The Massey Rating Method

An Introduction to Sports Team Rankings

Dr. Rebin Muhammad

Montgomery College

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Understanding the Massey Rating Method: Introduction

What are Ratings and Why are They Important?

- Ratings are numerical values assigned to entities (like sports teams) to rank them.
- They help in comparing performances and predicting future outcomes.
- Essential in fields like sports analytics and data science.

The Role of Linear Algebra in Ratings

How Does Linear Algebra Help in Ratings?

- Linear Algebra is a branch of mathematics dealing with vectors and matrices.
- It helps in organizing data and solving systems of equations, key in data analysis.
- In ratings, it's used to construct and solve rating models like the Massey method.

Massey Method: Overview

Basics of the Massey Method

- A system to rank teams based on their game performance.
- Uses points scored and conceded in games to calculate team ratings.
- Involves setting up and solving a system of linear equations.

Step 0: Gathering Data

	Duke	Miami	UNC	UVA	VT	Record	Point Differenti
Duke		7-52	21-24	7-38	0-45	0-4	-124
Miami	52-7		34-16	25-17	27-7	4-0	91
UNC	24-21	16-34		7-5	3-30	2-2	-40
UVA	38-7	17-25	5-7		14-52	1-3	-17
VT	45-0	7-27	30-3	52-14		3-1	90

Figure: Game score data for a small 5-team example

Step 1: Point Differential Vector (p)

Calculating Point Differentials

- Point Differential = Total points scored - Total points conceded.
- Example calculation for a team: Scored 100 points, Conceded 80 points, Differential = 100 - 80 = 20.
- For our 5-team example, the differentials are: -124 for Duke, 91 for Miami, -40 for UNC, -17 for UVA, and 90 for VT.
- Thus, $p = \begin{bmatrix} -124 \\ 91 \\ -40 \\ -17 \\ 90 \end{bmatrix}$ and $r = \begin{bmatrix} Duke \\ Miami \\ UNC \\ Thus \\ VT \end{bmatrix}$

Step 2: Constructing the Massey Matrix (M)

Building the Massey Matrix

- A square matrix where each element represents games between teams.
- Diagonal elements = Number of games played (4 for each team in our example).
- Off-diagonal elements = -1 (for each game played against other teams).
- Last row modified to ensure a unique solution (sum of ratings = 0).
- Example matrix:

$$M = \begin{bmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & -1 & 4 \end{bmatrix}$$

Resolving the Rank Deficiency in the System

- Encountered a challenge: Post several games, our matrix reached a rank of $n - 1$, leading to infinite solutions for $Mr = p$.
- Objective: To derive a unique solution for team ratings.
- Solution: Massey's approach involved altering the last matrix row to all ones and the corresponding point differential vector entry to zero.
- Outcome: This change secured the matrix's rank at n, allowing us to solve the system and ascertain team ratings.
- Example: The modified system is presented as follows:

Resolving the Rank Deficiency in the System

Original System:

$$M = \begin{bmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & -1 & 4 \end{bmatrix}, \quad p = \begin{bmatrix} -124 \\ 91 \\ -40 \\ -17 \\ 90 \end{bmatrix}$$

Updated System:

$$M_{\text{updated}} = \begin{bmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & -1 & 4 & -1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}, \quad p_{\text{updated}} = \begin{bmatrix} -124 \\ 91 \\ -40 \\ -17 \\ 0 \end{bmatrix}$$

Step 3: Solving the Linear System

Finding the Team Ratings

- Adjust point differentials: last element set to 0 to sum to zero.
- Solve the linear system $\bar{M}r = \bar{p}$ using matrix algebra.
- Ratings are found by solving this system.

Solving System of Equation

$$\begin{bmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & -1 & 4 & -1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -124 \\ 91 \\ -40 \\ -17 \\ 0 \end{bmatrix}$$

where x_1, x_2, x_3, x_4, x_5 represent the ratings of the teams Duke, Miami, UNC, Thus, and VT, respectively.

The Solution

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -24.8 \\ 18.2 \\ -8.0 \\ -3.4 \\ 18.0 \end{bmatrix}$$

- Teams with ratings 18.2 and 18.0 are rated as the strongest among the five.

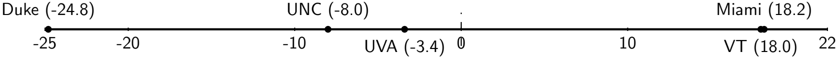
Interpreting the Ratings

Understanding the Results

- Higher ratings indicate stronger team performance.
- Negative ratings suggest weaker performance compared to others.
- Ratings provide a basis for comparison and analysis.

Massey Ratings: Line Rating Visualization

Visual Comparison of Team Strength



Interpretation of Ratings

Understanding the Results

- Higher ratings indicate stronger team performance.
- Negative ratings reflect relatively weaker performance compared to others.
- The visual line rating helps in understanding the relative strength between teams.

References

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