MTH20014 Mathematics 3B. Tutorial 9

1. Evaluate the following integrals by transforming them to polar form:

(a) $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} (3x + 2y^2)\,\mathrm{d}y\,\mathrm{d}x$, (b) $\displaystyle\int_{-2}^2 \int_0^{\sqrt{4-x^2}} (x^2 + y^2)\,\mathrm{d}y\,\mathrm{d}x$,

(c) $\displaystyle\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (2x + 3y)\,\mathrm{d}y\,\mathrm{d}x$, (d) $\displaystyle\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{2x-x^2}} (x^2 - y^2)\,\mathrm{d}y\,\mathrm{d}x$,

(e) $\displaystyle\int_{-2}^2 \int_0^{\sqrt{4-x^2}} (x^2 + 3y)\,\mathrm{d}y\,\mathrm{d}x$, (f) $\displaystyle\int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (2x^2 - y^2)\,\mathrm{d}y\,\mathrm{d}x$.

2. C is the triangular path consisting of the straight line segments running from $(0,0)$ to $(1,0)$ to $(1,1)$ and back to $(0,0)$. Evaluate $\oint_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$ (i) as a line integral and (ii) using Stokes' theorem if

(a) $\mathbf{F} = (y, x^2)$, (b) $\mathbf{F} = (y^3 - y, 3xy^2 - x)$,
(c) $\mathbf{F} = (3x^2y - y^2, x^3 - 2xy)$, (d) $\mathbf{F} = (x^2y + y^2, x^3 + 4xy)$.

3. Evaluate the flux integral $\iint_S \mathbf{G} \cdot \mathbf{n}\,\mathrm{d}A$, where $\mathbf{G} = (2y, -z, 2y - 1)$ and S is the parabolic bowl $z = x^2 + y^2$ for $0 \le z \le 4$ (take the normal to S to point away from the z axis).

4. Using the Divergence theorem, find $\iint_S \mathbf{F}\cdot\mathbf{n}\,\mathrm{d}A$, where the region of integration T is the rectangular parallelepiped $0 \le x \le 1$, $0 \le y \le 3$, $0 \le z \le 2$ and $\mathbf{F} = (x^2, z, y)$.

Answers

1. (a) $1+\dfrac{\pi}{8}$, (b) 4π, (c) $\dfrac{32}{3}$, (d) $\dfrac{\pi}{2}$, (e) $2(8+\pi)$, (f) $\dfrac{81\pi}{4}$.
2. (a) $\dfrac{1}{6}$, (b) 0, (c) 0, (d) $\dfrac{5}{6}$.
3. 4π.
4. $\nabla \cdot \mathbf{F} = 2x$ and we need to evaluate the integral

$$\int_0^1 \int_0^3 \int_0^2 2x \, \mathrm{d}z \, \mathrm{d}y \, \mathrm{d}x = 6\,.$$