Diagnostic Medical Image Processing Reconstruction – Basic Principles of Tomography

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Topics

Tomography

Projection

Image Reconstruction

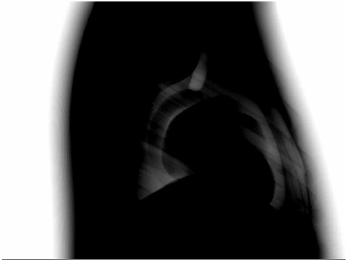
Backprojection

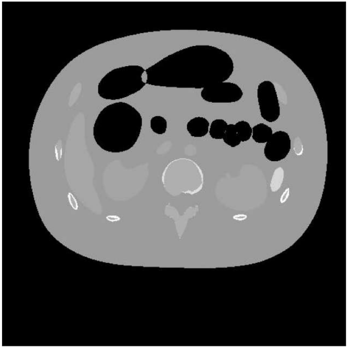
Short History of CT

Current State-of-the-art Developments

Basic Principles of Tomography

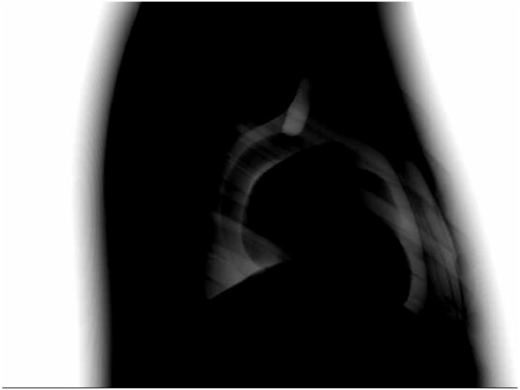
- $\pi o \mu o \sigma$ = tomos = slice





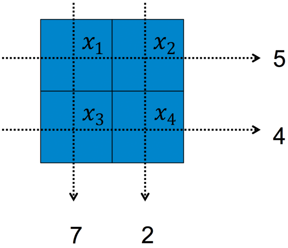
Basic Principles of Tomography (2)

- Idea: Observe object of interest from multiple sides



Basic Principles of Tomography

- Solve the puzzle



Basic Principles of Tomography

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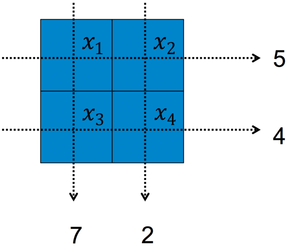
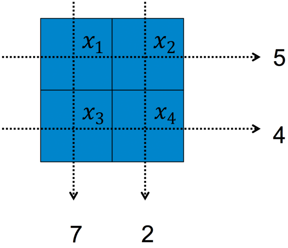


$$x_1 + x_3 = 7$$
$$x_2 + x_4 = 2$$
$$x_1 + x_2 = 5$$
$$x_3 + x_4 = 4$$

Basic Principles of Tomography

- Solve the puzzle



$$x_1 + x_3 = 7$$
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$$x_3 + x_4 = 4$$

$$x_1 = 3$$
$$x_2 = 2$$
$$x_3 = 4$$
$$x_4 = 0$$

Basic Principles of Tomography

- Solve the puzzle
- Problem size is usually 512 x 512 x 512 = 134 217 728
- How can this problem be solved?

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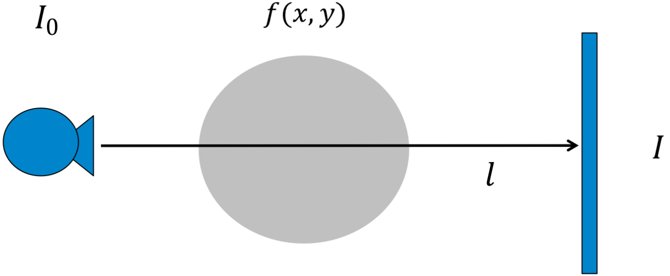
Projection – Physical Observations

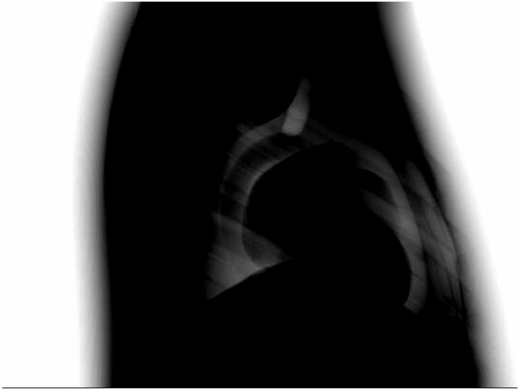


X-ray Attenuation: $I = I_0 e^{-(\int f(x,y) \mathrm{d}l)}$

Projection – Physical Observations (2)

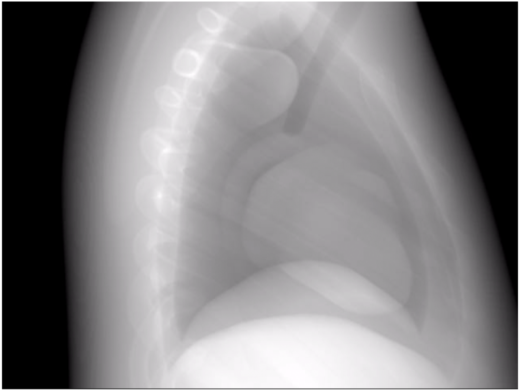
- X-ray Attenuation: $I = I_0 e^{-(\int f(x,y) \mathrm{d}l)}$

Projection – Physical Observations (3)



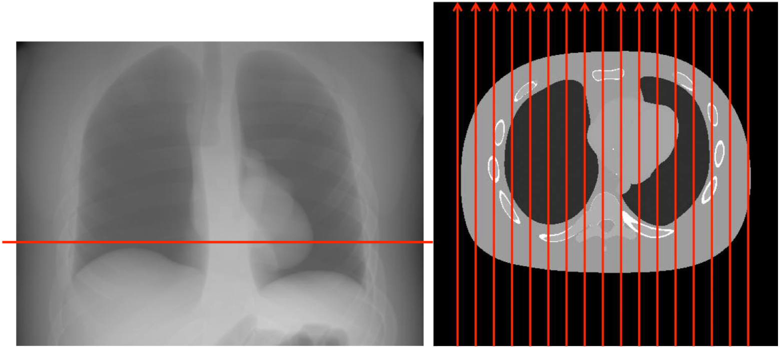
Observed Signal

Projection – Physical Observations (4)



Line Integral Data

Projection Formation



Projection – Mathematical Formulation

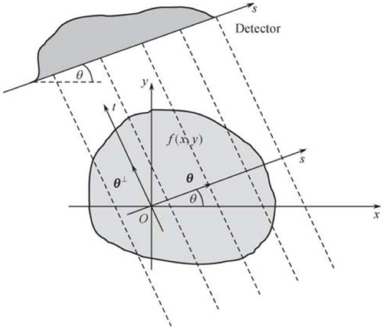


Image: Zeng, 2009

$$p(s,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(x\cos\theta + y\sin\theta - s)\mathrm{d}x, \mathrm{d}y$$

Projection – Example Point Object

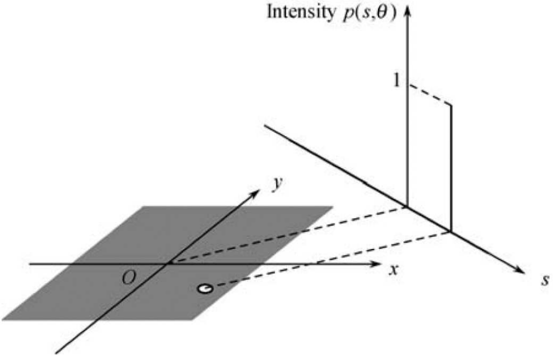
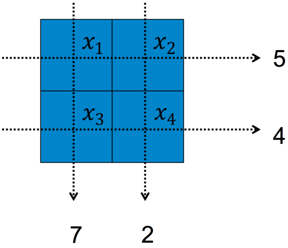


Image: Zeng, 2009

Topics

Reconstruction – Simple Example

- Solve the puzzle



Reconstruction – Simple Example

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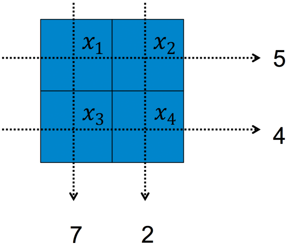

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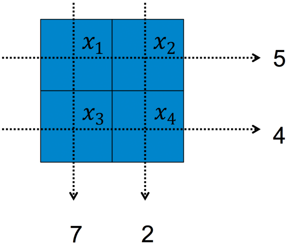

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$$x_1 = 3$$
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Reconstruction – Simple Example (2)

- Projection can be formulated in matrix notation

$$\boldsymbol{P} = \boldsymbol{A}\boldsymbol{X}$$

$$\boldsymbol{P} = \begin{pmatrix} 7 \\ 2 \\ 5 \\ 4 \end{pmatrix}, \qquad \boldsymbol{A} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}, \qquad \boldsymbol{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

Reconstruction – Simple Example (2)

- Solve with matrix inverse?

$$\boldsymbol{A}^{-1}\boldsymbol{P} = \boldsymbol{X}$$

- Common problem size:

$$\boldsymbol{A} \in \mathbb{R}^{512^3 \times 512^2 \times 512}$$

$$512^6 \cdot 4 \text{ Byte} = 2^{9 \cdot 6} \cdot 2^2 \text{ B} = 2^6 \cdot 2^{50} \text{ B}$$

$$= 64 \text{ PB} = 65536 \text{ TB}$$

Reconstruction – Example Projection

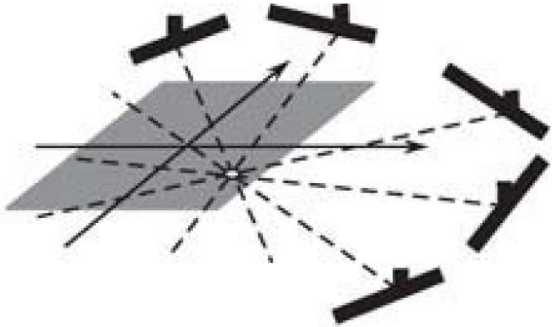


Image: Zeng, 2009

Reconstruction – Example Backprojection

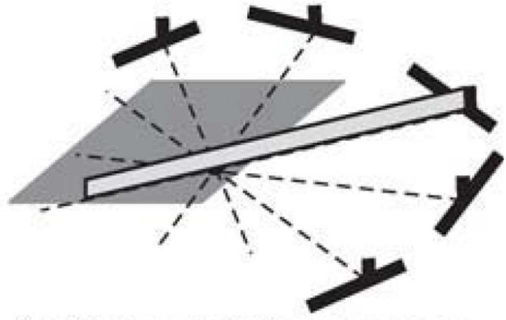


Image: Zeng, 2009

Reconstruction – Example Backprojection (2)

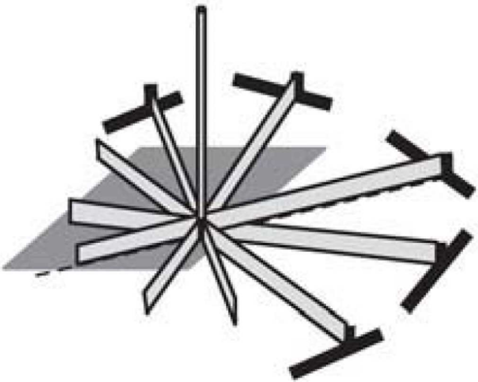


Image: Zeng, 2009

Reconstruction – Example Backprojection (3)

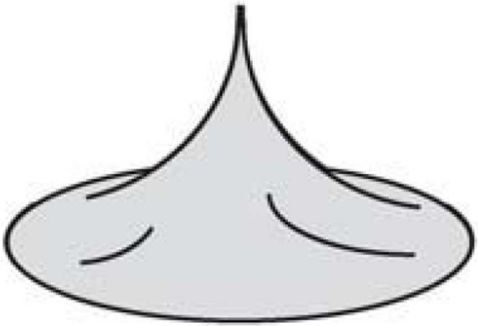


Image: Zeng, 2009

Reconstruction – Example "Negative Wings"

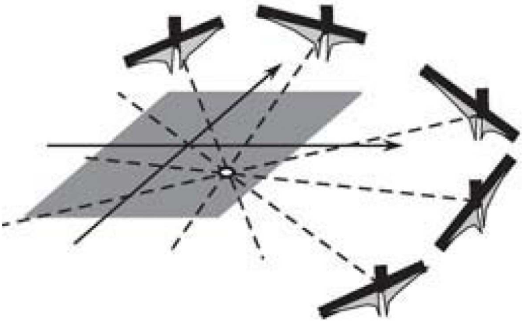
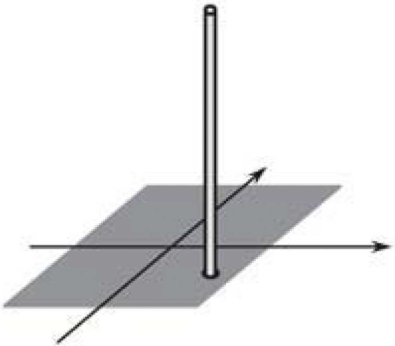


Image: Zeng, 2009

Reconstruction – Example Reconstruction



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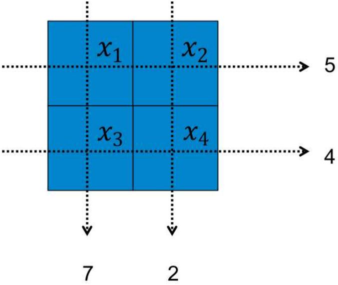
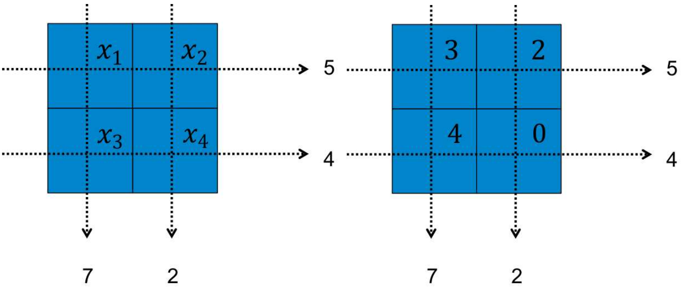
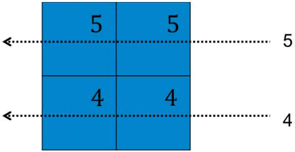
Backprojection – Example



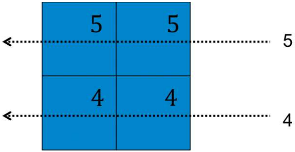
Backprojection – Example



Backprojection – Example (2)



Backprojection – Example (2)



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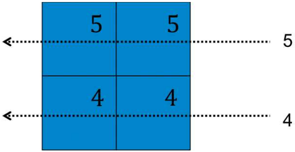

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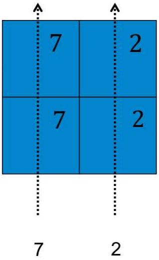
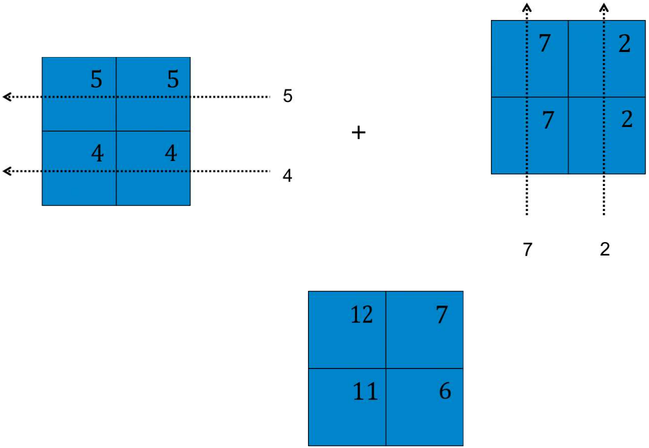


Backprojection – Example (2)



Backprojection – Example (3)

- Backprojection is not the inverse of Projection!
- In matrix notation, it is simply the matrix transpose:

$$\boldsymbol{B} = \boldsymbol{A}^T \boldsymbol{P}$$

$$\boldsymbol{A}^T = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}, \boldsymbol{P} = \begin{pmatrix} 7 \\ 2 \\ 5 \\ 4 \end{pmatrix}$$

$$\boldsymbol{B} = \begin{pmatrix} 12 \\ 7 \\ 11 \\ 6 \end{pmatrix}$$

Backprojection – Mathematical Formulation

- In matrix notation, it is simply the matrix transpose
- The following equivalent formulations are employed in literature:

$$b(x, y) \quad = \quad \int_0^{\pi} p(s, \theta)|_{s=x\cos\theta + y\sin\theta} \mathrm{d}\theta$$

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$$b(x,y) = \int_0^\pi p(s,\theta)|_{s=\boldsymbol{x}\cdot\boldsymbol{\theta}}\,\mathrm{d}\theta$$

Backprojection – Mathematical Formulation

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$$b(x, y) = \int_0^\pi p(\boldsymbol{x} \cdot \boldsymbol{\theta}, \theta) \mathrm{d}\theta$$

Backprojection – Mathematical Formulation

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$$b(x, y) = \int_0^{\pi} p(\boldsymbol{x} \cdot \boldsymbol{\theta}, \theta) \mathrm{d}\theta$$

$$b(x, y) = \frac{1}{2} \int_0^{2\pi} p(x \cos\theta + y \sin\theta, \theta) \mathrm{d}\theta$$

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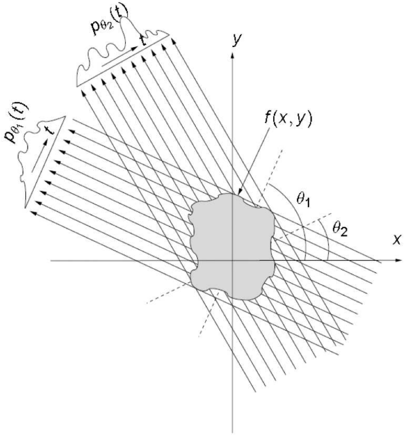
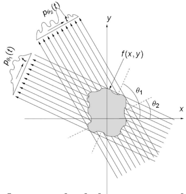
Parallel Beam Geometry



- Earliest Acquisition Geometry
- Principle: Rotate & Translate

Parallel Beam Geometry

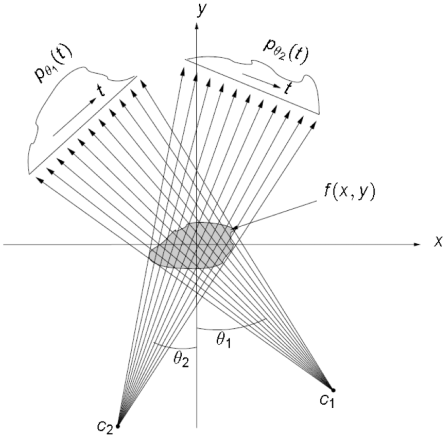


- Acquisition took 5 Minutes
- Reconstruction took 30 Minutes
- Slice resolution was 80 x 80 pixels

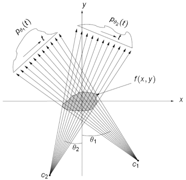
First CT Scanner: EMI (1971)

Image: Wikipedia

Fan Beam Geometry



Fan Beam Geometry



- Fan beam Scanners became available in 1975 (20s / slice)
- Fast rotations became possible 1987 with slip rings (300ms / slice)

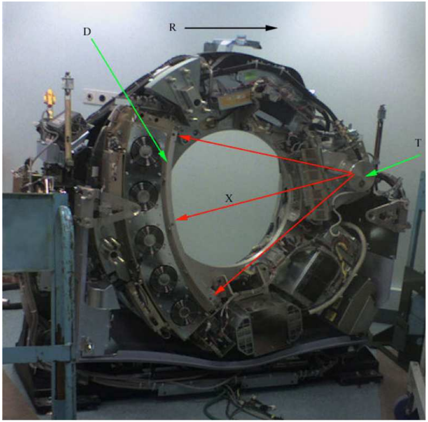
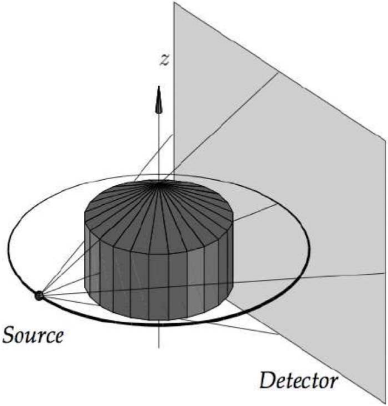
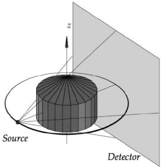


Image: Wikipedia

Cone Beam Geometry



Cone Beam Geometry



320 Row Scanner: Toshiba (2007)

Image: Toshiba

- Further increase in the number of rows did not take place so far
- Physical effects such as scattered radiation currently limit the number of detector rows in CT
- Flat panel detector technologies have even larger cone angles

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Highlight 1: 3-D Reconstruction in dual CT

Image: Siemens AG

- Dual source CT introduced 2005
- Fast scanning (75 ms)
- Material decomposition possible

Highlight 2: 3-D Reconstruction in Dental Medicine

Image: http://www.planmeca.com

- Introduced in October 2006

Highlight 3: 3-D Reconstruction in the Angio Lab

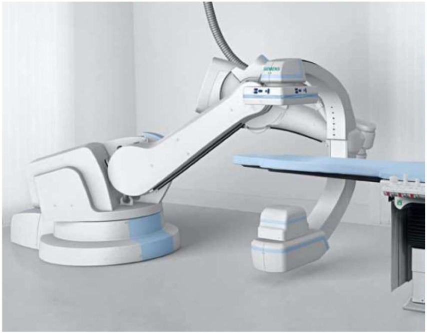

Image: Siemens AG

- C-arm mounted on a robot system (November 2007)

Highlight 4: 3-D Reconstruction in the Neuro Lab

Image: Siemens AG

- C-arm biplane device

Further Readings

- Gengsheng Lawrence “Larry” Zeng. "Medical Image Reconstruction – A Conceptual Tutorial". Springer 2009
- Avinash C. Kak, Malcolm Slaney. "Principles of Computerized Tomographic Imaging". Society for Industrial Mathematics 2001. http://www.slaney.org/pct/
- Thorsten M. Buzug. "Computed Tomography: From Photon Statistics to Modern Cone-Beam CT". Springer 2008
- Willi A. Kalender. "Computed Tomography: Fundamentals, System Technology, Image Quality, Applications". Wiley 2011

Questions?