# MTH20014 Mathematics 3B. Tutorial 1

## Revision of previously studied material

1. Solve the following systems of equations by using (a) Gaussian elimination, (b) Cramer's rule and (c) matrix inversion.

$$\text{(i)} \begin{cases} x + 2y + 2z &= 3 \\ 2x + 2y - 2z &= 1 \\ 3x + 4y - z &= 3 \end{cases} ; \qquad \text{(ii)} \begin{cases} a + 2b &= 1 \\ 2a + 3b + 2c &= 3 \\ a + b + 3c &= 4 \end{cases} .$$

2. Solve the following systems of equations by Gaussian elimination and state the geometrical meaning of the obtained solutions.

$$\text{(i)} \begin{cases} x + 2y + 2z &= 3 \\ 2x + 2y - 2z &= 0 \\ 3x + 4y &= 3 \end{cases} ; \qquad \text{(ii)} \begin{cases} a + 2b - c &= 1 \\ 2a + 3b + 2c &= 3 \\ a + b + 3c &= 4 \end{cases} .$$

3. Show that the vectors $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{e}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\mathbf{e}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ are linearly independent.

4. Show that the vectors $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{e}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ and $\mathbf{e}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ are linearly dependent.

5. Find the determinants of the following matrices:

$$\text{(a)} \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix} ; \quad \text{(b)} \begin{bmatrix} 20 & 0 & 1 \\ 0 & 5 & 4 \\ -4 & 4 & 3 \end{bmatrix} ; \quad \text{(c)} \begin{bmatrix} 2 & 0 & 1 \\ -1 & 4 & -1 \\ -1 & 2 & 0 \end{bmatrix} .$$

6. Invert the above matrices or state why this is impossible.

## Answers

1. (i) $(x, y, z) = \left(2, -\dfrac{1}{2}, 1\right)$; (ii) $(a, b, c) = (-5, 3, 2)$;

2. (i) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 0 \end{bmatrix} + t \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$; three planes intersect along the same straight line in a three-dimensional space.
   (ii) No solution, three planes do not have a common intersection.

5. (a) 6; (b) 0; (c) 6.

6. (a) $\dfrac{1}{6} \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$; (b) Impossible; (c) $\dfrac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ 1 & 1 & 1 \\ 2 & -4 & 8 \end{bmatrix}$.