Volatility forecast of financial returns with explanatory variables of different frequencies

Louis Geist

16/11/2023

- Financial risk measurement
- Daily volatility forecast can be useful to build confidence interval of the returns forecast
- Understand impact of long term explanatory variable on financial returns volatility
- Focus on S&P 500 and NASDAQ-100
- C. Conrad and O. Kleen. "Two are better than one: Volatility forecasting using multiplicative component GARCH-MIDAS models", 2020. [3]

1. GARCH-MIDAS model
2. Data
3. Empirical results

1. GARCH-MIDAS model
2. Data
3. Empirical results

GARCH-MIDAS Model Definition

A process $(\varepsilon_{i,t})_{t\in\mathbb{Z},i\in I_t}$ follows a **GARCH-MIDAS model** if :
$\exists(\alpha,\beta,\gamma)\in\mathbb{R}^3$, $\exists(Z_{i,t})_{t\in\mathbb{Z},i\in I_t}\sim WN, \forall t\in\mathbb{Z}, \forall i\in I_t$:

$$\boxed{\varepsilon_{it}=\sqrt{g_{it}\tau_t}Z_{i,t}}$$

with :

- $g_{i,t}=(1-\alpha-\frac{\gamma}{2}-\beta)+(\alpha+\gamma\mathbb{1}_{\varepsilon_{i-1,t}<0})\frac{\varepsilon_{i-1,t}^2}{\tau_t}+\beta g_{i-1,t}$
- τ_t is a fixed function of a low-frequency explanatory process X
- I_t, the list of values that can take "i" during the period t.

GARCH-MIDAS Model Definition - τ definition

- τ expresses the influence of the low-frequency variable X

$$\forall t \in \mathbb{N}, \boxed{\tau_t = \exp(m + \theta \sum_{k=1}^{K} \varphi_k X_{t-k})}$$

GARCH-MIDAS Model Definition - τ definition

- τ expresses the influence of the low-frequency variable X

$$\forall t \in \mathbb{N}, \boxed{\tau_t = \exp(m + \theta \sum_{k=1}^{K} \varphi_k X_{t-k})}$$

where :

- K is the number of lags of the variable X
- m and θ have to be estimated

GARCH-MIDAS Model Definition - τ definition

- τ expresses the influence of the low-frequency variable X

$$\forall t \in \mathbb{N}, \boxed{\tau_t = \exp(m + \theta \sum_{k=1}^{K} \varphi_k X_{t-k})}$$

where :

- K is the number of lags of the variable X
- m and θ have to be estimated
- φ_k is the weighting scheme $\forall k \in [1, K]$

$$\varphi_k = \lambda \left[\left(\frac{k}{K+1} \right)^{w_1 - 1} \left(1 - \frac{k}{K+1} \right)^{w_2 - 1} \right]$$

where λ is defined so that $\sum_{k=1}^{K} \varphi_k = 1$.

Results of estimation: Weighting Schemes

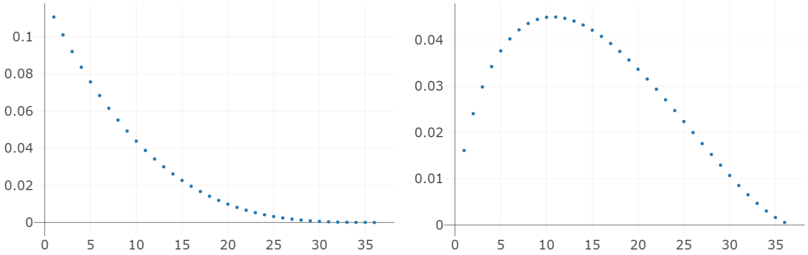

Figure 1: Examples of Weighting Schemes (restricted and unrestricted)

Results of estimation: τ

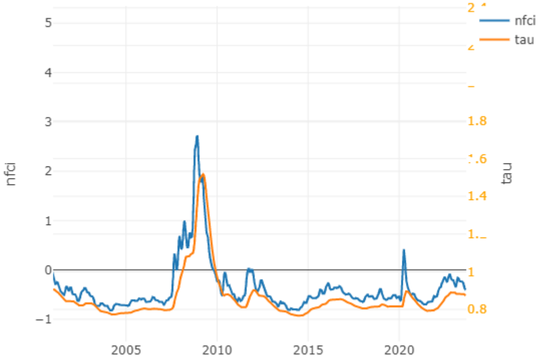

Figure 2: Example of a τ transformation

GARCH-MIDAS Forecast Formula

$$\forall s \in \mathbb{N}^*, \ \boxed{\hat{h}_{k,t+s|t} = \tau_{t+1}\left(1 + \delta^{n_h}(g_{1,t+1} - 1)\right)}$$

where :

- $\delta = \alpha + \frac{\gamma}{2} + \beta$
- $n_h = \# I_{t+1} + \ldots + \# I_{t+s-1} + k - 1$ (horizon)

1. GARCH-MIDAS model
2. Data
3. Empirical results

Table of Daily Quotation Availability

Index / Data Series	Start Date
S&P 500 (SPX)	05/01/1971
NASDAQ-100 (NDX)	02/10/1985
VIX	02/01/1990
RVOL22	03/02/1971
VRP	02/01/1990
NFCI	04/01/1971
NAI	01/02/1959
IP	01/02/1959
HOUST	01/02/1959

Table 1: Availability of Daily Quotations at Closing

Table of Daily Quotation Availability

Index / Data Series	Start Date
S&P 500 (SPX)	05/01/1971
NASDAQ-100 (NDX)	02/10/1985
VIX	02/01/1990
RVOL22	03/02/1971
VRP	02/01/1990
NFCI	04/01/1971
NAI	01/02/1959
IP	01/02/1959
HOUST	01/02/1959

Table 1: Availability of Daily Quotations at Closing

Realized volatility availibility :

- S&P 500 : 2000 - 2019 and 01/06/2023 - today
- NASDAQ-100 : 01/06/2023 - today

Index Plots

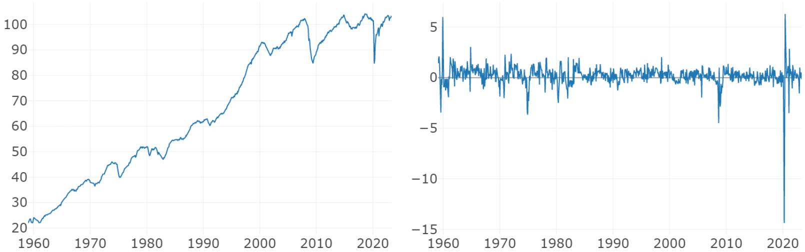

Figure 3: Raw Series of Industrial Production Index (IP) & Logarithmic Differences Transformation of Industrial Production Index (IP) Over Time

1. GARCH-MIDAS model
2. Data
3. Empirical results

Evaluating Volatility Prediction

- daily volatility : a theoretical quantity $\rightarrow$ no realization

Evaluating Volatility Prediction

- daily volatility : a theoretical quantity $\rightarrow$ no realization
- estimator of the daily volatility based on the 5 minutes intraday data of the index

Volatility point predictions of S&P500

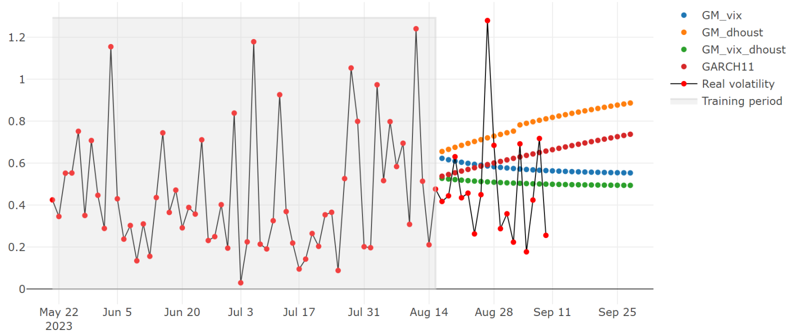

Figure 4: Prediction of S&P 500 daily volatility with an origin date of 15/08/2023. "GM" stands for GARCH-MIDAS.

Confidence Interval

Algorithm 1 Estimation of a Confidence Interval for a forecast of horizon h

Require:

- $(X_t)_{t\in[1,N]}$ target values
- $(\hat{X}_t)_{t\in[1,N]}$ predictions
- $\hat{X}_\nu$ for $\nu > N$, which is the prediction for which we want the confidence interval

Ensure: q_- and q_+, the bounds of the confidence interval of $\hat{X}_\nu$ at level α.

```
for i in [1, N] do
    γ_i ← X_i / X̂_i
end for
Sort γ in ascending order.
Calculate n_− ← ⌊((1−α)/2) N⌋ & n_+ ← ⌈(1 − (1−α)/2) N⌉
Calculate q_− ← γ_{n_−} X̂_ν & q_+ ← γ_{n_+} X̂_ν
    return q_− and q_+
```

for i in $[1, N]$ **do**

$\gamma_i \leftarrow \frac{X_i}{\hat{X}_i}$

end for

Sort γ in ascending order.

Calculate $n_- \leftarrow \lfloor \frac{1-\alpha}{2} N \rfloor$ & $n_+ \leftarrow \lceil (1 - \frac{1-\alpha}{2}) N \rceil$

Calculate $q_- \leftarrow \gamma_{n_-} \hat{X}_\nu$ & $q_+ \leftarrow \gamma_{n_+} \hat{X}_\nu$

return q_- and q_+

Confidence Intervals

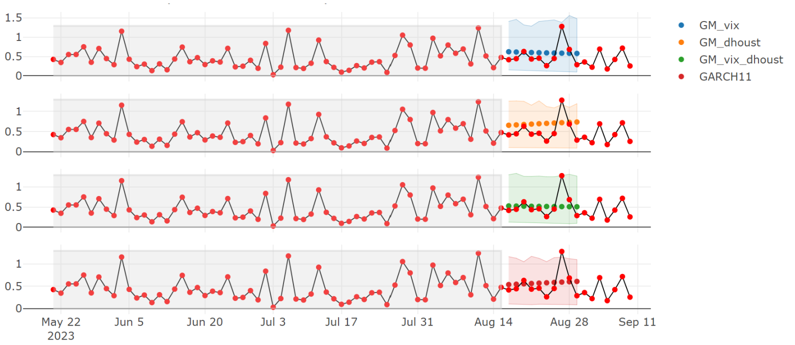

Figure 5: 90% confidence intervals for volatility predictions on S&P 500 from horizon 1 to 10 with an origin date of 15/08/2023.

Model Comparison

Loss function

- σ^2 the variance
- h its prediction

$$QLIKE(\sigma^2, h) = \log\left(\frac{h}{\sigma^2}\right) + \frac{\sigma^2}{h} - 1$$

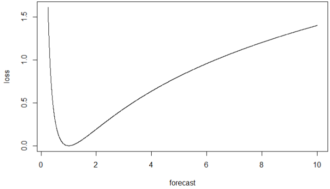

Figure 6: QLIKE Loss for $\sigma^2 = 1$.

Model Comparison

Horizon	1	2	5	10	22	44	66
GM_dhoust	0.29	0.26	0.36	0.42	0.40	0.33	0.29
GM_ip	0.32	0.28	0.35	0.39	0.36	0.29	0.23
GM_nai	0.29	0.26	0.34	0.39	0.36	<u>0.29</u>	<u>0.23</u>
GM_nfci	0.26	0.22	0.31	<u>0.37</u>	<u>0.35</u>	0.30	0.27
GM_Rvol22	0.27	0.23	0.32	0.38	0.36	0.29	0.24
GM_vix	<u>0.20</u>	<u>0.16</u>	<u>0.28</u>	0.40	0.46	0.45	0.44
GM_vrp	0.31	0.27	0.35	0.40	0.38	0.31	0.26
GM_vix_dhoust	0.23	0.21	0.34	0.43	0.47	0.45	0.44
GM_vix_ip	0.23	0.21	0.33	0.43	0.46	0.45	0.42
GM_vix_nai	0.23	0.21	0.32	0.41	0.42	0.39	0.37
GM_vix_nfci	0.22	0.20	0.33	0.43	0.47	0.46	0.44
GARCH(1,1)	0.43	0.42	0.49	0.48	0.43	0.36	0.29

Table 2: Cumulative Mean Error of S&P 500 Volatility Predictions - Training period: 1991 - 2014. $n = 250$. For each horizon, the underlined value is the <u>minimum error</u>.

Graphical User Interface

Main plot

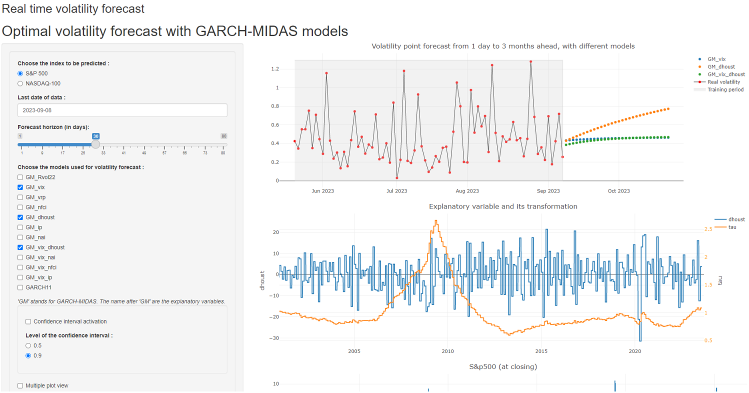

Figure 7: RShiny app - Part 1

Graphical User Interface

Average error

Models evaluation

Select another table number :

1

The following parameters were used:

Main index :	spx
Number of forecasts:	250
Training period :	1991-01-05/2015-01-01
Cumulative evaluation :	TRUE

QLIKE mean error

	1	2	5	10	22	44	66
GM_dhoust	0.30	0.26	0.36	0.42	0.40	0.33	0.29
GM_ip	0.32	0.28	0.35	0.39	0.36	0.29	0.23
GM_nai	0.30	0.26	0.34	0.39	0.36	0.29	0.23
GM_nfci	0.26	0.22	0.31	0.37	0.35	0.30	0.27
GM_Rvol22	0.27	0.23	0.33	0.39	0.37	0.30	0.24
GM_vix	0.20	0.16	0.28	0.40	0.46	0.45	0.44
GM_vrp	0.31	0.27	0.35	0.40	0.38	0.31	0.26
GM_vix_dhoust	0.23	0.21	0.34	0.43	0.47	0.46	0.44
GM_vix_ip	0.23	0.21	0.33	0.43	0.46	0.45	0.42
GM_vix_nai	0.24	0.21	0.32	0.41	0.43	0.40	0.37
GM_vix_nfci	0.22	0.20	0.33	0.43	0.47	0.46	0.44
GARCH(1,1)	0.43	0.42	0.49	0.49	0.43	0.36	0.29

Minimum mean error

	1	2	5	10	22	44	66
GM_dhoust							
GM_ip							
GM_nai						True	True
GM_nfci				True	True		
GM_Rvol22							
GM_vix	True	True	True				
GM_vrp							
GM_vix_dhoust							
GM_vix_ip							
GM_vix_nai							
GM_vix_nfci							
GARCH(1,1)							

Figure 8: RShiny - Part 2

Conclusion

- Utilizing explanatory variables in a GARCH-MIDAS model enhances prediction accuracy compared to a classical GARCH model.
- However, the effectiveness depends on selecting the appropriate explanatory variables for the appropriate horizon.
- Not all GARCH-MIDAS models consistently outperform the GARCH(1,1) model in certain test periods.

[1] Ole E. Barndorff-Nielsen and Neil Shephard. "Econometric Analysis of Realized Volatility and Its Use in Estimating Stochastic Volatility Models". In: *Journal of the Royal Statistical Society. Series B (Statistical Methodology)* 64.2 (2002), pp. 253–280. ISSN: 13697412, 14679868. URL: http://www.jstor.org/stable/3088799 (visited on 07/24/2023).

[2] Winston Chang et al. *shiny: Web Application Framework for R*. R package version 1.7.4. 2022. URL: https://CRAN.R-project.org/package=shiny.

[3] C. Conrad and O. Kleen. "Two are better than one: Volatility forecasting using multiplicative component GARCH-MIDAS models". In: *J Appl Econ.* 35 (2020), pp. 19–45. URL: https://doi.org/10.1002/jae.2742.

[4] Francis X Diebold and Robert S Mariano. "Comparing Predictive Accuracy". In: *Journal of Business & Economic*

Statistics 20.1 (2002), pp. 134–144. DOI: 10.1198/073500102753410444. eprint: https://doi.org/10.1198/073500102753410444. URL: https://doi.org/10.1198/073500102753410444.

[5] Ghysels E. Engle R. F. and Sohn B. "Stock market volatility and macroeconomic fundamentals". In: *Review of Economics and Statistics* 95 (2013), pp. 776–797.

[6] Alexios Ghalanos. *rugarch: Univariate GARCH models*. R package version 1.4-9. 2022.

[7] Onno Kleen. *alfred: Downloading Time Series from ALFRED Database for Various Vintage*. R package version 0.2.1. URL: https://github.com/onnokleen/alfred/.

[8] Onno Kleen. *mfGARCH: Mixed-Frequency GARCH Models*. R package version 0.2.1. 2021. URL: https://github.com/onnokleen/mfGARCH/.

[9] Carson Sievert. *Interactive Web-Based Data Visualization with R, plotly, and shiny*. Chapman and Hall/CRC, 2020. ISBN: 9781138331457. URL: https://plotly-r.com.