The Analysis Of Ship Mooring System Design Optimization

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Abstract

In this paper, mathematical model is established to solve the problem of optimal design of ship mooring system. When the inclination angle of steel bucket, steel pipe, the shape of anchor chain, and the draft and buoy area are as small as possible, the problem can be simplified into two dimensions physical and geometric problems in plane. The link chain can be regarded as a catenary, and the distribution of the chain and its force are calculated by deducing the catenary equation. The nodal masses are given by the concentrated-mass method, and the mechanical equations of the node are written by recursive method. In this paper, LINGO and MATLAB are used to solve the problem. what's more, multi-objective evaluation method and ideal solution are used to analyze the influence degrees of each factor to achieve the optimal solution.

For the problem One, the stress analysis of the mooring system is an important step to study its design optimization problem. In this paper, the nodal quality is given by the concentrated mass method, and the mechanics equations of the node are written by recursive method. The mechanics analysis of the anchor chain part is solved by deducing the catenary equations. Considering the effect of the wind force on the suspended state of anchor chain, the critical condition of mopping and all suspensions of anchor chain are calculated and solved, and the mathematic model of anchor chain suspension is modified to make it universal.

For the second problem,under the condition of the inclination angle of the steel bucket and the angle between the anchoring point and the seabed in a certain range of fluctuation, selecting the appropriate mass of the weight ball to make the angle of steel bucket buoy draft and the swimming area as small as possible, this problem can be transformed into a multi-objective optimization design problem. The optimal solution and the threshold solution are used to obtain the Compromise solutions and the satisfactory solutions. The influence of different parameters on the dependent variable is considered synthetically.

For the third problem, a mapping system of 6 input parameters and 4 output parameters is built on the basis of the model 1, and then 80 sets of system parameters are simulated in 4 environments. The optimal parameters of the mooring system are obtained by restraining the inclination angle of the steel bucket, the angle of the anchor chain at the point and the sea-bed, and the draft of the buoy. The weight of the anchor chain is at the angle between the point and the seabed, The optimal parameters of the mooring system are obtained.

Key words: Mooring system; Concentrated mass method; Catenary ideal solution; Multi-objective programming

Contents

1 Introduction

1.1 Background of the Problem

Buoy technology is widely used in marine environmental survey. Even under harsh marine environment conditions,it can also conduct a long-term,continuous, synchronized, automatic, comprehensive monitoring of the elements such as Ocean,hydrology and meteorology.The State Oceanic Administration has proposed,till 2020,China will build a comprehensive marine observation network,the initial formation of three-dimensional observation of the marine environment.The ocean observation network is of great value in marine hydrology and meteorological observation,resource exploitation,environmental pollutant diffusion and so on.The detection of the near-shallow sea environment is an important part of ocean observation network.The mooring system makes the transmission node of the observing network have the ability to resist the complex and changeable environment.,and ensure the operational requirement in the design environment and maintain the stability of the ocean observation network.

The purpose of this paper is to determine the type,length of the anchor chain and weight of the ball,so that the tilt angle of the bucket and its swimming area are as small as possible in different seawater velocity,wind speed and water depth,to reach optimization of mooring system design purposes.

1.2 Restatement of the Problem

Mooring system consists of steel pipe,steel drum,weight ball,anchor and welding chain composition,the use of anchor chain is the ordinary chain link without chain,in the appendix we can find: the anchor chain commonly used models and their parameters,steel bucket and anchor linking at the junction of the weight of the ball handing uncertainly,underwear acoustic communication system installed in steel drums. In order to optimize the operation of underwater acoustic communication equipment,the tangent direction between the end of anchor chain and anchor should be no more than $16°$ with the seabed and the inclination angle of steel drum should not exceed *5°*.

This paper needs to address the following issues:

(1)We should establish the mathematical model of change and analysis the law of various parameters. When the surface wind speed is *12m/s* and *24m/s*,we may seek answers to tilt angles of steel drums and sections of steel pipe,anchor chain shape,buoy draft and swimming area.

(2)We should establish the mathematical model to determined the minimum wind speed in which the anchor chains are suspended in seawater, and the inclination angle of the anchor chain and the swimming area of the buoy are calculated when the sea surface wind speed is *36m/s*.

(3)The model is established such that the inclination angle $\theta_5 < 5^\circ$ of the steel bucket,the anchor chain and the seabed at the anchor angle $\varphi < 16^\circ$ to determine the mass of the ball to achieve multi-objective satisfactory planning and solving the quality of the weight ball.

(4)Under the consideration of wind force,water flow and water depth, the mooring system with water depth between *16m* and *20m*, sea water velocity up to *1.5m/s* and wind speed up to *36m/s* is solved.

2 Model Assumptions And Analysis Before Modeling

- **Ignore the volume of anchor,anchor chain and weight ball,and the buoyancy of the sea they have withstood.**

Buoyancy formula: $\rho g v_{liquid}$ and they suffered the size of the weight of mg,then we can get:

$$\frac{\rho g v_{liquid}}{mg} = \frac{v_{liquid}}{solid}$$

As the mooring system use steel for hot-rolled steel bar,the density is $7.85\times 10^3 kg/m^3$,which is 7.66 times of the the sea water density of 1.0125×10^3,so it can be ignored.

- **The density of Steel pipe is uniform.**
- **The anchor chain is considered as an idealized Catenary model.**

The anchor chain is regarded as an ideal catenary model,ignoring the influence of hydrodynamics,elastic elongation of anchor chain and self vibration on the results.

- **The anchor chain doesn't have elastic extension force.**

Metal anchor chain is the rigid body,we ideally believed that after the force,the internal point of the relative position is same as before,it does not have the internal parts of the body between the role of elastic force.

- **Ignore the friction resistance of water.**

The flow resistance of the chain and steel pipe is

$$F_D = C_D \frac{1}{2} \rho V^2 A$$

;by stokes law on the ball in a viscous fluid motion,we can get:for a sphere with radius r,the total resistance in the fluid with viscous coefficient η at velocity υ is:

$$f = 6\pi\eta r\upsilon$$

- **The whole system is in the vertical two-dimensional plane,in order words,we only consider the height and the width of this system and the three-dimensional deformation isn't considered.**
- **The anchor is not blown by the wind.**
- **Ignore the inertia of the flow.**
- **Ignore the title of the float.**

3 Terminology

Symbols	Dedinitions
ρ	The density of sea water
g	Gravity constant
m_1	Buoy quality
v_{11}	Total Buoy Volume
v_1	Immersed Buoy Volume
h_{i1}	Immersed Buoy depth
h_1	Buoy height
m_2	Quality of steel pipes
v_2	Steel pipes volume
h_2	Steel pipes height
m_3	Steel drum quality
v_3	Steel drum volume
h_3	Steel drum height
m_4	Quality of weight ball
m_5	Quality of anchor chain
R_B	The radius of the buoy swimming
F	Horizontal wind power
ω	The weight of the length of an anchor chain
H	Water Depth

4 Analysis And Solutions Of The Problem

4.1 Problem One

4.1.1 Analysis and Modeling of Force of Steel Tube Part

The connecting nodes of each part are taken as the object of study. The stress analysis graphs are shown in **Figure 1**. Node 1 connects the end of iron and the weight ball, node 2 connects steel pipes and iron bucket, nodes 3 to 5 are connected with two steel pipe respectively, and node 6 connects buoy with the last steel pipe.

With the concentrated mass method, each node has half the mass of the connecting part on both sides. For example, the mass m_1 of the node 6 is equal to half of the buoy and the mass of the fourth steel pipe. Similarly, each node also has the half of adjacent buoyancy.In addition to the buoyancy and gravity that each node has, each node also has a pull from the object in contact with it. The first node is subjected to the force of the steel drum and the gravitational sphere, and the second node is subjected to the first root Steel pipe and drum

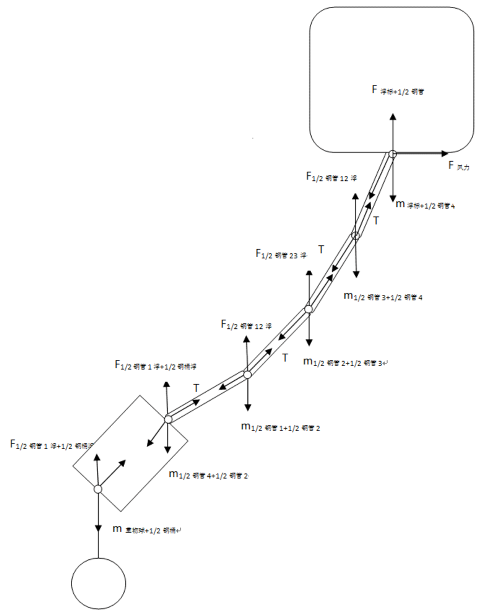

Figure 1: **Figure of partial stress analysis of steel pipe**

tension, the first 3 to 5 nodes on both sides are subject to steel pipe tension,with the sixth node by the buoy and the fourth root of the tensile force.

The tensile force on the node along the rod, gravity, buoyancy acting in the vertical direction, the wind pressure of the 6th node is horizontal, the rest of the tension can be decomposed into the horizontal and vertical directions, which is easy to get : the horizontal force or force components are equal to each other, the size of the sea breeze is the size of the wind.

Using the recursive method, the mechanical equilibrium equations of the six nodes are as follows:

For the first node:

$$F\tan\varphi + (\frac{m_3}{2} + m_4)g = F\tan\theta_1 + \rho g(\frac{v_3}{2}) \tag{1}$$

θ_i represents the angle between the (i-1)th steel pipe of the horizontal direction and the steel pipe clamped in the direction of the horizontal direction,$(1 \leq i \leq 5, i \in \mathbf{Z})$,with m_2 is the mass of the steel tube, m_3 is the mass of the steel drum, v_2 is the volume of the steel pipe, and v_3 is the volume of the iron drum.

For the second node:

$$F\tan\theta_1 + (\frac{m_2}{2} + \frac{m_3}{2})g = F\tan\theta_2 + \rho g(\frac{v_2}{2} + \frac{v_3}{2}) \tag{2}$$

For the third node:

$$F\tan\theta_2 + (m_2)g = F\tan\theta_3 + \rho g(v_2) \tag{3}$$

For the fourth node:

$$F\tan\theta_3 + (m_2)g = F\tan\theta_4 + \rho g(v_2) \tag{4}$$

For the fifth node:

$$F\tan\theta_4 + (m_2)g = F\tan\theta_5 + \rho g(v_2) \tag{5}$$

For the sixth node:

$$F\tan\theta_1 + (\frac{m_2}{2} + m_1 g = \rho g(v_1 + \frac{v_2}{2}) \tag{6}$$

$(\frac{m_3}{2} + m_4)$ g is the mass of nodal 1 under the concentrated mass method, which is equivalent to the addition of half of mass of the iron drum and weight of ball, and F is the horizontal sea wind, φ is the angle between the chain tangent and the horizontal direction at the connection of the anchor chain and the half of iron bucket , ρ is the density of the sea water, g is the acceleration of gravity, and v_3 is the volume of the iron bucket.

4.1.2 Stress Analysis and Modeling of Anchor Chain and Derivation of Anchor Chain Equation

Catenary is a non-extended homogeneous chain or lock hanging at two points on the formation of the curve,due to sea water flow and other errors, the general activities of the anchor chain don't exactly match the real world, but under the constraints of this problem, we can still use the catenary to describe the anchor chain, ignoring the influence of other factors such as dynamic vibration.

We derive the catenary equation using the micro-element method. The micro-element of the catenary is shown in Figure 2.

L is the length of the catenary, T is the tension of the line,φ is the direction angle of the tension, dφ is the change of the internal tension of the micro element, ω is the weight of the unit length of the anchor chain in the water.

The analysis are as follows, in the normal direction of the chain element:

$$T\mathrm{d}x = (\omega\cos\varphi)\mathrm{d}l \tag{7}$$

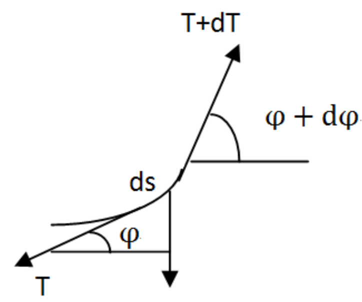

Figure 2: **Figure of Stress Analysis of micro-element**

In the vertical direction of the catenary:

$$\mathrm{d}T = (\omega \cos\varphi)\mathrm{d}l \tag{8}$$

Divide the two types can be:

$$\frac{\mathrm{d}T}{T} = \tan\varphi \mathrm{d}\varphi \tag{9}$$

(9) in the interval (φ_0, φ) within the range of integration:

$$\ln\frac{T}{T_0} = \mathrm{d}y \tag{10}$$

T_0 for the anchor, anchor link anchor chain tension, φ_0 for the angle of anchor, anchor chain link with the sea bed.

Formula is simplified as

$$T = T_0\frac{\cos\varphi_0}{\cos\varphi} \tag{11}$$

Where T_0 is the catenary tension in the φ_0, which can be obtained in the catenary of the various parts of the horizontal tension equal to the level of the buoy at the water level is equal to the level of tension that F.

(11) is substituted into (7) and the catenary is integrated from the origin to the point where the chain length is 1. Assuming that the angles between the catenary and the horizontal in the two points is φ_0, φ, then it has:

$$\int_{-\infty}^{\infty} f(x)\mathrm{d}x \tag{12}$$

According to the catenary to:

$$\mathrm{d}z = \sin\varphi \mathrm{d}l \tag{13}$$

(4.7) can be obtained by integrating:

$$z = \frac{1}{\omega}\int_{\varphi_0}^{\varphi} \frac{T_0 \cos\varphi_0 \sin\varphi}{\cos\varphi^2} \mathrm{d}\varphi = \frac{F}{\omega}(\frac{1}{\cos\varphi} - \frac{1}{\cos\varphi_0}) \tag{14}$$

Similarly available, we can get:

$$\mathrm{d}x = \sin\varphi \mathrm{d}l \tag{15}$$

Bring into (7), there are:

$$x = \frac{1}{\omega}\int_{\varphi_0}^{\varphi} \frac{T_0 \cos\varphi_0 \sin\varphi}{\cos\varphi^2} \mathrm{d}\varphi = \frac{F}{\omega}[(\ln\frac{1}{\cos\varphi} + \tan\varphi) - (\ln\frac{1}{\cos\varphi_0} + \tan\varphi_0)] \tag{16}$$

Equation (4.12) shows the length of any two points on the chain, equation (14) represents the magnitude of the height direction component of the distance between any two points on the chain. Equation (4.16) represents the level of the distance between any two points on the chain Component size.

4.1.3 Solving Process

- (1) **Geometric relationship**

According to the differential equation of the chain equation (14):

$$z = \frac{1}{\omega}(\frac{1}{\cos\varphi} - \frac{1}{\cos\varphi_0}) \tag{17}$$

According to the (12),we can get:

$$l = \frac{1}{\omega}(\tan\varphi - \tan\varphi_0) \tag{18}$$

The length of the chain in the horizontal direction is:

$$x = \frac{F}{\omega}(\frac{1}{\cos\varphi} - \frac{1}{\cos\varphi_0}) \tag{19}$$

The height of the whole system is:

$$H = z + s(\sin\theta_1 + \sin\theta_2 + \sin\theta_3 + \sin\theta_4 + \sin\theta + \sin\theta_5) + \frac{v_1}{\pi R^2} \quad (20)$$

The radius of the buoy swimming is:

$$R_B = x + s(\cos\theta_1 + \cos\theta_2 + \cos\theta_3 + \cos\theta_4 + \cos\theta_5) \quad (21)$$

- (2) **Mechanical relationship**

Wind force is:

$$F = 0.625 \times Sv^2 \quad (22)$$

Among them:

$$S = h_1(\frac{v_{11} - v_1}{\pi R^2}) \quad (23)$$

From the above two equations and equations (1) to (6), the minimum wind speed of the chain without dragging (all suspended in seawater) is 24.516m/s, which is greater than the two wind speeds given by Problem 1, which explained that v at 12m/s and 24m/s, the chain part is mooring, ,so that $\varphi_0 = 0°$, and we modify the catenary model:

$$\frac{x\omega}{F} = \ln(\frac{1 + \sin\varphi}{\cos\varphi}) \quad (24)$$

(4.19) is written as:

$$\coth\frac{x\omega}{F} = \tan\varphi \quad (25)$$

then we can get:

$$z = \frac{F}{\omega}(\frac{1}{\cos\varphi} - 1) = \frac{1}{\omega}(\coth\frac{x\omega}{F} - 1) \quad (26)$$

Simplify (17) and (18) again, we can get:

$$l = \frac{F}{\omega}\sinh\frac{x\omega}{F} \quad (27)$$

(27) is the shape equation of the suspended part of the chain.

4.2 Problem Two

4.2.1 Analysis of Problem Two

According to the model established in this paper, the sea surface wind speed v is 36m/s, which is larger than the minimum wind speed of the anchor chain without dragging the ground 24.51608 m/s. Therefore, the anchor chain is suspended in the sea water. Using the first model, we can get the solution of the pitch angle of each steel pipe section, the shape of the chain and the swimming area of the buoy. The problem which adjusts the mass of the weight ball so that the inclination angle of the steel bucket does not exceed 5 °and the angle of the anchor chain at the anchor point to the seabed does not exceed 16 °is converted to a multi-objective inclination angle $\theta_5 \leq 5°$, the anchoring point and the sea bed in the anchor angle $\varphi \leq 16°$conditions, the use of the ideal solution **[3]** and the threshold constraint method **[3]** to solve the compromise solution and satisfactory solution, taking the impact of different parameters on the dependent variable into account, and the selection of the appropriate weight ball makes the steel drum ,the section of steel pipe tilt angle,anchor chain shape and buoy swimming area of multiple targets range as small as possible a optimization design problem.

4.2.2 Modeling of Problem Two By using the sea surface wind speed v = 36m/s

The second part of the problem is based on the first problem, and after determining the wind speed of 36m/s making the anchor chain completely suspended, the known condition will be brought into the mathematical model of Problem 1 to get the solution.

4.2.3 Modeling of solving weight ball quality

The effective solution of the model is hard to find, we consider the ideal solution, that is, the compromise solution of the model, consider the following optimization problem:

In this paper,the multi-objective programming method is used to establish the ideal solution model as follows:

$$E = \min u_1(\frac{R_B^2}{R_{BM}^2})^2 + u_2(\frac{Y_1^2}{Y_{1M}^2})^2 + u_3(\frac{V_B^2}{V_{1M}^2})^2 \tag{28}$$

$$\boldsymbol{s.t.} \begin{cases} 18.849 \leq R_B \leq 19.502 \\ 2.918 \leq Y_1 \leq 4.279 \\ 0.032 \leq V_1 \leq 0.091 \\ m_4 \leq 5380 \\ \varphi_1 \leq 0.28 \\ 1.48 \leq \theta_1 \leq 1.57 \end{cases} \tag{29}$$

Among them:

$$u_1 + u_2 + u_3 = 1 \tag{30}$$

$$Y_1 = \cos\theta_1 \tag{31}$$

R_B, R_{BM} andV_M are reference values of R_B, Y_1, and V_1.

The constraint equations ofR_{BM}minimum are as follows:

$$\textbf{\textit{s.t.}} \begin{cases} z = \frac{1}{\omega}(\frac{1}{\cos\varphi} - \frac{1}{\cos\varphi_0}) \\ F\tan\varphi + (\frac{m_3}{2} + m_4)g = F\tan\theta_1 + \rho g(\frac{v_3}{2}) \\ F\tan\theta_1 + (\frac{m_2}{2} + \frac{m_3}{2})g = F\tan\theta_2 + \rho g(\frac{v_2}{2} + \frac{v_3}{2}) \\ F\tan\theta_2 + (m_2)g = F\tan\theta_3 + \rho g(v_2) \\ F\tan\theta_3 + (m_2)g = F\tan\theta_4 + \rho g(v_2) \\ F\tan\theta_4 + (m_2)g = F\tan\theta_5 + \rho g(v_2) \\ F\tan\theta_1 + (\frac{m_2}{2} + m_1 g = \rho g(v_1 + \frac{v_2}{2}) \\ l = \frac{1}{\omega}(\tan\varphi - \tan\varphi_0) \\ x = \frac{F}{\omega}(\frac{1}{\cos\varphi} - \frac{1}{\cos\varphi_0}) \\ H = z + s(\sin\theta_1 + \sin\theta_2 + \sin\theta_3 + \sin\theta_4 + \sin\theta + \sin\theta_5) + \frac{v_1}{\pi R^2} \\ R_B = x + s(\cos\theta_1 + \cos\theta_2 + \cos\theta_3 + \cos\theta_4 + \cos\theta_5) \\ F = 0.625 \times Sv^2 \\ S = h_1(\frac{v_{11} - v_1}{\pi R^2}) \\ \varphi_1 \leq 0.28 \\ 1.48 \leq \theta_1 \leq 1.57 \end{cases} \tag{32}$$

Similarly, the maximum value R_{BM} and the threshold values Y_{1M} and V_{1M} can be obtained.

4.3 Problem Three

4.3.1 Analysis of Mooring System under the influence of Tide and Other Factors

On the basis of model 1, the effect of water flow is considered and a mapping system of 6 input parameters and 4 output parameters is established. The output parameters are immersed buoy depth h_{i1}, buoy swimming area radius R_B, mooring line end inclination angle φ, drum inclination angle θ_4 and so on as small as possible. Input parameters can be divided into two categories: environmental factors (sea surface wind speed F, sea water depth H, sea water velocity v), mooring system parameters (weight of the weight of the ball m_4, chain model $model_5$, chain length l).

The design problem of the mooring system is to determine the type and length of the chain and the mass of the weight ball so that the buoy's draft,the angle between the anchor and the seabed,and the inclination of the steel bucket are as small as possible. The change of environmental factors will affect the parameters of mooring system, so the design of mooring system needs to take the changes of wind, water flow and water depth into account. In this problem, the layout of the sea water depth measured between 16m and 20m. The maximum speed of sea water can reach 1.5m/s and wind speed can reach 36m/s.

In this paper, 80 systems are designed, which correspond to 80 sets of system parameters (chain types, length and mass of the weight ball). In four different environments (high winds, high currents, high winds, low winds and low winds), the buoyancy depths of the 80 systems are solved. The anchors are anchored to the sea bed of the angle and steel drum tilt angle. Finally, three optimal solutions are obtained, and further considering the water depth, an optimal solution is obtained. The optimal mooring system has strong ability of resisting environmental changes and satisfies the angle constraint in all four environments.

4.3.2 Modeling and results of solving Problem three

The model is as follows:

$$\begin{cases} z = \frac{1}{\omega}(\frac{1}{\cos\varphi} - \frac{1}{\cos\varphi_0}) \\ F\tan\varphi + (\frac{m_3}{2} + m_4)g = F\tan\theta_1 + \rho g(\frac{v_3}{2}) \\ F\tan\theta_1 + (\frac{m_2}{2} + \frac{m_3}{2})g = F\tan\theta_2 + \rho g(\frac{v_2}{2} + \frac{v_3}{2}) \\ F\tan\theta_2 + (m_2)g = F\tan\theta_3 + \rho g(v_2) \\ F\tan\theta_3 + (m_2)g = F\tan\theta_4 + \rho g(v_2) \\ F\tan\theta_4 + (m_2)g = F\tan\theta_5 + \rho g(v_2) \\ F\tan\theta_1 + (\frac{m_2}{2} + m_1 g = \rho g(v_1 + \frac{v_2}{2}) \\ l = \frac{1}{\omega}(\tan\varphi - \tan\varphi_0) \\ x = \frac{F}{\omega}(\frac{1}{\cos\varphi} - \frac{1}{\cos\varphi_0}) \\ H = z + s(\sin\theta_1 + \sin\theta_2 + \sin\theta_3 + \sin\theta_4 + \sin\theta + \sin\theta_5) + \frac{v_1}{\pi R^2} \\ R_B = x + s(\cos\theta_1 + \cos\theta_2 + \cos\theta_3 + \cos\theta_4 + \cos\theta_5) \\ F = 0.625 \times Sv^2 \\ S = h_1(\frac{v_{11} - v_1}{\pi R^2}) \\ \varphi_1 \leq 0.28 \\ 1.48 \leq \theta_1 \leq 1.57 \\ v_1 = \pi R^2 h_{i1} \end{cases} \tag{33}$$

Similar to model 1, model 3 establishes the mapping of the buoy's draft to depth h_{i1}. A given depth of water H to determine the draft depth h_{i1}, and then get the mooring system of the other parameters, thus establishing a 6 input parameters, 4 output parameters of the mapping system.

5 Conclusion

5.1 Evaluation and results of Problem one

The data given in question 1 is replaced by the type II anchor chain which is 22.05 m, the mass of the weight ball m_4 is 1200 kg, the water depth h is 18 m, the sea water density ρ is 1.025 $\times 10^3$ kg/m_3 and the sea surface winds v are 12 m/s and 24m/s, using LINGO to get the solution for the results:

When v=12m/s,we can get that x=7.397m, F=227.74N, R_B=8.483m, h_{v1}=0.735m, l=15.23m.

Table 1: Wind speed of 12m / s steel drum, steel pipe and the horizontal angle

θ_1	θ_2	θ_3	θ_4	θ_5
1.553198 rad	1.553432 rad	1.553535 rad	1.553636 rad	1.553737 rad
88.986°	89.005°	89.011°	89.016°89.022 °	

The shape of the chain is given by the following x and z curves:

$$z=\begin{cases} 0 & x \leq 6.82 \\ 3.32\cosh(\frac{x-6.82}{3.32}-1) & 6.82 \leq x \leq 14 \end{cases} \tag{34}$$

The float range of the buoy is a circle with the radius of R_B + 22.05 - l = 15.305m, where the vertical line of the anchor is at the intersection with sea level and the draft is h_{v1} = 0.735m.

When v=24m/s,we can get that x=16.780m, F=900.78N, R_B=18.11m, h_{v1}=0.749m, l=21.3m.

Table 2: Wind speed of 12m / s steel drum, steel pipe and the horizontal angle

θ_1	θ_2	θ_3	θ_4	θ_5
1.503605 rad	1.504465 rad	1.504845 rad	1.505220 rad	1.505591 rad
89.150°	86.199°	86.221°	86.243°86.264°	

The shape of the chain is given by the following x and z curves:

$$z=\begin{cases} 0 & x \leq 6.82 \\ 3.32\cosh(\frac{x-6.82}{3.32}-1) & 6.82 \leq x \leq 14 \end{cases} \tag{35}$$

The float range of the buoy is a circle with the radius of R_B + 22.05 - l = 18.43m, where the vertical line of the anchor is at the intersection with sea level and the draft is h_{v1} = 0.749m.

5.2 Evaluation and results of Problem Two

5.2.1 Evaluation of Problem Two By using the sea surface wind speed v = 36m/s

When we put the sea wind velocity v which is 36m / s into the problem in a model, we can solve that : x=18.035m, R_B=19.72m, h_{v1} = 0.760 m, θ_0 = 0.3127 rad = 17.916°, F=1993N.

Note that at this time $\varphi_0 > 16$ °, according to the requirements of the anchor should be dragged, but we can assume that the anchor is fixed, there are : buoy swimming range for a anchor vertical line and the sea level at the intersection, the radius R_B = 18.43 m.

Table 3: Wind speed of 36m / s steel drum, steel pipe and the horizontal angle

θ_1	θ_2	θ_3	θ_4	θ_5
1.429934 rad	1.431636 rad	1.432388 rad	1.43313 rad	1.433868 rad
81.929°	82.027°	82.070°	82.112°	82.155°

The shape of the chain is given by the following x and z curves:

$$z = 29.05\cosh(\frac{x}{29.05} - 1) \quad 0 \leq z \leq 18.025 \tag{36}$$

The lingo procedure is shown in Appendix II.

5.2.2 Evaluation of solving weight ball quality

First of all to determine the range of m_4 to meet the requirements, so $v_1 = v_{11}$,which brought into the question in the given data: II type anchor chain is 22.05m, water depth h is 18m, sea water density ρ =1.025×103kg/m^3.We can solve the m_4 maximum value of 5380kg; Let θ_1 = 1.48, then we can solve the m_4 minimum value of 1730kg.

To determine the optimal solution of m_4, we should first determine the reference value: Y_{1M} take the maximum value of b_1 = 85°when the cosine value is 0.0907,the buoy volume V_{1M} is 6.283 m_2, R_{BM} for the reference radius, should take the radius of the vicinity of the value in the range, may wish to take as 20m.

When we get the ideal solution, the analysis can be obtained when the weight is u_3 in the overall proportion of 50% - 66.7% is the most reasonable, which can be the ideal solution, so we can take u_1: u_2: u_3 = 1: 1: 3 weight Assignment calculation. The weight of the ball is $m_4 = 2518$kg.

If we take a value of m_4, such that R_B, Y_1, and V_1 are within the respective thresholds, the m_4 is considered to be a satisfactory value for each of R_B, Y_1, and V_1, and then return to the original model solution. For the R_B, the range of change is not large, this paper will be the threshold value of 20, for Y_1, the range of θ_1 in 85°to 90 °, this paper take $\theta_1 = 87$ °cosine value for the threshold, that is 0.052336, In the case of V_1, the draft volume at the draft of 1.2 is 3.770 .

Comparing the R_B, y_1, v_1 data of the compromise solution of the ideal solution with the threshold value, we find that the compromise solution is a satisfactory solution. So the value of m_4 at this time a possible answer to the problem.

The lingo procedure is shown in Appendix III.

5.3 Evaluation and results of Problem Three

Similar to model 1, model 3 establishes the mapping of the buoy's draft to depth h. Given depth h of water H to determine the depth of draft and then get the other parameters of the mooring system which are obtained, so that a mapping system of 6 input parameters and 4 output parameters are established.

Table 4: Water depth,wind speed and flow rate correspond to different environments

Environments	Water depth	Wind speed	Flow rate
High winds high flow high tide	20	32	1.3
High winds high flow low tide	16	32	1.3
Low wind bottom flow high tide	20	12	0.5
Low wind bottom flow low tide	16	12	0.5

The weight of the weight ball is divided into four values (1200, 2000, 3000, 4000). The mooring chain type is divided into five categories, that is(20,22,24,26), these three parameters constitute 80 groups of mooring system parameters (parameter numbers see appendix), in the four kinds of different environments, the 80 systems to solve their buoy draft depth, the anchor chain in the anchor point and the seabed of the angle, steel drum tilt Angle, the data obtained in the appendix IV.

In 4 different environments, respectively, to select the steel drum to meet the tilt angle of not more than 5°, the anchor chain at the anchor point and seabed folder Angle of not more than 16 degrees of the system parameters, and then take the intersection to get three groups of feasible system parameters, they can be in 4 environments to maintain the above conditions, the parameter number is 52,64,72, which represents the system parameters, see Table 6.

Table 5: The weight of the ball, the type of chain and the chain length corresponding to Different parameter numbers

Different parameter numbers	Chain type	weight of ball	Chain length
52	3000	III	26
64	4000	II	26
72	4000	III	26

If we further consider the depth of the water into the water as small as possible, and the second question in the assumptions, so that the water depth ¡1.5, can be a group.The optimum mooring system parameter (parameter number 52), which maintains water depths below 1.5 in all four cases,Angle of not more than 5 degrees, the anchor chain in the anchor point and the seabed of the angle does not exceed 16 degrees. This can be No. 52 parameters for the optimal solution, in addition 64 and 72 are suboptimal solutions.

6 Error Analysis

- **(1)** There are two cases of catenary part:one is that the catenary partially suspended in the seawater(the other part of it mopping),the other is that the catenary all suspended in the seawater.As the situation is unknown,so the result will have some errors.
- **(2)** The deduction of the catenary line ignores the pressure of the water and the buoyancy of the anchor,weight ball and anchor chain,causing a certain error.
- **(3)** The shape of the catenary with links is approximated as a smooth curve without inelastic ductility modeling analysis,there is a certain error.
- **(4)** The hydraulic force actually acts on each section of the anchor chain,but when the model is built,it's simplified as the force on the whole model.
- **(5)** In the case of multi-objective programming, errors may occur due to the difference in the selected reference values during the dimensionless processing and linear weighting function construction.

7 Evaluations of solutions

7.1 Strengths

- **(1) Reasonable Assumptions:** By studying a large number of documents,this paper has established a series of scientific and reasonable assumptions,which can greatly reduce the complexity of the problem and obtain good medeling effect,irrespective of the variables with less influence on the dependent variable.
- **(2) Scientific Modeling:** This paper studied the static state of the mooring system in the water.When the seawater is at rest,there is no relative motion between the seawater and the anchor system,so the anchor system is not affected by the hydrodynamic and can be simplified into catenary form.
- **(3) Generalizing Model:** In the second and third solutions,this paper proposes an algorithm to achieve the best working effect,that is,to make the draft of the buoy as small as possible,and make the feasible solution model.The model has certain expansibility.
- **(4) high Efficiency of Evaluation:** In this paper,the time complexity of the optimal solution of the multi-objective programming problem is small,and the result can be calculated in a short time,which is convenient and quick.

7.2 Weaknesses

- **(1)** Threshold must be set reasonably. If the threshold is set incorrectly,the result may have more errors.
- **(2)** In setting up the model,the chain volume is not taken into account,which resulting in errors.
- **(3)** The use of catenary cable instead of chain analysis,neglects some of the less affected force,the result of a certain error.

8 References

[1] GB/T20848-2007Mooring chain steel.

[2] Henian LIU.Fluid mechanics[M].Beijing:China Building Industry Press.2004.

[3] Baodong LIU,Jie Su,Jianliang Chen.Mathematical Modeling Basic Course[M].Beijing:Higher Education Press,2015.

9 appendix

9.1 appendix I

```
data:
H=18;
v11=6.2832;
m1=1000;!;
m2=10;!;
m3=100;!;
s=1;!;
m4=1200;!;
m5=600;!;
R=1;!;
h1=2;!;
h2=1;!;
r2=0.025;!;
r3=0.15;!;
h3=1;!;
p=1025;!;
v=12;!12,24;
g=9.8;
w=68.6;
v2=0.0019635;
v3=0.07068578;
a0=0;
enddata
F=0.625*2/3.1415926*(v11-v1)*v^2;%*424,*9;
F*(@tan(a)-@tan(a0))=l*w;
y=@tan(A);
z=F/68.6*(1/@cos(a)-1/@cos(a0));
H=z+@sin(b1)+@sin(b2)+@sin(b3)+@sin(b4)+@sin(b5)+v1/(3.1415926*R^2);
F*@tan(a)+(m3/2+m4)*g=F*@tan(b1)+p*g*v3/2;
F*@tan(b1)+(m2/2+m3/2)*g=F*@tan(b2)+p*g*(v3/2+v2/2);
F*@tan(b2)+m2*g=F*@tan(b3)+p*g*v2;
F*@tan(b3)+m2*g=F*@tan(b4)+p*g*v2;
F*@tan(b4)+m2*g=F*@tan(b5)+p*g*v2;
F*@tan(b5)+(m1+m2/2)*g=p*g*(v1+v2/2);
x=F/w*(@log(1/@cos(a)+@tan(a))-@log(1/@cos(a0)+@tan(a0)));
y1=@cos(b1);
b1<=1.57;
b1>=1.48;
rb=x+@cos(b1)+@cos(b2)+@cos(b3)+@cos(b4)+@cos(b5)+r;
```

9.2 appendix II

```
data:
H=18;
v11=6.2832;
m1=1000;!;
m2=10;!;
m3=100;!;
s=1;!;
m4=1200;!;
m5=600;!;
R=1;!;
h1=2;!;
h2=1;!;
r2=0.025;!;
r3=0.15;!;
h3=1;!;
p=1025;!;
v=36;!12,24;
l=22.05;!;
g=9.8;!;
w=68.6;!;
v2=0.0019635;!;
v3=0.07068578;!;
enddata
F=0.625*2/3.1415926*(v11-v1)*v^2;!*424,*9;
F*(@tan(a)-@tan(a0))=l*w;
y=@tan(A);
z=F/68.6*(1/@cos(a)-1/@cos(a0));
H=z+@sin(b1)+@sin(b2)+@sin(b3)+@sin(b4)+@sin(b5)+v1/(3.1415926*R^2);
F*@tan(a)+(m3/2+m4)*g=F*@tan(b1)+p*g*v3/2;
F*@tan(b1)+(m2/2+m3/2)*g=F*@tan(b2)+p*g*(v3/2+v2/2);
F*@tan(b2)+m2*g=F*@tan(b3)+p*g*v2;
F*@tan(b3)+m2*g=F*@tan(b4)+p*g*v2;
F*@tan(b4)+m2*g=F*@tan(b5)+p*g*v2;
F*@tan(b5)+(m1+m2/2)*g=p*g*(v1+v2/2);
x=F/w*(@log(1/@cos(a)+@tan(a))-@log(1/@cos(a0)+@tan(a0)));
```

9.3 appendix III

```
data:
H=18;
v11=6.2832;
m1=1000;!;
m2=10;!;
m3=100;!;
```

```
s=1;!;
m5=600;!;
R=1;!;
h1=2;!;
h2=1;!;
r2=0.025;!;
r3=0.15;!;
h3=1;!;
p=1025;!;
v=36;!12,24;
l=22.05;
g=9.8;
w=68.6;
v2=0.0019635;
v3=0.07068578;
Y1m=0.09067162;
V1m=6.2831852;
RBm=20;
enddata
F*@tan(a)+(m1+4*m2+m3+m4)*g=p*g*(v1+4*v2+v3);
F=0.625*2/3.1415926*(v11-v1)*v^2;!*424,*9;
F*(@tan(a)-@tan(a0))=l*w;
y=@tan(a);
y1=@cos(b1);
z=F/68.6*(1/@cos(a)-1/@cos(a0));
H=z+@sin(b1)+@sin(b2)+@sin(b3)+@sin(b4)+@sin(b5)+v1/(3.1415926*R^2);
F*@tan(a)+(m3/2+m4)*g=F*@tan(b1)+p*g*v3/2;
F*@tan(b1)+(m2/2+m3/2)*g=F*@tan(b2)+p*g*(v3/2+v2/2);
F*@tan(b2)+m2*g=F*@tan(b3)+p*g*v2;
F*@tan(b3)+m2*g=F*@tan(b4)+p*g*v2;
F*@tan(b4)+m2*g=F*@tan(b5)+p*g*v2;
F*@tan(b5)+(m1+m2/2)*g=p*g*(v1+v2/2);
x=F/w*(@log(1/@cos(a)+@tan(a))-@log(1/@cos(a0)+@tan(a0)));
rb=x+@cos(b1)+@cos(b2)+@cos(b3)+@cos(b4)+@cos(b5)+r;
min=((rb^2/rbm^2)^2+(y1/y1m)^2+3*(v1/v1m)^2);!;
m4<=5380;
a0<=0.28;
-b1<=-1.48;
b1<=1.57;
```

9.4 appendix IV

```
clc
clear
tic
close all
```

```
%
H=16;%
v=32;%
u=1.3;%
%
m=[1200 2000 3000 4000];%
kinde=[1 2 3 4 5];%
Le=[20,22,24,26];%
index=1;
data=[];
h=waitbar(1/80,'ing...');
for i=1:4
data(index,1)=m(i);
for j=1:5
data(index,2)=kinde(j);
for k=1:4
data(index,3)=Le(k);
[data(index,5),data(index,6),data(index,7),data(index,8)]=...
main(m(i),v,H,kinde(j),Le(k),u);
index=index+1;
waitbar(index/80,h);
end
end
end
%[h0,L,t1,t2]=main(m,v,H,kinde,Le,u)
\clearpage
```