MEMS1082

Chapter 6 Digital Circuit 6-2

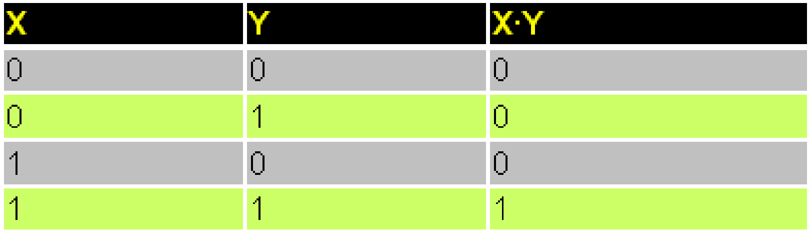
Logic Gates and Boolean Algebra

- Boolean algebra derives its name from the mathematician George Boole. Symbolic Logic uses values, variables and operations :
  - True is represented by the value 1.
  - False is represented by the value 0.
- Variables are represented by letters and can have one of two values, either 0 or 1. Operations are functions of one or more variables.
  - AND is represented by $X \cdot Y$
  - OR is represented by $X + Y$
  - NOT is represented by $\overline{\mathbf{X}}$

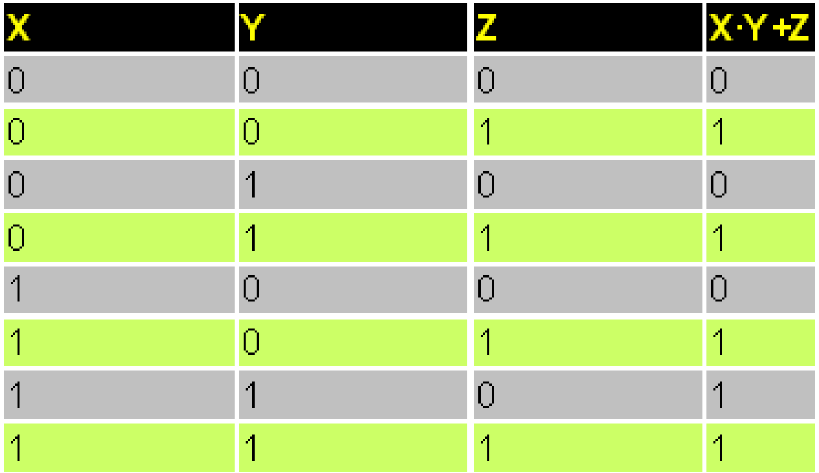
Truth Table

- Truth tables are a means of representing the results of a logic function using a table. They are constructed by defining all possible combinations of the inputs to a function, and then calculating the output for each combination in turn.

AND

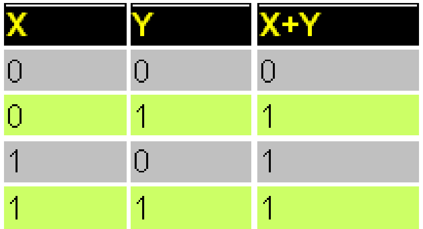


X	Y	X·Y
0	0	0
0	1	0
1	0	0
1	1	1



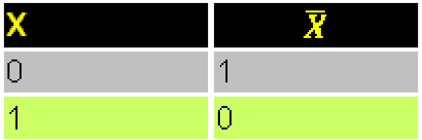
X	Y	Z	X·Y +Z
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

OR



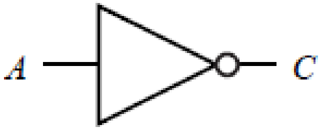
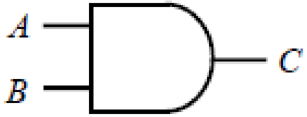
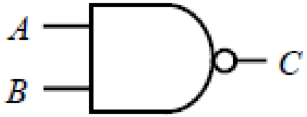
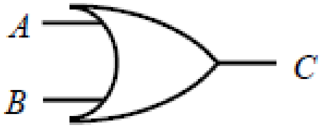
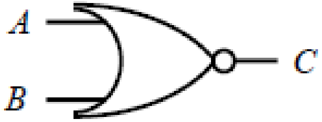
X	Y	X+Y
0	0	0
0	1	1
1	0	1
1	1	1

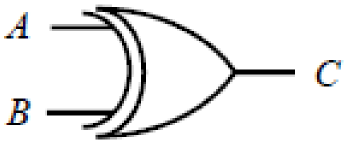
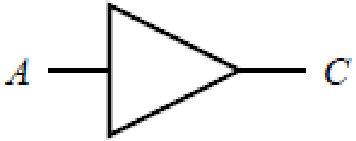
NOT



X	$\overline{X}$
0	1
1	0

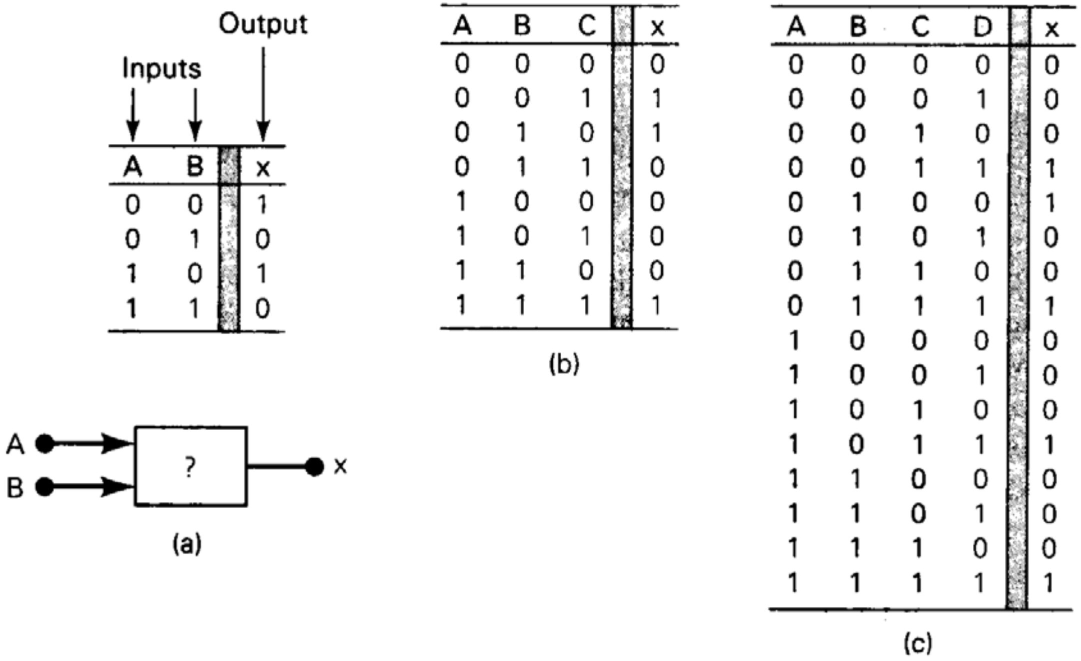
Table 6.3 Combinational logic operations

Gate	Operation	Symbol	Expression	Truth table
Inverter (INV, NOT)	Invert signal (complement)		$C = \overline{A}$	A C 0 1 1 0
AND gate	AND logic		$C = A \cdot B$	A B C 0 0 0 0 1 0 1 0 0 1 1 1
NAND gate	Inverted AND logic		$C = \overline{A \cdot B}$	A B C 0 0 1 0 1 1 1 0 1 1 1 0
OR gate	OR logic		$C = A + B$	A B C 0 0 0 0 1 1 1 0 1 1 1 1
NOR gate	Inverted OR logic		$C = \overline{A + B}$	A B C 0 0 1 0 1 0 1 0 0 1 1 0

XOR gate	Exclusive OR logic		$C = A \oplus B$	A B C 0 0 0 0 1 1 1 0 1 1 1 0
Buffer	Increase output signal current		$C = A$	A C 0 0 1 1

True Tables

Other Examples of truth tables



Output
Inputs
A
B
?
x

A	B	x
0	0	1
0	1	0
1	0	1
1	1	0

(a)

A	B	C	x
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

(b)

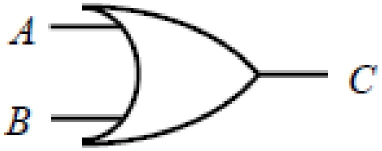
A	B	C	D	x
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

(c)

Example truth tables for (a) two-input, (b) three-input, and (c) four-input circuits.

OR operation

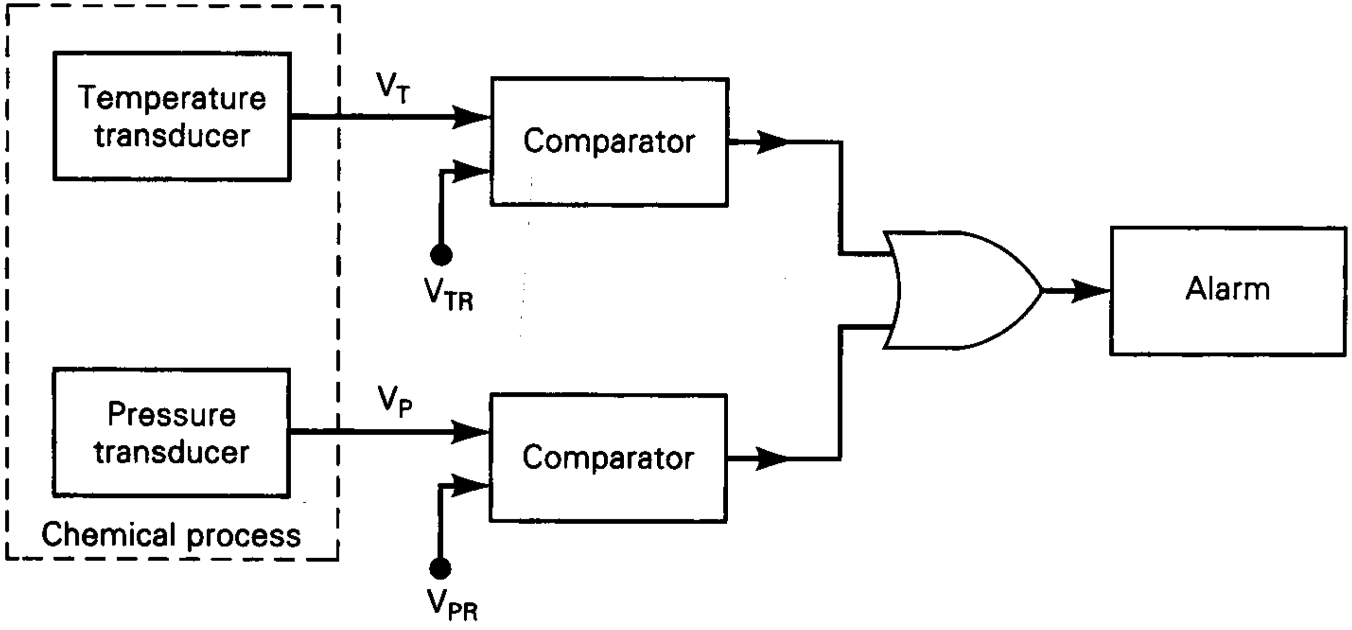
OR gate OR logic



$C = A + B$

A	B	C
0	0	0
0	1	1
1	0	1
1	1	1

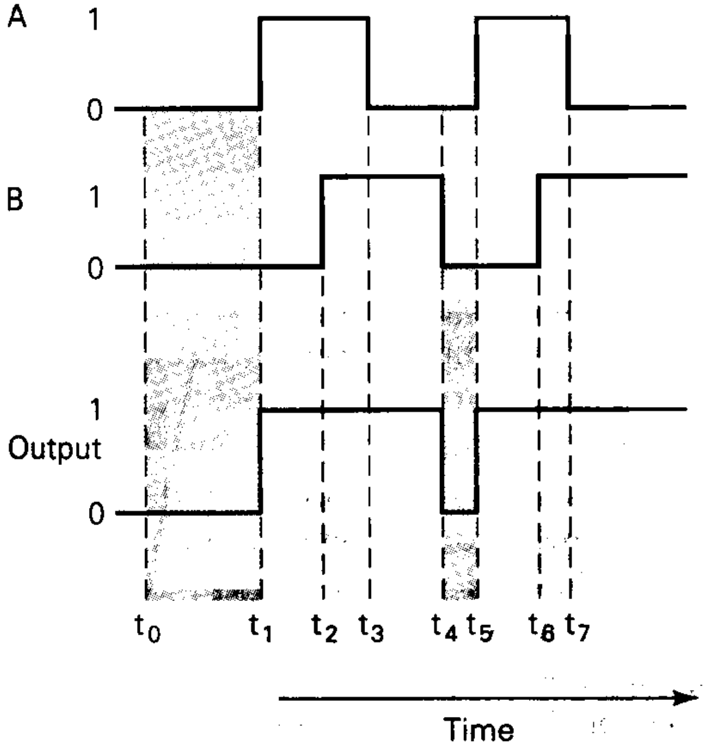
Example of the use of OR gate in alarm system.

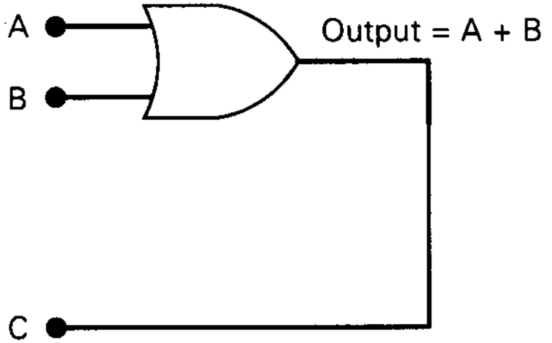


OR operation

EXAMPLE

Determine the OR gate output in Figure 3-5. The OR gate inputs A and B are varying according to the timing diagrams shown. For example, A starts out LOW at time t_0, goes HIGH at t_1, back LOW at t_3, and so on.

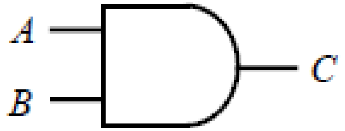




AND operation

For two input

AND gate AND logic

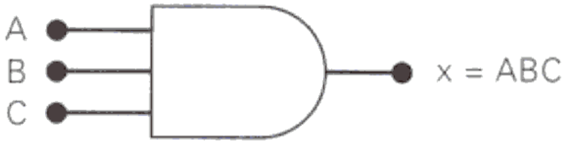


$C = A \cdot B$

A	B	C
0	0	0
0	1	0
1	0	0
1	1	1

For three input

A	B	C	x = ABC
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

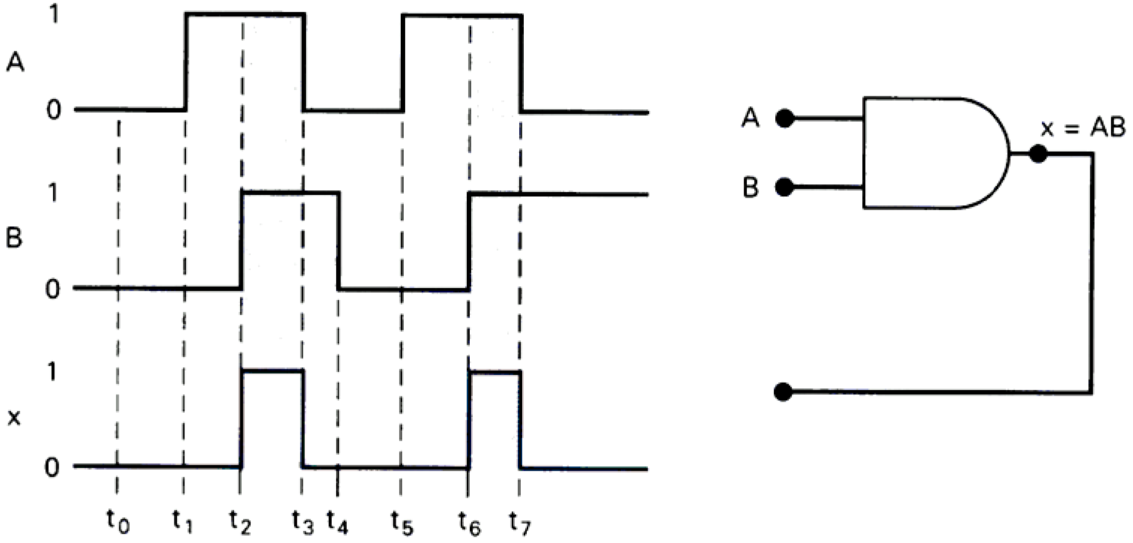


Summary of the AND Operation

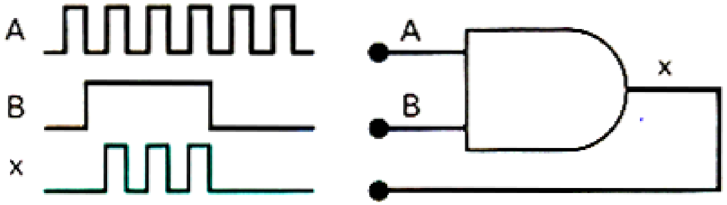
1. The AND operation is performed exactly like ordinary multiplication of 1s and 0s.
2. An output equal to 1 occurs only for the single case where all inputs are 1.
3. The output is 0 for any case where one or more inputs are 0.

AND operation: example

Determine the output *x* from the AND gate in Figure for the given input waveforms.

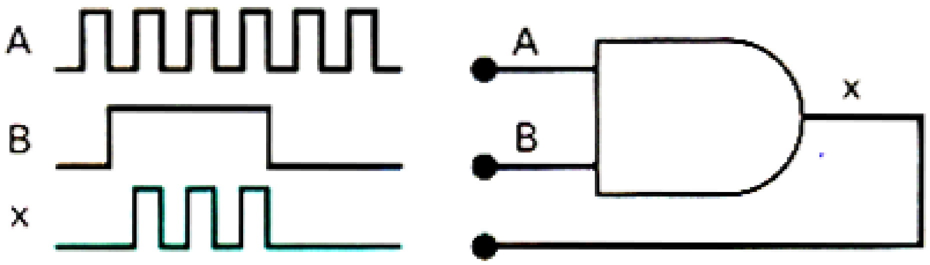


Determine the output waveform for the AND gate

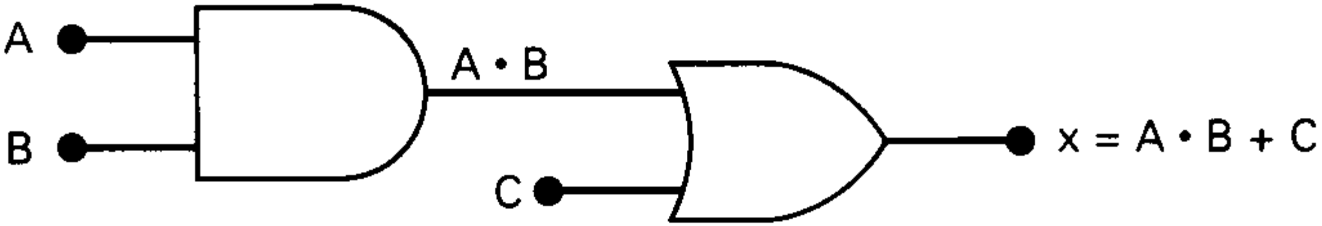


AND operation: example

Determine the output waveform for the AND gate

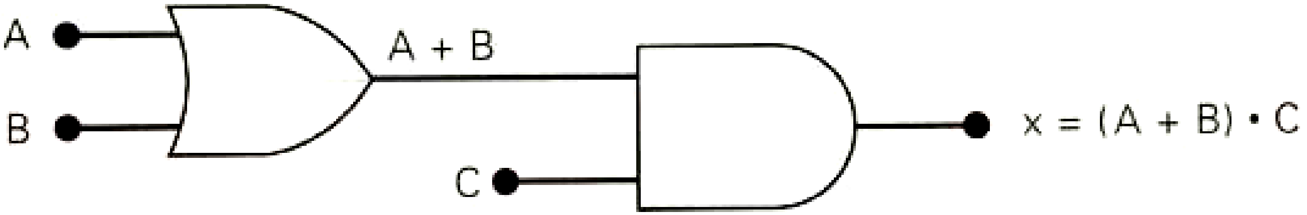


Logic circuit with its Boolean expression.



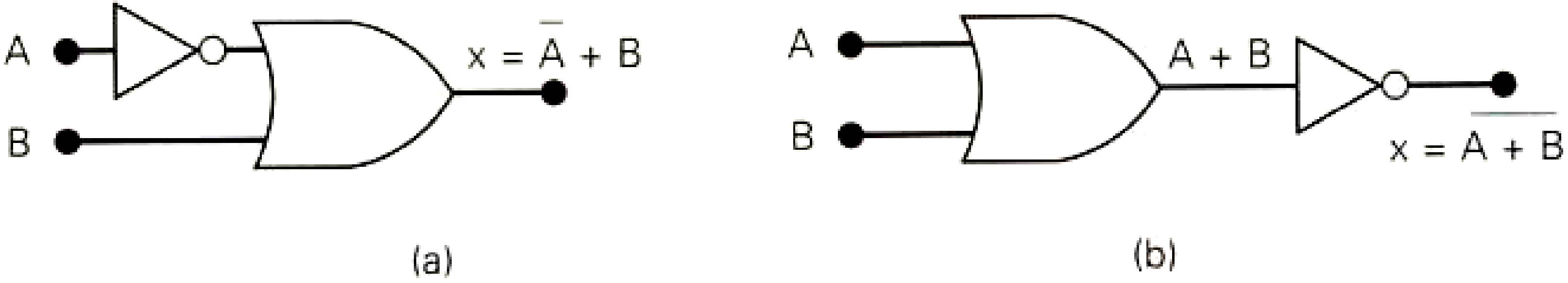
Logic Circuits

,

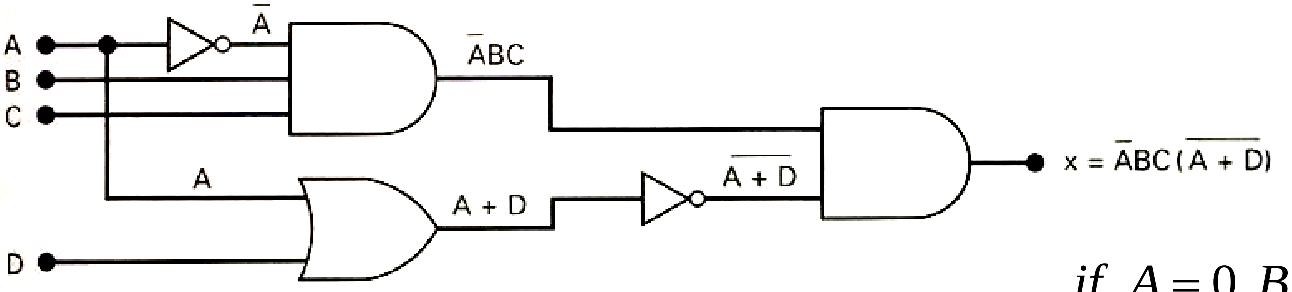


Logic circuit whose expression requires parentheses.

Circuits using INVERTERs.



Logic Circuits

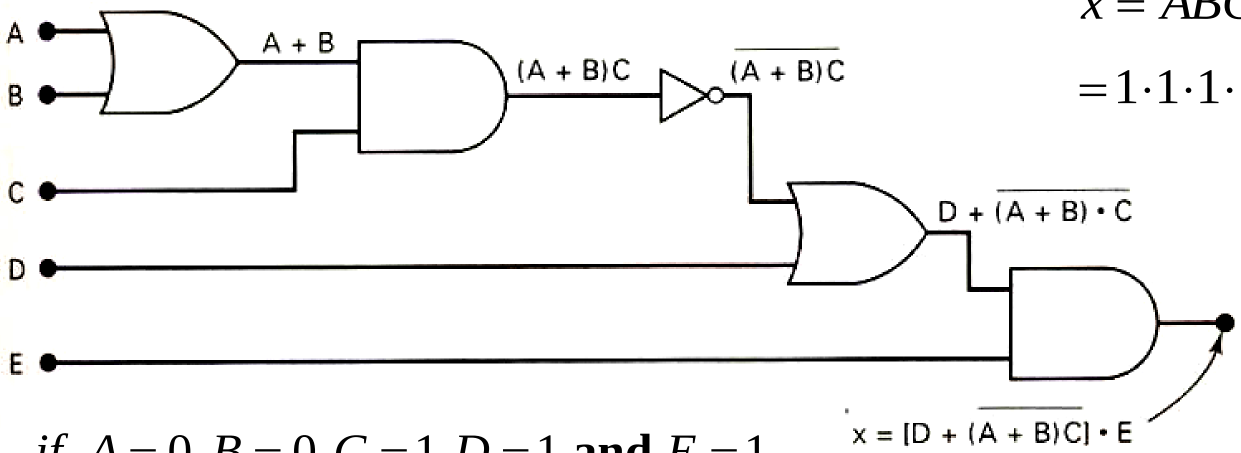


(a)

if $A=0, B=1, C=1, D=1$

$$x = \bar{A}BC\left(\overline{A+D}\right) = \bar{0}\cdot 1\cdot 1\cdot\left(\overline{0+1}\right)$$

$$= 1\cdot 1\cdot 1\cdot\left(\overline{0+1}\right) = 1\cdot 1\cdot 1\cdot 0 = 0$$


A
B
C
D
E
A + B
(A + B)C
$\overline{(A+B)C}$
$D + \overline{(A+B)\cdot C}$
$x = [D + \overline{(A+B)C}]\cdot E$

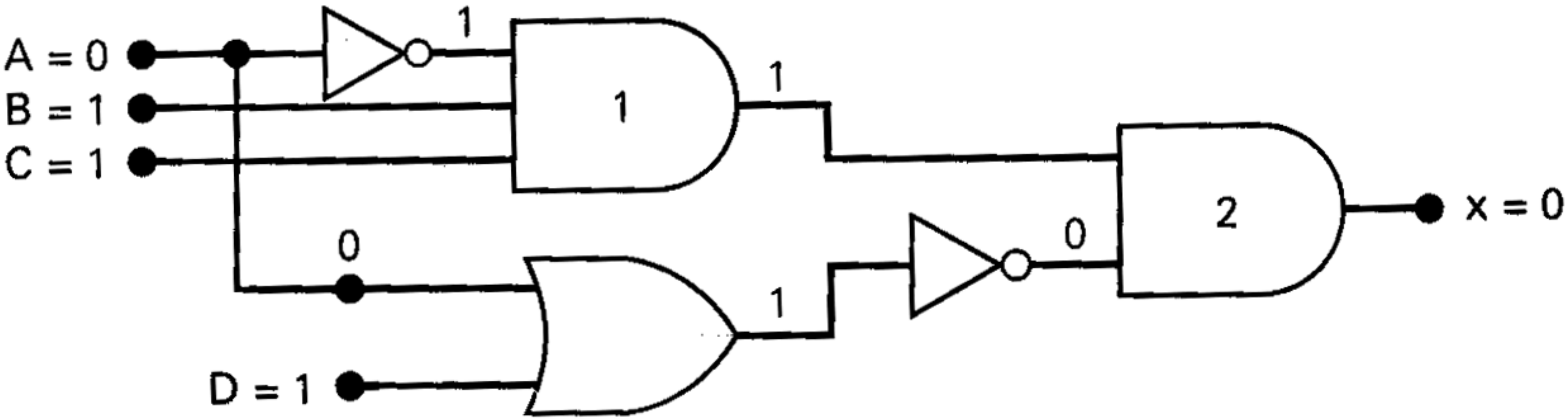
if $A=0, B=0, C=1, D=1,$ **and** $E=1$

$$x = \left[D + \overline{(A+B)C}\right]\cdot E = \left[1 + \overline{(0+0)\cdot 1}\right]\cdot 1$$

$$= \left[1 + \overline{0\cdot 1}\right]\cdot 1 = [1+1]\cdot 1 = 1\cdot 1 = 1$$

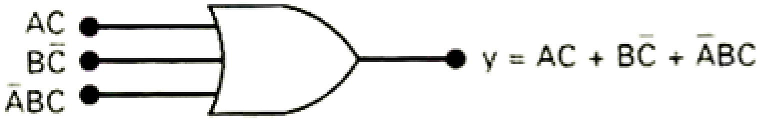
Logic Circuits

Determining output level from circuit diagram.

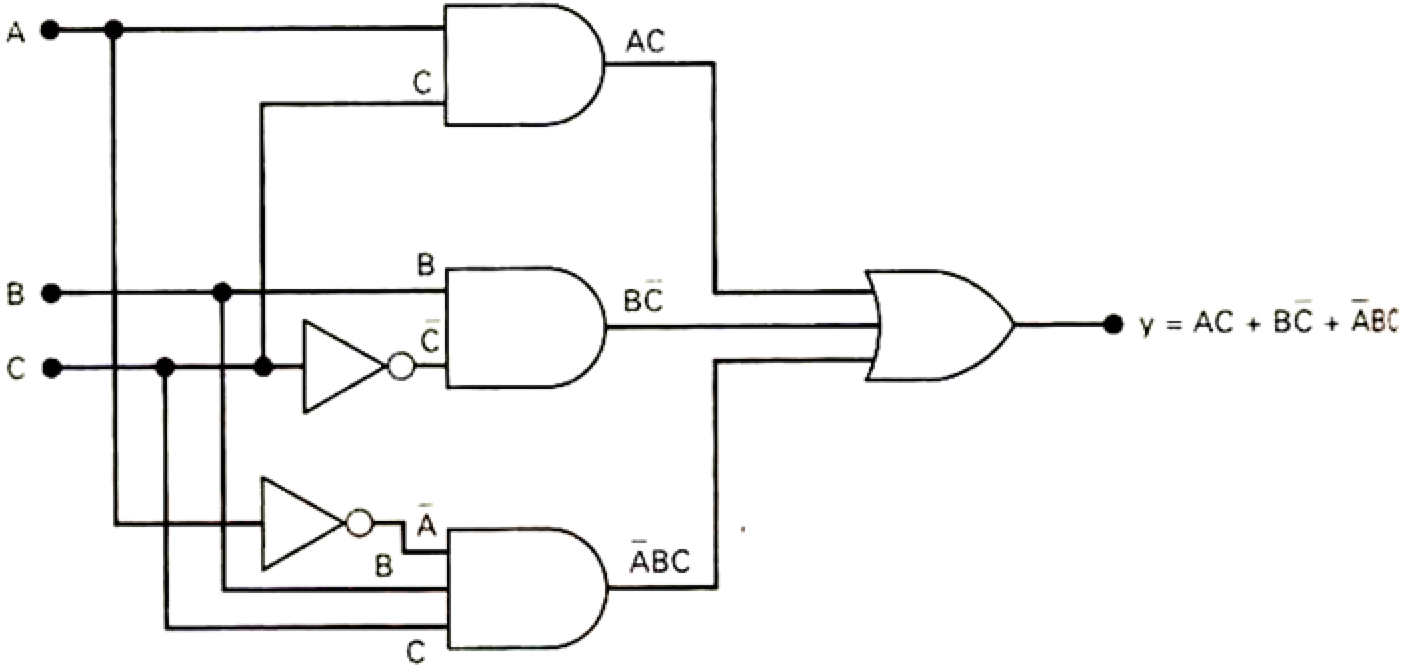


Implementing Logic Circuits from Boolean expressions

$$y = AC + B\overline{C} + \overline{A}BC$$

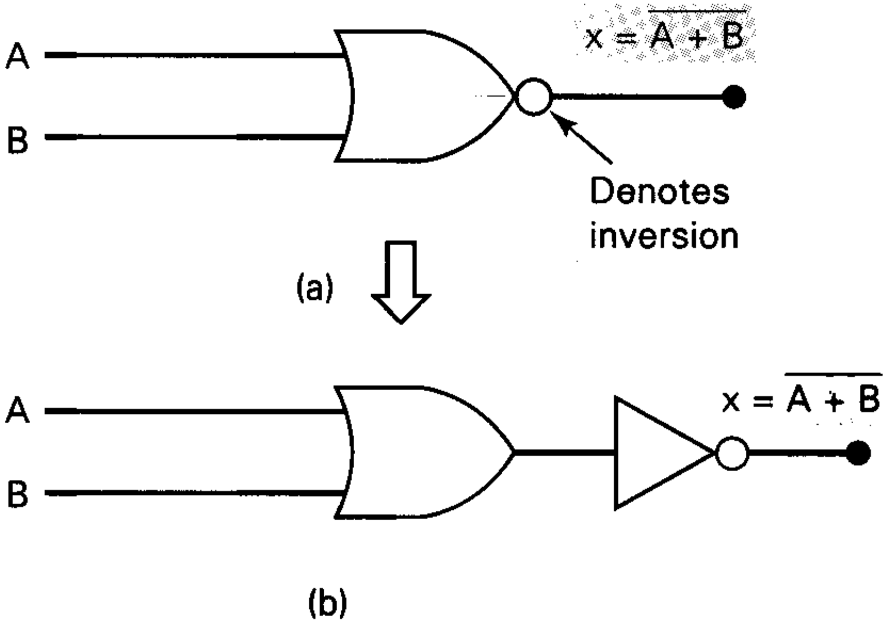


(a)



(b)

NOR Gate



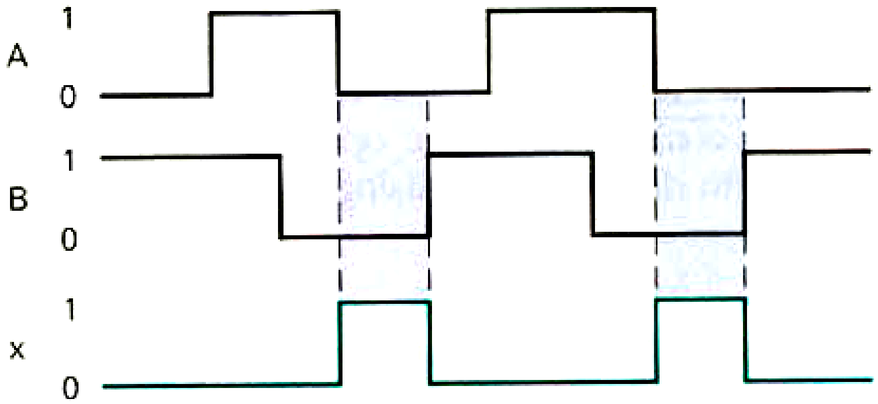
		OR	NOR
A	B	$A + B$	$\overline{A + B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

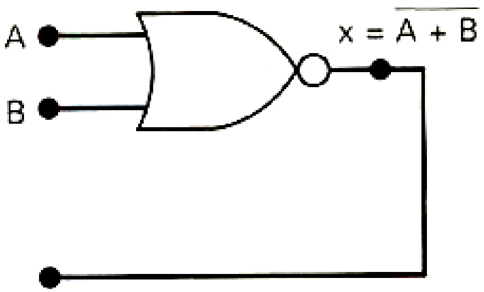
(c)

(a) NOR symbol; (b) equivalent circuit;
(c) truth table.

NOR Gate

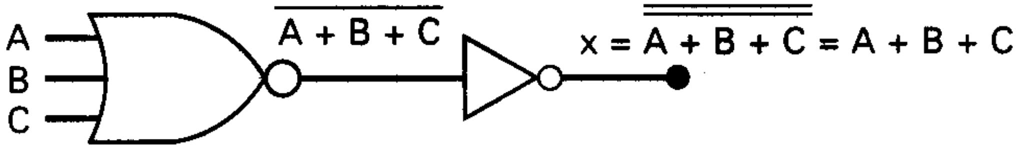
Determine the waveform at the output of a NOR gate for the input waveforms



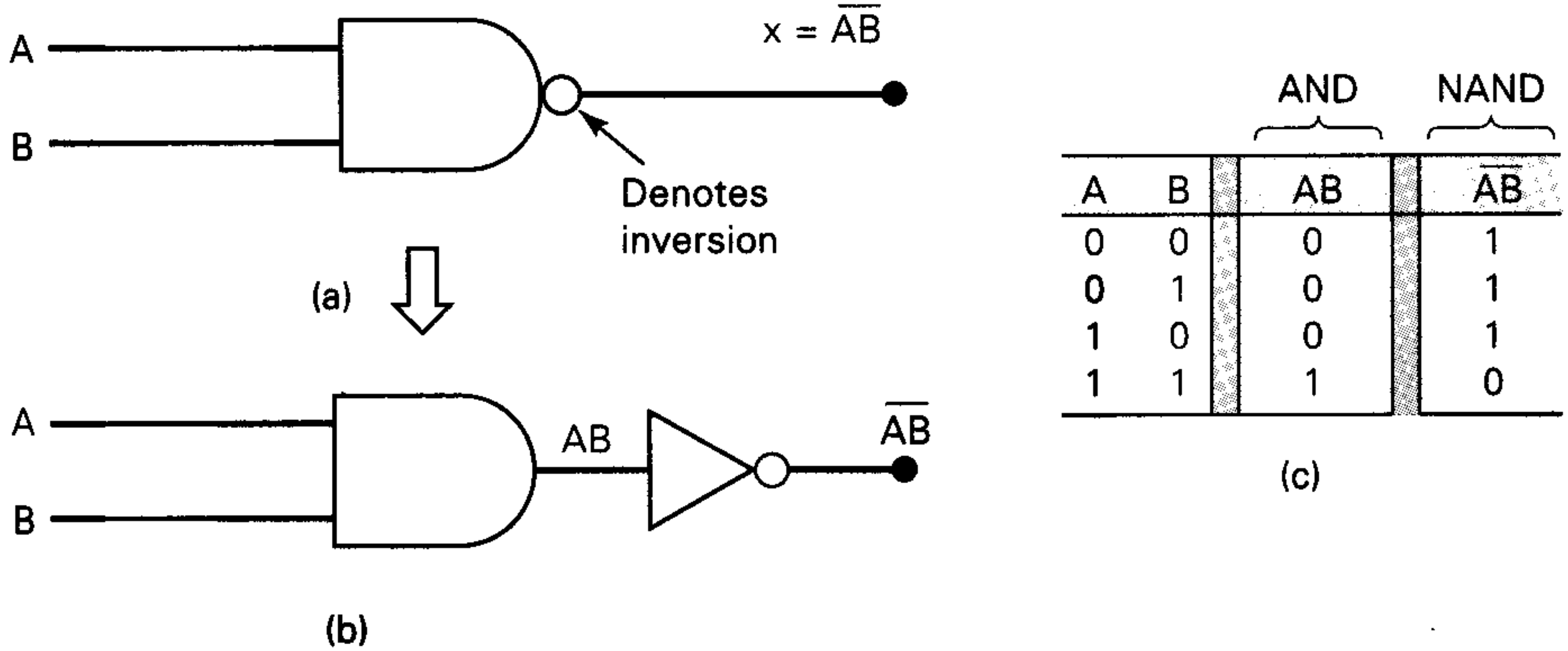


The inputs are all LOW, making the output HIGH

Three inputs NOR gate followed by an INVERTER



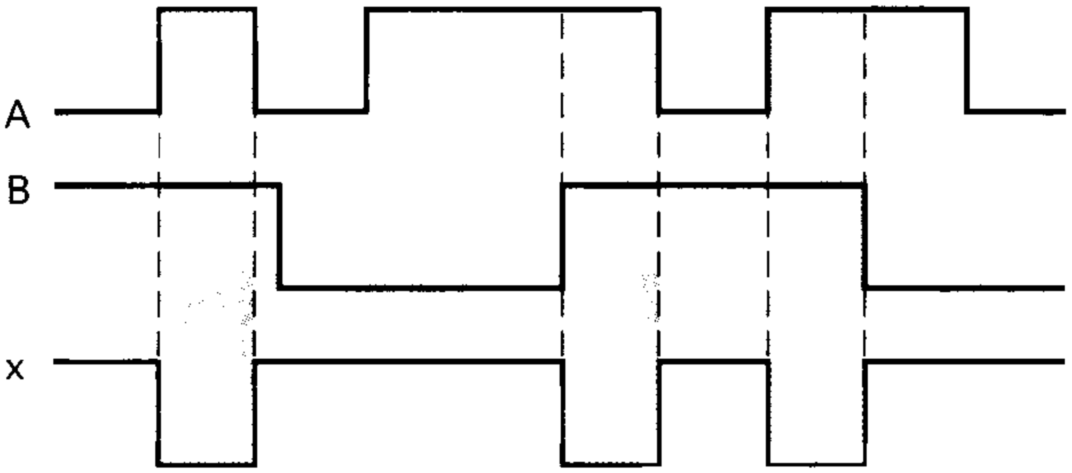
NAND Gate

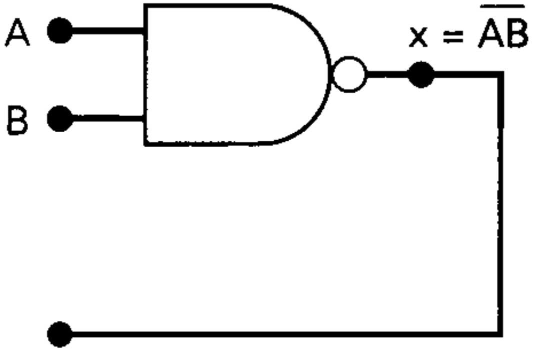


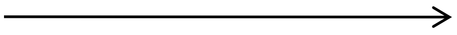
A	B	AB	$\overline{AB}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

(a) NAND symbol; (b) equivalent circuit; (c) truth table.

NAND Gate



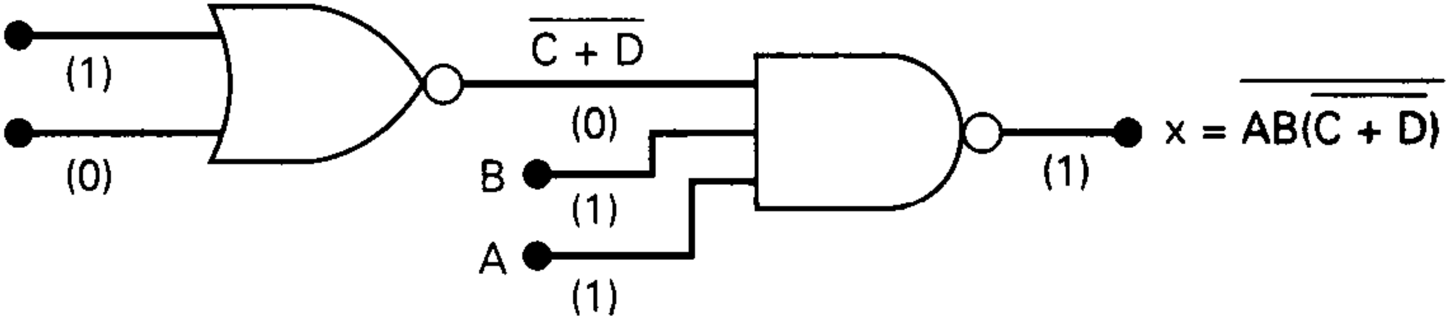




The inputs are all HIGH, making the output LOW

NOR and NAND Gates : Example

Implement the logic circuit that has the expression $x = \overline{AB \cdot (\overline{C + D})}$ using only NOR and NAND gates.



for $A = B = C = 1$ and $D = 0$.

$$
\begin{aligned}
x &= \overline{AB(\overline{C + D})} \\
&= \overline{1 \cdot 1 \cdot (\overline{1 + 0})} \\
&= \overline{1 \cdot 1 \cdot (\overline{1})} \\
&= \overline{1 \cdot 1 \cdot 0} \\
&= \overline{0} = 1
\end{aligned}
$$

Boolean Algebra

Boolean Algebra Laws and Identities

Fundamental Laws

OR	AND	NOT
$A + 0 = A$	$A \cdot 0 = 0$	
$A + 1 = 1$	$A \cdot 1 = A$	
$A + A = A$	$A \cdot A = A$	$\overline{\overline{A}} = A$
$A + \overline{A} = 1$	$A \cdot \overline{A} = 0$	

Commutative Laws

$$A + B = B + A$$

$$A \cdot B = B \cdot A$$

Associative Laws

$$(A + B) + C = A + (B + C)$$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

Distributive Laws

$$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$$

$$A + (B \cdot C) = (A + B) \cdot (A + C)$$

Boolean Algebra

De Morgan's Laws

$$\overline{A + B + C + \cdots} = \bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \cdots$$

$$\overline{A \cdot B \cdot C \cdot \cdots} = \bar{A} + \bar{B} + \bar{C} + \cdots$$

Other Useful Identities

$$A + (A \cdot B) = A$$

$$A \cdot (A + B) = A$$

$$A + (\bar{A} \cdot B) = A + B$$

$$(A + B) \cdot (A + \bar{B}) = A$$

$$(A + B) \cdot (A + C) = A + (B \cdot C)$$

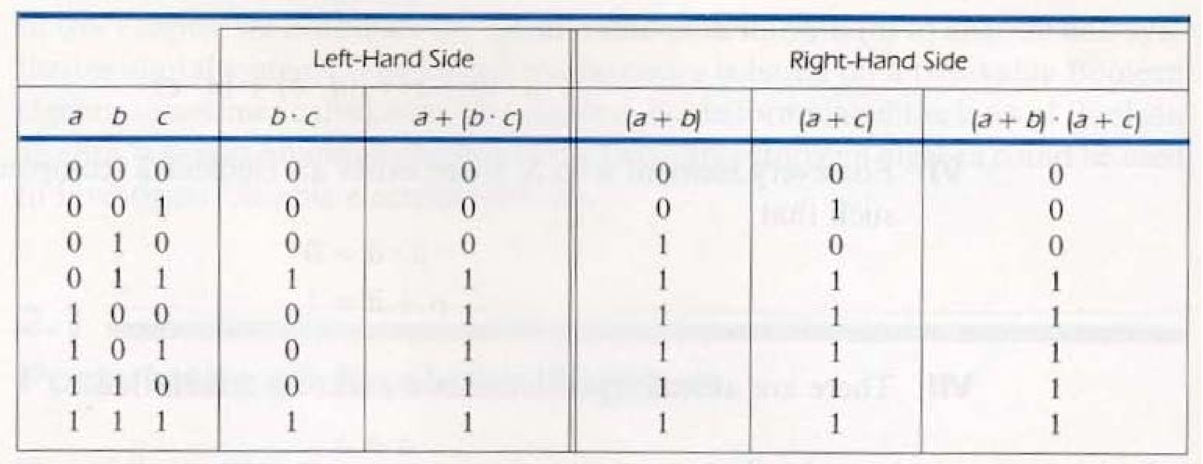
$$A + B + (A \cdot \bar{B}) = A + B$$

$$(A \cdot B) + (B \cdot C) + (\bar{B} \cdot C) = (A \cdot B) + C$$

$$(A \cdot B) + (A \cdot C) + (\bar{B} \cdot C) = (A \cdot B) + (\bar{B} \cdot C)$$

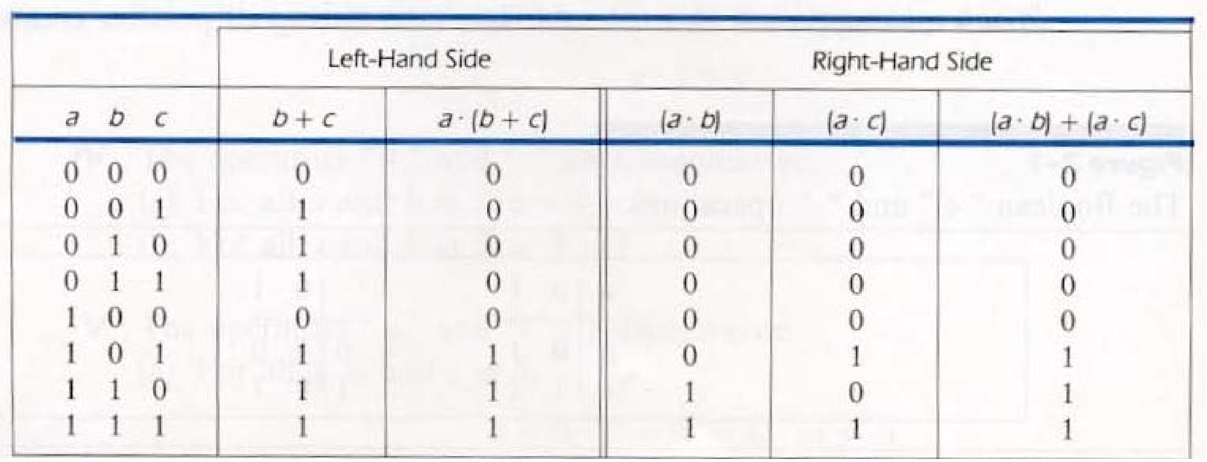
Boolean Algebra

$$a+(b\cdot c)=(a+b)\cdot(a+c)$$



			Left-Hand Side		Right-Hand Side		
a	b	c	$b\cdot c$	$a+(b\cdot c)$	$(a+b)$	$(a+c)$	$(a+b)\cdot(a+c)$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

$$a\cdot(b+c)=(a\cdot b)+(a\cdot c)$$



			Left-Hand Side		Right-Hand Side		
a	b	c	$b+c$	$a\cdot(b+c)$	$(a\cdot b)$	$(a\cdot c)$	$(a\cdot b)+(a\cdot c)$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

Examples using DeMorgan’s Law

$$\overline{A+B}=\overline{A}\cdot\overline{B} \qquad \overline{AB}=\overline{A}+\overline{B}$$

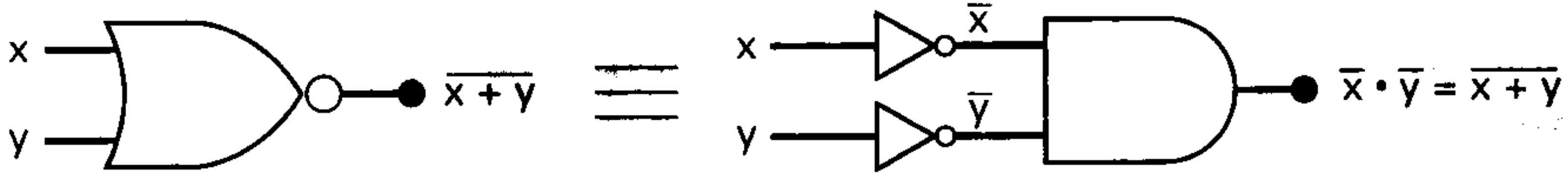
1. $$\begin{aligned} z &= \overline{A+\overline{B}\cdot C} \\ &= \overline{A}\cdot(\overline{\overline{B}\cdot C}) \\ &= \overline{A}\cdot(\overline{\overline{B}}+\overline{C}) \\ &= \overline{A}\cdot(B+\overline{C}) \end{aligned}$$

2. $$\begin{aligned} \omega &= \overline{(A+BC)\cdot(D+EF)} \\ &= (\overline{A+BC})+(\overline{D+EF}) \\ &= (\overline{A}\cdot\overline{BC})+(\overline{D}\cdot\overline{EF}) \\ &= [\overline{A}\cdot(\overline{B}+\overline{C})]+[\overline{D}\cdot(\overline{E}+\overline{F})] \\ &= \overline{A}\,\overline{B}+\overline{A}\,\overline{C}+\overline{D}\,\overline{E}+\overline{D}\,\overline{F} \end{aligned}$$

NOR Gate

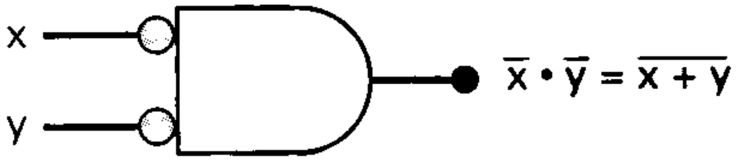
(a) Equivalent circuits

(b) alternative symbol for the NOR function.



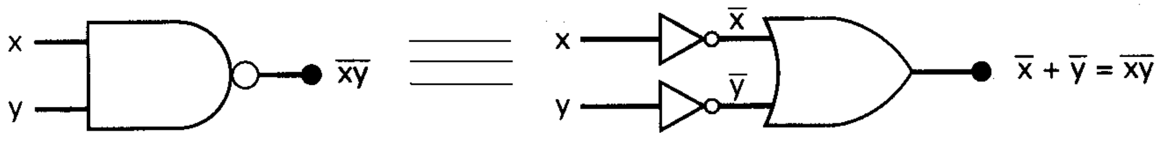
(a)

$$\overline{A+B} = \overline{A} \cdot \overline{B}$$

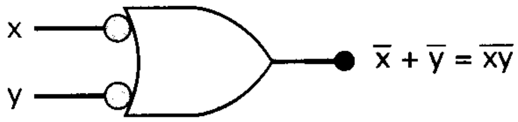


(b)

NAND Gate



(a)

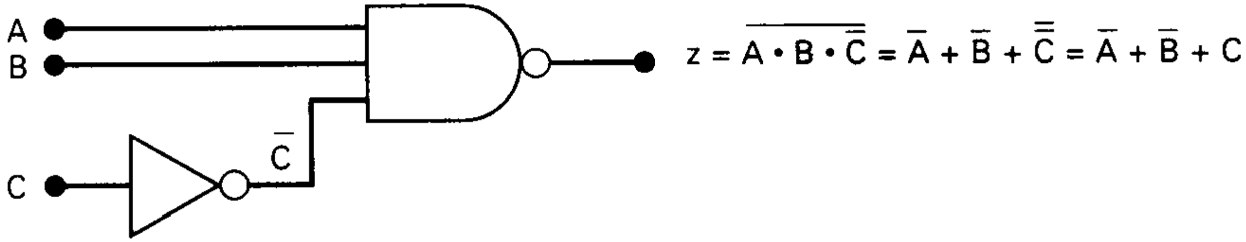


(b)

$$\overline{A+B} = \overline{A} \cdot \overline{B}$$

(a) Equivalent circuits
(b) alternative symbol for the NAND function.

example


A
B
C
$\overline{C}$
$z = \overline{A \cdot B \cdot \overline{C}} = \overline{A} + \overline{B} + \overline{\overline{C}} = \overline{A} + \overline{B} + C$