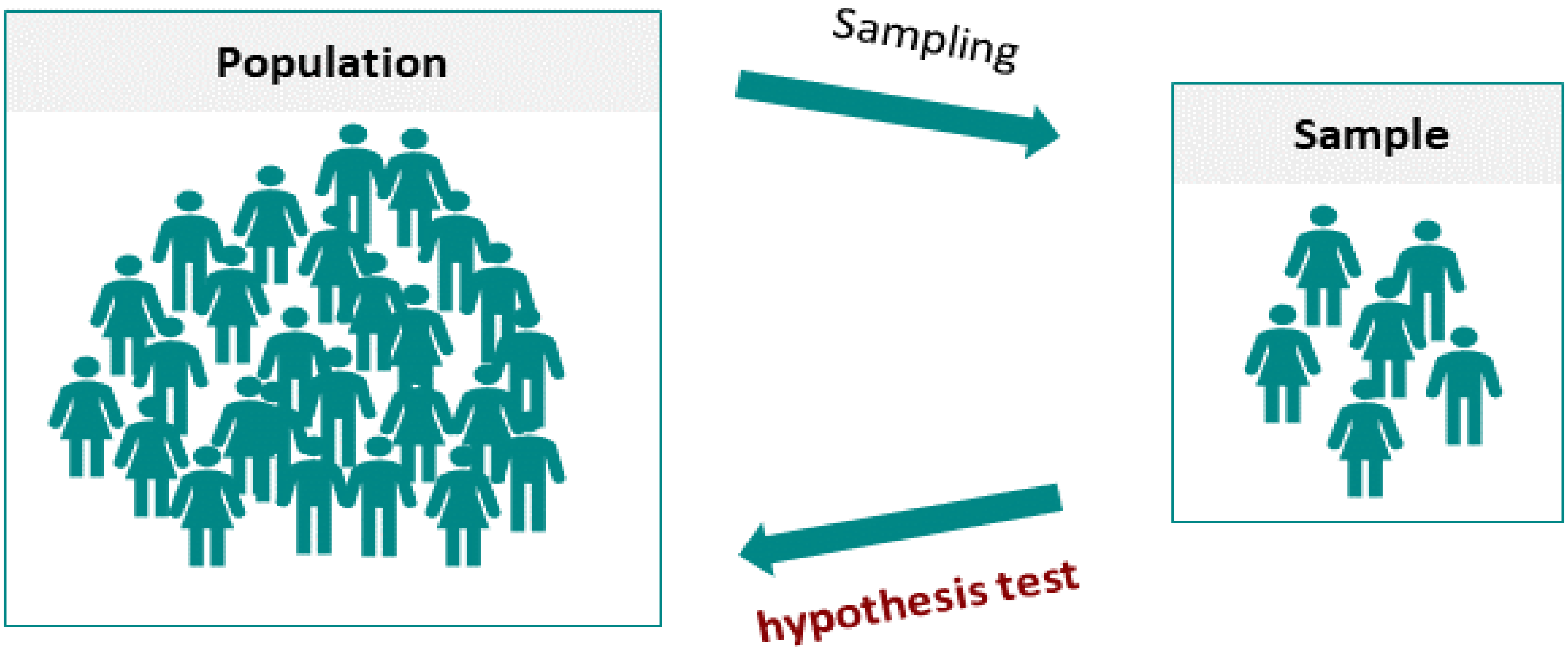
Hypothesis Testing

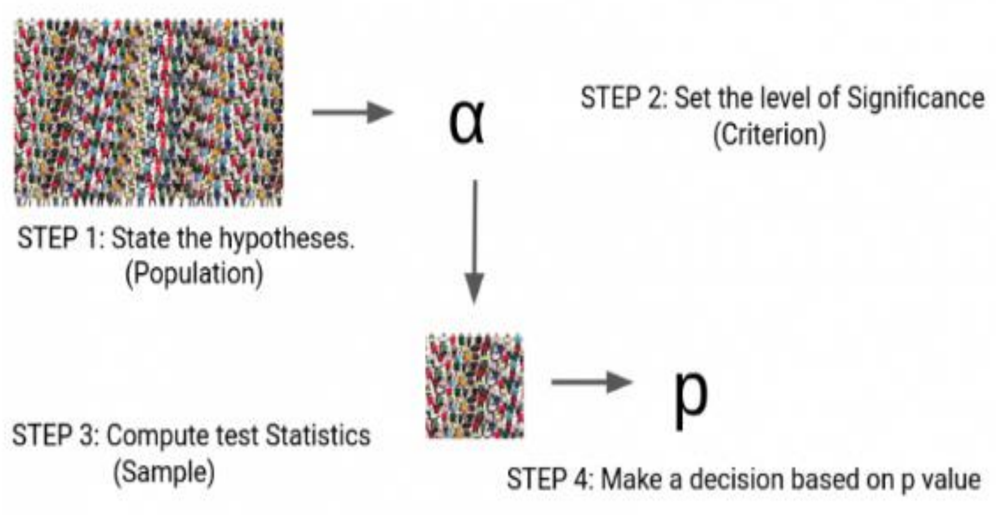
- A hypothesis is an assumption that is neither proven nor disproven. In the research process, a hypothesis is made at the very beginning and the goal is to either reject or not reject the hypothesis.
- Hypothesis test is used whenever you want to test a hypothesis about the population with the help of a sample.

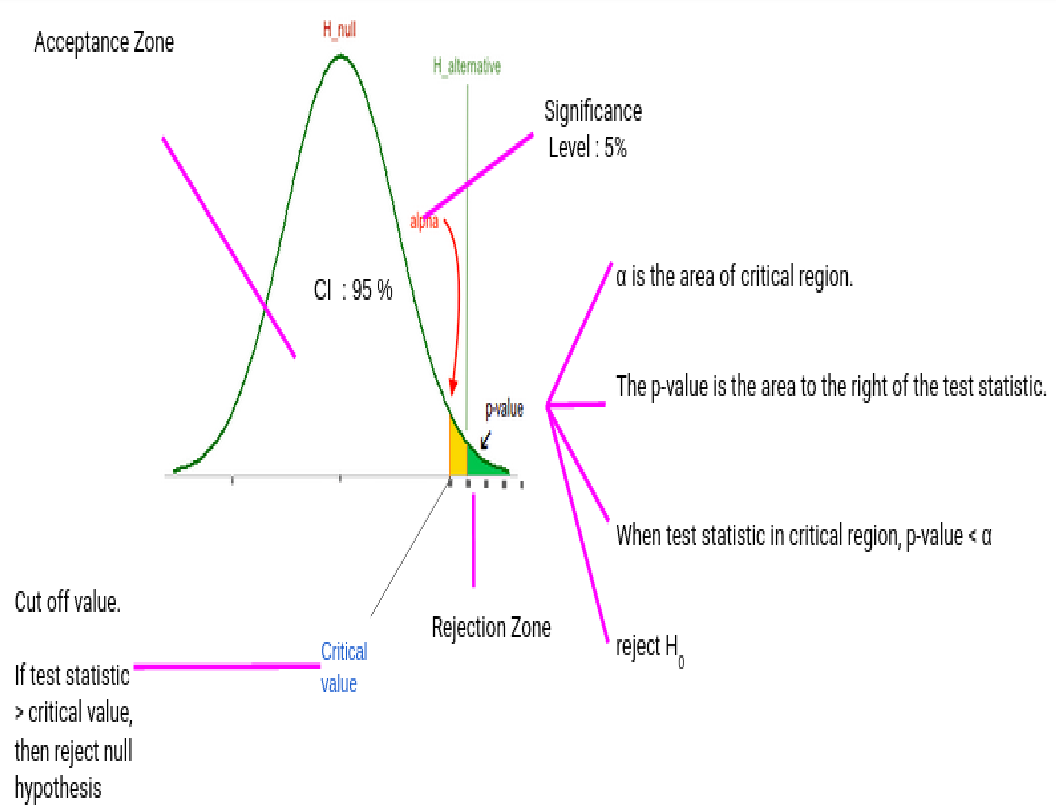


- Hypothesis can be of two types: Null Hypothesis (H0) and Alternate Hypothesis (H1).
- In a hypothesis test, only the null hypothesis can be tested; the goal is to find out whether the null hypothesis is rejected or not.
- We consider **the Null Hypothesis** to be true until we find strong evidence against it. Then. we accept the **Alternate Hypothesis**. We also determine the **Significance Level (α)**, if α is smaller, it will require more evidence to reject the Null Hypothesis.
- **Steps to Perform Hypothesis testing:**

  There are four steps to perform Hypothesis Testing:

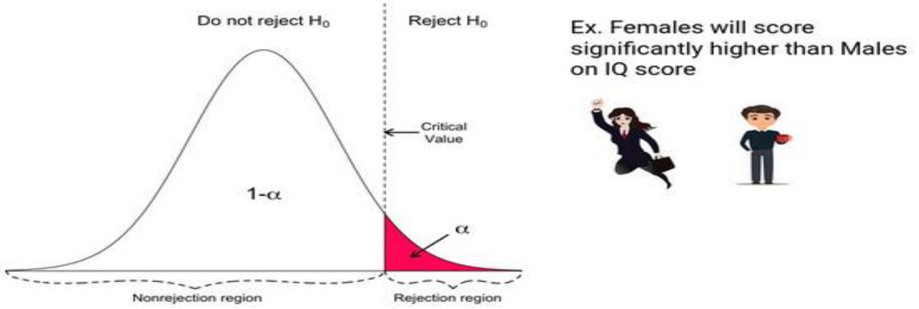
  1. Set the Hypothesis
  2. Set the Significance Level, Criteria for a decision
  3. Compute the test statistics
  4. Make a decision



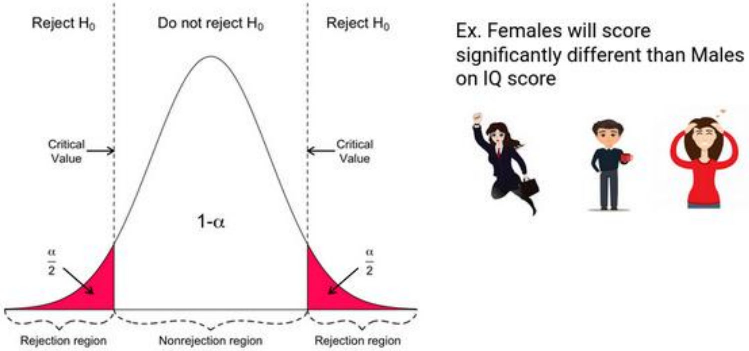


- Typically, we set the Significance level at 5%. If our test score lies in the Acceptance Zone we fail to reject the Null Hypothesis. If our test score lies in the critical zone, we reject the Null Hypothesis and accept the Alternate Hypothesis.
- Critical Value is the cut off value between Acceptance Zone and Rejection Zone.

- In the Directional Hypothesis, the null hypothesis is rejected if the test score is too large .Thus, the rejection region for such a test consists of one part, which is right from the center.



- In a Non-Directional Hypothesis test, the Null Hypothesis is rejected if the test score is either too small or too large. Thus, the rejection region for such a test consists of two parts: one on the left and one on the right.

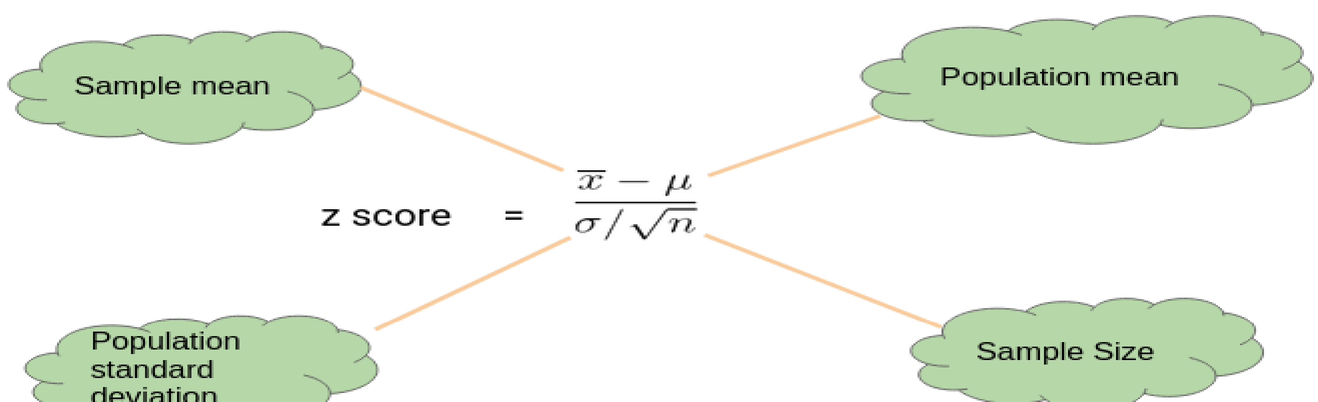


Z Test

- Z tests are a statistical way of testing a hypothesis when either:
  - We know the population variance, or
  - We do not know the population variance but our sample size is large $n \geq 30$
- If we have a sample size of less than 30 and do not know the population variance, then we must use a t-test.

One-Sample Z test

We perform the One-Sample Z test when we want to compare **a sample mean with the population mean.**



- Let’s say we need to determine if girls on average score higher than 600 in the exam. We have the information that the standard deviation for girls’ scores is 100. So, we collect the data of 20 girls by using random samples and record their marks. Finally, we also set our α value (significance level) to be 0.05.

  In this example:

  Mean Score for Girls is 641

  The size of the sample is 20

  The population mean is 600

  Standard Deviation for Population is 100

$$\text{z score} = \frac{\overline{x} - \mu}{\sigma / \sqrt{n}} = \frac{641 - 600}{100/\sqrt{20}} = 1.8336$$

p value = .033357.

Critical Value = 1.645

Z score > Critical Value

P value < 0.05

$H_0 : \mu \leq 600$

$\boxed{H_1 : \mu > 600}$

Since the P-value is less than 0.05, we can reject the null hypothesis and conclude based on our result

We perform a Two Sample Z test when we want to compare **the mean of two samples.**

Difference bw
Sample mean
$\bar{x}_1 - \bar{x}_2$

Difference bw
population mean
$\mu_1 - \mu_2$

$$\text{z score} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Population
standard
deviation σ, σ

Sample Size
n_1, n_2

Two Sample Z Test

- Here, let's say we want to know if Girls on average score 10 marks more than the boys. We have the information that the standard deviation for girls' Score is 100 and for boys' score is 90. Then we collect the data of 20 girls and 20 boys by using random samples and record their marks. Finally, we also set our α value (significance level) to be 0.05.
- In this example:

  Mean Score for Girls (Sample Mean) is 641

  Mean Score for Boys (Sample Mean) is 613.3

  Standard Deviation for the Population of Girls' is 100

  Standard deviation for the Population of Boys' is 90

  Sample Size is 20 for both Girls and Boys

  Difference between Mean of Population is 10

$$\text{z score} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{(641 - 613.3) - (10)}{\sqrt{\frac{100^2}{20} + \frac{90^2}{20}}}$$

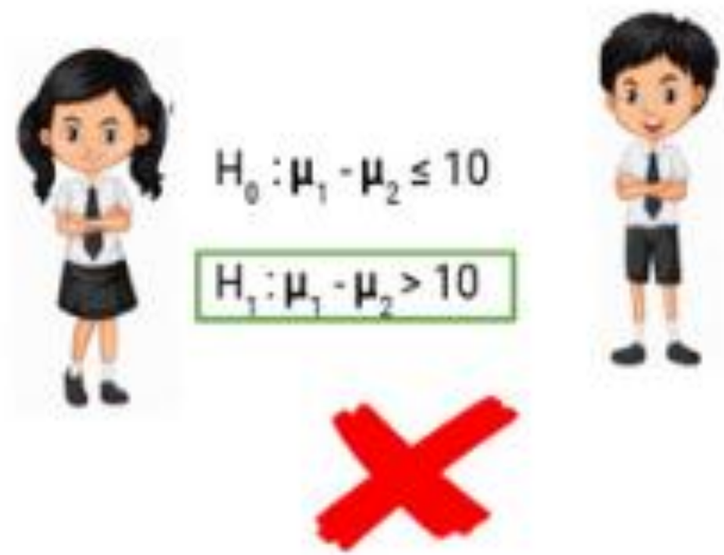
$$= 0.588$$

$$\text{P value} = 0.278$$

Critical Value = 1.645

Z score < Critical Value

P value > 0.05



Thus, we can **conclude based on the P-value that we fail to reject the Null Hypothesi**s.

T-Test

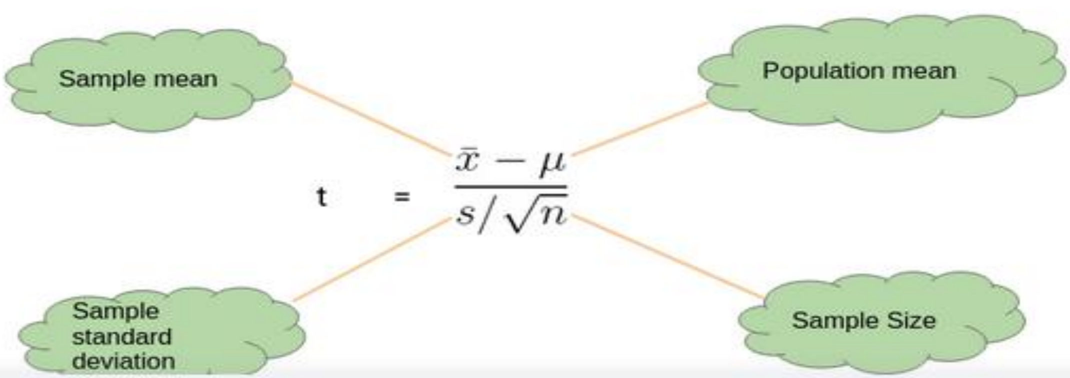
- t-tests are a statistical way of testing a hypothesis when:

  We do not know the population variance

  Our sample size is small, $n < 30$

One-Sample t-Test

We perform a One-Sample t-test when we want to **compare a sample mean with the population mean**. The difference from the Z Test is that we do **not have the information on Population Variance** here. We use the **sample standard deviation** instead of population standard deviation in this case.



Example to Understand a One Sample t-Test

- Let's say we want to determine if on average girls score more than 600 in the exam. We do not have the information related to variance (or standard deviation) for girls' scores. To a perform t-test, we randomly collect the data of 10 girls with their marks and choose our α value (significance level) to be 0.05 for Hypothesis Testing.
  - Mean Score for Girls is 606.8
  - The size of the sample is 10
  - The population mean is 600
  - Standard Deviation for the sample is 13.14

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{606.8 - 600}{13.14/\sqrt{10}} = 1.64$$

Critical Value = 1.833

t score < Critical Value

P value = 0.0678

P value > 0.05

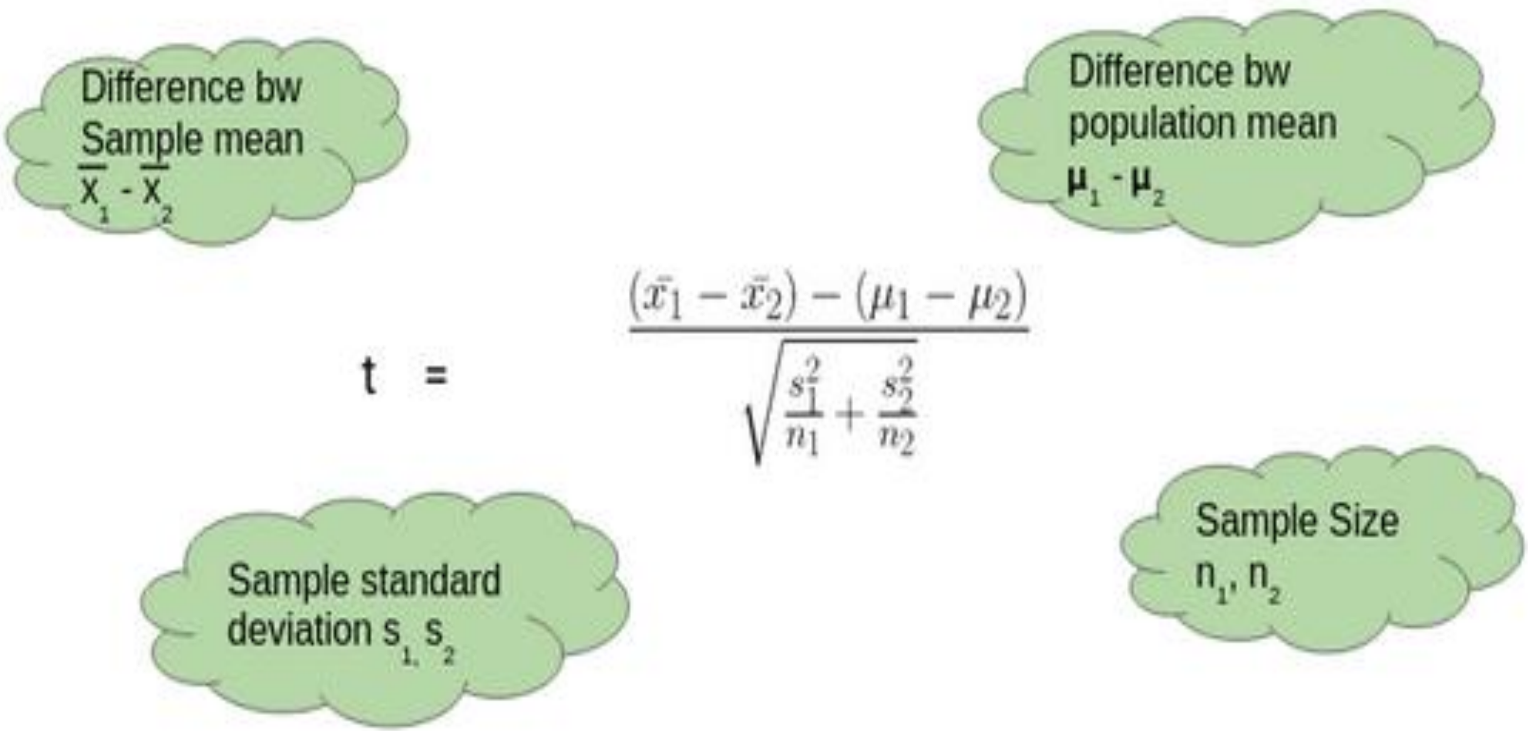
$H_0 : \mu \leq 600$

$\boxed{H_1 : \mu > 600}$

P-value is greater than 0.05 thus we fail to reject the null hypothesis and don't have enough evidence to support the hypothesis that on average, girls score more than 600 in the exam.

Two-Sample t-Test

- We perform a Two-Sample t-test when we want to compare the mean of two samples.



Example to Understand a Two-Sample t-Test

- Here, let’s say we want to determine if on average, boys score 15 marks more than girls in the exam. We do not have the information related to variance (or standard deviation) for girls’ scores or boys’ scores. To perform a t-test. we randomly collect the data of 10 girls and boys with their marks. We choose our α value (significance level) to be 0.05 as the criteria for Hypothesis Testing.

- In this example:

  Mean Score for Boys is 630.1

  Mean Score for Girls is 606.8

  Difference between Population Mean 15

  Standard Deviation for Boys’ score is 13.42

  Standard Deviation for Girls’ score is 13.14

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$\frac{(630.1 - 606.8) - (15)}{\sqrt{\frac{(13.42)^2}{10} + \frac{(13.14)^2}{10}}}$$

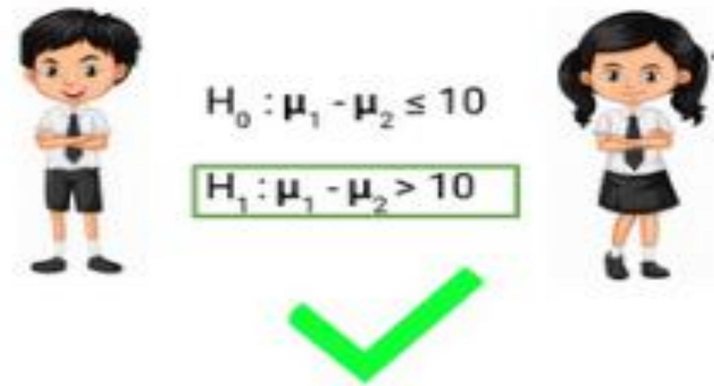
Critical Value = 1.833

t = 2.23

P value = 0.019

Critical Value > t score

P value < 0.05



Thus, **P-value is less than 0.05 so we can reject the null hypothesis** and conclude that on average boys score 15 marks more than girls in the exam.