for the current year. Annual valuations are required for accounting and statutory purposes; however, this is the final step in implementing a funding policy. The first step involves the use of valuations in performing sensitivity analyses of the different assumptions and methodologies that might be used to implement the funding policy. As a result of the advances in micro computer technology in recent years, management can quickly and cost-effectively observe the current year's financial implications of a wide range of actuarial assumptions and methodologies, a useful process in narrowing the range of possibilities.

A series of valuations, however, is not sufficient for implementing a funding policy because no information is provided on the plan's financial status beyond the current year. Deterministic projections allow management to explore the behavior of alternative funding policies under differing economic and demographic scenarios. For example, if management believes there is a high probability of substantial early retirements in future years, deterministic forecasts can show the corresponding financial implications under several proposed funding policies which may be useful in selecting the appropriate policy.

The distinguishing feature of a deterministic projection is that management sets forth the economic and demographic scenario for each year of the projection. The advantage of this process is that the expected financial effect of each change in the economic or demographic scenario can be quantified. These types of projections are useful in testing alternative sets of assumptions and funding methods in establishing the plan's funding policy.

As an alternative to specifying future scenarios on a year-by-year basis, the plan sponsor can use stochastic projections. Under this methodology, both the expected value and the expected volatility of one or more assumptions is specified. For example, plan assets might be specified as having an expected return of 8 percent with a standard deviation of 15 percent. Multiple asset return scenarios, consistent with these inputs, are then developed by the computer. The number of computer-generated scenarios should be sufficient to cover the entire range of possibilities, with 500 to 1,000 trials generally being adequate, depending on the degree of volatility specified and the length of the projection. The final step is to perform deterministic projections on each of the several hundred scenarios, the result being a stochastic projection of the plan's financial condition. Stochastic projections are particularly useful in studying the interrelationship of the funding policy with the plan's asset allocation policy.

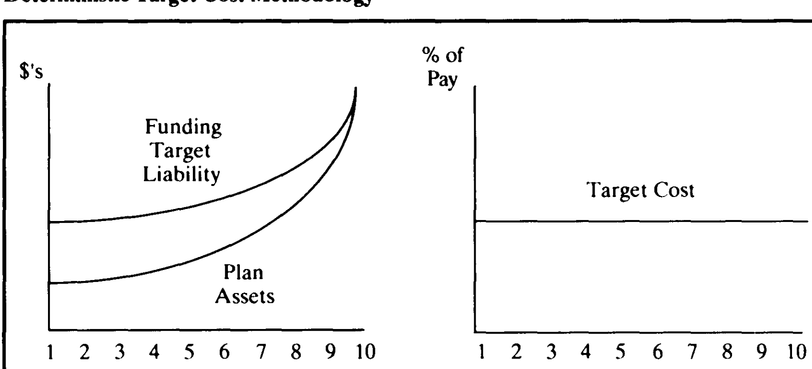
In summary, valuations are a useful first step in narrowing down alternatives available in establishing a funding policy. Deterministic projections are useful in testing the long-term validity of alternative funding policies, while stochastic projections are useful in exploring the impact of fluctuating experience, such as asset returns, on the proposed funding policy.[1]

TARGET COST METHODOLOGY

The so-called *target cost* methodology, which can be based on either deterministic or stochastic projections, is a useful step in the development of a funding policy. Based on the funding objectives set forth previously, Figure 14–3 illustrates this methodology deterministically. The funding target is projected to the end of the planning period, in this case 10 years. A level percent of the projected payroll is then calculated such that plan assets accumulate to the funding liability by the end of 10 years. This represents the ideal pattern of both contributions and accounting expense under the funding objectives previously stated, i.e., reaching a designated funding goal with contributions equal to a level percent of payroll. With this cost guideline, management can proceed to explore alternative assumptions and methodologies that produce contributions and accounting costs approximately equal to these levels.

The deterministic methodology produces target costs that have an equal chance of falling below or above the ultimate fund-

FIGURE 14–3
Deterministic Target Cost Methodology



[1]This discussion implies that the asset allocation policy is established independently from the funding policy, which may in fact be the case for many plan sponsors. A more correct procedure, however, is to establish these two policies jointly. The stochastic methodology is an ideal tool for this task.

ing goal.[2] Management may wish to build into the target cost calculation a higher confidence factor. For example, the funding objectives stated previously specified a 70 percent probability that assets will equal or exceed the funding target after 10 years. One method of injecting a higher than 50-50 probability into the target cost calculation is to fund toward some multiple of the funding liability, e.g., 125 percent. This approach, however, does not provide information on the *probability* of success, other than the fact that the probability exceeds 50 percent. Another approach is to use the stochastic projection methodology illustrated in Figure 14–4.

The stochastic methodology produces a range of target costs, shown in Figure 14–4 as percentiles of 50, 60, 70, 80, and 90 percent, based on a stochastic projection of the funding target. This methodology is typically implemented by management selecting distributions for inflation and various asset classes, along with structural relationships among such distributions. Distributions for various decrements can also be used; however, for a population with more than a few thousand employees, the financial variability introduced by stochastic decrements is small.

The stochastic methodology allows management to develop a target cost that explicitly meets the fourth funding objective, namely, to achieve the funding goal with a given confidence level.[3] If the confidence factor is 70 percent, as in the previously stated funding objectives, then the cost level associated with the 70th percentile is the appropriate guideline to be used in establishing the plan's funding policy.

As a practical matter, a plan sponsor would not perform a target cost analysis and then passively observe the results over the ensuing planning horizon. The analysis should be updated

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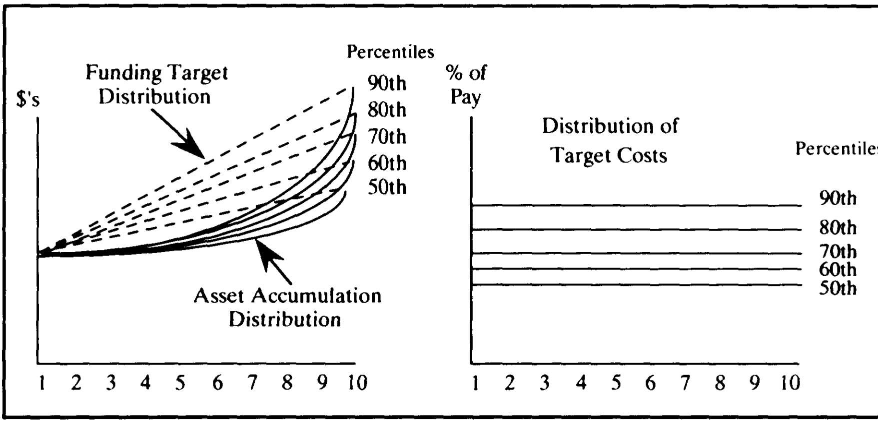
[2]This assumes, of course, that management has chosen the median value for each assumption's distribution in the target cost methodology. While this can not be done with precision, the point is that if management selects best estimate assumptions, then assets, by definition, have a 50–50 chance of being under or over the funding target at the end of 10 years.

[3]There is nothing inherently inappropriate about a 50 percent confidence level; however, most corporations using this methodology have opted for a confidence level falling between 60 and 80 percent. In only one company that the author is aware of did management select a factor less than 50 percent. In this case, 40 percent was selected for an hourly plan, representing management's view that, if the plan reached an overfunded status, the negotiation process for benefit increases would be compromised.

FIGURE 14–4
Stochastic Target Cost Methodology



periodically, especially if unusual events have occurred, such as a benefit change, or if the sponsor's views on capital market returns have change substantially.

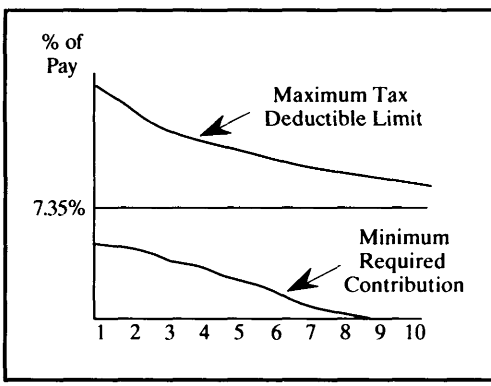
The largest source of financial variability in the stochastic target cost analysis typically comes from asset returns. Consequently, a complete analysis requires the asset allocation dimension to be analyzed in tandem with target costs. As noted previously, it is assumed for this analysis that the asset allocation decision has been preselected; however, this subject is studied in Chapter 15.

Based on the funding objectives specified previously in Table 14–2, the target cost for the model pension plan, based on a 70 percent confidence factor, is 7.35 percent of covered payroll.

STATUTORY METHODS AND ASSUMPTIONS

The plan sponsor, in the illustrative case, desires to contribute 7.35 percent of payroll, barring any new developments and/or until it is deemed appropriate to recalibrate the target cost guidelines. The funding policy articulated previously also calls for as much stability and flexibility as possible in annual contributions. Ideally, the statutory funding method would develop as wide a band around the target cost as possible. Figure 14–5 illustrates an ideal funding policy. The upper limit is drawn as a downward sloping line because, as the plan increases its current statutory funded position, the maximum contribution decreases and the full funding limits (FFLs) are more likely to be reached. Full funding limits are particularly troublesome in implementing a funding policy because generally they are either "on or off" in a given year, implying that contributions are either zero or approximately equal to the plan's normal cost. Thus, as the plan approaches its FFLs, contributions may fluctuate from zero to the plan's normal cost from one year to the next, creating a planning nightmare.

FIGURE 14-5
Ideal Relation Between Statutory Limits and Plan Contributions



Other things being equal, the upper statutory limit should be as high as possible to avoid spurious full funding "hits" when the plan has not yet reached management's funding target. Conversely, the lower limit should be as low as possible. If the minimum required contribution can be set below the target cost, then the excess contributions will develop a credit balance in the funding standard account and drive the minimum down to zero. By the same token, the larger the difference between the minimum and maximum limits, the greater will be the year-to-year stability in annual contributions. In fact, contributions can be estimated with virtual certainty for at least a few years, since the employer will simply contribute the target costs. Finally, a corollary to establishing stability is that it also develops flexibility. Clearly, if the minimum and maximum limits are wide, the employer can vary contributions in the upcoming year to either boundary.[4]

The cost prorate (entry age normal) method is the ideal funding method to be used for determining statutory limits, provided, of course, that actuarial assumptions can be selected such that the minimum required contribution is lower than the target cost. This method, which develops the largest actuarial liability and, hence, unfunded liability, provides the greatest gap between the minimum and maximum limit. Ironically, this method or some variation of it, was by far the most popular method used prior to the promulgation of SFAS 87, which mandated the benefit prorate method for pension expense. Although not logical, most plan sponsors likewise adopted this method for their contributions and statutory limits. It is unlikely that this represented the appropriate course of action for most corporations, especially if assets had not yet reached management's funding target.

If the funded status of the plan is deemed to be adequate, and especially if the plan develops a surplus, then it may be desirable to place the plan in full funding for a period of years until assets come back into balance with the funding target. This can best be accomplished by adopting an actuarial funding method for statutory purposes having a low full funding limit. In this case, the accrued benefit method would be ideal, with the constant dollar benefit prorate method being the second choice. Overfunded plans that adopted the benefit prorate method at the time SFAS 87 was promulgated may have done so with this strategy in mind. Nevertheless, when it is time to begin contributing again, it may be appropriate to change back to the cost prorate method.[5]

There are several forms of the cost prorate method: (1) explicit amortization of all unfunded liability (entry age method), (2) implicit amortization of all unfunded liabilities, (aggregate method), and (3) a combination of explicit and implicit amortiza-

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[4]There are many reasons for engineering flexibility into annual contribution ranges. For example, such flexibility would have been beneficial when TRA '86 lowered corporate tax rates. The astute plan sponsor would have maximized 1986 contributions and then lowered 1987 contributions such that the total during the two years was equal, but with a substantial after-tax cost savings. The opposite strategy should be adopted if tax rates are increased by future legislation.

[5]Funding methods for statutory purposes can be changed once every 3 years without IRS approval; consequently, such changes should be made only after careful study.

tions (frozen attained age method). The aggregate method provides very little contribution flexibility, with the maximum limit exceeding the minimum limit by one year's interest at the valuation rate. It is difficult, if not impossible, to rationalize this method being used as the statutory funding method. The entry age normal cost method was the logical choice prior to OBRA '87, which reduced the amortization period for actuarial gains and losses from 15 to 5 years. The frozen entry age method amortizes such losses over future working lifetimes (a period longer than 5 years); hence, this method would be ideal in many cases.

All of the actuarial assumptions can be selected with an eye toward developing wide statutory boundaries; however, each assumption must fall within a range that the actuary will certify as representing a "best-estimate" assumption.[6] Figure 14–6 illustrates the process by which one begins to search for an acceptable set of assumptions. In this illustration, only variations in the interest rate and salary rate assumptions are considered; however, all of the actuarial assumptions are candidates for such sensitivity analyses.[7] The funding method used is the frozen entry age method (the cost prorate method). These results indicate that an 8.5 percent interest rate and a 6 percent salary rate produce minimum required contributions and maximum tax deductible contributions that comfortably encompass the target cost.[8] Based on these results, the preliminary funding policy set forth in Table 14–3 can be tested for its long-term viability using one or more deterministic projections.

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[6]An actuary will generally recognize that, for each assumption, a range of values rather than a single point is consistent with professional standards and legal responsibilities under ERISA. In selecting a specific value in that range, it is reasonable to recognize the plan sponsor's goals. Unless management is abusing either tax deductions or the funded status of the plan, a rare occurrence in large corporations, assumptions should be selected to implement the target costs as efficiently as possible. The interest rate, which is by far the most powerful assumption, is also the most visible. Thus, even though a thoughtful and responsible target cost is being implemented, the use of an interest rate that does not fall within easily defensible boundaries should not be used. Salary scales graded by age, termination rates, and retirement rates provide opportunities to develop appropriate statutory boundaries.

[7]The indicated salary rate is in addition to the standard merit scale from Table 2-9.

[8]The 8 percent interest and 4.5 percent salary assumptions actually produce a somewhat wider range around the target cost than the 8.5 and 6 percent combination; however, the latter combination may have less audit risk for the plan sponsor.

FIGURE 14–6
Contribution Limits Under Alternative Actuarial Assumptions

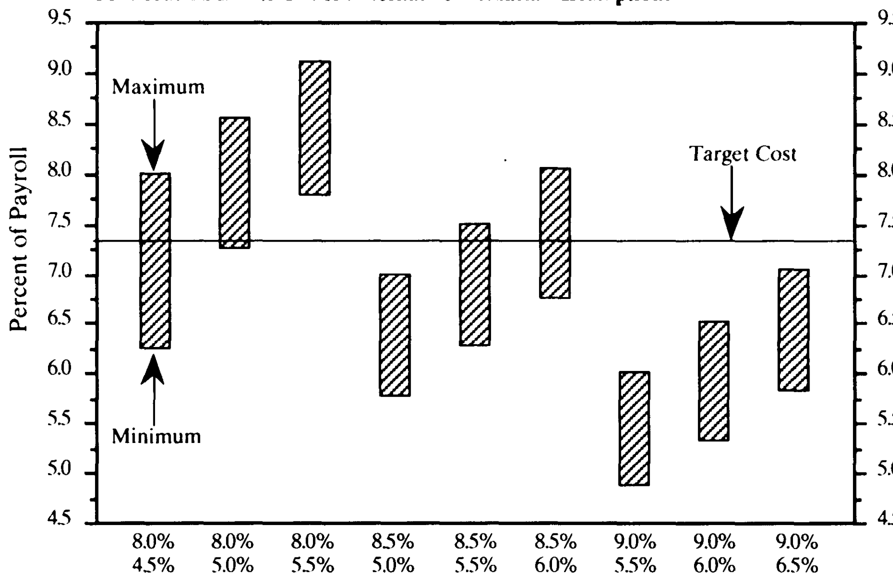


TABLE 14–3
Funding Policy Assumptions

Funding Method:	Frozen Entry Age Method
Interest Rate:	8.5%
Salary Rate:	6% plus merit scale
Decrement Rates:	Model assumptions

The next step is to test these assumptions over the planning horizon. Figure 14–7 shows the results of a 10-year deterministic projection and Figure 14–8 shows the results of a stochastic projection of the model pension plan under the preliminary set of assumptions. The deterministic projection shows that costs are expected to be a level percentage of pay equal to the target costs for the first 8 years of the projection, after which the maximum constraint lowers the contribution somewhat. Plan assets reach 100 percent of the economic liability after 5 years because of the 70 percent confidence factor specified in the funding policy. By the end of 10 years, the funded ratio is 112 percent of the economic liability and 96 percent of the actuarial liability. Based on these results, the preliminary funding policy appears to meet the funding policy objectives.

FIGURE 14–7
Deterministic Projection of Contribution Policy

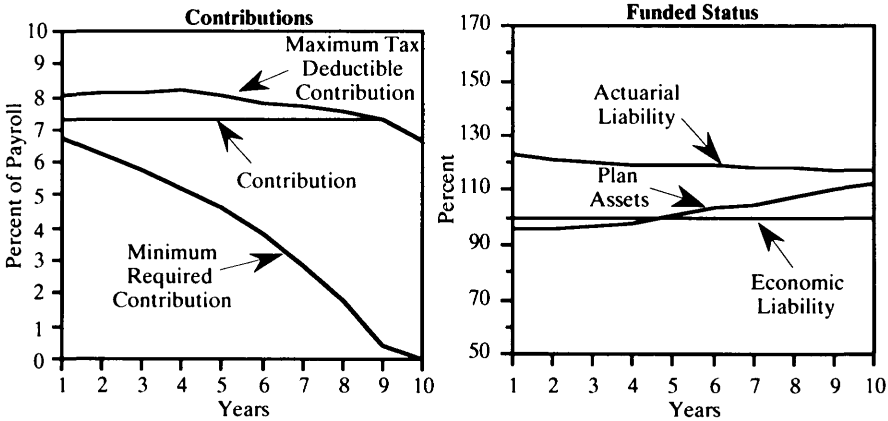
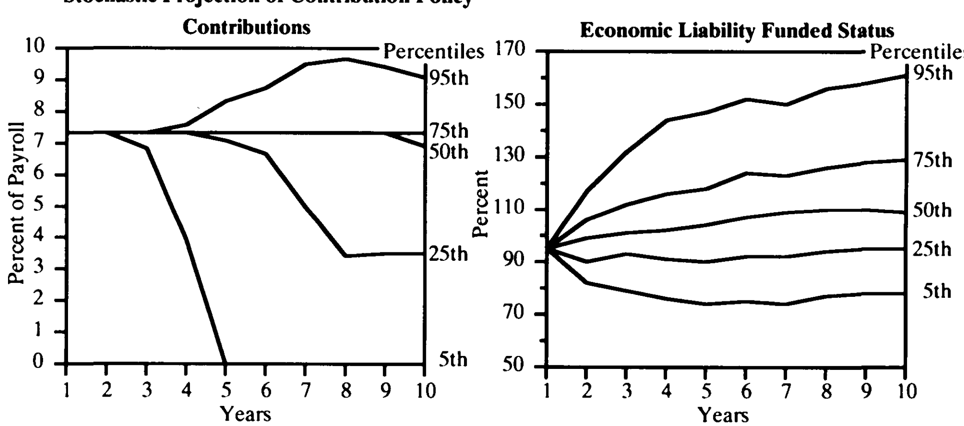


FIGURE 14–8
Stochastic Projection of Contribution Policy



The stochastic projection shown in Figure 14–8 indicates the full range of expected costs and funded ratios, and their corresponding probabilities of occurrence. The contribution projection indicates that, while contributions may be forced away from the 7.35 percent target cost, the "worst case" increase is less than 10 percent of payroll, as indicated by the 95th percentile. The probability that contributions will be forced downward from the target cost is greater than the probability of their being forced upward. The funded status, based on the economic liability, shows a low probability that the funded ratio will sink below 80 percent and fairly substantial probability that the funded ratio will greatly exceed 100 percent of the economic liability. Based on these results, the funding policy assumptions set forth in Table 14–3 appear to hold up satisfactorily during the 10-year planning horizon.

FASB ASSUMPTIONS

A preliminary contribution policy must be established before the SFAS 87 costs can be examined, since contributions affect such costs. In the case of the previously articulated funding policy, it is the plan sponsor's objective to have SFAS 87 costs approximately equal to target costs. Table 14–4 shows the net periodic pension cost under alternative interest discount rates and expected return on assets. Again, all of the actuarial assumptions used to determine SFAS 87 costs are candidates for these types of sensitivity analyses. Based on the results in Table 14–4, an interest assumption of 7.5 percent and an expected return on assets of 8 percent represent reasonable assumptions for initial testing.

TABLE 14–4

Net Periodic Pension Costs Under Alternative Assumptions

Interest Discount Rate	*Expected Return on Assets*	*Net Periodic Pension Cost*
7.0%	8%	8.82%
7.0%	9%	7.88%
7.0%	10%	6.95%
7.5%	8%	7.57%
7.5%	9%	6.64%
7.5%	10%	5.70%
8.0%	8%	6.46%
8.0%	9%	5.52%
8.0%	10%	4.59%

Figures 14–9 and 14–10 provide the deterministic and stochastic projections of the SFAS 87 policy. The net periodic pension cost decreases over the 10-year planning horizon, primarily because, as plan assets increase, the expected return on assets grows proportionately. The funded status indicates that assets will reach the PBO, which is somewhat higher than the economic liability, by year nine. The stochastic projections indicate that the

FIGURE 14-9
Deterministic Projection of SFAS 87 Expense and Funded Status

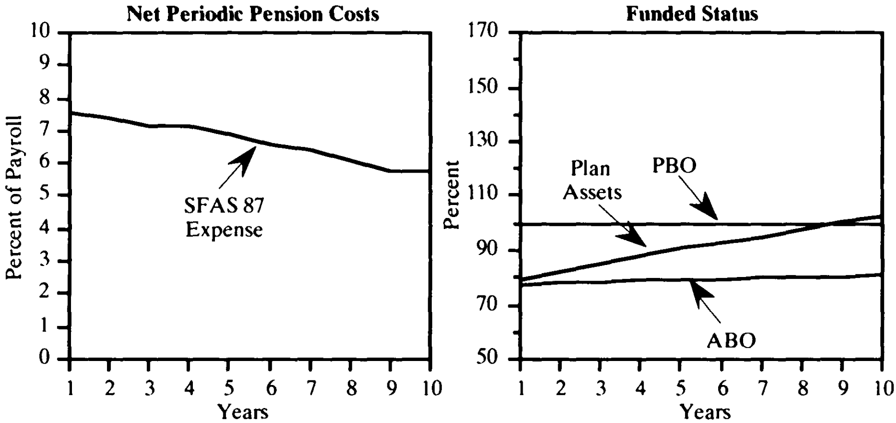
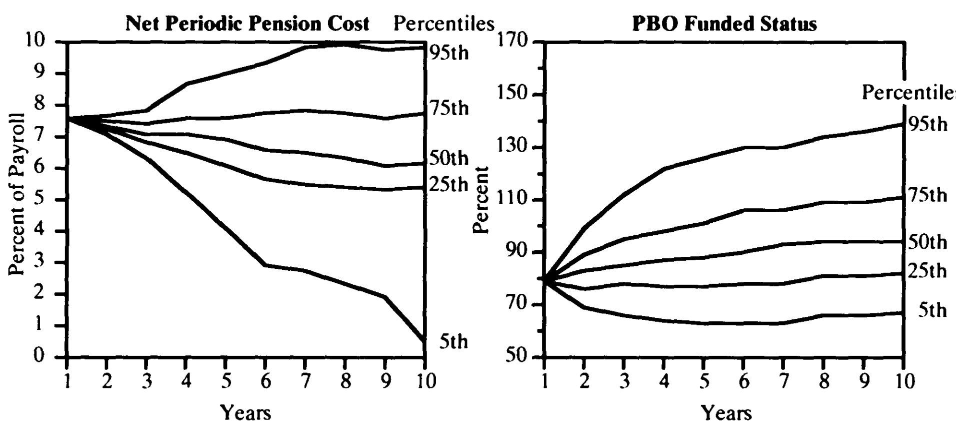


FIGURE 14-10
Stochastic Projection of SFAS 87 Expense and Funded Status



upside costs (or downside funded ratios) are well contained, with the most probable course of events being a reduction in costs and redundant funded ratios.

Based on these results, the preliminary assumptions appear to produce costs that are reasonably in line with the plan sponsor's objectives. In developing an overall funding policy, however, it may be necessary to revise the contribution policy and then reexamine the SFAS 87 policy and continue this process until satisfactory results are achieved.

Chapter 15

Investment Policy: Asset Allocation

The plan's investment policy includes the asset allocation decision, investment manager selection, and performance measurement. The purpose of this chapter is to discuss only the methodology for establishing the plan's asset allocation policy, since the other facets of the investment policy are beyond the scope of this book.

The plan's asset allocation is one of the most important, if not *the* most important, decision that the plan sponsor makes. While it is true that alternative actuarial assumptions and cost methods can have a significant impact on current year costs and the incidence of costs over time, they have no permanent impact on the economics of the pension plan. The asset allocation, on the other hand, has a direct impact on such economics. The ultimate cost of a pension plan can be represented as follows:

$$Ultimate\ Cost = B_t - d\ (Assets)_t \tag{15.1}$$

where

$$B_t = \text{total benefits paid in year } t$$

$$d = \frac{i}{1+i}.$$

The larger the level of assets, the lower the ultimate cost. Since the asset allocation decision has a direct bearing on the level of assets, its importance cannot be overemphasized.

Ironically, the asset allocation decision is given far less attention than other aspects of the plan's investment policy. Corporate America is notorious for spending large amounts of effort and money on investment manager selection and performance measurement, even though less than 10 percent of the plan's total return over an extended period of time is accounted for by this aspect of the investment policy. Conversely, 90 percent of the plan's return is due to the asset allocation decision.

More often than not, the asset allocation decision is improperly analyzed. The focus is often on the near-term volatility of alternative asset classes and, in particular, the possibility that equity returns will be poor for a period of time. In most cases, this is the wrong focus. The critical variables are the volatility of contributions, pension expense and, possibly, the funded ratio of the plan. Because of the various asset smoothing techniques employed in pension funding and accounting, most plans can withstand a significant near-term negative return on assets without having a dramatic increase in pension costs.

For example, if plan assets are valued on a 5-year moving average basis and investment gains and losses are amortized over 5 years, then approximately 5 percent of any year's market loss is recognized for cost purposes in the following year. The SFAS 87 pension cost is similarly insulated from sharp downturns in asset values. The point is that most pension plans can withstand significant near-term losses in investments without having a dramatic affect on costs; hence, the equity exposure can be higher than what management may believe to be prudent by only considering their risk/return characteristics.

If one believes that equities will outperform fixed income investments over the long term, and if the plan's costs and pension expense are sufficiently smoothed, then it is clear that a high percentage of equities (perhaps even 100 percent) is the logical asset allocation. This conclusion might not be appropriate, however, if the plan sponsor viewed the disclosure of the plan's funded status on a market value basis to be particularly important to its overall financial welfare. For example, if the plan were underfunded in the first instance and/or if the corporation's capital structure were highly leveraged, then it may be prudent to "leave some money on the table" with respect to the long-term return from equities as compared to fixed income investment, and to reduce the equity exposure, whether or not contributions and expenses are well insulated from gyrations in the equity market. Similarly, a corporation not paying taxes (or subject to the alternative minimum tax), may want to avoid an increase in near-term contributions by selecting a high percentage of short-term, fixed income investments even though the plan's long-term costs are likely to be higher as a result of this asset allocation decision.

Although it is important to consider the corporation's current circumstances in making the asset allocation decision, in most cases the end result will contain a far greater proportion of equities than currently exists.

CAPITAL MARKET SIMULATION

The first step in performing an asset allocation study is to simulate the returns of various asset classes. The conventional method is to establish, for each asset class, a real rate of return, the volatility of returns (standard deviation), and the covariability (or correlation) of returns among the classes. In addition, since inflation affects asset returns as well as liability values, this variable should also be included.

An equation for simulating inflation is given by

$$I_t = w\, I_{t-1} + (1 - w)\, I_\infty + {}^{Inf}e_t \tag{15.2}$$

where

I_t = inflation in year t

w = serial correlation of successive year's inflation

I_∞ = long-term rate of inflation

${}^{Inf}e_t$ = error term for unexpected inflation in year t, drawn from a log normal distribution with mean zero and specified standard deviation.

The value of w is typically between .60 and .75, with 2/3 being a reasonable choice. In effect, this coefficient says that 2/3 of the time last year's inflation rate is a good predictor of next year's, whereas 1/3 of the time next year's rate is significantly different from the previous year's. The simulation process is performed by drawing random variables from the error term distribution and adding these values to the weighted average of the previous year's simulated rate and long-run inflation.

The nominal return for a given asset class can be written as follows:

$$(NR)_t = (RR)_t + E(I_t) + {}^{AC}e_t \tag{15.3}$$

where

$(NR)_t$ = nominal return in year t

$(RR)_t$ = real return in year t

$E(I_t)$ = expected inflation in year t

$E(I_t)$ = $w\, I_{t-1} + (1 - w)\, I_\infty$ from equation (15.2)

${}^{AC}e_t$ = error term for asset class (AC) in year t, drawn from a log normal distribution with mean zero and specified standard deviation.

The error terms for each asset class and inflation are linked to each other by a specified covariance (or correlation).

There are a number of variations that one could make in the above equations, such as allowing the real returns, error terms, and/or correlations to follow a dynamic pattern over time rather than being stationary. The following numerical illustrations, however, are based on the previous equations without such embellishments.

Capital Market Statistics

Table 15–1 shows statistics on inflation and returns on five asset classes over the 20-year period 1971–1990.[1] Inflation is based on the Consumer Price Index for All Urban Consumers. The statistic shown under "nominal return" for inflation is the geometric average of the CPI over this period, whereas the standard deviation and correlations for inflation are based on an unexpected inflation variable.[2] T-Bills are based on 30-day bonds. Long-term bonds (L-Bonds) are based on Solomon Brothers Long-Term, High-Grade Corporate Bond Index, which includes nearly all bonds rated Aaa and Aa. The S&P 500 index is used for domestic stock returns. Small stocks (S-Stocks) are defined as the fifth (smallest) quintile on the NYSE, plus stocks on the AMEX and OTC with the same capitalization upper and lower bounds as the NYSE fifth quintile. World stocks (W-Stocks),

[1]The source of these data is Ibbotson Associates' Benchmark PLUS (Chicago, Illinois, 1991).

[2]There are a number of ways to define unexpected inflation from an inflation index. The method used in the above statistic was to determine the delta from a linear regression over the period from 1926 to 1990.

TABLE 15–1

Capital Market Statistics, 1971–1990
(Means and Standard Deviations in Percent)

	Nominal Return	*Real Return*	*Std. Dev.*	*Real Return Correlations*					
				*Inf**	*T-Bills*	*L-Bonds*	*S&P*	*S-Stocks*	*W-Stocks*
Inf*	6.26		2.64	1.00					
T-Bills	7.66	1.40	3.07	-.71	1.00				
L-Bonds	9.01	2.75	13.97	-.79	.68	1.00			
S&P	11.15	4.89	17.15	-.66	.41	.60	1.00		
S-Stocks	13.35	7.09	23.20	-.46	.10	.28	.71	1.00	
W-Stocks	15.79	9.53	27.13	-.30	.03	.30	.64	.41	1.00

*Standard deviations and correlations are based on unexpected inflation.

which exclude U.S. stocks, is based on the FT-Actuaries World Indices which includes data from 23 countries. The latter data are available for the past 10 years only.

The capital market simulation is parameterized by using the geometric mean rather than the arithmetic mean; hence, this statistic is shown for each index. The geometric mean is less than the arithmetic mean, the degree to which depends on the dispersion of yearly returns. As an example, the arithmetic mean of the S&P 500 nominal return is 12.49 percent over the 20-year period versus the geometric mean of 11.15 percent.

The real returns for all asset classes are negatively correlated with unexpected inflation, whereas the various asset classes are positively correlated to each other over this time period.

Illustrative Capital Market Parameters

Based on the data presented in Table 15–1, and to emphasize the point that the parameterization of a capital market simulation should be based, in part, on historical data and, in part, on judgment, the capital market parameters used in the simulations presented in this chapter are shown in Table 15–2. The long term inflation rate is set at 4 percent, with the incremental real returns being approximately equal to those shown in Table 15–1, but reduced for S-Stocks and W-Stocks. The standard deviations and correlations were selected to be "round numbers" reasonably consistent with historical data.

TABLE 15–2

Capital Market Assumptions
(Means and Standard Deviations in Percent)

	Nominal Return	*Real Return*	*Std. Dev.*	*Real Return Correlations* *Inf**	*T-Bills*	*L-Bonds*	*S&P*	*S-Stocks*	*W-Stocks*
Inf*	4.0		2.0	1.00					
T-Bills	5.0	1.0	2.0	–.70	1.00				
L-Bonds	7.0	3.0	10.0	–.75	.60	1.00			
S&P	9.0	5.0	18.0	–.55	.40	.50	1.00		
S-Stocks	10.0	6.0	25.0	–.35	.10	.20	.70	1.00	
W-Stocks	11.0	7.0	27.0	–.30	.00	.20	.60	.35	1.00

*Standard deviations and correlations are based on unexpected inflation.

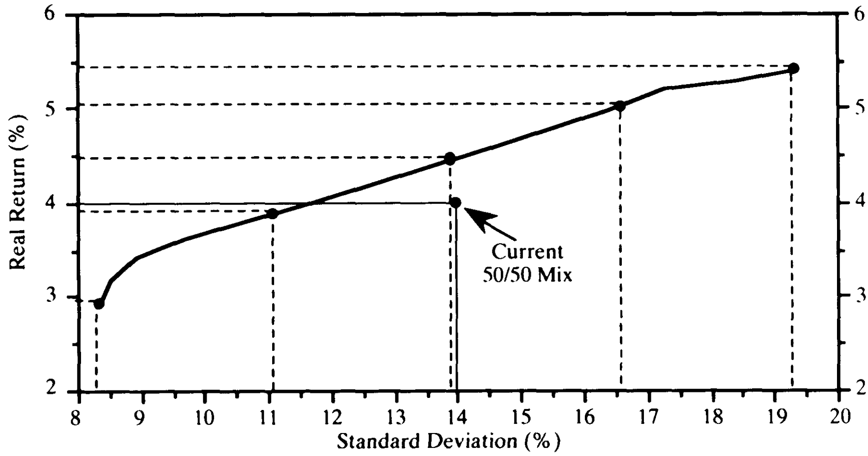
Efficient Frontier

There are an infinite number of asset mixes that one could derive from the five asset classes specified in the illustrative capital market simulation. The *efficient frontier* represents all combinations of these asset classes that have the highest return for a given risk or, conversely, the lowest risk for a given return. Figure 15–1 shows the efficient frontier for the illustrative assumptions, along with the five mixes to be studied in the asset allocation analysis. In deriving the efficient frontier, T-Bills were limited to a minimum of 5 percent and a maximum of 15 percent, and both S-Stocks (small stocks) and W-Stocks (world stocks) were limited to a maximum of 20 percent each. Under the assumptions and constraints used in this illustration, the efficient frontier is linear from Mix 2 to Mix 4 because the only change over this interval is the mix between the S&P and L-Bonds asset classes.

The current 50-50 stock-bond mix is also plotted in Figure 15–1. This mix is inefficient since has the same standard deviation as Mix 3, while having the same real return as Mix 2.

The five mixes selected from the efficient frontier range from the least risky to the most risky, with the intermediate mixes having a uniform difference in standard deviations. The graph shows that equal increases in the risk (standard deviation) results in proportionally smaller increases in real returns. The real returns

FIGURE 15–1
Efficient Frontier



	Mix 1	Mix 2	Mix 3	Mix 4	Mix 5
T-Bills	.15%	.05%	.05%	.05%	.05%
L-Bonds	.79	.65	.37	.08	.00
S&P	.00	.10	.38	.67	.55
S-Stocks	.00	.00	.00	.00	.20
W-Stocks	.06	.20	.20	.20	.20
	1.00	1.00	1.00	1.00	1.00
Real Return	2.9	3.9	4.5	5.0	5.4
Std. Dev.	8.4	11.1	13.9	16.6	19.3

and standard deviations for the various mixes are shown at the bottom of Figure 15–1. Since the long-term inflation rate is 4 percent, the nominal returns are equal to the indicated real returns plus 4 percent.

Because of the input assumptions for S&P and W-Stocks, both are preferred in the efficient frontier to the S-Stock asset class. Whether the inputs used in this illustration are reasonable will depend on the plan sponsor's analysis of these asset classes. The purpose of this chapter is simply to illustrate the asset allocation methodology, not to promote one or more specific asset classes over others.

Real Return Simulations

The 1-year, 5-year and 10-year simulated real returns for each of the five mixes are shown in Figures 15–2a through 15–2c, respectively. In each case, percentiles ranging from the

5th to the 95th are indicated. Note that the *y*-axis scale is different for each graph. Mix 5, for example, has simulated 1-year returns ranging from –21 to 40 percent, while the 5-year returns range from –4 to 19 percent and the 10-year returns range from –6 to 20 percent.

The attractiveness of alternative mixes depends on the time period under analysis. As the planning period is extended, the compression of compound real returns (known as time diversification), coupled with the higher expected returns, favors a higher exposure to equities. Since most pension plans have the luxury of a long planning horizon, a high equity exposure is often an appropriate asset allocation. Moreover, because of the asset smoothing that takes place in determining contributions and expense, high equity exposures make even more sense, as shown in the next section.

STOCHASTIC PENSION SIMULATIONS

A summary of the asset allocation results is presented in this section. Figure 15–3a shows the distribution of contributions for the 5th year of the stochastic projection under each of the five mixes, while Figure 15–3b shows the SFAS 87 expense and Figure 15–3c shows the economic liability funded ratio. Figures 15–4a, through 15–4c show these same results for the 10th year of the projection.

The distributions for employer contributions do not lend themselves to "floating bar charts" because they are often not continuous, due to the fact that (1) contributions often can be locked into the plan sponsor's target cost, provided such costs are within the ERISA minimum and maximum limits, and (2) because contributions cannot be negative, hence, a large probability of zero contributions can occur. Therefore, every 5th percentile of the contribution distribution is shown in Figure 15–3a. All of the mixes have a fairly substantial probability of allowing the 7.35 percent target cost to be contributed in the 5th year. This contribution for Mix 1 applies from the 35th to the 75th percentile, whereas for Mix 5 it applies from the 50th to the 75th percentile. There is very little chance that contributions will be zero in the 5th year under Mix 1, versus a 25 percent chance under Mix 5. On the other hand, the "worst case" (or 95th per-

FIGURE 15–2a
One-Year Real Returns

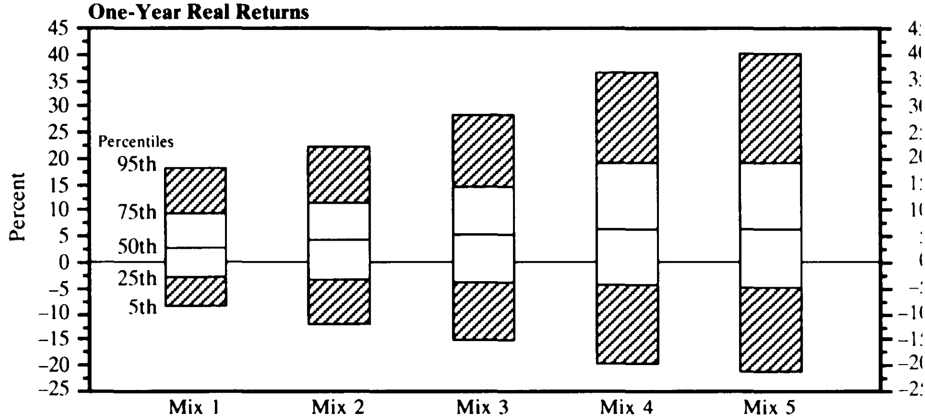


FIGURE 15–2b
Five-Year Compound Real Returns

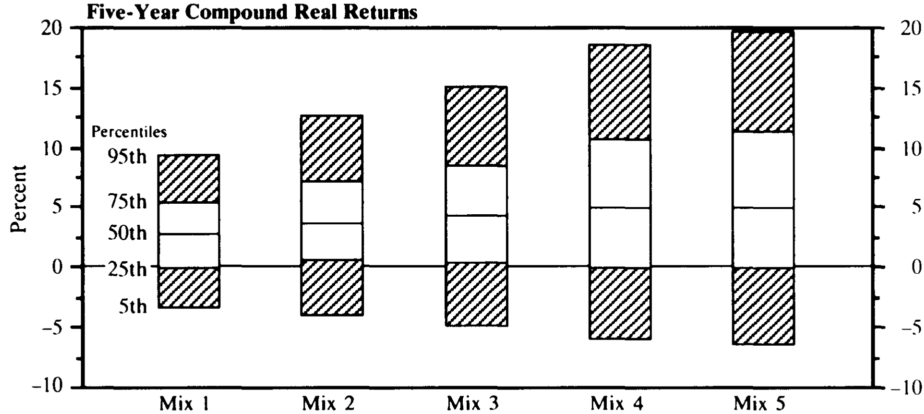


FIGURE 15–2c
Ten-Year Compound Real Returns

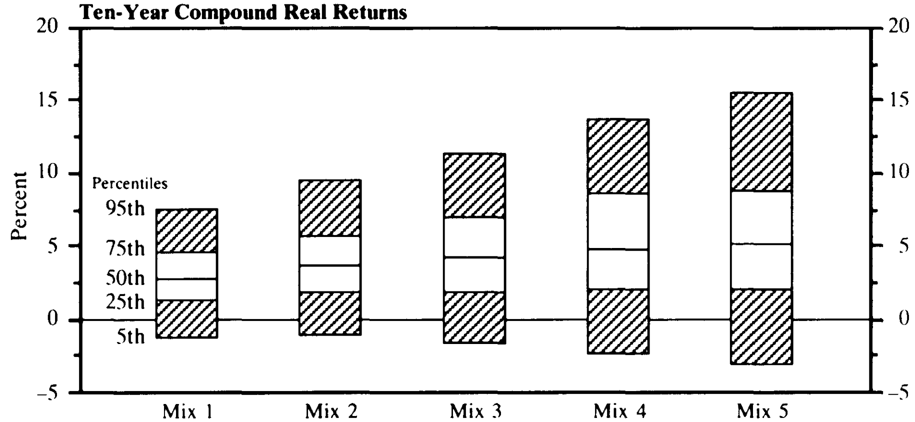


FIGURE 15–3a
Contributions in Year 5 Under Alternative Asset Mixes

Percentile	Mix 1	Mix 2	Mix 3	Mix 4	Mix 5
95th	7.90	8.23	8.62	9.78	9.53
85th	7.35	7.35	7.60	8.16	8.26
75th	7.35	7.35	7.35	7.35	7.35
65th	7.35	7.35	7.35	7.35	7.35
55th	7.35	7.35	7.35	7.35	7.35
50th	7.35	7.35	7.35	7.35	7.35
45th	7.35	7.35	7.35	7.24	7.09
35th	7.35	7.28	6.99	6.37	5.61
25th	7.23	6.64	6.39	2.09	0.00
15th	6.78	3.10	0.00	0.00	0.00
5th	0.85	0.00	0.00	0.00	0.00

FIGURE 15–3b
Pension Expense in Year 5 Under Alternative Asset Mixes

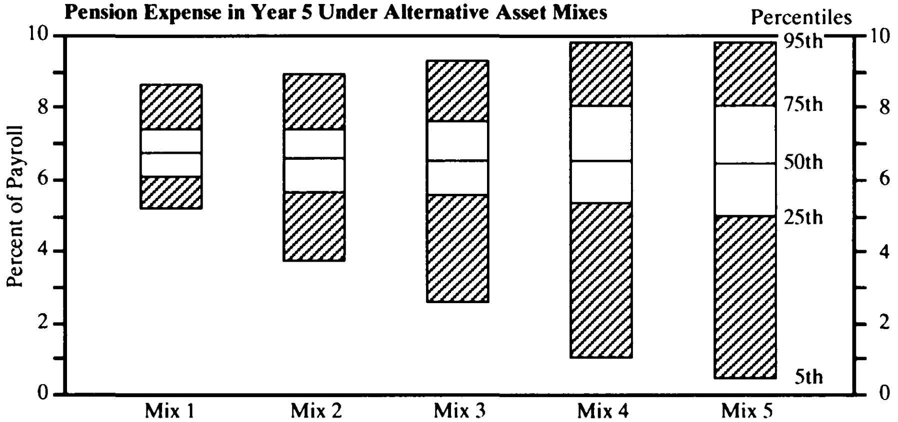


FIGURE 15–3c
Economic Liability Funded Ratio in Year 10 Under Alternative Asset Mixes

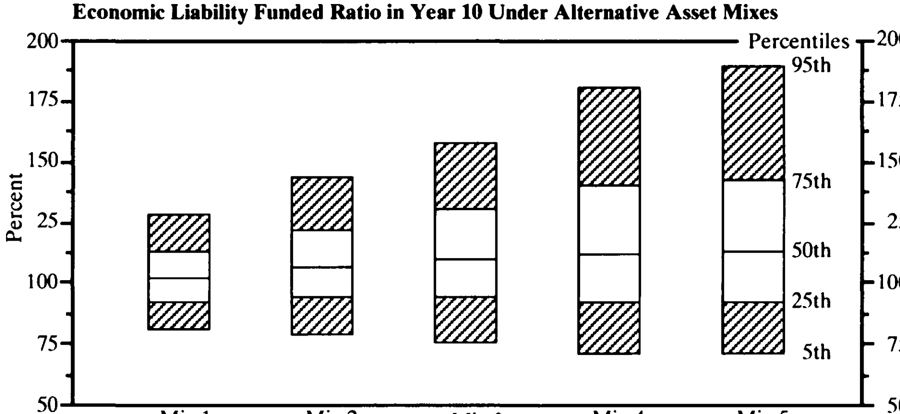


FIGURE 15–4a
Contributions in Year 10 Under Alternative Asset Mixes

Percentile	Mix 1	Mix 2	Mix 3	Mix 4	Mix 5
95th	7.88	7.94	8.32	9.49	9.50
85th	7.35	7.35	7.35	7.35	7.35
75th	7.35	7.35	7.35	7.35	7.35
65th	7.35	7.35	7.35	7.35	6.90
55th	7.35	6.73	6.24	5.91	5.63
50th	6.97	6.20	5.66	5.17	4.97
45th	6.58	5.69	5.24	4.72	0.00
35th	5.63	4.78	4.12	0.00	0.00
25th	4.93	2.71	0.00	0.00	0.00
15th	4.09	0.00	0.00	0.00	0.00
5th	0.00	0.00	0.00	0.00	0.00

FIGURE 15–4b
Pension Expense in Year 10 Under Alternative Asset Mixes

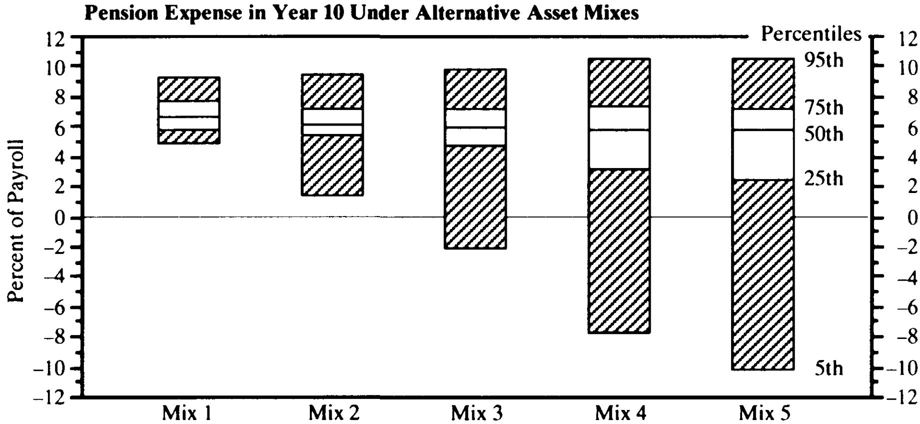
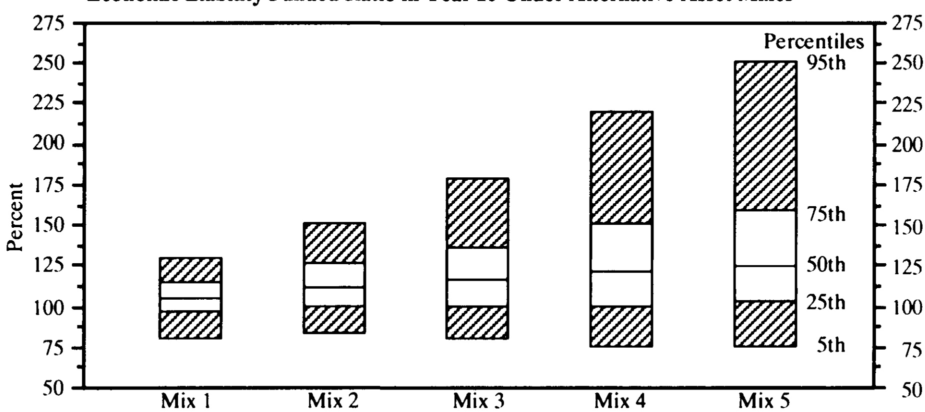


FIGURE 15–4c
Economic Liability Funded Ratio in Year 10 Under Alternative Asset Mixes



centile) for Mix 1 is 7.9 percent versus 9.53 for Mix 5 (note that Mix 4 has a higher "worst case" contribution in the 5th year). The "worst case" profile in the 10th year (see Figure 15–4a) is nearly identical to the "worst case" for the 5th year; however, the higher equity mixes have considerably lower expected contributions. Generally speaking, Mix 5 appears to be the superior mix except, perhaps, for those firms that place an extremely high priority on contribution predictability and stability.

There is little question that the SFAS 87 and economic funded ratio "floating bar charts" support Mix 5: the "worst case" is not any worse, yet the "best case" is considerably better for this mix than the others. This is true for both the 5th and 10th year of the analysis. Consequently, unless the "worst case" contributions under Mix 5 are of concern, which is unlikely for most plan sponsors, Mix 5 would be the preferred mix. It will be recalled that this mix is essentially an all-equity mix, having 55 percent of the S&P asset class, 20 percent each of S-Stocks and W-Stocks, and 5 percent of T-Bills (the latter representing the minimum required percent for this asset class).

Chapter 16

Funding and Accounting for Retiree Health Benefits

The funding and accounting for retiree health benefits is an important subject for many corporations. Although the focus of this book is on defined benefit pension plans, retiree health benefits is an ideal application of the actuarial concepts and mathematics presented in previous chapters.

Many corporations provide health benefits and, sometimes, life insurance benefits to employees in retirement. Unlike pension plans, where advance funding and accrual accounting have been required for decades, retiree health benefits have been funded and accounted for on a pay-as-you-go basis by nearly all employers. The increase in the number and percentage of retired employees, plus the persistent increases in health care costs, ranging from 10 to 25 percent per year, have created a financial burden for many companies. The cost associated with recovering from decades of funding and accounting on a pay-as-you-go basis will create an even more substantial financial burden in future years.

Unlike qualified pension plans, there is no legal requirement to prefund retiree health benefits. Moreover, the ability to advance fund retiree health liabilities with tax deductible contributions and tax-deferred interest earnings on plan assets was significantly curtailed by the passage of the Deficit Reduction Act (DEFRA) in 1984. Even with this constraint, however, employers can fund a substantial portion of this liability, yet there has been little movement in this direction. The one exception to this practice is employers that can pass the cost of prefunding on to their customers (e.g., utilities and government contractors).

In December of 1990, the FASB promulgated SFAS 106, *Employers' Accounting for Postretirement Benefits Other Than Pensions*, generally effective for fiscal years beginning after December 15, 1992. This accounting standard, whose form is similar to SFAS 87 for defined benefit pension plans, requires that employers adopt accrual accounting for retiree life and health benefits, as well as other non-pension benefits. The required adoption of SFAS 106 has caused many employers to rethink both the types and amounts of health benefits provided in retirement and the feasibility of prefunding.

This chapter covers both the funding and accounting aspects of retiree health benefits. First, however, the economic liability and cost of these benefits are set forth as background for the funding and accounting discussions.

ECONOMIC LIABILITIES AND COSTS

Health Benefits Cost Function

The starting point in formulating the economic liabilities and costs of retiree health benefits is to determine the total expected cost of such benefits at age x, which can be represented by the following:

$$^{Total}C_x = {}^{Hosp}C_x + {}^{Phy}C_x + {}^{Lab}C_x + {}^{Drgs}C_x + {}^{Other}C_x \tag{16.1}$$

where

$^{Total}C_x$ = total expected health benefit costs at age x

$^{Hosp}C_x$ = expected hospital costs

$^{Phy}C_x$ = expected physician costs

$^{Lab}C_x$ = expected laboratory costs

$^{Drgs}C_x$ = expected prescription drug costs

$^{Other}C_x$ = expected other charges (e.g., nursing home costs).

It is assumed that the health benefits cost associated with any spouse or other dependent coverage is included in (16.1) even though the equation is expressed in terms of an individual age x.

The employer's cost is reduced by: (1) payments made by the employee under the provisions of the plan (e.g., deductibles, copayments, annual maximums, lifetime maximums, and/or employee contributions), (2) Medicare reimbursements (Part A provides funds related to hospital charges; Part B provides funds related to physician and other costs), (3) Medicaid reimbursements (which provides funds for long-term nursing care as opposed to acute care), and (4) reimbursements through coordination with other insurance carriers. Thus, the employer's cost of health benefits is as follows:

$$^{ER}C_x = {}^{Total}C_x - {}^{EE}C_x - {}^{MC}R_x - {}^{Other}R_x \tag{16.2}$$

where

$^{ER}C_x$ = employer's expected health benefits cost for employee age x (minimum of 0)

$^{EE}C_x$ = employee's expected cost and/or contributions at age x

$^{MC}R_x$ = expected reimbursements from Medicare

$^{Other}R_x$ = expected reimbursements from other governmental programs and/or other private insurance programs.

There are three basic methods used to integrate Medicare reimbursements under a health care plan. The employer's cost for each method is given below.[1]

Carve-Out Method:

$$^{ER}C_x = {}^{ER}(CP)_t\left[{}^{Total}C_x - D_t\right] - {}^{MC}R_x - {}^{Other}R_x \tag{16.3a}$$

Exclusion Method:

$$^{ER}C_x = {}^{ER}(CP)_t\left[{}^{Total}C_x - D_t - {}^{MC}R_x\right] - {}^{Other}R_x \tag{16.3b}$$

[1]These formulas assume the underlying benefit plan is a simple comprehensive type plan with an up front deductible, constant coinsurance percentage paid after the deductible and no employee contributions. Although not explicitly stated, the minimum value in each case is zero.

Coordination Method:

$$ {}^{ER}C_x = Min \left\{ \begin{array}{c} {}^{ER}(CP)_t\left[{}^{Total}C_x - D_t\right] \\ {}^{Total}C_x - {}^{MC}R_x \end{array} \right\} - {}^{Other}R_x \qquad (16.3c) $$

where

$$ D_t = \text{employee deductible in year } t $$

$$ {}^{ER}(CP)_t = \text{employer copayment fraction (e.g., 80\%).} $$

Of these three methods, the carve-out method produces the lowest employer costs, while the coordination method produces the highest costs.

Actuarial Assumptions

All of the actuarial assumptions defined previously for pension plans are applicable to valuing retiree health benefits, except for any assumptions used in projecting Social Security benefits for integrated formulas. The financial impact of the various actuarial assumptions for retiree health benefits, however, may be different from the impact on pension plans, and there are a number of assumptions associated with projecting retiree health benefits that are not applicable to pension plans.

Decrement Assumptions. The four decrement assumptions are mortality, termination, disability, and retirement rates. Mortality rates for pension plans prior to retirement reduce retirement related liabilities but increase the liability associated with any lump-sum or survivor income benefits. Death benefits for active employees are not applicable in valuing health benefits; hence, pre-retirement mortality affects such liabilities more directly. On the other hand, if the plan provides health benefits to surviving spouses, then the liability reduction due to mortality would not be as great.

After retirement, mortality rates at older ages (e.g., beyond age 75) become relatively insignificant for pension liabilities because the value of the fixed (non-inflation adjusted) benefit is reduced substantially by the interest assumption. The interest factor for health benefits, however, is offset in part, or more than offset, by the increase in such benefits; hence, the benefits payable at older ages are more significant in the present value calculation than is the case for pension benefits. This implies that mortality rates at older ages are more important in valuing retiree health benefits than pension benefits.

Termination rates reduce retirement-related liabilities for pension plan valuations; however, vested termination benefits offset a portion of this reduction. Most retiree health benefit plans do not provide benefits to vested terminated employees after they reach retirement age; hence, termination rates affect health benefit liabilities more directly than pension liabilities.

Disability rates operate in the same manner as mortality and termination rates for pension valuations if the pension plan provides disability benefits. Such rates can either reduce or increase the plan's liabilities, depending on the nature of the disability benefit. Whereas the disability benefit is equal to or less than the retirement benefit for pension plans, this is not the case for health benefits. Such benefits are typically substantial for disabled employees. If such employees, prior to retirement, are considered members of the retiree health benefits plan, as opposed to members of the plan for active employees, then the disability assumption can be significant. For employers with significant retiree health benefits for disabled participants, the use of a disabled life mortality table that includes recovery from disability can be an extremely important assumption.

Retirement rates are much more critical for health benefit valuations than for retirement benefit valuations. Pension benefits are generally reduced for early retirement because of the service-related formula used by most plans and because benefits are often additionally reduced (by formula or actuarially) for retirement prior to the plan's defined normal retirement age. This is not the case for health benefits and, in fact, health benefit liabilities are substantially higher at retirement ages below 65 since Medicare reimbursements do not commence until that age.

Economic Assumptions. The primary economic assumptions for pension plans are the salary and interest rates. A salary assumption is not required in determining health benefit liabilities; however, it may be required for allocating such liabilities throughout the employee's active working lifetime in determining the plan's economic liability and, depending on the actuarial cost method used, it may be required in determining the statutory limits on tax deductible contributions.

The interest rate assumption for valuing pension plans will vary, depending on whether the economic, statutory, or accounting liabilities and costs are being calculated. The same is true for retiree health benefit liabilities; however, for economic and statutory calculations, an after-tax rate may be the appropriate assumption if investment returns on prefunding are taxed.

Inflation and Utilization. Each element of (16.1) must be projected for inflation and utilization in estimating the future health benefit costs applicable throughout retirement. Generally, the inflation assumption will decrease over a period of 10 to 20 years, beginning with a rate in the 10 to 15 percent range and ultimately reaching a level in the 5 percent range. This is an extremely important assumption and is generally based more on judgment than historical trends.

The utilization factor is three-dimensional: it includes gender, age, and time. Males in retirement tend to have higher health care utilization than females at the same age, and health care utilization increases with age for both males and females. The time dimension itself is two-dimensional: it includes technological changes and behavioral changes. Utilization may change in future years, increasing or decreasing, because of technological advances. The behavioral dimension reflects future changes in the propensity of retired employees to access health care services. Again, this assumption can be used to project either increases or decreases in utilization, and it is not necessarily independent of technological changes.

Employee Costs. Turning to equation (16.2), assumptions are required regarding any employee costs that reduce the employer costs of health benefits. For example, if the plan provides for employee contributions, then an assumption may be required to project future contributions, unless contributions are expressed as a percent of the employer's aggregate costs, in which case a separate assumption may not be required. If the plan currently provides for fixed contributions that are expected to be increased on an *ad hoc* basis, then it is appropriate to make such an assumption for determining the plan's economic liability and costs. Since contributory retiree health plans are elective on the part of retirees, some employees will not elect coverage, especially if they have benefits under a spouse's program. Consequently, a participation rate must be assumed with such plan designs. The participation rate may vary over time if the retiree's share of total plan costs is expected to change.

Similarly, if the plan provides for a fixed-dollar deductible that is expected to be increased periodically, then such an increase assumption is relevant. The same can be said for annual or lifetime maximums and, for that matter, copayment percentages, although the latter generally remains fixed (typically in the 80 percent range).

Medicare Reimbursements. The projection of future Medicare reimbursements in (16.2), or other governmental programs, also requires assumptions. While the relevant component increases in (16.1) represent the basis for projecting future cost reimbursements, an additional assumption reflecting a shift in costs from government programs to private employers (or vice versa) should be made. Typically, the Medicare reimbursements assumption will be somewhat less than the trend in health benefit costs, at least for a number of years. However, for Medicare eligible retirees, a large portion of their costs are controlled by Medicare through limits placed on physician and hospital billing. Therefore, this portion of plan costs can be expected to increase at the same rate as Medicare. If the employer pays the employee's Part B premium, then a premium increase assumption is required. This would typically equal the Medicare trend assumption, less any expected cost shifting from government programs to retirees.

If the plan provides for surviving spouse coverage or dependent coverage, additional assumptions are required. For example, the probability of having a spouse, and the probability that coverage under a contributory plan will be elected, must be estimated. In the case of dependent coverage, the probability of having dependents and the duration of such coverage (generally limited to a specified age) must be estimated.

Naturally, there may be other assumptions needed, depending on the nature of the health benefit program. The point, however, is that each component affecting the employer's cost of future retiree health benefits must have a corresponding assumption if the liabilities and costs of the program are to be properly valued.

Economic Liabilities

Assuming that the health benefit cost function is defined and projected with the appropriate assumptions, determining the economic liability is a straightforward application of the actuarial mathematics presented previously. The economic liability for an employee in retirement at age x is given by (16.4), where benefits are adjusted with interest to the end of the year:

$$^{HB}a_x = \sum_{t=0}^{\infty} {}^{ER}_{t}C_x \; {}_tp_x^{(m)} \; v^{t+1} \tag{16.4}$$

where

${}^{ER}_{t}C_x$ = employer's expected health benefits cost at age $x + t$ for a retiree currently age x.

The economic liability for an active employee is found by prorating the present value of future benefits to the age in question. This PVFB function for an employee age x is given by

$$^{HB}(PVFB)_x = \sum_{k=m}^{r''} {}_{k-x}^{\;\;r}p_x^{(T)} \; q_k^{(r)} \; v^{k-x} \; {}_{k-x|}^{\;HB}a_x \tag{16.5}$$

where

m = the greater of r' or x

${}_{k-x|}^{\;HB}a_x$ = deferred health benefits annuity, as defined in (16.4) at the point of retirement, with the deferral period reflecting increases in the employer's health benefits cost from age x to each specific retirement age, but not reflecting decrements and interest, which are explicit in (16.5).

The economic liability is equal to (16.5) with a salary proration factor included:

$$^{HB}(EL)_x = \sum_{k=m}^{r''} {}_{k-x}^{\;\;r}p_x^{(T)} \; q_k^{(r)} \; v^{k-x} \; \frac{S_x}{S_k} \; {}_{k-x|}^{\;HB}a_x \, . \tag{16.6}$$

As noted previously in discussing the economic liability of pension benefits, the salary-based proration is a logical choice if benefits in retirement, whether pension or health, are viewed as deferred wages. Nevertheless, some employers may wish to define the economic liability as a service proration, in which case the salary fraction in (16.6) would be replaced by $(x - y)/(k - y)$.

Economic Costs

Economic costs can be estimated either deterministically or stochastically, as described in the Target Cost Methodology section of Chapter 14 for pension plans. The plan sponsor selects a funding period, such as 20 years, and a cost pattern, such as a level percent of payroll. With these objectives established, the deterministic approach involves the calculation of a level percentage of payroll that will accumulate assets (either "hard" assets or accounting assets) to the economic liability by the end of the planning horizon.

The stochastic approach involves a distribution of level cost percentages that will accumulate assets to a distribution of economic liabilities by the end of the planning horizon. The employer can then select the target cost corresponding to a desired confidence level that assets will equal or exceed the economic liability by the end of the planning horizon.

Numerical Illustrations

Figures 16–1 and 16–2 show a sensitivity analysis of the economic liability and costs, respectively, for retiree health benefits. The plan design and actuarial assumptions used in the base case are set forth in Table 16–1. The health benefit costs and Medicare reimbursements are shown in Table 16–2, with all values expressed as a percent of the gross costs for females in the 55 to 59 age bracket.

Figure 16–1 indicates that a uniform 2 percentage point change in health benefits and Medicare trends has a dramatic impact on the economic liabilities, and Figure 16–2 shows that the percentage effect on economic costs is even greater. If only the health benefits trend is changed while holding the Medicare trend constant, the impact on liabilities and costs is somewhat larger. On the other hand, a 2 percentage point change in the Medicare trend itself has a reasonably modest impact, falling in the 10 to 20 percent range.

A 2 percentage point change in the interest rate assumption is, of course, significant and similar to the impact that such a change would have on pension plan costs and liabilities. Al-

TABLE 16–1

Retiree Health Benefit Plan and Assumptions

Plan Design	
Coverage:	Retirees and surviving spouses
Eligibility:	Age 55 and 10 years of service
Employee Contributions:	None
Employee Deductible:	$200 per year
Employee Copayment:	20%
Employee Maximums:	None
Medicare Integration:	Carve-out method
Decrement Rates	Same as those used with model pension plan
Economic Rates	
Salary Rate:	5% plus Merit (Table 2–10)
Interest Rate:	8%
Health Benefit Inflation:	10% grading down to 6% over 10 years
Medicare Inflation:	7% grading down to 5% over 10 years
Utilization:	5% grading down to 1% over 10 years
Demographic Assumptions:	80% married; females 3 years younger than males
Participation Rate	100%

TABLE 16–2

Health Benefits Cost Function
(As Percent of Gross Costs for Age 55 to Age 59 Females)

Age	Gross Costs		Medicare Offsets	
Range	Male	Female	Male	Female
55 to 59	105	100	0	0
60 to 64	130	120	0	0
65 to 69	145	130	55	55
70 to 74	155	140	65	65
75 to 79	165	150	75	75
80 to 84	175	160	85	85
85 to 89	185	170	95	95
90 & Over	195	180	95	95

though the effect on the economic liability is about the same as an equal, but opposite, change in the health benefits trend, economic costs are less sensitive to interest rate changes than to health benefits trend changes. The three plan design changes analyzed in Figures 16–1 and 16–2 have a comparatively small impact on liabilities and costs, falling in the 10 to 15 percent range.

FIGURE 16–1
Economic Liability Sensitivity Analysis

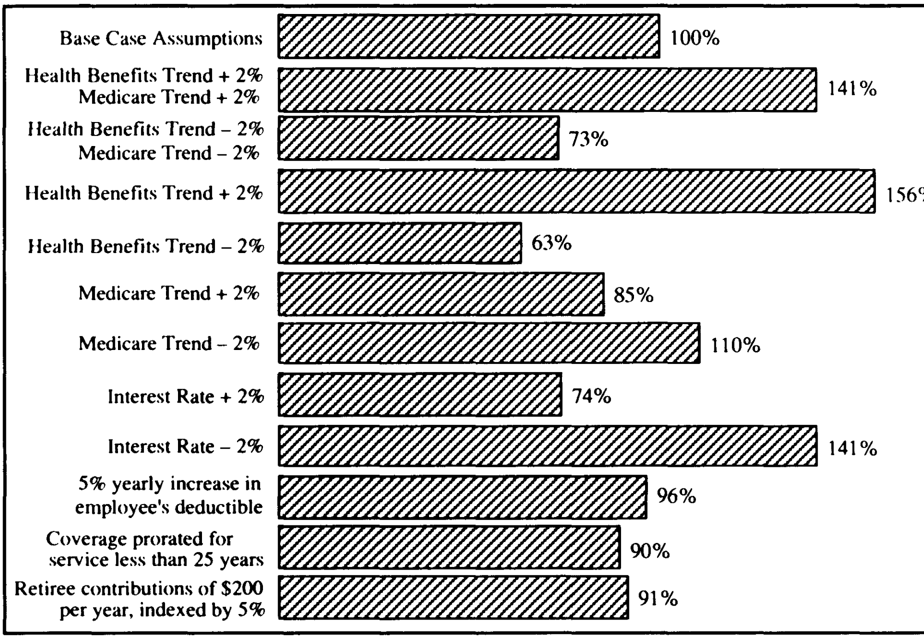
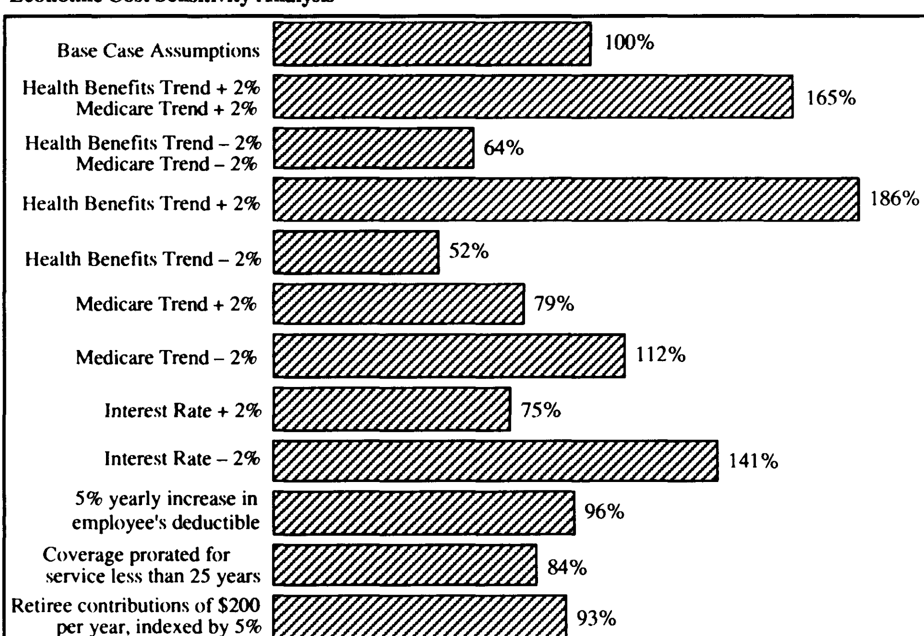


FIGURE 16–2
Economic Cost Sensitivity Analysis



FUNDING LIMITS

A Voluntary Employees' Beneficiary Association (VEBA) can be created for the establishment of a §501(c)(9) trust to prefund retiree health benefits.[2] VEBAs established for health benefits created under a collective bargaining agreement can be funded in essentially the same manner as qualified pension benefits, namely, with tax deductible contributions and tax-free interest earnings on trust assets. DEFRA placed limits on the prefunding of non-collectively bargained VEBAs by not allowing health care inflation to be used in projecting future health benefits for determining the funding liability, and by requiring the employer to pay tax on investment earnings, known as Unrelated Business Income Tax (UBIT).

Section 419 of the IRC sets forth the tax deductible contributions as follows:

$$(DC)_t = (QDC)_t + (\Delta QAA)_t - (ATI)_t \tag{16.7a}$$

where

$(DC)_t$ = tax deductible contributions in year t

$(QDC)_t$ = *qualified direct cost* in year t

$(\Delta QAA)_t$ = addition to a *qualified asset account* in year t, subject to the limitations of § 419A

$(ATI)_t$ = after-tax income in year t.

Qualified direct costs are equal to the benefits paid to plan members during the year plus related administrative expenses. A *qualified asset account* represents assets set aside to provide for the payment of health benefits. Tax deductible additions to this account are limited by §419A to (1) the amount reasonably and actuarially necessary to fund health claims incurred but unpaid as of the end of the taxable year plus related administrative expenses and (2) a reserve funded over the working lives of covered employees and actuarially determined on a level basis (using assumptions that are reasonable in the aggregate) to provide post-retirement health benefits (determined on the basis of current health benefit costs). Finally, the after-tax income in (16.7a) is

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[2] Alternatively, an irrevocable trust can be established to prefund post-retirement medical benefits which, under some circumstances, may be preferable to a §501(c)(9) trust.

defined as realized gross income (net of related expenses) less the Unrelated Business Income Tax (UBIT): [3]

$$\begin{aligned}(ATI)_t &= (NRI)_t - Tx\,(NRI)_t \\ &= (1 - Tx)\,(NRI)_t \end{aligned} \tag{16.7b}$$

where

$(NRI)_t$ = net realized income in year t

Tx = UBIT, equal to 31 percent federal tax rate plus applicable state tax.

Substituting (16.7b) into (16.7a) and representing qualified direct costs by the familiar B_t function, we have

$$(DC)_t = B_t + (\Delta QAA)_t - (1 - Tx)\,(NRI)_t\,. \tag{16.7c}$$

The safe harbor limit on the qualified asset account is 35 percent of the prior year's qualified direct costs, or B_{t-1}. Thus, the annual additions to the account can be defined as

$$(\Delta QAA)_t = \left[.35\,B_{t-1} - (Assets)_t\right]. \tag{16.7d}$$

The funding of retiree health benefits, however, can be based on a reasonable and consistently applied actuarial cost method, taking into account experience gains and losses, changes in assumptions, and other similar items, but can be no more rapid than funding on a level basis over the remaining working lifetimes of the current participants (reduced on the basis of reasonable turnover and mortality assumptions).

Since health care inflation is not permitted, selecting a funding method with the largest acceptable actuarial liability would help to offset this limitation. The actuarial liability under the level dollar cost prorate method (entry age normal method), or some variation of this method, would be the logical choice. The unfunded liability under this method is determined on an employee-by-employee basis and can be amortized over each employee's future working lifetime. This approach, which uses implicit supplemental costs, is known in pension funding as the individual level premium method. This method, which requires that assets be allocated to each plan member, is perhaps the most effective means of developing a large maximum tax deductible

[3]If the plan has employee contributions, such contributions would be added to (16.7b), which in turn is used as an offset to the tax deductible contributions otherwise determined.

contribution. A variation of this method, sometimes referred to as the *individual aggregate method*, is defined as follows:

$$(\Delta QAA)_x = \frac{{}^{HB'}(PVFB)_x - (Assets)_x}{\ddot{a}^{T}_{x:\overline{r-x}|}} \qquad (16.8a)$$

where

$(\Delta QAA)_x$ = addition to qualified asset account for employee age x

${}^{HB'}(PVFB)_x$ = present value of future health benefits (without health care inflation) for employee age x

$(Assets)_x$ = assets allocated to employee age x, with allocation proportional to ${}^{HB'}(PVFB)_x$

$\ddot{a}^{T}_{x:\overline{r-x}|}$ = temporary, employment-based, annuity running from attained age x to retirement age r.

This method not only meets the requirements of §419A but generates the largest contribution based on conventional actuarial cost methods. Annual actuarial losses associated with health care inflation, as well as other gains and losses, are amortized from the employee's attained age to retirement under this method. However, since §419A does not specify how actuarial gains and losses are to be treated, it may be possible to fund such losses immediately.

The amortization of the unfunded liability over "working lifetimes" is not well defined for non-active plan members. One interpretation is that the working lifetimes of actives are to be used for the unfunded liability of non-actives. A second possibility is that all VEBA members can be used, with zeros included in the average for the "working lifetimes" of retired members. In either case, an aggregation of the liability is required, and the corresponding contribution component would likewise involve an aggregate calculation. The frozen entry age method or the aggregate method, as defined in Chapter 10, could be used as a funding method for meeting these interpretations. However, since the §419A funding limitation does not require that the actuarial methodology combine plan members in making the calculation, a third interpretation is that the retiree liability can be funded, and deducted, in one year, since their "working lifetimes" is zero. In fact, the use of any individual-based cost method argues for this interpretation.

When (16.8a) is substituted for the first two terms in (16.7c), the tax deductible contribution is equal to

$$(DC)_t = \left[\sum \frac{{}^{HB'}(PVFB)_x - (Assets)_x}{\ddot{a}^T_{x:\overline{r-x}|}} \right] - (1 - Tx)\,(NRI)_t \quad (16.8b)$$

where the summation sign indicates a summation over all plan members.

A constraint on the application of (16.8b), or any funding method, is a full funding limit. The maximum tax deductible contribution is constrained by the qualified asset account limit:

$$^{QAA}(FFL)_{t+1} = Max \left\{ \begin{matrix} (AL)_{t+1} - (Assets)_{t+1} \\ 0 \end{matrix} \right\} \quad (16.9)$$

where

$^{QAA}(FFL)_{t+1}$ = full funding limit based on qualified asset account, applicable at end of year t (represented as $t + 1$)

$(AL)_{t+1}$ = actuarial liability at end of year t

$(Assets)_{t+1}$ = assets at end of year t.

The actuarial liability under the cost prorate method (entry age normal method) would be used if the funding method used in determining the maximum tax deductible contribution were the individual aggregate method.

Some experts believe the DEFRA requirement that health care inflation not be used in determining tax deductible contributions does not preclude the use of a utilization assumption over and above utilization based on age and gender. If this interpretation is valid, a utilization assumption would increase the qualified funding limit and, hence, the tax deductible contributions.

Alternative VEBA Investments

The UBIT on trust earnings could be eliminated by investing trust assets in financial instruments not subject to taxation. Two choices are available: (1) tax-free municipal bonds and (2) life insurance cash values. The yields on municipal bonds, to a large extent, are discounted in the marketplace because of their tax-free nature; hence, this choice produces low long-run expected returns.

The second choice is to invest in life insurance cash values. Investment earnings are not taxed and, if the policy is held until the death of the insured, such earnings are paid to the trust as tax-free death proceeds. Some insurance companies have developed financial instruments with the following characteristics to act as "wrappers" for VEBA assets.

- ***Separate Accounts***: VEBA assets are placed into a separate account, apart from the insurance carrier's general account. This eliminates the insurance company credit risk, since separate account assets cannot be attached by the carrier's general creditors in the event of insolvency.
- ***Asset Allocation***: A derivative of using separate accounts is that the VEBA trustee can make the asset allocation decision. Thus, plan assets can be invested in a manner similar to pension assets, with as high an equity exposure as dictated by the trustee. This is in contrast to having assets invested in the carrier's general account which consists primarily of fixed-income and real estate investments.
- ***Group Insurance Contracts***: Most carriers have developed group insurance contracts, as opposed to individual policies, for receiving VEBA assets. This allows carriers to provide differential pricing to VEBA trusts of different size, and to set the insurance company expenses at a level reflecting the purpose of the transaction. In addition, group contracts allow experience rating (i.e., if death payments during the year are less than the funds held back for this purpose, then the separate account is credited for the difference; similarly, if such holdbacks are inadequate, the separate account is charged for the shortfall in the following year).

A threshold issue for the use of this investment vehicle is whether or not the VEBA has an insurable interest in the lives of its members. The common law definition of insurable interest is that the owner of the policy must suffer a financial or emotional loss at the death of the insured. A VEBA established for funding retiree health benefits would be relieved of a financial burden at death of a member; hence, a VEBA would not have the capability of being the owner and beneficiary of insurance on its members. However, since a substantial portion of health care costs are frequently associated with the retiree's final illness, some believe that this death-related cost constitutes sufficient insurable interest. Moreover, a number of states have passed laws codifying a VEBA's insurable interest in its members.

Since the use of trust-owned life insurance (TOLI) avoids UBIT, it is a straightforward analysis to evaluate the potential advantage of this investment vehicle over the yield on tax-free municipal bonds or the after-tax return on other investments. The insurance carrier's expense charges generally fall in the range of 50 and 100 basis points, depending on the size of the transaction. Thus, a comparison of (1) the return on tax-free municipal bonds, versus (2) the after-tax return on conventional investments, versus (3) the net return from TOLI will provide guidance on the appropriate investment vehicle.

If one of these two investment vehicles is used, the last term in equations (16.7c) and (16.8b), $(1 - Tx)\,(NRI)_t$, is eliminated. This permits a somewhat larger tax deductible contribution. Thus, the maximum tax deductible contribution, based on the individual aggregate funding method, can be defined by

$$(DC)_t = \left[\sum \frac{(PVFB)_x - (Assets)_x}{\ddot{a}^{T}_{x:\overline{r-x}|}}\right]. \tag{16.10}$$

Numerical Illustrations

Figure 16–3 shows a 20-year deterministic projection of retiree health benefits under pay-as-you-go funding, VEBA funding with contributions defined by (16.8b) with UBIT on investment earnings, VEBA funding with contributions defined by (16.10) without UBIT (i.e., VEBA funding with TOLI), and economic costs. In each case, the retiree liability (without health care inflation) is contributed on a lump sum basis. Figure 16–4 shows the corresponding funded ratios based on the economic liability. The valuation and experience assumptions used in the projection are provided in Table 16–3. In determining the maximum tax deductible contributions, the size of the actuarial liability is maximized by using (1) the individual aggregate method, (2) an interest rate that grades from a current rate of 8 percent to a long-term rate of 6 percent over 5 years, and (3) a health care utilization rate of 5 percent grading down to 1 percent over 10 years.

TABLE 16–3

Valuation and Experience Assumptions

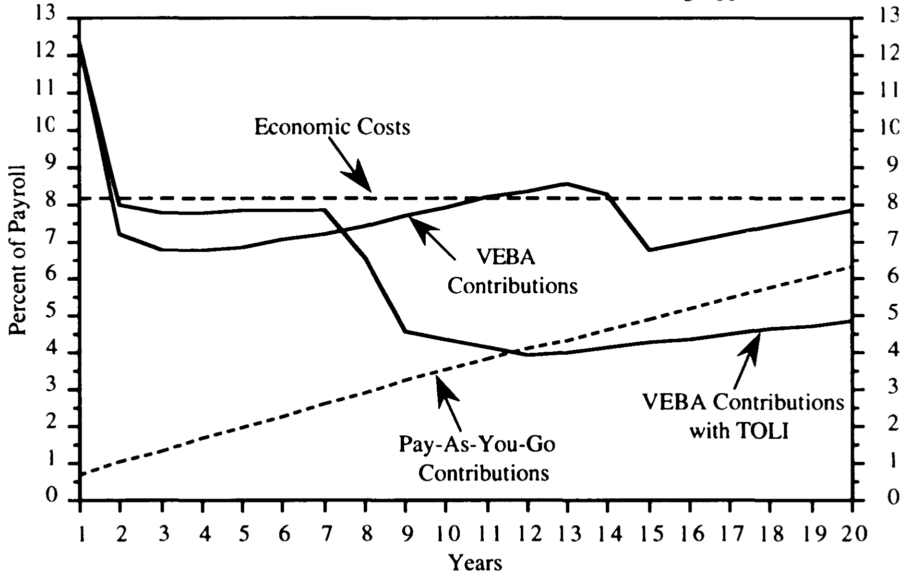
Valuation Assumptions for Contribution Limits:	
Decrement Rates:	See references provided in Table 16–1
Salary Increase:	Not applicable
Interest Rate:	8% grading down to 6% over 5 years
Health Care Inflation:	0%
Health Care Utilization:	5% grading down to 1% over 10 years
Medicare Inflation:	0%
Health Benefit Costs	Table 16–2
Asset Method:	Market value
Cost Method:	Individual Aggregate Method
Experience Assumptions:	
Decrement Rates:	See references provided in Table 16–1
Salary Increase:	See references provided in Table 16–1
Investment Return:	11%
TOLI Expenses	50 basis points
Trust Tax Rate:	35% (federal plus state taxes)
Health Benefits Inflation:	See Table 16–1
Utilization:	See Table 16–1
Medicare Inflation:	See Table 16–1
Health Benefit Costs	See references provided in Table 16–1

The economic costs are approximately 8 percent of payroll whereas the pay-as-you-go costs start out at .6 percent and increase to over 6 percent by the end of 20 years. The two prefunding scenarios have costs that begin at 12 percent due to the lump sum funding for retired members, then dropping to 7 and 8 percent in the next year. The VEBA funding with UBIT shows a near-term increasing contribution pattern while the VEBA funding with TOLI shows a level pattern. The full funding limit is reached in 9 years under VEBA funding with TOLI and in 15 years under VEBA funding with UBIT.

The funded status under the four funding scenarios is shown in Figure 16-4. The pay-as-you-go methodology, of course, produces no assets, whereas the economic cost methodology has assets that, by definition, accumulate to 100 percent of the economic liability by the end of the projection period. Both VEBA funding scenarios reach a funded ratio of over 70 percent; however, the VEBA with TOLI initially accumulates assets at a faster

FIGURE 16–3

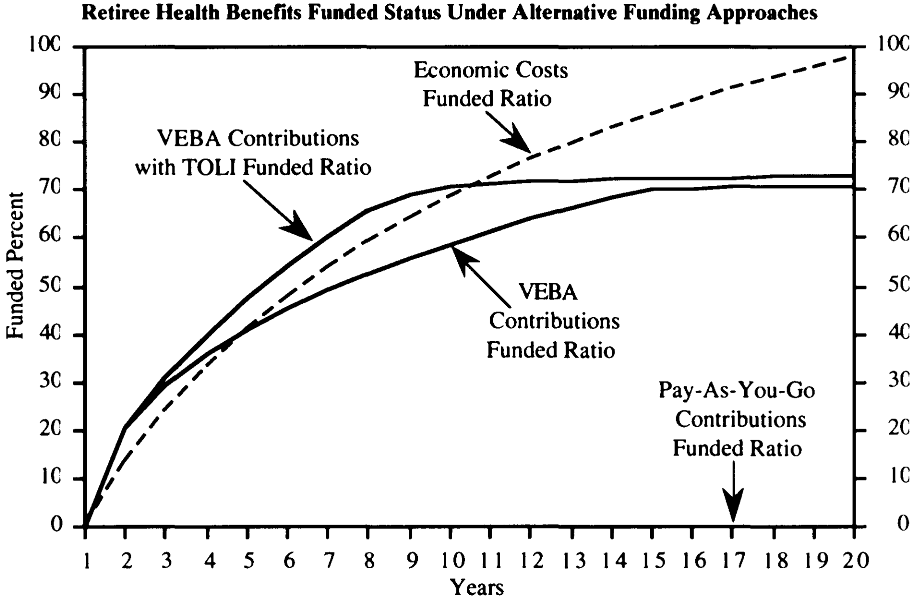
Retiree Health Benefits Contributions Under Alternative Funding Approaches



Years

FIGURE 16–4

Retiree Health Benefits Funded Status Under Alternative Funding Approaches



pace. These projections indicate that a substantial portion of the post-retirement health benefit liability can be funded and that VEBA with TOLI is a much less costly approach than a VEBA with UBIT. The impact of advance funding on accounting costs is examined in the following section.

ACCOUNTING REQUIREMENTS

SFAS 106 requires employers to adopt accrual accounting for retiree health benefits and other non-pension benefits provided in retirement, such as death benefits, dental benefits, long-term care, and so forth. The methodology required in determining the *net periodic postretirement benefit cost* and the various disclosure items to be included in the footnotes of the financial statement are similar to those of SFAS 87 for pension benefits, with some notable exceptions as discussed below. It is assumed for this discussion that the reader is familiar with the material presented in Chapter 11 on SFAS 87.

Liability Values

There are two liabilities defined by SFAS 106, the *expected postretirement benefit obligation* (EPBO) and the *accumulated postretirement benefit obligation* (APBO). These liabilities, which are identical for retired employees and employees eligible to retire, are equal to the present value of future retiree health benefits as set forth in (16.4) for a retired employee at age x.

The EPBO for an active employee age x ($x < r'$) is defined by equation (16.5) given previously. This liability represents the present value of all future expected retiree health benefits. The operative SFAS 106 liability for active employees, however, is the APBO which represents a pro rata portion of the EPBO. The proration is based on a fraction, the numerator of which is the employee's service to date and the denominator of which is the employee's service to the first age at which eligibility for full retiree health benefits is applicable, generally the employee's first eligible retirement age, r'. In other words, in the determination of the APBO, the EPBO is allocated from the employee's hire date (or, if later, the date at which benefits begin to accrue) to the date of full eligibility. This is the case even if the employee is expected to retire much later than the full eligibility date.

Net Periodic Postretirement Benefit Cost

The net periodic postretirement benefit cost (net cost) consists of the same components making up the net periodic pension cost under SFAS 87, as outlined in Table 11-1 of Chapter 11. These components are presented for retiree health benefits in Table 16-4.

TABLE 16-4
Net Periodic Postretirement Benefit Cost Components

Service Cost:	Normal cost under constant dollar benefit prorate method, adjusted with interest to end of year*
+	
Interest Cost:	APBO (adjusted for expected distributions during year) times discount rate
−	
Expected Return on Assets:	Market-related value of assets (adjusted for expected distributions and contributions during year) times expected rate of return on assets
+	
Amortization Costs:	Amortization Methodology:
Transition Obligation (Asset)	Straight line over average future service of plan participants expected to receive benefits or, optionally, 20 years, if greater
+	
Prior Service Cost	Fixed schedule over future service of plan participants expected to receive benefits*
+	
Net Loss (Gain)	Rolling schedule over future service of plan participants expected to receive benefits

*Future service extends only to full eligibility age, not expected retirement age.

The service cost is equal to the portion of the EPBO attributed to the employee's service during the current accounting period. The interest rate used in determining the interest cost must be based on the rates of return on high quality fixed income securities available at the measurement date (i.e., the date of the sponsor's financial statements or within three months prior to that date). This interest rate may be somewhat higher than the SFAS 87 rate, the latter representing the rate at which pension benefits could be settled through the purchase of insurance company annuities. The expected return on assets is based on the long-run expected investment return for the assets held in the trust. If assets are held in a taxable trust, then an *after-tax* expected return on assets must be used.

The transition obligation is defined in Chapter 11 by equation (11.10), with the APBO being used instead of the PBO. In many cases, the plan will not have assets or any prepaid or accrued expense; hence, the transition obligation is simply the APBO. The entire obligation can be recognized immediately or amortized on a straight line basis over the longer of 20 years or the remaining service (to date of expected retirement) of active employees expected to receive benefits under the plan. The future service for pension accounting was defined in Chapter 11 by (11.8), equal to (11.6d) divided by (11.7). The absence of vesting, disability, and death benefits may cause the future service to be somewhat longer for SFAS 106 purposes.

The prior service cost, equal to the obligation (or asset) attributed to plan changes, is to be amortized over the expected future service to the *full eligibility* date of active employees expected to receive benefits under the plan. This, of course, is a shorter period than "service to retirement." This period is defined in Chapter 11 by (11.11), with the previously noted adjustments for the absence of ancillary benefits.

Gains and losses are defined in the same manner as for SFAS 87, as set forth by equations (11.12) and (11.13), but with the PBO being replaced by the APBO. The amortization period is also the same, namely, the average remaining service to the date of expected retirement under the plan, again with the adjustment due to the lack of ancillary benefits.

Balance Sheet

Unlike SFAS 87 for pensions, there is no minimum liability that must be recognized on the balance sheet for retiree health benefit liabilities. Unless the plan is prefunded, the entire APBO will eventually be recorded on the balance sheet as an accrued expense. On the other hand, if cumulative contributions exceed cumulative expense, then the balance sheet will show a prepaid expense asset.

Footnote Disclosure

Table 16–5 outlines the various footnote disclosure items for retiree health benefits, with Tables 16–6 and 16–7 being referenced in Table 16–5.

TABLE 16–5

SFAS 106 Footnote Disclosure Items

- Plan Description: A description of the *substantive plan*, i.e., the legal plan plus expected changes in future benefits, employee cost sharing, and so forth, that are understood by the employer and employees.
- Net Periodic Postretirement Benefit Cost: The disclosure of net cost components, as shown in Table 16–6. As with pension cost disclosure, the actual return on assets is shown explicitly and then backed out as a component of net amortization and deferrals.
- Funded Status Reconciliation: This is similar to the SFAS 87 reconciliation, as shown in Table 16–7.
- Health Benefits Trend Rate: The first year's rate, along with a description of the rate used in future years.
- Interest Rate Assumption: The discount rate and the assumed rate of return on plan assets, along with the effective tax rate if such assets are subject to taxation.
- Health Care Benefit Sensitivity: The financial impact of a one percent increase in the health benefits trend rate on both the APBO and the sum of service costs plus interest costs.
- Plan Assets: The amount and types of employer securities, if any, included in plan assets.
- Alternative Amortization Methods: A description of any alternative amortization methods used.
- Settlement or Curtailments: The gain or loss recognized on any settlements or curtailments during the period.
- Special Termination Benefits: A description of any special health care related termination benefits provided (e.g., early retirement incentives) and related costs.

TABLE 16–6
Disclosure of Net Periodic Postretirement Benefit Cost

1. Service Cost	
2. Interest Cost	
3. Actual Return on Assets	Expected Return on Assets – Asset-Based Loss (Gain)
4. Net Amortization and Deferrals	Transition Obligation + Prior Service Cost + Loss (Gain) + Asset-Based Loss (Gain)
5. Net Periodic PB Cost	(1) + (2) – (3) + (4)

TABLE 16–7
Reconciliation of Funded Status

1. APBO	Year-end value (or projected year-end value)
2. Market Assets	Year-end value
3. Funded Status	(1) – (2)
4. Unrecognized Transition Obligation	Beginning of year value less current year's amortization amount
5. Unrecognized Prior Service Cost	Beginning of year value less current year's amortization amount
6. Unrecognized Net Loss (Gain)	(3) – (4) – (5) + {Prepaid (Accrued) Expense at B.O.Y. – Net Periodic PB Cost + Contributions}
7. Prepaid (Accrued) PB Cost	(3) + (4) + (5) + (6)

Numerical Illustrations

Figure 16–5 shows the results of a 20-year projection of SFAS 106 costs under three of the funding scenarios studied previously: (1) pay-as-you-go funding; (2) VEBA trust funding with UBIT; and (3) VEBA funding with TOLI. Figure 16–6 shows the APBO funded status.

FIGURE 16–5
SFAS 106 Expense Under Alternative Funding Scenarios

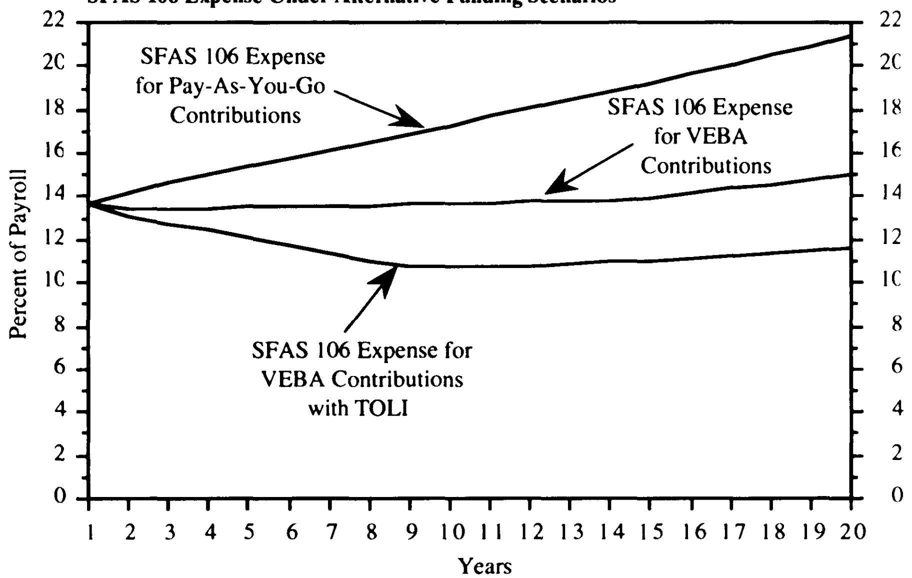
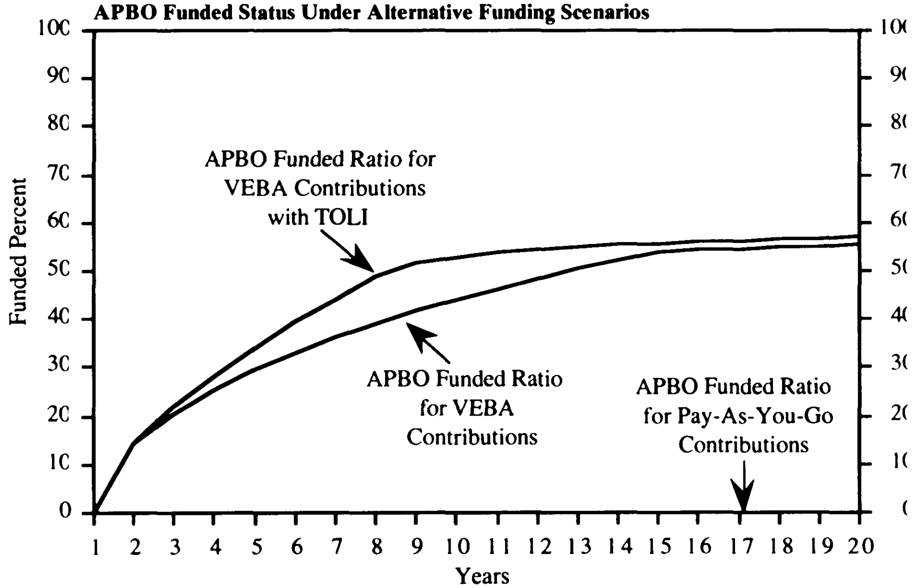


FIGURE 16–6
APBO Funded Status Under Alternative Funding Scenarios



Accounting expense under pay-as-you-go funding starts out at 14 percent of salary and escalates to over 21 percent by the end of the 20-year projection period. If the plan advance funds to the maximum extent using a VEBA with UBIT, the initial expense of 14 percent is maintained during the 20-year period. VEBA funding with TOLI reduces the expense to about 11 percent of payroll. The APBO funded ratio is, of course, zero under pay-as-you-go funding and reaches about 55 percent under either of the two VEBA funding scenarios.

CRITIQUE OF SFAS 106

The most significant aspect of SFAS 106 is that it requires employers to adopt accrual accounting for retiree health benefits, a long overdue change. As is the case for SFAS 87, there are deficiencies in the promulgation that, if changed, would result in an improved set of accounting procedures. These are briefly discussed in the order of their importance.

Salary vs. Benefit Proration

The service proration methodology used for the service cost and APBO should be changed to a proration based on salary, since this is consistent with the view that these benefits, like pension benefits, are deferred wages.

Proration Pattern and Period

The SFAS 87 promulgation was criticized in Chapter 11 for not using a linear proration to each potential retirement age, since prorating according to the benefit formula can produce inappropriate results for some formulas. This same proration is required for SFAS 106; however, since the benefits under most retiree health plans are not formula based, the default proration is, in fact, linear. If plans are amended in the future to provide service-related benefits, which is a growing trend, then this deficiency could emerge, hence, SFAS 106 should be changed to require linear proration.

A more serious problem with the retiree health benefit proration is the use of the *full eligibility* age instead of the expected retirement age. To give only one example of the absurdity of this requirement, consider a plan that permits early retirement after 20 years of service with a full actuarial reduction in pension benefits. Even though only a small fraction of employees will elect to retire prior to ages 55 or 60, the retiree health benefits liability must be fully accounted for by the end of 20 years of service for each employee. FASB's position of having the benefit fully accounted for at the point it is first available is neither sensible nor actuarially necessary. For a group of employees expected to retire at various ages, it is eminently reasonable to account for the expected benefits on a best-estimate basis of when such benefits will commence. The artificial requirement that it be accounted for at the first eligibility age will lead to unnatural contortions on the part of plan sponsors in changing their plan design to avoid this overly rapid accounting practice. The proration in SFAS 106 should be changed to be a linear proration to each expected retirement age.

Interest Cost

SFAS 87 was criticized for basing the interest rate on a so-called settlement rate, reflective of the cost of annuities, since annuity rates include insurance company risk, expense, and profit charges, which should not be part of the interest rate determination. The recommendation was that, if FASB wanted a market value rate, then the interest assumption should be keyed to the spot rate on investment-grade, long-term corporate bonds as of the measurement date. SFAS 106 does, in fact, require that this methodology be used; however, the author still believes that the discount assumption overemphasizes the termination or "wind up" contingency, and the use of a long-run, best-estimate assumption reflecting the plan's asset allocation would be preferable. In other words, the discount rate and the long-term expected return on assets would be identical.

Gain (Loss) Amount

It would be useful to show the effect of interest rate changes, as well as other actuarial assumption changes, separate from the effects of experience differing from the underlying assumptions. This is particularly important in light of the continually changing discount rate. This should be done whether or not the amortization of these two items is treated the same or differently. This is the same comment made in regard to SFAS 87.

Gain (Loss) Amortization

The 10 percent corridor around the larger of the APBO or market-related assets implies that, for those plans that begin to prefund their retiree health benefits liabilities, assets can fluctuate substantially without any amortization of the gains and losses. This is not a sensible result.

Again, as with pension expense, the shortcoming in the gain (loss) corridor is that it focuses on the wrong item. Since the ultimate objective is to adjust *costs* if gains and losses become too significant, on the one hand, without being overly sensitive, on the other, the corridor might be based on the service cost. For example, the amortization amount associated with the full unrecognized gain (loss) might be determined and, then, the portion of the amortization payment outside a 10 percent corridor, for example, of the service cost might be recognized.

Amortization Periods

SFAS 87 was criticized for not simply using "the future service of employees" instead of the future service of employees *expected to receive benefits*. Since retiree health benefit programs do not have ancillary benefits, the default calculation for SFAS 106 is, in fact, "the future service of employees." Nevertheless, SFAS 106 still errs in the selection of the amortization period for past service cost by requiring the use of the full eligibility age instead of expected retirement ages.

Terminology

As with SFAS 87, past service cost should be "past service obligation (asset)." Similarly, the transition obligation should be "transition obligation (asset)." Loss (gain) should be "experience loss (gain)," and changes in assumptions should be labeled "assumption change obligation (asset)." While these are only cosmetic changes, they would nevertheless facilitate the understanding of SFAS 106.

Glossary of Mathematical Notation

Notation		Definition
$\ddot{a}_{\overline{n}\rvert}$	=	Present value of an n-year annuity certain, with payments made at the beginning of the year (p. 47).
${}^{s}\ddot{a}_{\overline{m}\rvert}$	=	m-year period certain annuity, with payments increasing by the inflation and productivity components of the salary assumption (p. 102).
$\ddot{a}_x$	=	Present value, at age x, of a life annuity, with payments made at the beginning of the year (p. 46).
$\ddot{a}_{x:\overline{n}\rvert}$	=	Present value of an annuity payable until age n or the annuitant's death, whichever occurs first, with payments made at the beginning of the year (p. 47).
$\ddot{a}_{\overline{x:\overline{n}\rvert}}$	=	Present value of an n-year period certain life annuity, with payments made at the beginning of the year (p. 47).
${}_{n\rvert}\ddot{a}_x$	=	Present value of an annuity payable for life, with the first payment deferred n years (p. 47).
${}^{k}\ddot{a}_{xz}$	=	Present value of a joint and survivor annuity, paying \$1 while both annuitants are alive and \$$k$ to the survivor, with payments made at the beginning of the year (p. 47).

$^{k}\ddot{a}_{\underset{1}{x}z}$ = Present value of a joint life annuity paying \$1 per year while the life age x is alive and \$$k$ per year to the life age y if the life age x dies first (p. 48).

$^{HB}a_x$ = Economic liability for health benefits for retiree age x (p. 261).

$_{k-x|}^{HB}a_x$ = Deferred health benefits annuity at the point of retirement (p. 261).

$^{MCR}\ddot{a}_r$ = Present value of a modified cash refund annuity with lump sum death payment equal to the difference, if any, between the employee's pension contributions and the benefits received at date of death (p. 49).

$^{MIR}\ddot{a}_r$ = Present value of a modified installment refund annuity with payments at least until the employee's pension contributions are returned and thereafter until the annuitant's death (p. 49).

$\ddot{a}_x^{d}$ = Present value, at age x, of a life annuity based on disabled-life mortality (p. 116).

$\ddot{a}^{T}_{x:\overline{n}|}$ = Present value of an n-year employment-based annuity from age x, with payments made at the beginning of the year (p. 51).

$^{s}\ddot{a}^{T}_{x:\overline{n}|}$ = Present value of an n-year employment-based annuity from age x, with payments made at the beginning of the year equal in value to the employee's attained age salary, based on a unit salary at age x (p. 52).

$^{r'}\ddot{a}^{T}_{x:\overline{n}|}$ = Present value of an n-year employment-based annuity from age x, with payments made at the beginning of the year and assuming multiple retirement ages (p. 133).

${}^{r's}\ddot{a}^{T}_{x:\overline{n}|}$ = Present value of an n-year employment-based annuity from age x, with payments made at the beginning of the year equal in value to the employee's attained age salary, based on a unit salary at age x, and assuming multiple retirement ages (p. 133).

$(ABO)_t$ = Accumulated benefit obligation for all plan members at time t (p. 179).

$(ABO)_x$ = Accumulated benefit obligation for employee age x (p. 178).

$(AC)_k$ = Amortization cost during year k for prior service cost (p. 189).

$(AFC)_t$ = Additional funding contribution for year t (p. 151).

$(AFS)_t$ = Average future service of plan participants expected to receive benefits (p. 188).

$(AL)_t$ = Actuarial liability for plan at beginning of year t (p. 98).

${}^{r}(AL)_x$ = Actuarial liability under a specified actuarial cost method for individual age x assumed to retire at age r (p. 72).

${}^{AB\,r}(AL)_x$ = Actuarial liability under the accrued benefit method for individual age x assumed to retire at age r (p. 74).

${}^{BD\,r}(AL)_x$ = Actuarial liability under the benefit prorate (constant dollar) method for individual age x assumed to retire at age r (p. 74).

${}^{BD\,r'}(AL)_x$ = Actuarial liability under the benefit prorate (constant dollar) method for individual age x assuming multiple retirement ages (p. 132).

${}^{BP\,r}(AL)_x$ = Actuarial liability under the benefit prorate (constant percent) method for individual age x assumed to retire at age r (p. 74).

${}^{BP\,r'}(AL)_x$ = Actuarial liability under the benefit prorate (constant percent) method for individual age x assuming multiple retirement ages (p. 132).

${}^{CD\,r}(AL)_x$ = Actuarial liability under the cost prorate (constant dollar) method for individual age x assumed to retire at age r (p. 75).

${}^{CD\,r'}(AL)_x$ = Actuarial liability under the cost prorate (constant dollar) method for individual age x assuming multiple retirement ages (p. 133).

${}^{CD\,*r'}(AL)_x$ = Actuarial liability under the cost prorate (constant dollar) method for individual age x assuming multiple retirement ages with actuarially equivalent early retirement benefits (p. 134).

${}^{CD\,T}(AL)_x$ = Actuarial liability, including ancillary benefits, under the cost prorate (constant dollar) method for individual age x (p. 121).

${}^{CP\,r}(AL)_x$ = Actuarial liability under the cost prorate (constant percent) method for individual age x assumed to retire at age r (p. 75).

$(Assets)_t$ = Plan assets at beginning of year t (p. 98).

$(Assets)_x$ = Plan assets allocated to employee age x (p. 110).

$(ATI)_t$ = After-tax income in year t (p. 265).

$(AV)_t$ = Actuarial value of assets at beginning of year t (p. 172).

${}^{r}(AVPNC)_x$ = Accumulated value of past normal costs for employee age x (p. 82).

b_x = Benefit accrual during age x (p. 40).

$b_{x,y}$ = Benefit accrual during age x for an age-y entrant (p. 84).

b_x^T = Total benefit allocated during age x under benefit-based cost methods that include implicit supplemental costs (pp. 106–108).

${}^{CD}b_x$ = Constant dollar benefit accrual during age x (p. 42).

${}^{CP}b_x$ = Constant percent benefit accrual during age x (p. 43).

B_n = Benefits paid to participants during year n (p. 98).

B_x = Accrued benefit at beginning of age x (p. 40).

$B_{r,y}$ = Accrued benefit at age r for an age-y entrant (p. 84).

${}^{CD}B_x$ = Constant dollar accrued benefit at beginning of age x (p. 43).

${}^{CE}B_t$ = Current year's benefit payments associated with all contingent events (p. 155).

${}^{CP}B_x$ = Constant percent accrued benefit at beginning of age x (p. 43).

$(BV)_t$ = Book value of assets at beginning of year t (p. 172).

C_r = Accumulated employee contributions at retirement (p. 48).

${}^{Drgs}C_x$ = Expected prescription drug costs at age x (p. 255).

${}^{EE}C_x$ = Employee's expected cost and/or contributions at age x (p. 256).

${}^{ER}C_x$ = Employer's expected health benefit cost for employee age x (p. 256).

${}^{ER}_{\ t}C_x$ = Employer's expected health benefit cost at age $x + t$ for a retiree currently age x (p. 261).

${}^{Hosp}C_x$ = Expected hospital costs at age x (p. 255).

${}^{Lab}C_x$ = Expected laboratory costs at age x (p. 255).

${}^{Other}C_x$ = Expected other charges at age x (e.g., nursing home costs) (p. 255).

${}^{Phy}C_x$ = Expected physician costs at age x (p. 255).

${}^{Total}C_x$ = Total expected health benefit costs at age x (p. 255).

${}^{Qrt}C_{t-1}$ = Prior year's quarterly contributions (p. 165).

${}^{SC}C_{t-1}$ = Portion of prior year's total contribution applied to the maximum supplemental cost limits (p. 165).

${}^{SC_n}C_{t-1}$ = Portion of prior year's contribution applied to nth supplemental cost for determining maximum contributions (p. 166).

$(CC)_t$ = Cumulative contributions from plan inception to year t (p. 188).

$(CE)_t$ = Cumulative expense from plan inception to year t (p. 188).

$(CG)_t$ = Capital gains (or losses), both realized and unrealized, during year t (p. 173).

$(CL)_x$ = Current liability for employee age x (pp. 151–152).

$(Cont)_n$ = Employer contributions during year n (p. 99).

${}^{ER}(CP)_t$ = Employer copayment fraction (p. 257).

d = Rate of discount (i.e., $i\ v$) (p. 99).

$d_x^{(d)}$ = Number of employees becoming disabled during age x from a service table (p. 34).

$d_x^{(m)}$ = Number of employees dying during age x from a service table (p. 34).

$d_x^{(r)}$ = Number of employees retiring at age x from a service table (p. 34).

$d_x^{(t)}$ = Number of employees terminating during age x from a service table (p. 34).

$d_x^{(T)}$ = Number of employees leaving service during age x from a service table (p. 32).

D_t = Employee deductible in year t (p. 257).

$(DC)_t$ = Tax deductible contributions in year t (p. 265).

e_x = Curtate life expectancy (i.e., based on whole years only) at age x (p. 46).

${}^{AC}e_t$ = Error term for asset class in year t (p. 245).

${}^{Inf}e_t$ = Error term for unexpected inflation in year t, (p. 244).

$E_x^{(k)}$ = Function to determine if employee age x is eligible for benefit-type k (p. 187).

$E[(AL)_{n+1}]$ = Expected actuarial liability at year end, or the beginning of year $n + 1$ (p. 98).

$E[(Assets)_{n+1}]$ = Expected assets at year end, or the beginning of year $n + 1$ prior to any contributions (p. 98).

$E(B)$ = Expected early retirement benefit based on multiple retirement ages (p. 131).

$E(B)_t$ = Expected benefit payments during year t (p. 157).

$E(C)_t$ = Expected employer contributions during year t ((p. 185).

${}^{Min}(EC)_{t+1}$ = Minimum required contribution payable at the end of year t (p. 140).

${}^{Qrt}(EC)_t$ = Minimum required quarterly contributions in year t (p. 141).

$E(I_t)$ = Expected inflation in year t (p. 245).

${}^{HB}(EL)_x$ = Economic liability for health benefits for employee age x (p. 261).

$(ERB)_x$ = Function denoting whether the employee is expected to receive benefits (p. 188).

${}_k(ERCR)_x$ = Early retirement cost ratio: the cost (or liability) of an early retirement benefit to the cost (or liability) of a normal retirement benefit (p. 135).

${}_k^*(ERCR)_x$ = Early retirement cost ratio with actuarially equivalent early retirement benefits: the cost (or liability) of an early retirement benefit to the cost (or liability) of a normal retirement benefit (p. 135).

$(EROA)_t$ = Expected return on the market-related value of assets (p. 185).

$E[(UL)_{n+1}]$ = Expected unfunded liability at year end, or the beginning of year $n + 1$ prior to any contributions (p. 98).

${}^{d}F_k$ = Value of disability benefits payable at age k (p. 120).

${}^{r}F_r$ = Value of retirement benefits payable at age r (p. 120).

${}^{s}F_k$ = Value of surviving spouse benefits payable at age k (p. 120).

${}^{v}F_k$ = Value of vested termination benefits payable at age k (p. 120).

${}^{AL}(FFL)_t$ = Full funding limit based on the statutory funding method's actuarial liability for year t (p. 157).

${}^{CL}(FFL)_t$ = Full funding limit based on the current liability for year t (p. 159).

${}^{QAA}(FFL)_{t+1}$ = Full funding limit based on qualified asset account, applicable at end of year t (p. 268).

$(FR)_t$ = Funded ratio equal to actuarial value of assets less the FSA credit balance, all divided by the current liability (p. 154).

$(FS)_x$ = Future service for employee age x (p. 187).

$(FSA)_t$ = Funding standard account balance from end of prior year (i.e., at beginning of year t), with a credit balance representing a positive value and a funding deficiency representing a negative value (p. 140).

${}^{CB}(FSA)_t$ = Credit balance in the funding standard account at beginning of year t, i.e., from the end of prior year (p. 153).

${}^{FD}(FSA)_t$ = Funding deficiency in funding standard account at end of year, determined without regard to prior year credit balance and current year contribution (p. 162).

$g_x^{(d)}$ = Grading function equal to the proportion of accrued benefit provided if disability occurs during age x (p. 116).

$g_x^{(r)}$ = Grading function equal to the proportion of accrued benefit payable if retirement occurs at the beginning of age k (p. 129).

${}^*g_k^{(r)}$ = Grading function which, when applied to the participant's accrued benefit, produces actuarially equivalent benefits (p. 126).

$g_x^{(s)}$ = Grading function equal to the proportion of accrued benefit provided to a surviving spouse if death occurs during age x (p. 117).

$g_x^{(v)}$ = Grading function equal to the proportion of accrued benefit vested at age x (p. 115).

I = Rate of inflation (p. 38).

I_t = Inflation in year t (p. 244).

I_∞ = Long-term rate of inflation (p. 244).

i = Interest rate (p. 34).

i' = Current liability interest rate for statutory funding requirements (p. 158) and expected return on assets for accounting requirements (p. 186).

$(IC)_t$ = Interest cost component of net periodic pension cost (p. 185).

$l_x^{(T)}$ = Number of employees in active service at age x from a service table (p. 32).

$l_{x,y}$ = Number of age-y entrants currently age x (p. 84).

M = Probability that the participant has a surviving spouse at death (p. 117).

$(MC)_t$ = Miscellaneous credits, primarily reflecting full funding limits (p. 162).

$(MRA)_t$ = Market-related value of assets for year t (p. 185).

$(MV)_t$ = Market value of assets at beginning of year t (p. 172).

${}^r(NC)_x$ = Normal cost under a specified actuarial cost method for individual age x assumed to retire at age r (p. 80).

$(NC)_t$ = Normal cost under statutory funding method, including any implicit supplemental costs, for year t (p. 140).

${}^{AAB\,r}(NC)$ = Normal cost under the aggregate accrued benefit method for all participants, based on retirement at age r (p. 84).

${}^{AB\,r}(NC)_x$ = Normal cost under the accrued benefit method for individual age x assumed to retire at age r (p. 84).

${}^{AB\,r'}(NC)_x$ = Normal cost under the accrued benefit method for individual age x assuming multiple retirement ages (p. 131).

${}^{AB\,*r'}(NC)_x$ = Normal cost under the accrued benefit method for individual age x assuming multiple retirement ages with actuarially equivalent early retirement benefits (p. 131).

${}^{AB\,T}(NC)_x$ = Normal cost, including ancillary benefits, under the accrued benefit method for individual age x (p. 120).

${}^{ABD\,r}(NC)$ = Normal cost under the aggregate benefit prorate (constant dollar) method for all participants, based on retirement at age r (p. 85).

${}^{ABP\,r}(NC)$ = Normal cost under the aggregate benefit prorate (constant percent) method for all participants, based on retirement at age r (p. 86).

${}^{ACD\,r}(NC)$ = Normal cost under the aggregate cost prorate (constant dollar) method for all participants, based on retirement at age r (p. 88).

${}^{ACP\,r}(NC)$ = Normal cost under the aggregate cost prorate (constant percent) method for all participants, based on retirement at age r (p. 88).

${}^{BD\,r}(NC)_x$ = Normal cost under the benefit prorate (constant dollar) method for individual age x assumed to retire at age r (p. 85).

${}^{BD\,r'}(NC)_x$ = Normal cost under the benefit prorate (constant dollar) method for individual age x assuming multiple retirement ages (p. 132).

${}^{BD\,*r'}(NC)_x$ = Normal cost under the benefit prorate (constant dollar) method for individual age x assuming multiple retirement ages with actuarially equivalent early retirement benefits (p. 132).

${}^{BD\,T}(NC)_x$ = Normal cost, including ancillary benefits, under the benefit prorate (constant dollar) method for individual age x (p. 120).

${}^{BP\,r}(NC)_x$ = Normal cost under the benefit prorate (constant percent) method for individual age x assumed to retire at age r (p. 85).

${}^{BP\,r'}(NC)_x$ = Normal cost under the benefit prorate (constant percent) method for individual age x assuming multiple retirement ages (p. 132).

${}^{BP\,T}(NC)_x$ = Normal cost, including ancillary benefits, under the benefit prorate (constant percent) method for individual age x (p. 121).

${}^{CD\,r}(NC)_y$ = Normal cost under the cost prorate (constant dollar) method for individual age x assumed to retire at age r (p. 86).

${}^{CD\,r'}(NC)_x$ = Normal cost under the cost prorate (constant dollar) method for individual age x assuming multiple retirement ages (p. 133).

${}^{CD\,*r'}(NC)_x$ = Normal cost under the cost prorate (constant dollar) method for individual age x assuming multiple retirement ages with actuarially equivalent early retirement benefits (p. 134).

${}^{CD\,T}(NC)_x$ = Normal cost, including ancillary benefits, under the cost prorate (constant dollar) method for individual age x (p. 121).

${}^{CL}(NC)_t$ = Current liability normal cost at beginning of year t (p. 158).

${}^{CP\,r}(NC)_x$ = Normal cost under the cost prorate (constant percent) method for individual age x assumed to retire at age r (p. 87).

${}^{CP\,r'}(NC)_x$ = Normal cost under the cost prorate (constant percent) method for individual age x assuming multiple retirement ages (p. 133).

${}^{CP\,*r'}(NC)_x$ = Normal cost under the cost prorate (constant percent) method for individual age x assuming multiple retirement ages with actuarially equivalent early retirement benefits (p. 134).

${}^{PT}(NC)_x$ = Normal cost under the plan termination method for individual age x assumed to retire at age r (p. 95).

$(NR)_t$ = Nominal return in year t (p. 245).

$(NRI)_t$ = Net realized income in year t (p. 265).

P = Rate of productivity reflected in salary increases (p. 38).

$p_x^{(d)}$ = Probability of an employee not terminating from service from age x to age $x + 1$, excluding consideration of other decrements (p. 21).

${}_n p_x^{(d)}$ = Probability of an employee not terminating from service from age x to age $x + n$, excluding consideration of other decrements (p. 21).

$p_x^{(m)}$ = Probability of a life age x living to age $x + 1$ (p. 16).

${}_n p_x^{(m)}$ = Probability of a life age x living to age $x + n$ (p. 16).

${}^d p_x^{(m)}$ = Probability of a disabled life age x living to age $x + 1$ (p. 21).

$p_x^{(r)}$ = Probability of an employee retiring at the beginning of age x, excluding consideration of other decrements (p. 31).

$p_x^{(t)}$ = Probability of an employee not terminating employment from age x to age $x + 1$, excluding consideration of other decrements (p. 18).

${}_n p_x^{(t)}$ = Probability of an employee not terminating employment from age x to age $x + n$, excluding consideration of other decrements (p. 18).

$p_x^{(T)}$ = Probability of an employee surviving in service from age x to age $x + 1$ (p. 31).

${}_n p_x^{(T)}$ = Probability of an employee surviving in service from age x to age $x + n$ (p. 32).

${}^{r}_{n}p_x^{(T)}$ = Probability of surviving in employment n years, where retirement decrements are included with mortality, termination, and disability decrements (p. 129).

$(PBO)_x$ = Projected benefit obligation for employee age x (p. 181).

${}^{AB\,r}(PCL)_x$ = Plan continuation liability for accrued benefits for employee age x (p. 70).

$(PSC)_x$ = Prior service cost created at an employee's age x (p. 189).

$(PTL)_x$ = Plan termination liability for employee age x (p. 69).

${}^{d}(PVFB)_x$ = Present value of future disability benefits for employee age x (p. 117).

${}^{HB}(PVFB)_x$ = Present value of future health care benefits for employee age x (p. 261).

${}^{HB'}(PVFB)_x$ = Present value of future health benefits (without health care inflation) for employee age x (p. 267).

${}^{r}(PVFB)_x$ = Present value of future benefits for employee age x assumed to retire at age r (p. 72).

${}^{r}(PVFB)_{x,y}$ = Present value of future benefits at age x for an age-y entrant (p. 84).

${}^{r'}(PVFB)_x$ = Present value of future benefits for employee age x under multiple retirement ages (p. 129).

${}^{*r'}(PVFB)_x$ = Present value of future benefits for employee age x under multiple retirement ages with actuarially equivalent early retirement benefits (p. 130).

${}^{s}(PVFB)_x$ = Present value of future surviving spouse benefits for employee age x (p. 118).

${}^{v}(PVFB)_x$ = Present value of future vested termination benefits for employee age x (p. 115).

${}^{r}(PVFNC)_y$ = Present value of future normal costs for employee age y (p. 80).

$q_x'^{(d)}$ = Disability rate at age x (p. 21).

$q_x^{(d)}$ = Probability of decrementing from active service due to disability during age x (p. 31).

$q_x'^{(m)}$ = Mortality rate at age x (p. 16).

$q_x^{(m)}$ = Probability of decrementing from active service due to death during age x (p. 32).

${}^{d}q_x'^{(m)}$ = Mortality rate for a disabled life at age x (p. 21).

$q_x'^{(r)}$ = Retirement rate at age x (p. 25).

$q_x^{(r)}$ = Probability of retiring at the beginning of age x (p. 31).

$q_x'^{(t)}$ = Termination rate at age x (p. 18).

$q_x^{(t)}$ = Probability of decrementing from active service due to termination during age x (p. 31).

$q_x^{(T)}$ = Probability of decrementing from active service during age x (p. 32).

$(\Delta QAA)_t$ = Addition to a qualified asset account in year t (p. 265).

$(QDC)_t$ = Qualified direct cost in year t (p. 265).

r = Normal retirement age (p. 5).

r' = First age at which an employee becomes eligible for early retirement (p. 5).

r'' = Age by which all employees are assumed to be retired (p. 5).

$(RA)_t$ = Reconciliation account balance (p. 163).

$(RR)_t$ = Real return in year t (p. 245).

${}^{MC}R_x$ = Expected reimbursements from Medicare (p. 256).

${}^{Other}R_x$ = Expected reimbursements from other governmental programs and/or other private insurance programs (p. 256).

s_x = Current dollar salary for a participant age x (p. 38).

$s_{x,y}$ = Salary at age x for an age-y entrant (p. 86).

S_x = Cumulative salary from entry age y up to, but not including, age x (p. 38).

s_y = Entry age dollar salary (p. 38).

$S_{r,y}$ = Cumulative salary from entry age y to retirement age r (p. 86).

$s_{\overline{1|}}^{(4)}$ = Factor for accumulating quarterly contributions with interest to end of year (p. 141).

$(SS)_x$ = Merit salary scale at age x (p. 38).

$(\sum SC)_t$ = Sum of all explicit supplemental costs associated with prior increases or decreases in the unfunded liability that are not yet fully amortized (p. 140).

$(SC)_x$ = Service cost for employee age x (p. 184).

$(SC_n)_j$ = jth supplemental cost for the nth unfunded liability increment (p. 101).

${}^{AB}(SC_n)_x$ = Implicit supplemental cost at age x for nth unfunded liability change under the accrued benefit method (p. 105).

${}^{CD}(SC_n)_x$ = Implicit supplemental cost at age x for nth unfunded liability change under the cost prorate (constant dollar) method (p. 109).

${}^{CE}(SC)_t$ = Supplemental costs associated with contingent events (p. 151).

${}^{CE}_{7\text{-}pay}(SC)_t$ = Supplemental costs associated with amortizing contingent event liabilities over 7 years (p. 155).

${}^{CP}(SC_n)_x$ = Implicit supplemental cost at age x for nth unfunded liability change under the cost prorate (constant percent) method (p. 109).

${}^{I/PA}(SC)_t$ = Sum of all explicit supplemental costs for the initial unfunded actuarial liability and plan amendments that are not yet fully amortized (p. 151).

${}^{New}(SC)_t$ = Supplemental cost associated with the *new* unfunded current liability as of the current year, excluding any remaining *old* unfunded current and any unfunded contingent event liabilities (p. 151).

${}^{Old}(SC)_t$ = Supplemental cost associated with *old* unfunded current liability established in 1988 and amortized over the succeeding 18 years (p. 151).

${}^{AAB\,r}(TC)_t$ = Total cost (normal plus supplemental cost) under the aggregate accrued benefit method with implicit supplemental cost (p. 107).

$^{ABD\,r}(TC)_t$ = Total cost (normal plus supplemental cost) under the aggregate benefit prorate (constant dollar) method with implicit supplemental cost (p. 108).

$^{ABP\,r}(TC)_t$ = Total cost (normal plus supplemental cost) under the aggregate benefit prorate (constant percent) method with implicit supplemental cost (p. 108).

$^{ACD\,r}(TC)_t$ = Total cost (normal plus supplemental cost) under the aggregate cost prorate (constant dollar) method with implicit supplemental cost (p. 111).

$^{ACP\,r}(TC)_t$ = Total cost (normal plus supplemental cost) under the aggregate cost prorate (constant percent) method with implicit supplemental cost (p. 111).

$^{CD}(TC)_x$ = Total cost (normal plus supplemental cost) under the cost prorate (constant dollar) method with implicit supplemental cost (p. 109).

$^{d}(TC)_x$ = Term cost of disability benefits for an employee age x (p. 116).

$^{s}(TC)_x$ = Term cost of surviving spouse benefits for an employee age x (p. 117).

$^{v}(TC)_x$ = Term cost of vested termination benefits for an employee age x (p. 115).

$(TP)_t$ = Transition percentage, equal to 20 percent for 1993 and increasing by 10 percent per year to 100 percent by 2001 (p. 155).

Tx = Unrelated business income tax rate (p. 266).

u = Number of years (positive or negative) that, when added to the participant's age, yields an assumed age for the surviving spouse (p. 117).

$(UCL)_t$ = Unfunded current liability during year t (p. 152).

$(UL)_t$ = Unfunded liability for plan at beginning of year t (p. 98).

$(\Delta_n UL)$ = Unfunded liability (positive or negative) developed during year n (p. 98).

$(\Delta_n UL)_x$ = nth unfunded liability developed during a prior year (p. 101).

${}^{CE}(UL)_t$ = Unfunded liability associated with prior contingent events (p. 154).

${}^{Old}(UL)_t$ = Balance of unfunded *old* current liability (p. 153).

$(ULB)_t$ = Unamortized liability balance in year t (p. 162).

$(\Delta_n ULB)_j$ = Unfunded liability balance at the beginning of the jth year for the nth unfunded liability change (p. 101).

v^n = Present value of one dollar due in n years at an annual compound rate of interest equal to i (p. 36).

w = Waiting period before disability benefits commence (p. 116) and serial correlation of successive year's inflation (p. 244).

${}^{FD}(Waiver)_t$ = Funding deficiency waiver (p. 162).

x = Employee's attained age.

y = Employee's entry age.

$(Yrs)_n$ = Amortization years remaining for the nth supplemental cost (p. 167).