

$x_i$	$y_i$
2	3
1	-4
3	6
4	5
3	7

$$n = 5$$

$$\begin{aligned}
 (1) \quad d_{SSD} : (x, y) \mapsto \|x - y\|_2^2 &= \langle x - y, x - y \rangle = \sum_{i=1}^5 (x_i - y_i)^2 \\
 &= (2-3)^2 + (1-(-4))^2 + (3-6)^2 + (4-5)^2 + (3-7)^2 \\
 &= (-1)^2 + (5)^2 + (-3)^2 + (-1)^2 + (-4)^2 \\
 &= 1 + 25 + 9 + 1 + 16 \\
 &= 52
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad d_{MAE} : (x, y) \mapsto \frac{d_{SAD}}{n} &= \frac{\|x - y\|_1}{n} = \frac{1}{n} \sum_{i=1}^n |x_i - y_i| \\
 &= \frac{1}{5} (|2-3| + |1-(-4)| + |3-6| + |4-5| + |3-7|) \\
 &= \frac{1}{5} (1 + 5 + 3 + 1 + 4) \\
 &= \frac{1}{5} (14) \\
 &= \frac{14}{5} = 2.8
 \end{aligned}$$

$$(3) \quad d_{MSE} : (x, y) \mapsto \frac{d_{SSD}}{n} = \frac{52}{5} = 10.4$$

$$(4) \quad d_2 : (x, y) \mapsto \|x - y\|_2 = \sqrt{d_{SSD}} = \sqrt{52} = 7.21$$

$$\begin{aligned}
 (5) \quad d_1 \equiv d_{SAD} : (x, y) \mapsto \|x - y\|_1 &= \sum_{i=1}^5 |x_i - y_i| \\
 &= |2-3| + |1-(-4)| + |3-6| + |4-5| + |3-7| \\
 &= 1 + 5 + 3 + 1 + 4 \\
 &= 14
 \end{aligned}$$

$$\begin{aligned}
 (6) \quad d_\infty : (x, y) \mapsto \|x - y\|_\infty &= \lim_{p \rightarrow \infty} \left( \sum_{i=1}^n |x_i - y_i|^p \right)^{1/p} = \max_i |x_i - y_i| \\
 &= \max (1, 5, 3, 1, 4) \\
 &= 5
 \end{aligned}$$



$p=3$

$$(7) d_p: (x, y) \mapsto \|x - y\|_p = \left( \sum_{i=1}^n |x_i - y_i|^3 \right)^{1/3}$$

$$= (|2-3|^3 + |1-(-4)|^3 + |3-6|^3 + |4-5|^3 + |3-7|^3)^{1/3}$$

$$= (1^3 + 5^3 + 3^3 + 1^3 + 4^3)^{1/3}$$

$$= (1 + 125 + 27 + 1 + 64)^{1/3}$$

$$= (218)^{1/3}$$

$$= 6.02$$

$$(8) d_{\text{CAD}}: (x, y) \mapsto \sum_{i=1}^n \frac{|x_i - y_i|}{|x_i| + |y_i|} = \frac{|2-3|}{|2|+|3|} + \frac{|1-(-4)|}{|1|+|(-4)|} + \frac{|3-6|}{|3|+|6|} + \frac{|4-5|}{|4|+|5|} + \frac{|3-7|}{|3|+|7|}$$

$$= \frac{1}{5} + \frac{5}{5} + \frac{3}{9} + \frac{1}{9} + \frac{4}{10}$$

$$= \frac{6}{5} + \frac{4}{9} + \frac{2}{5}$$

$$= \frac{8}{5} + \frac{4}{9}$$

$$= \frac{92}{45}$$

$$= 2.04$$

$$(9) d_{\cos}: (x, y) \mapsto 1 - \frac{\langle x, y \rangle}{\|x\|_2 \|y\|_2} = 1 - \frac{\sum_{i=1}^n x_i y_i}{\sqrt{\sum_{i=1}^n x_i^2} \sqrt{\sum_{i=1}^n y_i^2}}$$

$$= 1 - \frac{(2 \cdot 3) + (1 \cdot (-4)) + (3 \cdot 6) + (4 \cdot 5) + (3 \cdot 7)}{\sqrt{2^2 + 1^2 + 3^2 + 4^2 + 3^2} \sqrt{3^2 + (-4)^2 + 6^2 + 5^2 + 7^2}}$$

$$= 1 - \frac{61}{\sqrt{39} \sqrt{135}}$$

$$= 1 - \frac{61}{\sqrt{5265}}$$

$$= 1 - .841$$

$$= .159$$

$$(10) d_{\text{Pearson}}: (x, y) \mapsto 1 - \text{Corr}(x, y) = 1 - .84 = .16$$

$$\text{Corr}(x, y) = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y} = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \mu_x)(y_i - \mu_y)}{\sigma_x \sigma_y} = \frac{3.36}{(1.02)(3.93)} = .84$$

$$\mu_x = \frac{1}{5} (2 + 1 + 3 + 4 + 3) = \frac{13}{5} = 2.6$$

$$\mu_y = \frac{1}{5} (3 + (-4) + 6 + 5 + 7) = \frac{17}{5} = 3.4$$

$$\sigma_x = \sqrt{\frac{1}{5} ((2-2.6)^2 + (1-2.6)^2 + (3-2.6)^2 + (4-2.6)^2 + (3-2.6)^2)} = \sqrt{\frac{1}{5} (5.2)} = 1.02$$

$$\sigma_y = \sqrt{\frac{1}{5} ((3-3.4)^2 + (-4-3.4)^2 + (6-3.4)^2 + (5-3.4)^2 + (7-3.4)^2)} = \sqrt{\frac{1}{5} (77.2)} = 3.93$$

$$\text{cov}(x, y) = \frac{1}{5} [(2-2.6)(3-3.4) + (1-2.6)(-4-3.4) + (3-2.6)(6-3.4) + (4-2.6)(5-3.4) + (3-2.6)(7-3.4)]$$

$$= \frac{1}{5} [-.24 + 11.84 + 1.04 + 2.24 + 1.44]$$

$$= \frac{1}{5} [16.8] = 3.36$$