Introduction to Markov Chains and Their Applications

Math 264

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What is a Markov Chain?

A Markov chain is a mathematical system that undergoes transitions from one state to another within a finite or countable number of possible states. It is characterized by the Markov Property, which states that the future state depends only on the current state and not on the sequence of events that preceded it.

Understanding State Transitions in Markov Chains

In a Markov chain, the transition from state i to state $i+1$ is determined by a set of probabilities. These probabilities are represented in a transition matrix, where each entry T_{ij} indicates the probability of moving from state i to state j.

If $\mathbf{x}_i$ represents the state vector at time i, then the state vector at time $i+1$ (denoted $\mathbf{x}_{i+1}$) is calculated using the equation $\mathbf{x}_{i+1} = T\mathbf{x}_i$. This formula allows us to predict the next state based on the current state and the transition probabilities.

Activity 1: Weather Prediction Model

This activity uses a Markov chain model to predict weather transitions.

States and Transitions

- States: Sunny (S), Cloudy (C), Rainy (R).
- Transition probabilities: (listed probabilities).

Transition Matrix

$$T = \begin{pmatrix} 0.7 & 0.2 & 0.1 \\ 0.3 & 0.5 & 0.2 \\ 0.2 & 0.2 & 0.6 \end{pmatrix}$$

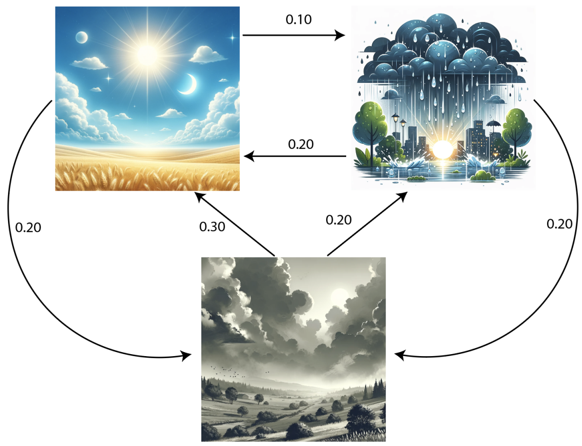

Figure 1: Annual percentage migration between city and suburbs

Activity Task

1. Assume the initial state is Sunny. Represent this as a state vector $\mathbf{v} = [1, 0, 0]$.
2. Calculate the weather probabilities for the next three days using the transition matrix T.
3. Discuss the probability trends observed over these days.

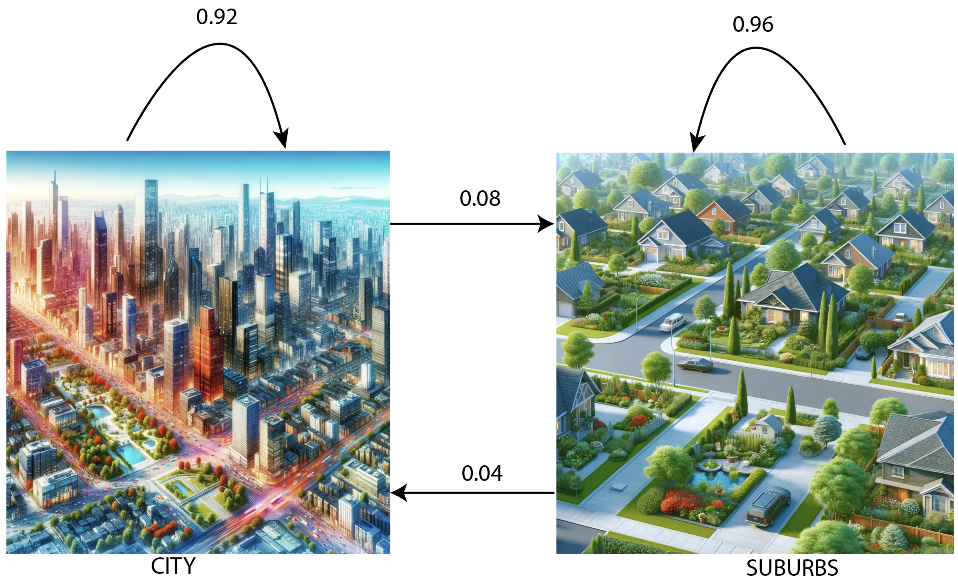

Figure 2: Annual percentage migration between city and suburbs

Activity 2: Urban and Suburban Population Dynamics

This activity models the annual population movements between a city and its suburbs using a Markov chain.

Scenario

We will model the annual migration of populations between a city and its surrounding suburbs.

Data

- 8% of the city's population moves to the suburbs annually.
- 4% of the suburban population moves to the city annually.

Model

- Initial population for 2022: $\mathbf{x}_0 = \begin{bmatrix} 600,000 \\ 400,000 \end{bmatrix}$.
- Transition matrix: $A = \begin{bmatrix} 0.92 & 0.04 \\ 0.08 & 0.96 \end{bmatrix}$.

Activity Task

1. Calculate the population vectors $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$, etc., for the years 2023, 2024, 2025, and so on, using the equation $\mathbf{x}_{k+1} = A\mathbf{x}_k$.
2. Predict the population of the city and suburbs for the year 2030.

3. Analyze the long-term trends and discuss their implications for urban planning and resource allocation.

Conclusion

This activity demonstrates the use of Markov chains in modeling more complex systems, showing how they can be applied to real-world problems in demography and urban planning.

Assignment: Analyzing Customer Choices Between Products Using Markov Chains

Background

In the realm of data science, particularly in consumer behavior analysis, it's crucial to understand how customers interact with different products. Markov chains offer a valuable method for analyzing and predicting customer choices.

Scenario

Consider a scenario where a company offers three main products: Product A, Product B, and Product C. Customers make choices between these products over time, and these choices can be modeled using a Markov chain.

Task

Your assignment is to develop a Markov chain model to analyze and predict customer behavior in terms of their product choices. The goal is to identify patterns in product preferences and predict future trends in customer choices.

Data and Assumptions

- Each customer initially considers Product A.
- Based on market research, the transition probabilities between products are known.
- Assume that at each step, customers can choose any of the three products (including the one currently considered) with certain probabilities.

Assignment Instructions

1. Define the states of the Markov chain (the products).
2. Construct the transition matrix using the provided probabilities.
3. Calculate the steady-state distribution to determine the most preferred product.
4. Analyze the results to provide insights into customer product preferences.
5. (Optional) Extend the model to include scenarios like new product introduction or discontinuation of a product.

Expected Outcomes

Students are expected to submit a report including:

- The defined states and the transition matrix.
- Steady-state distribution calculations.

- Analysis of the most preferred product among customers.
- Discussion on the implications of the findings for product strategy and marketing.