ENPHYS253
Lab 5: Bending Beams and Strain Gauges

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1 Theory

A cantilever beam that is fixed on one end and with forced displacement on the other has its longitudinal strain ϵ_l determined by this equation:

$$\epsilon_l = \frac{3a(L-x)}{2L^3}y \tag{1}$$

where x is the distance between the fixed point and the point of strain, a is thickness of beam and L is the distance between the fixed point and the applied force. If the force is known, the strain can be found by using:

$$\epsilon_l = \frac{6(L-x)}{a^2bY}W \tag{2}$$

where Y is the Young's modulus of the material, b is width of beam, and W is the applied force. If the beam is subject to uniaxial stress, it exhibits transversal strain ϵ_t. The Poisson's ratio, σ of the material can be found as the negative ratio between transverse and longitudinal stress:

$$\sigma = -\frac{\epsilon_t}{\epsilon_l} \tag{3}$$

For physically measuring the value of strain, a strain gauge is used. They typically consist of a layer of metal or semiconductor on a thin insulating film and are bonded directly to the beam. As the beam bends, the resistance changes as function of the strain. The calibration of the gauge called the gauge factor, defined as $GF = \delta R/R/\epsilon_l$, where $\delta R/R$ is the relative change in the resistance caused by the strain ϵ_l. A gauge is designed to be more sensitive when stretched in one direction than its perpendicular, and ratio of the two sensitivities is called the transverse sensitivity K_t. This means that for a transverse strain, the measured value contains a significant error. To correct for the error the following equation is used:

$$\epsilon_t = \epsilon_t^{'} - K_t\epsilon_l \tag{4}$$

where $\epsilon_t^{'}$ is the strain measured by the gauge.

2 Results and Analysis

The longitudinal strains caused by the micrometer for the three gauges in table 1 were plotted onto figures 1, 2 and 3. A linear regression was performed, resulting in slopes of $(1.32 \pm 0.01) * 10^{-4}\mathrm{m}^{-1}$, $(8.78 \pm 0.03) * 10^{-5}\mathrm{m}^{-1}$, $(4.55 \pm 0.01) * 10^{-5}\mathrm{m}^{-1}$ for gauge 1, gauge 2 and gauge 3 respectively. The theoretical values for these slopes were calculated using equation 4 and were determined as $(1.21 \pm 0.01) * 10^{-4}\mathrm{m}^{-1}$, $(8.10 \pm 0.02) * 10^{-5}\mathrm{m}^{-1}$ and $(4.12 \pm 0.07) * 10^{-5}\mathrm{m}^{-1}$. None of these values overlap, indicating a systematic error.

The longitudinal strains in gauge 1 caused by varying masses were recorded in table 2 and were plotted onto figure 4. A linear regression was performed, resulting in a slope of $(2.19 \pm 0.02) * 10^{-4}\mathrm{N}^{-1}$. This value was used with equation 5 (found in the appendix), determining the Young's modulus of the aluminum beam as 70 ± 1GPa. This value agrees with the theoretical value of 69GPafor aluminum [1].

The compression strains in gauge 1 caused by the micrometer were recorded in table 3 and were plotted on figure 5. After a linear regression was performed, the slope was determined as $(1.14 \pm 0.01) * 10^{-4}\mathrm{N}^{-1}$. Though this value of the slope was expected to be equal in magnitude to the slope of figure 1, the values do not overlap.

Using a different beam, the longitudinal and transverse strains on their respective gauges were measured onto table 4. The transverse strain data was corrected using equation 4 and was plotted on figure 7. The longitudinal strain data was plotted on figure 6. The transverse and longitudinal data points were plotted against each other on figure 8, and a linear regression determined the ratio between the two strains as -0.322 ± 0.002. Equation 3 was used with this value to determine that the Poisson's ratio of the aluminum beam was 0.322 ± 0.002 GPa, agreeing with the accepted value of 0.32 GPa [2].

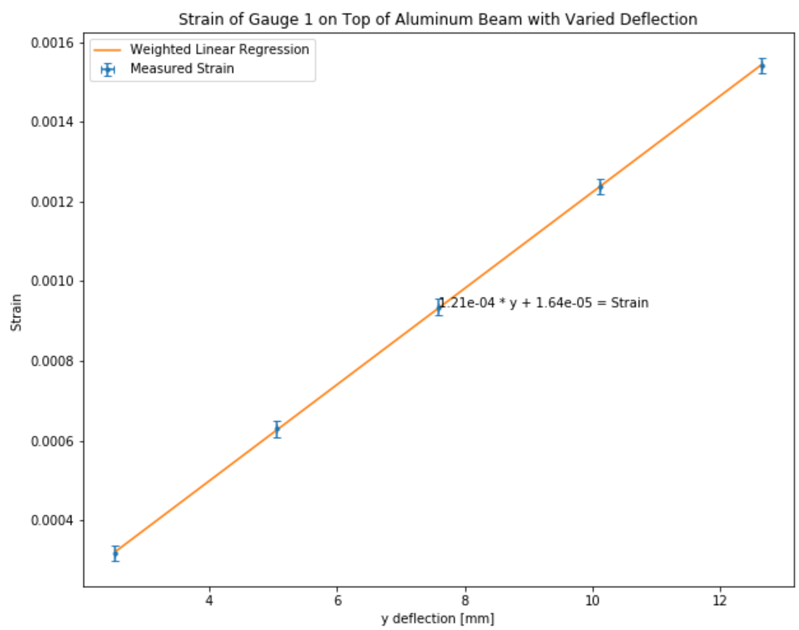

Figure 1: Strain of Gauge 1 on Top of Aluminum Beam with Varied Deflection with $L = 255 \pm 0.5$mm, $x = 25.5 \pm 0.01$mm and $GF = 2.0$

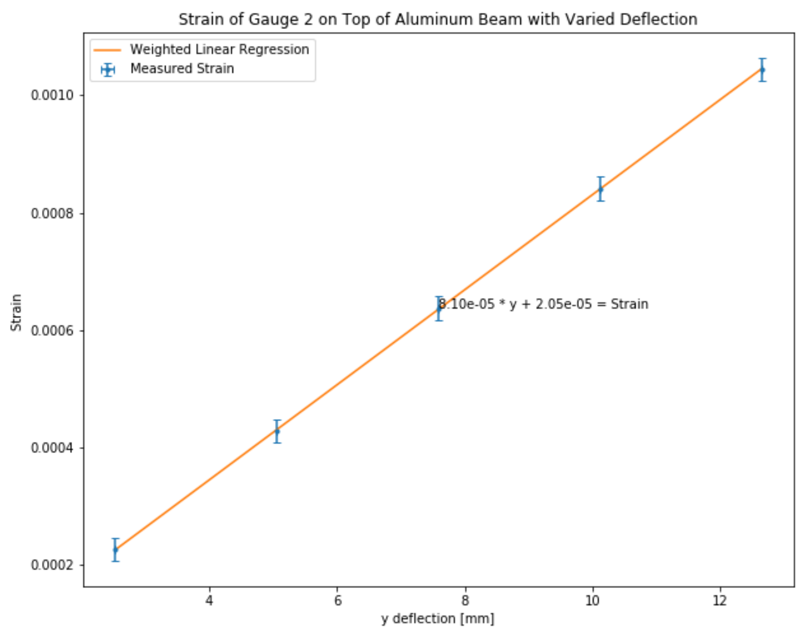

Figure 2: Strain of Gauge 2 on Top of Aluminum Beam with Varied Deflection with $L = 255 \pm 0.5$mm, $x = 101.66 \pm 0.01$mm and $GF = 2.0$

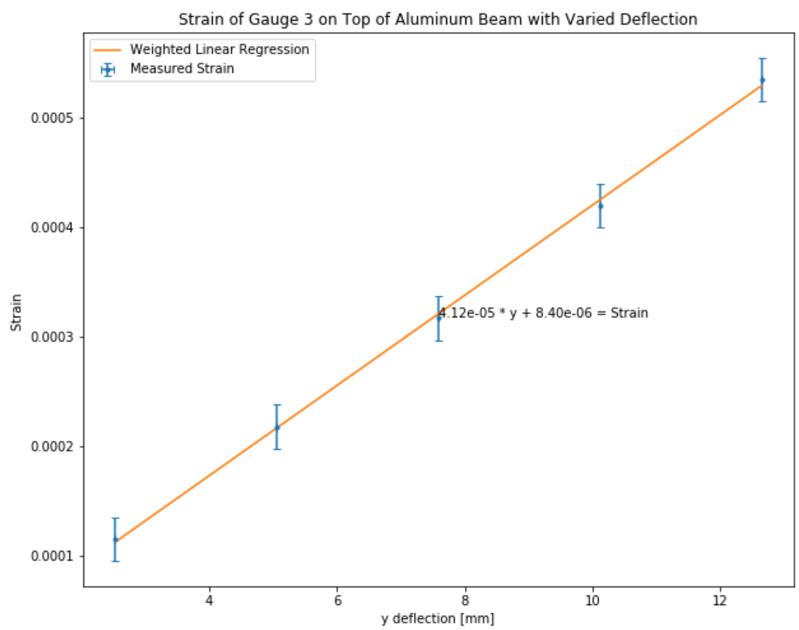

Figure 3: Strain of Gauge 3 on Top of Aluminum Beam with Varied Deflection with $L = 255 \pm 0.5$mm, $x = 175.5 \pm 0.01$mm and $GF = 2.0$

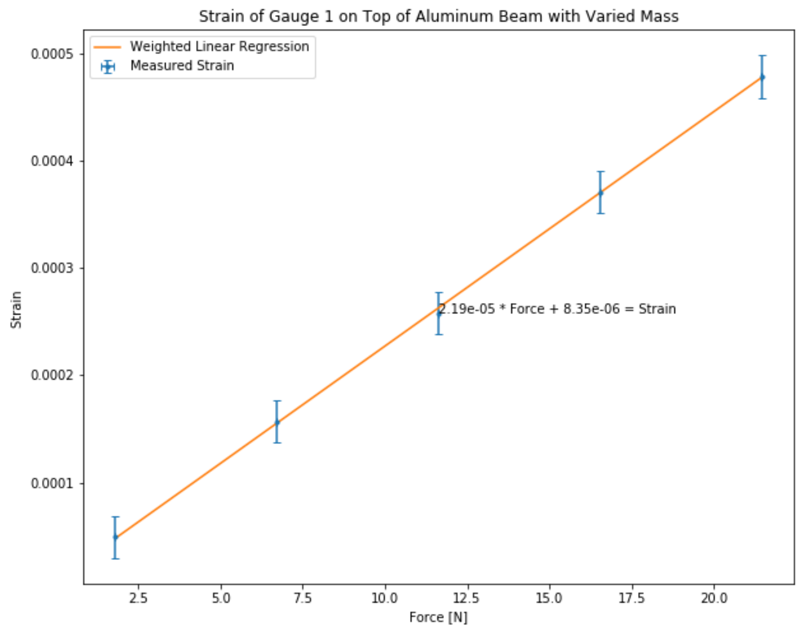

Figure 4: Strain of Gauge 1 on Top of Aluminum Beam with Varied Mass with $L = 286 \pm 0.5$mm, $x = 25.5 \pm 0.01$mm and $GF = 2.0$

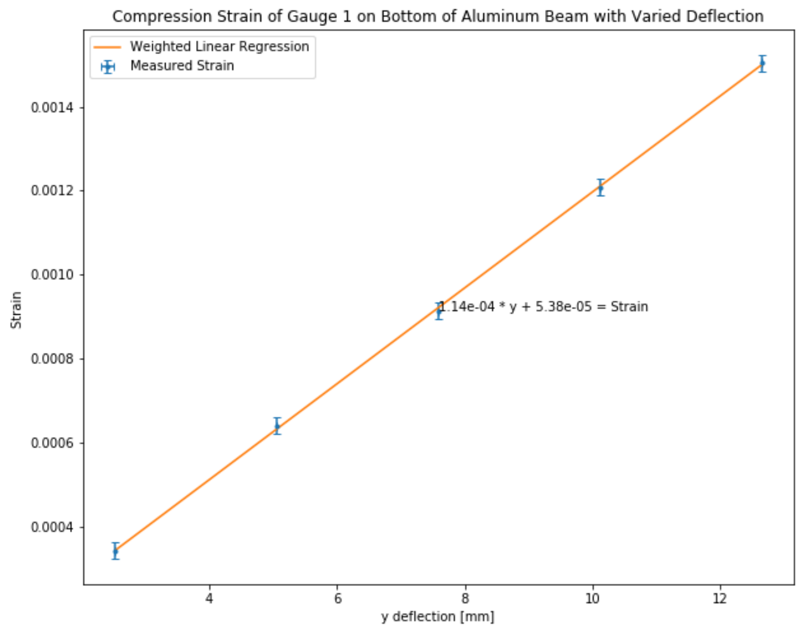

Figure 5: Compression Strain of Gauge 1 on Bottom of Aluminum Beam with Varied Deflection with $L = 286 \pm 0.5$mm, $x = 25.5 \pm 0.01$mm and $GF = 2.0$

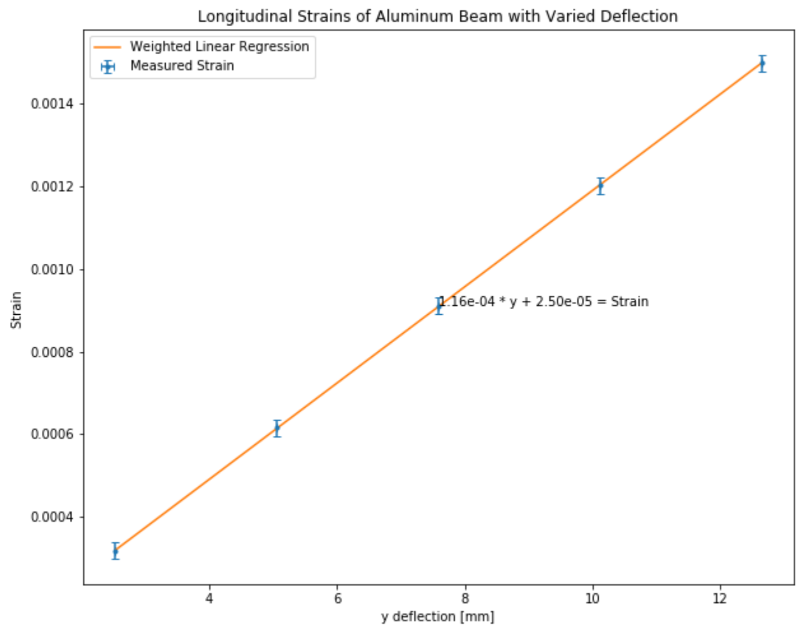

Figure 6: Longitudinal Strain of Gauge 1 on Bottom of Aluminum Beam with Varied Deflection with $L = 252 \pm 0.5$mm, $x = 25 \pm 0.5$mm and $GF = 2.0$

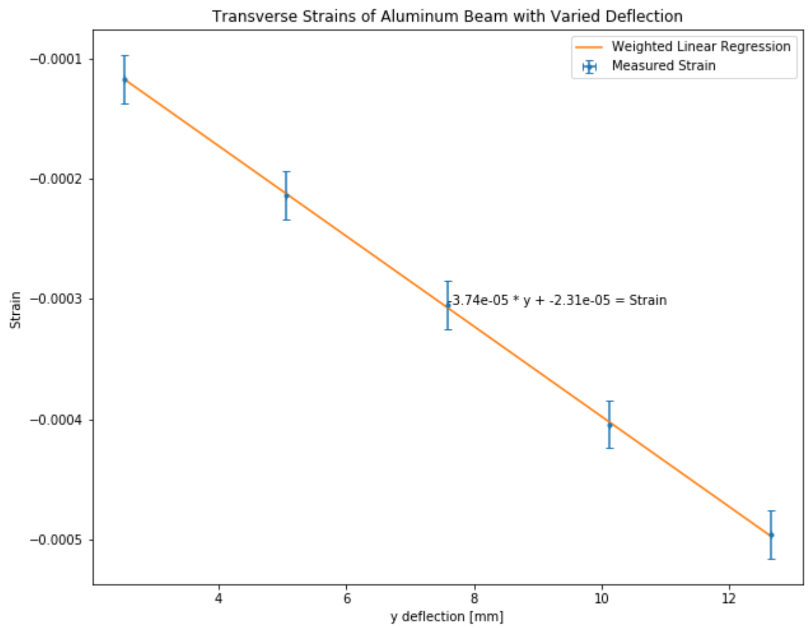

Figure 7: Transverse Strain of Gauge 1 on Bottom of Aluminum Beam with Varied Deflection with $L = 252 \pm 0.5$mm, $x = 26 \pm 0.5$mm and $GF = 2.0$

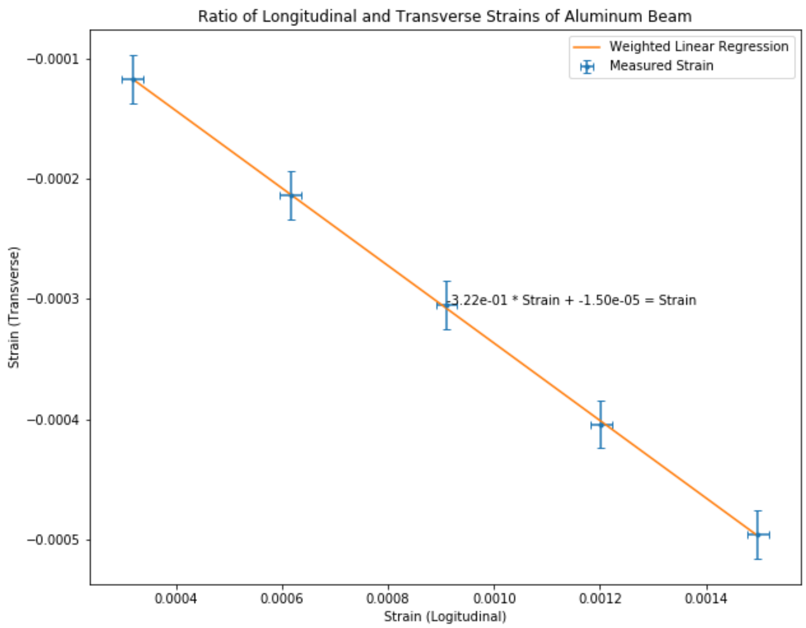

Figure 8: Ratio of Longitudinal and Transverse Strains of Aluminum Beam with $L = 252 \pm 0.5$mm and $GF = 2.0$

3 Appendix

3.1 Equation List

$$\epsilon_l = \frac{3a(L-x)}{2L^3}y \tag{1}$$

ϵ_l is longitudinal strain, a is thickness of beam, b is width of beam, x is the distance between the clamp and the gauge, y is the deflection length and L is the distance between the clamp and the force.

$$\epsilon_l = \frac{6(L-x)}{a^2bY}W \tag{2}$$

Y is Young's modulus, W is the force at the end of the beam.

$$\sigma = -\frac{\epsilon_t}{\epsilon_l} \tag{3}$$

σ is Poisson's ratio, ϵ_t is the transverse strain.

$$\epsilon_t = \epsilon_t^{'} - K_t\epsilon_l \tag{4}$$

ϵ_t is the corrected transverse strain, $\epsilon_t^{'}$ is the measured transverse strain and K_t is the transverse sensitivity.

$$Y = \frac{1}{\frac{a^2*b}{6(L-x)}\frac{\epsilon_l}{W}} \tag{5}$$

Note: this is equation 2 rearranged for Y.

3.2 Raw Data

Table 1: Strain Measurements for Aluminum Beam using Micrometer with with $L = 255 \pm 0.5$mm, $x_1 = 25.5 \pm 0.01$mm $x_2 = 101.66 \pm 0.01$mm $x_3 = 175.5 \pm 0.01$mm, $GF = 2.0$, $a = 6.33 \pm 0.01$mm, b =25.61 $\pm$ 0.01mm, and $K_t = 0.01$.

Deflection [in]	Gauge 1 Strain * 10ˆ-5 +/- 2 * 10ˆ-5	Gauge 2 Strain * 10ˆ-5 +/- 2 * 10ˆ-5	Gauge 3 Strain * 10ˆ-5 +/- 2 * 10ˆ-5
0.1	31.8	22.6	11.5
0.2	63	42.8	21.8
0.3	93.6	63.7	31.7
0.4	123.8	84.1	42
0.5	154.1	104.4	53.5

Table 2: Strain Measurements for Aluminum Beam using Mass with with $L = 286 \pm 0.5$mm, $x = 25.5 \pm 0.01$mm, $GF = 2.0$, $a = 6.33 \pm 0.01$mm, b =25.61 $\pm$ 0.01mm, and $K_t = 0.01$.

Mass [g]	Error in Mass [g]	Gauge Strain * 10ˆ-5 +/- 2 * 10ˆ-5
184.87	0.2	4.9
685.58	0.4	15.7
1186.43	0.4	25.8
1687.14	0.6	37.1
2187.84	0.4	47.9

Table 3: Compression Strain Measurements for Aluminum Beam using Mass with with $L = 255 \pm 0.5$mm, $x = 25.5 \pm 0.01$mm, $GF = 2.0$, $a = 6.33 \pm 0.01$mm, b =25.61 $\pm$ 0.01mm, and $K_t = 0.01$.

Deflection [in]	Strain * 10ˆ-5 +/- 2 * 10ˆ-5
0.1	34.2
0.2	64
0.3	91.3
0k.4	120.8
0.5	150.4

Table 4: Compression Strain Measurements for Aluminum Beam using Mass with with $L = 286 \pm 0.5$mm, $x_{longitudinal} = 25 \pm 0.5$mm, $x_{transverse} = 26 \pm 0.5$mm, $GF = 2.0$, $a = 6.0 \pm 0.5$mm, b =26.0 $\pm$ 0.5mm, and $K_t = 0.01$.

Deflection [in]	Longitudinal Strain* 10ˆ-5 +/- 2 * 10ˆ-5	Transverse Strain * 10ˆ-5 +/- 2 * 10ˆ-5
0.1	31.8	11.4
0.2	61.6	20.8
0.3	91	29.6
0.4	120.2	39.2
0.5	149.8	48.1

3.3 Sample Calculations

3.3.1 Theoretical Slope for Gauge 1

$= \frac{3a(L-x)}{2L^3}$

$= \frac{3*6.33\pm0.01\text{mm}(255\pm0.5\text{mm}-25.5\pm0.01\text{mm})}{2*(255\pm0.5\text{mm})^3}$

$= (1.32 \pm 0.01) * 10^{-4}\text{m}^{-1}$

3.3.2 Theoretical Slope for Gauge 2

$= \frac{3a(L-x)}{2L^3}$
$= \frac{3*6.33\pm0.01\text{mm}(255\pm0.5\text{mm}-101.66\pm0.01\text{mm})}{2*(255\pm0.5\text{mm})^3}$
$= (8.78 \pm 0.03) * 10^{-5}\text{m}^{-1}$

3.3.3 Theoretical Slope for Gauge 3

$= \frac{3a(L-x)}{2L^3}$
$= \frac{3*6.33\pm0.01\text{mm}(255\pm0.5\text{mm}-175.5\pm0.01\text{mm})}{2*(255\pm0.5\text{mm})^3}$
$= (4.55 \pm 0.01) * 10^{-5}\text{m}^{-1}$

3.3.4 Young's Modulus

$= \frac{1}{\frac{(6.33\pm0.01\text{mm})^2*25.61\pm0.01\text{mm}}{6(255\pm0.5\text{mm}-25.5\pm0.01\text{mm})}*(2.19\pm0.02)*10^{-4}\text{N}^{-1}}$
$= 70 \pm 1\text{GPa}$

3.3.5 Poisson's Ratio

$= -\frac{\epsilon_t}{\epsilon_l}$
$= -(2.19 \pm 0.02) * 10^{-4}\text{N}^{-1}$
$= 0.322 \pm 0.002\text{GPa}$

References

[1] MEMSnet, "Material: Aluminum (al)."

[2] Engineer's Edge, "Poisson's ratio metals materials chart."