HL2027 - Project 1
MRI: Image Reconstruction from Radial Sampling

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1 Introduction

1.1 Underlying problems

In magnetic resonance imaging (MRI) the received signal is the Fourier transform of the object's spin density. The conventional way to reconstruct the image from the k-space consists in acquiring an adequate number of samples in order to satisfy the Nyquist theorem with a rectilinear approach (Cartesian sampling). However, the time needed for a fully Cartesian acquisition is a big issue: long time scan could cause motion artifacts, discomfort for the patient and long queues while waiting to get checked. This is the reason why, researchers developed different strategies to reduce the scan time trying to affect the quality of the reconstructed image as little as possible.

1.2 Basic theories

An interesting approach to reduce the acquisition time consists in under-sampling the k-space following a radial trajectory called 'spikes'. This technique permits us to oversample the centre of the k-space (low frequencies) that contains the main information of the image and undersample the edges where details and noise are located.

2 Data Construction: Task 1

Starting from a fully sampled Cartesian k-space of our slice 1, it's easy to generate radial coordinates in the kspace through the function provided by sigpy.

$$coord = sigpy.mri.radial([spikes, 256, 2][256, 256]) \tag{1}$$

Let's highlight that setting a single 'spike' in the function provided above returns the coordinates related to points that go from the centre of the kspace to an edge. Since our purpose consists in creating lines that go from edges to edges (diametrical line and not radial line), it's important to set an even number of spikes ($n_{lines} = spikes/2$). Furthermore, in all cases, we selected 256 points per spike, which left us with 512 points per line.
Furthermore, there are two ways of creating spikes:

1. Golden = True. This is the default parameter that creates spikes according to the Golden Ratio method, so they are not equally distributed in the circle [2], as shown in figure 2.

2. Golden = False. Each spike is equally spaced from the others, shown below 3.

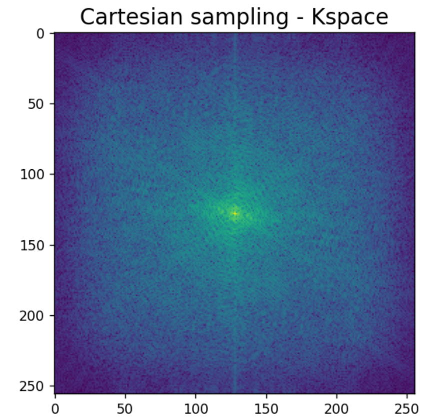


Figure 1: Central slice - cartesian sampling kspace

For all of the tasks, we are using the equally spaced spikes method as specified by the project description.
The acceleration factor R that we use to speed up the acquisition can be related to the number of spikes conforming the radial kspace 4. In particular, it can be defined with respect to the number of samples in the fully Cartesian k-space (N), and the samples in the radial k-space ($M = samples_{spike} * spikes$) as $R = \frac{N}{M}$.
Therefore, once the radial coordinates have been obtained thanks to the sigpy function, one can interpolate in the cartesian kspace and get the radial kspace to start the reconstruction task.

3 Quality Assessment: Task 2

The purpose of the project is to test and compare different reconstruction techniques.To asses the quality of the reconstruction, we chose some qualitative and quantitative tools. The quantitative metrics chosen are listed below:

1. SSIM: it quantifies the perceptual difference between two similar images.
2. PSNR: it represents a measure of the peak error and the function used provides a value in dB.
3. MSE: it represents the cumulative squared error between two images.

On the other side, the purpose of the qualitative analysis tools is to visually evaluate the correctness of the reconstruction.

1. Difference Map: the function (DiffMap in the code) displays the difference in intensity between the reference and the reconstructed image.
2. Difference Map of the gradient. The function (DiffGradient in the code) computes the gradient of the original and the reconstructed images and displays the difference. We chose this qualitative method to visually analyse the loss of details due to the undersampling of high frequencies in k-space. The gradient of the image is realized with Sobel filter. However, we have to be careful when using this metric, since Sobel filter is very sensitive to noise.

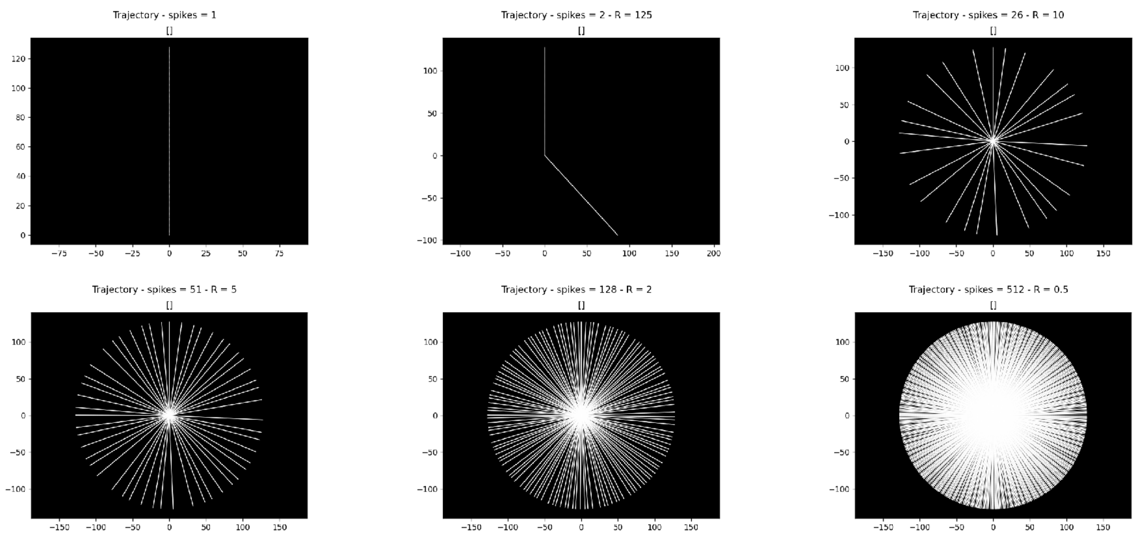


Figure 2: Radial Sampling Trajectories for different number of spikes - Golden angle

4 Solution Strategies Used

In this section, we will be talking about different strategies used to reconstruct the image when the data has been acquired via radial sampling. We can divide the reconstruction approaches into two subgroups: those methods using a Regridding technique and those using a Projection technique.

4.1 Regridding Reconstruction Methods

The main idea behind this approach is interpolating the measured spikes obtained with radial sampling in the kspace (Fourier domain) into a Cartesian grid. After the interpolation is done, a reconstruction approach based on the inverse FFT is used on the gridded data and the reconstructed image is obtained.
Before the gridding, a density compensation factor is applied on the radial kspace samples. This is done to compensate for the variable sample density caused by the radial sampling, since there's a higher number of sample-points in the center of the k-space than on the locations further away from the center.
We understood this dcf as a weighting function that assigns weights to those k-space samples in order to compensate for the non-uniform sampling density before interpolation in the cartesian kspace. In our case, we used a simple dcf based on the magnitude of the position vector (coordinates) of each sample in the radial trajectory, as shown in the equation below 2. Therefore, we assigned a higher weight to those points further away, compensating for the low density of the radial kspace on the outer part.

$$dcf = \sqrt{(coord_{radial_0})^2 + (coord_{radial_1})^2} \tag{2}$$

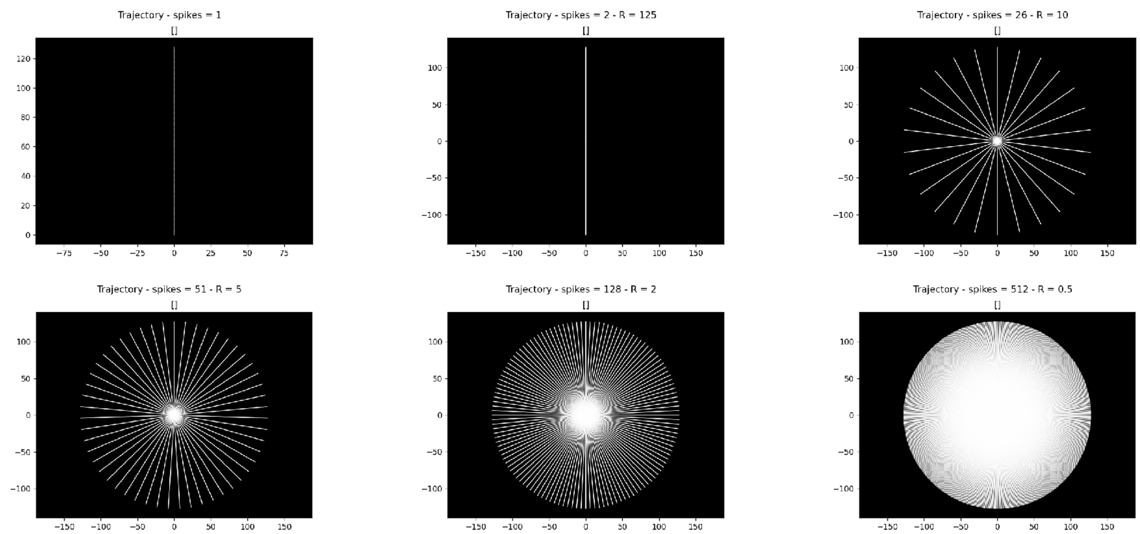


Figure 3: Radial Sampling Trajectories for different number of spikes - equally distributed in the circle

4.1.1 Traditional FFT and Non-Uniform Adjoint FFT (NUFFT)

The first approach to reconstruct the MRI slice is the standard inverse Fourier Transform. Initially, we tried without the compensation factor, directly gridding our radial coordinates on the cartesian grid, later using the standard iFFT. However, results where tremendously far from what we expected so the dcf was also used before regridding in this case. The kspace regridded with and without compensation are shown in figure 5.
After the density compensation, the gridding takes place and then a standard inverse FFT is applied, so the image can be recovered. In the case of the standard FFT approach, we used a spline kernel for the interpolation. In the literature, the kaiser-Bessel kernel is suggested as an alternative [2]. A more specific type of FFT that can be used in our case is the non-uniform FFT. This discrete Fourier transform is used for those cases where the sampling is not done at equally spaced frequencies. In the default setting, the Kaiser-Bessel kernel is used with a width equal to 4. This is a more application-specific approach and therefore it brings better results.

4.1.2 Other Reconstruction Methods: Wavelet and Total Variation Reconstruction

Another used approach is the compressed sensing reconstruction based on a wavelet method. This approach exploits the sparsity of MRI images in the Wavelet domain. The magnitude of the wavelet coefficients allow to differentiate the important information, standing out above the artifacts and interferences. This way, we can recover the object from our undersampled k-space.
So, with this L1 wavelet reconstruction method we are just solving the equation found in the sigpy documentation [3]. In this method, we have to set the sensitivity map. Knowing that we are working with a single coil, we can set this map to be maximum everywhere (equal to 1). Also, some parameters that need to be taken into consideration when applying this reconstruction method are the regularization parameter λ, and the type of wavelet used. We will see the effect of changing them in further sections in this report.

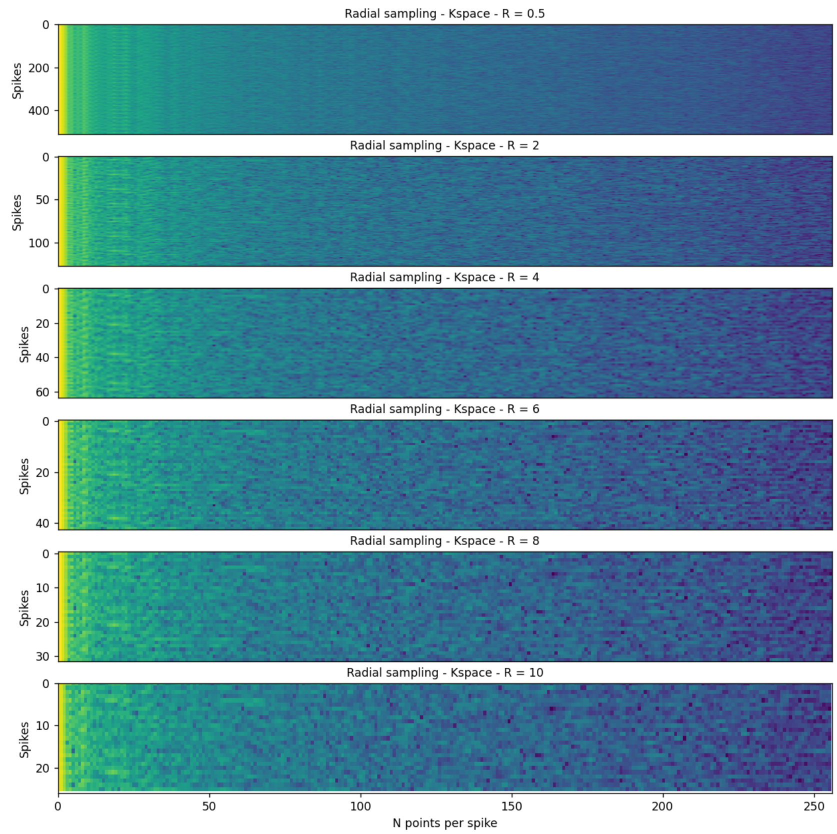


Figure 4: Radial kspace for different R factors - Number of spikes

A different approach used in this project was the Total Variation Reconstruction, which is based on an optimization problem that tries to solve the equation explained in the sigpy documentation [4]
Also in this case we can modify the values of λ to visualize different results.

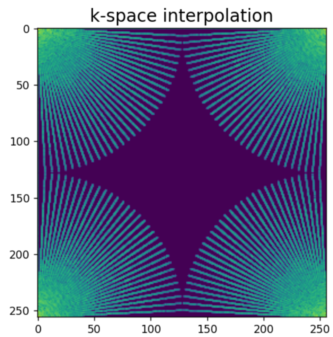
4.2 Projection Reconstruction Method: Task 5

The projection reconstruction method exploits the CT reconstruction method of filtered back-projection to reconstruct MRI images 3.

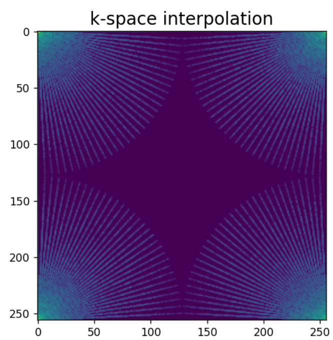
$$f(x,y) = \frac{1}{2} B F^{-1}[|\omega| F(Rf)(\omega,\theta)](x,y) \tag{3}$$

The connection between CT and Radial sampling MRI reconstruction is the Central Slice Theorem which claims the following:

$$F_2 f(\omega * \cos(\theta), \omega * \sin(\theta)) = F_1(Rf)(\omega,\theta) \tag{4}$$



(a) Gridding - radial sampling with applied dcf



(b) Gridding - radial sampling with no dcf

Figure 5: Kspace regridding - with and without compensation factor

From this theorem, we deduce that the Fourier transform of each couple of 'opposite' spikes in the radial k-space of the image can be seen as the Radon transform of the image (or the projection through the object) in an angle perpendicular to the direction of the spike.
Since the filtered back-projection technique starts from the sinogram of the image 3, by performing the inverse Fourier transform for each spike we moved to the Radon space. In this space, at each value of the x-axis, there is the angle at which the projection is done. To simplify the use of the pre-built function scikit-image.iradon to perform backprojection, we used a radial sampled kspace with equally spaced spikes (Golden = False) in order to obtain equally spaced projections in the Radon space.

5 Findings from the experiments

For tasks 1-6 we only used the central slice, shown in figure 6. From the slice, we calculated the cartesian kspace, and the radial coordinates to be able to simulate the kspace from radial sampling.

5.0.1 Findings: Task 3

As previously mentioned, we tried to reconstruct the image after regridding without the compensation function dcf. As can be seen in Figure 7, the results are not what we aim at obtaining, so we used the dcf function before gridding. Even though the dcf compensation was applied before using

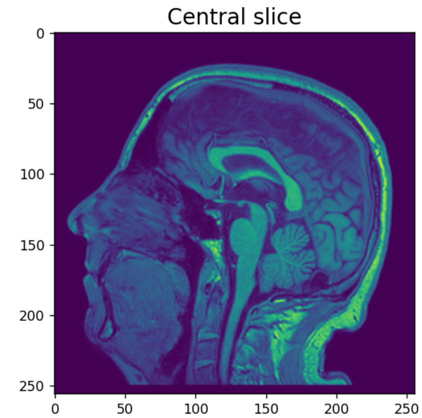


Figure 6: Central slice: 45

the standard iFFT, we still expected the NUFFT to be better, shown in the quantitative indicators on the figure.

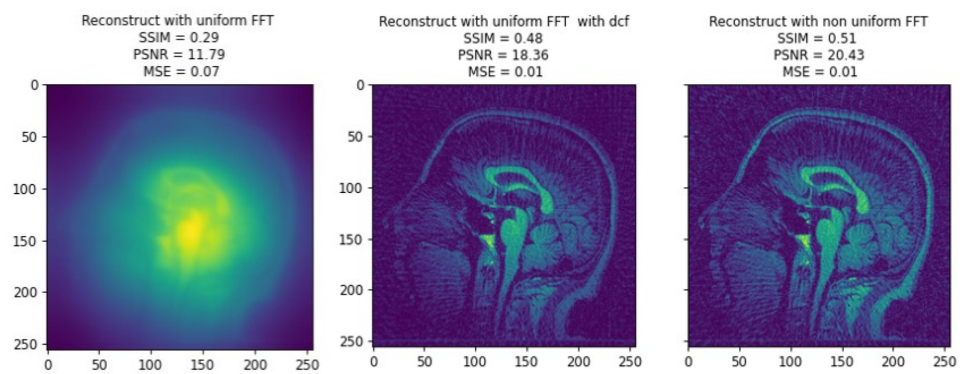


Figure 7: Comparison between NO dcf standard FFT, dcf standard FFT and NUFFT - 128 spikes

Another visual assessment that we could do is studying the gradient of our reconstructed image and the original, as shown in Figure 8. We thought this could be interesting, since with radial sampling we are oversampling low frequency areas and undersampling the high frequency part of the Fourier space. However, as said previously, Sobel operator is very sensitive to noise. As can be seen in the reconstructed figures, it seems that the radial sampling might introduce some noise in the reconstructed object so we should pay attention and not only rely on this measurement.
Furthermore, we run the NUFFT for different values of the acceleration factor R (related to number of spikes) to see the impact of it in the quality of the reconstruction. We see that the results improve with the decreasing of the acceleration factor. Of course, this takes us back to the commonly known trade-off in the MRI field between acquisition time and resolution.

5.0.2 Findings: Task 4

To assess the goodness of the other two reconstruction methods within the regridding techniques, we visually analyse the effect of different values of R in both methods: the L1 Wavelet and the

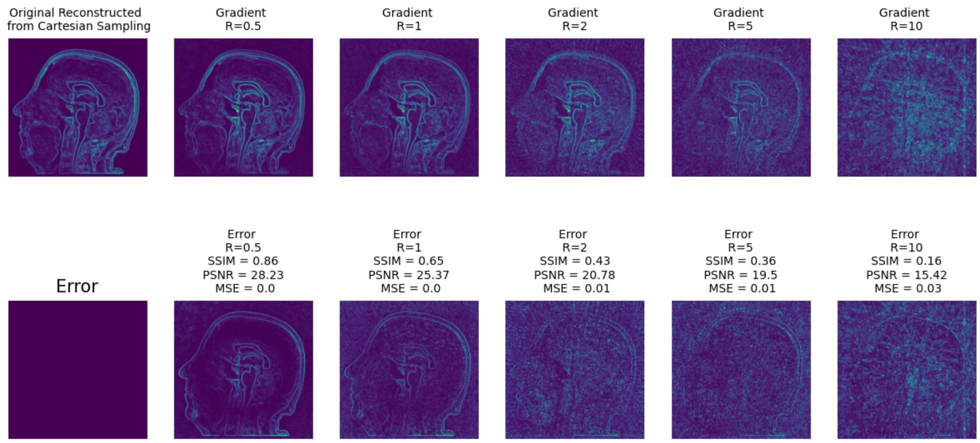


Figure 8: Magnitude of the Gradient - reconstructed vs original

Total Variation approach, while keeping the $lambda = 0.005$. The results can be seen in the figure below 10.
Furthermore, while keeping the $R = 2$, we varied the value of λ and visualized the impact of it on the quality of the reconstruction in the Wavelet-based method, as shown in Figure 11. However, it does not seem to have a great impact on our results. Same occurs when modifying the width of the kernel used during interpolation. We see a slight improvement on our quantitative indexes but nothing that we consider very relevant for our case.

5.0.3 Findings: Task 6

In the figure below 12, the radial kspace that we used to obtain the sinogram 13 is shown. In order to get this new kspace matrix, we had to reshape our existing kspace by concatening opposite spikes (half a line) before performing the 1D iFFT of each line.
As we expected, in the reconstructed image 14, it's possible to see the back-projections lines: by increasing the number of spikes this visual effect decreases 16. Furthermore, we realized that the error displayed in the reconstructed image is similar to the phase of the original image 15. From that, we assumed that we might be loosing information about the phase doing the reconstruction with back-projection algorithm. To overcome this issue, the backprojection of the real part and the complex part of the sinogram should be done separately[1].

6 Conclusion

From the qualitative and the quantitative analysis we can conclude that the best methods for radial sampling reconstruction of the k-space is the Non Uniform Fourier Transform based reconstruction method.
Consequently, in the last part of our code we applied this method on each slice of our image in order to reconstruct the 3D volume. With an acceleration factor of 5 and 10 we need respectively

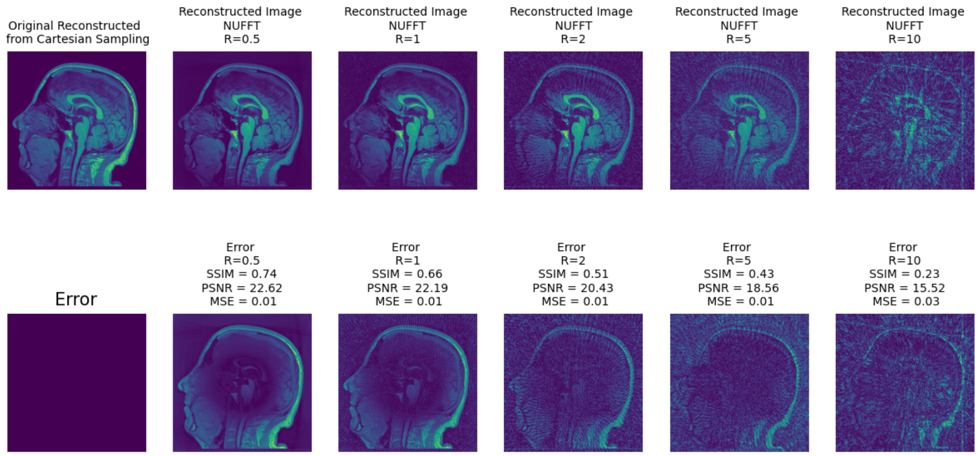


Figure 9: Qualitative and quantitative comparison for different R factors - NUFFT

52 spikes and 26 spikes. Our code allows to interactively modify the visualized slice on the 3D reconstructed volume.

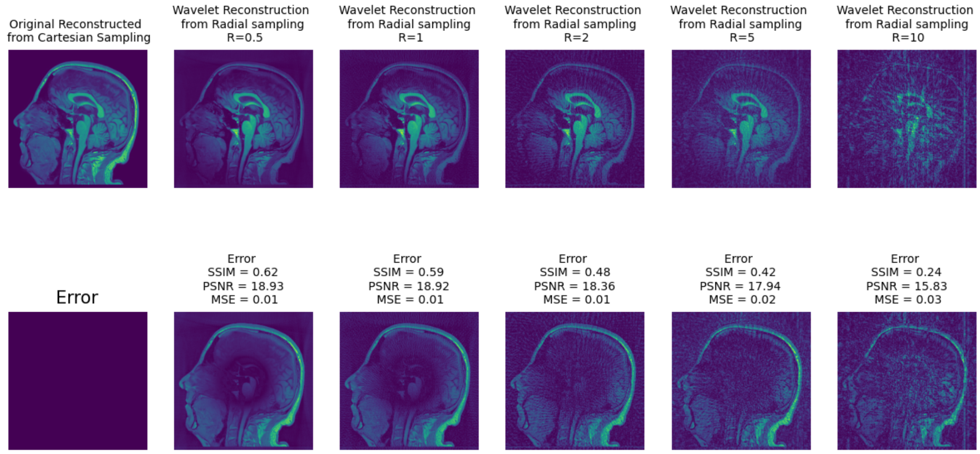


Figure 10: Qualitative and quantitative comparison for different R values - Wavelet

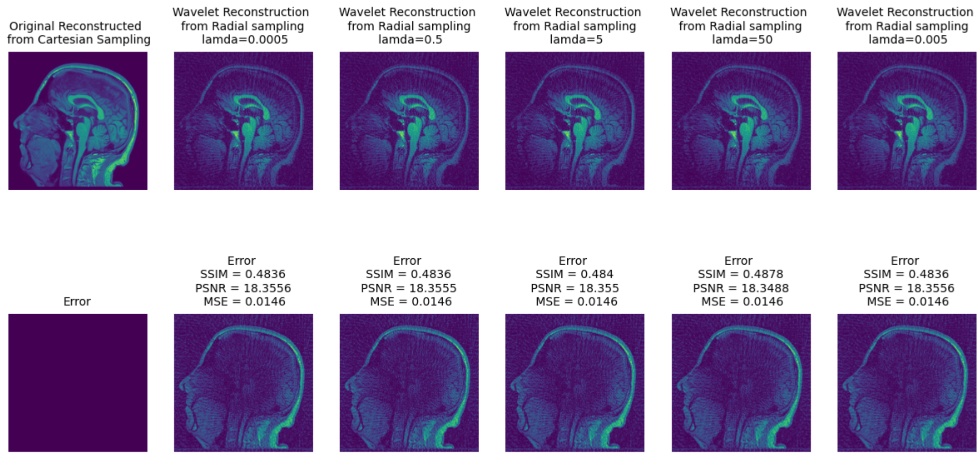


Figure 11: Qualitative and quantitative comparison for different lambda values - Wavelet

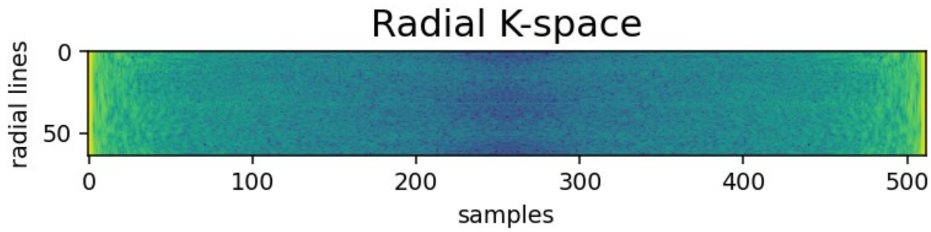


Figure 12: Radial k-space

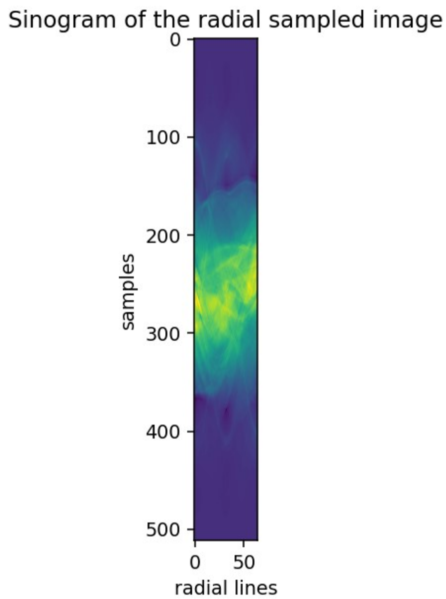


Figure 13: Sinogram of the radial sampled image

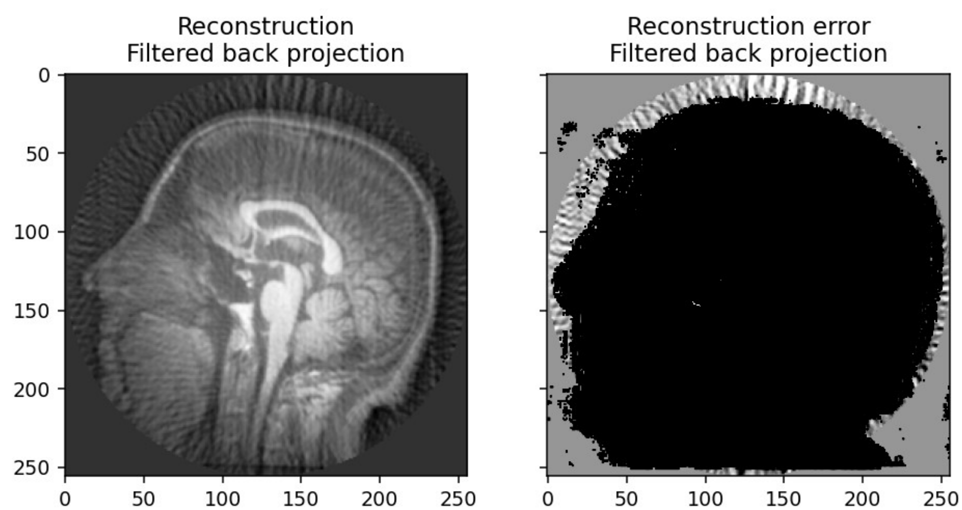


Figure 14: Recostructed imaged with the filtered backprojection algorithm

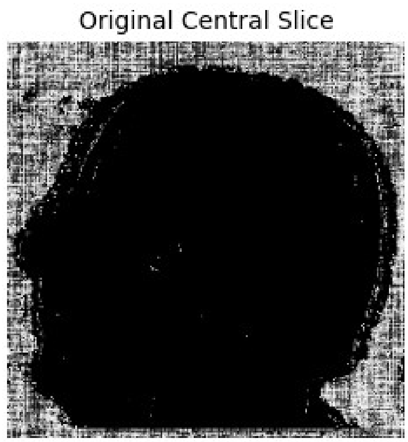


Figure 15: Phase of the central slice

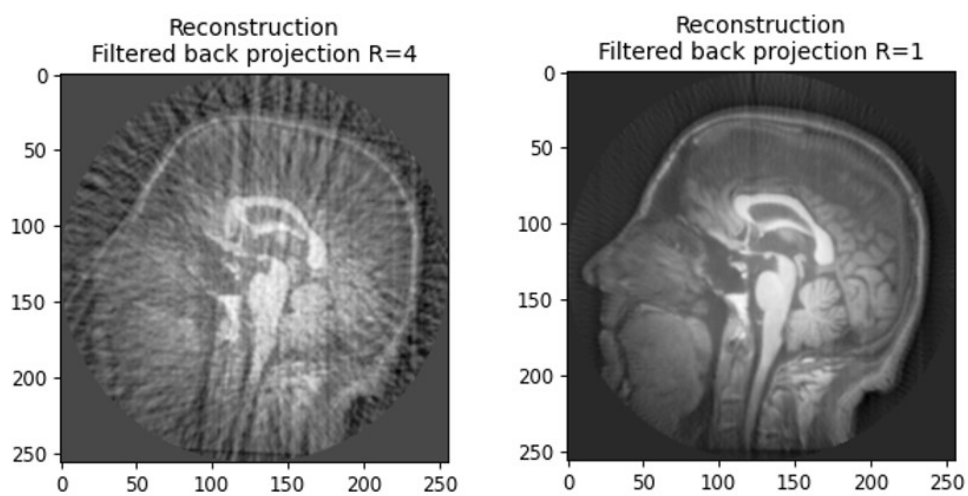


Figure 16: Back-projection lines with different acceleration factors

References

[1] Kai Tobias Block. "Advanced methods for radial data sampling in magnetic resonance imaging". PhD thesis. 2008.

[2] Kai Tobias Block, Martin Uecker, and Jens Frahm. "Undersampled radial MRI with multiple coils. iterative image reconstruction using a total variation constraint". In: *Magnetic Resonance in Medicine* 57.6 (2007), pp. 1086–1098. DOI: 10.1002/mrm.21236.

[3] *Sigpy.mri.app.l1waveletrecon¶*. URL: https://sigpy.readthedocs.io/en/latest/generated/sigpy.mri.app.L1WaveletRecon.html.

[4] *Sigpy.mri.app.TotalVariationRecon¶*. URL: https://sigpy.readthedocs.io/en/latest/generated/sigpy.mri.app.TotalVariationRecon.html.