

The Massey Rating Method

An Introduction to Sports Team Rankings

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Understanding the Massey Rating Method: Introduction

What are Ratings and Why are They Important?

- ▶ Ratings are numerical values assigned to entities (like sports teams) to rank them.
- ▶ They help in comparing performances and predicting future outcomes.
- ▶ Essential in fields like sports analytics and data science.

The Role of Linear Algebra in Ratings

How Does Linear Algebra Help in Ratings?

- ▶ Linear Algebra is a branch of mathematics dealing with vectors and matrices.
- ▶ It helps in organizing data and solving systems of equations, key in data analysis.
- ▶ In ratings, it's used to construct and solve rating models like the Massey method.

Massey Method: Overview

Basics of the Massey Method

- ▶ A system to rank teams based on their game performance.
- ▶ Uses points scored and conceded in games to calculate team ratings.
- ▶ Involves setting up and solving a system of linear equations.

Step 1: Point Differential Vector (p)

Calculating Point Differentials

- ▶ Point Differential = Total points scored - Total points conceded.
- ▶ Example calculation for a team: Scored 100 points, Conceded 80 points, Differential = $100 - 80 = 20$.
- ▶ For our 5-team example, the differentials are: -124 for Duke, 91 for Miami, -40 for UNC, -17 for UVA, and 90 for VT.

▶ Thus, $p = \begin{bmatrix} -124 \\ 91 \\ -40 \\ -17 \\ 90 \end{bmatrix}$ and $r = \begin{bmatrix} \text{Duke} \\ \text{Miami} \\ \text{UNC} \\ \text{Thus} \\ \text{VT} \end{bmatrix}$

Step 2: Constructing the Massey Matrix (M)

Building the Massey Matrix

- ▶ A square matrix where each element represents games between teams.
- ▶ Diagonal elements = Number of games played (4 for each team in our example).
- ▶ Off-diagonal elements = -1 (for each game played against other teams).
- ▶ Last row modified to ensure a unique solution (sum of ratings = 0).
- ▶ Example matrix:

$$M = \begin{bmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & -1 & 4 \end{bmatrix}$$

Resolving the Rank Deficiency in the System

- ▶ Encountered a challenge: Post several games, our matrix reached a rank of $n - 1$, leading to infinite solutions for $Mr = p$.
- ▶ Objective: To derive a unique solution for team ratings.
- ▶ Solution: Massey's approach involved altering the last matrix row to all ones and the corresponding point differential vector entry to zero.
- ▶ Outcome: This change secured the matrix's rank at n , allowing us to solve the system and ascertain team ratings.
- ▶ Example: The modified system is presented as follows:

Resolving the Rank Deficiency in the System

Original System:

$$M = \begin{bmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & -1 & 4 \end{bmatrix}, \quad p = \begin{bmatrix} -124 \\ 91 \\ -40 \\ -17 \\ 90 \end{bmatrix}$$

Updated System:

$$M_{\text{updated}} = \begin{bmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & -1 & 4 & -1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}, \quad p_{\text{updated}} = \begin{bmatrix} -124 \\ 91 \\ -40 \\ -17 \\ 0 \end{bmatrix}$$

Step 3: Solving the Linear System

Finding the Team Ratings

- ▶ Adjust point differentials: last element set to 0 to sum to zero.
- ▶ Solve the linear system $\bar{M}r = \bar{p}$ using matrix algebra.
- ▶ Ratings are found by solving this system.

Solving System of Equation

$$\begin{bmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & -1 & 4 & -1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -124 \\ 91 \\ -40 \\ -17 \\ 0 \end{bmatrix}$$

where x_1, x_2, x_3, x_4, x_5 represent the ratings of the teams Duke, Miami, UNC, Thus, and VT, respectively.

The Solution

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -24.8 \\ 18.2 \\ -8.0 \\ -3.4 \\ 18.0 \end{bmatrix}$$

- Teams with ratings 18.2 and 18.0 are rated as the strongest among the five.

Interpreting the Ratings

Understanding the Results

- ▶ Higher ratings indicate stronger team performance.
- ▶ Negative ratings suggest weaker performance compared to others.
- ▶ Ratings provide a basis for comparison and analysis.

Massey Ratings: Line Rating Visualization

Visual Comparison of Team Strength



Interpretation of Ratings

Understanding the Results

- ▶ Higher ratings indicate stronger team performance.
- ▶ Negative ratings reflect relatively weaker performance compared to others.
- ▶ The visual line rating helps in understanding the relative strength between teams.

References



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