MTH20014 Mathematics 3B. Tutorial 11

1. Find Maclaurin series of $f(z) = \dfrac{1}{1+az}$, where a is a complex constant, and determine its radius of convergence R.

2. Calculate directly the first two non-zero terms in the Taylor series expansion of $\sinh z$ about $z = \mathrm{i}\pi$.

3. Find Laurent series expansion for $f(z) = \dfrac{1}{z(z-1)^2}$ about (a) $z = 0$ and (b) $z = 1$. Specify the region of convergence in each case.

4. Find the Laurent series expansion for $f(z) = \dfrac{1}{(z+1)(2+\mathrm{i}z)}$ valid in the region (a) $|z| < 1$, (b) $1 < |z| < 2$ and (c) $2 < |z|$.

Answers

1. $\displaystyle\sum_{n=0}^{\infty}(-az)^n$, $R = \frac{1}{|a|}$.

2. $\sinh z = (\mathrm{i}\pi - z) + \dfrac{1}{6}(\mathrm{i}\pi - z)^3 + \cdots$.

3. (a) $\dfrac{1}{z} + 2 + 3z + 4z^2 + \cdots$, $0 < |z| < 1$;

   (b) $\dfrac{1}{(z-1)^2} - \dfrac{1}{z-1} + 1 - (z-1) + (z-1)^2 \cdots$, $0 < |z-1| < 1$.

4. (a) $\dfrac{1}{2} - \dfrac{2+\mathrm{i}}{4}z + \dfrac{3+2\mathrm{i}}{8}z^2 - \dfrac{6+3\mathrm{i}}{16}z^3 \cdots$;

   (b) $\dfrac{2+\mathrm{i}}{5}\left(\cdots + \dfrac{1}{z^3} - \dfrac{1}{z^2} + \dfrac{1}{z}\right) + \dfrac{1-2\mathrm{i}}{10}\left(1 + \dfrac{\mathrm{i}z}{2} - \dfrac{z^2}{4} + \dfrac{\mathrm{i}z^3}{8} + \cdots\right)$;

   (c) $-\dfrac{\mathrm{i}}{z^2} + \dfrac{2+\mathrm{i}}{z^3} - \cdots$.