

# Cheetah MPC

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## 1 Conventions

Notation	
$i$	ith leg
$\mathcal{B}$	left subscript for body coordinate system
$[\mathbf{x}]_{\times} \in \mathbb{R}^{3 \times 3}$	skew-symmetric matrix, $[\mathbf{x}]_{\times} \mathbf{y} = \mathbf{x} \times \mathbf{y}$
$\Theta = [\phi, \theta, \psi]^T$	orientation as ZYX Euler angles
$\mathbf{J}_i \in \mathbb{R}^{3 \times 3}$	foot Jacobain
$\Lambda_i \in \mathbb{R}^{3 \times 3}$	operational space inertia matrix
$\mathbf{K}_p, \mathbf{K}_d \in \mathbb{R}^{3 \times 3}$	diagonal positive definite pd gain matrices

## 2 State Space Model

$$\begin{aligned}
 \frac{d}{dt} \hat{\Theta} &= R_z(\psi) \hat{\Theta} \\
 \frac{d}{dt} \hat{\mathbf{p}} &= \hat{\mathbf{p}} \\
 \frac{d}{dt} \hat{\omega} &= \hat{\mathbf{I}}^{-1} \sum_{i=1}^n \mathbf{r}_i \times \mathbf{f}_i = \hat{\mathbf{I}}^{-1} ([\mathbf{r}_1]_{\times} \mathbf{f}_1 + \dots + [\mathbf{r}_n]_{\times} \mathbf{f}_n) \\
 \frac{d}{dt} \hat{\mathbf{p}} &= \frac{\sum_{i=1}^n \mathbf{f}_i}{m} - \mathbf{g} = \frac{\mathbf{f}_1 + \dots + \mathbf{f}_n}{m} - \mathbf{g}
 \end{aligned}$$

$$\begin{aligned}
 \frac{d}{dt} \begin{bmatrix} \hat{\Theta} \\ \hat{\mathbf{p}} \\ \hat{\omega} \\ \hat{\mathbf{p}} \\ \mathbf{g} \end{bmatrix} &= \begin{bmatrix} \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{R}_z(\psi) & \mathbf{0}_3 & \mathbf{0}_{1 \times 3} \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{I}_3 & \mathbf{0}_{1 \times 3} \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_{1 \times 3} \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & [0 \ 0 \ 1]^T \\ \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & 0 \end{bmatrix} \begin{bmatrix} \hat{\Theta} \\ \hat{\mathbf{p}} \\ \hat{\omega} \\ \hat{\mathbf{p}} \\ \mathbf{g} \end{bmatrix} \\
 &+ \begin{bmatrix} \mathbf{0}_3 & \dots & \mathbf{0}_3 \\ \mathbf{0}_3 & \dots & \mathbf{0}_3 \\ \hat{\mathbf{I}}^{-1} [\mathbf{r}_1]_{\times} & \dots & \hat{\mathbf{I}}^{-1} [\mathbf{r}_4]_{\times} \\ \mathbf{I}_3/m & \dots & \mathbf{I}_3/m \\ \mathbf{0}_{1 \times 3} & \dots & \mathbf{0}_{1 \times 3} \end{bmatrix} \begin{bmatrix} \mathbf{f}_1 \\ \vdots \\ \mathbf{f}_4 \end{bmatrix}
 \end{aligned}$$

## 3 QP Formulation

$$\begin{aligned}
 \mathbf{x}_k &= \mathbf{A}^k \mathbf{x}_0 + \sum_{i=0}^{k-1} \mathbf{A}^{k-1-i} \mathbf{B} \mathbf{u}_i \\
 \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_k \end{bmatrix} &= \begin{bmatrix} \mathbf{A}^1 \\ \mathbf{A}^2 \\ \vdots \\ \mathbf{A}^k \end{bmatrix} \mathbf{x}_0 + \begin{bmatrix} \mathbf{0} & & & & \mathbf{0} \\ \mathbf{B} & \mathbf{0} & & & \mathbf{0} \\ \mathbf{AB} & \mathbf{B} & \mathbf{0} & & \mathbf{0} \\ \vdots & & & \ddots & \vdots \\ \mathbf{A}^{k-1} \mathbf{B} & \mathbf{A}^{k-2} \mathbf{B} & \mathbf{0} & \dots & \mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{u}_0 \\ \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_{k-1} \end{bmatrix}
 \end{aligned}$$