Lab 2 Analysis

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ENGPHYS232

Results and Analysis

The time shift data was used with equation (5) to calculate the the phase and uncertainty of the experimental transfer function *H* over the given range of frequencies. These points were plotted on a log scale with the theoretical phase values from equation (2) onto Figure 1. Most of the experimental points fit on the theoretical curve within their errors except for the first, second and last points. The first two discrepancies can be explained by the fact that the initial time shift was difficult to discern from the oscilloscope using the naked eye while the last is due to the function generator having a greater uncertainty during high frequencies.

The voltage data was used with equation (4) to determine the experimental transfer function magnitude and was plotted with the theoretical curve using equation (1). All the experimental points fall well within the ranges of uncertainties of the theoretical curve. The cutoff frequency calculated from equation (3) is 4900 Hz ± 700 Hz, and it agrees with the expected -3 dB value predicted in equation (7).

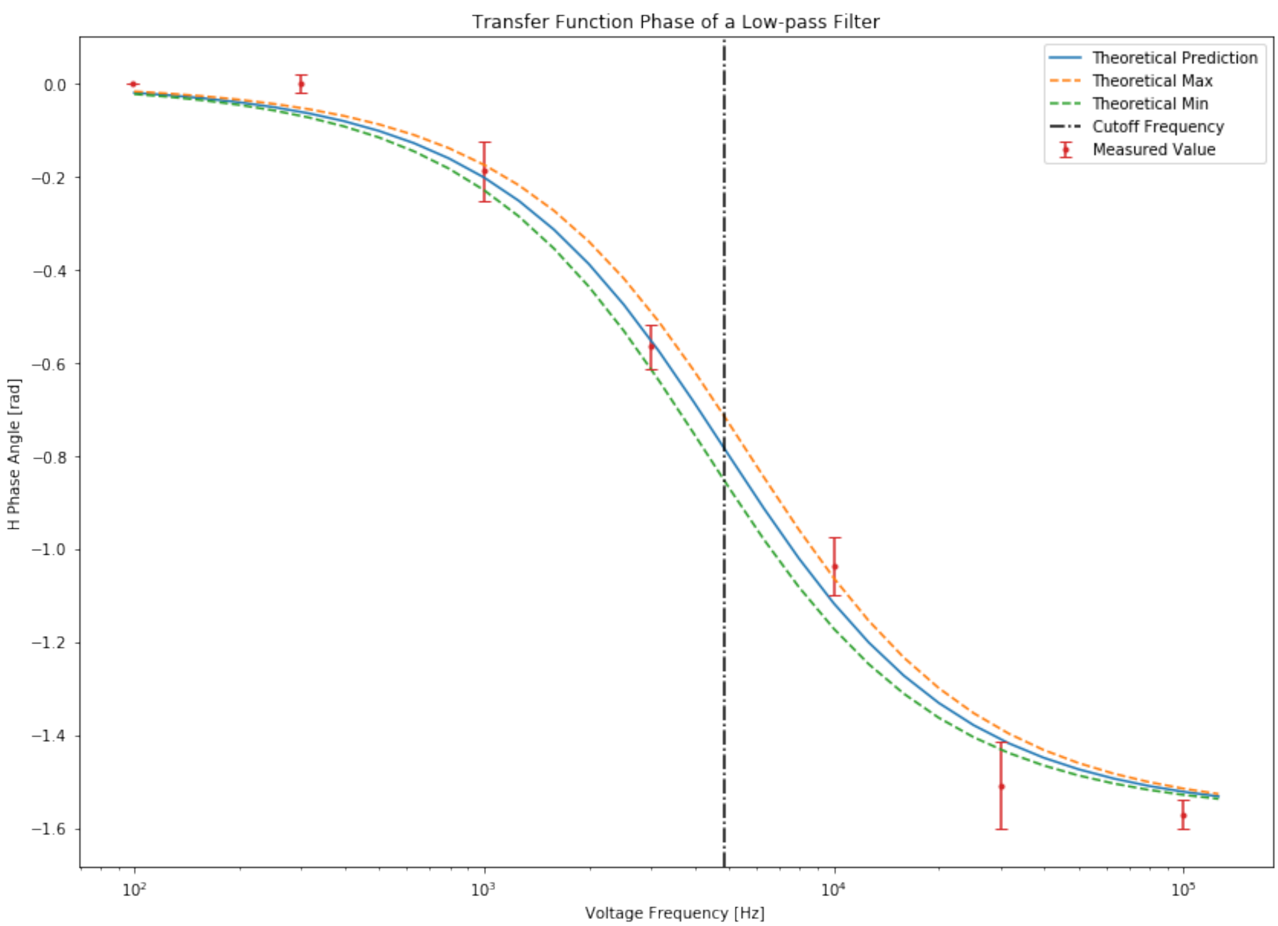

Figure 1: Low Pass Filter Phase - Experimental and Theoretical Curves for $R = 1505\ \Omega \pm 5\ \Omega$, $C = 2.2 \times 10^{-8}\ F \pm 0.3 \times 10^{-8}\ F$. *The min and max uncertainties are represented as dotted lines.*

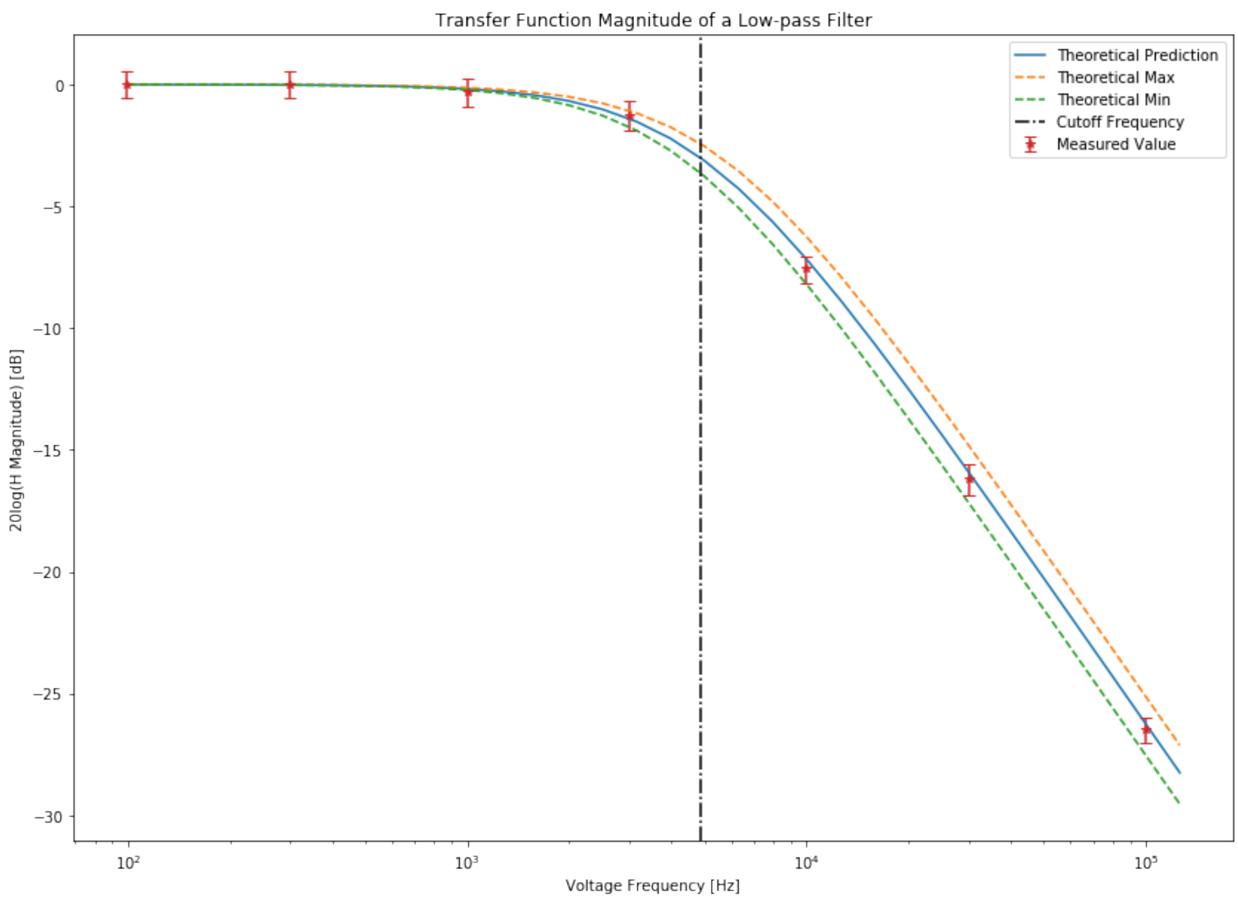

Figure 2: Low Pass Filter Magnitude - Experimental and Theoretical Curves for R = 1505 Ω ± 5 Ω, C = 2.2 x 10^{-8} F ± 0.3 x 10^{-8} F. *Note that the y axis is* 20*log10(|H|).

Appendix

Low Pass Filter: Experimental Values for $R = 1505\ \Omega \pm 5\ \Omega$ and $C = 2.2 \times 10^{-8}$ F $\pm\ 0.3 \times 10^{-8}$ F

Frequency [Hz]	Vout [V] +/– 0.5 [V]	Vin [V] +/– 0.5 [V]
99	11	11
300	11	11
1001	10.6	12
3000	9.5	13
10003	4.6	14
30000	1.7	15
100010	0.52	16
Time Shift [ms]	**Error in Time Shift [ms]**	**Vout/Vin**
0	0	1.0
0	0.01	1.0
0.03	0.01	0.964
0.03	0.003	0.864
0.017	0.001	0.418
0.008	0.0005	0.155
0.0025	0.00005	0.047
Error in Vout/Vin	**Phase Shift [rad]**	**Error in Phase Shift [rad]**
0.06	0.00	0.00
0.06	0.00	0.02
0.06	−0.19	0.06
0.06	−0.57	0.05
0.03	−1.04	0.06
0.01	−1.51	0.09
0.00	−1.57	0.03

List of Equations and Sample Calculations

$$|H| = \left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1+(\omega R\mathrm{C})^2}}$$

Equation 1 - Theoretical transfer function magnitude

Sample calculation:

`ω = 2π(99 Hz) rad/s, R = 1505 Ω ± 5 Ω, C = 2.2 ∗ 10⁻⁸ F ± 0.3 ∗ 10⁻⁸ F`

`H = 1.0 rad/s +/− 0.6 rad/s`

$$\phi = \mathrm{atan}(\omega R\mathrm{C})$$

Equation 2 - Theoretical phase shift

Sample calculation:

`ω = 2π(99 Hz) rad/s, R = 1505 Ω ± 5 Ω, C = 2.2 ∗ 10⁻⁸ F ± 0.3 ∗ 10⁻⁸ F`

`Φ = −2.0 x 10⁻² rad +/− 0.3 x 10⁻² rad`

$$f_c = \frac{1}{2\pi RC}$$

Equation 3 - Theoretical cutoff frequency

Sample calculation:

`R = 1505 Ω ± 5 Ω, C = 2.2 ∗ 10⁻⁸ F ± 0.3 ∗ 10⁻⁸ F`

`f_c = 4900 Hz +/− 700 Hz`

$$|H| = \left|\frac{Vout\ measured}{Vin\ measured}\right|$$

Equation 4 - Experimental transfer function magnitude

$$\phi = -2\pi\Delta t_{shift}$$

Equation 5 - Experimental Phase shift

$$\sigma f(a, b\ ...) = \left[\left(\frac{\partial f}{\partial a}\sigma a\right)^2 + \left(\frac{\partial f}{\partial b}\sigma b\right)^2 + ...\right]^{\frac{1}{2}}$$

Equation 6 - Derivative Method for Calculating Uncertainties. The uncertainties for equations 1, 2, 4 and 5 were symbolically computed using this method with Python

$$20\log\left(\frac{Vout}{Vin}\right) = -3dB$$

Equation 7 - The expected value of the magnitude of the transfer function for when it is plotted against frequency and evaluated at the cutoff frequency.