

This College Algebra sampler has items in several categories:

- 1, 2, 3) write an expression to fit a prose description
- 4) model with simple equation, solve and interpret
- 5) simplify product of powers (rules for exponents)
- 6) collection of True/False items about algebraic identities
- 7, 8) distinguish "implies" from "is equivalent to"
- 9, 10) example of instructor responding to student comments
- 11) match: interval with inequality
- 12) arithmetic with fractions
- 13) expand $(a - bx)^2$
- 14) convert Fahrenheit temperatures to Celsius
- 15) coordinate plane and distance formula
- 16, 17) lines and slopes
- 18) quadratic polynomial function: evaluate, substitute
- 19) interpret a graph
- 20) composition of functions
- 21) linear model, domain "in context"
- 22, 23, 24, 25) shift (translate) a graph

Problems 17, 19, 22-25 create and display a graph.

1. (1 pt) A caterer for a party buys 65 cans of soda and 25 bags of chips. Write an expression for the total cost if soda costs s dollars per can and chips cost c dollars per bag.

\$ _____

Answer(s) submitted:

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(incorrect)

2. (1 pt) (a) Write an expression for the total cost of buying 8 apples at \$ a each and 3 pears at \$ p each. Your expression should be in terms of a and p .

\$ _____

- (b) Find the total cost if apples cost \$0.45 each and pears cost \$0.90 each.

\$ _____

Answer(s) submitted:

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(incorrect)

3. (1 pt) (a) Suppose you pick two numbers 5 and 8. Find their sum and product, but do *not* enter it below. Then, find the average of their sum and product and enter it below.

- (b) Using the variables x and y to stand for the two numbers, write an algebraic expression that represents this calculation.

Answer(s) submitted:

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(incorrect)

4. (1 pt)

You buy a pot and its lid for a total of \$ 11. The sales person tells you that the pot by itself costs \$ 10 more than the lid. The price of the pot is \$ _____ and the price of the lid is \$ _____

Answer(s) submitted:

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(incorrect)

5. (1 pt) The expression $(3a^2b^4c^2)^2(2a^5b^3c^2)^3$ equals $na^rb^sc^t$

where n , the leading coefficient, is: _____

and r , the exponent of a , is: _____

and s , the exponent of b , is: _____

and finally t , the exponent of c , is: _____

Answer(s) submitted:

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(incorrect)

6. (1 pt)

For each statement below enter a T (true) the statement is true and an F (false) otherwise. In this problem you need to get everything correct before receiving credit. Whenever there is a division below, **we assume that the divisor is non-zero**.

___ For all real numbers a , b , and x

$$a - b(x - 1) = a - bx - b.$$

___ For all real numbers a , b , and x

$$a - b(x - 1) = a - bx + b.$$

___ For all real numbers a , b , and x

$$\frac{3 + bx}{3} = 1 + bx.$$

___ For all real numbers a , b , and x

$$(x - r)^2 = x^2 - r^2.$$

___ For all real numbers a , b , and x

$$(x - r)^2 = x^2 - 2rx + r^2.$$

___ For all real numbers a , b , and x

$$(x - r)(x + r) = x^2 - r^2.$$

Answer(s) submitted:

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-
-
-
-
-

(incorrect)

7. (1 pt) *Solving an equation* means figuring out which values of the variable make the equation true. The basic approach to equation solving consists of applying the same operation on both sides of the equation until we come up with another equation that has the variable by itself on one side and an expression not containing that variable on the other.

For example, in the (very simple) equation

$$x + 3 = 5$$

we subtract 3 on both sides and obtain the equation

$$x = 2.$$

The latter equation tells us the solution, and we verify that $x = 2$ does indeed solve the original equation by substituting 2 for x in $x + 3 = 5$. Since $2 + 3$ does equal 5 we have indeed found the solution.

The preceding two paragraphs are deceptively simple, but they describe one of the key ideas of algebra.

The following problems explore a subtlety of the concept of "applying the same operation on both sides of the equation". In the above example, the two equations involved are equivalent, i.e., one implies the other. If $x = 2$ then $x + 3 = 5$. Conversely, if $x + 3 = 5$ then x must be 2, there is no other solution. Sometimes, however, doing the same thing on both sides of an equation creates a new equation that is not equivalent to the old one.

For example, if $x = 3$, then squaring on both sides gives $x^2 = 9$. It is true that $x = 3$ implies that $x^2 = 9$. But the other direction does not hold, if $x^2 = 9$ then x may be -3 since the square of -3 also equals 9. The process of squaring introduces a *extraneous* (or sometimes called *spurious*) solution. The existence of such solutions is a major reason why **you always check your answer**.

In this and the next few problems you are asked to decide whether two equations are equivalent, one implies the other, or neither implies the other. Enter (without the quotation marks) " \implies " if the left equation implies the right, " \impliedby " if the right equation implies the left, " \iff " if either equation implies the other, and " \nRightarrow " if neither equation implies the other. An

equation A implies an equation B if B is true for all variables for which A is true.

For example,

$$x + 3 = 5 \iff x = 2$$

$$x = 2 \implies x^2 = 4$$

$$x^2 + 6 = 10 \iff x = 2$$

$$x + 3 = 5 \nRightarrow x^2 = 9$$

Use these statements in the following items:

$$x + 3 = 5 \text{ ____ } x = 2.$$

$$x = 2 \text{ ____ } x^2 = 4.$$

$$x^2 + 6 = 10 \text{ ____ } x = 2.$$

$$x + 3 = 5 \text{ ____ } x^2 = 9.$$

Answer(s) submitted:

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-
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(incorrect)

8. (1 pt)

In this problem, assume that x is positive (so we can take its square root).

Enter (without the quotation marks) " \implies " if the left equation implies the right, " \impliedby " if the right equation implies the left, " \iff " if either equation implies the other, and " \nRightarrow " if neither equation implies the other.

$$x - 5\sqrt{x} + 6 = 0 \text{ ____ } x^2 - 5x + 6 = 0.$$

$$x - 5\sqrt{x} + 6 = 0 \text{ ____ } 5\sqrt{x} = x + 6.$$

$$5\sqrt{x} = x + 6 \text{ ____ } 25x = x^2 + 12x + 36.$$

This last equation is an ordinary quadratic equation. Solve it to answer the remaining question: The smaller solution of $x - 5\sqrt{x} + 6 = 0$ is $x = \text{____}$, and the larger is $x = \text{____}$.

Answer(s) submitted:

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(incorrect)

9. (1 pt) The first five problems in this set were motivated by the feedback I received on problem 22 (that airplane problem...) of set 2. I thought of it as a routine problem, but it seems to have been quite difficult for a good number of people. These first five problems illustrate a **major** principle of problem solving:

If a problem is hard simplify it and first solve the simpler problem.

Put a little more loosely: *If at first you don't succeed, do something easier.*

To appreciate this lesson, before you start on this problem, try problem 5 on this set. You'll see it's tricky. But it will be a piece of cake after you solve the first four problems.

So if you find yourself again (in this class, or much beyond it) facing a hard problem, attack it by building a sequence of easier problems, the easiest being very easy, that lead to the difficult problem and, in the end, make it easy to solve the hard problem.

You and your friend part at an intersection. You drive off north at 50 mph, and your friend drives east at 50mph. After three hours the distance between you and your friend is _____ miles.

Hint: Use the Pythagorean Theorem.

Answer(s) submitted:

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(incorrect)

10. (1 pt) This problem differs from the preceding one in that the departure times are different. In building a hierarchy of problems it's a good idea to change one ingredient at a time.

You and your friend part at an intersection. Your friend drives away north at a constant speed. You linger at the intersection for an hour, and then drive off due east at a speed that is 11 miles per hour faster than your friend's speed. 2 hours after your friend's departure the distance between the two of you is 106 miles. Your friend is traveling at a speed of _____ miles per hours.

Answer(s) submitted:

•

(incorrect)

11. (1 pt) Match the statements defined below with the letters labeling their equivalent intervals.

You must get all of the answers correct to receive credit.

___1. $x \in (2, 6]$

___2. $x \in (2, 6)$

___3. $x \in [2, 6]$

___4. $x \in [2, 6)$

___5. $x \in [2, \infty)$

A. $2 < x \leq 6$

B. $2 \leq x \leq 6$

C. $2 \leq x$

D. $2 < x < 6$

E. $2 \leq x < 6$

Answer(s) submitted:

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(incorrect)

12. (1 pt)

The expression $\frac{6}{35} \div \frac{3}{7}$ is a fraction $\frac{a}{b}$ where b is positive, and a and b have no common factors.

Enter a=_____ and b=_____

Answer(s) submitted:

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•

(incorrect)

13. (1 pt) The expression $(4 - 7x)^2$ equals $Ax^2 + Bx + C$

where A equals: _____

and B equals: _____

and C equals: _____

Answer(s) submitted:

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•
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(incorrect)

14. (1 pt) Your friend from Paris arrives in New York and the forecast is for a low of 51 and a high of 73 degrees Fahrenheit.

What is the forecasted low temperature in Celsius? _____

What is the forecasted high temperature in Celsius? _____

Answer(s) submitted:

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(incorrect)

15. (1 pt) Find the perimeter of the triangle with the vertices at

(2, 0), (-3, 4), and (-3, -3).

Answer(s) submitted:

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(incorrect)

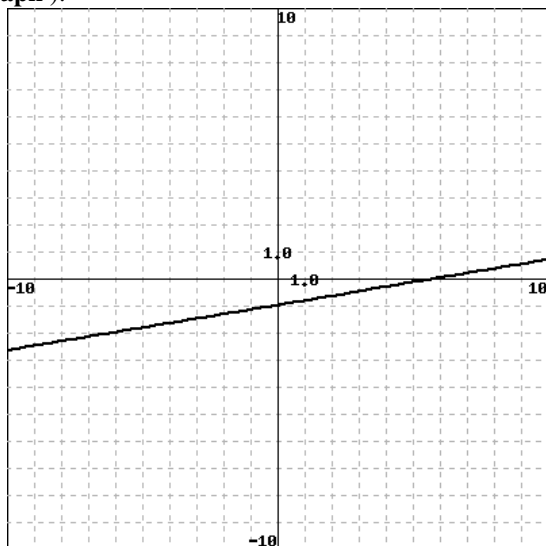
16. (1 pt) The equation of the line that goes through the point (9, 9) and is parallel to the line $2x + 2y = 2$ can be written in the form $y = mx + b$ where m is: _____ and where b is: _____

Answer(s) submitted:

•
•

(incorrect)

17. (1 pt) Find an equation $y = mx + b$ for the line whose graph is sketched (click on the graph to view an enlarged graph):



The number m equals ____;

The number b equals ____;

Answer(s) submitted:

•
•

(incorrect)

18. (1 pt) Let $f(x) = 5x^2 + 2x - 5$. Evaluate the following:

1. $f(0) =$ _____

2. $f(2) =$ _____

3. $f(-2) =$ _____

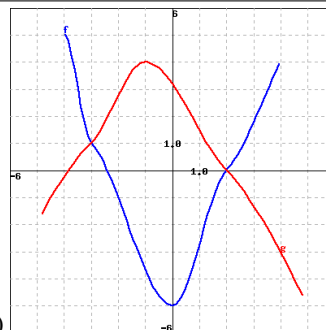
4. $f(x+1) =$ _____

5. $f(-x) =$ _____

Answer(s) submitted:

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(incorrect)



19. (1 pt)

Given the graphs of f (in blue) and g (in red) above, answer these questions:

- ___1. What is the value of f at -4 ?
- ___2. For what values of x is $f(x) = g(x)$? Separate answers by a comma.
- ___3. Estimate the solution of the equation $g(x) = -5$.
- ___4. What is the largest closed interval for which the above graph shows that the function f is decreasing? Enter your answer using interval notation.

Answer(s) submitted:

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(incorrect)

20. (1 pt) Let $f(x) = 4x - 9$ and $g(x) = 9 - x^2$. Evaluate the following:

1. $f(g(-2)) =$ _____

2. $g(f(-2)) =$ _____

Answer(s) submitted:

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•

(incorrect)

21. (1 pt) A movie theater is filled to capacity with 700 people. After the movie ends, people start leaving at the rate of 50 each minute.

(a) Find an equation for N , the number of people in the theater, as a function of t , the number of minutes after the movie ends. Enter your answer as an equation, such as $N = 5t - 1$.

(b) For what values of t does the equation make sense in practical terms.

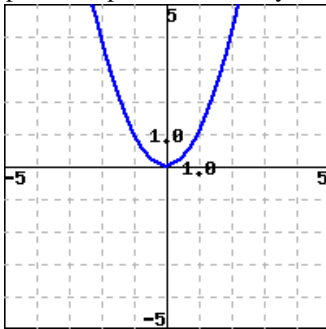
Domain: _____

Answer(s) submitted:

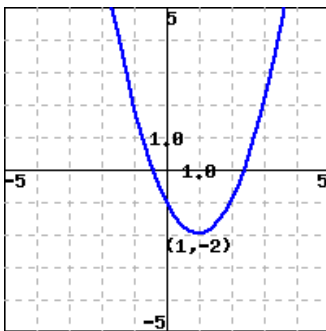
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(incorrect)

22. (1 pt) The graph of $y = x^2$ is given below. (To look at the graph in a separate window, you can click on it).



Find a formula for the function whose graph is given below.



$y =$ _____

Answer(s) submitted:

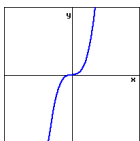
(incorrect)

23. (1 pt) Suppose $f(x) = x^3$ and the graph of $g(x)$ is the graph of $f(x)$ shifted vertically up 2 units and horizontally to the right 1 units.

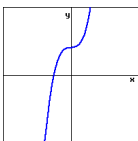
(a) Find a formula for $g(x)$.

$g(x) =$ _____

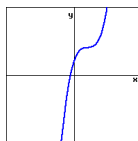
(b) Which graph A-C below could be a graph of $g(x)$? ☐ ?



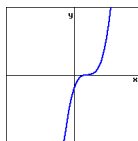
$f(x) = x^3$



A



B



C

(Click on a graph to enlarge it.)

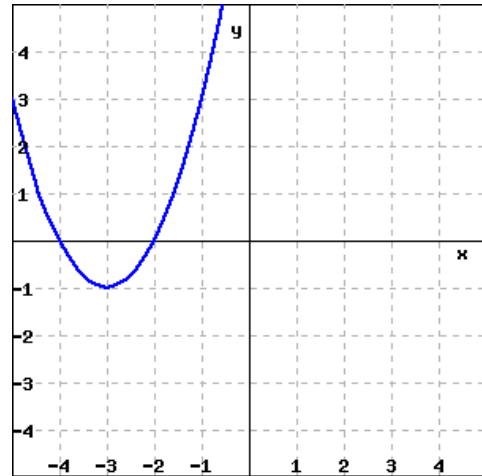
Answer(s) submitted:

(incorrect)

24. (1 pt)

The graph in the figure is a horizontal and/or a vertical shift of the graph of $y = x^2$. Find a formula for the function graphed in the figure.

$g(x) =$ _____



(Click on graph to enlarge)

Answer(s) submitted:

(incorrect)

25. (1 pt)

The graph of $y = f(x)$ is given in the figure.

Match each formula with its graph A-F below.

☐ 1. $y = f(x - 3) - 1$

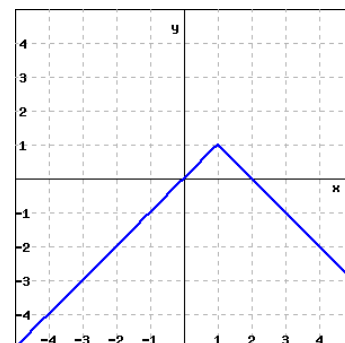
☐ 2. $y = f(x + 1) + 2$

☐ 3. $y = f(x) - 3$

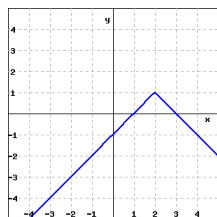
☐ 4. $y = f(x + 3)$

☐ 5. $y = f(x) + 2$

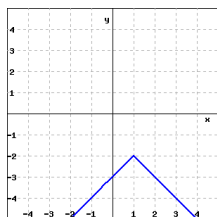
☐ 6. $y = f(x - 1)$



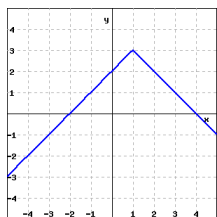
Graph of $y = f(x)$



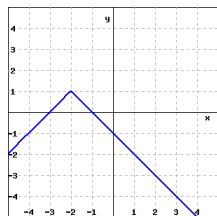
A



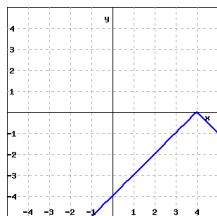
B



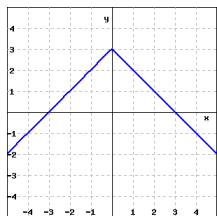
C



D



E



F

(Click on a graph to enlarge it)

Answer(s) submitted:

-
-
-
-
-

(incorrect)