# MTH20014 Mathematics 3B. Tutorial 3

1. Find the modal matrices $\mathbf{M}$ for

$$\text{(a) } \mathbf{A} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} ; \quad \text{(b) } \mathbf{A} = \begin{bmatrix} -1 & 3 & -1 \\ -3 & 5 & -1 \\ -3 & 3 & 1 \end{bmatrix} ; \quad \text{(c) } \mathbf{A} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & 0 \\ 2 & -4 & 2 \end{bmatrix}$$

   and check by direct matrix multiplication that $\mathbf{M}^{-1}\mathbf{A}\mathbf{M}$ results in the corresponding spectral matrices $\mathbf{\Lambda}$.

2. Use the results of Question 1 to compute $\mathbf{A}^4$.

3. Find the spectral matrix $\mathbf{\Lambda}$ and the corresponding modal matrix $\mathbf{M}$ for

$$\mathbf{A} = \begin{bmatrix} 5 & 2 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 5 & -2 \\ 0 & 0 & -2 & 2 \end{bmatrix}$$

   given that the eigenvalues of $\mathbf{A}$ are 6, 6, 1 and 1.

4. Compute $\mathbf{A}^{20}$ using the appropriate corollary of Cayley-Hamilton theorem if $\mathbf{A}$ is

$$\text{(a) } \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} ; \quad \text{(b) } \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} ; \quad \text{(c) } \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix} ; \quad \text{(d) } \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix} .$$

   **Hint:** leave large powers of numbers in an exponential form, e.g. $3^{20}$.

## Answers

1. (a) $\mathbf{M} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, $\mathbf{\Lambda} = \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}$;

   (b) $\mathbf{M} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 3 & 1 \end{bmatrix}$, $\mathbf{\Lambda} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$;

   (c) $\mathbf{M} = \begin{bmatrix} -1 & 0 & -1 \\ -1 & 0 & 0 \\ 2 & 1 & 2 \end{bmatrix}$, $\mathbf{\Lambda} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$;

2. (a) $\begin{bmatrix} 41 & 40 \\ 40 & 41 \end{bmatrix}$; (b) $\begin{bmatrix} -29 & 45 & -15 \\ -45 & 61 & -15 \\ -45 & 45 & 1 \end{bmatrix}$; (c) $\begin{bmatrix} 1 & 80 & 0 \\ 0 & 81 & 0 \\ 30 & -160 & 16 \end{bmatrix}$;

3. $\mathbf{\Lambda} = \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $\mathbf{M} = \begin{bmatrix} 2 & 0 & 1 & 0 \\ 1 & 0 & -2 & 0 \\ 0 & -2 & 0 & 1 \\ 0 & 1 & 0 & 2 \end{bmatrix}$

4. (a) $\dfrac{1}{2}\begin{bmatrix} 3^{20}+1 & 3^{20}-1 \\ 3^{20}-1 & 3^{20}+1 \end{bmatrix}$; (b) $\begin{bmatrix} 1 & 20 \\ 0 & 1 \end{bmatrix}$; (c) $\begin{bmatrix} 2(3^{20}-2^{19}) & 2^{20}-3^{20} \\ 2(3^{20}-2^{20}) & 2^{21}-3^{20} \end{bmatrix}$;
   (d) $\dfrac{1}{4}\begin{bmatrix} 3\cdot 5^{20}+1 & 5^{20}-1 \\ 3(5^{20}-1) & 5^{20}+3 \end{bmatrix}$.