CS 5320

Assignment 1

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1. (f)

$\rho = (p1, Purchase)$

$\rho = (p2, Purchase)$

$\rho = (pr, Product)$

$\rho = (c, Customer)$

$\rho = (temp, \pi_{p1.cid}(\sigma_{p1.pid=pr.pid \wedge c.cid=p1.cid \wedge p1.price<pr.msrp} p1 \bowtie pr \bowtie c))$

$\pi_{c.cid,c.cname} p2 \bowtie c - temp$

1. (g)

$\rho = (p1, Purchase)$

$\rho = (p2, Purchase)$

$\rho = (temp1, \pi_{count(*)\rightarrow count}(\sigma_{p1.cid=p2.cid \wedge p1.pid=p2.pid} p1 \bowtie p2))$

$\rho = (temp2, \pi_{p1.cid,p1.pid,temp1.count} p1 \bowtie temp1)$

$\pi_{temp2.cid,temp2.pid}(\sigma_{temp2.count=2} temp2)$

1. (h)

$\rho = (p1, Purchase)$

$\rho = (p2, Purchase)$

$\rho = (temp, \pi_{min(p1.price)\rightarrow min}(\sigma_{p1.pid=p2.pid} p1 \bowtie p2))$

$\pi_{p2.pid,temp.min} p2$

2. (a) True

First, prove that $\delta(\sigma_c(R)) \subseteq \sigma_c(\delta(R))$ for arbitrary R.

Fix an arbitrary R and suppose $t{:}k \in \delta(\sigma_c(R))$. From the assumption, we know that t:k is in R and that C holds on t after duplicate elimination. So, there should be at least one t in $\sigma_c(R)$, which means t satisfies C and there is at least one t in R. Since there is at least one t in R, there is one t in $\delta(R)$. Since t satisfies C, $t{:}k \in \sigma_c(\delta(R))$. So, $\delta(\sigma_c(R)) \subseteq \sigma_c(\delta(R))$ is proved to be true.

Next, prove that $\sigma_c(\delta(R)) \subseteq \delta(\sigma_c(R))$ for arbitrary R.

Fix an arbitrary R and suppose $t{:}k \in \sigma_c(\delta(R))$. From the assumption, we know that t:k is in $\delta(R)$ and that C holds on t. Since t:k is in $\delta(R)$, there is at least one t in R. t satisfies C and t is in R, so there is at least one t $\sigma_c(R)$. Then t:k is in the result of $\delta(\sigma_c(R))$, $t{:}k \in \delta(\sigma_c(R))$. So, is proved to be true.

As result, the statement $\delta(\sigma_c(R)) \equiv \sigma_c(\delta(R))$ is true.

2. (b) True

First, prove that $\delta(\pi_A(R)) \subseteq \pi_A(\delta(R))$ for arbitrary R.

Fix an arbitrary R and suppose $t{:}k \in \delta(\pi_A(R))$. From the assumption, we know that there is at least one tuple containing attribution A of t. $\delta(R)$ eliminates duplicates, but there is still one tuple containing attribution A of t. After projection of A on $\delta(R)$, one t is still there, $t{:}k \in \pi_A(\delta(R))$. So, $\delta(\pi_A(R)) \subseteq \pi_A(\delta(R))$ is proved to be true.

Next, prove that $\pi_A(\delta(R)) \subseteq \delta(\pi_A(R))$ for arbitrary R.

Fix arbitrary R and suppose $t{:}k \in \pi_A(\delta(R))$. From the assumption, we know that t with attribution A is in the duplicate-eliminated R, which means there is at least one tuple containing attribution A of t in R. So, t satisfies $\pi_A(R)$. After duplicate elimination, t is still in the relation $\delta(\pi_A(R))$, which means $t{:}k \in \delta(\pi_A(R))$. So $\pi_A(\delta(R)) \subseteq \delta(\pi_A(R))$ is proved to be true.

As result, the statement $\delta(\pi_A(R)) \equiv \pi_A(\delta(R)$ is true.

2. (c) True

First, prove that $\delta(R\times S) \subseteq \delta(R)\times\delta(S)$ for arbitrary R.

Fix an arbitrary R and suppose $t{:}k \in \delta(R\times S)$. From the assumption, we know that there is at least one t in the cross product of R and S, $t{:}k \in R\times S$. So, the entries of t can be found in R and S, which means the entries in t appears in $\delta(R)$ and $\delta(S)$. When doing cross join between $\delta(R)$ and $\delta(S)$, t should in the cross product of it, $t{:}k \in \delta(R)\times\delta(S)$. So, $\delta(R\times S) \subseteq \delta(R)\times\delta(S)$ is proved to be true.

Next, prove that $\delta(R)\times\delta(S) \subseteq \delta(R\times S)$ for arbitrary R.

Fix an arbitrary R and suppose $t{:}k \in \delta(R)\times\delta(S)$. From the assumption, we know that t is in the cross product of duplicate-eliminated R and S. Let a and b be the entries of t from R and S that means (a, b) denotes t. There is at least one a in R and one b in S. Based on this, when doing cross join between R and S, (a, b) is in $R \times S$, which mean t satisfies $R \times S$. Since $t \in R\times S$, t also satisfies $\delta(R\times S)$. So, $\delta(R)\times\delta(S) \subseteq \delta(R\times S)$ is proved to be true.

As result, the statement $\delta(R\times S) \equiv \delta(R)\times\delta(S)$ is true.

2. (d) True

First, prove that $\delta(R \bowtie_c S) \subseteq \delta(R) \bowtie_c \delta(S)$.

Fix an arbitrary R and suppose $t{:}k \in \delta(R \bowtie_c S)$. From the assumption, we know that t satisfies C and $t \in \delta(R \bowtie_c S)$. So, $t \in R \bowtie_c S$. Let a be the entries in t that correspond to attributes of R, and b be the entries in t that correspond to attributes of S. So a and b satisfy C, and $a \in R$ and $b \in S$. Also, $a \in \delta(R)$ and $b \in \delta(S)$. If join $\delta(R)$ and $\delta(S)$ under condition C, the tuple containing a and b should in the relation, which is $t{:}k \in \delta(R) \bowtie_c \delta(S)$. Therefore, $\delta(R \bowtie_c S) \subseteq \delta(R) \bowtie_c \delta(S)$ is proved to be true;

Next, prove that $\delta(R) \bowtie_c \delta(S) \subseteq \delta(R \bowtie_c S)$.

Fix an arbitrary R and suppose $t{:}k \in \delta(R) \bowtie_c \delta(S)$. Let a be the entries in t that correspond to attributes of R, and b be the entries in t that correspond to attributes of S. So $a \in \delta(R)$ and $b \in \delta(S)$, and a and b satisfy C, then we have $a \in R$ and $b \in S$, and a and b satisfy C. Then, since a and b are the entries of t, t should be in the relation of $R \bowtie_c S$. So, t is also in $\delta(R \bowtie_c S)$, which is $t \in \delta(R \bowtie_c S)$. Therefore, $\delta(R \bowtie_c S) \subseteq \delta(R) \bowtie_c \delta(S)$ is proved to be true.

As result, the statement $\delta(R \bowtie_c S) \equiv \delta(R) \bowtie_c \delta(S)$ is true.

2. (e) False

First, prove that $\delta(R) \cup_B \delta(S) \subseteq \delta(R \cup_B S)$.

Fix an arbitrary R and suppose $t{:}k \in \delta(R) \cup_B \delta(S)$. Let a be the entries of t that corresponds to attributes of R and S in common. So $a \in \delta(R)$ and $a \in \delta(S)$. When bag union $\delta(R)$ and $\delta(S)$, there will two a appear in the processed relation, $t{:}2 \in \delta(R) \cup_B \delta(S)$. For $\delta(R \cup_B S)$, no matter how many a is in $R \cup_B S$, after duplicate elimination, there is only one a in $\delta(R \cup_B S)$. So, $t{:}1 \in \delta(R \cup_B S)$. Since the set of $\delta(R) \cup_B \delta(S)$ is bigger than that of $\delta(R \cup_B S)$, $\delta(R) \cup_B \delta(S) \subseteq \delta(R \cup_B S)$ is proved to be false.

Since the first condition fails, the statement $\delta(R) \cup_B \delta(S) \equiv \delta(R \cup_B S)$ is false.

2. (f) True

First, prove that $\delta(R \cap_B S) \subseteq \delta(R) \cap_B \delta(S)$.

Fix an arbitrary R and suppose $t{:}k \in \delta(R \cap_B S)$. From the assumption, we know that $t{:}k \in R \cap_B S$ as well and there is only one t in $\delta(R \cap_B S)$. So, t is the tuple that R and S share in common, which means $t \in R$ and $t \in S$. Then, we know $t \in \delta(R)$ and $t \in \delta(S)$. When performing bag intersection to $\delta(R)$ and $\delta(S)$, t should appear in the processed relation only once. So, $\delta(R \cap_B S) \subseteq \delta(R) \cap_B \delta(S)$ is proved to be true.

Next, prove that $\delta(R) \cap_B \delta(S) \subseteq \delta(R \cap_B S)$.

Fix an arbitrary R and suppose $t{:}k \in \delta(R) \cap_B \delta(S)$. From the assumption, we know that there is only one t in $\delta(R) \cap_B \delta(S)$, $t \in \delta(R)$, and $t \in \delta(S)$. Then $t \in R$ and $t \in S$. So, there is at least one t in $R \cap_B S$. After duplicate elimination, no matter how many t is in $R \cap_B S$, there is only one t in $\delta(R \cap_B S)$. So, $\delta(R) \cap_B \delta(S) \subseteq \delta(R \cap_B S)$ is proved to be true.

As result, the statement $\delta(R \cap_B S) \equiv \delta(R) \cap_B \delta(S)$ is true.

2. (g) False

First, prove that $\delta(R -_B S) \subseteq \delta(R) -_B \delta(S)$.

Fix an arbitrary R and suppose $t{:}k \in \delta(R -_B S)$. Let t be the tuple appears $n + 1$ times in R and n times in S where $n \neq 0$. So $t \in R$ and $t \in S$. In the case of $\delta(R -_B S)$, when performing $R -_B S$, there is one $(n + 1 - n = 1)$ t remaining in the relation, so there is one t in $\delta(R -_B S)$ as well. However, in the case of $\delta(R) -_B \delta(S)$, there is only one t in both $\delta(R)$ and $\delta(S)$. After perfoming bag difference to $\delta(R)$ and $\delta(S)$, t is no longer in the processed relation. So, $\delta(R -_B S) \subseteq \delta(R) -_B \delta(S)$ is proved to be false.

Since the first condition fails, the statement $\delta(R -_B S) \equiv \delta(R) -_B \delta(S)$ is false.