TRIBHUVAN UNIVERSITY
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LALITPUR, NEPAL

A
LAB REPORT
ON

Simulation of Mass Spring Damper System (Continuous System)

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SUBMITTED TO:

SIMULATION AND MODELLING

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OBJECTIVE:

1. To develop the mathematical modeling of the (continuous system) mass spring damper system.
2. To determine the state of the system i.e. x, distance moved at different points of time.

THEORY

Continuous uniformly distributed random numbers mean the set of random numbers where the probability of any number in any integral within a certain range of values is proportional to the ratio of the interval size to the range. There are many different methods to obtain random numbers.
They are:
1. Random numbers from the table.
2. From hardwired device.
3. Using pseudo Number generation.

So in our lab, we focused on the generation of random numbers using pseudo-random and Chi-square test to verify their acceptance.

Generation of pseudo-random numbers (congruence Method):
In this method, we use one initial number (r0 called seed) and a few constants. Using the seed and the formula,

$$r_{i+1} = (r_i \times a + b) \text{ (modulo } P)$$

Where ‘a’ and ‘b’ are constant and P is that number which is the upper limit of the required random number. This finds the number in the closed interval [0,P-1]

Conditions:
1. If a=1 the method is called additive.
2. If b=0 the method is called multiplicative.

3. Otherwise the method is called mixed.

Test for random number:

Frequency tests, Runs test, correlation test, Gap test, and Poker test are some of the well-known tests. Here we describe the frequency test (chi-square test). Chi-square test uses the sample statistics

$$\chi 2 = \sum_{i=1}^{N} \frac{(Oi - Ei)2}{Ei}$$

Where,

Ei = N/n is the expected number and

Where N is the Number of observations and n is the Number of Classes.

Oi = is the observed number

For a given confidence level and degree of freedom, acceptance value is found out from the table. If the calculated value is greater than the tabular value, then the null hypothesis is rejected that means the distribution of generated random number is not uniform. Else the null hypothesis is not rejected that means the distribution of the generated number is uniform.

SOURCE CODE FOR SIMULATION

Now I've used python programming language with framework numpy, scipy, and matplotlib libraries to simulate the above mathematical model to generate the different random numbers:

```
# Import Libraries
import numpy as np
from scipy.stats import chi2
import matplotlib.pyplot as plt
%matplotlib inline
```

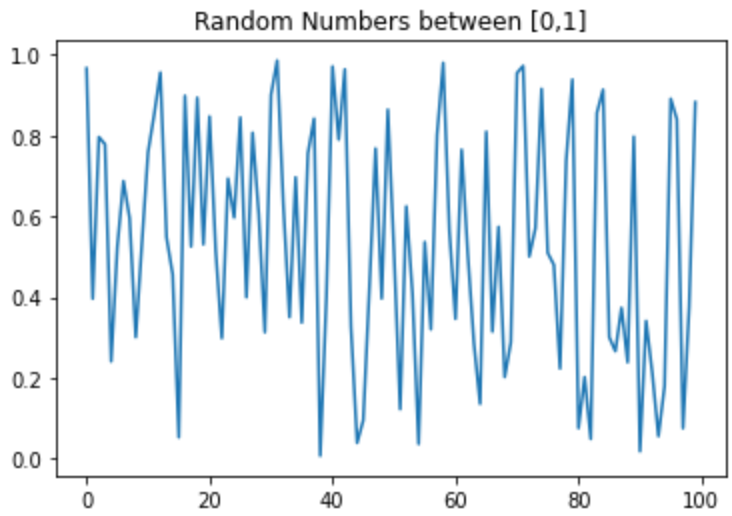
```
a = 16805
c = 0
m = 2147483647
```

```
r0 = 123457
sample_size = 100



def lcg(a, r0, c, m):
 for i in range(sample_size):
   rn = (a * r0 + c) % m
   yield rn/m
   r0 = rn
```

```
rn = np.array(list(lcg(a, r0, c, m)))
print(rn)
```

```
[0.96610509 0.39606047 0.79617841 0.77817559 0.2408524  0.52458716
 0.68721186 0.59525742 0.30089454 0.53274375 0.75869397 0.85218422
 0.95581959 0.54825685 0.45629593 0.05307338 0.89821631 0.52513499
 0.89345614 0.53048778 0.84709999 0.51534057 0.29822634 0.6935791
 0.59683692 0.84447483 0.39951245 0.80676022 0.60549319 0.31299617
 0.90062395 0.98545845 0.62929785 0.35035386 0.69653894 0.33690901
 0.75595607 0.84183323 0.00746203 0.39940321 0.970889   0.78970328
 0.96366123 0.32698836 0.03939873 0.09559201 0.42378863 0.7678903
 0.39642154 0.86399951 0.51175975 0.12261974 0.62471936 0.40880908
 0.03662817 0.53646654 0.32019052 0.80160546 0.97974203 0.56479299
 0.346252   0.76486203 0.50635439 0.28557785 0.13584108 0.80942068
 0.3145834  0.57406738 0.20233791 0.28860186 0.95429765 0.97200242
 0.50059928 0.57089649 0.91547216 0.50957928 0.47981096 0.22325126
 0.73745543 0.9384752  0.0757038  0.20232366 0.04914346 0.85587107
 0.91331754 0.30117622 0.26631203 0.373653   0.23872641 0.79732332
 0.01840769 0.34122064 0.21279714 0.05600595 0.17999946 0.89085629
 0.83993309 0.07553559 0.37559549 0.88228252]
```

```
plt.plot(rn)
plt.title("Random Numbers between [0,1]")
```




```
# Chi-Square Test
def data_to_freq(data, no_of_classes):
 class_size = 1 / no_of_classes
 count = np.zeros(shape=(no_of_classes))
  for i in range(no_of_classes):
   for no in data:
     if (class_size * i <= no < class_size * (i+1)):
       count[i] += 1

 return count
```

```
def chi_square_test(data, m, no_of_classes):

 obs_freq = data_to_freq(data, no_of_classes)
 # obs_freq = np.array([8, 8, 10, 9, 12, 8, 10, 14, 10, 11])
 exp_freq = np.full(shape= (no_of_classes), fill_value= m / no_of_classes)
 chisquare_value = (((obs_freq - exp_freq)**2) / exp_freq).sum()

 return chisquare_value
```

```
alpha = 0.05
no_of_classes = 10
dof = no_of_classes - 1
```

```
cal_chisquare = chi_square_test(rn, sample_size, no_of_classes)
critical_chisquare = chi2.isf(df= dof, q= alpha)


print(f"Calculated Chi-Square value : {cal_chisquare}")
print(f"Critical Chi-Square value : {critical_chisquare}")
if (cal_chisquare <= critical_chisquare):
 print("Null Hypothesis is not rejected i.e. The generated sequence of Random Number is distributed Uniformly.")
else:
 print("Null Hypothesis is rejected i.e. The generated sequence of Random Number isnot distributed Uniformly.")
```

```
Calculated Chi-Square value : 19.4
Critical Chi-Square value : 16.91897760462045
Null Hypothesis is rejected i.e. The generated sequence of Random Number isnot distributed Uniformly.
```

DISCUSSION AND CONCLUSION

From this lab session, we became familiar with the concept of a Random Number. We also learned to generate the pseudo-random number using the linear congruential method. We also explored the variations of linear congruential methods i.e. Additive, Multiplicative, and Mixed. We generated random numbers between [0, 1] using a python implementation of Linear-Congruential Generator. I've taken 100 samples from the generated random number and tested the frequency distribution of it using the Chi-Square test. From various experiments, I concluded that the greater the interval of classes taken, the higher is the chance of the null hypothesis not being rejected which means the generated distribution of random number is uniformly distributed.