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Source: *The Review of Economics and Statistics*, July 2013, Vol. 95, No. 3 (July 2013), pp. 776-797

Published by: The MIT Press

Stable URL: <https://www.jstor.org/stable/43554794>

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# STOCK MARKET VOLATILITY AND MACROECONOMIC FUNDAMENTALS

Robert F. Engle, Eric Ghysels, and Bumjean Sohn\*

**Abstract**—We revisit the relation between stock market volatility and macroeconomic activity using a new class of component models that distinguish short-run from long-run movements. We formulate models with the long-term component driven by inflation and industrial production growth that are in terms of pseudo out-of-sample prediction for horizons of one quarter at par or outperform more traditional time series volatility models at longer horizons. Hence, imputing economic fundamentals into volatility models pays off in terms of long-horizon forecasting. We also find that macroeconomic fundamentals play a significant role even at short horizons.

## I. Introduction

WE have made substantial progress on modeling the time variation of volatility. Unfortunately, progress has been uneven. We have a better understanding of forecasting volatility over relatively short horizons, ranging from one day ahead to several weeks. A key ingredient is volatility clustering, a feature and its wide-range implications, first explored in the seminal paper on ARCH models by Engle (1982). We also bridged the gap between discrete time models, such as the class of ARCH models, and continuous time models, such as the class of stochastic volatility (SV) models with close links to the option pricing literature.<sup>1</sup> As a by-product, we moved ahead on linking discrete time volatility prediction and option pricing. We are now much more comfortable with the notions of objective and risk-neutral probability measures and know how to empirically implement them compared to, say, fifteen years ago.<sup>2</sup>

Despite the impressive list of areas where we made measurable and lasting progress, we are still struggling with some basic issues. For example, Schwert (1989) wrote a paper with the pointed title, “Why Does Stock Market Volatility Change over Time?” Schwert tried to address the relation between stock volatility and (a) real and nominal macroeconomic volatility, (b) the level of economic activity, as well as (c)

financial leverage.<sup>3</sup> Roughly around the same time, Fama and French (1989) and Ferson and Harvey (1991) documented the empirical regularity that risk premiums are countercyclical. This finding prompted research on asset pricing models that provide rational explanations for countercyclical stock market volatility and risk premiums.

In this paper we revisit modeling the economic sources of volatility. The progress of the past fifteen years allows us to approach this question with various new insights, matured during these two decades of research on volatility. We start from the observation that volatility is not just volatility, as we have come to understand that there are different components to volatility and gains to modeling these components separately. This insight enables us also to shed new light on the link between stock market volatility and economic activity. In recent years, various authors have advocated the use of component models for volatility. Engle and Lee (1999) introduced a GARCH model with a long- and short-run component. Several others have proposed related two-factor volatility models, including Ding and Granger (1996), Gallant, Hsu, and Tauchen (1999), Alizadeh, Brandt, and Diebold (2002), Chernov et al. (2003), and Adrian and Rosenberg (2008).<sup>4</sup> While the principle of multiple components is widely accepted, there is no clear consensus on how to specify the dynamics of each of the components.

The purpose of this paper is to suggest several new component model specifications with direct links to economic activity. It is important to note, however, that our models remain reduced-form models, not directly linked to any structural model of the macroeconomy. The empirical regularity that risk premiums are countercyclical has led to a number of structural models. Examples include the time-varying risk-aversion model of Campbell and Cochrane (1999) with external habit formation; the prospect theory approach of Barberis, Huang, and Santos (2001) generates similar countercyclical variations in risk premiums. Countercyclical stock market volatility also relates to the so-called feedback effect—the effect by which asset returns and volatility are negatively correlated (see Campbell & Hentschel, 1992, among others). Along different lines, Bansal and Yaron (2004) and Tauchen (2005) argue that investors with a preference for early resolution of uncertainty require compensation, thereby inducing negative comovements between ex post returns and volatility. Some of the models on limited stock market participation, such as Basak and Cuoco (1998), are also able to generate

Received for publication January 9, 2010. Revision accepted for publication March 22, 2012.

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We are most grateful to Bill Schwert for providing us with data used in his 1989 paper and Gonzalo Rangel for help with the Spline-GARCH estimations. We thank Chris Carroll, Frank Diebold, Xavier Gabaix, Chris Jones, Andrew Karolyi, Oliver Linton, Stijn Van Nieuwerburgh, Robert Whitelaw, and seminar participants at the 2008 American Finance Association Meetings, the 2007 North American Meetings of the Econometric Society, Johns Hopkins University, London School of Economics, New York University, Ohio State University, Oxford University, University of North Carolina, and University of Pittsburgh for helpful comments.

<sup>1</sup> For surveys of the ARCH literature, see Bollerslev, Engle, and Nelson (1994). For a survey of SV models, see Garcia, Ghysels, and Renault (2010) and Shephard (2005).

<sup>2</sup> On the topic of discrete time ARCH and continuous time diffusions, see Nelson (1990), Foster and Nelson (1996), and Drost and Werker (1996), among others. The subject of option pricing and volatility prediction is covered in many papers, two of them worth mentioning: Bates (1996) and Garcia et al. (2010).

<sup>3</sup> Before Schwert, Officer (1972) related changes to volatility to macroeconomic variables, whereas many authors have documented that macroeconomic volatility is related to interest rates.

<sup>4</sup> Chernov et al. (2003) examine quite an exhaustive set of diffusion models for the stock price dynamics and conclude quite convincingly that at least two components are necessary to adequately capture the dynamics of volatility.

asymmetric stock market volatility movements. These theories are important, for they highlight the main mechanisms linking stock market volatility to macroeconomic factors.

Practically speaking, the research pursued in this paper is inspired by two recent contributions. The first is Engle and Rangel (2008), who introduce a spline-GARCH model where the daily equity volatility is a product of a slowly varying deterministic component and a mean reverting unit GARCH. Unlike conventional GARCH or stochastic volatility models, this model permits unconditional volatility to change over time. Engle and Rangel use an exponential spline as a convenient nonnegative parameterization. A second goal of their paper is also to explain why this unconditional volatility changes over time and differs across financial markets. The model is applied to equity markets for fifty countries for up to fifty years of daily data, and the macroeconomic determinants of volatility are investigated. Engle and Rangel find that volatility in macroeconomic factors, such as GDP growth, inflation, and short-term interest rates, are important explanatory variables. There is evidence that high inflation and slow growth of output are also positive determinants. This analysis draws on the cross-sectional behavior of the spline component across fifty countries.

In this paper, we focus instead on long historical time series, similar to Schwert (1989). While the spline specification could still be used, we explore a very different approach that allows us to better handle the links between stock market data, observed on a daily basis, and macroeconomic variables that are sampled monthly or quarterly. For example, Schwert (1989) aggregates daily data to monthly realized volatilities, which are then used to examine the link between stock market volatility and economic activity. If there are several components to volatility, monthly realized volatility may not be a good measure to consider. Rather, we use the long-term component. To do so, we adopt a framework that is suited to combine data that are sampled at different frequencies. The new approach is inspired by the recent work on mixed data sampling, or MIDAS. In the context of volatility, Ghysels, Santa-Clara, and Valkanov (2005) studied the traditional risk-return trade-off and used monthly data to proxy expected returns while the variance was estimated using daily squared returns.

We use a MIDAS approach to link macroeconomic variables to the long-term component of volatility. Hence, the new class of models is called GARCH-MIDAS, since it uses a mean reverting unit daily GARCH process, similar to Engle and Rangel (2008), and a MIDAS polynomial that applies to monthly, quarterly, or biannual macroeconomic or financial variables. Having introduced the GARCH-MIDAS model that allows us to extract two components of volatility, one pertaining to short-term fluctuations and the other to a long-run component, we are ready to revisit the relationship between stock market volatility and economic activity and volatility. The first specification we consider uses exclusively financial series. The GARCH component is based on daily (squared) returns, whereas the long-term component is based

on realized volatilities computed over a monthly, quarterly or biannual basis. In some sense, Schwert's original work comes closest to this specification, as the long-term component is a filtered realized volatility process, whereas Schwert uses raw realized volatilities (on a monthly basis). The GARCH-MIDAS model with a long-run component based on realized volatility will be a benchmark model against which we can measure the success of empirical specifications involving macroeconomic variables. The GARCH-MIDAS model with a long-run component based on realized volatility will also be compared to existing component models, including the spline-GARCH.

The GARCH-MIDAS model also allows us to examine directly the macrovolatility links, avoiding the two-step procedure Schwert used. Indeed, we can estimate GARCH-MIDAS models where macroeconomic variables directly enter the specification of the long-term component. The fact that the macroeconomic series are sampled at a different frequency is not an obstacle, again due to the advantages of the MIDAS weighting scheme. Hence, compared to the original work of Schwert, our approach has the following advantages: (a) we separate short- and long-run components of volatility and (b) use either a filtered realized variances or a direct approach imputing macroeconomic time series to capture the economic sources of stock market volatility. Since we handle long historical time series, our choice is limited and therefore focus on two key economic variables: inflation and industrial production growth.

The main findings of the paper can be summarized as follows. In terms of forecasting, we find that the new class of models driven by economic variables is roughly at par with time series volatility models at the quarterly horizon and at par or outperforms them at the semiannual horizon. Hence, imputing economic fundamentals—inflation and industrial production growth—into volatility models pays off in terms of long-horizon forecasting. We also find that at a daily level, industrial production and inflation account for roughly between 10% and 35% of expected one-day-ahead volatility in most of subsamples. Hence, macroeconomic fundamentals play a significant role even at short horizons. Unfortunately, all the models—purely time series ones as well as those driven by economic variables—feature structural breaks over the entire sample spanning roughly a century and a half of daily data. This is not entirely unexpected as the long span of data covers fundamental changes in the economy, although the spline-GARCH and GARCH-MIDAS models are designed to capture fundamental shifts. Our results suggest they do not fully capture this. Consequently, our analysis also focuses on subsamples: pre-World War I, the Great Depression era, and post-World War II (also split to examine the so-called Great Moderation). Our findings are robust across subsamples except the pre-World War I subsample, presumably plagued by poor measurement of inflation and industrial production.

Section II describes the new class of component models for stock market volatility, followed by sections III and IV,

which cover the empirical implementation of the new class and revisit the relationship between stock market volatility and macrovariables. Conclusions appear in section V.

## II. A New Class of Component Models for Stock Market Volatility

Different news events may have different impacts on financial markets, depending on whether they have consequences over short or long horizons. A conventional framework to analyze this is the familiar log linearization of Campbell (1991) and Campbell and Shiller (1988), which states that

$$r_{i,t} - E_{i-1,t}(r_{i,t}) = (E_{i,t} - E_{i-1,t}) \sum_{j=0}^{\infty} \rho^j \Delta d_{i+j,t} - (E_{i,t} - E_{i-1,t}) \sum_{j=1}^{\infty} \rho^j r_{i+j,t}, \quad (1)$$

where we deliberately write returns in terms of days of the month, namely,  $r_{i,t}$  is the log return on day  $i$  during month  $t$ ,  $d_{i,t}$  the log dividend on that same day, and  $E_{i,t}(\cdot)$  the conditional expectation given information at the same time. Following Engle and Rangel (2008), the left-hand side of equation (1), or unexpected returns, can be rewritten as

$$r_{i,t} - E_{i-1,t}(r_{i,t}) = \sqrt{\tau_t} \times g_{i,t} \varepsilon_{i,t}, \quad (2)$$

where volatility has at least two components:  $g_{i,t}$  which accounts for daily fluctuations that are assumed short-lived, and a long-run component  $\tau_t$ .<sup>5</sup> The main idea of equation (2) is that the same news, say, better-than-expected dividends, may have different effects depending on the state of the economy. For example, unexpected poor earnings should have an impact during expansion different from that during recession. The component  $g_{i,t}$  is assumed to relate to the day-to-day liquidity concerns and possibly other short-lived factors (Chordia, Roll, & Subrahmanyam, 2002, document quite extensively the impact of liquidity on market fluctuations). In contrast, the component  $\tau_t$  relates first and foremost to the future expected cash flows and future discount rates, and macroeconomic variables are assumed to tell us something about this source of stock market volatility.

Various component models for volatility have been considered—Engle and Lee (1999), Ding and Granger (1996), Gallant et al. (1999), Alizadeh et al. (2002), Chernov et al. (2003), and Adrian and Rosenberg (2008), among many others. The contributions of our work pertain to modeling  $\tau_t$  and are inspired by recent work on mixed data sampling, or MIDAS, discussed in a context similar to the one used here

<sup>5</sup> Note that the specification in equation (2) is slightly different from that in Engle and Rangel (2008) in that the  $\tau$  component in equation (2) is assumed constant throughout the month, quarter, or half-year, an assumption made here for convenience. Later, we will introduce a specification where this restriction is removed and the  $\tau$  component varies daily.

(volatility filtering) by Ghysels et al. (2005). Generically we will call the new class of models GARCH-MIDAS component models. The distinct feature of the new class is that the mixed data sampling allows us to link volatility directly to economic activity (data that are typically sampled at the different frequency than daily returns). Practically speaking, there will be two cases studied in this paper: (a) the component  $\tau_t$  does not change for a fixed time span and involves low-frequency financial or macroeconomic data and (b) the component  $\tau_t$  changes daily and involves rolling windows of financial data.

The easier case is the fixed window case, and it is therefore the first we cover, in section IIA. We also cover the rolling window specification in the same section. In section IIB, we cover alternative specifications involving macrovariables directly.

### A. Models with Realized Volatility

We start again with equation (2) but consider the return for day  $i$  of any arbitrary period  $t$  (which may be a month, quarter, or longer and has  $N_t$  days), which may vary with  $t$ . Since the timescale is not important for the exposition of the model, we will treat  $t$  as a month, so that  $r_{i,t}$  is the return on day  $i$  of month  $t$ . It will matter empirically which frequency to select, and one of the advantages of our approach is that  $t$  will be a choice variable that will be selected as part of the model specification. For the moment, we do not discuss this yet, and therefore we let  $t$  be fixed at the monthly frequency, but readers can keep in mind that  $t$  is a fixed window that will be determined by empirical model selection criteria. The return on day  $i$  in month  $t$  is written as (assuming for notational convenience it is not the first day of the period)

$$r_{i,t} = \mu + \sqrt{\tau_t} \times g_{i,t} \varepsilon_{i,t}, \quad \forall i = 1, \dots, N_t, \quad (3)$$

where  $\varepsilon_{i,t} | \Phi_{i-1,t} \sim N(0, 1)$  with  $\Phi_{i-1,t}$  is the information set up to day  $(i-1)$  of period  $t$ . Following Engle and Rangel (2008), we assume the volatility dynamics of the component  $g_{i,t}$  is a (daily) GARCH(1,1) process:

$$g_{i,t} = (1 - \alpha - \beta) + \alpha \frac{(r_{i-1,t} - \mu)^2}{\tau_t} + \beta g_{i-1,t}. \quad (4)$$

The first specification of the  $\tau$  component for GARCH-MIDAS builds on a long tradition going back to Merton (1980), Schwert (1989), and others of measuring long-run volatility by realized volatility over a monthly or quarterly horizon. In particular, consider monthly realized volatility, denoted  $RV_t$ . Unlike the previous work, however, we do not view the realized volatility of a single quarter or month as the measure of interest. Instead, we specify the  $\tau_t$  component



by smoothing realized volatility in the spirit of MIDAS regression and MIDAS filtering:

$$\tau_t = m + \theta \sum_{k=1}^K \varphi_k(\omega_1, \omega_2) RV_{t-k}, \quad (5)$$

$$RV_t = \sum_{i=1}^{N_t} r_{i,t}^2. \quad (6)$$

Note also that the  $\tau$  component is predetermined,

$$E_{t-1}[(r_{i,t} - \mu)^2] = \tau_t E_{t-1}(g_{i,t}) = \tau_t, \quad (7)$$

assuming the beginning-of-period expectation of the short-term component,  $E_{t-1}(g_{i,t})$ , to be equal to its unconditional expectation,  $E_{t-1}(g_{i,t}) = 1$ . To complete the model, we need to specify the weighting scheme for equation (5),

$$\varphi_k(\omega) = \begin{cases} \frac{(k/K)^{\omega_1-1} (1-k/K)^{\omega_2-1}}{\sum_{j=1}^K (j/K)^{\omega_1-1} (1-j/K)^{\omega_2-1}} \text{Beta} \\ \omega^k / (\sum_{j=1}^K \omega^j) \text{Exp. Weighted,} \end{cases} \quad (8)$$

where the weights in equation (8) sum up to 1. The weighting function or smoothing function in equation (8) is either the beta lag structure discussed in Ghysels, Sinko, and Valkanov (2006) or the commonly used exponentially weighting. The beta lag, based on the beta function, is very flexible to accommodate various lag structures. It can represent a monotonically increasing or decreasing weighting scheme. It can also represent a hump-shaped weighting scheme, although it is limited to unimodal shapes.<sup>6</sup>

Equations (3) to (6) and (8) form a GARCH-MIDAS model for time-varying conditional variance with fixed time span RVs and parameter space  $\Theta = \{\mu, \alpha, \beta, m, \theta, \omega_1, \omega_2\}$ . This first model has a few nice features. First, the number of parameters are fixed, and it is parsimonious relative to the existing component volatility models, which typically are not parsimonious. Second, since the number of parameters are fixed, we can compare various GARCH-MIDAS models with different time spans. Indeed,  $t$  can be a month, quarter, or semester. Therefore, we can vary  $t$  and profile the log-likelihood function to maximize with respect to the time span covered by  $RV$ . Moreover, the number of lags in MIDAS can vary as well, again while keeping the parameter space fixed. This is a feature that we exploit at the stage of empirical model selection. Note that we can take, say, a monthly  $RV$ , and take twelve lags, or a quarterly  $RV$  with four lags. Both involve the same daily squared returns, yet the application of the weighting scheme in equation (8) implies different weights across the year.

Another interpretation of our approach is to view the GARCH-MIDAS model as a filter. We know from work by

<sup>6</sup> See Ghysels et al. (2006) for further details regarding the various patterns one can obtain with beta lags.

Barndorff-Nielsen and Shephard and by Jacod (1994) that the monthly realized volatilities are a very noisy measure of volatility. One answer to improve precision is to use high-frequency data. However, we have on record only roughly fifteen years of such data. For longer data spans, we need to rely on filtering, and in this respect we can view equation (5) as a filter of  $RV_t$ . The estimation procedure, discussed later, will allow us to obtain appropriate weights for the volatility filter.

The filtering interpretation allows us also to explain the differences between our approach and that of Schwert (1989). Using a VAR model, Schwert (1989) investigates the causality relations between monthly  $\sqrt{RV_t}$  and volatility of PPI inflation rate, MB growth rate, and IP growth rate. To streamline the argument, let us consider a bivariate model involving  $\sqrt{RV_t}$  and a macrovariable, say,  $X_t$ . Moreover, for simplicity, let us restrict ourselves to a VAR(1),

$$\begin{aligned} \sqrt{RV_t} &= a_{rr} \sqrt{RV_{t-1}} + a_{rx} X_{t-1} + \eta_t^r, \\ X_t &= a_{xr} \sqrt{RV_{t-1}} + a_{xx} X_{t-1} + \eta_t^x, \end{aligned} \quad (9)$$

and suppose that the proper equations would in fact involve the long-run component of volatility instead, denoted as in the paper by  $\tau_t$ ,

$$\begin{aligned} \sqrt{\tau_t} &= a_{\tau\tau} \sqrt{\tau_{t-1}} + a_{\tau x} X_{t-1} + \tilde{\eta}_t^\tau, \\ X_t &= a_{x\tau} \sqrt{\tau_{t-1}} + a_{xx} X_{t-1} + \tilde{\eta}_t^x, \end{aligned} \quad (10)$$

where  $RV$  is a noisy measure of the long-run component of volatility. The econometric analysis of equations (9) versus (10) tells us that the measurement error in  $RV$  will (a) deteriorate the fit of the first equation in the system (9)—and therefore dilute the inference with respect to  $a_{rx}$ —and, more seriously, (b) bias the estimate of  $a_{xr}$  (as well as  $a_{rr}$ ). The illustrative example tells us that not filtering the  $RV$  series to extract its long-run component will adversely affect the econometric analysis of the causal patterns.<sup>7</sup>

Unfortunately, the argument of measurement error involving  $RV$  also applies to situations where we replace  $\tau_t$  by an estimate  $\hat{\tau}_t$ , such as an estimate obtained from the GARCH-MIDAS model. For this reason, we refrain from using the two-step approach consisting of measuring volatility and estimating VAR models and conducting Granger causality tests with volatility proxies. Instead, the GARCH-MIDAS model will allow us to study the interactions between stock market volatility and the macroeconomy using a single step procedure, as we will explain later.

Next we consider a rolling window specification for the MIDAS filter: we remove the restriction that  $\tau_t$  is fixed for month  $t$ , which makes  $\tau$  and  $g$  both change at the daily frequency. We do this by introducing the rolling window  $RV$  as opposed to the fixed-span  $RV$  specification.

<sup>7</sup> It is important to note that there is no direct connection between the linear VAR models in equations (9) and (10) and the models proposed in this paper. The class of models we propose circumvents the issues one faces with VAR models without there being an explicit mapping.

A GARCH-MIDAS model with rolling window RV can be defined as

$$RV_i^{(rw)} = \sum_{j=1}^{N'} r_{i-j}^2, \quad (11)$$

where we use the notation  $r_{i-j}$  to indicate that we roll back the days across various periods  $t$  without keeping track of it. When  $N' = 22$ , we call it a monthly rolling window RV, while  $N' = 65$  and  $N' = 125$ , amount to, respectively, quarterly rolling and biannual rolling window RV. Furthermore, the  $\tau$  process can be redefined accordingly:

$$\tau_i^{(rw)} = m^{(rw)} + \theta^{(rw)} \sum_{k=1}^K \varphi_k(\omega_1, \omega_2) RV_{i-k}^{(rw)}. \quad (12)$$

Finally, we drop  $t$  from equations (3) and (4) (since everything is of daily frequency now), and, together with equations (8), (11), and (12), they form the class of GARCH-MIDAS models with rolling window RV. Note that it still maintains all the nice features from GARCH-MIDAS with fixed span RV that previously mentioned.

In addition to the multiplicative specifications presented so far, we also consider a log version of the GARCH-MIDAS: for the fixed time span case, we replace equation (5) by

$$\log \tau_t = m + \theta \sum_{k=1}^K \varphi_k(\omega_1, \omega_2) RV_{t-k}, \quad (13)$$

and its rolling sample counterpart is defined similarly. We consider a log version as it matches the class of models involving macroeconomic variables introduced next.

The appeal of component models is their ability to capture complex dynamics via a parsimonious parameter structure. In general, this also brings about more complexity regarding stationarity conditions for component models. For some component models, like the restricted GARCH(2,2) model of Engle and Lee (1999), which consist of two GARCH(1,1) components, it is relatively easy to characterize nonstationarity issues of the components since the behavior of the components is well understood. For the type of component models we consider here, as well as of Engle and Rangel (2008), the issues are more involved. Ghysels and Wang (2011) study the regularity conditions such that GARCH-MIDAS model appearing in equations (3) through (5) admits covariance stationary or strictly stationary ergodic solutions, or satisfy  $\beta$ -mixing properties. The standard approach is to link the underlying model with a multivariate stochastic difference equation (see, in particular, Bougerol & Picard, 1992, and Glasserman & Yao, 1995). Although there is a vast literature on the general theory of the existence and uniqueness of strictly stationary ergodic solution, the technical challenge is how to evaluate theoretically the top Lyapunov exponent of the stochastic difference equation. Ghysels and Wang (2011) tackle this in various ways. Given the empirical nature of

this paper, we refrain here from the technical complexities involved. It should be noted, however, that these conditions do not apply to the logarithmic specification of the model appearing in equation (13). In the empirical applications, we will consider long historical time series—for which it is probably not warranted that the model is stationary. In fact, the idea behind the component models in Engle and Rangel (2008) and our paper is to address such nonstationarities typical of long historical time series. Obviously this nonstationarity has implications for the statistical inference that we conduct. The fact that we have a time-varying long-run component that is nonstationary does not preclude us from doing proper maximum likelihood inference (see Dahlhaus & Rao, 2006, for more details).

### B. Incorporating Macroeconomic Information Directly

We now turn to volatility models that directly incorporate macroeconomic time series. The class of GARCH-MIDAS models, so far, involving realized volatility, allows us to do this. The GARCH-MIDAS models discussed so far, were based on one-sided MIDAS filters, therefore yielding prediction models. In this section, we present GARCH-MIDAS models with one-sided filters, involving past macroeconomic variables. Also, for comparison, at the end of the section, we introduce two-sided filters involving macroeconomic variables.

We will consider various specifications going from specific to general. Moreover, we consider fixed span specifications and take a quarterly frequency,

$$\log \tau_t = m_l + \theta_l \sum_{k=1}^{K_l} \varphi_k(\omega_{1,l}, \omega_{2,l}) X_{l,t-k}^{mv}, \quad (14)$$

where  $X_{l,t-k}^{mv}$  is the level (hence the subscript  $l$ ) of a macrovariable 'mv'. The macroeconomic variables of interest are industrial production growth rate (IP) and producer price index inflation rate (PPI). As explained later, when we provide the details of the data configurations, by level we mean inflation and IP growth. Hence, we are dealing with two models with a single series explaining the long-run component.

Besides the level of  $mv$ , we also consider volatility—inflation and IP growth volatility—which will be measured using the approach of Schwert (1989), using innovations from autoregressive models. This yields the next two GARCH-MIDAS models featuring macroeconomic volatility,

$$\log \tau_t = m_v + \theta_v \sum_{k=1}^{K_v} \varphi_k(\omega_{1,v}, \omega_{2,v}) X_{v,t-k}^{mv}, \quad (15)$$

where  $X_{v,t-k}^{mv}$  represents the volatility that will be characterized later. Note that we use different weighting schemes for levels and volatility—hence, the subscripts  $l$  and  $v$  to the weighting scheme parameters.

We also consider a model that combines the level and volatility of each series:

$$\log \tau_t = m_{lv} + \theta_l \sum_{k=1}^{K_l} \varphi_k(\omega_{1,l}, \omega_{2,l}) X_{l,t-k}^{mv} + \theta_v \sum_{k=1}^{K_v} \varphi_k(\omega_{1,v}, \omega_{2,v}) X_{v,t-k}^{mv} \quad (16)$$

Hence, we now have two models—one for level/vol of IP growth and one for level/vol of PPI. We also estimated a general model specification that combines all four series. Such a model involves a lot more parameters, since the weighting schemes for both volatility and the level of IP and PPI differ and therefore double the parameter space. More specifically, the  $\tau$  component in this case involves thirteen parameters compared to the single-variable models in equation (14), which involve four parameters (in both cases, not counting the GARCH parameters). The results for the general model are available on request but not reported here.

In a sense, one can think of equations (14) through (16) as regression models with a latent regressand, which we are able to estimate through the maximization of the likelihood function. In particular, if we denote in equation (3) the conditional variance  $\sigma_{it}^2 = \tau_t \cdot g_{it}$ , we can write in the general case (combining all the series)

$$\log \sigma_{it}^2 = m_{lv2} + \sum_{mv=IP,PPI} \theta_{l,mv} \sum_{k=1}^{K_{l,mv}^{mv}} \varphi_k(\omega_{1,mv,l}, \omega_{2,mv,l}) X_{l,t+k}^{mv} + \sum_{mv=IP,PPI} \theta_{v,mv} \sum_{k=1}^{K_{v,mv}^{mv}} \varphi_k(\omega_{1,mv,v}, \omega_{2,mv,v}) X_{v,t+k}^{mv} + \log g_{it}$$

where the residual is  $\log g_{it}$ , that is, the GARCH(1,1) component. The comparison with regression models is not entirely accurate, however, since we do not impose orthogonality of the regressors with the residuals, that is, the orthogonality between  $g$  and  $\tau$ . Nevertheless, it is useful to think of these models as having explanatory variables.

The comparison with regressions leads us to GARCH-MIDAS models with two-sided filters, where the latter involve both past and future macroeconomic variables. This provides us with a tool to assess how much market volatility dynamics relates to both past and future macroeconomic activity. The specification we consider (taking again a quarterly frequency and a fixed span) for a single series—levels and volatility—is

$$\log \tau_t = m_2 + \sum_{k=-K_l^f}^{K_l^f} \varphi_k(\omega_1, \omega_2) \theta_l^{(k)} X_{l,t+k}^{mv} + \sum_{k=-K_v^f}^{K_v^f} \varphi_k(\omega_3, \omega_4) \theta_v^{(k)} X_{v,t+k}^{mv} \quad (17)$$

where we allow for different slope coefficients for leads and lags, namely,

$$\theta_{l/v}^{(k)} = \begin{cases} \theta_{l/v}^f & \forall k, k \geq 0; \\ \theta_{l/v}^p & \forall k, k < 0 \end{cases} \quad (18)$$

hence, the impact on volatility of past as opposed to future realizations of macroeconomic variables is allowed to differ.<sup>8</sup>

It should also be noted that the two-sided model specification in equation (17) is in the spirit of causality tests proposed by Sims (1972). Being able to examine potential causal forward-looking behavior of volatility is particularly important since stock market volatility, being countercyclical, tends to lead economic activity.<sup>9</sup>

In the remainder of the paper, we will not use the two-sided filters for the evaluation of forecasts, as this would not entail a fair forecasting exercise, but instead use them for the purpose of appraising the impact of anticipated economic movements on the stock market.

### C. Spline-GARCH Component Volatility Model

There are other two component GARCH models besides the ones proposed in this paper. The direct antecedent of GARCH-MIDAS is the spline-GARCH model of Engle and Rangel (2008), which shares features with the models we have already discussed. Many other component models have been suggested, as noted before. We stay within the class of multiplicative models, however, which means we focus exclusively on the spline-GARCH. Both the spline-GARCH and our models provide a multiplicative decomposition of conditional variance, and both specify the short-run component as a unit GARCH(1,1) process.<sup>10</sup> In fact, the specification shares equations (3) and (4). The only difference comes from the  $\tau$  specification, which is as follows,

$$\tau_t = c \exp \left( w_0 t + \sum_{k=1}^K w_k ((t - t_{k-1})_+)^2 \right), \quad (19)$$

where  $\{t_0 = 0, t_1, t_2, \dots, t_K = T\}$  denotes a partition of the time horizon  $T$  in  $(K+1)$  equally spaced intervals with the number of knots selected via the BIC criterion.<sup>11</sup> We will estimate and compare the performance of both types of models.

<sup>8</sup> Note that the filter weights are constructed via one single beta polynomial for each series across leads and lags. While this yields a lot of smoothness conditions, it has the advantage that the two-sided scheme remains parsimonious.

<sup>9</sup> See Sheppard (2003) for recent evidence regarding equity (co)variation and economic activity.

<sup>10</sup> One could possibly consider an additive GARCH-MIDAS class of models as well, but this is beyond the scope of this paper. See, however, Ghysels and Wang (2011).

<sup>11</sup> See Engle and Rangel (2008) for further details.



### III. Estimation Results

This is the first of two empirical sections. In this section, we cover the estimation of GARCH-MIDAS volatility models, looking first at models with realized volatility and then those involving macroeconomic variables.

#### A. Model Selection and Estimation of GARCH-MIDAS Models with Realized Volatility

We take the conventional approach to estimate GARCH-type models, QMLE. From Schwert's website, we obtained daily U.S. stock returns over the period from February 16, 1885 to July 2, 1962.<sup>12</sup> We extended these data up to December 31, 2010, using the CRSP value-weighted return series. Similarly, we also use macroeconomic series that start in 1884.<sup>13</sup> Details regarding the macroseries appear in section IIIB. Due to the concern of potential structural breaks, we will consider various subsamples and also formally test for breaks. The choice of subsamples follows Schwert (1989), except for the most recent subsample. We consider a split in 1984 to address the so-called Great Moderation, pertaining to the recent decline (up to the financial crisis) in macrovolatility. Kim and Nelson (1999), McConnell and Perez-Quiros (2000), Blanchard and Simon (2001), and Stock and Watson (2002) find evidence of a regime shift to lower volatility of real macroeconomic activity. Stock and Watson (2002) find the break occurred around 1984, and they conclude that the decline in volatility has occurred in employment growth, consumption growth, inflation, and sectoral output growth, as well as in GDP growth in domestic and international data. Since the financial crisis occurred at the end of our sample and did not contain sufficient observations to estimate our models separately, we conducted our analysis with series ending prior to the 2007 events, as well as the full sample ending in 2010.

There are two variations of GARCH-MIDAS models with RV: GARCH-MIDAS with (a) fixed span and (b) rolling window RV. Furthermore, for each variation, we can consider a large class of models by varying two features. The first feature is the number of years, which we will henceforth call MIDAS lag years, spanned in each MIDAS polynomial specification for  $\tau_t$ .<sup>14</sup> The second feature along which we distinguish models is the computation of RV, weighted by the MIDAS polynomial. In case of fixed-span RV,  $t$  in equation (6) can be a month, or a quarter, or a half-year. As  $t$

varies, the time span that  $\tau_t$  is fixed also changes. Similarly, for the rolling window RV, we can change  $N'$  in equation (11). Finally, in each case, we have a level and a log specification for  $\tau$ .

We start with the beta lag structure for the weights in equation (8) and the case where we model  $\tau$ . The log-likelihood function can be written as

$$LLF = -\frac{1}{2} \sum_{t=1}^T \times \left[ \log(2\pi) + \log g_t(\Phi) \tau_t(\Phi) + \frac{(r_t - \mu)^2}{g_t(\Phi) \tau_t(\Phi)} \right]. \quad (20)$$

Figure 1 displays the estimated lag weights of GARCH-MIDAS with fixed span RV for three to five MIDAS lag years. The figure shows that optimal weights decay to 0 around thirty months of lags regardless of the choice of  $t$  and length of MIDAS lag year. Plots of the likelihood function (not reported here because of space constraints) reveal that for both fixed-span RV and rolling window RV, the optimal value of the log likelihood reaches its plateau for the same MIDAS lag years. Hence, it is enough to take four MIDAS lag years to capture reasonable dynamics of  $\tau_t$  for both GARCH-MIDAS with fixed-span RV and rolling window RV. In addition, the quarterly time span appears to dominate, again in terms of the likelihood profile, and others at most MIDAS lag years. Consequently we choose quarterly time spans and four MIDAS lag years for the GARCH-MIDAS model over the full sample period. Another noteworthy feature is that the fixed-span RV and rolling window RV models level off at roughly the same value for the log-likelihood function. This indicates that holding  $\tau$  constant for some short periods (quarterly) or letting it vary every day does not make much of a difference in terms of likelihood behavior. The fact that we are able to compare these two different specifications is an attractive feature of our specification.

As noted earlier, studying long historical samples invariably raises questions about structural breaks. While we will conduct tests for structural breaks, we will also study various subsamples, assumed to be homogeneous. When we later look into the relationship between stock market volatility and macroeconomic variables, we will also look at subsamples as well as the full sample. Of course, one could argue that the GARCH-MIDAS models accommodate structural breaks via the movements in  $\tau$ . One can indeed view this as an alternative to segmentation of the sample either by eras, as in Schwert's analysis, or by testing for structural breaks.<sup>15</sup> We will turn to the issue of testing for breaks after we report estimates of the various models.

Figures 2 and 3 show the volatility components of GARCH-MIDAS with fixed span RV and rolling window

<sup>12</sup> For detailed information about this return series, see Schwert (1990).

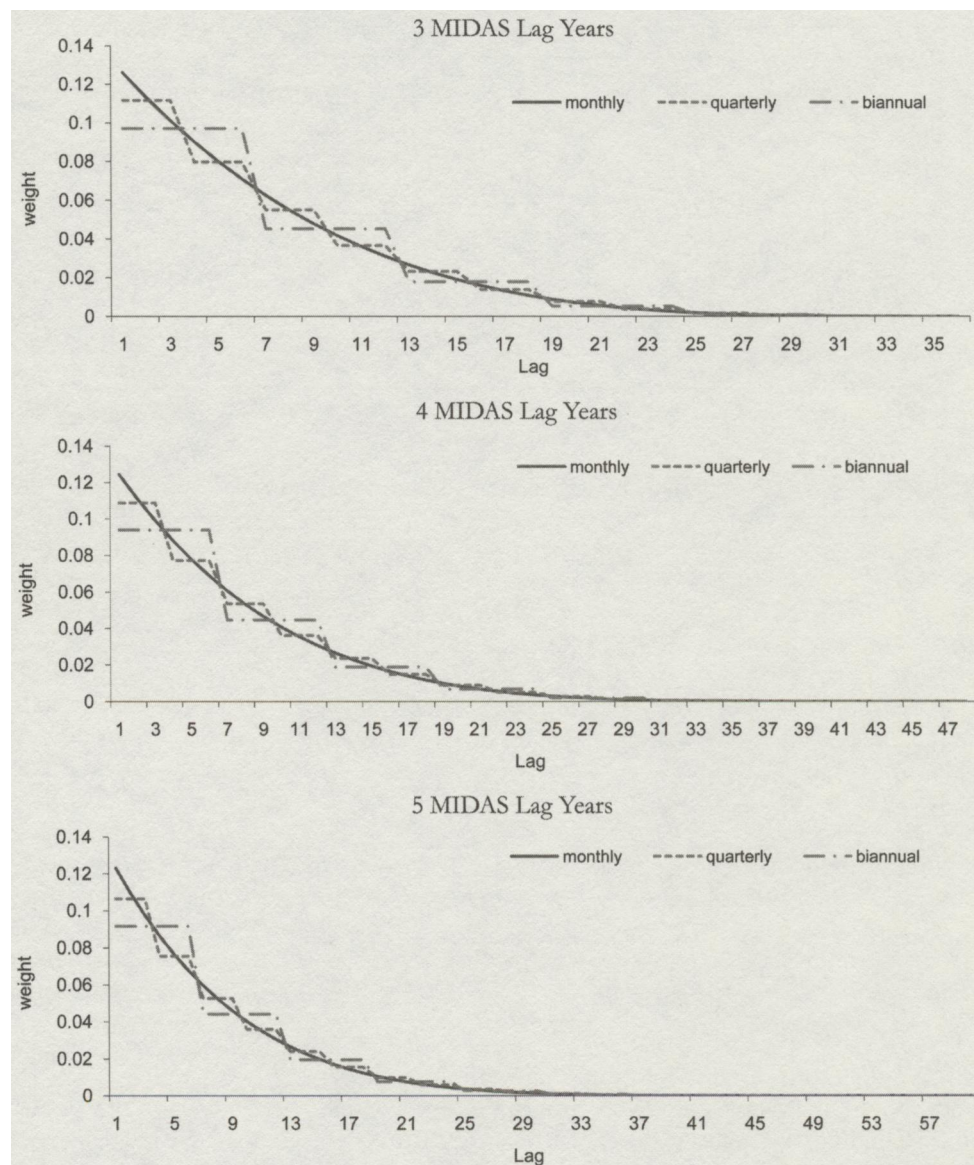
<sup>13</sup> The data estimates of macroeconomic volatility start from the third quarter of 1885.

<sup>14</sup> Note that this is not the number of lags ( $K$ ) in equation (5) or (12). For example, in the case of GARCH-MIDAS with a quarterly fixed-span RV ( $t$  is a quarter),  $\tau_t$  is fixed for each quarter and two MIDAS lag years for this model refers to eight quarters spanned by eight lagged quarterly RVs in the MIDAS filter ( $K = 8$ ). On the other hand, the GARCH-MIDAS model with a quarterly rolling window RV has a  $\tau$  component that varies on a daily basis with a window length of a quarter (65 trading days) for the rolling window of RVs. For this model, two MIDAS lag years refer to 500 trading days spanned by 500 lagged quarterly rolling window RVs in the MIDAS filter ( $K = 500$ ).

<sup>15</sup> For evidence on breaks in (a) volatility, see Lamoureux and Lastrapes (1990), Andreou and Ghysels (2002), Horvath, Kokoszka, and Zhang (2006); (b) for the shape of the option smile, see Bates (2000); and (c) for the equity premium, see Pastor and Stambaugh (2001) and Kim, Morley, and Nelson (2005).



FIGURE 1.—OPTIMAL WEIGHTING FUNCTIONS



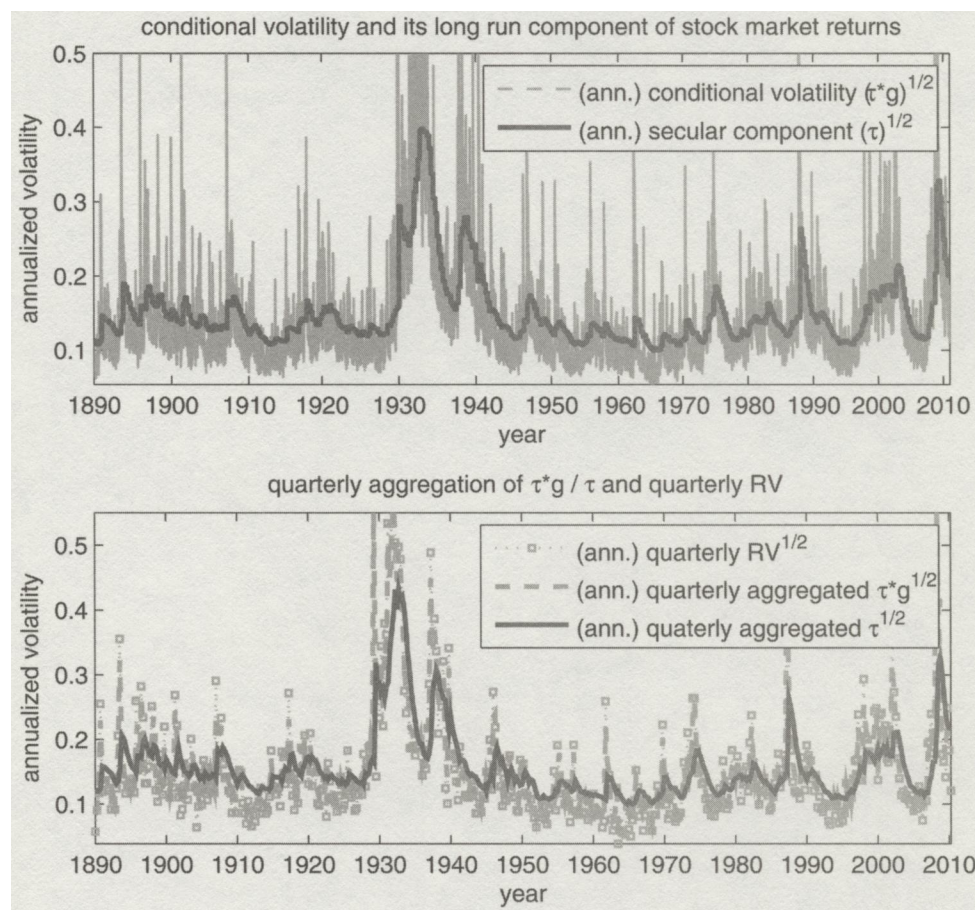
The figure shows the estimated optimal lag weights for variations of GARCH-MIDAS with fixed-span RV over the full sample period. MIDAS lag year is the number of years spanned in MIDAS regression for  $\tau$ , and it determines the number of lagged RVs in MIDAS filter. For example, GARCH-MIDAS with monthly fixed-span RV and three MIDAS lag years uses 36 lagged monthly RVs in MIDAS regression for  $\tau$ . Three choices of regressors (monthly/quarterly/biannual RV) are considered and three choices of number of lags (3, 4 and 5 MIDAS lag years). The horizontal axis of the figure is lag period in months. Hence, weights for GARCH-MIDAS with quarterly fixed-span RV show shapes of step functions. Weights for GARCH-MIDAS with quarterly fixed-span RV shown in the figure are constant for three months. The biannual case can be understood in the similar sense.

RV respectively. Since the  $\tau$  component is of quarterly frequency in figure 2 and of daily frequency in figure 3, the latter obviously looks more smooth. Table 1 provides parameter estimates for GARCH-MIDAS with quarterly fixed-span RV and quarterly rolling window RV. Although we do not report them in the table, we also explored both GARCH-MIDAS specifications with monthly and biannual RV. In some subsamples, the model with monthly RV or biannual RV offers the best fit, but the quarterly RV case always follows the best model quite closely. Therefore, to keep consistency and comparability with the full sample case, we choose models with quarterly RV throughout our analysis. The results in the table show that almost all parameters are significant. Most of all,

$\theta$  is strongly significant. Another interesting feature of the GARCH-MIDAS model appearing in the table is that sums of  $\alpha$  and  $\beta$  are 0.9697 and 0.9595 for the fixed-span RV and rolling window RV cases for the full sample, respectively. These numbers are noticeably less than 1, while in a standard GARCH model, the sum is typically 1. The same finding is also reported in Engle and Rangel (2008). All models for subsamples appearing in table 1 share the same features as the full sample case:  $\theta$  is strongly significant across all specifications in subsamples, and the sums of  $\alpha$  and  $\beta$  are noticeably smaller than 1.

We should also mention that exponential weights instead of the beta weights in equation (8) yield for all practical purposes

FIGURE 2.—GARCH-MIDAS WITH FIXED SPAN RV, 1890–2010



The two panel shows the estimated conditional volatility and its long-run component of the GARCH-MIDAS model with quarterly fixed RV and 4 MIDAS lag years of RVs (or 16 lagged quarterly RVs) in the MIDAS filter. The estimated parameters are shown in the first row of table 2. In the bottom panel, these conditional variances and long-run components are summed over quarters to show quarterly aggregated conditional variance and quarterly aggregated long-run components with quarterly RV for comparison. The plots display standard deviations on an annualized scale.

the same  $\tau$  dynamics. We therefore refrain from reporting all the results with both weighting schemes. It is reassuring, however, that the empirical findings are robust to the choice of MIDAS weights. Since both of our parameterizations involve a single parameter, one can select either one.<sup>16</sup>

To conclude we briefly turn our attention to the log  $\tau$  specification, which is reported in table 2. Overall the results are similar to the previous specification, except that we typically find lower levels of likelihoods, although the BIC criteria are extremely close.

#### B. Estimation of GARCH-MIDAS Models with Macroeconomic Variables

How much does volatility relate to the macroeconomy, and, in particular, how much does volatility anticipate the

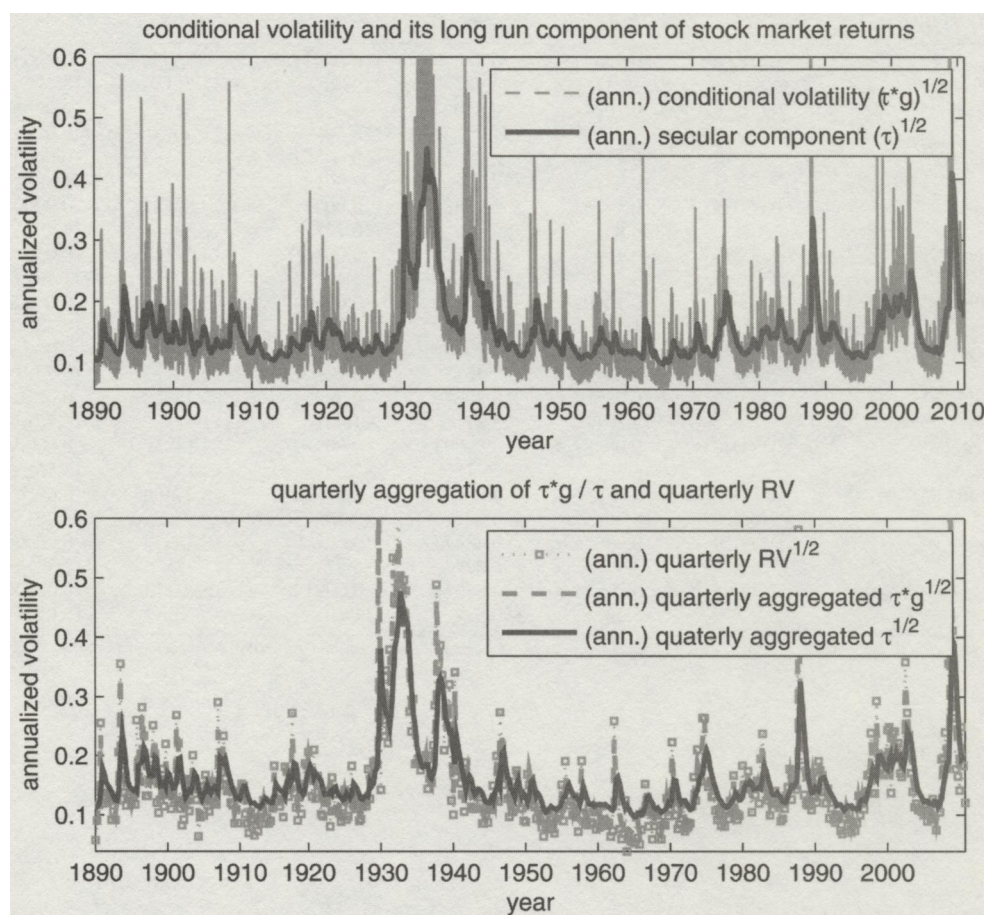
future? This is an important question we try to answer. The macroeconomic series we use are drawn from a long historical data set constructed by Schwert (1989), which we augmented with recent data. The series we use are monthly PPI (producer price index) inflation rate and IP (industrial production) growth rate. They are the same series used in Schwert (1989) to see the link between stock market volatility and macroeconomic volatility. Compared to Schwert (1989), we do not include the monetary base since the models we estimated with it yielded results very similar to the models with inflation. We also did not use interest data as we wanted to use exclusively real economy as opposed to financial series.

Schwert (1989) investigates the relationship between monthly stock market volatility and monthly macroeconomic variables. We decided to stay with a quarterly frequency since the log-likelihood profile of GARCH-MIDAS models with fixed-span RV suggested that the quarterly frequency offers both good fit and stability. Hence, we construct quarterly macroeconomic series from the monthly data using a geometric mean of the monthly growth rates. Table 3 provides the summary statistics of the quarterly macroeconomic series.

<sup>16</sup>Note that the original specification for beta lag structure shown in equation (8) involves two parameters. However, for both GARCH-MIDAS models with RV, optimal  $\omega_1$  is always 1 such that the weights are monotonically decreasing over the lags. Hence, for the GARCH-MIDAS models with RV, we set  $\omega_1 = 1$ , which makes the resulting beta lag structure involve a single parameter.



FIGURE 3.—GARCH-MIDAS WITH ROLLING WINDOW RV, 1890–2010



The top panel shows the estimated conditional volatility and its long-run component of GARCH-MIDAS model with quarterly rolling window RV and four MIDAS lag years of RVs in the MIDAS filter. They are all shown in standard deviation and annualized scale. The estimated parameters are shown in the second row of table 2. In the bottom panel, these conditional variance and long-run components are summed over quarters to show quarterly aggregated conditional variance and quarterly aggregated long-run components with quarterly RVs for comparison. As in the top panel, these are shown in standard deviation and annualized scale.

In addition to the levels of quarterly macroeconomic data, we are also interested in linking stock market volatility to the volatility of these quarterly macroeconomic series. In order to estimate volatility of quarterly macroeconomic series, we follow the approach taken by Schwert (1989).<sup>17</sup> We fit the following autoregressive model with four quarterly dummy variables  $D_{jt}$  to estimate quarterly macroeconomic volatility. In particular,  $(\hat{\varepsilon}_t)^2$  from the following regression is used to estimate quarterly macroeconomic volatility (for any macrovariable  $X$ ):

$$X_t = \sum_{j=1}^4 \alpha_j D_{jt} + \sum_{i=1}^4 \beta_i X_{t-i} + \varepsilon_t. \quad (21)$$

To appreciate the time series pattern of the series that enter our model specification, we provide plots of the macroeconomic series in figures 4 and 5. The former shows macroeconomic level variables, whereas the latter shows macroeconomic

<sup>17</sup> We also used a GARCH(1,1) specification to model quarterly volatility of macroeconomic variables and found similar results. Details are available on request.

volatility variables used in the GARCH-MIDAS specification. We have referred to the issue of structural breaks. Figures 4 and 5 clearly reveal why this is a concern. As far as the levels go, we note remarkable changes across time, something already noted, for instance, by Romer (1986). Romer also points out that these changes are in part due to data quality. Macroeconomic series were not very well measured in the early parts of our sample. In a sense, our paper is dealing with noisiness of volatility measures, but not with noisiness in macroeconomic series—an issue much harder to deal with as it largely relates to data collection. In figure 5 we turn our attention to the volatility of the macroeconomic series, as computed in equation (21). Recall that we mentioned the recent work on the Great Moderation. Clearly we see that the volatility of IP has been dramatically reduced as part of the Great Moderation, followed at the end of the sample by the financial crisis with increased volatility. The choice of our subsamples will partly deal with the issue of breaks that are clearly present in the macroeconomic series. In the next section, we also look more explicitly at testing for structural breaks.



TABLE 1.—PARAMETER ESTIMATES FOR GARCH-MIDAS WITH REALIZED VARIANCE

Sample	MIDAS Regressor	$\mu$	$\alpha$	$\beta$	$\theta$	$\omega$	$m$	LLF/BIC
1890–2010	Fixed RV	0.00058 (13.53)	0.10471 (13.71)	0.86499 (79.94)	0.00911 (19.90)	4.70059 (2.75)	0.00003 (14.28)	111,624.6 –6.7286
	Rolling RV	0.00058 (2.79)	0.10944 (7.76)	0.85004 (51.61)	0.01054 (28.54)	6.62953 (11.40)	0.00003 (10.01)	111,628.9 –6.7288
1890–1919	Fixed RV	0.00054 (7.26)	0.15368 (9.57)	0.78035 (39.35)	0.00462 (–7.96)	236.68483 (30.32)	0.00005 (14.58)	30,603.5 –6.9161
	Rolling RV	0.00052 (7.12)	0.15732 (13.86)	0.75886 (34.45)	0.00741 (10.77)	29.82684 (1.59)	0.00004 (12.49)	30,612.4 –6.9181
1920–1952	Fixed RV	0.00076 (9.59)	0.10643 (8.74)	0.84806 (53.67)	0.00901 (–13.43)	14.61377 (49.00)	0.00004 (11.40)	31,251.3 –6.4200
	Rolling RV	0.00076 (9.26)	0.10499 (11.15)	0.85280 (62.16)	0.01102 (15.10)	3.73909 (3.44)	0.00003 (7.30)	31,270.8 –6.4240
1953–2010	Fixed RV	0.00052 (7.36)	0.08508 (6.93)	0.89882 (52.40)	0.00950 (6.27)	3.07853 (0.83)	0.00004 (6.77)	49,800.6 –6.8176
	Rolling RV	0.00052 (5.07)	0.08553 (6.30)	0.89679 (25.81)	0.00953 (11.90)	2.93222 (111.67)	0.00004 (0.88)	49,799.6 –6.8175
1953–1984	Fixed RV	0.00047 (5.30)	0.08482 (6.30)	0.89993 (43.07)	0.00720 (–3.09)	4.15859 (2.15)	0.00004 (3.56)	28,571.6 –7.0980
	Rolling RV	0.00048 (1.06)	0.09256 (6.20)	0.88041 (33.07)	0.01094 (3.58)	6.24408 (36021.77)	0.00002 (843.32)	28,576.6 –7.0992
1985–2010	Fixed RV	0.00064 (6.38)	0.09056 (4.59)	0.88175 (35.62)	0.00746 (6.57)	10.27918 (1.52)	0.00006 (7.32)	21,242.4 –6.4703
	Rolling RV	0.00065 (6.45)	0.09624 (5.32)	0.85684 (36.68)	0.00918 (11.70)	16.42114 (1.70)	0.00004 (6.97)	21,241.5 –6.4700

GARCH-MIDAS models with various specifications are fitted using QMLE. The model specification has different interpretations for GARCH-MIDAS model with fixed-span RV and the one with rolling window RV. The former is:

$$r_{i,t} = \mu + \sqrt{\tau_t} \cdot g_{i,t} \varepsilon_{i,t}, \quad \forall i = 1, \dots, N_t,$$

$$g_{i,t} = (1 - \alpha - \beta) + \alpha \frac{(r_{i-1,t} - \mu)^2}{\tau_t} + \beta g_{i-1,t},$$

$$\tau_t = m + \theta \sum_{k=1}^K \varphi_k(\omega_1, \omega_2) RV_{t-k} \quad \text{where} \quad RV_t = \sum_{i=1}^{N_t} r_{i,t}^2,$$

and the rolling window RV has

$$\tau_t^{(rw)} = m^{(rw)} + \theta^{(rw)} \sum_{k=1}^K \varphi_k(\omega_1, \omega_2) RV_{t-k}^{(rw)} \quad \text{where} \quad RV_t^{(rw)} = \sum_{j=1}^{N'} r_{t-j}^2.$$

The Qtr/4yr model with fixed span RV sets its long-run component  $\tau$  fixed at a quarterly frequency and uses sixteen lagged quarterly RVs (RVs spanning past four years) to model the  $\tau$  filter. In contrast the GARCH-MIDAS model with rolling window RV uses quarterly rolling window RVs, that is, sum of 65 (approximate number of days in a quarter) squared daily returns that cover the past four years to model the  $\tau$ . For various sample choices and GARCH-MIDAS with (fixed span/rolling RV), the specification of Qtr/4yr is commonly taken. The  $\omega$  in the table is  $\omega_2$  as the optimal  $\omega_1$  is 1; such that the optimal weights are monotonically decreasing over the lags. The numbers in parentheses are robust  $t$ -statistic computed with HAC standard errors. LLF is the optimal log-likelihood function value, and BIC is the Bayesian information criterion.

We start with the specifications involving the single series, PPI and IP, for either the level or variance. We focus first on the one-sided filters. The parameter estimates appear in table 4 for PPI and table 5 for IP. In each case we took four years of lags, or sixteen lags.

The most interesting parameters are the slope parameters  $\theta_{l/v}$  for level/volatility (l/v) specifications of the MIDAS filter. Consider first the parameter estimates of  $\theta_l$  for the PPI series. They range from 0.2264 in the 1920–1952 sample to 1.0962 for the 1953–1984 sample. Hence, in all cases, the parameters are positive—and in all but one case (the post-1985 sample), they are statistically significant. This means that more inflation leads to high stock market volatility. For the full sample, the parameter estimate is 0.2696 with a  $t$ -statistic of 6.35. Since the weighting function with  $\omega_1 = 20.83$  and  $\omega_2 = 4.23$  puts 0.1193 on the first lag and 0.3120 (which is the maximum of the weights) on the second lag of PPI level, we find that a 1% increase of inflation during the current quarter would increase the next quarter market volatility by  $e^{0.27 \cdot 0.1193/3} - 1 \approx 0.0108$ , or 1.08%. For example, the

average of the annualized  $\tau$  in the full sample is 0.0324 (or, equivalently, the average of the annualized  $\sqrt{\tau}$  is 18%). If the current quarter inflation increases by 1 percentage point (say, from 3% to 4%), the next quarter market volatility would rise by 1.08% (say, from 0.0324 to 0.0328 in annualized  $\tau$  or from 18% to 18.2% in annualized  $\sqrt{\tau}$ ). Similarly, if the previous quarter's inflation increased by 1 percentage point, we would see  $e^{0.27 \cdot 0.3120/3} - 1 \approx 0.0285$ , or 2.85%, increase in market volatility next quarter. Hence, the cumulative effect after two quarters of a 1% inflation increase would raise stock market volatility from 18% to 18.5%. For the 1953–1984 sample, the optimal weighting function is characterized by  $\omega_1 = 7.40$  and  $\omega_2 = 2.67$  and puts 0.0640 on the first lag and 0.1726 (the maximum weight) on the fourth lag. In this case, a 1 percentage point increase in current quarter inflation would lead to  $e^{1.10 \cdot 0.0640/3} - 1 \approx 0.0237$ , or 2.37% increase in market volatility next quarter. With the similar computations, we would see 6.53% increase in market volatility at the current quarter when there was 1 percentage point increase in inflation a year ago. This sample, of course, covers

TABLE 2.—PARAMETER ESTIMATES FOR GARCH-MIDAS WITH REALIZED VARIANCE—LOG SPECIFICATION

Sample	MIDAS Regressor	$\mu$	$\alpha$	$\beta$	$\theta$	$\omega$	$m$	LLF/BIC
1890–2010	Fixed RV	0.00057 (12.82)	0.10108 (13.88)	0.87731 (91.10)	48.58598 (5.74)	3.20609 (7.19)	−9.63991 (−123.42)	111,599.6 −6.7271
	Rolling RV	0.00057 (12.60)	0.10108 (13.61)	0.87726 (95.77)	53.32615 (15.44)	2.18552 (3.10)	−9.64472 (−111.30)	111,596.1 −6.7269
1890–1919	Fixed RV	0.00054 (7.60)	0.15233 (9.32)	0.78688 (36.43)	46.68378 (5.19)	35.41077 (1.37)	−9.79615 (−92.38)	30,599.8 −6.9153
	Rolling RV	0.00053 (7.67)	0.15472 (10.23)	0.77506 (37.59)	74.41938 (6.10)	18.66361 (2.53)	−9.91757 (−90.69)	30,606.2 −6.9167
1920–1952	Fixed RV	0.00075 (9.47)	0.10398 (10.87)	0.86611 (70.43)	43.89592 (13.47)	3.90938 (4.15)	−9.58621 (−72.64)	31,260.1 −6.4218
	Rolling RV	0.00075 (9.50)	0.10383 (10.33)	0.86633 (69.17)	49.23552 (12.60)	2.55245 (4.48)	−9.57460 (−72.56)	31,259.5 −6.4217
1953–2010	Fixed RV	0.00052 (7.55)	0.08040 (7.46)	0.90928 (80.74)	82.20873 (3.81)	0.89499 (2.70)	−9.72069 (−42.57)	49,796.3 −6.8170
	Rolling RV	0.00052 (7.55)	0.08021 (7.47)	0.90944 (80.16)	59.56938 (3.08)	1.12917 (2.20)	−9.61780 (−43.85)	49,792.4 −6.8165
1953–1984	Fixed RV	0.00047 (5.61)	0.08361 (7.63)	0.90223 (53.76)	102.03849 (1.39)	3.60318 (2.50)	−10.01656 (−23.28)	28,571.6 −7.0980
	Rolling RV	0.00047 (5.81)	0.08691 (7.99)	0.89428 (60.77)	130.72881 (4.31)	4.30043 (2.96)	−10.19076 (−42.30)	28,574.2 −7.0986
1985–2010	Fixed RV	0.00063 (5.26)	0.08426 (4.20)	0.90090 (42.60)	39.38826 (7.42)	2.32297 (1.36)	−9.32029 (−37.33)	21,237.5 −6.4688
	Rolling RV	0.00063 (5.60)	0.08344 (4.09)	0.90233 (40.98)	34.78767 (2.37)	1.68767 (1.30)	−9.28606 (−34.74)	21,236.3 −6.4684

The specifications are the same as in table 1, with the difference that the long-run component  $\tau$  is specified in terms of logs as in equation (13). The numbers in parentheses are robust  $t$ -statistic computed with HAC standard errors. LLF is the optimal log-likelihood function value, and BIC is the Bayesian information and criterion.

TABLE 3.—SUMMARY STATISTICS FOR U.S. DAILY STOCK RETURNS AND QUARTERLY MACROECONOMIC-LEVEL VARIABLES

Sample	Variable	Mean	SD	Skewness	Kurtosis	Variable	Mean	SD	Skewness	Kurtosis
1890–2010	Daily stock returns	0.00033	0.01048	−0.11	20.63	PPI	0.00185	0.01029	−1.21	15.70
	IP	0.00318	0.01759	0.14	13.15					
1884–1919	Daily stock returns	0.00027	0.00846	−0.32	9.99	PPI	0.00198	0.01161	0.71	3.87
	IP	0.00461	0.02067	0.33	7.44					
1920–1952	Daily stock returns	0.00038	0.01315	0.29	16.24	PPI	0.00025	0.01395	−1.84	13.77
	IP	0.00314	0.02453	−0.10	8.31					
1953–2010	Daily stock returns	0.00034	0.00975	−0.66	25.62	PPI	0.00269	0.00606	−1.52	21.50
	IP	0.00233	0.00685	−1.11	5.95					
1953–1984	Daily stock returns	0.00031	0.00774	0.06	7.03	PPI	0.00329	0.00474	1.74	7.14
	IP	0.00280	0.00814	−1.04	4.85					
1985–2010	Daily stock returns	0.00038	0.01176	−0.87	25.38	PPI	0.00195	0.00733	−2.34	19.93
	IP	0.00174	0.00479	−1.68	7.57					

Daily U.S. stock return series from 1885 to 2010 were constructed from William Schwert's website and the CRSP data set. The macroeconomic variables are producer price index inflation rate (PPI) and industrial production growth rate (IP). Quarterly macroeconomic rates are obtained by taking geometric means of monthly rates.

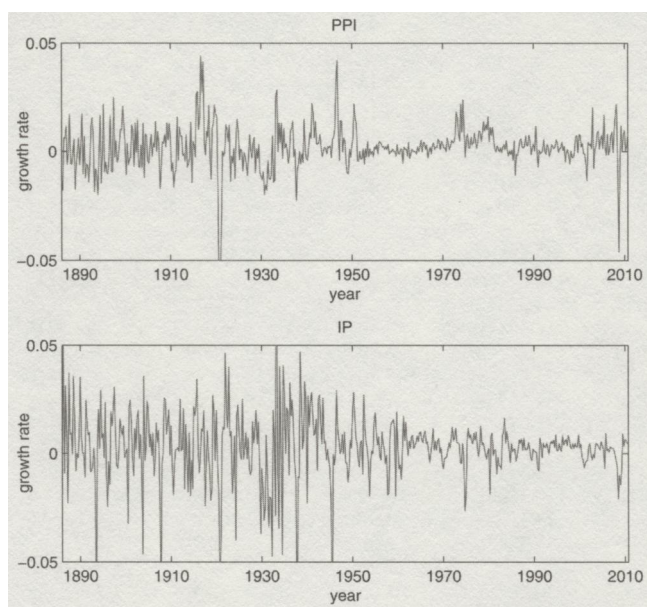
the Volcker and Greenspan years with very little inflation. The lower panel of table 4 reports the impact of inflation uncertainty on stock market volatility. For the full sample, the impact is insignificant, and looking at the subsamples, we observe that the evidence is quite mixed. Notably, during the Great Depression era and the post-1985 samples, the impact is insignificant, while it is strongly significant and positive during the 1890–1919 and 1953–1984 subsamples. We note again the large parameter estimates for the 1953–1984 sample. It is interesting to note that in terms of economic magnitude, the impact of inflation uncertainty is about the same as the impact of the actual inflation level.

Next we turn to table 5, which covers IP. The parameter estimates of  $\theta_i$  range from  $-0.9385$  to  $-0.0966$ . Hence, increases in industrial production decrease volatility—the well-known countercyclical pattern notably reported in Officer (1972) and Schwert (1989). The effect is statistically

significant, although for the full sample and Great Depression era only marginal. For the full sample, the parameter estimate is  $-0.30$ , implying that a 1% increase of IP at the current quarter would decrease the next quarter stock market volatility by 0.1 percentage points. For the post-World War II era, those impacts are larger—roughly 0.6% for the 1953–2010 sample. From the lower panel of table 5 we note that IP volatility has a significant positive impact on stock market volatility: business cycle uncertainty matters.<sup>18</sup>

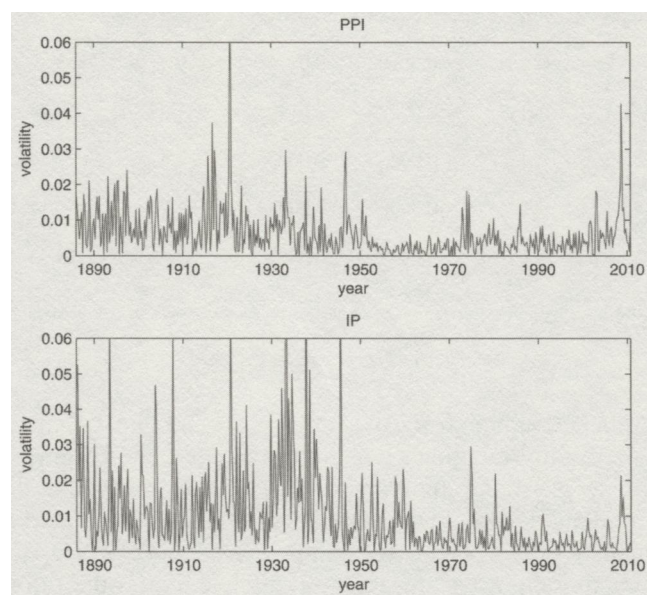
<sup>18</sup> Some of the estimates reported in table 5 suffer from an undesirable estimation problem. Namely, we put an upper bound on the MIDAS beta polynomial parameters that is equal to 300 as values above that tend to create numerical instability. Most of these issues appear in sub samples and may possibly be viewed as a small sample problem. The boundary issue has some econometric implications, but they do not affect the main conclusions of our analysis.

FIGURE 4.—QUARTERLY MACROECONOMIC LEVEL VARIABLES, 1886–2010



These figures show macroeconomic-level variables used in the GARCH-MIDAS with macroeconomic variables as specified in equation (14). PPI and IP represent producer price index inflation rate and industrial production growth rate, respectively. The original data set consists of monthly series of these variables. For PPI and IP, we obtained quarterly series by taking geometric means of three months, that is, a quarter, of these series.

FIGURE 5.—QUARTERLY MACROECONOMIC VOLATILITY, 1886–2010



These figures show macroeconomic volatility variables used in the GARCH-MIDAS with macroeconomic variables as specified in equation (15). PPI and IP represent producer price index inflation rate and industrial production growth rate, respectively. For these quarterly macroeconomic series, we adopt a variant of Schwert's (1989) approach, as in equation (21), to measure macroeconomic volatility, which is shown in these figures. Note that the GARCH-MIDAS with macroeconomic variables as in equation (15) uses macroeconomic variance as an input, and it is a squared term of volatility.

The parameter estimates of the model that combines the level and volatility of each series, namely, models described by equation (16), appear in table 6. In all cases, we observe that the point estimates are quite similar to those obtained with each single series. Yet the standard errors have increased

in the joint empirical model, and most of the measured impacts are no longer statistically significant. This suggests that there is either evidence of colinearity among the series, or that the volatility models are overparameterized and difficult to identify.

To conclude, we also cover the two-sided specifications described by equation (17). The parameter estimates appear in table 7. The top panel pertains to PPI inflation.<sup>19</sup> With some minor exceptions we find that more inflation—past and future—and more inflation volatility—again, past and future—increase stock market volatility. This effect appears significant in the first subsample, where  $\theta_{l/v}^b$  is significant, after World War II, where anticipated future inflation and past and future inflation volatility enter significantly. However, in the subsample pertaining to the post-1984 period, we find the wrong sign for the effect of inflation on volatility: a negative (albeit insignificant) sign for  $\theta_l^b$ . This subsample is a period of relatively low inflation in addition to the stock market crash of 1987, and it is also a relatively small sample—smaller than the one-sided models reported in the prior tables. The two combined—the crash unrelated to fundamentals and a short sample—indeed appear to produce anomalous results. We also should note that  $\theta_l^f$  with the two-sided filters in some cases yields parameter estimates that take on very large values. This result emerges because the forward-looking part of the two-sided filter weighting scheme is very small in all such cases. Hence, the product of  $\theta_l^f$  and the sum of the filter weights are actually small. Fortunately, we find this in only a few cases, mostly occurring with the PPI series. It is also worth noting that the parameters  $\theta_v^{b/f}$  in the 1953–2010 subsample are highly significant, while they are not in the two subsamples. One should note, however, that the MIDAS filter weights are very different across the subsamples.

The second panel of table 7 confirms, with two-sided filters, the countercyclical nature of stock market volatility, as parameter estimates of  $\theta_l^b$  are negative. Future (anticipated) IP has a more ambiguous sign, but when it is significant, it is clearly negative as well. This result also indicates two-way causality between stock market volatility and real activity. Table 7 also shows that this two-sided causality applies to IP volatility.

It is also worth examining some plots of sample paths. Figures 6 through 8 display the one-sided as well as the two-sided IP GARCH-MIDAS models—full sample as well as the Great Depression and post-World War II subsamples. The top panel contains the time series paths of  $\tau$  and  $g \times \tau$ . The lower panel contains the lag-lead weights for level and volatility of IP in the  $\tau$  component according to equation (17). When we consider the models estimated over the respective subsamples, we get a better close-up picture. Figure 7 covers the interwar period, while figure 8 covers the last subsample from 1985 on. In particular, in the latter case we see that the October 1987 crash was not driven by these economic

<sup>19</sup>Due to the leads, and lags, the sample sizes are no longer the same, since those filters involve four years of leads and lags.



TABLE 4.—PARAMETER ESTIMATES OF ONE-SIDED GARCH-MIDAS WITH PPI

Sample	$\mu$	$\alpha$	$\beta$	$\theta_l$	$\omega_{l,1}$	$\omega_{l,2}$	$m$
Level of PPI							
1890–2010	0.00056 (11.58)	0.09409 (3.51)	0.89604 (134.97)	0.26962 (6.35)	20.83365 (21.62)	4.23304 (3.07)	–9.11428 (–4.77)
1890–1919	0.00053 (7.10)	0.14355 (8.73)	0.81297 (37.41)	0.24431 (2.44)	13.23874 (1.14)	3.57186 (0.95)	–9.54223 (–84.40)
1920–1952	0.00073 (8.59)	0.09379 (8.59)	0.89822 (79.16)	0.22639 (2.32)	16.33749 (1.03)	2.52484 (0.97)	–8.72243 (–30.69)
1953–2010	0.00051 (3.75)	0.07593 (0.61)	0.91692 (7.72)	0.66796 (25.19)	29.71031 (3.42)	6.82835 (4.85)	–9.35581 (–2.81)
1953–1984	0.00047 (4.93)	0.08202 (7.80)	0.90040 (58.92)	1.09618 (4.50)	7.39798 (1.89)	2.67376 (3.10)	–10.03831 (–48.36)
1985–2010	0.00062 (7.20)	0.07868 (4.61)	0.91255 (52.99)	0.78060 (1.26)	8.85670 (1.22)	13.79158 (1.36)	–9.03349 (–25.27)
Variance of PPI							
1890–2010	0.00056 (12.65)	0.09371 (13.60)	0.89675 (122.82)	0.01671 (1.58)	299.85850 (358.36)	237.56383 (187.41)	–9.05655 (–41.29)
1890–1919	0.00053 (7.16)	0.14346 (8.59)	0.81063 (36.21)	0.19544 (3.16)	8.31992 (1.15)	1.40548 (0.91)	–9.74430 (–74.32)
1920–1952	0.00073 (3.71)	0.09337 (6.92)	0.89955 (84.13)	–0.03908 (–0.12)	11.09956 (0.95)	1.00000 (0.05)	–8.57764 (–9.32)
1953–2010	0.00051 (8.15)	0.07566 (6.83)	0.91746 (81.06)	0.12898 (1.17)	88.52420 (0.47)	52.19322 (0.48)	–9.22556 (–38.00)
1953–1984	0.00047 (6.12)	0.07970 (9.31)	0.90643 (88.04)	1.27777 (3.80)	19.68184 (1.31)	4.95236 (1.62)	–9.87795 (–47.34)
1985–2010	0.00062 (6.04)	0.07804 (3.83)	0.91263 (42.77)	0.07110 (1.34)	300.00000 (8.35)	182.00279 (8.84)	–8.94072 (–28.00)

GARCH-MIDAS models with various specifications are fitted via QMLE. The specifications appear in equation (14) for the level and equation (15) for the variance. The quarterly macroeconomic-level variable is obtained by taking the geometric mean of monthly rates. The corresponding variance is estimated from equation (21), a similar approach to that of Schwert (1989). For both specifications with macroeconomic level and variance in the MIDAS filter, sixteen lags are taken to model  $\log \tau_t$ .  $\theta_l$  and  $\theta_v$  are rescaled by multiplication of  $10^{-2}$  and  $10^{-4}$  to make the macrolevel variables represented in percentage units. The numbers in parentheses are robust  $t$ -statistics computed with HAC standard errors.

TABLE 5.—PARAMETER ESTIMATES OF ONE-SIDED GARCH-MIDAS WITH IP

Sample	$\mu$	$\alpha$	$\beta$	$\theta_l$	$\omega_{l,1}$	$\omega_{l,2}$	$m$
Level of PPI							
1890–2010	0.00056 (13.44)	0.09397 (13.54)	0.89593 (120.39)	–0.29580 (–1.91)	2.39016 (1.80)	2.71319 (1.14)	–8.96616 (–61.88)
1890–1919	0.00054 (7.15)	0.14157 (9.41)	0.81879 (45.61)	–0.09659 (–2.05)	40.18934 (0.94)	140.04966 (0.87)	–9.42615 (–78.56)
1920–1952	0.00073 (9.22)	0.09488 (9.53)	0.89602 (84.88)	–0.18956 (–1.90)	2.70598 (1.03)	3.19719 (1.33)	–8.65862 (–33.44)
1953–2010	0.00052 (8.28)	0.07800 (7.83)	0.91400 (92.65)	–0.93852 (–2.53)	4.81398 (0.74)	2.94544 (0.56)	–8.98368 (–36.56)
1953–1984	0.00048 (6.09)	0.08116 (7.76)	0.90720 (78.79)	–0.91602 (–2.43)	5.64629 (0.83)	3.85090 (0.88)	–9.36597 (–45.23)
1985–2010	0.00063 (6.14)	0.08252 (5.97)	0.90601 (67.87)	–0.64475 (–4.65)	36.33068 (4.12)	6.27684 (43.37)	–8.82381 (–28.51)
Variance of IP							
1890–2010	0.00056 (14.76)	0.09549 (17.22)	0.89255 (135.41)	0.07313 (2.18)	2.76912 (26.22)	1.52251 (7.12)	–9.30757 (–117.35)
1890–1919	0.00054 (7.57)	0.13932 (9.62)	0.81980 (45.86)	0.02446 (2.15)	243.02613 (93.86)	299.99971 (143.60)	–9.54998 (–73.31)
1920–1952	0.00073 (8.39)	0.09794 (9.60)	0.88783 (80.70)	0.06086 (4.39)	8.02994 (1.56)	2.52323 (2.89)	–9.20466 (–40.88)
1953–2010	0.00052 (6.99)	0.07644 (1.38)	0.91724 (15.21)	0.10151 (2.41)	300.00000 (0.10)	212.21824 (0.09)	–9.20055 (–30.45)
1953–1984	0.00047 (6.19)	0.07946 (8.93)	0.91062 (90.93)	0.31918 (1.84)	2.54411 (1.11)	2.07558 (3.51)	–9.81497 (–51.99)
1985–2010	0.00062 (6.04)	0.07925 (3.90)	0.91139 (43.95)	0.00087 (5.21)	13.12098 (3.15)	6.67859 (1.33)	–8.89593 (–27.94)

GARCH-MIDAS models with various specifications are fitted via QMLE. The specifications appear in equations (14) for the level and (15) for the variance. The quarterly macroeconomic-level variable is obtained by taking the geometric mean of monthly rates. The corresponding variance is estimated from equation (21), a similar approach to that of Schwert (1989). For both specifications with macroeconomic level and variance in the MIDAS filter, sixteen lags are taken to model  $\log \tau_t$ .  $\theta_l$  and  $\theta_v$  are rescaled by multiplication of  $10^{-2}$  and  $10^{-4}$  to make the macrolevel variables represented in percentage units. The numbers in parentheses are robust  $t$ -statistics computed with HAC standard errors.

fundamentals; in all model specifications, the large spike in market volatility is picked up by the  $g$  component. In great contrast, the Great Depression era was clearly a turbulent time, with market volatility linked to economic sources. The

weighting schemes that are displayed in the lower panels are also interesting. They show that a great deal of the weight is attributed to the future—which is expected as it reflects the anticipation of economic fundamentals by the stock market.

TABLE 6.—PARAMETER ESTIMATES OF ONE-SIDED GARCH-MIDAS WITH LEVEL + VARIANCE

Sample	$\mu$	$\alpha$	$\beta$	$\theta_l$	$\omega_{l,1}$	$\omega_{l,2}$	$\theta_v$	$\omega_{v,1}$	$\omega_{v,2}$	$m$
Level and Variance of PPI										
1890–2010	0.00056 (198.10)	0.09422 (46.26)	0.89568 (128.96)	0.28204 (1.76)	16.01767 (0.71)	3.60525 (0.99)	0.03471 (0.77)	27.58141 (0.61)	15.87336 (0.49)	–9.15590 (–46.94)
1890–1919	0.00053 (0.23)	0.14380 (0.32)	0.80822 (1.21)	0.18369 (0.02)	2.99299 (0.00)	35.93817 (0.00)	0.21308 (2.16)	5.35999 (0.00)	1.00001 (0.00)	–9.79149 (–10.93)
1920–1952	0.00073 (28.87)	0.09353 (16.06)	0.89787 (77.44)	0.24101 (1.51)	23.80903 (1.16)	3.46200 (1.24)	0.03687 (0.91)	300.00000 (0.95)	51.89784 (0.96)	–8.80583 (–38.64)
1953–2010	0.00051 (79.64)	0.07559 (14.62)	0.91683 (74.27)	0.66459 (0.63)	26.30214 (0.33)	6.29305 (0.45)	0.10276 (18.60)	124.53981 (0.72)	72.97437 (0.66)	–9.41325 (–19.70)
1953–1984	0.00047 (59.40)	0.08029 (20.34)	0.90318 (39.42)	0.42313 (1.87)	300.00000 (0.77)	149.99514 (0.81)	0.99909 (13.98)	28.59527 (1.85)	6.90855 (1.72)	–9.99398 (–39.95)
1985–2010	0.00063 (54.52)	0.07845 (14.26)	0.91395 (51.61)	0.49884 (0.35)	6.98951 (0.61)	11.73499 (0.97)	0.18630 (0.82)	32.38214 (2.74)	300.00000 (2.95)	–8.98403 (–17.28)
Level and Variance of IP										
1890–2010	0.00056 (65.31)	0.09534 (60.16)	0.89244 (113.07)	–0.17021 (–0.63)	2.77555 (2.14)	4.09762 (1.08)	0.05299 (1.54)	9.99761 (0.57)	2.81425 (1.10)	–9.20577 (–39.65)
1890–1919	0.00054 (36.82)	0.13974 (35.91)	0.81747 (49.03)	–0.06005 (–1.92)	300.00000 (1.65)	91.48020 (1.73)	0.02705 (1.41)	187.06963 (0.11)	230.57839 (0.10)	–9.55180 (–149.28)
1920–1952	0.00073 (35.50)	0.09769 (38.37)	0.88804 (92.17)	–0.05322 (–1.45)	300.00000 (1.15)	192.31363 (1.20)	0.07561 (2.96)	1.33973 (0.82)	1.00000 (1.34)	–9.28313 (–36.78)
1953–2010	0.00052 (26.97)	0.07790 (21.41)	0.91409 (96.52)	–0.91539 (–1.65)	4.98455 (0.96)	3.11553 (0.87)	0.01785 (0.20)	300.00000 (1.04)	50.83816 (0.99)	–9.00308 (–26.44)
1953–1984	0.00050 (36.84)	0.08415 (31.54)	0.90157 (96.57)	–1.02798 (–1.94)	5.20349 (1.47)	3.56877 (1.37)	0.15256 (2.40)	111.58383 (0.79)	300.00000 (0.77)	–9.50151 (–57.46)
1985–2010	0.00063 (1.74)	0.08065 (5.58)	0.90756 (22.88)	–0.77126 (–1.78)	6.86196 (0.23)	1.48281 (0.07)	0.10126 (4.05)	300.00000 (0.77)	31.97666 (0.17)	–8.83710 (–33.72)

GARCH-MIDAS models with past IP or PPI level and volatility series are fitted via QMLE. The specification appears in equation (16). The quarterly macroeconomic-level variable is obtained by taking the geometric mean of monthly rates. The corresponding variance is estimated from equation (21), a similar approach to that of Schwert (1989). For both macroeconomic level and variance in the MIDAS filter, sixteen lags are taken to model  $\log \tau$ .  $\theta_l$  and  $\theta_v$  are rescaled by multiplication of  $10^{-2}$  and  $10^{-4}$  to make the macrolevel variables represented in percentage units. The numbers in parentheses are robust  $t$ -statistics computed with HAC standard errors.

TABLE 7.—PARAMETER ESTIMATES OF TWO-SIDED GARCH-MIDAS WITH PPI/IP

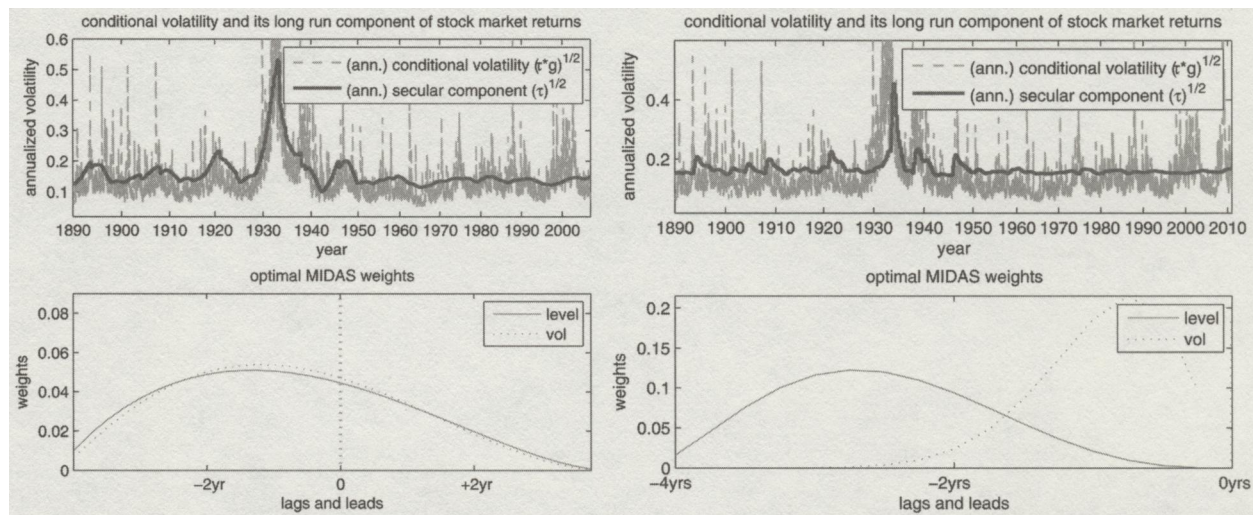
	$\mu$	$\alpha$	$\beta$	$\theta_l^p$	$\theta_l^i$	$\theta_v^p$	$\theta_v^i$	$\omega_1$	$\omega_2$	$\omega_3$	$\omega_4$	$m$
Level and Variance of PPI												
1890–2010	0.00056 (58.00)	0.095 (46.44)	0.894 (129.28)	0.25585 (1.13)	0.32970 (0.76)	0.08106 (0.74)	0.18429 (0.70)	25.06 (0.40)	32.63 (0.38)	8.99 (0.76)	10.86 (1.12)	–9.27 (–48.55)
1890–1919	0.00053 (35.24)	0.144 (53.01)	0.806 (47.06)	0.28414 (2.19)	0.90884 (0.25)	0.21854 (1.92)	0.00151 (1.65)	52.40 (1.10)	71.92 (1.03)	3.03 (0.54)	40.61 (0.39)	–9.84 (–63.55)
1920–1952	0.00073 (35.76)	0.094 (22.55)	0.898 (68.70)	1.09287 (0.53)	0.35375 (0.56)	0.05589 (0.62)	0.15051 (0.79)	17.40 (0.83)	13.23 (0.64)	58.25 (0.77)	63.90 (0.66)	–8.82 (–16.61)
1953–2010	0.00051 (50.39)	0.076 (19.00)	0.917 (83.64)	0.68539 (1.67)	1.98336 (4.69)	0.10291 (417.58)	0.01032 (479.33)	140.63 (1.16)	195.22 (1.16)	241.76 (14.05)	299.14 (19.70)	–9.47 (–29.52)
1953–1984	0.00047 (29.43)	0.082 (33.26)	0.902 (53.70)	1.05254 (1.93)	2918.61284 (0.81)	–0.51883 (–0.80)	1.15017 (0.81)	23.30 (0.88)	55.80 (1.42)	86.09 (1.42)	300.00 (1.49)	–9.98 (–36.18)
1985–2010	0.00063 (26.14)	0.075 (10.38)	0.913 (44.42)	–1.68342 (–1.32)	1.13580 (1.09)	0.00092 (29.46)	–0.19537 (–1.04)	37.34 (1.09)	10.50 (1.05)	86.54 (1.29)	8.34 (1.17)	–9.22 (–26.33)
Level and Variance of IP												
1890–2010	0.00057 (31.33)	0.097 (35.63)	0.885 (84.34)	–0.91538 (–1.36)	–0.96349 (–2.28)	0.09947 (0.90)	0.19671 (0.46)	1.94 (2.02)	2.70 (1.23)	2.18 (0.33)	3.00 (0.95)	–9.32 (–47.30)
1890–1919	0.00054 (47.59)	0.145 (44.15)	0.801 (32.73)	–0.12499 (–2.00)	1.50459 (0.69)	0.09650 (2.06)	0.13656 (1.29)	31.10 (1.17)	226.78 (1.10)	28.67 (0.88)	27.39 (1.03)	–9.81 (–66.68)
1920–1952	0.00074 (124.61)	0.100 (46.48)	0.863 (67.36)	–0.42589 (–3.06)	0.30435 (0.79)	0.14860 (2.65)	0.15721 (5.13)	1.71 (3.26)	1.82 (1.44)	4.93 (2.88)	3.39 (3.25)	–9.89 (–65.37)
1953–2010	0.00053 (64.60)	0.081 (52.50)	0.900 (78.09)	–2.66147 (–4.25)	–3.83689 (–4.32)	–0.12132 (–1.11)	–0.01272 (–0.83)	3.85 (3.47)	4.61 (2.71)	164.10 (0.63)	300.00 (0.55)	–8.61 (–43.84)
1953–1984	0.00053 (36.70)	0.092 (15.41)	0.870 (43.14)	–3.36394 (–4.42)	–4.21650 (–5.42)	0.17340 (1.18)	0.00009 (2.78)	3.01 (5.07)	3.23 (4.09)	51.64 (0.29)	300.00 (0.28)	–8.85 (–46.19)
1985–2010	0.00064 (40.13)	0.079 (12.00)	0.908 (51.00)	–2.53382 (–1.55)	–2.57622 (–0.99)	0.01006 (253.51)	–0.11379 (–0.38)	12.04 (1.33)	11.87 (1.84)	300.00 (0.40)	37.75 (0.39)	–8.49 (–15.38)

GARCH-MIDAS models with past and future PPI or IP level and variance series are fitted using QMLE. The specification appears in equation (17). The quarterly macroeconomic-level variable is obtained by taking the geometric mean of monthly rates. The corresponding variance is estimated from equation (21), a similar approach to that of Schwert (1989). For both macroeconomic level and variance in the MIDAS filter, sixteen lags and sixteen leads are taken to model  $\log \tau$ .  $\theta_l^p$  and  $\theta_l^i$  are rescaled by multiplication of  $10^{-2}$  and  $10^{-4}$  to make the macrolevel variables represented in percentage units. The numbers in parentheses are robust  $t$ -statistics computed with HAC standard errors.

Visual inspection of figures 6 through 8 reveals that the two-sided models provide a better fit. For example, in figure 6, we see that the one-sided model misses the volatility surge of the Great Depression. This is even clearer in

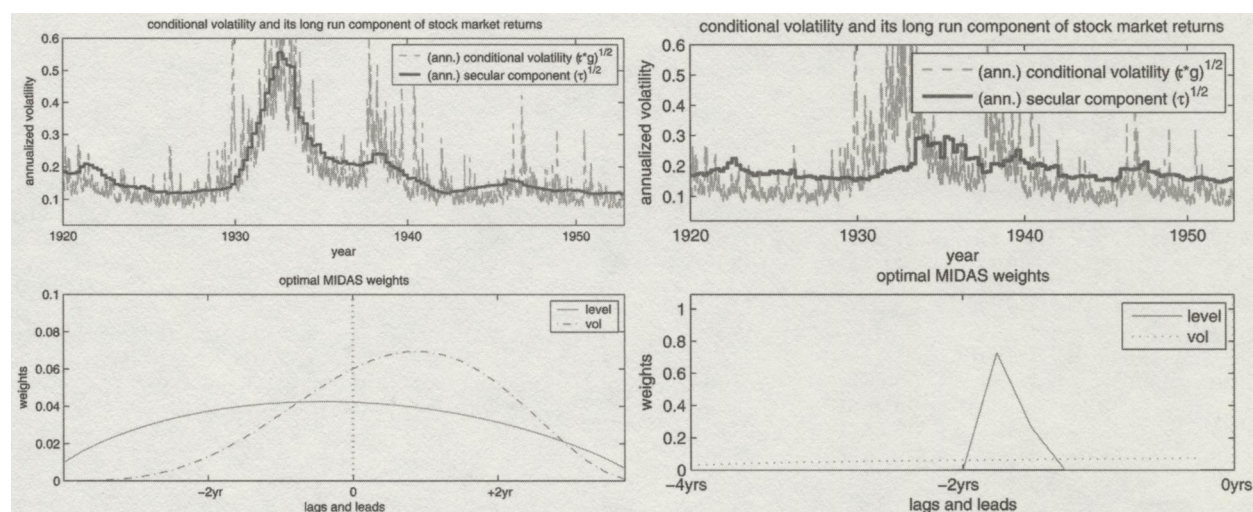
the next figure. We will appraise the forecast performance of (one-sided) models in the next section and revisit this issue. To conclude, we report the parameter estimates of the spline-GARCH models. The parameter estimates appear in

FIGURE 6.—GARCH-MIDAS WITH IP: TWO-SIDED VERSUS ONE-SIDED MODEL, 1890–2010



The figure pertains to the two-/one-sided IP level/volatility GARCH-MIDAS models for the full sample. Sixteen lags (and sixteen leads in the case of the two-sided model) of both quarterly IP level and variance are filtered by MIDAS filter to model the long-run component  $\tau$ . The corresponding parameter estimates are shown in tables 6 and 7. The top panels contain the time series paths of  $\tau$  and  $g \times \tau$ . They are shown in standard deviation and annualized scale. The lower panels contain the lag-lead weights for level and volatility of IP in the  $\tau$  component according to equation (17). Note that in the case of the two-sided model, the stock returns fitted end four years earlier than the one-sided model since the last four years of return data cannot be modeled with the next four years of IP data.

FIGURE 7.—GARCH-MIDAS WITH IP: TWO-SIDED VERSUS ONE-SIDED, 1920–1952



The figure pertains to the two-/one-sided IP level/volatility GARCH-MIDAS models for the interwar subsample, which includes the Great Depression period. Sixteen lags (and sixteen leads, in the case of the two-sided model) of both quarterly IP level and variance are filtered by MIDAS filter to model the long-run component  $\tau$ . The corresponding parameter estimates are shown in tables 6 and 7. The top panels contain the time series paths of  $\tau$  and  $g \times \tau$ . They are shown in standard deviation and annualized scale. The lower panels contain the lag-lead weights for level and volatility of IP in the  $\tau$  component according to equation (17). Note that in case of the two-sided model, the stock returns fitted end four years earlier than the one-sided model since the last four years of return data cannot be modeled with future four years of IP data.

table 8. The drawback of the spline-GARCH model selection approach is that the likelihood tends to fluctuate as one increases the number of knots since the position of the knots changes as the number increases. This issue appears to be particularly critical in long time spans, as illustrated in figure 9. The figure compares the long-run components as measured by  $\tau$  in the spline-GARCH model fitted over the full sample and each of the subsamples. The optimal number of knots, with the lowest BIC, for the full sample (1890–2010) is six while those of subsamples are one

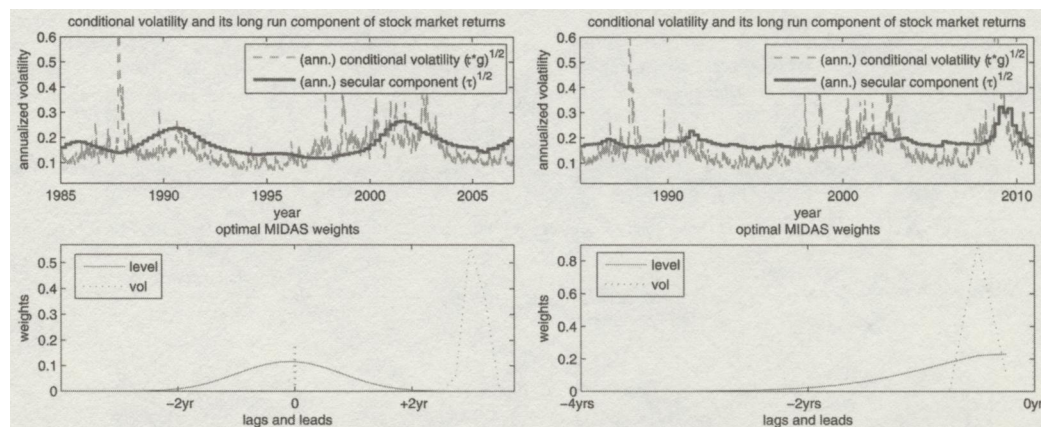
(1890–1919), eight (1920–1952), and eight (1953–2010), respectively.

#### IV. Appraising the Models and Analyzing the Economic Sources

In this section we analyze the economic content of volatility models using various new approaches. Section IVA deals with correlation and structural breaks. In section IVB, we study the forecasting performance of the models we



FIGURE 8.—GARCH-MIDAS WITH IP: TWO-SIDED VERSUS ONE-SIDED, 1985–2010



The figure pertains to the two-/one-sided IP level/volatility GARCH-MIDAS models for the most recent subsample, which includes the 1987 crash. Sixteen lags (and sixteen leads in the case of the two-sided model) of both quarterly IP level and variance are filtered by the MIDAS filter to model the long-run component  $\tau$ . The corresponding parameter estimates are shown in tables 6 and 7. The top panels contain the time series paths of  $\tau$  and  $g \times \tau$ . They are all shown in standard deviation and annualized scale. The lower panels contain the lag-lead weights for level and volatility of IP in the  $\tau$  component according to equation (17). Note that in the case of the two-sided model, the stock returns fitted end four years earlier than the one-sided model since the last four years of return data cannot be modeled with future four years of IP data.

TABLE 8.—PARAMETER ESTIMATES OF SPLINE-GARCH

	$\mu$	$\alpha$	$\beta$	$c$	$\omega_0$						
1890–2010	0.00057 (15.66)	0.09899 (31.91)	0.88415 (242.73)	0.00005 (7.78)	15.25 (5.24)						
1890–1919	0.00053 (7.66)	0.14219 (17.02)	0.80760 (61.86)	0.00005 (78.87)	4.77 (60.31)						
1920–1952	0.00076 (9.22)	0.09899 (30.50)	0.85998 (115.91)	0.00010 (58.14)	–0.63 (–0.93)						
1953–2010	0.00054 (9.30)	0.08573 (11.86)	0.89167 (113.87)	0.00007 (4.22)	8.22 (2.66)						
1953–1984	0.00051 (7.37)	0.09079 (13.01)	0.87796 (88.93)	0.00005 (58.39)	7.42 (35.95)						
1985–2010	0.00064 (6.43)	0.08195 (12.43)	0.88142 (92.24)	0.00005 (5.15)	15.88 (6.92)						
	$\omega_1$	$\omega_2$	$\omega_3$	$\omega_4$	$\omega_5$	$\omega_6$	$\omega_7$	$\omega_8$	$\omega_9$	LLF/BIC	
1890–2010	–81.57 (–5.64)	139.82 (6.45)	–88.81 (–8.57)	–14.45 (–6.15)	140.42 (13.31)	–170.64 (–12.01)	143.88 (7.04)			111622.52 –6.7266	
1890–1919	–7.78 (–35.31)	15.83 (72.32)								30599.92 –6.9143	
1920–1952	–41.86 (–36.09)	137.65 (75.29)	–71.98 (–53.07)	–157.77 (–32.61)	201.12 (31.84)	–134.15 (–15.71)	174.21 (20.05)	–183.80 (–57.51)	55.79 (13.77)	31297.15 –6.4219	
1953–2010	–100.00 (–6.98)	182.16 (5.53)	–69.14 (–0.61)	–43.16 (–0.26)	60.22 (0.53)	–115.53 (–9.83)	191.04 (5.82)	–200.00 (–3.72)	200.00 (2.59)	49858.06 –6.8202	
1953–1984	–33.15 (–35.87)	66.11 (22.06)	–47.95 (–54.92)	22.67 (13.06)						28615.09 –7.1055	
1985–2010	–55.32 (–9.41)	8.31 (2.50)	139.35 (9.73)	–121.89 (–8.27)	–39.49 (–4.04)	172.45 (9.33)	–200.00 (–4.73)			21283.56 –6.4748	

Spline-GARCH-MIDAS models are fitted via QMLE. The specification appears in equation (19). For a given sample choice, the number of knots is selected using the BIC. For empirical implementation, we normalized  $\tau$  in equation (19) by dividing it by the total number of days in the sample. This makes our spline parameters typically bigger than those shown in Engle and Rangel (2008). The numbers in parentheses are robust  $t$ -statistic computed with HAC standard errors.

estimated. Finally, we measure the contribution of economic sources to expected volatility.

#### A. Structural Breaks and Correlations

We cover two topics: (a) how the models handle structural breaks and (b) how similar the components are across models. As noted earlier, there is considerable evidence suggesting structural breaks in volatility dynamics (see the references in note 15).

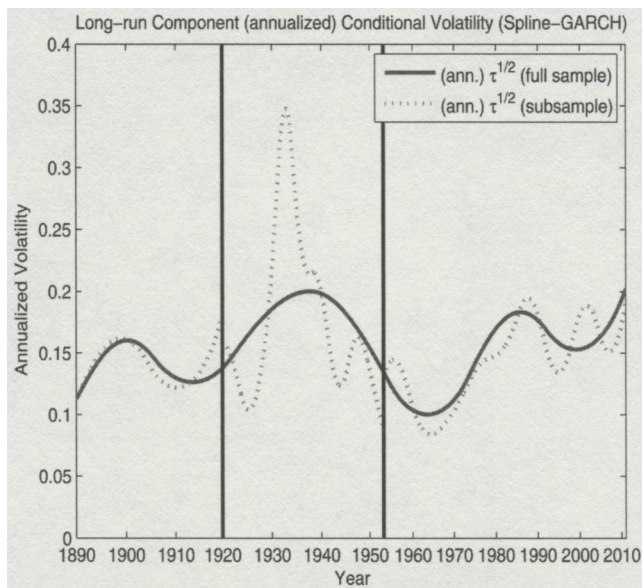
So far we have considered subsamples to guard against possible breaks in the volatility models. Here, we study whether

in fact full-sample models are immune to breaks. To address the structural break question, we compute a likelihood ratio statistic, comparing the log-likelihood function for the full sample with those of the subsamples. In particular,

$$-2 \left[ LLF_{full} - \sum_{i=sub-samples} LLF_i \right] \sim \chi^2(df),$$

where  $df$  is the number of parameters times 1 minus than the number of subsamples, which corresponds to the number of restrictions. Since the number of parameters differs across models, we adjust the degrees of freedom accordingly. This

FIGURE 9.—COMPARISON OF  $\tau$  COMPONENT FROM SPLINE-GARCH MODEL FITTED OVER THE FULL SAMPLE AND THE SUBSAMPLES



Long-run components as measured by  $\tau$  in spline-GARCH model fitted over the full sample and each of subsamples are compared. The full sample period (1890–2010) is divided into three subsample periods: 1890–1919, 1920–1952, and 1953–2010. The optimal number of knots, with lowest BIC, for the full sample is six, while those of subsamples are one, eight, and eight, respectively. All are shown in standard deviation and annualized scale.

analysis is confined to GARCH-MIDAS models. It does not include spline-GARCH since the latter involves a different number of knots in the various subsamples, and therefore these models are nonnested. The results are easy to summarize, and therefore we do not report the details in a table. The full-sample models are not immune to breaks. Hence, the class of models in this paper still leaves room for improvement as far as structural stability goes. It also explains why the empirical results involving individual macroeconomic series differ so much across the various subsamples.

Next, we study the correlations among the various components. Again, due to space limitations, we do not report correlations in a table, but rather briefly describe their salient features, focusing exclusively on the full sample. The highest correlations between *RV* and any of the estimated macrovariable's long-run components is achieved with the two-sided IP level/variance model, which at .36 is slightly less than the spline-GARCH. The long-run component based on inflation yields a somewhat odd negative value, albeit it is very small. The inflation-based long-run component also correlates negatively with the IP one. In general, all the IP-based component models feature the highest correlations with any of the *RV*-based models.

The results in this section tell us that there is room for improvement. For example, we do not have models that are stable for the full sample. Nevertheless, as we show in the next sections, the models, we have so far already perform quite well in comparison to existing models, and we will also show that the long-run component constitutes an important part of volatility forecasts.

## B. Forecast Comparisons

Table 9 displays the comparison of forecasting performance over a month, quarter, and semester horizon of the two-component volatility models discussed so far using full-sample QMLE parameter estimates. The measure of forecasting performance is the mean squared error of conditional variance forecasts compared to realized variance. All cases cover pseudo-out-of-sample forecasts and pertain to nonoverlapping samples of forecasts, either monthly, quarterly, or biannual. The results reported in table 9 pertain to one-sided models only. For comparison, the GARCH-MIDAS model with rolling window *RV* is chosen as a benchmark. All forecasts are reported as ratios relative to the latter model's MSE, and a ratio below 1 means an improvement on the rolling window *RV* model. For the GARCH-MIDAS with fixed-span *RV*, the spline-GARCH models, and the GARCH-MIDAS with macroeconomic variables, we keep the  $\tau$  component fixed at the level of the last observation prior to prediction. For the GARCH-MIDAS with rolling window *RV*, we can easily make a day-forward forecast using  $g$  and predetermined  $\tau$ , yielding  $g\tau$ , which can be substituted into the MIDAS filter. This process can be iterated forward over the entire prediction horizon. The comparison in table 9 between GARCH-MIDAS with fixed-span *RV* and rolling window *RV* reveals that the former is very imprecise (relatively speaking) at short horizons (monthly horizons), but the disadvantage disappears at longer horizons and ultimately is typically at par or even below par with the latter in terms of MSE's over-biannual forecast horizons.

Let us focus first on the full-sample forecasting evaluation results, ignoring for the moment the evidence of structural break tests. Moreover, we also focus mostly on the log *RV* version because this is directly comparable with the models driven by macroeconomic variables. For the full sample, it is clear that GARCH-MIDAS with rolling log *RV* is the most attractive two-component model for one-month-ahead forecasts. Moreover, the fixed-span models perform poorly in comparison (again at the one month horizon).

When we increase the forecast horizons, we observe that other models start to improve on the rolling *RV* specification. First, it is interesting to note that the fixed-span specification does better than the rolling *RV* one. At the six-month horizon, the best model is the GARCH-MIDAS with IP level/variance, as well as the fixed-span *RV* specification. Moreover, models involving IP typically fare better than the models with PPI. For the intermediate horizon (one quarter ahead), we observe for the full sample that the best *RV*-based models and the best models driven by macroeconomic variables are roughly at par.

The first subsample ending in 1919 is disastrous for the models involving macroeconomic data. A plausible explanation is that the macroeconomic data may not be of good quality to produce good forecasts. Another explanation is that the full-sample parameter estimates simply do not fit this subsample. As we will show, it is the latter that appears to be the case, as the forecasting results with the subsample estimates will show.

TABLE 9.—COMPARISON OF FORECASTING PERFORMANCE OF TWO-COMPONENT VOLATILITY MODELS USING FULL SAMPLE ESTIMATES

Forecasting Horizon	MSE Ratio Relative to GARCH-MIDAS with Rolling Window RV					
	1890–2010	1890–1919	1920–1952	1953–2010	1953–1984	1985–2010
GARCH-MIDAS with rolling window RV						
Month	0.000015	0.000005	0.000027	0.000014	0.000001	0.000030
Quarter	0.000094	0.000024	0.000195	0.000073	0.000008	0.000153
Semester	0.000355	0.000064	0.000774	0.000265	0.000028	0.000558
GARCH-MIDAS with rolling window RV (Log)						
Month	1.04	1.11	1.07	1.00	1.12	0.99
Quarter	0.91	1.24	0.92	0.85	1.10	0.83
Semester	0.89	1.04	0.83	0.97	1.03	0.97
GARCH-MIDAS with fixed-span RV (Log)						
Month	1.31	1.17	1.32	1.33	1.12	1.34
Quarter	0.96	1.39	1.05	0.76	1.13	0.74
Semester	0.85	1.23	0.74	0.98	1.08	0.97
Spline-GARCH						
Month	1.03	1.14	1.07	0.97	1.09	0.96
Quarter	0.94	1.41	0.99	0.78	1.13	0.76
Semester	0.96	1.21	0.98	0.91	1.01	0.90
GARCH-MIDAS with PPI level						
Month	1.08	1.23	1.10	1.02	1.18	1.02
Quarter	0.93	1.92	0.95	0.72	1.38	0.68
Semester	0.95	1.88	0.90	0.92	1.36	0.89
GARCH-MIDAS with PPI variance						
Month	1.08	1.27	1.10	1.03	1.19	1.02
Quarter	0.93	1.89	0.94	0.75	1.38	0.71
Semester	0.95	2.04	0.88	0.93	1.42	0.90
GARCH-MIDAS with PPI (level+variance)						
Month	1.08	1.22	1.10	1.02	1.16	1.01
Quarter	0.93	1.96	0.95	0.72	1.36	0.68
Semester	0.95	1.89	0.89	0.92	1.35	0.89
GARCH-MIDAS with IP level						
Month	1.08	1.22	1.09	1.05	1.18	1.04
Quarter	0.91	1.83	0.91	0.75	1.39	0.71
Semester	0.94	1.96	0.87	0.93	1.42	0.90
GARCH-MIDAS with IP variance						
Month	1.08	1.27	1.10	1.03	1.19	1.02
Quarter	0.92	1.73	0.94	0.76	1.26	0.73
Semester	0.92	1.99	0.83	0.93	1.35	0.91
GARCH-MIDAS with IP (level+variance)						
Month	1.07	1.23	1.10	1.02	1.18	1.01
Quarter	0.93	1.71	0.96	0.76	1.28	0.73
Semester	0.85	1.84	0.73	0.93	1.41	0.90

Various GARCH-MIDAS models and spline-GARCH model from Engle and Rangel (2008) are fitted over the full sample, and the forecasting performance of these models over the subsamples is compared. Note that all the models considered are two-component volatility models. However, GARCH-MIDAS with fixed-span RV and GARCH-MIDAS with macrolevel/volume fix the long-run component for a certain period, whereas the others do not. For those with a fixed component, the models are estimated separately for each forecasting horizon to match the fixing term with the forecasting horizon. Also, for these models, the number of lags used in MIDAS filter is determined such that the MIDAS regressors span the past four years of data (for example, sixteen lags for quarterly RV). To be consistent with others, Qtr/4yr GARCH-MIDAS with rolling window RV as described in tables 1 and 2 are used. The parameters for each model are estimated using the full sample, and the forecasts for next month/quarter/semester are computed assuming that today is the last day of a month/quarter/semester. The mean squared errors (MSE) of the forecasts are calculated with respect to monthly, quarterly, and half-year realized variance computed from daily stock return series. For comparison, the GARCH-MIDAS model with rolling window RV is chosen as a benchmark. Except for this case, all the other MSEs are presented as a ratio to the base MSE from the forecasts of GARCH-MIDAS with rolling window RV for the corresponding forecasting horizon and the subsample.

For the Great Depression subsample, there are clearly two models that forecast best at the six-month horizon: (a) the fixed-span RV and (b) the GARCH-MIDAS involving IP level/variance. It is also interesting to note that all models involving macroeconomic series are at par with the statistical models at the one-quarter horizon, while all models involving macroeconomic series are at par or tend to outperform statistical models at the six-month horizon. The improvements are roughly 10% in terms of MSE in the longer horizon case.

The 1953–2010 and 1985–2010 subsamples share similar features: at the one-quarter-ahead horizon, the models involving economic data perform as well as, if not better than, the best RV-driven models. This is also true at the six-month horizon.

For more robust findings, we also computed one-semester-horizon forecasts using the subsample estimates instead of the

full-sample estimates. The results (not reported here but available on request) clearly show that our main findings remain. The weakest results appear to be for the 1953–1984 subsample (although for this subsample, the models involving PPI do comparatively well), as this is the era of the oil price shock. It is also worth noting that the 1890–1919 subsample shows very poor forecast results for all the models driven by macroeconomic variables. Hence, it is clearly the case that for this subsample, the full sample estimates are highly inadequate.

### C. Measuring the Contribution of Economic Sources

How much of expected volatility can be explained by economic variables? To answer this question, we compute, the ratio  $\text{Var}(\log(\tau_i^{[M]}))/\text{Var}(\log(\tau_i^{[M]}g_i^{[M]}))$ , where  $M$  refers



TABLE 10.—SUMMARY TABLE FOR VARIANCE RATIOS

Model	1890–2010	1890–1919	1920–1952	1953–2010	1953–1984	1985–2010
GM with rolling window RV	45.93	24.57	54.09	29.57	29.39	44.15
GM with rolling window RV (Log)	30.15	17.44	45.89	12.68	19.04	8.75
GM with fixed-span RV	38.29	14.82	52.05	25.53	9.06	27.91
	37.97	14.60	51.23	25.26	8.94	27.17
GM with fixed-span RV (Log)	29.94	11.20	45.35	14.66	8.74	9.89
	29.68	10.96	45.34	14.52	8.66	9.64
Spline-GARCH	21.21	12.87	53.67	38.19	54.55	43.15
	21.15	12.45	52.87	37.23	56.97	41.55
GM with macro series						
PPI level	5.59	6.89	5.96	11.64	35.10	5.76
	5.55	6.72	5.97	11.47	35.54	5.68
Variance	0.23	9.51	0.45	2.62	22.36	1.93
	0.22	9.26	0.45	2.56	22.19	1.88
Level+variance	5.50	13.50	5.41	13.35	35.89	4.31
	5.46	13.30	5.41	13.13	36.23	4.33
IP level	2.88	4.12	2.44	10.95	17.04	9.45
	2.85	4.02	2.42	10.88	17.11	9.35
Variance	10.11	3.66	15.38	1.87	5.56	0.00
	10.04	3.54	15.26	1.84	5.51	0.00
Level+variance	10.43	5.91	14.20	10.96	25.09	11.85
	10.36	5.72	14.05	10.88	25.45	11.73

The first variance ratio is  $\text{Var}(\log(\tau_t^{[M]}))/\text{Var}(\log(\tau_t^{[M]}g_t^{[M]}))$ , where  $M$  refers to a specific model: GARCH-MIDAS with rolling window RV, with fixed-span RV, with macrovariables, and finally Spline-GARCH. Except for GARCH-MIDAS with rolling RV, each model has a second line that refers to a variance ratio normalized by  $\text{Var}(\log(\tau_t^{[gm-rollRV]}g_t^{[gm-rollRV]}))$ .

to a specific model: GARCH-MIDAS with rolling window RV, with fixed-span RV, with macro-volatility, level, and finally spline-GARCH. We also consider a second ratio,  $\text{Var}(\log(\tau_t^{[M]}))/\text{Var}(\log(\tau_t^{[gm-rollRV]}g_t^{[gm-rollRV]}))$ , where all ratios now have the same denominator, the GARCH-MIDAS with rolling RV. The choice of this particular expected volatility is motivated by the fact it yields the best predictions and is therefore a good choice as a common target. The variance ratio results appear in table 10, where we cover the full sample as well as the subsamples 1890–1919, 1920–1952, 1953–2010, 1953–1984 and 1985–2010.

The full-sample estimates tell us that the GARCH-MIDAS model with rolling RV has the most important long-run component contribution—over 50% during the Great Depression era. Among the models involving economic time series, we observe that the IP-level model contributes to more than 15% to total volatility in the post-World War II subsample, while it is the IP variance model: output uncertainty is clearly a great source of market volatility during the Great Depression era. If we combine level and variance of IP into one model, it is not surprising that we see the largest contribution—over 25% even in some subsamples. In contrast, inflation is the great source of the long-run component during the 1953–1984 sample: over 35% of the variance is due to the long-run inflation-driven component.

The results show that there is clearly room for improvement in terms of explaining volatility with economic variables. Yet with the two historical series we have, there is already a quite significant fraction of variation in expected volatility that can be attributed to economic sources. Obviously the framework we introduced here allows us to consider other series. Because we used long historical series, our hands were tied due to a small set of available series.

## V. Summary and Conclusion

In this paper we introduced a new, versatile class of component volatility models combining the insights of spline-GARCH and MIDAS filters. This new class allowed us to distinguish short- and long-run sources of volatility and link them directly to economic variables. The new model specifications also relate to the long-established use of realized volatility yet refines these measures through MIDAS filtering.

The approach we propose to measure the contribution of economic variables can be viewed as regression through filtering. Our analysis focused on long historical time series. The long time span limited the set of macroeconomic series available. The class of GARCH-MIDAS models can easily handle any set of variables. With more recent data, we could consider liquidity-related series, an event-related dummy variable (such as announcement effects), and others. Hence, our analysis of GARCH-MIDAS models is not confined to macroeconomic variables, as one could conceivably incorporate other economic variables. We leave this for future research.

To assess the economic content, we suggest a variance ratio measuring the contribution of economic sources to expected volatility. The results reveal that for the full sample, the long-run component typically accounts for roughly half of predicted volatility. For the most recent period, the results show roughly a 30% contribution. When the long-run component is driven by economic variables, the numbers are not so high, except for specific subsamples such as the Great Depression and some of the post-World War II era.

What is most encouraging is our findings regarding long-term forecasting. We find that models with the long-term component driven by inflation and industrial production

growth are at par in terms of out-of-sample prediction for horizons of one quarter and outperform pure time series statistical models at longer horizons. The significance of this finding is important and is mostly attributable to the ability of our new models to incorporate macroeconomic variables directly into the specification of volatility dynamics.

Finally, it should also be noted that the idea of component models, short and long run, that are driven by economic sources can potentially be extended to multivariate settings—correlation that is. A step in that direction is the work of Colacito, Engle, and Ghysels (2011).

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