MTH20014 Mathematics 3B. Tutorial 7

1. Check if the following vector fields have a scalar potential and, if yes, find it:

   (a) $\mathbf{F} = e^{xy}(y, x)$,
   (b) $\mathbf{F} = (3x^2y - y^2, x^3 - 2xy)$,
   (c) $\mathbf{F} = (3x^2 + xy, -x^3 - y)$,
   (d) $\mathbf{F} = (e^{-x} + y, x + e^y)$,
   (e) $\mathbf{F} = (x^2 - y^2, y^2 - 2xy)$,
   (f) $\mathbf{F} = (\sin(xy), \cos(xy))$,
   (g) $\mathbf{F} = (y^3 - y, 3xy^2 - x)$,
   (h) $\mathbf{F} = (y\cos(xy), x\cos(xy))$.

2. Calculate $\boldsymbol{\nabla} \times \mathbf{F}$ for the following vector fields:

   (a) $\mathbf{F} = (x, 2y, 3z)$,
   (b) $\mathbf{F} = (3x^2 - y^2, x^2 - 2xy, 3z)$,
   (c) $\mathbf{F} = (3x^2, y^2, z^2)$,
   (d) $\mathbf{F} = (xy, yz, xz)$,
   (e) $\mathbf{F} = (yz, -zx, xy)$,
   (f) $\mathbf{F} = (e^{-x}, xe^{-y}, xy)$,
   (g) $\mathbf{F} = (2xy + 3x^2z, x^2 - 3z^2, x^3 - 6yz)$,
   (h) $\mathbf{F} = (3x^2y + z^2, x^3 + 6y^2z, 2xz + 2y^3)$.

3. Given that the vector field $\mathbf{F} = (x^2+3y+az, bx-3y-z, 4x+cy+2z^2)$ is irrotational find a, b and c.

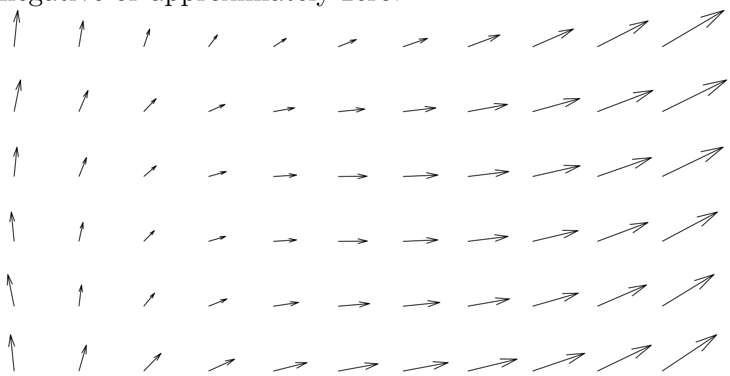
4. Check if the following vector fields have a scalar potential and, if yes, find it:

   (a) $\mathbf{F} = (y, -2x, x)$,
   (b) $\mathbf{F} = (4xyz, 2x^2z + 3, 2x^2y)$,
   (c) $\mathbf{F} = (yz, xz, xy)$,
   (d) $\mathbf{F} = (2x - z^3, 2yz, y^2 - 3xz^2)$.

5. Calculate the Laplacian $\nabla^2\phi$ for the following functions:

   (a) $\phi = x^2 + y^2 + z^2$,
   (b) $\phi = 3x^2y - 3xy^2 + 2z^2$,
   (c) $\phi = 2x^2yz + 3zy^2 - 2xyz^2$,
   (d) $\phi = e^x \sin y - ze^x \cos y$,
   (e) $\phi = \frac{1}{x^2+y^2+z^2}$,
   (f) $\phi = \exp(-r)$, $r = \sqrt{x^2 + y^2 + z^2}$,
   (g) $\phi = x^2y^2z^3$,
   (h) $\phi = \frac{1}{r}$, $r = \sqrt{x^2 + y^2 + z^2}$.

6. For the following vector field indicate regions where you expect the curl to be positive, negative or approximately zero:



Answers

1. (a) $\phi = e^{xy} + C$, (b) $\phi = x^3y - xy^2 + C$, (c) no potential function exists, (d) $\phi = e^y - e^{-x} + xy + C$, (e) $\phi = \frac{1}{3}(x^3 + y^3) - xy^2 + C$, (f) no potential function exists, (g) $\phi = xy^3 - xy + C$, (h) $\phi = \sin(xy) + C$.

2.

	$\nabla \times \mathbf{F}$
(a)	$\mathbf{0}$
(b)	$(0, 0, 2x)$
(c)	$\mathbf{0}$
(d)	$(-y, -z, -x)$
(e)	$(2x, 0, -2z)$
(f)	$(x, -y, e^{-y})$
(g)	$\mathbf{0}$
(h)	$\mathbf{0}$

3. $a = 4$, $b = 3$, $c = -1$.

4. (a) no potential function exists, (b) $\phi = 2x^2yz + 3y + C$, (c) $\phi = xyz + C$, (d) $\phi = x^2 + y^2z - xz^3 + C$.

5. (a) 6;
   (b) $6y - 6x + 4$;
   (c) $-4xy + 6z + 4yz$;
   (d) 0;
   (e) $\frac{2}{(x^2+y^2+z^2)^2}$;
   (f) $e^{-r}\left(1 - \frac{2}{r}\right)$;
   (g) $2z^3(x^2 + y^2) + 6x^2y^2z$;
   (h) 0.

6.