ENPHYS253
Lab 3: Electrical Impedance

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Abstract

The impedances of an inductor and capacitor at 1000 Hz were experimentally determined by measuring the RMS values of the components in various circuit configuration with a digital multimeter. AC circuits with either one inductor or capacitor were constructed with varying input voltage frequencies. The RMS current and voltage values were used to create a series of linear regressions which determined that $R_L = 27.7\Omega \pm 5 * 10^{-2}\Omega$, $L = 1.12 * 10^{-2}H \pm 9 * 10^{-6}H$ and $R_C = 24\Omega \pm 6\Omega$, $C = 4.56*10^{-7}F \pm 3*10^{-10}F$. A parallel and series LC circuit was created at 1000Hz with varying input voltage frequency from the function generator. By finding a linear relationship between I_{rms} and V_{rms} through linear regression, an impedance value was determined. The capacitance and inductance were used with theoretical equations for impedance and to compare with experimentally derived impedance. All the predictions from the theoretical impedances were within error of the physically measured impedance, verifying that the impedance values at 1000Hz were $Z_L = 27.7\Omega + 70.5j\Omega$ and $Z_C = -349j\Omega$.

Results and Analysis

The voltage and current data for the single inductor with $R_L = 26.7\Omega \pm 0.1\Omega$ circuit from Table 1 was graphed according to Equation 3 onto Figure 1. A weighted linear regression was performed, giving a result of $R_L = 27.7\Omega \pm 5 * 10^{-2}\Omega$ and $L = 1.12*10^{-2}H \pm 9*10^{-6}H$. The derived value for R_L agrees with the $26.7\Omega \pm 0.1\Omega$ reading obtained by using a multimeter on the inductor as it lies within the error range and has a percentage difference of 3.8% Using the data in Table 2 with Equation 4 where a single capacitor circuit with $C = 0.454\mu F \pm 0.002\mu F$ was used, a weighted linear regression was created onto Figure 2. The regression gave that $R_C = 24\Omega \pm 6\Omega$ and $C = 4.56 * 10^{-7}F \pm 3 * 10^{-10}F$. The value of C agrees with the value of the measurement $C = 0.454\mu F \pm 0.002\mu F$ obtained from the multimeter as the values lie within their margins of error and have a percentage difference of 0.47%.

Using Equation 2, the V_{rms} and I_{rms} values in Table 3 measured from the Parallel LC circuit were plotted against each other in Figure 3, and a linear regression was used to determine that $|Z_{Parallel}| = 95.3\Omega \pm 0.3\Omega$. Similarly, the values from the series LC on Table 4 were plotted with linear regression to determine that $|Z_{series}| = 284\Omega \pm 1\Omega$. Using the test frequency of $1000Hz$ and equations 5 and 6 with the R_L, L and C values obtained earlier, the impedances were determined to be $Z_L = 27.7 + 70.5j\Omega$ and $Z_C = -349j\Omega$. Using equations 7 and 9 to combine the impedances in parallel and series, $|Z'_{Parallel}|$ and $|Z'_{Series}|$ values were determined as $94.5\Omega \pm 0.9$ and $280\Omega \pm 2\Omega$. These values lie within the range of the expected values, and have percentage difference of only 0.9% and 1.42%.

The values of $|Z'_{Parallel}|$ and $|Z'_{Series}|$ were verified by using them with Equation 2 to predict an expected V_{rms} and I_{rms} by comparing it to a measured value. The I_L and I_C values were physically measured at max voltage in the parallel LC circuit on Table 3 as $I_L = 77.2mA \pm 0.37mA$ and $I_C = 16.8mA \pm 0.84mA$. Using Equation 2 with Z_L and Z_C, the expected values are $I_C = 77.3mA \pm 0.3mA$ and $I_L = 16.8mA \pm 0.4mA$ values within the range of the measured values, having a percentage difference of 0.03% and 0% respectively. In the series LC circuit, the V_{rms} values were measured as $V_L = 1.89V \pm 0.94V$ and $V_C = 9.36V \pm 0.47V$ on Table 4. Using Equation 2, the expected values values are $V_L = 2.02 \pm 0.07$ and $V_C = 9.31 \pm 0.04$. The Voltage values all lie within the range of uncertainty and have a respective percentage difference of 7.0% and 0.47%

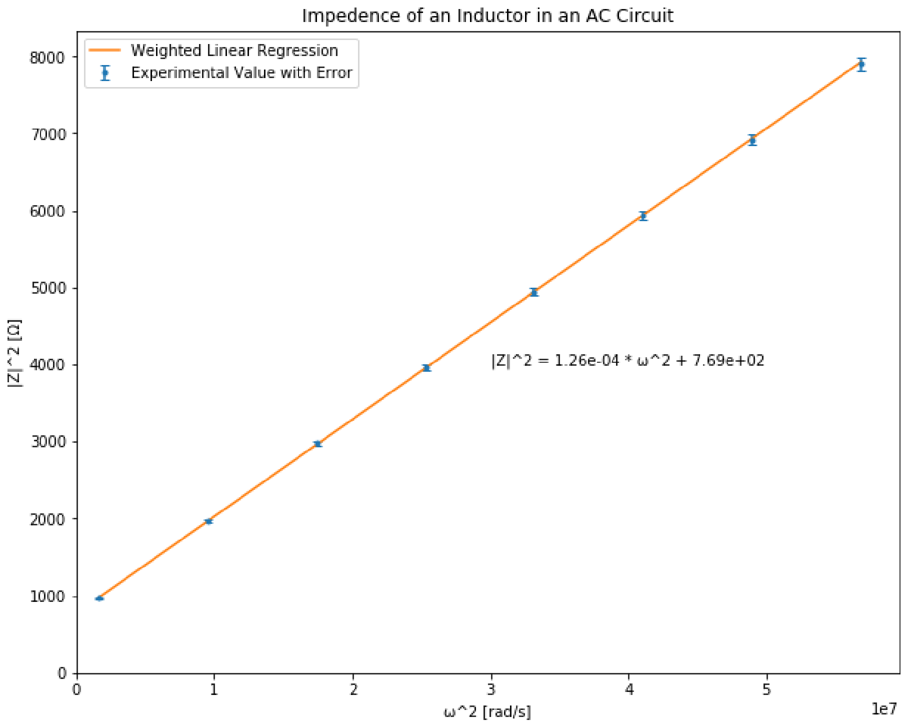

Figure 1: Impedance of an Inductor in an AC Circuit with Weighted Linear Regression where $R_L = 26.7\Omega \pm 0.1\Omega$

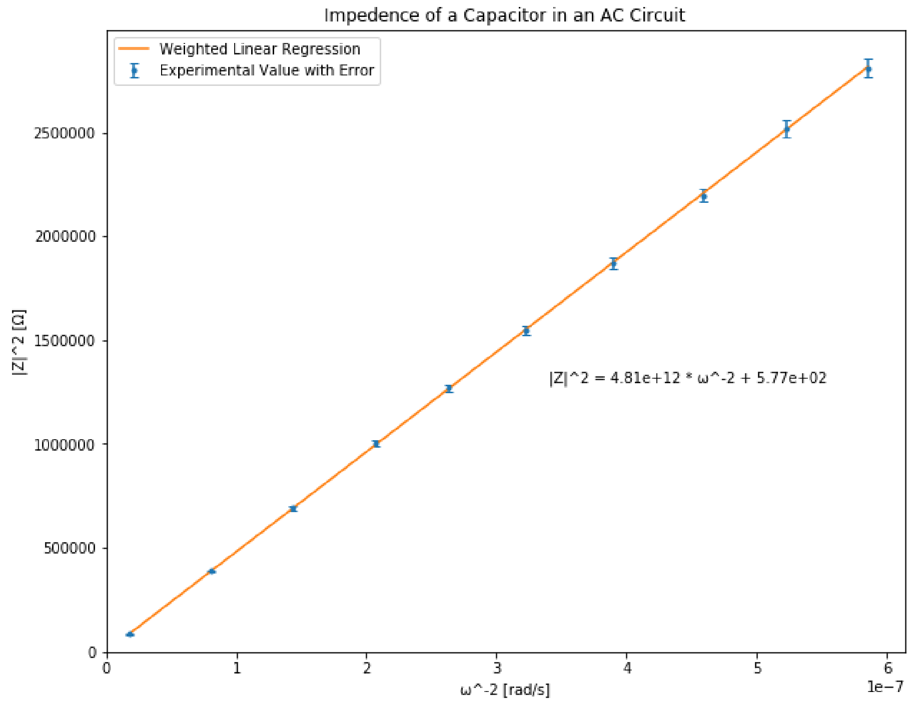

Figure 2: Impedance of an Capacitor in an AC Circuit with Weighted Linear Regression where $C = 0.454\mu F \pm 0.002\mu F$

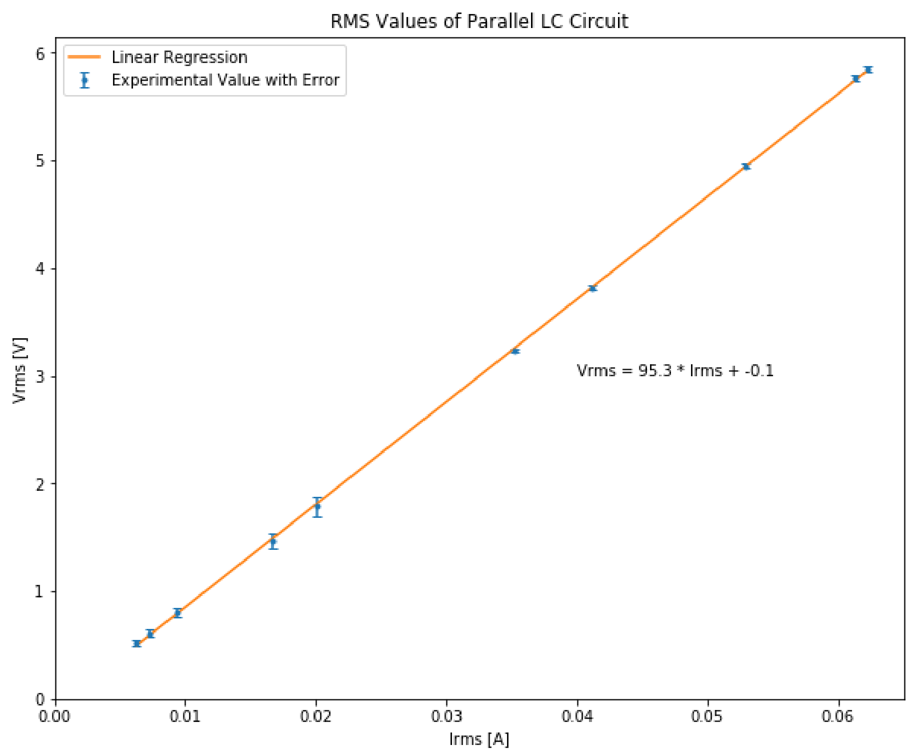

Figure 3: RMS Values of Parallel LC Circuit with Linear Regression where $f = 1000Hz$

Note: The input voltage amplitude was varied from the minimum to the maximum setting on the function generator.

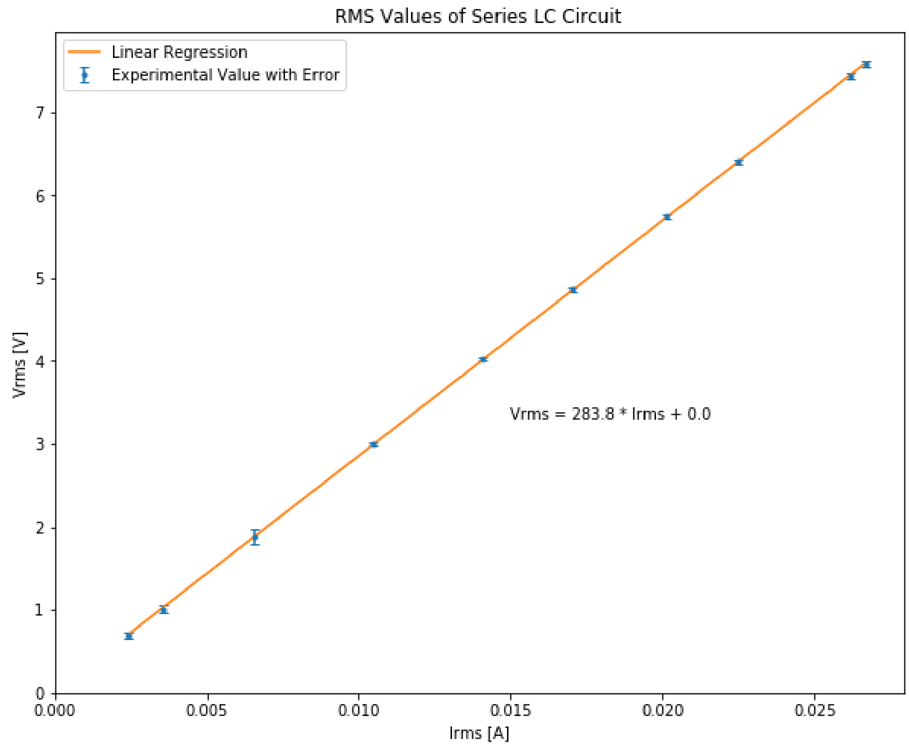

Figure 4: RMS Values of Series LC Circuit with Linear Regression where $f = 1000Hz$

Note: The input voltage amplitude was varied from the minimum to the maximum setting on the function generator.

Appendix

Equation List

Z is defined as impedance, V is voltage, I is current, f is Voltage frequency, L is inductance, C is capacitance, and R is resistance.

$$f = \frac{\omega}{2\pi} \tag{1}$$

$$|Z| = V_{rms}/I_{rms} \tag{2}$$

$$|Z| = R_L^2 + \omega^2 L^2 \tag{3}$$

$$|Z| = R_C^2 + \frac{1}{\omega^2 C^2} \tag{4}$$

$$Z = R_L + j\omega L \tag{5}$$

$$Z = \frac{1}{j\omega C} \tag{6}$$

$$Z_{series} = \sum Z_i \tag{7}$$

$$Z_{parallel}^{-1} = \sum \frac{1}{Z_i} \tag{8}$$

Raw Data and Sample Calculations

Measured values:

- $R_L = 26.7\Omega \pm 0.1\Omega$
- $C = 0.454\mu F \pm 0.002\mu F$

Table 1: Readings and Calculations of AC Circuit with single Inductor with measured $R_L = 26.7\Omega \pm 0.1\Omega$

Frequency[Hz]	Vrms [V] (± 0.75% ± 2 digits)	Irms [mA] (± 0.75% ± 2 digits)	Z [Ω]	Error in Z [Ω]
200	3.02	97.2	31.07	0.16
490	3.97	89.5	44.36	0.23
665	4.64	85	54.59	0.27
801	5.06	80.4	62.94	0.31
916	5.38	76.5	70.33	0.36
1020	5.63	73.1	77.02	0.39
1114	5.83	70.1	83.17	0.42
1200	6	67.5	88.89	0.45

Table 2: Readings and Calculations of AC Circuit with single Capacitor with measured $C = 0.454\mu F \pm 0.002\mu F$

Frequency [Hz]	Vrms [V] (± 0.75% ± 2 digits)	Irms [mA] (± 0.75% ± 2 digits)	Z [Ω]	Error in Z [Ω]
208	7.76	4.63	1.676	0.015
220	7.76	4.89	1.587	0.014
235	7.75	5.23	1.482	0.013
255	7.75	5.67	1.367	0.012
280	7.74	6.22	1.244	0.011
310	7.74	6.87	1.127	0.010
350	7.73	7.72	1.001	0.009
420	7.71	9.28	0.831	0.008
560	7.78	12.48	0.623	0.006
1200	7.69	26.36	0.292	0.001

Table 3: Readings and Calculations of Parallel LC circuit with measured $f = 1000Hz$, $C = 0.454\mu F \pm 0.002\mu F$, $f = 1000Hz$

Vrms [V] (± 0.75% ± 2 digits)	Irms [mA] (± 0.75% ± 2 digits)
0.52	6.27
0.61	7.31
0.8	9.4
1.47	16.73
1.79	20.1
3.23	35.2
3.82	41.2
4.95	52.9
5.76	61.3
5.85	62.2

The input voltage amplitude was varied from the minimum to the maximum setting on the function generator. At the maximum setting, the I_{rms} values of the individual component were measured as: $I_L = 77.2mA \pm 0.37mA$
$I_C = 16.8mA \pm 0.84mA$

Table 4: Readings and Calculations of Series LC circuit with measured $f = 1000Hz$, $C = 0.454\mu F \pm 0.002\mu F$, $f = 1000Hz$

Vrms [V] (± 0.75% ± 2 digits)	Irms [mA] (± 0.75% ± 2 digits)
0.69	2.44
1.01	3.55
1.88	6.58
3	10.5
4.03	14.1
4.86	17.03
5.74	20.15
6.4	22.5
7.44	26.2
7.58	26.7

The input voltage amplitude was varied from the minimum to the maximum setting on the function generator. At the maximum setting, the V_{rms} values of the individual component were measured as: $V_L = 1.89V \pm 0.94V$
$V_C = 9.36V \pm 0.47V$