Data Structures and Algorithms
Lecture notes: Trees

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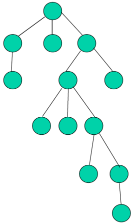
Outline

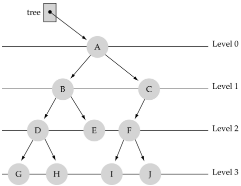
What is a tree

- At this point we look at trees as a data structure like arrays, stacks, queues
- More generally, a tree is a graph, i.e. a mathematical abstraction, thus a tree data structure has nodes and edges
- Nodes in a tree data structure, like nodes in a queue, have a data field and have pointer field(s).



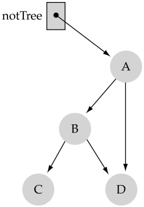
What is a tree

- Like in queues, nodes are connected by pointers to form a sequence (called levels in a tree)
- Usually the beginning of the sequence is the node at the lowest depth (level 0) in the tree
- This node is called the root of the tree (node A in this tree), there is an external pointer to this node for finding the tree
- Unlike queues, nodes in a tree can branch out in several directions from level i to level $i + 1$



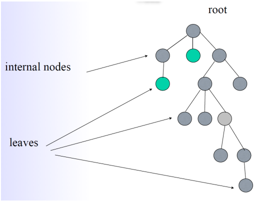
Not a tree

- A tree has a unique path from the root to every other node
- Thus the graph on the right is not a tree as there exists two paths from the root A to the leaf D



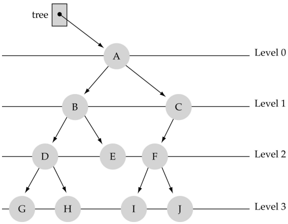
Types of nodes in a tree

- Again, the first node in a tree is called the root of the tree
- The nodes at the end of a branch in the tree are called leaves
- The other nodes are named internal nodes



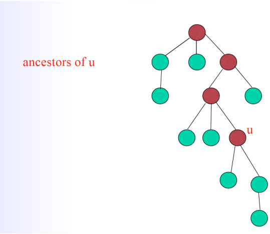
Tree : parent, child, siblings

- A node y at level i connected to a node x at level $i-1$ is said to be the child of node x and x is the parent of node y
  - Node F is the child of node C
  - Node C is the parent of node F
- The root node has no parent
- Leaf nodes have no child
- The child nodes w, y, z of a same parent node x are said to be siblings with respect to each other
  - Nodes G and H have the same parent, node D, thus they are siblings



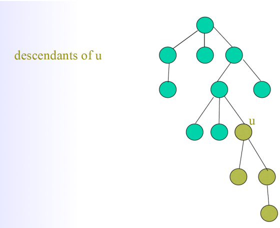
Tree : ancestors, height, depth

- The ancestors of a node u are all the nodes on the path from u to the root
- According to CLRS textbook, u is his own ancestor
- The root has only itself as ancestor
- The height of a node u in a tree is the length of the longest path from u to any leaf
- The height of u in the tree is 2
- The height of a leaf is 0
- The height of a tree is the height of its root
- The depth of tree is the number of levels in the tree, the present tree has depth 5
- The depth of a node is the level of that node in the tree. The depth of the root is 0, the depth of node u in the tree is 3



Tree : descendants and subtrees

- The descendants of a node u are the nodes on all the paths from node u to leaves
- According to CLRS textbook, u is his own descendant
- The descendants of a node u form of subtree rooted at node u
- If n is the number of nodes in a tree, there are $n-1$ subtrees
- A subtree where the root is a leaf has only itself as descendant
- The degree of a node u is the number children of u (thus node u has degree 2)
- The degree of a tree is equal to the largest degree of its nodes



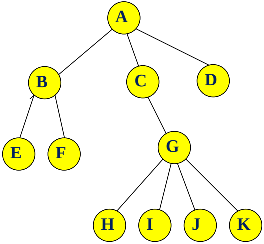
Node declaration in C

Nodes in a tree are declared in the same way as for nodes of a queue. Nodes have two pointers, one for the leftmost child and one for the right-sibling of the leftmost child

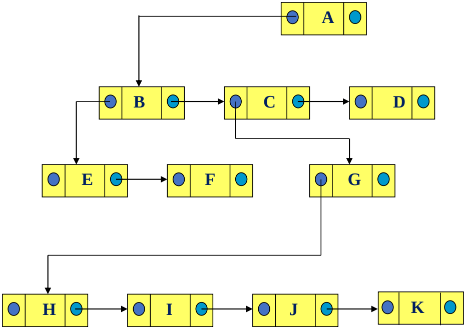
Data	
Leftmost Child	Right-sibling

```
typedef struct
{
 int data;  // data of each node
 struct treeNode  * leftmost_child;
 struct treeNode  * right_sibling;
}treeNode;
treeNode * Root;
```

Tree example

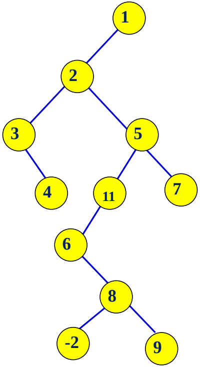


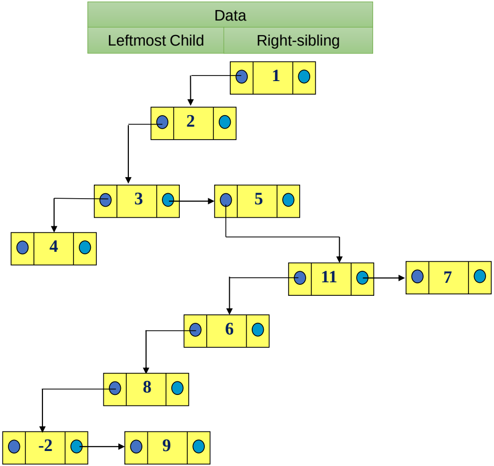
Data	
Leftmost Child	Right-sibling




```
typedef struct
{
  char data;  // data of each node
  struct treeNode  * leftmost_child;
  struct treeNode  * right_sibling;
}treeNode;
treeNode * Root;
```

Tree second example



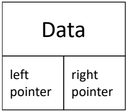


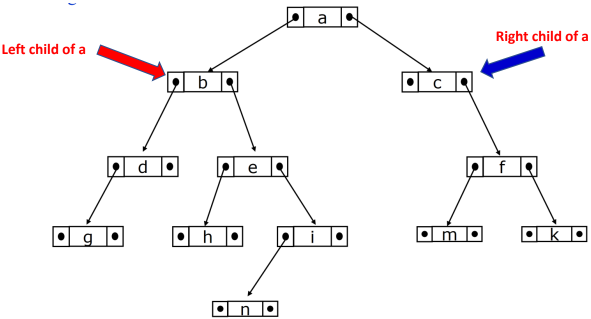

```
typedef struct
{
  int data;  // data of each node
  struct treeNode  * leftmost_child;
  struct treeNode  * right_sibling;
}treeNode;
treeNode * Root;
```

Binary trees

A binary tree is a tree such that

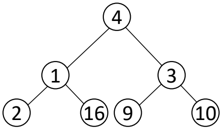
- every node has at most 2 children
- each node is labeled as being either a left child or a right child





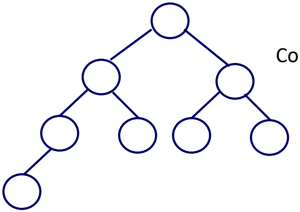
Full versus complete binary trees

- ***Full*** binary tree: a binary tree in which every node has two children except for the leaves (which, by definition, have no children)

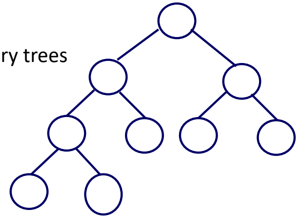


Full binary tree

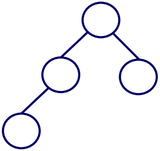
- ***Complete*** binary tree: a binary tree in which
  - every level is full except possibly the deepest level
  - if the deepest level isn't full, leaf nodes are as far to the left as possible



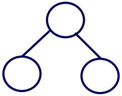
Complete binary trees



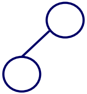
Examples



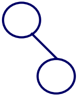
Complete binary tree



Full and complete

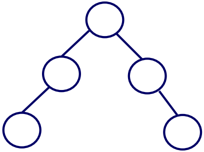


Complete



Neither full nor complete

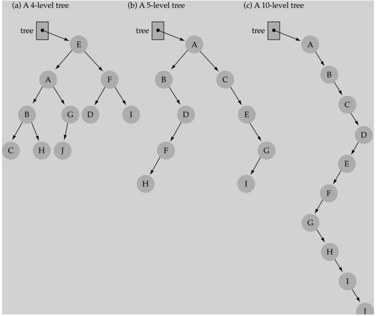
Full and complete



Neither full nor complete

Height of binary tree

- The maximum height of a binary tree with n nodes is the same as the length of a link list with n nodes, i.e. n
- The minimum height of a binary tree with n nodes is $\lceil \log(n+1) \rceil - 1$
- Complete binary trees have minimum height



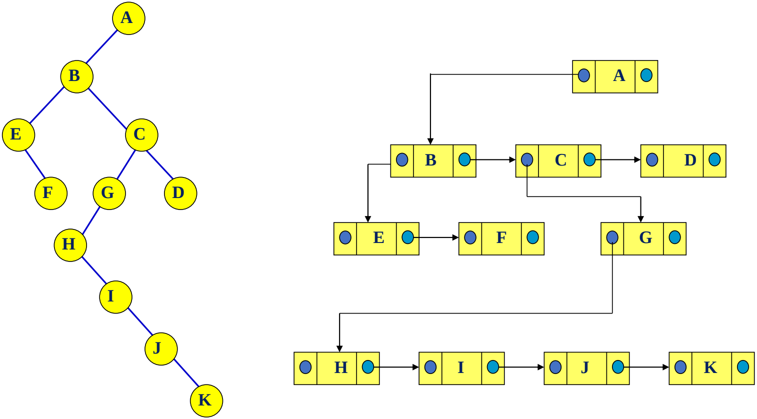
Binary tree representation

Binary trees are represented using pointers in a similar ways as ordinary trees :

- Each node contains the address of the left child and the right child
- If any node has its left or right child empty then it will have in its respective pointer a null value
- A leaf has null value in both of its pointers

```
typedef struct
{
   DataType data; /*data of node; DataType: int, char, double..*/
   struct node *left ;   /* points to the left child */
   struct node *right;   /* points to the right child */
}node;
```

Example



Binary tree traversal

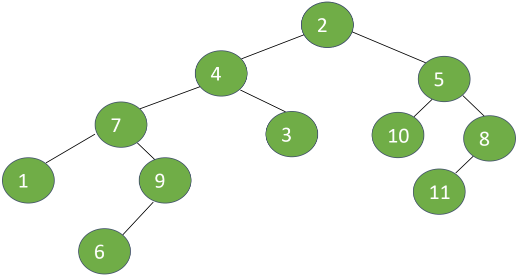
A traversal is a systematic way to visit all nodes of a graph, a binary tree in the present case

There are two very common traversals :

- Breadth First
- Depth First

Breadth First : In a breadth first traversal all of the nodes on a given level are visited and then all of the nodes on the next level are visited. Usually in a left to right fashion

On the tree below $\mathit{Breadth} - \mathit{first} - \mathit{search}(2)$ visits the nodes in this order : 2, 4, 5, 7, 3, 10, 8, 1, 9, 11, 6



Depth first traversals

In a depth first traversal all the nodes of a subtree are visited prior to visit another subtree

There are three common depth first traversals

- Inorder
- Preorder
- Postorder

Please observes the next tree depth first traversal procedures, these are recursive algorithms as they call themselves inside their own code (rather then calling another function)

Inorder tree traversal

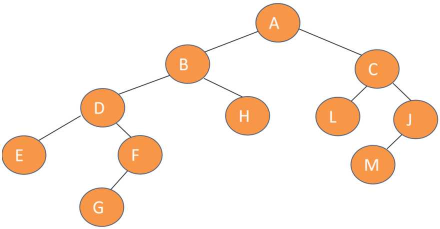
Traverse the left subtree ; Visit the root ; Traverse the right subtree

InorderTreeWalk(x)
  if $x \neq NIL$
    InorderTreeWalk(x.left) ;
    print(x.key) ;
    InorderTreeWalk(x.right) ;

Call: InorderTreeWalk(A);

E D G F B H A L C M J

left subtree   right subtree



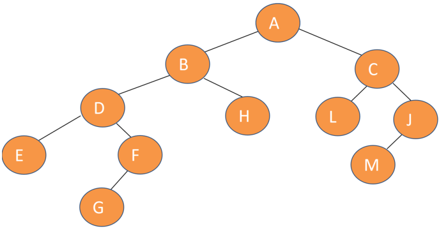
Preorder tree traversal

Visit the root ; Traverse the left subtree ; Traverse the right subtree

```
PreorderTreeWalk(x)
if x ≠ NIL
   print(x.key) ;
   PreorderTreeWalk(x.left) ;
   PreorderTreeWalk(x.right) ;
```

Call: PreorderTreeWalk(A);

$\mathbf{A}\ \underbrace{\mathbf{B\ D\ E\ F\ G\ H}}_{\text{left subtree}}\ \underbrace{\mathbf{C\ L\ J\ M}}_{\text{right subtree}}$



Postorder tree traversal

Traverse the left subtree ; Traverse the right subtree ; Visit the root

PostorderTreeWalk(x)
if $x \neq NIL$
    PostorderTreeWalk(x.left) ;
    PostorderTreeWalk(x.right) ;
    print(x.key) ;

`Call: PostorderTreeWalk(A);`

$\underbrace{\mathbf{E\ G\ F\ D\ H\ B}}_{\text{left subtree}}\ \underbrace{\mathbf{L\ M\ J\ C}}_{\text{right subtree}}\ \mathbf{A}$