

- 1) translate inequality statements into interval notation
  - 2) limits for a piecewise-linear function
  - 3) unconventional phrasing for a tangent-line task
  - 4) tangent to cubic
  - 5) derivative of  $\cos(x+a)$
  - 6) identify function and its derivatives in a graph
  - 7)  $\lim_{x \rightarrow 0} \frac{\sin(ax)}{bx}$
  - 8) figure to motivate l'Hopital's rule
  - 9) implicit differentiation [problem is static]
  - 10) slope of tangent to  $2(x^2 + y^2)^2 = 25(x^2 - y^2)$
  - 11) derivative of  $a \arcsin(x^n)$
  - 12) derivative of  $a \cos(b \ln(x))$
  - 13) intervals of increase and decrease for cubic
  - 14) confirm Rolle's Theorem for a quadratic
  - 15)  $\lim_{x \rightarrow 0^+} x^{a \sin(x)}$
  - 16)  $\int \frac{a}{x \cdot \ln(bx)} dx$
  - 17) maximize area of Norman window with perimeter P
  - 18) solve IVP:  $f''(x) = ax + b$
  - 19) differentiate  $g(x) = \int_{ax}^{bx} \frac{u+c}{u-d} du$
  - 20) definite integral: interval additivity, linear operator
- Problems 2, 6, 8 create and display individualized graphs.

1. (1 pt) Sketch the following sets on a piece of paper and write them in interval notation. Enter the interval in the answer box. You may use "infinity" for  $\infty$  and "-infinity" for  $-\infty$ . For example, you may write (-infinity, 5] for the interval  $(-\infty, 5]$ .

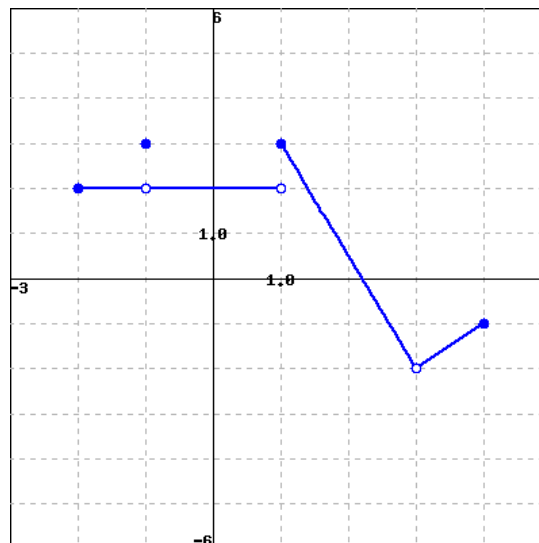
$22 \leq x \leq 25$  \_\_\_\_\_  
 $13 < x \leq 18$  \_\_\_\_\_  
 $9 < x < 13$  \_\_\_\_\_  
 $-4 \leq x < 1$  \_\_\_\_\_

Answer(s) submitted:

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(incorrect)

2. (1 pt) Let F be the function below.



Evaluate each of the following expressions.

Note: Enter 'DNE' if the limit does not exist or is not defined.

- a)  $\lim_{x \rightarrow -1^-} F(x) = \underline{\hspace{2cm}}$
- b)  $\lim_{x \rightarrow -1^+} F(x) = \underline{\hspace{2cm}}$
- c)  $\lim_{x \rightarrow -1} F(x) = \underline{\hspace{2cm}}$
- d)  $F(-1) = \underline{\hspace{2cm}}$
- e)  $\lim_{x \rightarrow 1^-} F(x) = \underline{\hspace{2cm}}$
- f)  $\lim_{x \rightarrow 1^+} F(x) = \underline{\hspace{2cm}}$
- g)  $\lim_{x \rightarrow 1} F(x) = \underline{\hspace{2cm}}$
- h)  $\lim_{x \rightarrow 3} F(x) = \underline{\hspace{2cm}}$
- i)  $F(3) = \underline{\hspace{2cm}}$

Answer(s) submitted:

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3. (1 pt) For what values of  $a$  and  $b$  is the line  $-4x + y = b$  tangent to the curve  $y = ax^3$  when  $x = -4$ ?

$a = \underline{\hspace{2cm}}$   
 $b = \underline{\hspace{2cm}}$

Answer(s) submitted:

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(incorrect)

4. (1 pt) Let  $h(x) = 7 - 3x^3$ ,  
 $h'(3) =$  \_\_\_\_\_

Use this to find the equation of the tangent line to the curve  $y = 7 - 3x^3$  at the point  $(3, -74)$  and write your answer in the form:

$y = mx + b$ , where  $m$  is the slope and  $b$  is the y-intercept.

\_\_\_\_\_

Answer(s) submitted:

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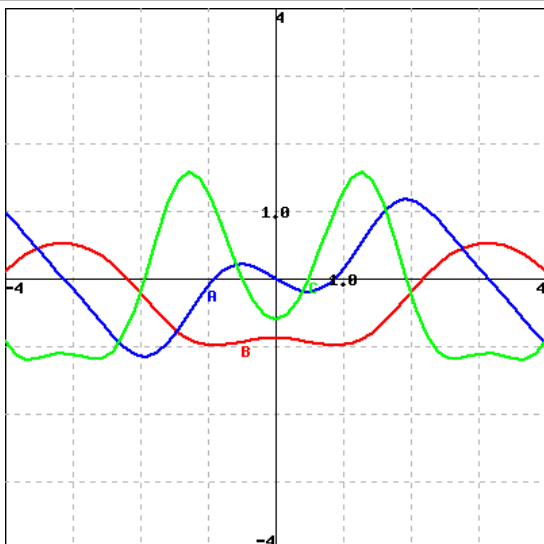
(incorrect)

5. (1 pt) Let  $f(x) = \cos(x+5)$ ,  
 $f'(1) =$  \_\_\_\_\_

Answer(s) submitted:

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(incorrect)



6. (1 pt)

Identify the graphs A (blue), B (red) and C (green) as the graphs of a function and its derivatives:

\_\_\_\_\_ is the graph of the function

\_\_\_\_\_ is the graph of the function's first derivative

\_\_\_\_\_ is the graph of the function's second derivative

Answer(s) submitted:

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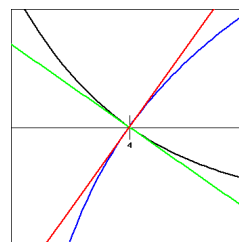
7. (1 pt) Evaluate the limit  $\lim_{x \rightarrow 0} \frac{\sin(4x)}{6x} =$  \_\_\_\_\_

Answer(s) submitted:

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(incorrect)

8. (1 pt) The functions  $f$  and  $g$  and their tangent lines at  $(4, 0)$  are shown in the figure below.



$f$  is shown in blue,  $g$  in black, and the tangent line to  $f$  is  $y = 1.8(x - 4)$ , and is graphed in green, and the tangent line to  $g$  is  $y = -0.9(x - 4)$ , and is graphed in red.

Find the limit

$\lim_{x \rightarrow 4} \frac{f(x)}{g(x)} =$  \_\_\_\_\_

Answer(s) submitted:

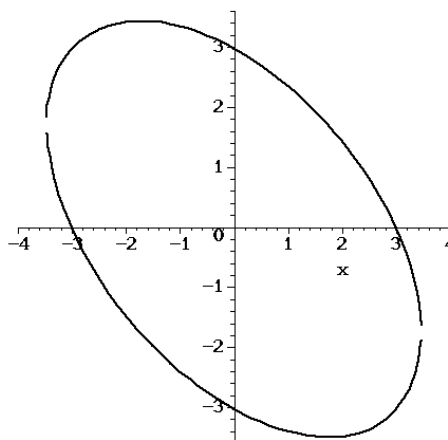
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9. (1 pt) The graph of the equation

$$x^2 + xy + y^2 = 9$$

is a slanted ellipse illustrated in this figure:



Think of  $y$  as a function of  $x$ . Differentiating implicitly and solving for  $y'$  gives:

$y' =$  \_\_\_\_\_. (Your answer will depend on  $x$  and  $y$ .)

The ellipse has two horizontal tangents. The upper one has the equation

$y =$  \_\_\_\_\_.

The right most vertical tangent has the equation

$x =$  \_\_\_\_\_.

That tangent touches the ellipse where

$y =$  \_\_\_\_\_.

**Hint:** The horizontal tangent is of course characterized by  $y' = 0$ . To find the vertical tangent use symmetry, or think of  $x$  as a function of  $y$ , differentiate implicitly, solve for  $x'$  and then set  $x' = 0$ .

Answer(s) submitted:

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(incorrect)

**10.** (1 pt) Find the slope of the tangent line to the curve (a lemniscate)

$$2(x^2 + y^2)^2 = 25(x^2 - y^2)$$

at the point  $(3, 1)$ .

$m =$  \_\_\_\_\_

Answer(s) submitted:

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(incorrect)

**11.** (1 pt) Let

$$f(x) = 7\sin^{-1}(x^3)$$

$f'(x) =$  \_\_\_\_\_

NOTE: The webwork system will accept  $\arcsin(x)$  or  $\sin^{-1}(x)$  as the inverse of  $\sin(x)$ .

Answer(s) submitted:

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(incorrect)

**12.** (1 pt) If  $f(x) = 2\cos(2\ln(x))$ , find  $f'(x)$ .

Find  $f'(5)$ .

Answer(s) submitted:

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(incorrect)

**13.** (1 pt) The function

$$f(x) = 6x^3 - 63x^2 - 144x - 2$$

is decreasing on the interval \_\_\_\_\_.

Enter your answer using the interval notation for open intervals.

It is increasing on the interval(s) \_\_\_\_\_.

The function has a local maximum at \_\_\_\_\_.

Answer(s) submitted:

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(incorrect)

**14.** (1 pt) Consider the function  $f(x) = x^2 - 4x + 3$  on the interval  $[0, 4]$ . Verify that this function satisfies the three hypotheses of Rolle's Theorem on the interval.

$f(x)$  is \_\_\_\_\_ on  $[0, 4]$ ;

$f(x)$  is \_\_\_\_\_ on  $(0, 4)$ ;

and  $f(0) = f(4) =$  \_\_\_\_\_.

Then by Rolle's theorem, there exists a  $c$  such that  $f'(c) = 0$ . Find the value  $c$ .

$c =$  \_\_\_\_\_

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**15.** (1 pt)

Evaluate the limit using L'Hospital's rule if necessary.

$$\lim_{x \rightarrow 0^+} x^{4\sin(x)}$$

Answer: \_\_\_\_\_

Answer(s) submitted:

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(incorrect)

**16.** (1 pt) Find **one** indefinite integral (anti-derivative):

$$\int \frac{6}{x \ln(8x)} dx$$

Answer(s) submitted:

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(incorrect)

**17.** (1 pt) A Norman window has the shape of a semicircle atop a rectangle so that the diameter of the semicircle is equal to the width of the rectangle. What is the area of the largest possible Norman window with a perimeter of 33 feet?

Answer(s) submitted:

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(incorrect)

**18.** (1 pt) Suppose  $f''(x) = 4x + 0$  and  $f'(-3) = -2$  and  $f(-3) = -2$ .

$f'(x) =$  \_\_\_\_\_

$f(2) =$  \_\_\_\_\_

Answer(s) submitted:

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(incorrect)

19. (1 pt) Find the derivative of

$$g(x) = \int_{5x}^{9x} \frac{u+2}{u-3} du$$

Answer(s) submitted:

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(incorrect)

20. (1 pt) Suppose  $\int_8^{12.5} f(x)dx = 4$ ,  $\int_8^{9.5} f(x)dx = 5$ ,  $\int_{11}^{12.5} f(x)dx = 10$ .

$$\int_{9.5}^{11} f(x)dx = \underline{\hspace{2cm}}$$

$$\int_{11}^{9.5} (4f(x) - 5)dx = \underline{\hspace{2cm}}$$

Answer(s) submitted:

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(incorrect)