# Written Report

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## 1 Difference between Minimum Hellinger Distance and Minimum Hellinger Distance Estimation

Hellinger Distance measures the distance between two probability distributions and is given by:

$$H(p,q) = \frac{1}{\sqrt{2}} \left( \int \left( \sqrt{p(x)} - \sqrt{q(x)} \right)^2 dx \right)^{\frac{1}{2}}$$

where $p(x)$ and $q(x)$ are the two probability distributions.

Minimum Hellinger Distance (MHD) refers to the actual value of the Hellinger Distance between a reference distribution and the closest distribution found through a minimization process while Minimum Hellinger Distance Estimation (MHDE), on the other hand, is obtained by minimizing the Hellinger Distance between an assumed parametric model and a nonparametric estimation of the model.

## 2 Interest in Minimum Hellinger Distance Estimation

I am particularly interested in exploring Minimum Hellinger Distance Estimation (MHDE) due to its application in various statistical models, especially in the context of mixture models.

## 3 Semi-Parametric Location-Shifted Mixture Model

A semi-parametric location-shifted mixture model combines the precision of parametric models with the flexibility of non-parametric models while accounting for variations in location across different components of the mixture.

The mixture model can be expressed as:

$$g(x) = \sum_{i=1}^{k} \pi_i f(x - \mu_i)$$

where:

- $k$ is the number of mixture components.
- $\pi_i$ are the mixing proportions, with $\sum_{i=1}^{k} \pi_i = 1$.
- $f(x)$ is the base distribution, which might be parametric or non-parametric.
- $\mu_i$ are the location parameters for each component, shifting the base distribution $f(x)$.

# 4 Statistical Computation

Statistical Computation is a branch of computational mathematics that focuses on developing computational techniques for situations involving randomness. It can also be defined as the application of computer science to statistics.

# 5 Methodology: MHDE within Semi-Parametric Location-Shifted Mixture Models

The method used in this context refers to "Minimum Hellinger Distance Estimation (MHDE) within a semi-parametric location-shifted mixture model."

# 6 Review of Related Literature

I reviewed the paper by Sijia Xiang on "Minimum Hellinger Distance Estimation in a Semiparametric Mixture Model" and noted that the Expectation-Maximization (EM) algorithm is a common estimation procedure to find parameters in such models.

# 7 Integration of MHDE and EM Algorithm in Semi-Parametric Location-Shifted Mixture Models

This work focuses on the application of Minimum Hellinger Distance Estimation (MHDE) within a semi-parametric location-shifted mixture model. The Expectation-Maximization (EM) algorithm is commonly employed to estimate the parameters of such models. However, given that both MHDE and the EM algorithm are estimation procedures, their simultaneous use within the same framework may introduce unnecessary complexity.

# 8 Efficient Estimation

a. Parameter estimation is involves the determination of parameter values that best fit the observed data.This process presents several challenges that can impact the accuracy and reliability of the estimates. Some challenges are;

- Model Complexity
- Computational Complexity
- Identifiability Issues

b. Efficient estimation refers to an estimator achieving the lowest possible variance among all unbiased estimators for a given parameter.
[1]. [2].

# References


[1] JH Westcott. The parameter estimation problem. *IFAC Proceedings Volumes*, 1(1):789–797, 1960.

[2] Jingjing Wu and Xiaofan Zhou. Minimum hellinger distance estimation for a semiparametric location-shifted mixture model. *Journal of Statistical Computation and Simulation*, 88(13):2507–2527, 2018.