ENPHYS253
Lab 6: Young's Modulus of Steel

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1 Procedure

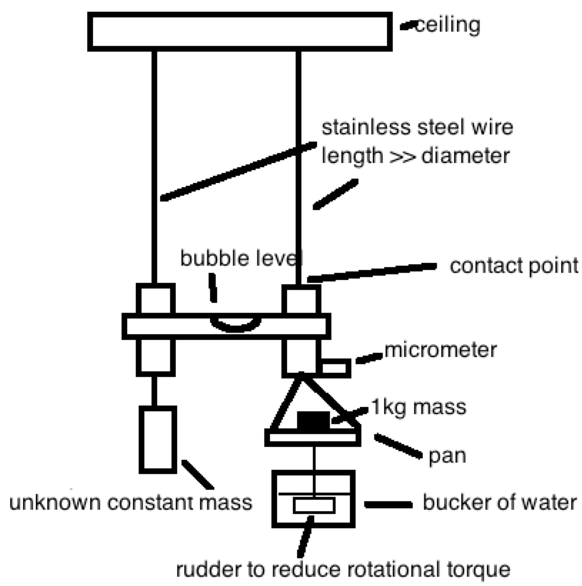

Figure 1: Static Apparatus Diagram

The static apparatus in figure 1 was set up. The micrometer (each subdivision representing 10 µm) was adjusted so that the bubble level was centered, and the micrometer reading was recorded as the zero value. The length of the wire was measured with a two-meter stick by summing the distances from the halfway point of the wire to the ceiling and to the and contact point. The length was determined to be 2.83 ± 0.01m. The width of the wire was measured by a different micrometer at three points and was averaged to 0.52 ± 0.01mm. A 100 gram weight was added to the pan and the micrometer was readjusted so that the bubble level was centered. The new micrometer reading and the added weight (not including the initial kilogram) were recorded. The change length for the wire is the zero value subtracted from the micrometer reading. This process was repeated until the sum of added weights reached 1000 grams. Weights were then removed 50 grams at a time until only the initial 1kg weight was left. After each removal, the same process of adjusting the micrometer and recording the measurements was repeated. The uncertainty in the weights were considered negligible, and the uncertainty in the micrometer reading was half the smallest measurement. The largest source of error was in the bubble level, as there is no way to precisely center it.

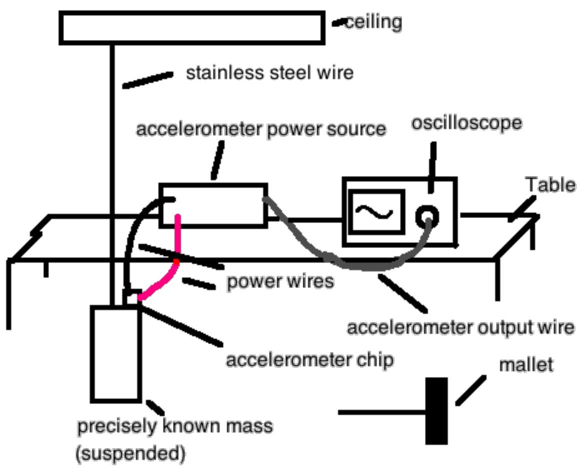

Figure 2: Static Apparatus Diagram

The dynamic apparatus in figure 2 was set up and the oscilloscope and accelerometer power source turned on. The accelerometer has a sensitivity of $400\frac{\mathrm{mV}}{\mathrm{g}}$. Using the same methods as the static apparatus, the length of the wire was measured as 2.81 ± 0.01m and the width was measured as $(78 \pm 1) * 10^{-2}$mm. The power wires in the accelerometer have slack, allowing that when the mass to oscillate freely. The mass was firmly struck with the mallet, creating a voltage reading on the oscilloscope. After the accelerometer readings had stabilized into a simple harmonic waveform, the oscilloscope signal was frozen. The horizontal cursors in the oscilloscope were used to measure the pk-pk voltage amplitude of the wave, and the vertical cursors were used to measure the time between ten full periods. The voltage and time readings were recorded. This process was repeated two more times. The mass was struck again, but the oscilloscope was frozen after the reading had started damping. The horizontal cursors were used to find the points where the pk-pk amplitudes had dampened by a factor of one half, and the vertical cursors were used to find the time difference. The time difference was recorded. This process was repeated two more times for the same change in amplitude. Since the voltage and time measurements were taken multiple times, their uncertainty is their standard error of the mean.

2 Results and Analysis

The static apparatus load data from table 1 and table 2 was plotted onto figure 3 and a linear regression was performed. The slopes were determined to be $0.697 \pm 0.010\text{mm}\,\text{kg}^{-1}$ and $0.679 \pm 0.009\text{mm}\,\text{kg}^{-1}$, which are identical within experimental error. Residual plots were created on figures 4 and 5, with both having no discernible pattern. The two datasets were combined and plotted onto figure 6, while the residuals were plotted onto figure 7. The slope from the linear regression was determined to be $0.686 \pm 0.007\text{mm}\,\text{kg}^{-1}$. This slope was used with equation 1 to calculate the Young's modulus of the stainless steel wire as 193 ± 8GPa, agreeing with the accepted range of Young's modulus of stainless steel of 180GPa to 200GPa[1]. Using the second-last data point in table 1 with equation 1, the maximum stress was calculated as $(9.5 \pm 0.4) * 10^3$kPa and the maximum strain was calculated as $(2.555 \pm 0.008) * 10^{-4}$.

The dynamic apparatus time data from table 3 was used to calculate the average frequency of oscillate as 10.70 ± 0.01Hz. This frequency was used with equation 2 to determine the Young's modulus for stainless steel as $(185 \pm 5) * 10^9$GPa. This value also lies within the expected range of 180GPa to 200GPa[1]. By calculating the average voltage as while knowing that the chip sensitivity was $400\frac{\text{mV}}{\text{g}}$, the maximum acceleration of the mass was calculated as $3.0 \pm 0.2\text{m/s}^2$. From equation 3, it was determined that the maximum tension in the wire occurred when the magnitude of the acceleration was at a maximum and its direction opposed gravity. The maximum tension was calculated as 89 ± 2N, resulting in a maximum stress of $(186 \pm 6) * 10^3$kPa. The maximum acceleration was used with equation 4 to determine the amplitude of vibration x_o. Since the acceleration was a maximum, the cosine term in equation 4 equalled 1, making it possible the amplitude of vibration as $(66 \pm 5) * 10^{-5}$m. Knowing that the maximum strain occurs at x_o, it was calculated as $(23 \pm 2) * 10^{-5}$. The times it took for the amplitude to decrease by a factor of 0.5 from table 4 was averaged and used with equation 5 in order to calculate the quality factor as $(6.2 \pm 0.2) * 10^2$.

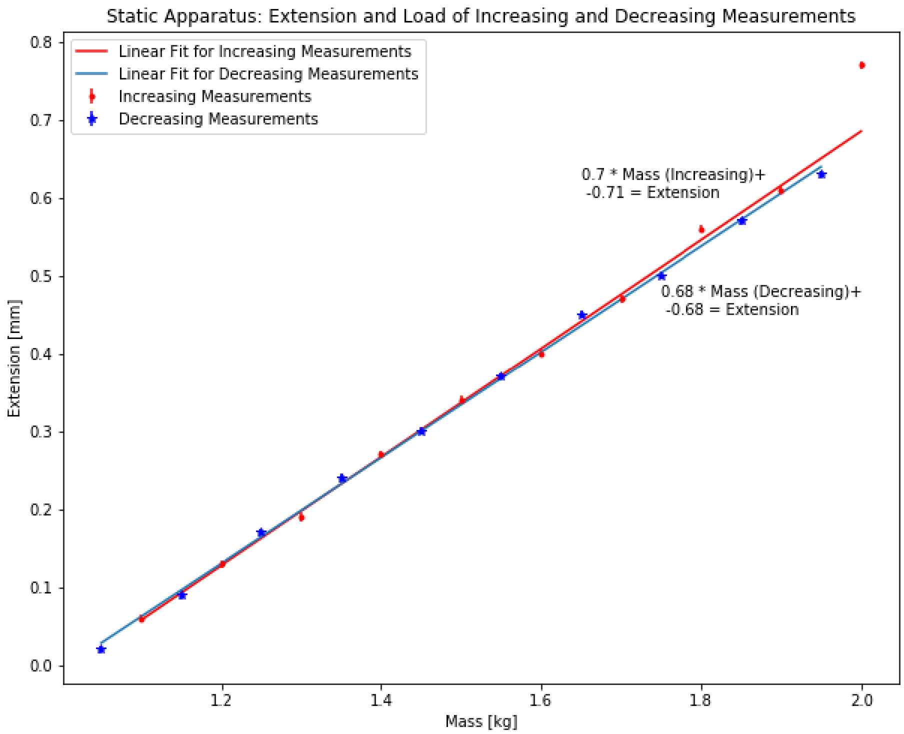

Figure 3: Static Apparatus: Extension and Load of Increasing and Decreasing Measurements with stainless steel wire length 2.83 ± 0.01m and diameter 0.52 ± 0.01mm
Note: The last data point in the increasing loads was ignored from the linear fit and subsequent plots/calculations as it appears as an outlier.

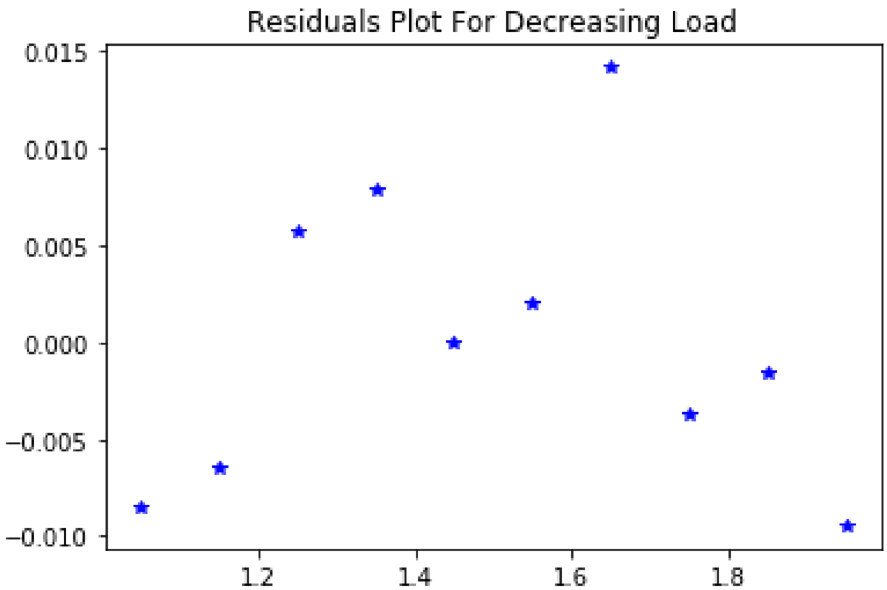

Figure 4: Residuals Plot For Increasing Load

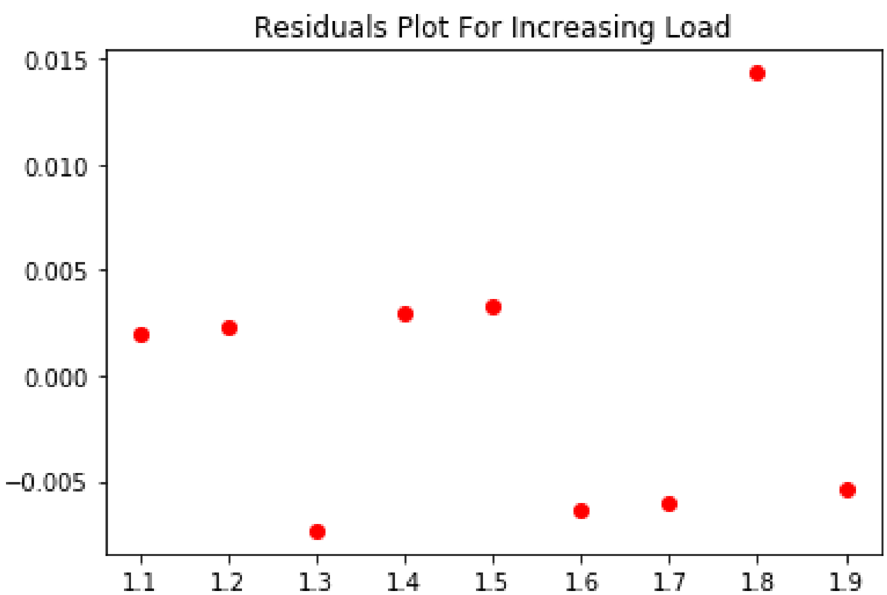

Figure 5: Residuals Plot For Decreasing Load

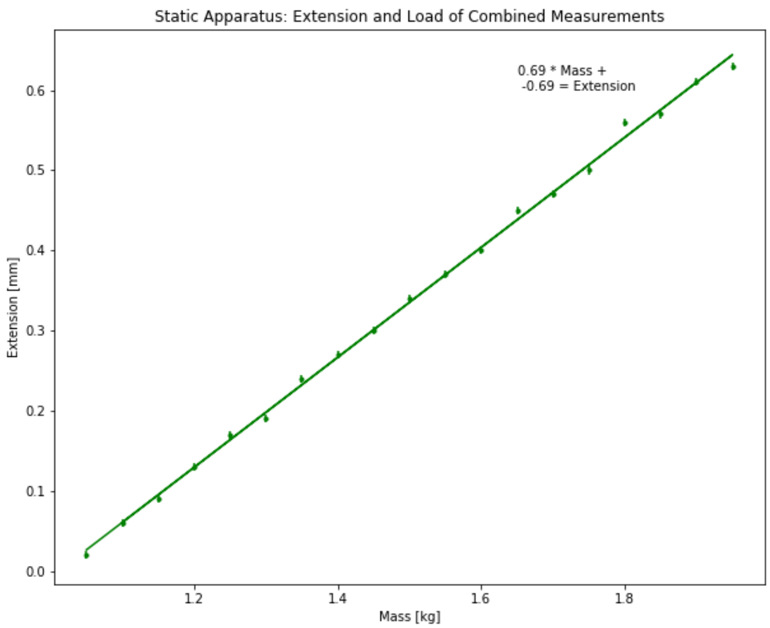

Figure 6: Static Apparatus: Extension and Load of Increasing and Decreasing Measurements with stainless steel wire length 2.83 ± 0.01mand diameter 0.52 ± 0.01mm

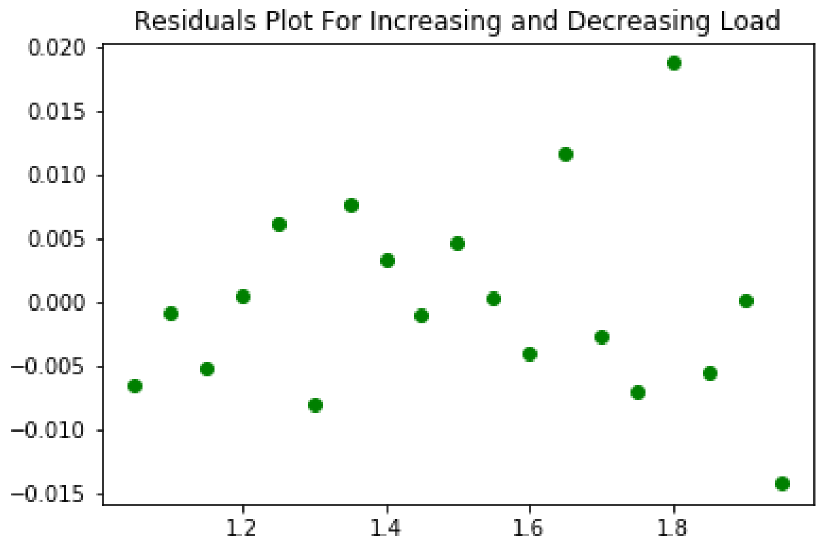

Figure 7: Residuals Plot For Increasing and Decreasing Load

3 Appendix

3.1 Equation List

3.1.1 Static Case

$$E = \frac{stress}{strain} = \frac{Fl}{A\Delta l} = \frac{mgl}{A\Delta l} \tag{1}$$

E is Young's modulus, F is the tensile force acting over the cross sectional area A, l is the length, Δl is the change in length, m is mass and g is gravitational acceleration.

3.1.2 Dynamic Case

T is the tension in the wire. These are the forces acting upon the mass.

$$E = (2\pi f_{avg})^2 Ml/A \tag{2}$$

f_{avg} is the frequency of oscillation, M is the mass.

$$M\ddot{x} = T - Mg \tag{3}$$

$\ddot{x}$ is the acceleration of the mass, T is the tension in the wire. These are the forces acting upon the mass.

$$\ddot{x} = -(2\pi f)^2 x_o \cos(2\pi ft + \phi) \tag{4}$$

t is time, x_o is the amplitude of vibration.

$$Q = \pi t_{1/2} f / \ln(2) \tag{5}$$

Q is the quality factor, $t_{1/2}$ is time for the amplitude to decay by a factor of 1/2.

3.2 Raw Data

Table 1: Extension for Increasing Loads in Static Apparatus with Stainless Steel Wire with Length 2.83 ± 0.01m, Diameter 0.52 ± 0.01mm and Zeroed Extension at 0.170 ± 0.005mm

Mass (kg)	Micrometer Extension [mm] +/- 0.005 [mm]
0.1	0.23
0.2	0.3
0.3	0.36
0.4	0.44
0.5	0.51
0.6	0.57
0.7	0.64
0.8	0.73
0.9	0.78
1	0.94

Table 2: Extension for Decreasing Loads in Static Apparatus with Stainless Steel Wire with Length 2.83 ± 0.01m, Diameter 0.52 ± 0.01mm and Zeroed Extension at 0.170 ± 0.005mm

Mass (kg)	Micrometer Extension [mm] +/- 0.005 [mm]
0.95	0.8
0.85	0.74
0.75	0.67
0.65	0.62
0.55	0.54
0.45	0.47
0.35	0.41
0.25	0.34
0.15	0.26
0.05	0.19

Table 3: Time for 10 Periods of Dynamic Apparatus with Stainless Steel Wire with Length 2.81 ± 0.01m, Diameter $(78 \pm 1) * 10^{-2}$mm, mass 6.949 ± 0.001kg and voltage sensitivity $400 \frac{\mathrm{mV}}{\mathrm{g}}$

Time [ms]	Amplitude [mV]
936	248
934	250
934	232

Table 4: Time for Amplitude to go from 140mV to 70mV with Stainless Steel Wire with Length 2.81 ± 0.01m, Diameter $(78 \pm 1) * 10^{-2}$mm, mass 6.949 ± 0.001kg and voltage sensitivity $400\frac{\mathrm{mV}}{\mathrm{g}}$

Time [s] +/- 0.5 [s]
12.9
12
12.97
12.16
14.16

3.3 Sample Calculations

3.3.1 Young's Modulus (Static)

Using equation 1 and the slope from 6, the following equation can be used: $(\frac{slope}{g})^{-1}\frac{l}{A}$

$$= \frac{(2.83\pm0.01\mathrm{m}/9.81)^{-1}*2.83\pm0.01\mathrm{m}}{\pi(0.52\pm0.01\mathrm{mm}/2)^2}$$
$$= 193 \pm 8\mathrm{GPa}$$

3.3.2 Maximum Stress (Static)

$$\text{Stress} = F_{\max}/A$$
$$= (2kg * 9.81m/s^2)/\pi(0.52 \pm 0.01\mathrm{mm}/2)^2$$
$$= (9.5 \pm 0.4) * 10^3\mathrm{kPa}$$

3.3.3 Maximum Strain (Static)

$$\Delta l/l$$
$$2.155 * 10^4/2.83 \pm 0.01\mathrm{m}$$
$$\text{Strain} = (2.555 \pm 0.008) * 10^{-4}$$

3.3.4 Young's Modulus (Dynamic)

$$E = (2\pi f_{avg})^2 Ml/A$$
$$= (2\pi 10.70 \pm 0.01\mathrm{Hz})^2 6.949 \pm 0.001\mathrm{kg}2.81 \pm 0.01\mathrm{m}/(4.8 \pm 0.1) * 10^{-9}\mathrm{m}^2$$
$$= (185 \pm 5) * 10^9\mathrm{GPa}$$

3.3.5 Maximum Acceleration (Dynamic)

$$\ddot{x}_{\max} = 1/400\frac{\mathrm{mV}}{\mathrm{g}} * V_{avg} * 9.81\mathrm{m/s}^2$$
$$= 1/400\frac{\mathrm{mV}}{\mathrm{g}} * 122 \pm 9\mathrm{mV} * 9.81\mathrm{m/s}^2$$
$$= 3.0 \pm 0.2\mathrm{m/s}^2$$

3.3.6 Maximum Stress (Dynamic)

$$
\begin{aligned}
\text{Stress} &= T/A \\
&= M(a+g)/A \\
&= 6.949 \pm 0.001\text{kg}(3.0 \pm 0.2\text{m/s}^2 + 9.81\text{m/s}^2)/(4.8 \pm 0.1) * 10^{-9}\text{m}^2 \\
&= (186 \pm 6) * 10^3\text{kPa}
\end{aligned}
$$

3.3.7 Amplitude of Vibration (Dynamic)

$$
\begin{aligned}
\ddot{x}_{\max} &= -(2\pi f)^2 x_o \cos(2\pi f t + \phi) \\
x_o &= \ddot{x}/(2\pi f)^2 \\
x_o &= 3.0 \pm 0.2\text{m/s}^2/(2\pi 10.70 \pm 0.01\text{Hz})^2 \\
x_o &= (66 \pm 5) * 10^{-5}\text{m}
\end{aligned}
$$

3.3.8 Maximum Strain (Dynamic)

$$
\begin{aligned}
\text{Strain} &= x/l \\
&= (66 \pm 5) * 10^{-5}\text{m}/2.81 \pm 0.01\text{m} \\
&= 6.949 \pm 0.001\text{kg}(3.0 \pm 0.2\text{m/s}^2 + 9.81\text{m/s}^2)/(4.8 \pm 0.1) * 10^{-9}\text{m}^2 \\
&= (23 \pm 2) * 10^{-5}
\end{aligned}
$$

References

[1] The Engineering ToolBox, "Modulus of elasticity or young's modulus - and tensile modulus for common materials."