Particle Acceleration Hands-On

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Motion equation for charged particle in electromagnetic field

$$\frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$\vec{p} = \gamma m \vec{v}$	- particle relativistic momentum
$m,\ q$	- particle mass and charge
$\vec{E} = -\vec{u} \times \vec{B} + \eta \vec{J}$	- electric field
$\vec{B},\ \vec{J} = \frac{1}{\mu_0} \nabla \times \vec{B}$	- magnetic field vector and current density
$\vec{v},\ \vec{u}$	- particle and fluid velocity vectors
$\gamma = \left[1 - (v/c)^2\right]^{1/2}$	- Lorentz factor
$\eta,\ \mu_0$	- electric resistivity coefficient and permeability

Ignoring the plasma resistivity, the motion equation can be rewritten as

$$\frac{d}{dt}\left(\gamma\vec{v}\right) = \frac{q}{m}\left[\left(\vec{v} - \vec{u}\right) \times \vec{B}\right]$$

How to find particle position?

$$\frac{d\vec{r}}{dt} = \vec{v}$$

Combining both equation together we can write the motion equation as one vector equation

$$\frac{d}{dt}\left\{\begin{array}{c}\gamma v_x\\ \gamma v_y\\ \gamma v_z\\ x\\ y\\ z\end{array}\right\} = \left\{\begin{array}{c}\frac{q}{m}\left[\left(v_y-u_y\right)B_z-\left(v_z-u_z\right)B_y\right]\\ \frac{q}{m}\left[\left(v_z-u_z\right)B_x-\left(v_x-u_x\right)B_z\right]\\ \frac{q}{m}\left[\left(v_x-u_x\right)B_y-\left(v_y-u_y\right)B_x\right]\\ v_x\\ v_y\\ v_z\end{array}\right\}$$

Or simply

$$\frac{d\vec{y}}{dt}=\vec{f}$$

We have to integrate the motion equation numerically.

$$\frac{d\vec{y}}{dt} = \vec{f}$$

Simple choice, 4[th] order Runge-Kutta method:

$$y_{n+1} = y_n + \frac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right), \qquad t_{n+1} = t_n + h$$

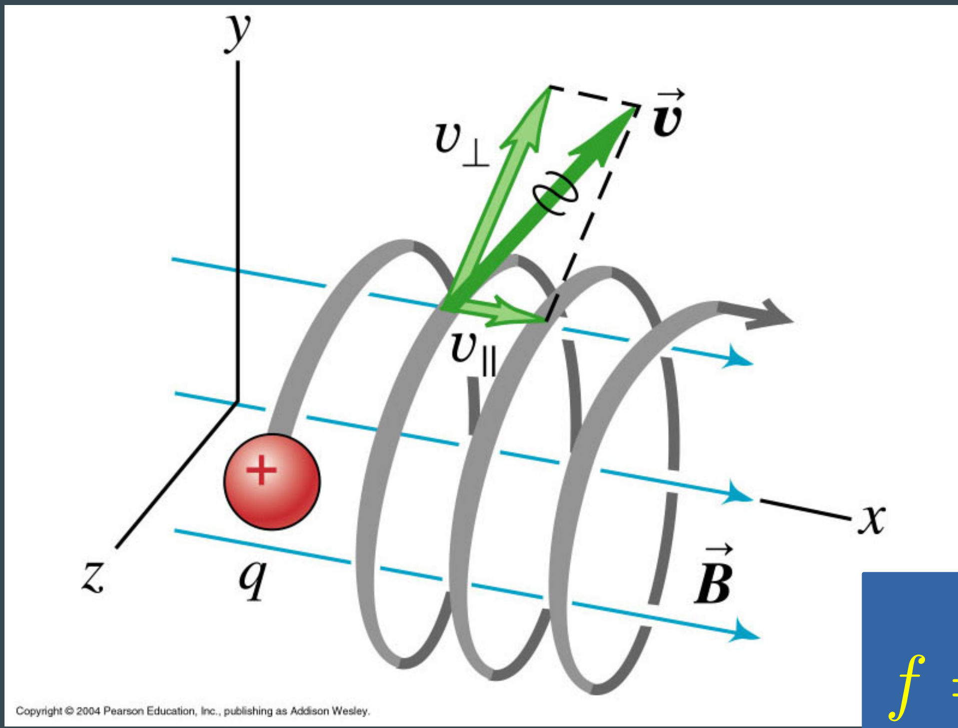
$$k_1 = f(t_n, y_n)$$

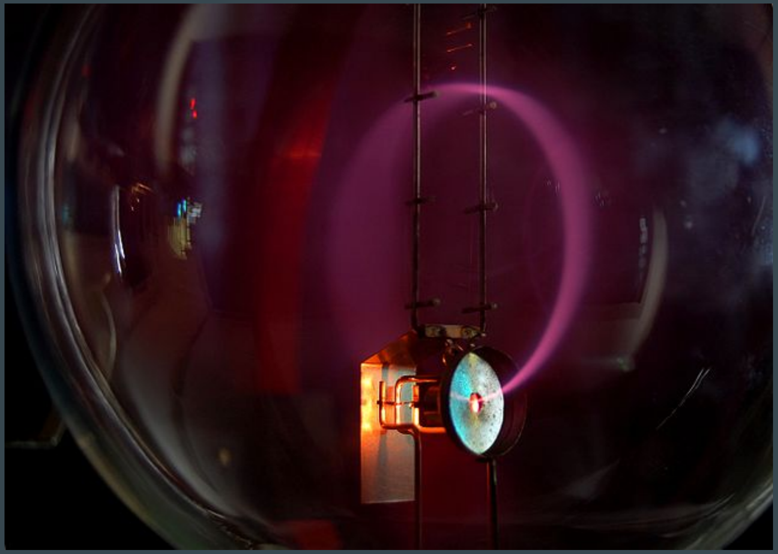
$$k_2 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1)$$

$$k_3 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2)$$

$$k_4 = f(t_n + h, y_n + hk_3)$$

Motion of test particle in uniform magnetic field





Gyroperiod:

$$T = \frac{2\pi}{\omega} = \frac{2\pi r}{v} = \frac{2\pi\gamma m}{|q|B}$$

Gyrofrequency:

$$f = \frac{1}{T} = \frac{|q|B}{2\pi\gamma m} \Rightarrow \omega = 2\pi f = \frac{|q|B}{\gamma m}$$

Gyro- or Larmor radius:

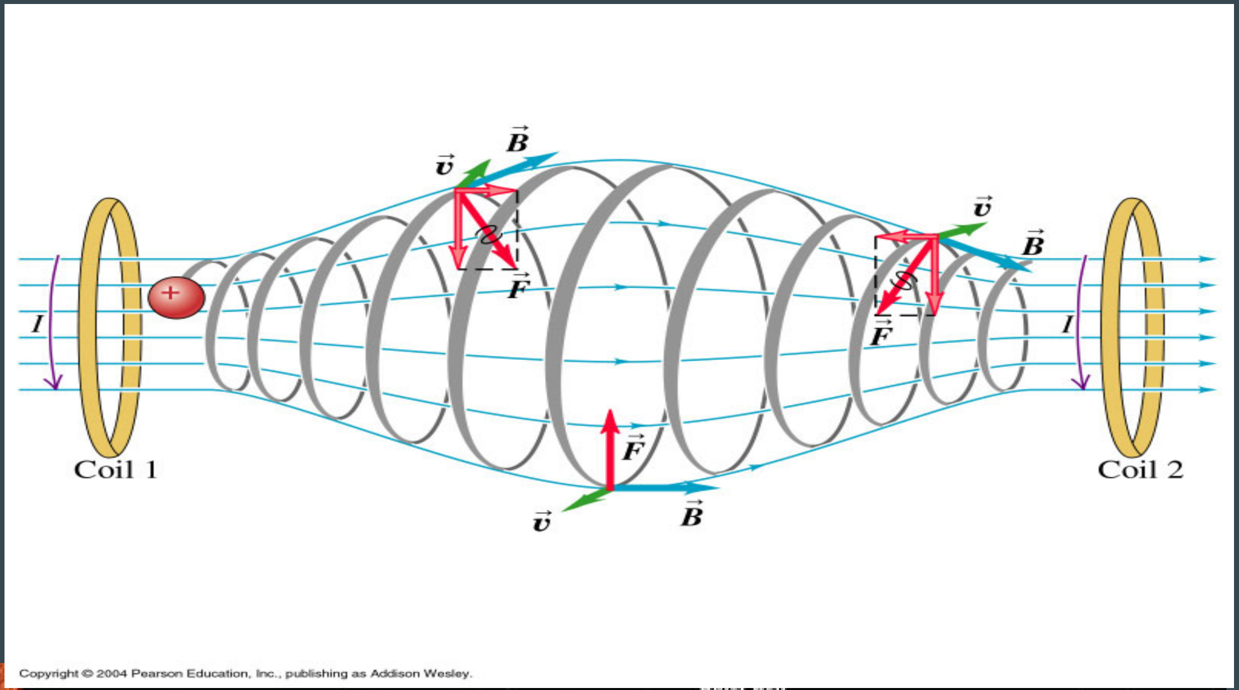
$$r_L = \frac{\gamma m v_\perp}{|q|B}$$

Motion of test particle in non-uniform magnetic field

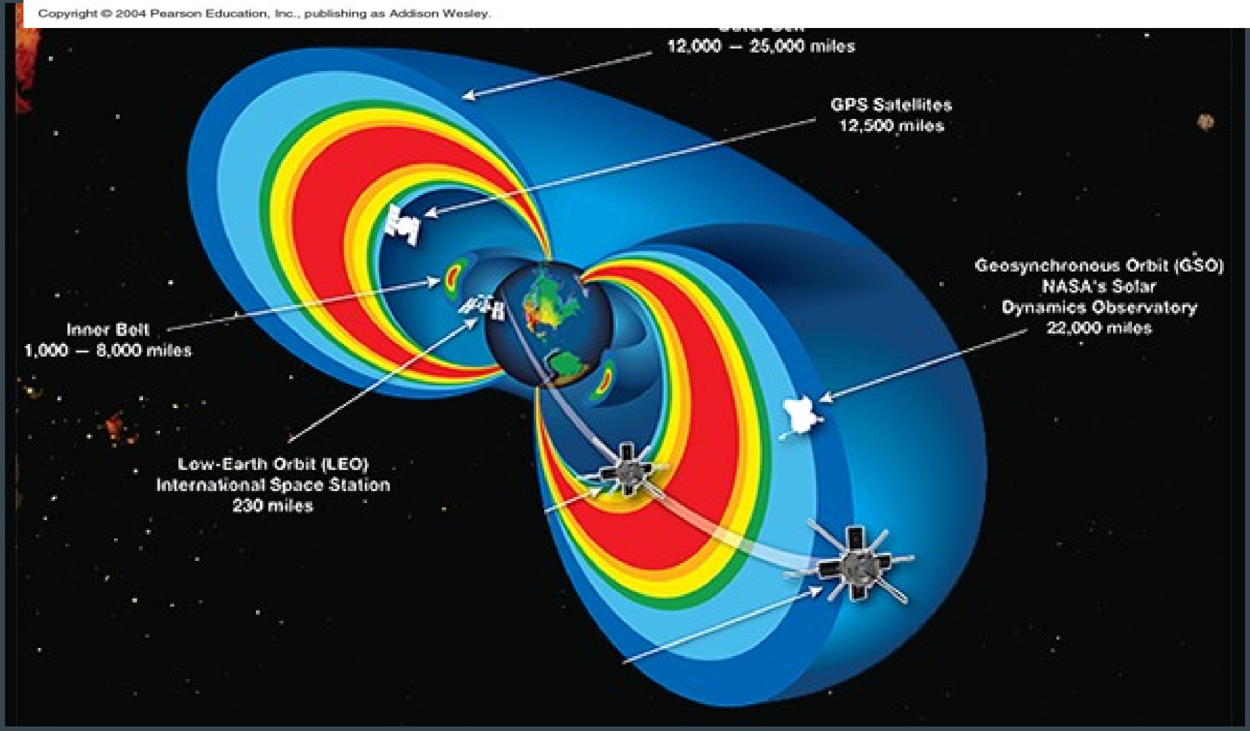
Magnetic mirror:

$$\mu = \frac{mv_\perp^2}{2B}$$

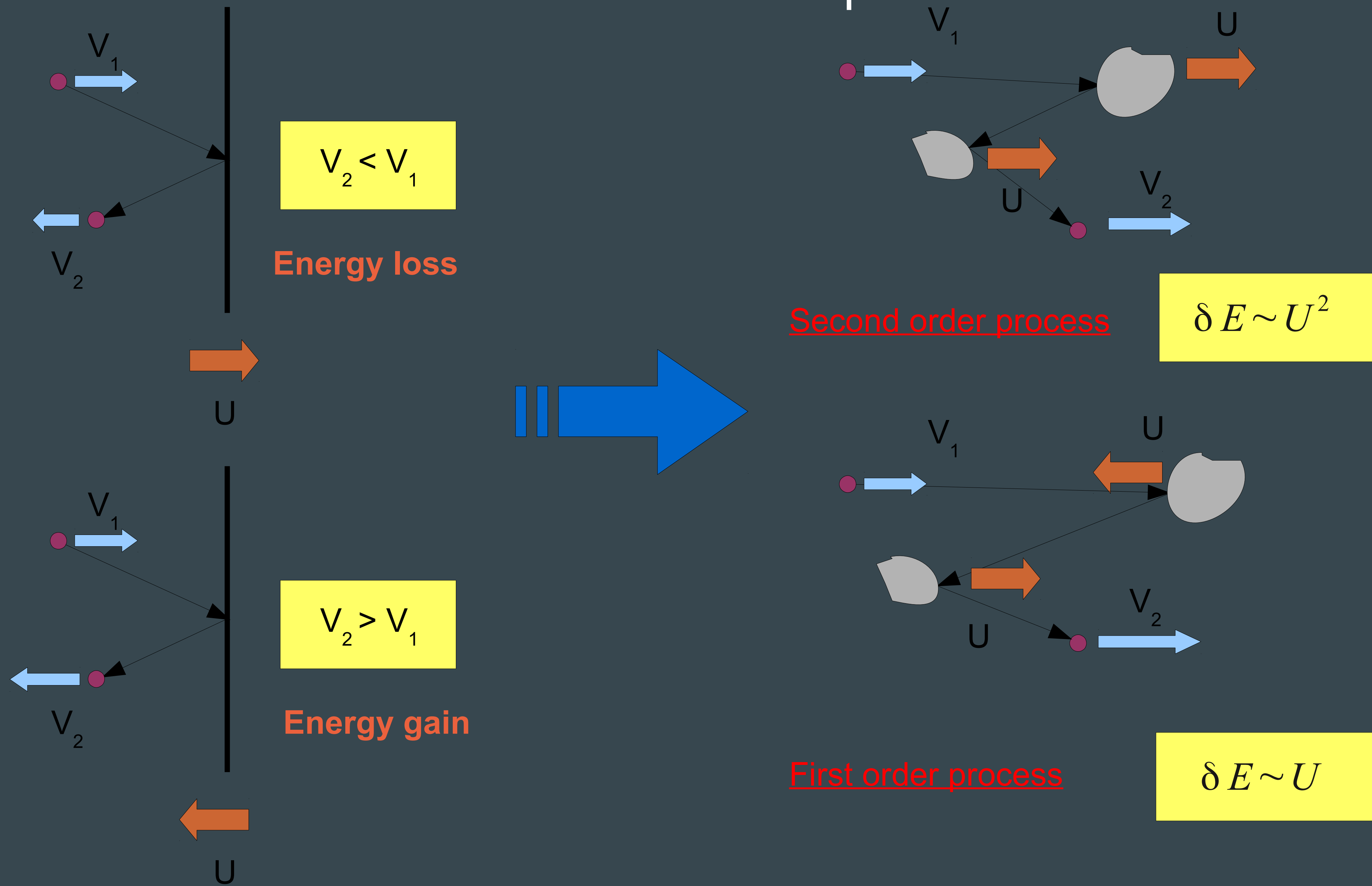
$$E_{kin} = \text{const}$$



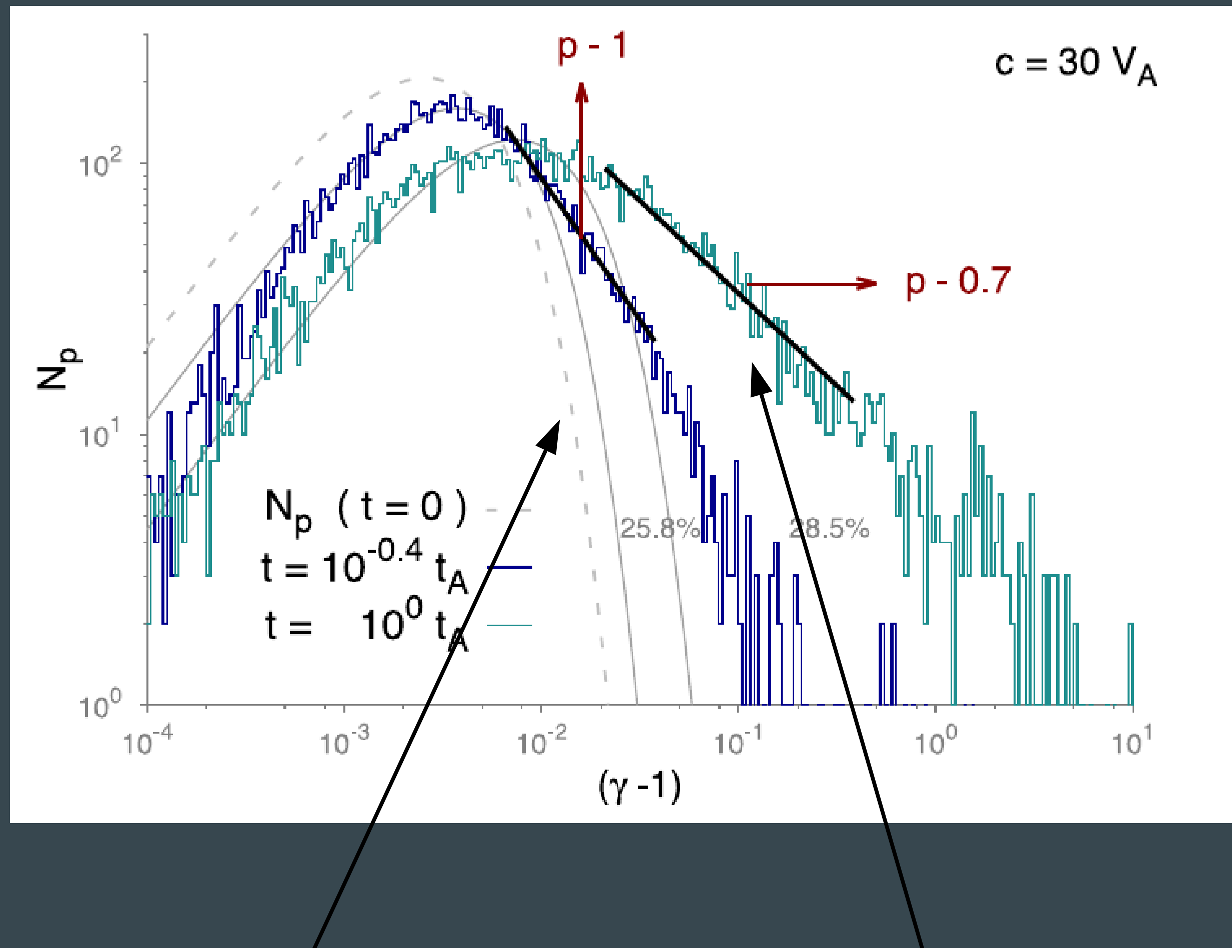
Van Allen belts



Motion of test particles in turbulent medium: 1st and 2nd order Fermi processes



Power spectra of energetic test particles



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POWER-LAW