ECE 438 - Laboratory 8
Number Representation and Waveform Quantization

Last Updated on May 5, 2022

Date:3/23/2023
Section:

Name	Signature	Time spent outside lab
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Student Name #2 [---%]		

	Below expectations	Lacks in some respect	Meets all expectations
Completeness of the report			
Organization of the report			
Quality of figures: *Correctly labeled with title, x-axis, y-axis, and name(s)*			
Understanding and implementation of uniform quantizer (45 pts): *Image: original and quantized images, comparison, questions. Audio: Python figures, questions*			
Understanding of error analysis (35 pts): *Error histograms, correlation, PSNR, rate-distortion curve, questions*			
Understanding of max quantizer (20 pts): *Histograms, PSNR, comparison with uniform quantizer*			

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import soundfile as sf
import IPython.display as ipd
from helper import xcorr, lloyds
```

In [2]:
```
# make sure the plot is displayed in this notebook
%matplotlib inline
# specify the size of the plot
plt.rcParams['figure.figsize'] = (16, 6)

# for auto-reloading extenrnal modules
%load_ext autoreload
%autoreload 2
```

Exercise 3.3: Image Quantization

1. Load the image and display it using the following sequence of commands.

```
image = plt.imread("fountainbw.tif")
plt.imshow(image, cmap='gray', vmin=0, vmax=255)
plt.axis('image')
plt.show()
```

In [7]:
```
image = plt.imread("fountainbw.tif")
plt.imshow(image, cmap='gray', vmin=0, vmax=255)
plt.axis('image')
plt.show()
```

2. Print the data type of this image, then convert the image matrix to type `float` , and print the data type of this image again.

- Use `image.dtype` to get the data type of `image` .

In [8]:
```
print(image.dtype)

image = image.astype(float)

print(image.dtype)
```

```
uint8
float64
```

3. Complete the function below which will uniformly quantize an input array `X` (either a vector or a matrix) to an `numBits` -bit array.

In [9]:
```
def Uquant(X, numBits):
    """
    Parameters
    ---
    X: the input array to be quantized
    numBits: the number of bits. The number of quantization levels will be

    Returns
    ---
    Y: the quantized array
    """
    N = 2**numBits
    delta = (np.max(X) - np.min(X))/(N-1)
    X_min = np.min(X)
    X = np.round((X - np.min(X))/delta)
    Y = X * delta + X_min

    return Y
```

4. Use this function to quantize the fountain image to $7, 6, 5, 4, 3, 2, 1$ b/pel, and display and observe the output images. Don't forget the titles of the images.

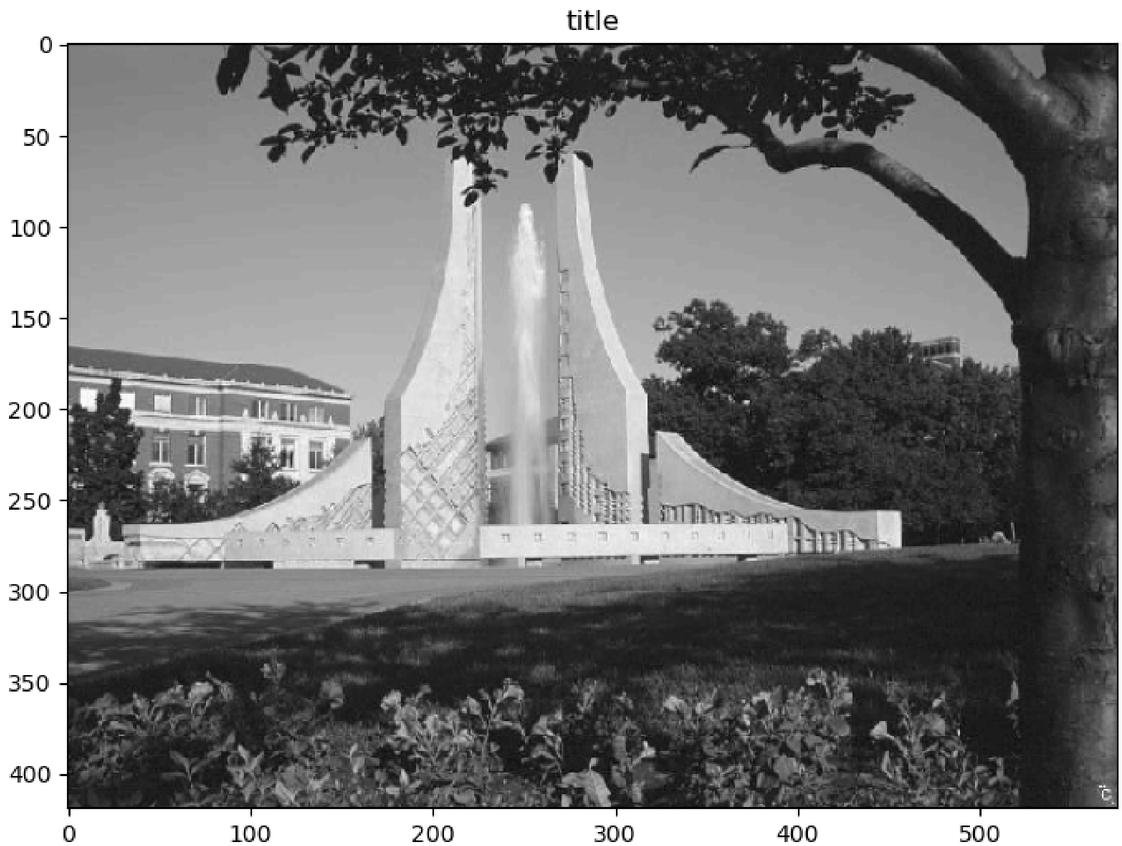
- To display a grayscale image `image` , use the following commands:

```
plt.imshow(image.astype(np.uint8), cmap='gray', vmin=0, vmax=25
5)
plt.title("title")
plt.show()
```

In [15]:
```
image7 = Uquant(image, 7)
plt.imshow(image7.astype(np.uint8), cmap='gray', vmin=0, vmax=255)
plt.title("title")
plt.show()

image6 = Uquant(image, 6)
plt.imshow(image6.astype(np.uint8), cmap='gray', vmin=0, vmax=255)
plt.title("title")
plt.show()

image5 = Uquant(image, 5)
plt.imshow(image5.astype(np.uint8), cmap='gray', vmin=0, vmax=255)
plt.title("title")
plt.show()
```


title
0
50
100
150
200
250
300
350
400
0 100 200 300 400 500

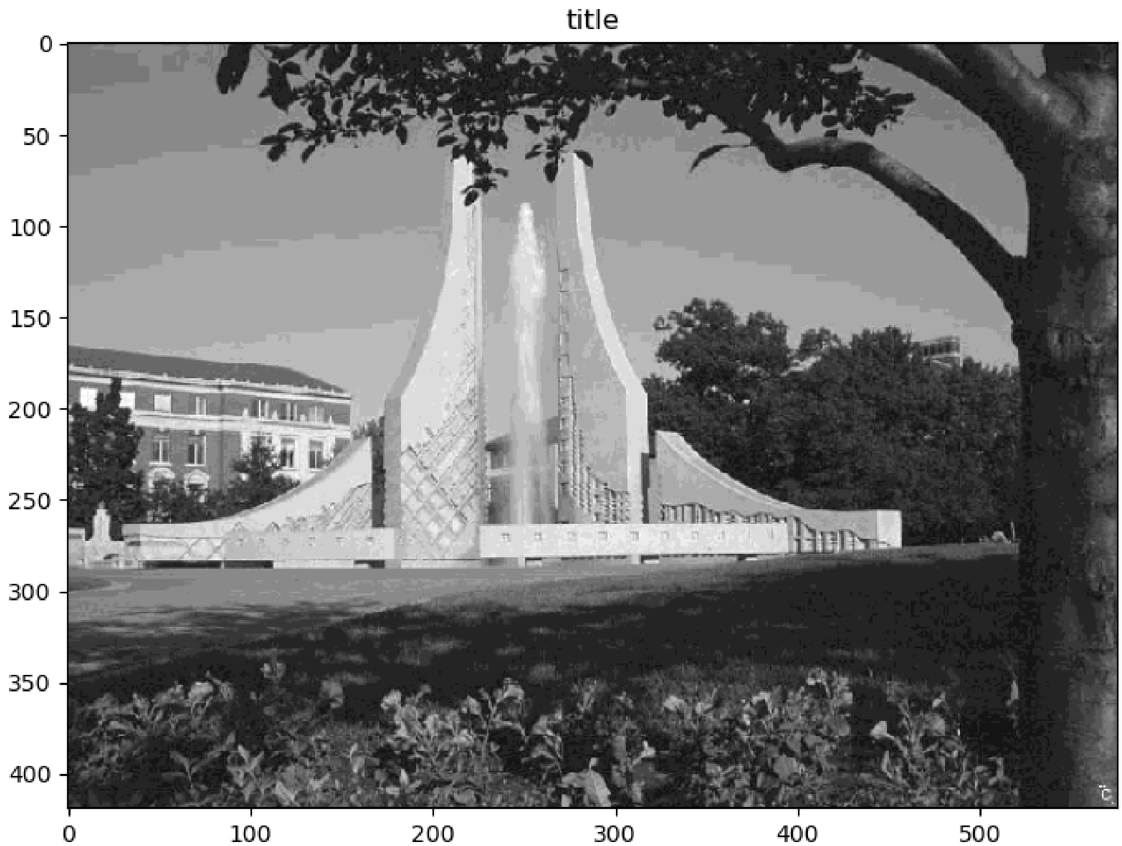

title
0
50
100
150
200
250
300
350
400
0 100 200 300 400 500

In [17]:

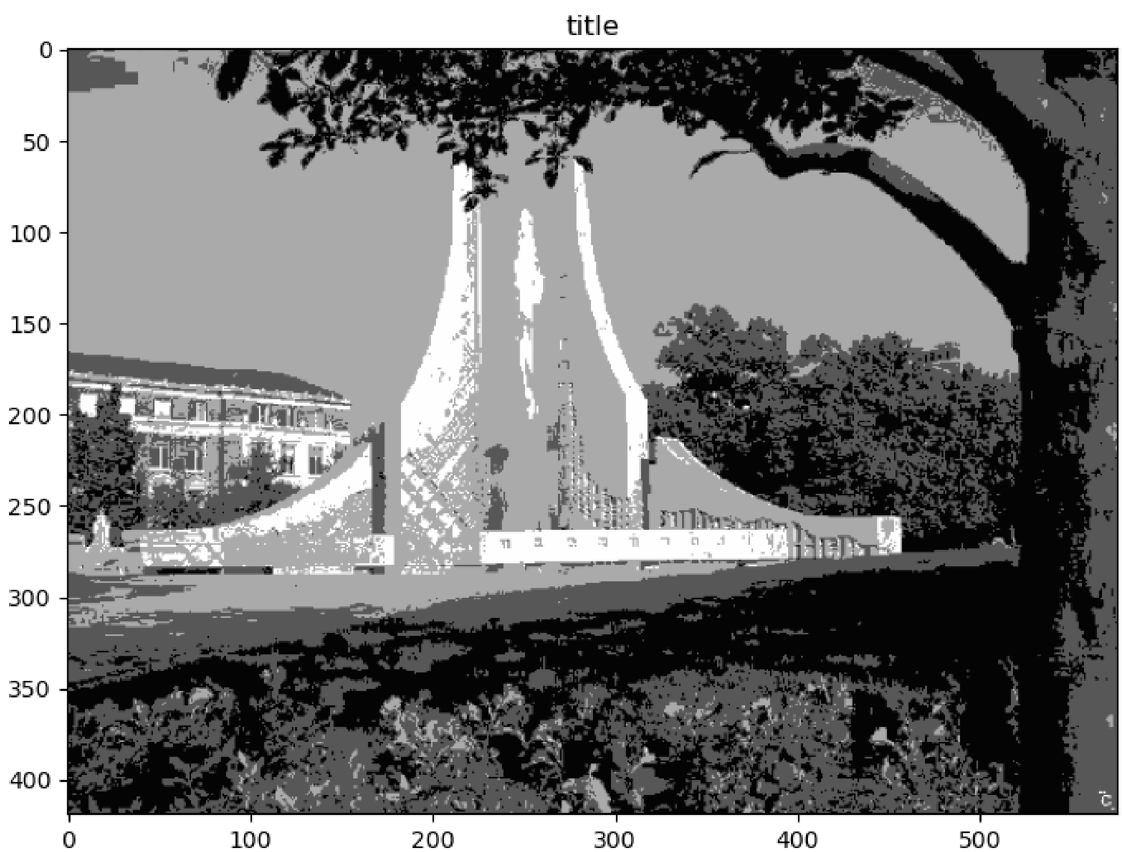
```
image4 = Uquant(image, 4)
plt.imshow(image4.astype(np.uint8), cmap='gray', vmin=0, vmax=255)
plt.title("title")
plt.show()

image3 = Uquant(image, 3)
plt.imshow(image3.astype(np.uint8), cmap='gray', vmin=0, vmax=255)
plt.title("title")
plt.show()

image2 = Uquant(image, 2)
plt.imshow(image2.astype(np.uint8), cmap='gray', vmin=0, vmax=255)
plt.title("title")
plt.show()
```

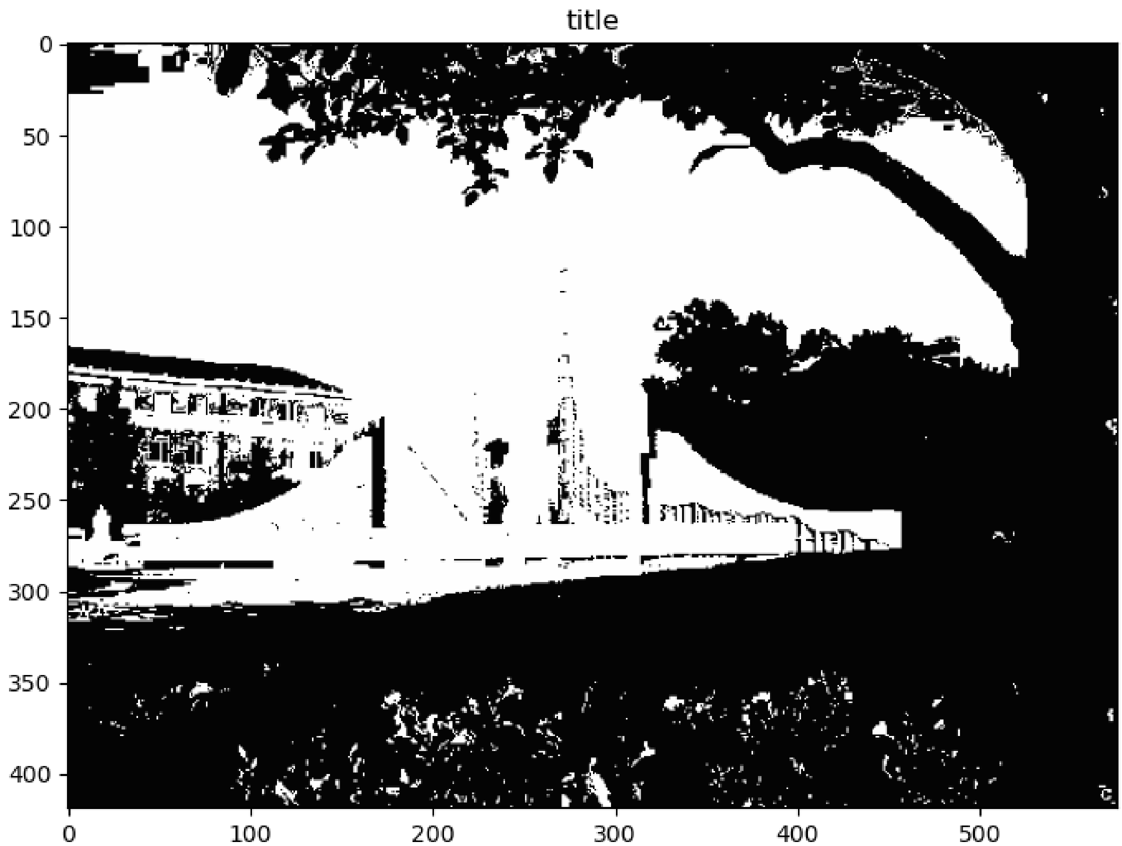



title
0
50
100
150
200
250
300
350
400
0
100
200
300
400
500


title
0
50
100
150
200
250
300
350
400
0
100
200
300
400
500

In [18]:

```
image1 = Uquant(image, 1)
plt.imshow(image1.astype(np.uint8), cmap='gray', vmin=0, vmax=255)
plt.title("title")
plt.show()
```



5. Describe the artifacts (errors) that appear in the image as the number of bits is lowered.

The artifacts(errors) increase as the number of bits is lowered

6. Note the number of b/pel at which the image quality noticeably deteriorates.

1

7. Compare each of four quantized images ($7, 4, 2$ and 1 b/per) to the original.

For 7b/per, the difference is not obvious. For 4b/per, the sky is quantized into smaller color blocks. For 2b/per, there are essentially three color, black and white and grey. For 1b/per, there are only black and white

Exercise 3.4: Audio Quantization

1. Use your function `Uquant()` to quantize each of these signals: `speech.au` and `music.au` to 7, 4, 2 and 1 bits/sample. Listen to the original and quantized signals.

- To read an audio file:

```
speech, fs = sf.read("speech.au")  # speech is the signal vecto
r, and fs is the sampling frequency
```

- To play a signal

```
ipd.Audio(speech, rate=fs)
```

In [22]:
```
speech, fs = sf.read("speech.au")
music, fm = sf.read("music.au")
ipd.Audio(speech, rate=fs)
```

Out[22]:

0:02 / 0:02

In [23]:
```
speech7 = Uquant(speech, 7)
ipd.Audio(speech7, rate=fs)
```

Out[23]:

0:02 / 0:02

In [24]:
```
speech4 = Uquant(speech, 4)
ipd.Audio(speech4, rate=fs)
```

Out[24]:

0:02 / 0:02

In [25]:
```
speech2 = Uquant(speech, 2)
ipd.Audio(speech2, rate=fs)
```

Out[25]:

0:02 / 0:02

In [26]:
```
speech1 = Uquant(speech, 1)
ipd.Audio(speech1, rate=fs)
```

Out[26]:

0:02 / 0:02

In [27]:
```
ipd.Audio(music, rate=fm)
```

Out[27]:

0:10 / 0:10

In [28]:
```
music7 = Uquant(music, 7)
ipd.Audio(music7, rate=fm)
```

Out[28]:

0:10 / 0:10

In [29]:
```
music4 = Uquant(music, 4)
ipd.Audio(music4, rate=fm)
```

Out[29]:

0:10 / 0:10

In [30]:
```
music2 = Uquant(music, 2)
ipd.Audio(music2, rate=fm)
```

Out[30]:

0:10 / 0:10

In [31]:
```
music1 = Uquant(music, 1)
ipd.Audio(music1, rate=fm)
```

Out[31]:

0:00 / 0:10

2. For each signal, describe the change in quality as the number of b/sample is reduced.

For each signal, the quality decrease as the number of b/sample is reduced. The result of quality reduction sound like white noise

3. For each signal, is there a point at which the signal quality deteriorates drastically? At what point (if any) does it become incomprehensible?

At 2b/sample, the signal quality deteriorates drastically, and at 1b/sample, the output signal is incomprehensible.

4. Which signal's quality deteriorates faster as the number of levels decreases?

The speech signal

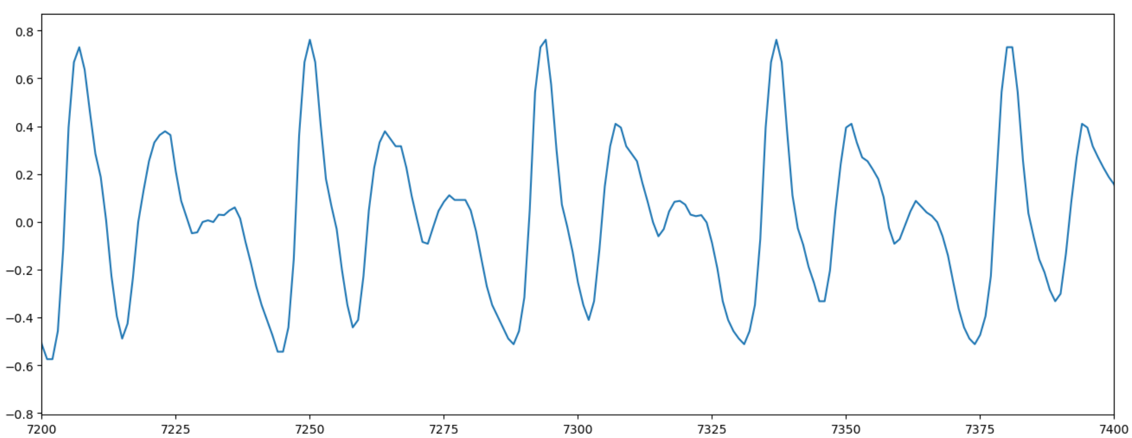
5. Do you think 4 b/sample is acceptable for telephone systems? What about 2 b/sample?

4b/sample is acceptable under certain circumstances. 2b/sample is not reliable.

6. Plot the four quantized `speech` signals over the index range $[7200 : 7400)$. Generate a similar figure for the `music` signal, using the same indices.

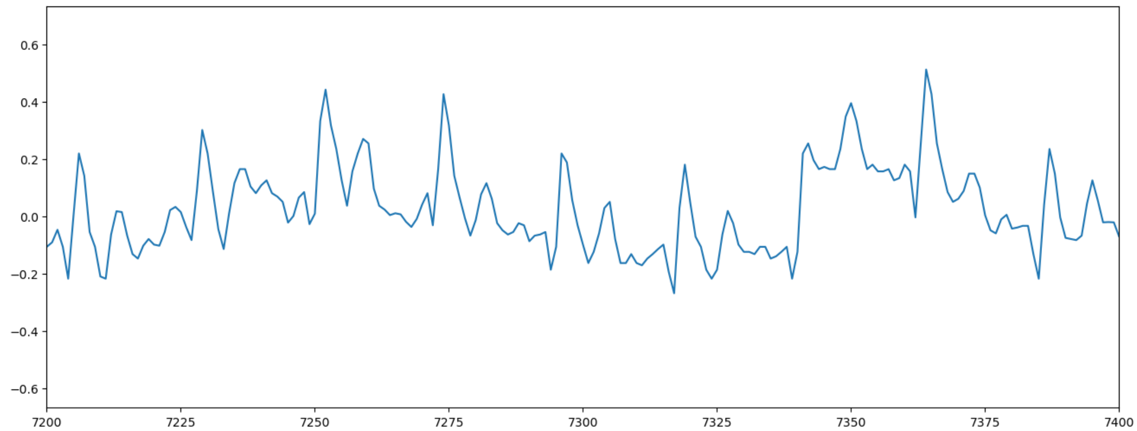
In [75]:
```
plt.xlim(7200,7400)
plt.plot(speech)
```

Out[75]: [<matplotlib.lines.Line2D at 0x2cc45ef87c0>]



In [35]:
```
plt.xlim(7200,7400)
plt.plot(music)
```

Out[35]: [<matplotlib.lines.Line2D at 0x2cc44f62df0>]



Exercise 3.5. Error Analysis

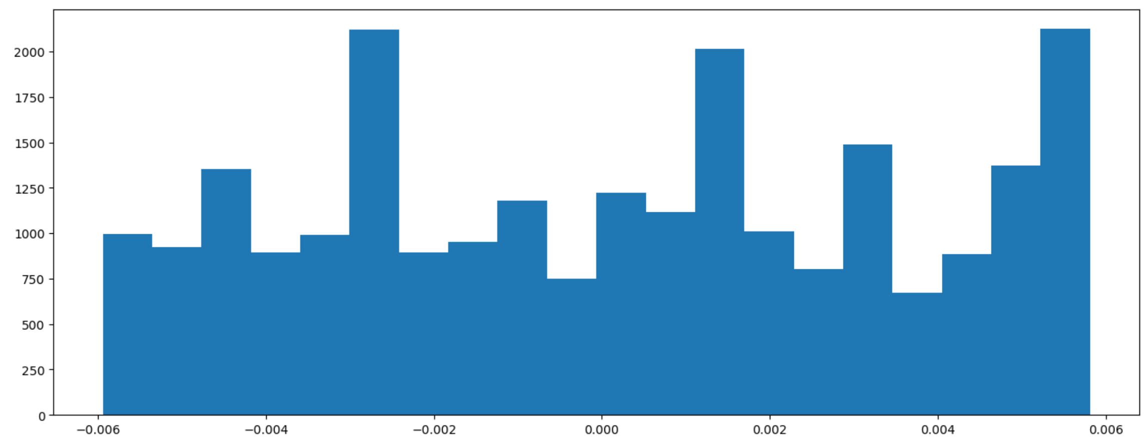
1. Compute the error signal for the quantized speech for $7, 4, 2$ and 1 b/sample.

In [37]:
```
E7 = speech7 - speech
E4 = speech4 - speech
E2 = speech2 - speech
E1 = speech1 - speech
```

2. Use the command [`plt.hist(E, bins=20)` (https://matplotlib.org/3.2.1/api/_as_gen/matplotlib.pyplot.hist.html)](https://matplotlib.org/3.2.1/api/_as_gen/matplotlib.pyplot.hist.html) to generate 20-bin histograms for each of the four error signals.

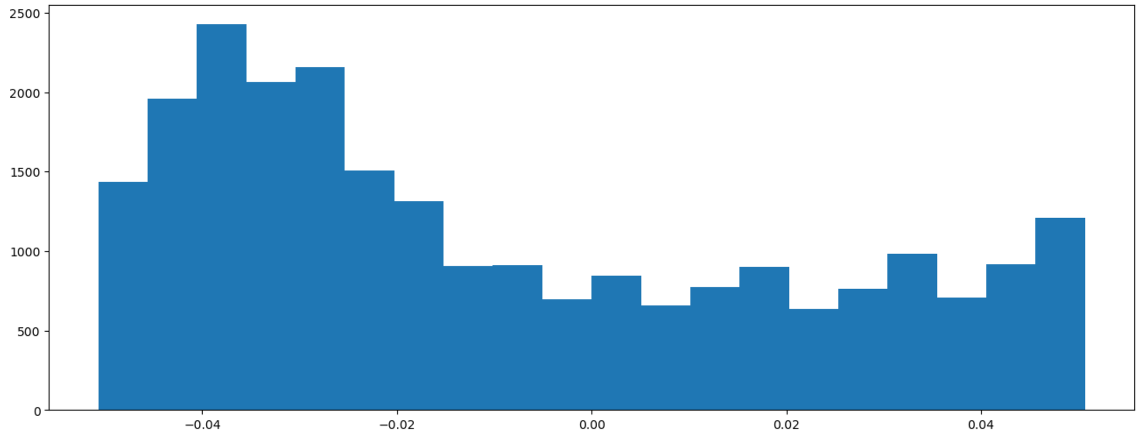
In [40]:
```
plt.hist(E7, bins=20)
```

Out[40]:
```
(array([ 998.,  921., 1353.,  895.,  991., 2122.,  894.,  950., 1178.,
         748., 1225., 1115., 2013., 1011.,  804., 1486.,  673.,  887.,
        1371., 2124.]),
 array([-5.94588152e-03, -5.35782942e-03, -4.76977731e-03, -4.18172521e-0
3,
        -3.59367311e-03, -3.00562100e-03, -2.41756890e-03, -1.82951679e-0
3,
        -1.24146469e-03, -6.53412586e-04, -6.53604823e-05,  5.22691622e-0
4,
         1.11074373e-03,  1.69879583e-03,  2.28684793e-03,  2.87490004e-0
3,
         3.46295214e-03,  4.05100424e-03,  4.63905635e-03,  5.22710845e-0
3,
         5.81516056e-03]),
 <BarContainer object of 20 artists>)
```



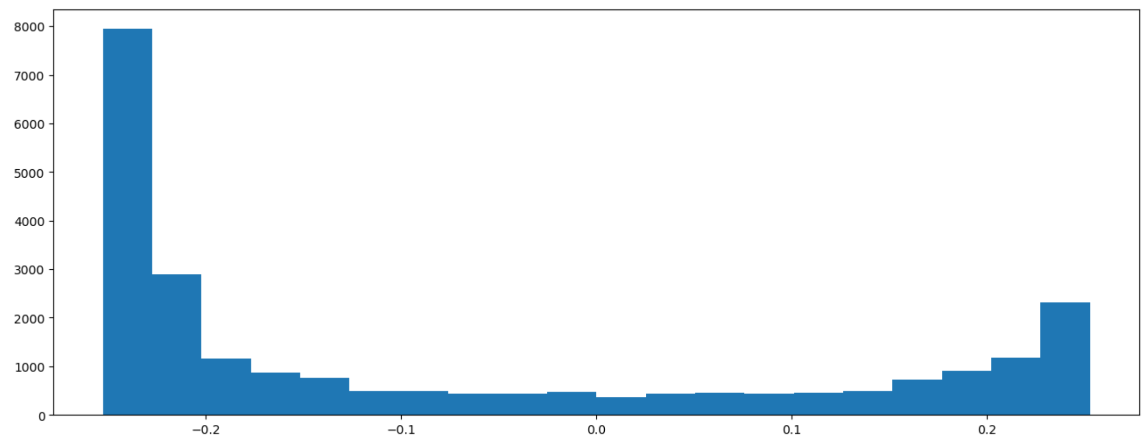
In [41]:
```
plt.hist(E4, bins=20)
```

Out[41]:
```
(array([1435., 1957., 2426., 2061., 2159., 1507., 1312.,  905.,  913.,
         697.,  847.,  657.,  773.,  898.,  637.,  760.,  983.,  706.,
         917., 1209.]),
 array([-0.05068359, -0.04561523, -0.04054687, -0.03547852, -0.03041016,
        -0.0253418 , -0.02027344, -0.01520508, -0.01013672, -0.00506836,
         0.        ,  0.00506836,  0.01013672,  0.01520508,  0.02027344,
         0.0253418 ,  0.03041016,  0.03547852,  0.04054687,  0.04561523,
         0.05068359]),
 <BarContainer object of 20 artists>)
```



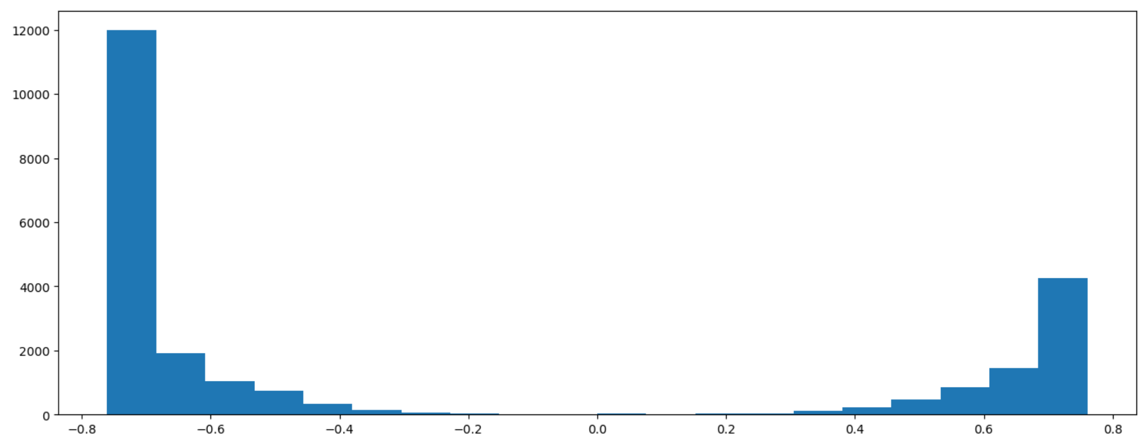
In [42]:
```
plt.hist(E2, bins=20)
```

Out[42]:
```
(array([7943., 2886., 1162.,  878.,  766.,  501.,  486.,  440.,  433.,
         478.,  361.,  446.,  454.,  447.,  465.,  487.,  720.,  911.,
        1171., 2324.]),
 array([-2.52766927e-01, -2.27478027e-01, -2.02189128e-01, -1.76900228e-0
1,
        -1.51611328e-01, -1.26322428e-01, -1.01033529e-01, -7.57446289e-0
2,
        -5.04557292e-02, -2.51668294e-02,  1.22070312e-04,  2.54109701e-0
2,
         5.06998698e-02,  7.59887695e-02,  1.01277669e-01,  1.26566569e-0
1,
         1.51855469e-01,  1.77144368e-01,  2.02433268e-01,  2.27722168e-0
1,
         2.53011068e-01]),
 <BarContainer object of 20 artists>)
```



In [43]:
```
plt.hist(E1, bins=20)
```

Out[43]:
```
(array([1.1985e+04, 1.9070e+03, 1.0420e+03, 7.5200e+02, 3.3700e+02,
        1.3700e+02, 6.9000e+01, 3.3000e+01, 4.0000e+00, 1.1000e+01,
        2.6000e+01, 1.8000e+01, 3.0000e+01, 4.4000e+01, 1.0500e+02,
        2.2600e+02, 4.7400e+02, 8.4100e+02, 1.4540e+03, 4.2640e+03]),
 array([-7.60498047e-01, -6.84436035e-01, -6.08374023e-01, -5.32312012e-0
1,
        -4.56250000e-01, -3.80187988e-01, -3.04125977e-01, -2.28063965e-0
1,
        -1.52001953e-01, -7.59399414e-02,  1.22070312e-04,  7.61840820e-0
2,
         1.52246094e-01,  2.28308105e-01,  3.04370117e-01,  3.80432129e-0
1,
         4.56494141e-01,  5.32556152e-01,  6.08618164e-01,  6.84680176e-0
1,
         7.60742188e-01]),
 <BarContainer object of 20 artists>)
```



3. How does the number of quantization levels seem to affect the shape of the distribution?

Less quantization levels gives bigger error. In higher quantization, the error looks like it is randomly distributed.

4. Explain why the error histograms you obtain might not be uniform?

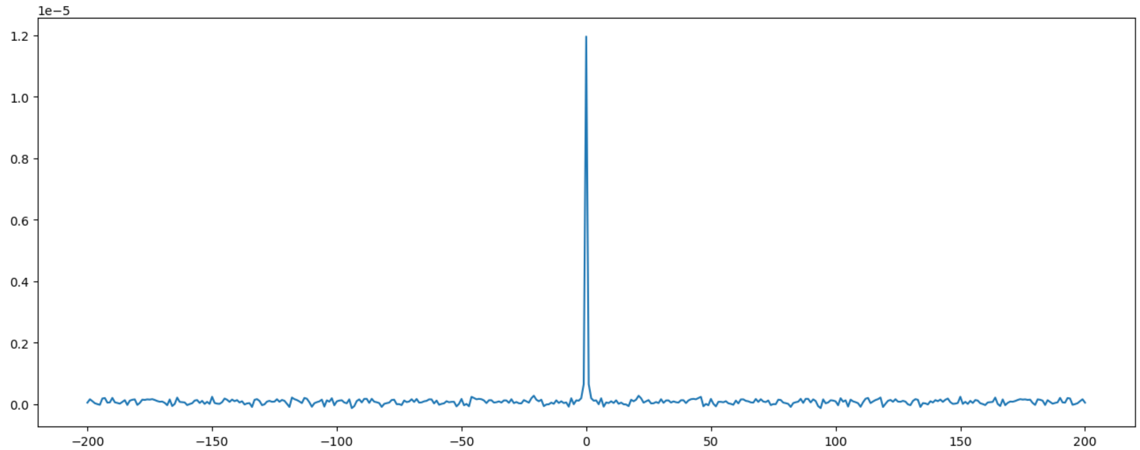
The error is not uniform if the error is mostly due to quantization

5. Compute and plot an estimate of the autocorrelation function for each of the four error signals using the following commands:

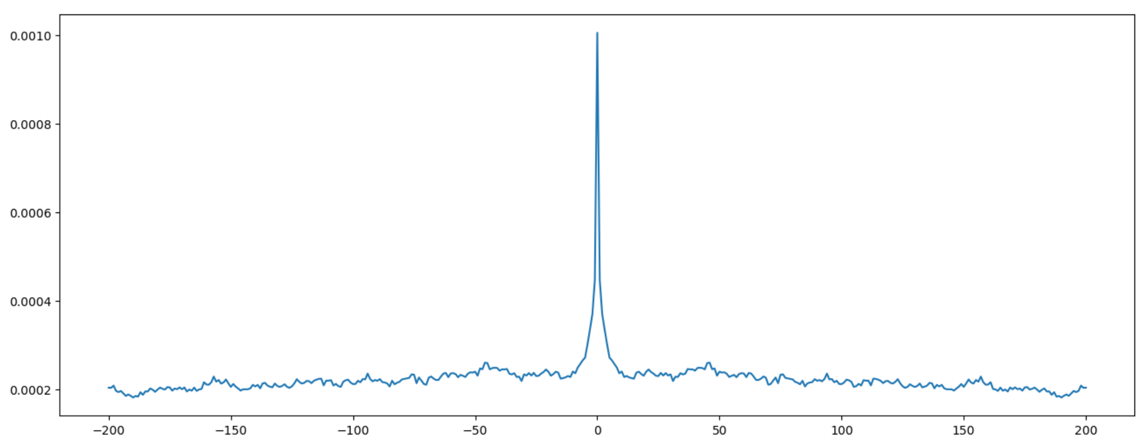
```
lags, r = xcorr(E, maxlags=200)
plt.plot(lags, r)
plt.show()
```

Hint: function `xcorr` is provided in the file `helper.py`

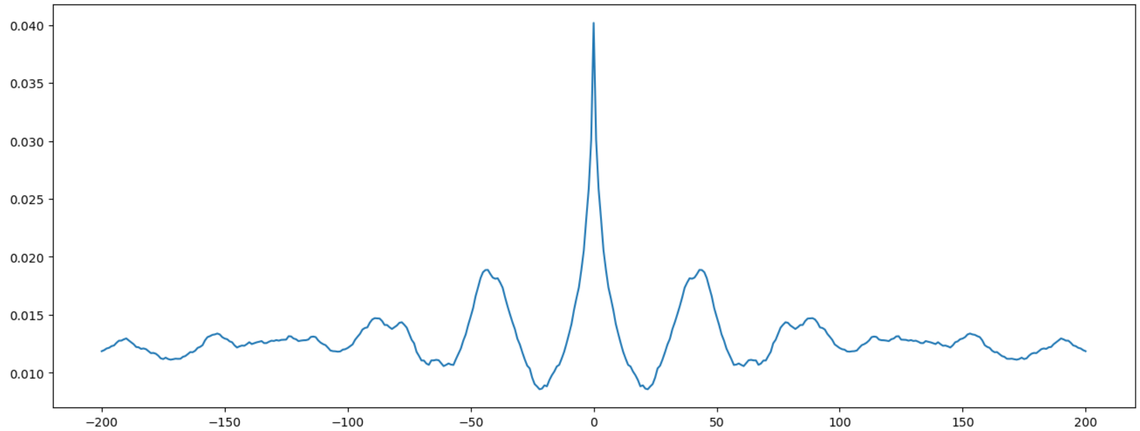
In [46]:
```
lags, r = xcorr(E7, E7, maxlags=200)
plt.plot(lags, r)
plt.show()
```



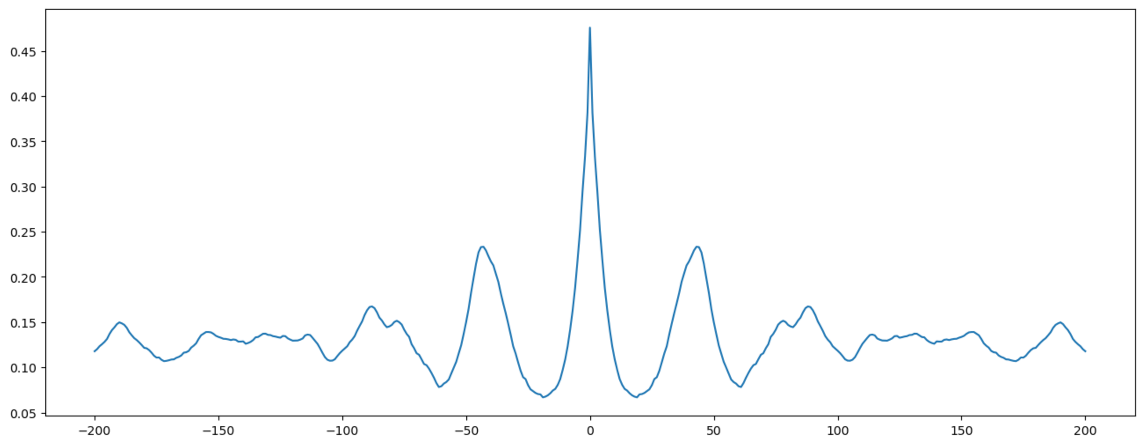
In [47]:
```
lags, r = xcorr(E4,E4, maxlags=200)
plt.plot(lags, r)
plt.show()
```



In [48]:
```
lags, r = xcorr(E2,E2, maxlags=200)
plt.plot(lags, r)
plt.show()
```



In [49]:
```
lags, r = xcorr(E1,E1, maxlags=200)
plt.plot(lags, r)
plt.show()
```

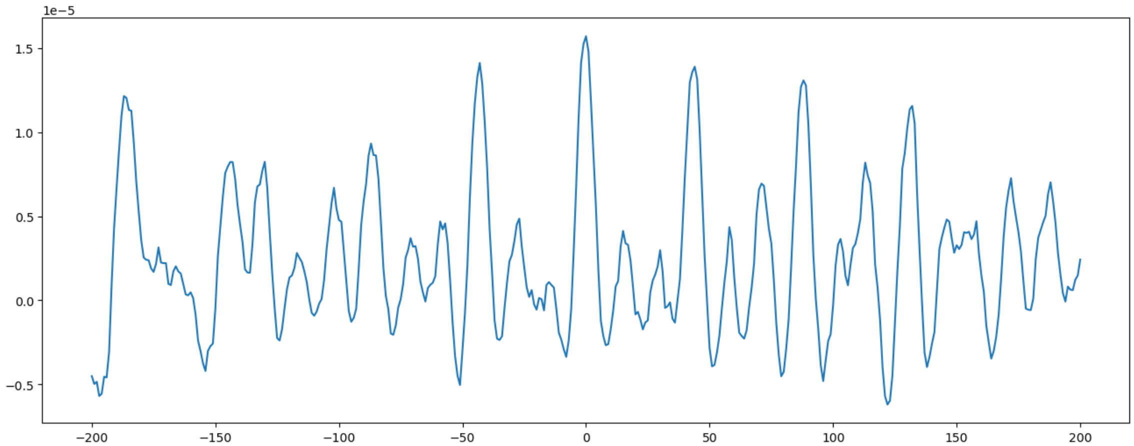


6. Now compute and plot an estimate of the cross-correlation function between the quantized speech Y and each error signal E using

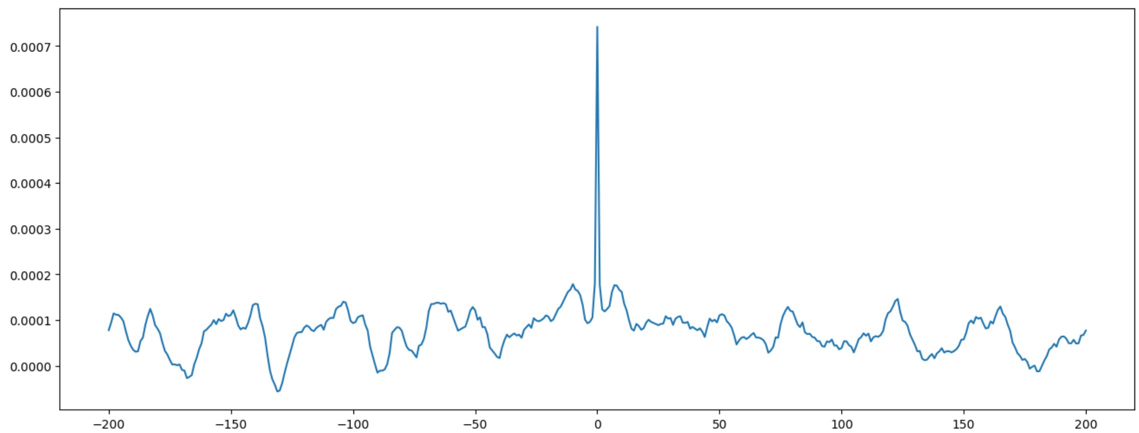
```
lags, r = xcorr(E, Y, maxlags=200)
plt.plot(lags, r)
plt.show()
```

Hint: function `xcorr` is provided in the file `helper.py`

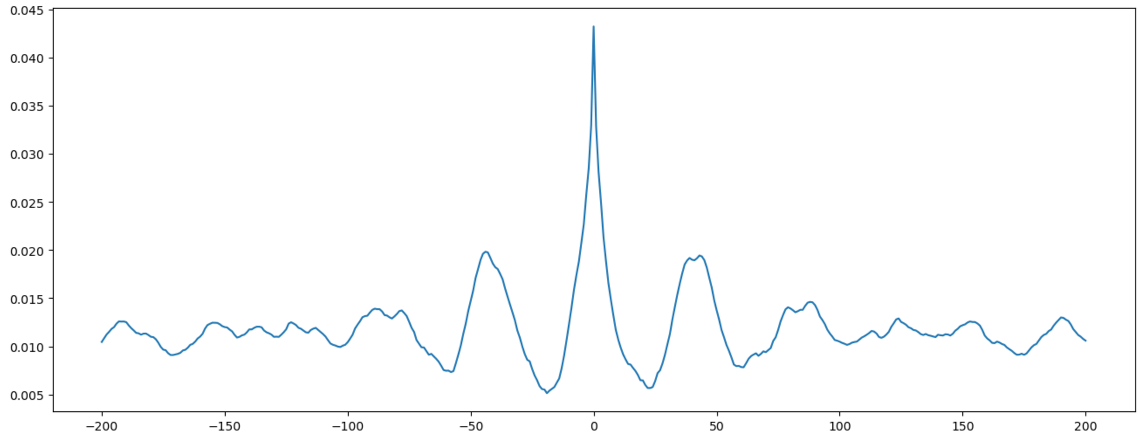
In [51]:
```
lags, r = xcorr(E7, speech7, maxlags=200)
plt.plot(lags, r)
plt.show()
```



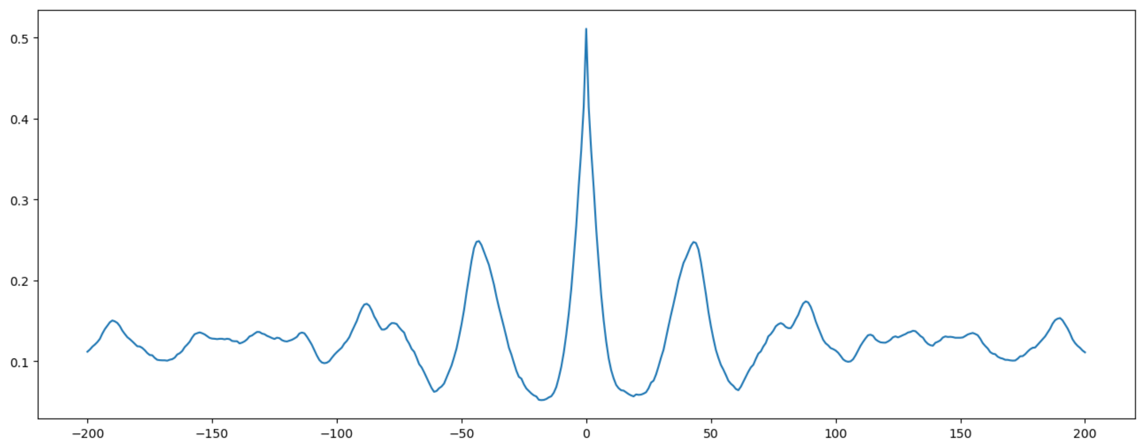
In [52]:
```
lags, r = xcorr(E4, speech4, maxlags=200)
plt.plot(lags, r)
plt.show()
```



In [53]:
```
lags, r = xcorr(E2, speech2, maxlags=200)
plt.plot(lags, r)
plt.show()
```



In [54]:
```
lags, r = xcorr(E1, speech1, maxlags=200)
plt.plot(lags, r)
plt.show()
```



7. Is the autocorrelation influenced by the number of quantization levels? Do samples in the error signal appear to be correlated with each other?

Yes. Samples in the error signal is correlated with each other.

8. Does the number of quantization levels influence the cross-correlation?

Yes, the more quantization, the weaker is the cross-correlation.

Exercise 3.6: Signal to Noise Ratio

1. Complete the function below that calculates the power of a sampled signal `x` .

In [57]:
```
def get_power(x):
    """
    Parameters
    ---
    x: the input signal

    Returns
    ---
    P: the power of the signal
    """

    P = np.sum(x[1:]**2)/len(x)
    return P
```

2. Compute the PSNR for the four quantized speech signals from the previous section.

In [76]:
```
PSNR7 = get_power(speech7)/get_power(E7)
PSNR4 = get_power(speech4)/get_power(E4)
PSNR2 = get_power(speech2)/get_power(E2)
PSNR1 = get_power(speech1)/get_power(E1)
```

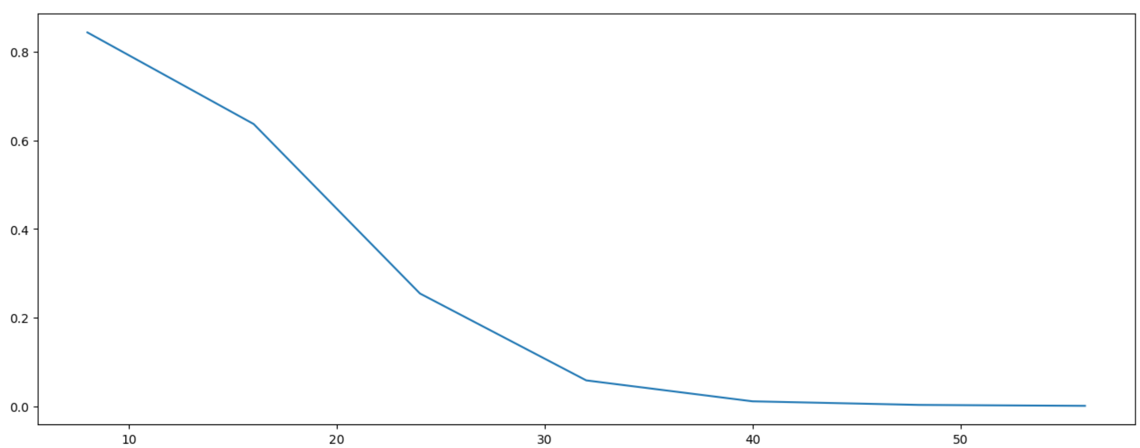
3. Assuming that the speech is sampled at 8kHz, plot the rate distortion curve using $\frac{1}{\text{PSNR}}$ as the measure of distortion. Generate this curve by computing the PSNR for $7, 6, 5, \ldots, 1$ bits/sample. Make sure the axes of the graph are in terms of *distortion* and *bit rate*.

In [77]:
```
signal_distortion = [0]*7
bit_rate = [0]*7
speech6 = Uquant(speech, 6)
speech5 = Uquant(speech, 5)
speech3 = Uquant(speech, 3)
E6 = speech6 - speech
E5 = speech5 - speech
E3 = speech3 - speech
PSNR6 = get_power(speech6)/get_power(E6)
PSNR5 = get_power(speech5)/get_power(E5)
PSNR3 = get_power(speech3)/get_power(E3)

signal_distortion[0] = 1/PSNR7
bit_rate[0] = 8 * 7
signal_distortion[1] = 1/PSNR6
bit_rate[1] = 8 * 6
signal_distortion[2] = 1/PSNR5
bit_rate[2] = 8 * 5
signal_distortion[3] = 1/PSNR4
bit_rate[3] = 8 * 4
signal_distortion[4] = 1/PSNR3
bit_rate[4] = 8 * 3
signal_distortion[5] = 1/PSNR2
bit_rate[5] = 8 * 2
signal_distortion[6] = 1/PSNR1
bit_rate[6] = 8 * 1

plt.plot(bit_rate, signal_distortion)
```

Out[77]: [<matplotlib.lines.Line2D at 0x2cc454a8880>]

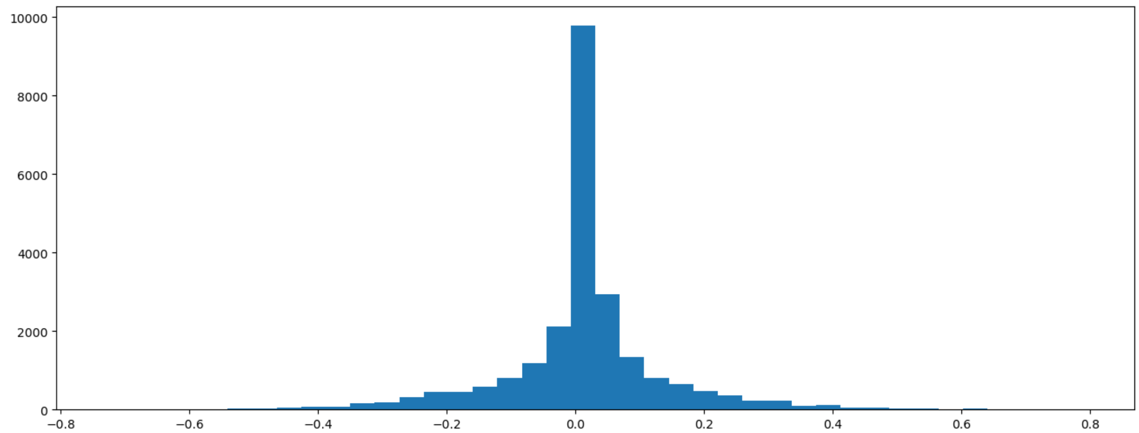


Exercise 3.8

1. First plot a 40-bin histogram of this speech signal using `plt.hist(speech, bins=40)`, and make an initial guess of the four optimal quantization levels. Print out the histogram and the initial guess of the quantization levels.

In [82]:
```
plt.hist(speech,bins=40)
print
```

Out[82]: <function print>



2. Use the function `lloyds()` to compute an optimal 4-level codebook using `speech.au` as the training set.

In [68]:
```
partition,codebook=lloyds(speech,[-0.3, -0.1, 0.1, 0.3])
print(partition)
print(codebook)
```

```
[-0.17775693 -0.02570996  0.15095067]
[-0.27381915 -0.0816947   0.03027478  0.27162657]
```

3. Once the optimal codebook is obtained, use the `codebook` and `partition` vectors to quantize the speech signal.

- This may be done with a *for* loop and *if* statements.

In [79]:
```
speech_q = np.zeros(len(speech))

for i in range(len(speech)):
    if speech[i] < -0.178:
        speech_q[i] = -0.274
    elif speech[i] < -0.026:
        speech_q[i] = -0.082
    elif speech[i] < 0.151:
        speech_q[i] = 0.03
    else:
        speech_q[i] = 0.272
```

4. Compute the error signal and PSNR.

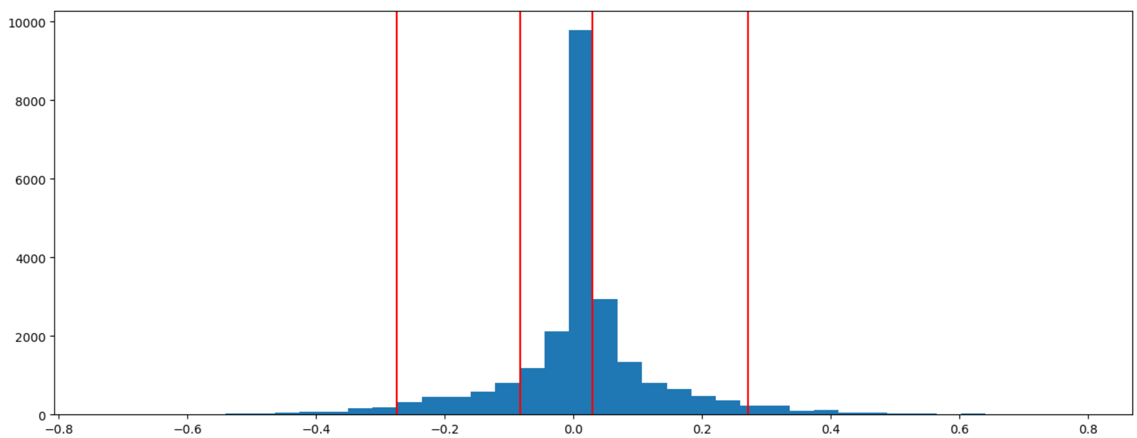
In [80]:
```
E = speech_q - speech
PSNR = get_power(speech_q)/get_power(E)
```

5. Plot the histogram in Q1 again. However, on this histogram plot, also mark where the optimal quantization levels fall along the x-axis.

- To draw a vertical line, use `plt.axvline(x=0.8, color='r')` to plot a vertical line $x = 0.8$ of red color.

In [83]:
```
plt.hist(speech,bins=40)
plt.axvline(x=-0.274, color='r')
plt.axvline(x=-0.082, color='r')
plt.axvline(x=0.03, color='r')
plt.axvline(x=0.272, color='r')
```

Out[83]: <matplotlib.lines.Line2D at 0x2cc44e3ffa0>



6. Play the quantized audio, and compare the sound quality of the uniform- and max-quantized signals.

In [85]:
```
speech4 = Uquant(speech, 4)
ipd.Audio(speech4, rate=fs)
```

Out[85]:

0:02 / 0:02

In [84]:

```
ipd.Audio(speech_q, rate=fs)
```

Out[84]:

0:02 / 0:02

7. If the speech signal was uniformly distributed, would the two quantizers be the same? Explain your answer.

Yes, the density function is uniformly distributed so qk is uniformly spaced.