PART 1:

For the first part of the project the position and orientation was considered. The following general equation sets were used to encode Matlab functions and derive the plots. The Prediction step was non linear, thus Extended Kalman Filter had to be used. The equations are as follows.

Prediction Step:

$$\bar{\mu}_t = \mu_{t-1} + \delta t * f(\mu_{t-1}, u_t, 0)$$

$$\bar{\Sigma}_t = F_t * \Sigma_{t-1} * F_t^T + V_t * Q_d * V_t^T$$

Where,

$$\dot{x} = f(x, u, n) \quad n_t \sim N(0, Q_t) \quad A_t = \frac{\partial f}{\partial x}\bigg|\mu_{t-1}, u_t, o \qquad U_t = \frac{\delta f}{\delta n}\bigg|\mu_{t-1}, u_t, o$$

$$F_t = I + \delta t * A_t \qquad V_t = U_t \qquad Q_d = Q_t * \delta t$$

Unlike the Prediction step, the Update was linear. That is why Kalman Filter was used instead of Extended Kalman Filter. The equations are as follows.

Update Step:

$$\mu_t = \bar{\mu}_t + K_t * (z_t - C_t * \bar{\mu}_t)$$

$$\Sigma_t = \bar{\Sigma}_t - K_t * C_t * \bar{\Sigma}_t$$

$$K_t = \bar{\Sigma}_t * {C_t}^T (C_t \bar{\Sigma}_t {C_t}^T + R_t)^{-1}$$

Where,

$$z_t = C_t * x_t + v_t \qquad v_t \sim N(0, R_t)$$

The state and process models were taken as follows,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} position \\ orientation \\ linear\ velocity \\ gyroscope\ bias \\ accelerometer\ bias \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} x_3 \\ G(x_2)^{-1}(\omega_m - x_4 - n_g) \\ g + R(x_2)(a_m - x_5 - n_a) \\ n_{bg} \\ n_{ba} \end{bmatrix}$$

$$R = Rotation\ Matrix_{(Euler(Z-X-Y))}(x_2) \quad Ang\ Velocity\ in\ body\ frame = G(x2) * \dot{x}_2$$

The input u_t was provided to us in the form of ω_m and a_m. Gravity was taken as a 3✕1 matrix where the third row was taken as -9.81. The process model 'x' and state model $\dot{x}$ were both 15✕1 matrices. Thenafter the Jacobians were derived, where $A_t$was 15✕15 matrix and $U_t$was a 15✕12 matrix. To find out their values, noise was considered to be zero, as the Prediction step of the Extended Kalman filter suggests. Thenafter F_t and V_t were derived. The matrix Q_t is of 12✕1. And it consists of the noise n_g, n_a, n_{bg} and n_{ba}. These noises were to be tuned for the results to match the Actual Vicon data.

For the Update step, using Kalman Filter we can estimate a value of the R_t as a 6✕6 matrix. This value was also there to be tunes. But, it does not lead upto vast differences in the plots. For the C_t matrix, it was taken as 6✕15 matrix, which when multiplied would only give the output of the Position and Orientation of the Update step. The K_t matrix comes out as a 15✕6 matrix. And finally, the z value only updates the Position and Orientation part.

For both the parts, the R_t matrix was taken as identity matrix of the said dimension and multiplied by 0.0001. And this was kept constant even for the second part as advised.

Following the n_g, n_a, n_{bg} *and* n_{ba} values were taken as

$$n_{ba} = n_g = \begin{bmatrix} 0.0000001 \\ 0.0000001 \\ 0.0000001 \end{bmatrix}$$

$$n_a = \begin{bmatrix} 0.5 \\ 0.01 \\ 0.01 \end{bmatrix} n_g = \begin{bmatrix} 0.01 \\ 0.01 \\ 0.004 \end{bmatrix}$$

The deviations in the datasets for the various datas are different. And also there is an inverse taken of the Covariance matrix estimated. Thus, the white noises determined by us also become a part of the covariance matrix. In turn it affects the Mean. Thus, different values were taken by trial and error method, for a fixed value of R_t matrix.

The plots and certain details about them are given as follows.

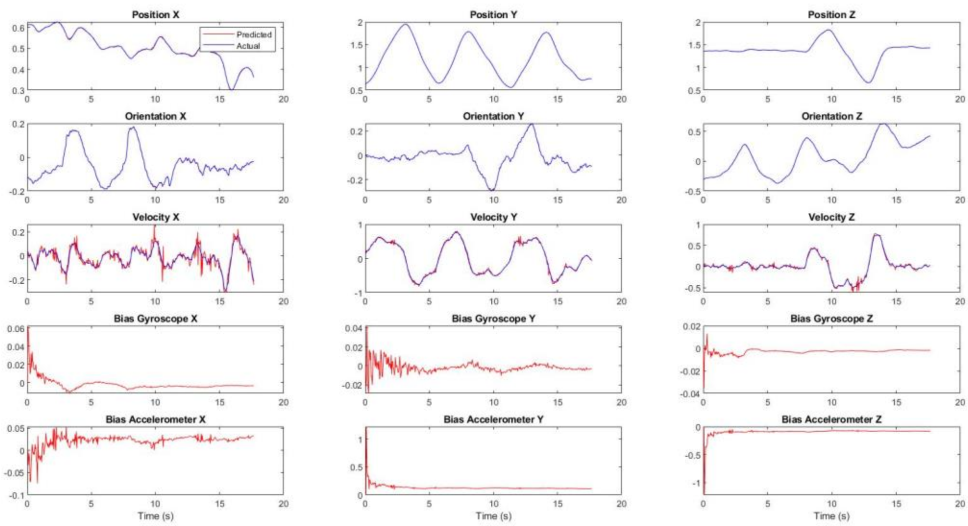

Fig.1: Plot of Model 1 for Dataset 1

For Student Dataset 1, the plots seem to be matching with the actual Vicon data. That immplies there was not much variation of noise. Moreover I tuned the R_t matrix based on Part 1. Also the biases seem to converge, so there was not much of a difficulty with Dataset 1. Though the graphs could have been better and could have had an exact match with further tuning. The Velocity of X, Y and Z does not seem so fine tuned. That is because in the C_t matrix I am not counting them in. The matrix along with R_t matrix and the input to z is designed in such a way that it can only modify the position and orientation values coming from $\bar{\mu}_t$ and $\overline{\Sigma}_t$. Thus the R_t matrix cannot have an effect on the velocity. So that is why, it looks less tuned in the plot.

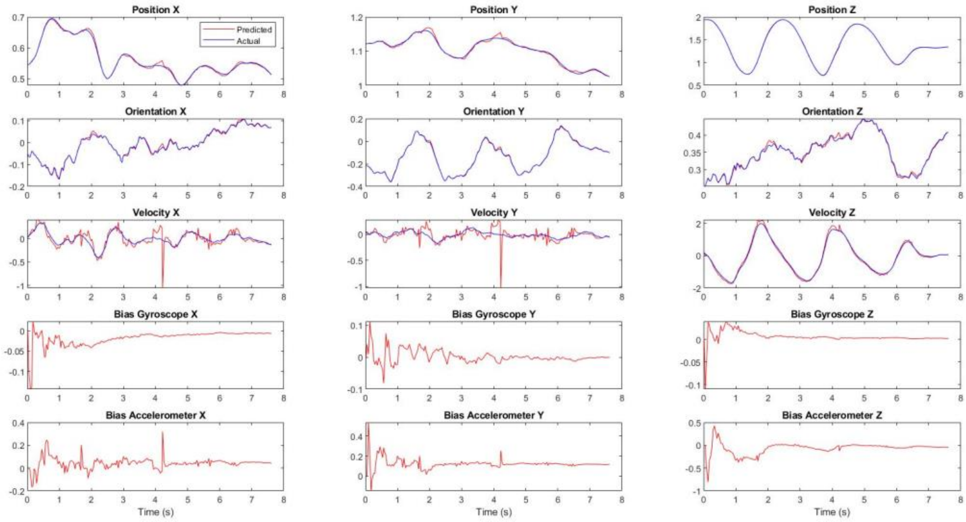

Fig.2: Plot of Model 1 for Dataset 4

For the Student Dataset 4, we have a little more diversion in the position and orientation. The biases seem to converge, so the problem should have been in the tuning. The orientation of Z value seems to go a bit off the actual value, so that could be tuned keeping the rest of the values constant. For the velocity, we have more deviation in this plot. So tuning the n_a values here can give a more accurate output. As mentioned earlier, the noise tuning from the Update step cannot affect the velocity. Thus, if the position and orientation values are close to actual, the velocity is not likely to be as close as they are. Further tuning would be required of the white noise generated in Prediction step.

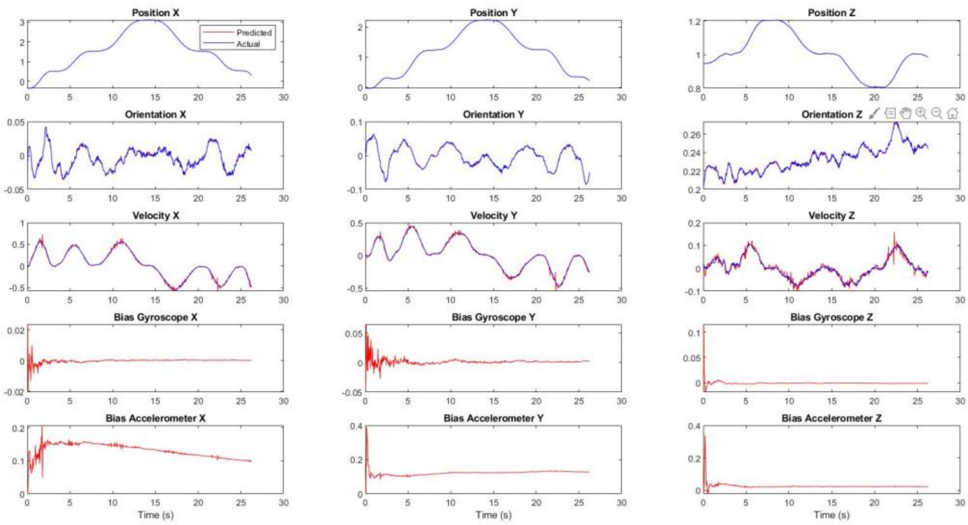

Fig.3: Plot of Model 1 for Dataset 9

For Student Dataset 9, the output seems to come out pretty fine. Not much deviation is seen for the Position and orientation parts. The biases also seem to converge. Yet, the Velocity of Z does not. Further tuning is required for this for the same reasons mentioned above. The position and orientation has little distortion in them, but they mostly match with the Vicon data.

Accounting it all, for the First part the plots were quite all right. Some distortions due to tuning were observed in Dataset 2 in Z Orientation. With better tuning of n_g and R_t it could be resolved.

PART 2:

For the second part of the project, only velocity was considered. The following general equation sets were used to encode Matlab functions and derive the plots. The Prediction step was nonlinear, thus Extended Kalman Filter was used. The equations are as follows.

Prediction Step:

$$\overline{\mu_t} = \mu_{t-1} + \delta t * f(\mu_{t-1}, u_t, 0)$$

$$\overline{\Sigma_t} = F_t * \Sigma_{t-1} * F_t^T + V_t * Q_d * V_t^T$$

Where,

$$\dot{x} = f(x, u, n) \quad n_t \sim N(0, Q_t) \quad A_t = \frac{\partial f}{\partial x}\Big|\mu_{t-1}, u_t, o \qquad U_t = \frac{\delta f}{\delta n}\Big|\,\mu_{t-1}, u_t, o$$

$$F_t = I + \delta t * A_t \qquad V_t = U_t \qquad Q_d = Q_t * \delta t$$

Unlike the Prediction step, the Update was linear. That is why Kalman Filter was used instead of Extended Kalman Filter. The equations are as follows.

Update Step:

$$\mu_t = \overline{\mu_t} + K_t * (z_t - C_t * \overline{\mu_t})$$

$$\Sigma_t = \overline{\Sigma_t} - K_t * C_t * \overline{\Sigma_t}$$

$$K_t = \overline{\Sigma_t} * {C_t}^T (C_t \overline{\Sigma_t} {C_t}^T + R_t)^{-1}$$

Where,

$$z_t = C_t * x_t + v_t \qquad v_t \sim N(0, R_t)$$

The state and process models were taken as follows,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} position \\ orientation \\ linear\ velocity \\ gyroscope\ bias \\ accelerometer\ bias \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} x_3 \\ G(x_2)^{-1}(\omega_m - x_4 - n_g) \\ g + R(x_2)(a_m - x_5 - n_a) \\ n_{bg} \\ n_{ba} \end{bmatrix}$$

$$R = Rotation\ Matrix_{(Euler(Z-X-Y))}(x_2) \quad Ang\ Velocity\ in\ body\ frame = \ G(x2) * \dot{x}_2$$

The input u_t was provided to us in the form of ω_m *and* a_m. Gravity was taken as a 3✕1 matrix where the third row was taken as -9.81. The process model 'x' and state model $\dot{x}$ were both 15✕1 matrices. Thenafter the Jacobians were derived, where A_t was 15✕15 matrix and U_t was a 15✕12 matrix. To find out their values, noise was considered to be zero, as the Prediction step of the

Extended Kalman filter suggests. Thenafter F_t and V_t were derived. The matrix Q_t is of 12X1. And it consists of the noise n_g, n_a, n_{bg} and n_{ba}. These noises were to be tuned for the results to match the Actual Vicon data.

For the Update step, using Kalman Filter we can estimate a value of the R_t as a 3X3 matrix. This value was also there to be tuned, but it was resembled as the first part, thus taken as an identity matrix and multiplied by 0.0001. But, it does not lead upto vast differences in the plots. For the C_t matrix, it was taken as 3X15 matrix, which when multiplied would only give the output of the Position and Orientation of the Update step. The K_t matrix comes out as a 15X3 matrix. And finally, the z value only updates the Position and Orientation part.

Following the n_g, n_a, n_{bg} and n_{ba} values were taken as

$$n_{ba} = n_g = \begin{bmatrix} 0.0000001 \\ 0.0000001 \\ 0.0000001 \end{bmatrix}$$

$$n_a = \begin{bmatrix} 0.5 \\ 0.01 \\ 0.01 \end{bmatrix} n_g = \begin{bmatrix} 0.01 \\ 0.01 \\ 0.004 \end{bmatrix}$$

The deviations in the datasets for the various datas are different. And also there is an inverse taken of the Covariance matrix estimated. Thus, the white noises determined by us also become a part of the covariance matrix. In turn it affects the Mean. Thus, different values were taken by trial and error method, for a fixed value of R_t matrix.

The plots and certain details about them are given as follows.

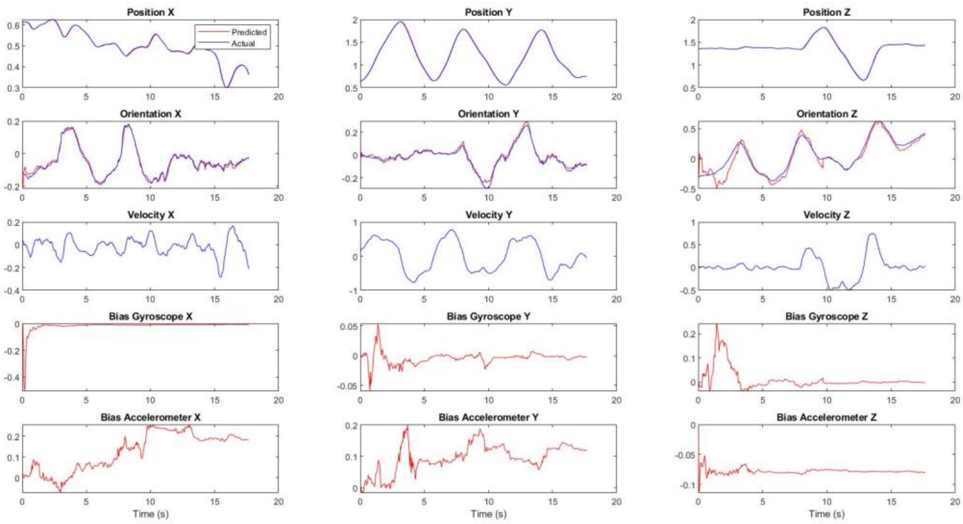

Fig.4: Plot of Model 2 for Dataset 1

For the Student dataset 1 of the Model 2, the plot is Velocity oriented. Therefore, as per the actual data, the plot of the Velocity looks fine. In fact, the position also looks good. However, it could be better tuned. In this case, the C_t matrix only lets the identified noise affect the Velocity. Thus, we can see that the Velocity is properly tuned. On the other hand, the Orientation Z is seems to be deviated compared to the Vicon Data. The bias also seems to converge by the end of it. Therefore, it is not that much a big defect. Nevertheless, the plot should readily converge. From this case, we can say that the values of the Vicon inputs are close to perfect. Thus, there is not much deviation. For the latter parts, we will see further deviations taking place due to absence of proper tuning. If estimated further tuning of n_g would be of use.

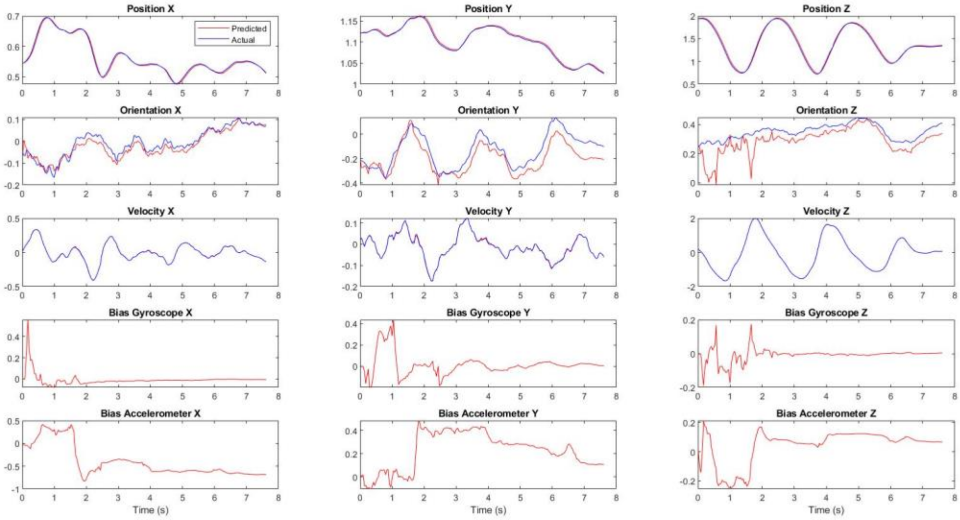

Fig.5: Plot of Model 2 for Dataset 4

For the Dataset 4 of Model 2, we can see that the predicted velocity matches pretty well with the Vicon data. On the other hand, we see drifting of the Actual data of Orientation Z from the Vicon Data. Especially in Orientation Z. In addition, the deviation has a lot of noise. Therefore, we can say it is not properly tuned. The Vicon inputs seem to be all right for the Dataset, as we can see the trend of the predicted graph matching the graph from Vicon data. The accelerometer biases converge. Therefore, there is not much of an error there. Further tuning of n_g could be useful. Changing the R_t value also gives better results. But as advised, only the C_t matrix was changed. The output of the position seems to be fine. Further tuning should reveal a better output.

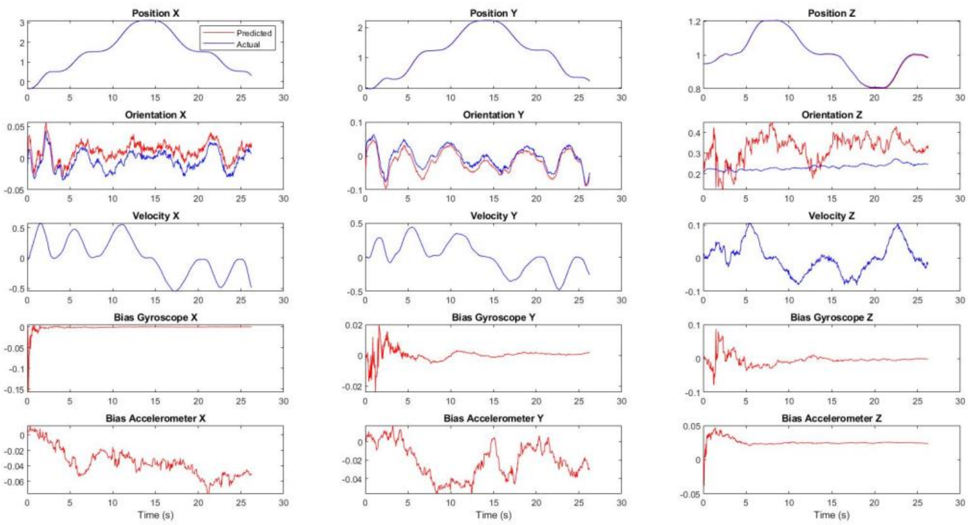

Fig.6: Plot of Model 2 for Dataset 9

The Student Dataset 9 was hard one to deal with. The Accelerometer Bias X and Y have distortions in them. However, by the end they somewhat converge. The distortions in the Orientation is huge, especially that of Z. The trends also do not quite match. This symbolizes that there was something wrong with the tuning of n_g. In addition, the R_t matrix was useless in tuning the Orientation. Changing it could have made a small difference. The velocity quite matches the plots from the Vicon Data. The position plots seem all right though. The velocities of the X and Y also seem to follow the trend but not converge with the plot from Vicon data.

Accounting it all, for the Second part for the third data set there has been some deviations and distortions found, due to lack of tuning in the orientation plots. With better tuning of n_g and R_t it could be resolved.