Part 1: An electromagnetic velocity sensor

A) Explain the Biot-Savart law.

The Biot-Savart law is gives the magnetic field of a steady line current and is analogous to Coulomb's law in electrostatics:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{I}\times\hat{\mathbf{r}'}}{r'^2}dl = \frac{\mu_0}{4\pi}I\int \frac{d\mathbf{l}\times\hat{\mathbf{r}'}}{r'^2}dl$$

Where $\mathbf{I}$ is the electric current, $\mathbf{r}'$ is the vector from the source to the point $\mathbf{r}$, dl is an element along the wire in the direction of the current and $\mu_0 = 4\pi\times10^{-7}N/A^2$ is the permeability of free space. The units for the magnetic field are newtons per ampere-meter (N/ (A m)) or teslas (T) and the integral is along the current path in the same direction as the flow.

Source: Griffiths, David. Introduction to Electrodynamics. Fourth Edition

B) Explain Onsager's reciprocal theorem

Onsager's reciprocal theorem involves the reciprocity of coupled electrical and thermal systems. Lars Onsager's 1931 paper uses the following notation:

X_1 and X_2 are driving electrical and thermodynamic forces, respectively and, if they were independent, could be written as:

$$X_1 = R_1J_1$$
$$X_2 = R_2J_2$$

where R_1 and R_2 are the electrical resistivity and thermal resistance, respectively. J_1 and J_2 are electrical and thermal current.

However, these systems are coupled since electrical current is not independent of the temperature. Therefore we can add in this dependency using cross coefficients R_{12} and R_{21} so that:

$$X_1 = R_{11}J_1 + R_{12}J_2,$$

and

$$X_2 = R_{21}J_1 + R_{22}J_2.$$

My understanding of this is that the electrical force is equal to electrical resistance times current density plus the current modified scaled by a coeffecient that represents the coupling of the thermal heat on the electrical system.

Similarly, the thermodynamic force is equal to the thermal resistance times thermal current density plus the electrical current density scaled by a coeffecient representing the coupling of the electrical system on the thermal. the reciprocity theorem states that these two coefficients are equal,

$R_{12} = R_{21}$

Sources:

Onsager,Lars. Reciprocal Relations in Irreversible Processes. I. Physical Review, Vol 37. 1931

http://www.iue.tuwien.ac.at/phd/holzer/node24.html

C)How would you compute l given a coil with radius r and number of coils n?

$l = 2\pi rn$

D) Derive equation 12.49

First list the forces by looking at Figure 12.15.

1. Force of gravity on the mass
2. Force from the spring
3. Magnetic force

So using Newton, we have: $F = F_{gravity} - F_{spring} - F_{mag}$

We saw in class that the force from the spring ($K(z - l_0)$) and the force of gravity ($Mg = k(z - l_0)$) combine and can be written as kz(t). We also saw that there are two sources of acceleration, from the seismometer and from the ground, so that we have:

$$M\frac{\partial^2}{\partial t^2}(z(t) + u(t)) = -kz(t) - F_{mag}$$

(1)

Rearranging and dividing by M:

$$z'' + u'' = -\omega w_0^2 z - \frac{1}{M}Fmag$$

(2),

where $w_o^2 = \frac{k}{m}$.

Aki and Richards give the magnetic force (from the Lorentz force law) as:

$F = IlB$,

where I is the current, l is the length of the wire and B is the flux density. The mechanical power is the rate that work is done so they multiply both sides by the velocity of the moving mass, or z'. so that :

$Fz' = IlBz'$

Then they say that the mechanical power must be consumed by the resistance and since electrical power is equal to $P = VI$ by ohm's law they obtain

$V = lBz'$.

Now they define lB as G so $V = Gz'$. Using ohm's law and the total resistance equal to $R_0 + R$, $I(R_0 + R) = Gz'$ and simple division yields

$$I = \frac{Gz'}{R_0 + R}. \tag{3}$$

Now $F = GI$ and we can substitute in equation **??** to get

$F = \frac{G^2 z'}{R_0+R}$.

Now we can substitute this back into equation **??** so that:

$$z'' + u'' = -\omega_0^2 z - \frac{G^2}{R_0 + R}\frac{z'}{M} \tag{4}$$

or

$$z'' + \omega_0^2 z = -u'' - \frac{G^2}{R_0 + R}\frac{z'}{M}. \tag{5}$$

In class we saw that for a similar system with a dashpot instead of a magnet we get:

$$z'' + \omega_0^2 z = -u'' - 2\epsilon z'. \tag{6}$$

and comparing the two equations:

$$2\epsilon = \frac{G^2}{R_0 + R}\frac{1}{M} \tag{7}$$

This equation relates the two dampening terms in the equation of motion for both systems. If there is also mechanical attenuation, ϵ_0 then

$$\epsilon = \epsilon_0 + \frac{G^2}{2(R_0 + R)}\frac{1}{M} \tag{8}$$