Cheetah MPC

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1 Conventions

Notation	
i	ith leg
$_{\mathcal{B}}$	left subscript for body coordinate system
$[\mathbf{x}]_\times \in \mathbb{R}^{3\times3}$	skew-symmetric matrix, $[\mathbf{x}]_\times \mathbf{y} = \mathbf{x} \times \mathbf{y}$
$\mathbf{\Theta} = [\phi, \theta, \psi]^T$	orientation as ZYX Euler angles
$\mathbf{J}_i \in \mathbb{R}^{3\times3}$	foot Jacobain
$\mathbf{\Lambda}_i \in \mathbb{R}^{3\times3}$	operational space inertia matrix
$\mathbf{K}_p, \mathbf{K}_d \in \mathbb{R}^{3\times3}$	diagonal positive definite pd gain matrices

2 State Space Model

$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\hat{\mathbf{\Theta}} = R_z(\psi)\hat{\mathbf{\Theta}} \\
&\frac{\mathrm{d}}{\mathrm{d}t}\hat{\mathbf{p}} = \hat{\dot{\mathbf{p}}} \\
&\frac{\mathrm{d}}{\mathrm{d}t}\hat{\omega} = \hat{\mathbf{I}}^{-1}\sum_{i=1}^{n}\mathbf{r}_i \times \mathbf{f}_i = \hat{\mathbf{I}}^{-1}\left([\mathbf{r}_1]_\times \mathbf{f}_1 + \cdots + [\mathbf{r}_n]_\times \mathbf{f}_n\right) \\
&\frac{\mathrm{d}}{\mathrm{d}t}\hat{\dot{\mathbf{p}}} = \frac{\sum_{i=1}^{n}\mathbf{f}_i}{m} - \mathbf{g} = \frac{\mathbf{f}_1 + \cdots + \mathbf{f}_n}{m} - \mathbf{g}
\end{aligned}
$$

$$
\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} \hat{\mathbf{\Theta}} \\ \hat{\mathbf{p}} \\ \hat{\omega} \\ \hat{\dot{\mathbf{p}}} \\ \mathbf{g} \end{bmatrix} = \begin{bmatrix} \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{R}_z(\psi) & \mathbf{0}_3 & \mathbf{0}_{1\times3} \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{I}_3 & \mathbf{0}_{1\times3} \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_{1\times3} \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & [0\ 0\ 1]^T \\ \mathbf{0}_{1\times3} & \mathbf{0}_{1\times3} & \mathbf{0}_{1\times3} & \mathbf{0}_{1\times3} & 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{\Theta}} \\ \hat{\mathbf{p}} \\ \hat{\omega} \\ \hat{\dot{\mathbf{p}}} \\ \mathbf{g} \end{bmatrix}
+ \begin{bmatrix} \mathbf{0}_3 & \cdots & \mathbf{0}_3 \\ \mathbf{0}_3 & \cdots & \mathbf{0}_3 \\ \hat{\mathbf{I}}^{-1}[\mathbf{r}_1]_\times & \cdots & \hat{\mathbf{I}}^{-1}[\mathbf{r}_4]_\times \\ \mathbf{I}_3/m & \cdots & \mathbf{I}_3/m \\ \mathbf{0}_{1\times3} & \cdots & \mathbf{0}_{1\times3} \end{bmatrix} \begin{bmatrix} \mathbf{f}_1 \\ \vdots \\ \mathbf{f}_4 \end{bmatrix}
$$

3 QP Formulation

$$
\boldsymbol{x_k} = \mathbf{A}^k \boldsymbol{x}_0 + \sum_{i=0}^{k-1}\mathbf{A}^{k-1-i}\mathbf{B}\mathbf{u}_i
$$

$$
\begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \vdots \\ \boldsymbol{x}_k \end{bmatrix} = \begin{bmatrix} \mathbf{A}^1 \\ \mathbf{A}^2 \\ \vdots \\ \mathbf{A}^k \end{bmatrix} \boldsymbol{x}_0 + \begin{bmatrix} \mathbf{0} & & & \cdots & \mathbf{0} \\ \mathbf{B} & \mathbf{0} & & \cdots & \mathbf{0} \\ \mathbf{AB} & \mathbf{B} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & & & \ddots & \vdots \\ \mathbf{A}^{k-1}\mathbf{B} & \mathbf{A}^{k-1}\mathbf{B} & \mathbf{0} & \cdots & \mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{u}_0 \\ \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_{k-1} \end{bmatrix}
$$