Michael Gage

Calculus I — sample problems

1) translate inequality statements into interval notation

- 2) limits for a piecewise-linear function
- 3) unconventional phrasing for a tangent-line task
- 4) tangent to cubic
- 5) derivative of cos(x+a)
- 6) identify function and its derivatives in a graph
- 7) $\lim_{x\to 0} \frac{\sin(ax)}{bx}$ 8) figure to motivate l'Hopital's rule
- 9) implicit differentiation [problem is static]
- 10) slope of tangent to $2(x^2 + y^2)^2 = 25(x^2 y^2)$
- 11) derivative of $a \arcsin(x^n)$
- 12) derivative of $a \cos(b \ln(x))$
- 13) intervals of increase and decrease for cubic
- 14) confirm Rolle's Theorem for a quadratic
- 15) $\lim x^{a \sin(x)}$ $\int \frac{a}{x \cdot \ln(bx)} \, dx$
- 17) maximize area of Norman window with perimeter P
- 18) solve IVP: f''(x) = ax + b
- 19) differentiate $g(x) = \int_{ax}^{bx} \frac{u+c}{u-d} du$ 20) definite integral: interval additivity, linear operator
- Problems 2, 6, 8 create and display individualized graphs.

1. (1 pt) Library/Rochester/setAlgebra01RealNumbers/lhp1_31-34_mo.pg

Sketch the following sets on a piece of paper and write them in interval notation. Enter the interval in the answer box. You may use "infinity" for ∞ and "-infinity" for $-\infty$. For example, you may write (-infinity, 5] for the interval $(-\infty, 5]$.

$$16 \le x \le 19$$

$$3 < x \le 8$$
 _______ 25 < $x < 29$ ______

Answer(s) submitted:

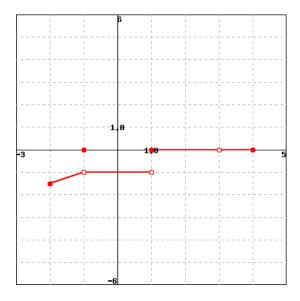
- [16,19]
- (3,81
- (25,29)
- [12,17)

(correct)

2. (1 pt) Library/Rochester/setLimitsRates1_5Graphs/ur_lr_1-

5_1_mo.pg

Let F be the function below.



Evaluate each of the following expressions.

Note: Enter 'DNE' if the limit does not exist or is not defined.

a)
$$\lim_{x \to 1^{-}} F(x) =$$

b)
$$\lim_{x \to -1^+} F(x) =$$

c)
$$\lim_{x \to -1} F(x) =$$

d)
$$F(-1) =$$

e)
$$\lim_{x \to 1^{-}} F(x) =$$

f)
$$\lim_{x \to 1^+} F(x) =$$

g)
$$\lim_{x \to \infty} F(x) = \underline{\hspace{1cm}}$$

h)
$$\lim_{x \to 2} F(x) =$$

i)
$$F(3) =$$

Answer(s) submitted:

- −1
- −1

- 0
- dne
- 0
- dne

(correct)

3. Library/Rochester/setDerivatives1_5Tangents-/ur_dr_1_5_16_mo.pg

For what values of a and b is the line -4x + y = b tangent to the curve $y = ax^3$ when x = -5?

Answer(s) submitted:

(incorrect)

4. Library/Rochester/setDerivatives1_5Tangents-/ur_dr_1_5_2a_mo.pg

Let
$$h(x) = 4 - 4x^3$$
,

$$h'(3) =$$

Use this to find the equation of the tangent line to the curve $y = 4 - 4x^3$ at the point (3, -104) and write your answer in the

y = mx + b, where m is the slope and b is the y-intercept.

Answer(s) submitted:

(incorrect)

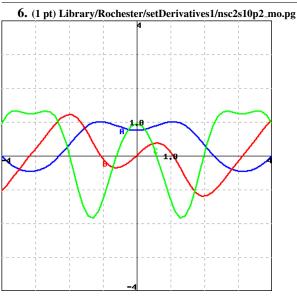
5. (1 pt) Library/Rochester/setDerivatives1/s2_1_23_mo.pg

Let
$$f(x) = \cos(x+5)$$
,

$$f'(3) = \cos(x+3),$$

Answer(s) submitted:

(incorrect)



Identify the graphs A (blue), B(red) and C (green) as the graphs of a function and its derivatives:

- ___ is the graph of the function
- ___ is the graph of the function's first derivative
- ___ is the graph of the function's second derivative Answer(s) submitted:

(score 0.333333333333333)

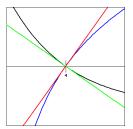
7. (1 pt) Library/Rochester/setDerivatives4Trig/s2_4_7_mo.pg

Evaluate the limit $\lim_{x\to 0} \frac{\sin(2x)}{7x} =$ Answer(s) submitted:

(incorrect)

8. (1 pt) Library/Michigan/Chap4Sec7/Q15.pg

The functions f and g and their tangent lines at (4,0) are shown in the figure below.



f is shown in blue, g in black, and the tangent line to f is y = 1(x-4), and is graphed in green, and the tangent line to g is y = -0.5(x-4), and is graphed in red.

Find the limit

$$\lim_{x \to 4} \frac{f(x)}{g(x)} = \underline{\qquad}$$

SOLUTION: (Instructor solution preview: show the student solution after due date.)

SOLUTION

Observe that both f(4) and g(4) are zero. Also, f'(4) = 1and g'(4) = -0.5, so by l'Hopital's rule,

$$\lim_{x \to 4} \frac{f(x)}{g(x)} = \frac{f'(4)}{g'(4)} = \frac{1}{-0.5} = -2.$$

Answer(s) submitted:

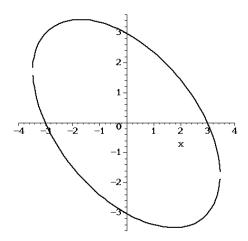
(incorrect)

(1 pt) Library/Utah/Calculus_I/set5_The_Derivative/1210s5p17-/1210s5p17.pg

The graph of the equation

$$x^2 + xy + y^2 = 9$$

is a slanted ellipse illustrated in this figure:



Think of y as a function of x. Differentiating implicitly and solving for y' gives:

y' = ______. (Your answer will depend on x and y.)

The ellipse has two horizontal tangents. The upper one has the equation

v = ____.

The right most vertical tangent has the equation

 $x = \underline{\hspace{1cm}}$

That tangent touches the ellipse where

 $y = \underline{\hspace{1cm}}$.

Hint: The horizontal tangent is of course characterized by y' = 0. To find the vertical tangent use symmetry, or think of x as a function of y, differentiate implicitly, solve for x' and then set x' = 0.

SOLUTION: (Instructor solution preview: show the student solution after due date.)

Solution: Differentiating implicitly in

$$x^2 + xy + y^2 = 9$$

gives

$$2x + y + xy' + 2yy' = 0.$$

Solving for y' gives

$$y' = -\frac{2x + y}{x + 2y}.$$

Setting

$$y' = 0$$

and solving for x gives

$$2x + y = 0 \implies x = -\frac{y}{2}.$$

Substituting in the original equation gives

$$\frac{y^2}{4} - \frac{y^2}{2} + y^2 = 9.$$

Thus

$$\frac{3y^2}{4} = 9$$

or

$$y^2 = 12$$
.

Hence the horizontal asymptotes have the equations

$$y = \pm \sqrt{12}$$
.

The plus sign gives the upper asymptote, the minus sign the lower. By symmetry the vertical asymptotes have the equations

$$x = \pm \sqrt{12}$$
.

The rightmost vertical asymptote with the equation

$$x = \sqrt{12}$$

touches the ellipse in the point where

$$y = -\frac{x}{2} = -\frac{\sqrt{12}}{2} = -\sqrt{3}.$$

Answer(s) submitted:

- .
- •
- .

(incorrect)

10. (1 pt) Library/Rochester/setDerivatives2_5Implicit/s2_6_25a_mo.pg Find the slope of the tangent line to the curve (a lemniscate)

$$2(x^2+y^2)^2 = 25(x^2-y^2)$$

at the point (3, -1).

 $m = \underline{\qquad}$

Answer(s) submitted:

• (incorrect)

 $\begin{tabular}{lll} \bf 11. & (1 & pt) & Library/maCalcDB/setDerivatives6InverseTrig-/sc3_6_25_mo.pg \end{tabular}$

Let

$$f(x) = 4\sin^{-1}(x^2)$$

NOTE: The webwork system will accept $\arcsin(x)$ or $\sin^- 1(x)$ as the inverse of $\sin(x)$.

Answer(s) submitted:

(incorrect)

12. (1 pt) Library/Rochester/setDerivatives7Log/sc3_7_4_mo.pg If $f(x) = 5\cos(6\ln(x))$, find f'(x).

Find f'(3).

Answer(s) submitted:

•

13. (1 pt) Library/Rochester/setDerivatives10MaxMin/c3s3p1_mo.pg The function

$$f(x) = 4x^3 + 6x^2 - 240x - 7$$

is decreasing on the interval __

Enter your answer using the interval notation for open intervals.

It is increasing on the interval(s) _

The function has a local maximum at ____

SOLUTION: (Instructor solution preview: show the student solution after due date.)

Solution:

To find the intervals of increase and decrease, we have to find the intervals where the derivative is positive and where it is neg-

$$f'(x) = 12x^2 + 12x - 240.$$

The derivative is 0 when $12x^2 + 12x - 240 = 0$.

$$12(x^2 + 1x - 20) = 0$$

$$12(x+5)(x-4)=0$$

We have two roots: x = -5 and x = 4.

It is easy to check that f'(x) is negative (and therefore f(x) is decreasing) on the interval (-5,4).

f'(x) is positive (and therefore f(x) is increasing) on the interval $(-\infty, -5)$ and on the interval $(4, \infty)$.

Since f'(x) changes from positive to negative at -5, f(x) has a local maximum at -5.

Answer(s) submitted:

(incorrect)

14. (1 pt) Library/ma122DB/set7/s4_2_1_mo.pg

Consider the function $f(x) = x^2 - 4x + 6$ on the interval [0,4]. Verify that this function satisfies the three hypotheses of Rolle's Theorem on the inverval.

$$f(x)$$
 is _____ on [0,4];
 $f(x)$ is _____ on (0,4);

and
$$f(0) = f(4) =$$

Then by Rolle's theorem, there exists a c such that f'(c) = 0. Find the value *c*.

Answer(s) submitted:

(incorrect)

15. (1 pt) Library/ma122DB/set8/s4_4_55_mo.pg

Evaluate the limit using L'Hospital's rule if necessary.

$$\lim_{x\to 0^+} x^{6\sin(x)}$$

Answer: _

Answer(s) submitted:

(incorrect)

16. (1 pt) Library/Rochester/setIntegrals14Substitution-/sc5_5_25_mo.pg

Find **one** indefinite integral (anti-derivative):

$$\int \frac{7}{x \ln(6x)} \, dx$$

Answer(s) submitted:

(incorrect)

17. (1 pt) Library/Rochester/setDerivatives10_5Optim/s3_8_26_mo.pg

A Norman window has the shape of a semicircle atop a rectangle so that the diameter of the semicircle is equal to the width of the rectangle. What is the area of the largest possible Norman window with a perimeter of 21 feet?

Answer(s) submitted:

(incorrect)

18. pt) Library/Rochester/setDerivatives20Antideriv-

/c3s10p1_mo.pg

Suppose
$$f''(x) = 4x + 1$$
 and $f'(0) = -3$ and $f(0) = 4$.

$$f'(x) = \underline{\qquad}$$

$$f(1) = \underline{\qquad}$$

Answer(s) submitted:

(incorrect)

19. (1 pt) Library/maCalcDB/setIntegrals4FTC/sc5_4_19_mo.pg Find the derivative of

$$g(x) = \int_{3x}^{6x} \frac{u+2}{u-6} du$$

HINT: (Instructor hint preview: show the student hint after 1 attempts:)

Hint:

$$\int_{3x}^{6x} \frac{u+2}{u-6} du = \int_{0}^{6x} \frac{u+2}{u-6} du + \int_{3x}^{0} \frac{u+2}{u-6} du$$

•

(incorrect)

20. (1 pt) Library/Rochester/setIntegrals0Theory/ur_in_0_13_mo.pg Suppose
$$\int_{-4}^{3.5} f(x) dx = 3$$
, $\int_{-4}^{-1.5} f(x) dx = 9$, $\int_{1}^{3.5} f(x) dx = 8$.

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$$\int_{-1.5}^{1} f(x)dx = _{---}$$

$$\int_{1}^{-1.5} (3f(x) - 9)dx = _{---}$$
Answer(s) submitted:

•

Michael Gage

Calculus II — sample problems

1) $\int \sin^n(ax) \cdot \cos(ax) dx$

2)
$$\int \frac{ax-b}{ax^2-Bx+c} dx \text{ with } B = 2b$$

$$3) \int_{1}^{e^n} \frac{dx}{x \cdot (1 + \ln(x))}$$

- 4) compute area between 2 curves: integrate wrt x, then wrt y
- 5) solid of revolution: rotate about y = H
- 6) work to lift a leaking bucket
- 7) $\int e^{ax} \cdot \sin(bx) dx$
- 8) $\int ax \cdot \ln(bx) dx$
- 9) $\int x^a \cdot \cos(x^b) dx$ with a = 2b 1
- 10) partial fractions for $\frac{\text{cubic}}{x^2 a^2}$
- 11) partial fractions for $\frac{\text{quadratic}}{(x-a)\cdot(\text{quadratic})}$ with x-a being a factor of the denominator quadratic
- 12) evaluate $\int_{-\infty}^{\infty} x \cdot e^{bx} dx$ if it converges (b is negative)

$$13) \int \frac{dx}{x^2 \cdot \sqrt{x^2 + a^2}} \, dx$$

- 14) match: sequence, convergence/divergence behavior
- 15) geometric series: evaluate if convergent
- 16) match: series, convergence/divergence behavior
- 17) 2-phase match: series, behavior, test technique
- 18, 19) convergence interval for a power series
- 20) coefficients c_0, \ldots, c_4 of Maclaurin series for $\frac{a}{1-r^3}$; radius of convergence
- 21) coefficients c_0, \ldots, c_4 of Taylor series centered at A for ln(x); interval of convergence
- 22) Do you believe Maclaurin series for cosine is a function?
- 23) Taylor polynomial for x^3 centered at A
- 24) match: graphs of curves, parametric equations
- 25) Cartesian equation for polar curve: $r = \frac{a}{b\cos(\theta) + c\cos(\theta)}$

1. (1 pt) Library/Union/setIntSubstitution/an6_3_07.pg

Evaluate the indefinite integral.

$$\int \sin^7(5x)\cos(5x) dx = \underline{\qquad} +C.$$
Answer(s) submitted:

(incorrect)

2. (1 pt) Library/Union/setIntSubstitution/sc5_5_33a.pg Evaluate the indefinite integral.

$$\int \frac{2x-2}{\left(2x^2-4x+1\right)^4} dx = \underline{\qquad} +C.$$
Answer(s) submitted:

(incorrect)

3. (1 pt) Library/Utah/Calculus_II/set15_Practice/osu_in_14_7.pg Evaluate the definite integral.

$$\int_{1}^{e^5} \frac{dx}{x(1+\ln x)}$$

Answer(s) submitted:

(incorrect)

4. (1 pt) Library/maCalcDB/setIntegrals19Area/osu_in_19_14.pg Consider the area between the graphs x + 5y = 7 and $x + 7 = y^2$. This area can be computed in two different ways using integrals.

First of all it can be computed as a sum of two integrals

$$\int_{a}^{b} f(x) dx + \int_{b}^{c} g(x) dx$$

with the following values:

a = _____

b =_____

c =

$$c = \underline{\hspace{1cm}}$$

$$f(x) = \underline{\hspace{1cm}}$$

$$g(x) = \underline{\hspace{1cm}}$$

Alternatively this area can be computed as a single integral

$$\int_{\alpha}^{\beta} h(y) \, dy$$

with the following values:

 $\alpha = \underline{\hspace{1cm}}$

 $\beta =$

h(y) =_____

Either way we find that the area is ______ Answer(s) submitted:

5. (1 pt) Library/ma123DB/set1/s6_2_14.pg

Find the volume of the solid obtained by rotating the region bounded by the given curves about the line y = -2.

$$y = \frac{1}{x^4}$$
, $y = 0$, $x = 3$, $x = 8$

Answer: _____

Answer(s) submitted:

(incorrect)

6. (1 pt) Library/Rochester/setIntegrals23Work/S06.04.Work.PTP09.pg

A bucket that weighs 5 pounds and a rope of negligible weight are used to draw water from a well that is 84 feet deep. The bucket is filled with 39 pounds of water and is pulled up at a rate of 3 feet per second, but water leaks out of a hole in the bucket at a rate of 0.1 pounds per second. Find the work done pulling the bucket to the top of the well. Your answer must include the correct **units**. (You may enter *lbf* or *lb*ft* for *ft-lb*.)

Work = _____

HINT: (Instructor hint preview: show the student hint after 1 attempts:)

Hint: Calculate the work done lifting the bucket (without the water) and the water (without the bucket) separately and add them to find the total work done. Find a function that describes the weight of the water in the bucket when the bucket is x feet above its starting position.

Answer(s) submitted:

(incorrect)

7. (1 pt) Library/Union/setIntByParts/bennyparts1.pg

Evaluate the indefinite integral.

$$\int e^{4x} \sin(2x) dx = \underline{\qquad} +C.$$
Answer(s) submitted:

(incorrect)

8. (1 pt) Library/Union/setIntByParts/sc5_6_04.pg

Evaluate the indefinite integral.

$$\int 5x \ln(6x) dx = \underline{\qquad} +C.$$
Answer(s) submitted:

•

(incorrect)

9. (1 pt) Library/Union/setIntByParts/sc5_6_26.pg

Evaluate the indefinite integral.

$$\int x^{13} \cos\left(x^7\right) \, dx = \underline{\qquad} +C.$$

Hint: First make a substitution and then use integration by parts to evaluate the integral.

Answer(s) submitted:

•

(incorrect)

${\bf 10.} \qquad (1\ \ pt) \ \ Library/Rochester/setIntegrals 25 Rational Functions/osu_in_25_7.pg$

Note: You can get full credit for this problem by just entering the final answer (to the last question) correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.

Consider the indefinite integral
$$\int \frac{4x^3 + 4x^2 + 3x - 5}{x^2 - 1} dx$$

Then the integrand decomposes into the form

$$ax+b+\frac{c}{r-1}+\frac{d}{r+1}$$

where

a = _____

b = _____

c = _____

d =____

Integrating term by term, we obtain that

$$\int \frac{4x^3 + 4x^2 + 3x - 5}{x^2 - 1} dx = +C$$

Answer(s) submitted:

- •
- •
- •
- •

(incorrect)

$\begin{tabular}{ll} \bf 11.~(1~pt)~Library/Union/setIntPartialFractions/ur_in_25_16.pg\\ Evaluate~the~indefinite~integral. \end{tabular}$

$$\int \frac{7x^2 - 39x + 4}{(x - 4)(x^2 - 3x - 4)} dx =$$
+C.

Answer(s) submitted:

incorrec

12. (1 pt) Library/maCalcDB/setIntegrals18Improper/ur_in_18_3.pg Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If not, give the answer -1.

$$\int_{4}^{\infty} xe^{-3x} dx$$

Answer(s) submitted:

(incorrect)

13. (1 pt) Library/Union/setIntInverseTrigSub/an8_4_01.pg Evaluate the indefinite integral.

$$\int \frac{dx}{x^2 \sqrt{x^2 + 81}} = \frac{1}{Answer(s) \ submitted:} + C.$$

(incorrect)

14. (1 pt) Library/Utah/Calculus_II/set7_Infinite_Series/set7_pr7.pg Match each sequence below to statement that BEST fits it.

STATEMENTS

- Z. The sequence converges to zero;
- I. The sequence diverges to infinity;
- F. The sequence has a finite non-zero limit;
- D. The sequence diverges.

SEQUENCES

- $\underline{\hspace{1cm}}$ 1. $n\sin\left(\frac{1}{n}\right)$

- $\underline{\hspace{1cm}}$ 5. $\frac{n!}{n^{1000}}$
- $-6. \frac{(\ln(n))}{(\ln(n))}$
- $\underline{}$ 7. $\ln(\ln(\ln(n)))$
- $\underline{\hspace{1cm}}$ 8. arctan(n+1)

Answer(s) submitted:

(incorrect)

15. (1 pt) Library/maCalcDB/setSeries4Geometric/ur_sr_4_1.pg

The following series are geometric series.

Determine whether each series converges or not.

For the series which converge, enter the sum of the series. For the series which diverges enter "DIV" (without quotes).

- (d) $\sum_{n=5}^{\infty} \frac{4^n}{5^n} =$ _______,

Answer(s) submitted:

(incorrect)

16. (1 pt) Library/Rochester/setSeries7AbsolutelyConvergent-/eva8_3_2.pg

Match each of the following with the correct statement.

- A. The series is absolutely convergent.
- C. The series converges, but is not absolutely convergent.
- D. The series diverges.

___1.
$$\sum_{n=1}^{\infty} \frac{(-6)^n}{n^3}$$

Answer(s) submitted:

17. (1 pt) Library/maCalcDB/setSeries6CompTests/ur_sr_6_17.pg

For each of the series below select the letter from a to c that best applies and the letter from d to k that best applies. A possible answer is af, for example.

- A. The series is absolutely convergent.
- B. The series converges, but not absolutely.
- C. The series diverges.
- D. The alternating series test shows the series converges.
- E. The series is a *p*-series.
- F. The series is a geometric series.
- G. We can decide whether this series converges by comparison with a *p* series.
- H. We can decide whether this series converges by comparison with a geometric series.
- I. Partial sums of the series telescope.
- J. The terms of the series do not have limit zero.
- K. None of the above reasons applies to the convergence or divergence of the series.

Answer(s) submitted:

•

(incorrect)

18. (1 pt) Library/Rochester/setSeries8Power/eva8_5a_6.pg

Find all the values of x such that the given series would converge.

$$\sum_{n=1}^{\infty} \frac{x^n}{(2)^n(\sqrt{n}+2)}$$

The series is convergent

from x =___, left end included (enter Y or N): ___ to x =___, right end included (enter Y or N): ___ Answer(s) submitted:

- •
- •
- •

(incorrect)

19. (1 pt) Library/ma123DB/set12/s11_8_13.pg

Find all the values of x such that the given series would converge.

$$\sum_{n=1}^{\infty} \frac{x^n}{\ln(n+4)}$$

Answer: _____

Note: Give your answer in <u>interval notation</u>

Answer(s) submitted:

(incorrect)

20. (1 pt) Library/ma123DB/set12/s11_9_5.pg

Suppose that

$$\frac{2}{(1-x^3)} = \sum_{n=0}^{\infty} c_n x^n$$

Find the following coefficients of the power series.

$$c_0 =$$

$$c_1 =$$

$$c_2 =$$

$$c_3 =$$

$$c_4 =$$

Find the radius of convergence *R* of the power series.

R =

Answer(s) submitted:

- •
- •
- •
- •

(incorrect)

21. (1 pt) Library/ma123DB/set13/s11_10_12.pg

The Taylor series of function $f(x) = \ln(x)$ at a = 5 is given by:

$$f(x) = \sum_{n=0}^{\infty} c_n (x-5)^n$$

Find the following coefficients:

$$c_0 =$$

 $c_3 =$ ___

 $c_4 =$ _____

Determine the interval of convergence:

Note: Give your answer in interval notation

Answer(s) submitted:

(incorrect)

22. (1 pt) Library/Union/setSeriesTaylor/benny_taylor2.pg

Let $f(x) = \frac{\cos(4x^2) - 1}{x^2}$. Evaluate the 6th derivative of f at x = 0. $f^{(6)}(0) = 0$

Hint: Build a Maclaurin series for f(x) from the series for $\cos(x)$.

Answer(s) submitted:

(incorrect)

23. (1 pt) Library/Rochester/setSeries9Taylor/S11.10.TaylorPolynomials.PTP01.pg The fourth degree Taylor polynomial for $f(x) = x^3$ centered at a = 4 is $T_4(x) = c_0 + c_1(x-4) + c_2(x-4)^2 + c_3(x-4)^3 + c_4(x-4)^4 + c_4(x$ $4)^{4}$.

Find the coefficients of this Taylor polynomial.

 $c_1 =$ _____

 $c_2 =$ _____

 $c_3 =$ _____

 $c_4 =$ _____

? The function $f(x) = x^3$ equals its fourth degree Taylor polynomial $T_4(x)$ centered at a = 4. Hint: Graph both of them. If it looks like they are equal, then do the algebra.

Answer(s) submitted:

(incorrect)

(1 pt) Library/maCalcDB/setParametric1Curves/ur_pa_1_13-/ur_pa_1_13.pg

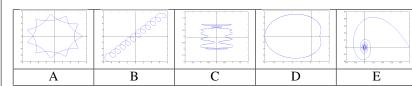
Assume t is defined for all time. Enter the letter of the graph below which corresponds to the curve traced by the parametric equations. Think about the range of x and y, and whether there is periodicity and or symmetry.

$$1. x = \sin(t)(3 - 2\sin(t)); y = \cos(t)(3 - 2\sin(t))$$

$$2. x = \sin(t + \sin(7t)); y = \cos(t)$$

$$3. x = t + \sin(5t); y = t + \cos(5t)$$

$$4. x = \frac{1}{1+t^2}\cos(t^2); y = \frac{1}{1+t^2}\sin(t^2)$$



Answer(s) submitted:

(incorrect)

25. (1 pt) Library/Utah/Calculus_II/set12_Conics_and_Polar_Coordinates-/set12_pr6.pg

A curve with polar equation

$$r = \frac{3}{4\sin\theta + 9\cos\theta}$$

represents a line. This line has a Cartesian equation of the form

y = mx + b, where m and b are constants. Give the formula for y in terms of x. For example, if the line had equation y = 2x + 3 then the answer would be 2 * x + 3.

Answer(s) submitted:

1. (1 pt) setOrientation/prob01.pg

Understanding WeBWorK Problem Pages

The WeBWorK screen is divided into several areas, each used for a different purpose. You will need to understand these in order to use WeBWorK effectively.

At the upper left are the navigation buttons that allow you to move from problem to problem. The "Next" and "Previous" buttons, naturally, send you to the next and previous problems. The "Prob. List" button takes you back to the opening page for the homework set (the one that lists all the problems and gives the instructions for the homework set).

The area below the navigation buttons is where WeBWorK tells you about your score for the current problem. When you have submitted your answers, this is where you will be given information about what answers you got right and wrong. (It will be shown on a grey backgound to help make it stand out; you will see this later.) This area also shows you how many points a problem is worth.

The main part of the page is the text of the problem you are trying to answer, including blank boxes for you to enter your answers. (There aren't any such boxes on this page, because you are not being asked any questions here, but usually there will be one or more answer blanks on a page.)

Below the problem text is a message area where you may be informed about how partial credit is handled in multi-part problems. Other information also may appear there, such as a message indicating that the due date is passed, or that answers are available.

In the red panel at the left, instead of the list of homework sets, you now have a list of the problems within this assignment. You can go to any problem just by clicking on it. The checkboxes at the bottom of the panel are discussed in the next problem.

In the center of the top red panel is an indication of how much time you have left to work on the problems for this set. Note that this is computed at the moment you downloaded the problem page; if you use the "Back" and "Forward" buttons on your browser, this does not reload the page, so the times will not be updated. Reload the page using your browser's Reload or Refresh buttons to get the current time.

The buttons at the bottom of the screen, including the "Submit Answers" button, are discussed in the next problem. At this point, you can get credit for Problem 1 by pressing the "Submit Answers" button at the bottom of the page (even though there was no answer to submit), and then pressing the "Next" button at the top of the screen to go on to the next problem.

2. (1 pt) setOrientation/prob02.pg

Controlling WeBWorK

The buttons at the bottom of the screen are what cause WeB-WorK to process your answers. Nothing that you type will have any effect until you press one of these buttons.

The "Submit Answers" button causes WeBWorK to check your answers and report your score for the problem. You can continue to work on a problem until you get it right, so don't be afraid to submit your answer even if you have only finished parts of the problem or are not sure of the correctness of your answer. If the due date is passed and the answers are available, you can click the "Show Correct Answers" button before pressing "Submit". If you do, the correct answer(s) will be displayed in the answer area at the top of the screen along with the answers you have provided.

If you have typed in a complicated answer, or are being told your answer is incorrect when you think it's right, you may want to use the "Preview Answer" button. This will ask WeBWorK to display at the top of the page its interpretation of what you have entered. This can be used to help spot errors in your typing, and verify that WeBWorK understands your answer the way you intend it to. (This is discussed further in a later problem.)

At the bottom of the red panel at the left is a box that allows you to change how the problem is displayed. The equations within the problem can be represented in three different ways:

- "images" mode is the default choice (what WeBWorK uses unless you tell it otherwise). It produces the accurate mathematical notation, with the disadvantages of being slightly slower, and not printing well if you want to print out a single problem.
- "formatted-text" mode produces results quickly, but more complicated mathematics may be difficult to read in this mode. For example, some fractions and integrals are not easy to read in formatted-text mode.
- "jsMath" mode uses a JavaScript program and Cascading Style Sheets (CSS) to display the mathematics. This produces good results, and is particularly nice if you need to change the size of the fonts in your browser or wish to print out a single problem. For best results, however, you should download and install some fonts that jsMath uses; there is a link to the fonts on the jsMath control panel, which you can activate using the small jsMath button that will appear at the lower right of the screen when you activate this mode. The first problem you view using jsMath may take a little longer to

1

appear, as the JavaScript program is being loaded into your browser, but things should run faster after that.

Choose whichever mode is most comfortable for you. You can always select a different mode if a particular problem needs it. Here is a sample of some simple mathematics, $x^2 + 3$, and a more complicated expression, $\frac{x(1-x)}{2x+1}$. Try changing the display mode by clicking the "formatted-text" or "jsMath" radio button and then pressing the "Apply Options" button to cause the page to be reformatted. Then change the display mode to back to "images" mode for the rest of the homework set.

The "Show saved answers" checkbox tells whether you want WeBWorK to fill in the answer blanks with your previous answers or not. (If you like, you can test this out on the next problem, since there are no answer blanks in this one.)

You are now ready to learn how to enter answers into WeB-WorK. Press the "Submit Answers" button to get credit for this problem, and then press the "Next" button at the top of the page to go on to the next one.

3. (1 pt) setOrientation/prob03.pg

Typing in Your Answers

Here are the standard symbols that WeBWorK, along with most other computer software, uses for arithmetic operations:

Symbol	Meaning	Example
+	Addition	3+4 = 7
-	Subtraction	3-4 = -1
*	Multiplication	3 * 4 = 12
/	Division	3/4 = .75
^ or **	Exponentiation	$3^4 = 81 \text{ or } 3**4 = 81$

Sometimes WeBWorK will insist that you calculate the value of an expression as a single number before you enter it. For example, calculate the value of 4(-5-1)-(4-4) and enter it in the following blank. (Here you have to enter a single integer; the question is testing whether you can do the operations correctly.)

$$4(-5-1)-(4-4) =$$

Most often you will not have to simplify your answer, but can let WeBWorK do this for you. The following blanks are all expecting the value 16. Try entering it several different ways, such as 7+9, 18-2, 8*2, 32/2, and 4^2. Note: pressing the "Tab" key on your keyboard will move you from one answer box to the next.

WeBWorK also understands that quantities written next to each other are supposed to be multiplied. For example, you can enter (9) (7) instead of 63. Most often this is used when one quantity is a number and the other a variable or function. For instance, 2x means 2*x, while $3\sin(5x)$ means $3*\sin(5*x)$. The following blank is expecting the value 100; try entering it as 4(30-5).

When you are ready, don't forget to press the "Submit Answers" button to ask WeBWorK to check your work. Once you get the answers correct, press "Next" to go on.

Answer(s) submitted:

- •
- •
- •
- •

(incorrect)

4. (1 pt) setOrientation/prob04.pg

Rules of Precedence

The rules of precedence determine the order in which the mathematical operations are performed by WeBWorK. It is essential for you to understand these so that you know how WeBWorK interprets what you type in. If there are no parentheses and no functions (such as sin or log), then WeBWorK computes the value of your answer by performing exponentiation first, followed by multiplication and division (from left to right), and finally addition and subtraction (from left to right).

If there are expressions within parentheses, those expressions are simplified first. We'll talk about functions (and give a more complete list of rules) in a later problem.

Examples:

- 4*3/6 = 12/6 = 2 (multiplications and divisions are done from left to right), and 2*7 = 14, so 4*3/6-2*7+10 = 2 14 + 10 = -2.
- 12/3/2 = 4/2 = 2 (multiplications and divisions are done from left to right).
- 12/(3/2) = 12/1.5 = 8 (expressions inside parentheses are calculated before anything else).
- 2*4^2 = 2*16 = 32 (exponentiation is done before multiplication), so 2*4^2 3*4 = 2*16 3*4 = 32 12 = 2

To practice these rules, completely simplify the following expressions. Because the point of this problem is for you to do the numerical calculations correctly, WeBWorK will only accept sufficiently accurate decimal numbers as the answers to these problems. It will not simplify any expressions, including fractions.

$$\begin{array}{rcl}
2+3*5 & = & & \\
2/3*5 & = & & \\
3*3-2/5*5+16 & = & & \\
2^3+1 & = & & \\
2^3+1) & = & & \\
\end{array}$$

Answer(s) submitted:

- •
- •
- •

5. (1 pt) setOrientation/prob05/prob05.pg

Common Errors to Avoid

Many of the answers you enter into WeBWorK will be expressions that involve variables. Here are some important things to know.

• It matters what letter you use. For example, if you are asked for a function using the variable x, then it won't work to enter the function with the variable t. Also, WeBWorK considers upper- and lower-case letters to be different, so don't use the capital letter X in place of the lower-case letter x. The following blank is expecting the function x^3 , which you would enter as x^3 or x**3. Instead, try entering t^3 and submitting your answer.

You should get an error message informing you that t is not defined in this context. This tells you that WeB-WorK did not receive the correct variable and doesn't know how to check your answer. Now enter x^3 and resubmit to get credit for this part of the problem.

• WeBWorK requires that you be precise in how you think about and present your answer. We have just seen that you need to be careful about the variables that you use. You must be equally careful about how the rules of precedence apply to your answers. Often, this involves using parentheses appropriately.

For example, you might write 1/x + 1 on your paper when you meant $\frac{1}{x+1}$, but that is actually incorrect. The expression 1/x + 1 means $\frac{1}{x} + 1$, according to the rules of precedence. WeBWorK will force you to be exact in what you are thinking and in what you are writing, because it must interpret your answers according to the standard rules. If you want to enter something that means $\frac{1}{x+1}$, you must write 1/(x+1). This also is true in written work, so making a habit of being precise about this will improve your written mathematics as well as your ability to enter answers quickly and correctly in WeBWorK.

Now enter the following functions:

$$\frac{1}{2t+6} = \underline{\qquad \qquad }$$

$$\frac{1}{2(x-5)} = \underline{\qquad \qquad }$$

$$(2x-3)^4 = \underline{\qquad \qquad }$$

Answer(s) submitted:

(incorrect)

6. (1 pt) setOrientation/prob06.pg

Using Parentheses Effectively

One of the hardest parts about using parentheses is making sure that they match up correctly. Here are a couple of hints to help you with this:

- Several types of parentheses are allowed: (), [], and {}. When you need to nest parentheses inside other parentheses, try using a different type for each so that you can see more easily which ones match up.
- When you type a left parenthesis, type the corresponding right parenthesis at the same time, then position your cursor between them and type the expression that goes inside. This can save you a lot of time hunting for mismatched parentheses.
- When you have a complicated answer, type a template for the structure of your result first. For example, suppose that you are planning to enter the fraction

$$\frac{2x^2 - 5}{(x+1)(3x^{3x} - 22)}.$$

A good way to start would be to type in ()/[()*()]. This shows a template of one number divided by the product of two other numbers. (Note that ()/()*() would not be a good way to start; do you see why?) Now when you fill in the expressions, you will be sure your parentheses balance correctly.

Although WeBWorK understands that numbers written next to each other are meant to be multiplied (so you do not have to use * to indicate multiplication if you don't want to), it is often useful for you to include the * anyway, as it helps you keep track of the structure of your answer.

- To see how WeBWorK is interpreting what you type, enter your answer and then click the "Preview Answers" button, which is next to the "Submit Answers" button below. WeBWorK will show you what it thinks you entered (the preview appears in your answer area at the top of the page). Previewing your answer does not submit it for credit; that only happens when you press the "Submit Answers" button.
- When division or exponentiation are involved, it is a good idea to use parentheses even in simple situations, rather than relying on the order of operations. For example, 1/2x and (1/2)x both mean the same thing (first

3

divide 1 by 2, then multiply the result by x), but the second makes it easier to see what is going on. Likewise, use parentheses to clarify expressions involving exponentiation. Type $(e^{x})^{2}$ if you mean $(e^{x})^{2}$, and type $e^{(x^{2})}$ if you mean $e^{(x^{2})}$.

Now enter the following functions:

$$\frac{x^{2x-1}}{(x^2-x)(3x+5)} = \underline{\hspace{1cm}}$$

Start with the template $[x^{()}]/[()^{*}()]$.

$$\frac{(y+3)(y^3+y+1)}{(2y^2-2)(5y+4)} = \underline{\hspace{1cm}}$$

Start by putting in an appropriate template. This means that you should begin by looking at the function and thinking about how many pieces are used to construct it and how those pieces are related. Once you have entered your answer, try using the "Preview" button to see how WeBWorK is interpreting your answer.

$$\left(\frac{x+1}{x-2}\right)^4 = \underline{\hspace{1cm}}$$

Start by putting in an appropriate template. *Answer(s) submitted:*

• • (incorrect)

7. (1 pt) setOrientation/prob07.pg

Constants and Functions in WeBWorK

WeBWorK knows the value of π , which you can enter as pi, and the value of e (the base of the natural logarithm, $e \approx 2.71828$), which you can enter simply as the letter e.

WeBWorK also understands many standard functions. Here is a partial list. Notice that all the function names start with a lower-case letter. Capitalizing the function will lead to an error message.

• WeBWorK knows about sin(x), cos(x), tan(x), arcsin(x), arccos(x), arctan(x) and the other trigonometric functions and their inverses. WeBWorK *always* uses radian mode for these functions.

WeBWorK will evaluate trigonometric functions for you in many situations. For example, the following blank is expecting the value -1. Remember that $\cos(\pi) = -1$, so enter $\cos(\pi)$ and submit it.

$$_{---} = -1$$

• The square root \sqrt{x} is represented by the function sqrt(x) or by $x^{(1/2)}$.

- The function $\log(x)$ means the *natural* logarithm of x (the logarithm with base e), not the common logarithm (the logarithm with base 10, sometimes written \log_{10}). You can also write $\ln(x)$ for the natural logarithm of x, so $\log(x)$ and $\ln(x)$ mean the same thing. Use $\log \log(x)$ for the base 10 logarithm of x.
- The exponential function with base e can be entered as e^x or exp(x). The second notation is convenient if you have a long, complicated exponent.
- The absolute value function, |x|, should be entered as |x| or abs (x).
- The inverse sine function, $\sin^-1(x)$, is written $\arcsin(x)$ or $\sin(x)$ or $\sin^-(-1)$ (x) in WeBWorK. Note that this is *not* the same as $(\sin(x))^-(-1)$, which means $\frac{1}{\sin(x)}$. The other inverse functions are handled similarly.

Now enter the following functions:

$$\frac{1}{\tan(x)} = \frac{1}{\sin^{-1}(t+1)} = \frac{\sin(x) - \cos(x)}{\sqrt{2x - 7}} = \frac{1}{\cos(x) - \cos(x)}$$

Answer(s) submitted:

(incorrect)

8. (1 pt) setOrientation/prob08.pg

Rules of Precedence (Again)

At this point, we can give the complete rules of precedence for how WeBWorK computes the value of a mathematical formula. The operations are handled in the following order:

- (1) Evaluate expressions within parentheses.
- (2) Evaluate functions such as sin(x), cos(x), log(x), sqrt(x).
- (3) Perform exponentiation (from right to left).
- (4) Perform multiplication and division, (from left to right).
- (5) Perform addition and subtraction, (from left to right).

This can get a little subtle, so be careful. The following are some typical traps for WeBWorK users.

• WeBWorK interprets $\sin 2x$ to mean $(\sin 2) *x$

Explanation: Rule 2 tells you that WeBWorK does evaluation of functions (like sin) before multiplication. Thus WeBWorK first computes sin 2, and then multiplies the result by x.

4

Moral: You must type $\sin(2x)$ for the sine of 2x, even though we often write it as $\sin 2x$. Get in the habit of using parentheses for all your trigonometric functions.

Now enter the following function:

The cosine of 5x is entered as _____.

• WeBWorK interprets $\cos t^3$ to mean $(\cos t)^3$

Explanation: Rule 2 tells you that WeBWorK does evaluation of functions (like cos) before exponentiation. Thus WeBWorK first computes cost and then raises the result to the power 3.

Moral: You must type in $cos(t^3)$ if you mean the cosine of t^3 , even though we sometimes write it as $cost^3$.

Now enter the following function:

The tangent of y^4 is entered as _____.

In mathematics, we often write sin²x to mean (sinx)².
 WeBWorK will let you write sin²2 (x) for this, though it is probably better to type (sin(x))²² instead, as this makes your intention clearer. Note that a power of −1, as in sin² (-1) (x), is a special case; it indicates the *inverse* function arcsin(x) rather than a power.

Now enter the following function:

$$\sin^2 x + \cos^3 x = \underline{\hspace{1cm}}$$

• e^3x means $(e^3)x$ and not $e^{(3x)}$

Explanation: Rule 3 says that WeBWorK does exponentiation before multiplication. Thus WeBWorK first computes e^3 , with the result e^3 , and then multiplies the result by x.

Moral: Always put parentheses around an exponent. Type $e^{(3x)}$ if you want e^{3x} .

Now enter the following function:

$$2^{4x^3} =$$

Answer(s) submitted:

- •
- •

(incorrect)

9. (1 pt) setOrientation/prob09.pg

Non-Numeric Answers

Sometimes you will be asked to enter answers that are not numbers. For example, if you are asked to determine a limit, the answer may be that the limit does not exist, so you might have to type DNE to indicate this (the problem should tell you what word to use). Note that upper- and lower-case letters are not the same to WeBWorK, so you will need to enter the answer exactly as indicated in the problem. (Well written problems will allow the answer to be entered either way.)

Please enter "DNE" here: _____

Other problems may require you to enter ∞ , which you do using the word "INFINITY" (in upper- or lower-case) or "INF" for short. The problem should remind you of how to do this. Note that most operations are not defined on infinity, so you can't add or multiply something by infinity. You can, however, indicate $-\infty$ by "-INFINITY", or "-INF".

Try entering $-\infty$ here: _____.

One common place where you use ∞ is as an endpoint of an interval. WeBWorK allows you to enter intervals using standard interval notation, including infinite endpoints. For example, [-2,5) represents an interval that is closed on the left and open on the right, while [2,inf) is an interval that extends infinitely to the right.

Write the interval of points that are less than 2:

Several intervals can be combined into one region using the "set union" operation, \cup , which is represented as "U" in WeB-WorK. For example, [-2,0] U (8, inf) represents the points from -2 to 0 together with everything bigger than 8.

Write the set of points from -7 to 0 but excluding -2 and 0 as a union of intervals: _____.

Answer(s) submitted:

- •
- •
- •

(incorrect)

10. (1 pt) setOrientation/prob10.pg

Points and Vectors

Some problems will ask you to enter an answer that is a point rather than a number. You enter points in WeBWorK just as you would expect: by separating the coordinates by commas and enclosing them all in parentheses. So (2, -3) represents the point in the plane that has an *x*-coordinate of 2 and *y*-coordinate of -3.

What point is halfway between (3,2) and (7,6)?

Other problems require you to provide a vector as your answer. WeBWorK allows you to enter vectors either as a list of coordinates enclosed in angle braces, \langle and \rangle , or as a sum of multiples of the coordinate unit vectors, i, j and k, which you enter as i, j and k. For example, $\langle 1, 3, -2 \rangle$ represents the same vector as i+3j-2k.

What vector points from the origin to the point (0,-4,0)?

Just as you can enter a number by giving an equation that reduces to it, WeBWorK allows you to enter points and vectors by giving equations for the individual coordinates, or by using a vector-valued equation that reduces to your answer. For example,

$$<1-(-3)$$
, $2-sqrt(4)$, $6/2>$ and $[1-(-3)]i + [2-sqrt(4)]j + (6/2)k$

both represent the vector $\langle 4,0,3 \rangle$, while $\langle 1,0,-1 \rangle + \langle 2,-2,3 \rangle$ could be used to answer a question that asks for the vector $\langle 3,-2,2 \rangle$.

Write
$$\langle -3,2,5 \rangle$$
 as a sum of two vectors:

Answer(s) submitted:

•

(incorrect)

11. (1 pt) setOrientation/prob11.pg

Multiple Answers in One Blank

You may sometimes be asked to provide more than one answer in a single answer blank. For example, you may need to enter all the values where a function is not defined. In this case, you should separate your answers by commas. Such an answer is called a *list* in WeBWorK. Note that you need not enter multiple answers for a list; a single number is a legal answer (there might only be one point where the function is undefined, for instance).

The function
$$f(x) = \frac{1}{x^2 - 25}$$
 is not defined at these *x* values:

When you are asked for a list of numbers, another possible answer is that there are *no* numbers that satisfy the requirements. In that case, you should enter "NONE" as your answer.

The function
$$f(x) = \frac{1}{x^2 + 25}$$
 is not defined at these x values: ______.

Answer(s) submitted:

(incorrect)

12. (1 pt) setOrientation/prob12.pg

True/False Questions in WeBWorK

Enter a T or an F in each answer space below to indicate whether the corresponding statement is true or false.

$$1. -6 \le -6$$

 $2. \pi \ge 3.2$

$$_{--}3. -4 < -7$$

$$-4. -3 < -3$$

In most multipart problems, if one or more of your answers is wrong, then WeBWorK tells you which ones they are. For True/False or multiple-choice questions, however, WeBWorK

usually only tells you whether *all* the answers are correct. It won't tell you which ones are right or wrong.

Answer(s) submitted:

- •
- •

(incorrect)

13. (1 pt) setOrientation/prob13.pg

Matching Lists in WeBWorK

Match the statements defined below with the letters labeling their equivalent expressions.

- 1. The distance from x to -2 is at most 1
- 2. The distance from x to -2 is more than 1
- $\underline{\hspace{1cm}}$ 3. x is greater than or equal to -2
- ____ 4. *x* is any real number
- $\underline{\hspace{1cm}}$ 5. x is less than -2

A.
$$|x+2| > 1$$

B.
$$x \ge -2$$

C.
$$-\infty < x < \infty$$

D.
$$x < -2$$

E.
$$|x+2| \le 1$$

Usually with matching problems like this, WeBWorK only tells you whether *all* your answers are correct or not. If they are not all correct, WeBWorK will not tell you which ones are right and which are wrong.

Answer(s) submitted:

- •
- -
- •
- •

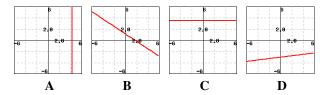
(incorrect)

14. (1 pt) setOrientation/prob14/prob14.pg

Matching Graphs in WeBWorK

Enter the letter of the graph that corresponds to each statement:

- ____1. The line is the graph of an increasing function
- ____ 2. The line is the graph of a constant function
- ____ 3. The line is not the graph of a function
- ____4. The line is the graph of a decreasing function



As with the previous matching problems, you will not be told which of your answers are correct when you submit your answers to this problem. WeBWorK will only tell you if *all* your answers are correct or not.

Some WeBWorK problems display a link to additional information or a <u>hint</u>. Follow this link for a hint about graphs in WeBWorK.

Answer(s) submitted:

- •
- •
- •

(incorrect)

15. (1 pt) setOrientation/prob15.pg

When You're Stuck...

The goal of the WeBWorK software is to help you learn mathematics by giving you immediate feedback on the correctness of your answer to a problem. It is not designed to be a tutorial or to replace humans in explaining the material to you. As with any learning tool, it is up to you to make efficient and effective use of the software.

Here are some things you can try when you are stuck on a problem.

- Reread the problem carefully to see if there are any instructions that you did not notice.
- Check carefully for directions on the Problem List page. You can get to this page by pressing the "Prob. List" button at the top of any problem page.

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- Look in the textbook for similar problems or relevant methods.
- Talk to your instructor during office hours.
- Ask a fellow student for help.
- Use the Calculus Help Center. (Be sure to take a printout of the problem with you. The tutors will need the *exact* wording of the problem.)
- Use the "Email instructor" button at the bottom of the problem page to send e-mail to your instructor. Include in your message the details of what you have tried so far. If you are having a software problem, include details about the error messages you are getting.

When you are truly stuck on a WeBWorK problem, you should turn to other sources (humans or books) for help, because it is not in your best interest to guess repeatedly instead of thinking about what you might be doing wrong.

To get credit for this problem, you must click the "Submit Answers" button. Then you can use the "Prob. List" button at the top of the page to return to the problem list page. You will see that the problems you have done have been labeled as correct or incorrect, and you can go back and do problems you skipped or couldn't get right the first time. Once you have done a problem correctly, it is *always* listed as correct even if you go back and do it incorrectly later. This means you can use WeBWorK to review course material without any danger of changing your grade.