MTH20014 Mathematics 3B. Tutorial 8

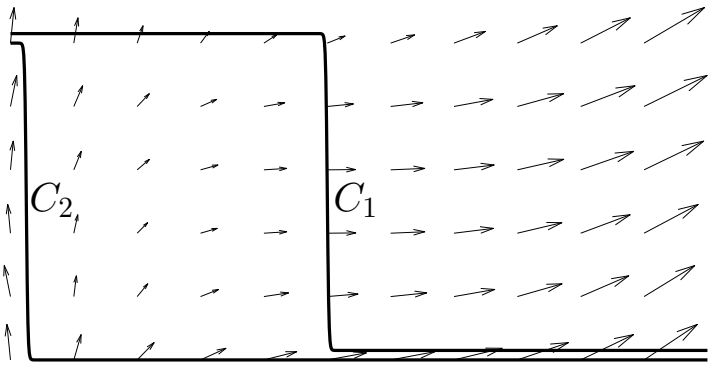
1. For the following vector fields $\mathbf{F}$ show that they are irrotational, then find a potential function for $\mathbf{F}$ and evaluate the line integral $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$, where C is a path running from the origin to the point indicated.

   (a) $\mathbf{F} = (y^2 + 2xz^2, 2xy, 2x^2z)$, $P(2, 1, 3)$;
   (b) $\mathbf{F} = (x + y^2 + 4z, 2xy - 3y - z, 4x - y + 2z)$, $P(3, -2, 3)$;
   (c) $\mathbf{F} = (4xyz, 2x^2z + 3, 2x^2y)$, $P(2, 2, 3)$;
   (d) $\mathbf{F} = (6xy - 4yz, 3x^2 - 4xz + z^3, 3z^2y - 4xy + 1)$, $P(3, -1, 2)$.

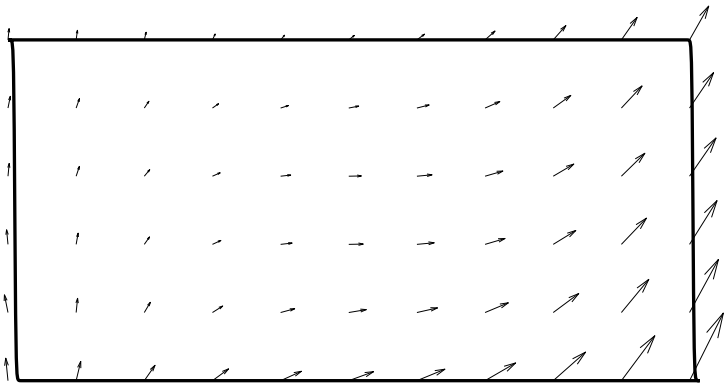
2. Determine the sign of a work integral

   (a) along an arbitrary path from the bottom left to top right corner,
   (b) along path C_1 from the top left to bottom right corner,
   (c) along path C_2 from the top left to bottom right corner

   of the vector field image shown below.



3. Determine the sign of circulation of the vector field along the closed path shown below.



4. Find the circulation $\oint_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$ of the following vector fields along a circular path $C : x^2 + y^2 = 4$.
   (a) $\mathbf{F} = (y, -x)$, (b) $\mathbf{F} = (xy, xy)$, (c) $\mathbf{F} = (3x, -2y)$.

5. The following integrals cannot be written in terms of standard elementary functions. Evaluate them by reversing the order of integration. In each case start with sketching the region of integration.
   (a) $\displaystyle\int_0^2 \int_y^2 e^{x^2} \,\mathrm{d}x \,\mathrm{d}y$, (b) $\displaystyle\int_0^\pi \int_x^\pi \frac{\sin y}{y} \,\mathrm{d}y \,\mathrm{d}x$.

Answers

1. (a) $\phi = xy^2 + x^2z^2 + C$, 38;

   (b) $\phi = \frac{1}{2}x^2 + xy^2 + 4xz - \frac{3}{2}y^2 - yz + z^2 + C$, $\dfrac{123}{2}$;

   (c) $\phi = 2x^2yz + 3y + C$, 54;

   (d) $\phi = 3x^2y - 4xyz + yz^3 + z + C$, -9.

2. (a) positive, (b) positive, (c) cannot say.

3. Positive.

4. (a) -8π, (b) 0, (c) 0.

5. (a) $\dfrac{1}{2}(e^4 - 1)$, (b) 2.