ІПС-22

Гончаренко Ліза

$i = 32$

① $(x - 2iy)dx + (x - 2iy)dy = 0 \quad i > 1$

$(x - 32y)dx + (x - 64y)dy = 0$

$(x - 2iy)dy = (iy - x)dx$ Рівняння однорідне якщо $M(kx, ky) = k^n M(x, y)$

$kx - 2ik^2 y = iky - kx \to k$

Заміна $u = \frac{y}{x} \qquad y = ux$

$dy = u\,dx + x\,du$

$(1 - 2iu)x(u\,dx + x\,du) = (iu - 1)x\,dx \quad | :x$

$(1 - 2iu)(u\,dx + x\,du) = (iu - 1)dx$

$(x - 2iux)du = (2iu^2 + iu - u - 1)dx$

$(1 - 2iu)x\,du = (2iu^2 + iu - u - 1)dx \quad \Big| \begin{matrix} x \\ 2iu^2 + iu + iu - u - 1 \end{matrix}$

$= \frac{(2iu - 1)du}{2iu^2 + (i - 1)u - 1} = \frac{dx}{x} \quad | \int$

$\int \frac{2iu - 1}{2iu^2 + (i - 1)u - 1}\,du = \int \frac{1}{x}\,dx$

$\int - \frac{\ln|2iu^2 + (i-1)u - 1|}{2} - \frac{(2i - 2)\arctan\left(\frac{4\sqrt{2}iu + \sqrt{2}i + \sqrt{2}}{2\cdot\sqrt[4]{-1} + \sqrt{3}}\right)}{\sqrt[4]{-1}\sqrt{2}\sqrt{3}} = \ln x + C$

$- \frac{\ln\left(\frac{2iy^2}{x^2} + \frac{(i-1)y}{x} - 1\right)}{2} - \frac{(2i - 2)\arctan\left(\frac{4\sqrt{2}\frac{y}{x} + \sqrt{2}i + \sqrt{2}}{2\cdot\sqrt[4]{-1} + \sqrt{3}}\right)}{\sqrt[4]{-1}\sqrt{2}\sqrt{3}} = \ln x + C$

$- \frac{\ln(2iy^2 + (i-1)xy - x^2)}{2} - \frac{i\arctan\left(\frac{4\sqrt{2}y + (\sqrt{2}i + \sqrt{2})x}{2\cdot\sqrt[4]{-1}\sqrt{3}x}\right)}{\sqrt[4]{-1}\sqrt{2}\sqrt{3}} + \frac{\arctan\left(\frac{-4\sqrt{2}y + (\sqrt{2}i + \sqrt{2})x}{2\sqrt[4]{-1}\sqrt{3}x}\right)}{\sqrt[4]{-1}\sqrt{2}\sqrt{3}} = C$

$- \frac{\ln(64y^2 + 31xy - x^2)}{2} - 32\arctan\left(\frac{4\sqrt{2}y(8 + \sqrt{2})x}{2\cdot\sqrt[4]{-1} + \sqrt{3}x}\right) + \frac{\arctan\left(\frac{-4\sqrt{2}y + (8 + \sqrt{2})x}{2\sqrt[4]{-1}\sqrt{3}y}\right)}{\sqrt[4]{-1}\sqrt{2}\sqrt{3}} = C$

②

$$x y' - y = i \tan \frac{y}{x}$$

$$x y' = i \tan\left(\frac{y}{x}\right) + y$$

$$\frac{x\,dy}{dx} = i \tan\left(\frac{y}{x}\right) + y \quad | \cdot dx$$

$$x\,dy = \left(i \tan\left(\frac{y}{x}\right) + y\right) dx \quad | \cdot x^2$$

$$\frac{dy}{x} + \left(-\frac{i \tan\frac{y}{x}}{x^2} - \frac{y}{x^2}\right) dx = 0$$

$$d\left(\frac{y}{x}\right) - \frac{i \tan\frac{y}{x}\,dx}{x^2 \cdot x} = 0$$

$$\frac{y\,dx - x\,dy}{y^2} = d\left(\frac{x}{y}\right)$$

замена $u = \frac{y}{x}$ $\quad du = d\left(\frac{y}{x}\right)$

$$du - \frac{i \tan(u)\,dx}{x^2} = 0$$

$$du = \frac{i \tan(u)\,dx}{x^2} \quad | \cdot \tan u$$

$$\cot(u)\,du = \frac{i\,dx}{x^2} \quad | \int$$

$$\int \cot(u)\,du = \int \frac{i}{x^2}\,dx$$

$$\ln|\sin(u)| = C - \frac{i}{x}$$

$$\ln\left|\sin\left(\frac{y}{x}\right)\right| = C - \frac{i}{x}, \quad y = 0$$

$$\ln\left|\sin\left(\frac{y}{x}\right)\right| = C - \frac{3i}{x}$$

(4)

$y = 2xy' - i\ln y'$

Зу вимагаємо параметризацію

$y = 2px - i\ln p$

$dy = 2p dx + 2x dp - \frac{i dp}{p}$ заміна $dy = p dx$

$p dx = 2p dx + 2x dp - \frac{i dp}{p}$

$p dx = 2p dx + 2x dp - \frac{i dp}{p}$

$-p dx = \left(2x - \frac{i}{p}\right) dp$

перехід до однорідного

заміна $x = \frac{1}{z}$, $dx = -\frac{dz}{z^2}$

заміна $p = z$

$p \frac{p dz}{z^2} = \left(\frac{2}{z} - \frac{i}{p}\right) dp$

$-2z^2 dz = \left(2z^2 - \frac{i}{z}\right) dz$

$\alpha = -1$

функція $M(z,p)$ однорідного виду

$M(kz, kp) = k^n M(z,p)$

тобто: $\frac{p}{kz^2} = \frac{2}{kz} - \frac{i}{kp} \to \frac{1}{k}$

заміна $u = \frac{z}{p}$, $z = pu$

$dz = p du + u dp$

$\frac{p du + u dp}{p u^2} = \frac{\left(\frac{2}{u} - i\right) dp}{p} \Big| \cdot pu^2$

$p du + u dp = \left(\frac{2}{u} - i\right) dp$

$p du = (u - iu^2) dp$ $\Big| : p(u - iu^2)$

$\frac{du}{iu^2 - u} = \frac{dp}{p} \Big| \int$

$\int -\frac{1}{iu^2 - u} du = \int \frac{1}{p} dp$

$\ln\left(\frac{u}{iu - 1}\right) = \ln(p) + C$

$e^{\ln x} = x$

$\frac{u}{iu - 1} = e^C p$

$u = \frac{z}{p}$

$\frac{z}{p\left(\frac{iz}{p} - 1\right)} = Cp$

$\frac{1}{p\left(\frac{i}{p x} - 1\right)x} = Cp$ $z = \frac{1}{i}$

$x = \frac{i}{p} + \frac{C}{p^2}$

$p = \frac{\pm\sqrt{4Cx - 1}}{2x} + \frac{i}{2x}$

$y = i\ln\left(\frac{\pm\sqrt{4Cx - 1} + i}{2x}\right) \pm \sqrt{4Cx - 1} + i$

$y = i\ln\left(\pm\sqrt{4Cx - 1} + i\right) + i\ln(x) \pm \sqrt{4Cx - 1} + i\ln 2 + i$

$y = 32\ln\left(\pm\sqrt{4Cx - 1} + 32\right) + 32\ln(x) \pm \sqrt{4Cx - 1} + 3 + 32\cdot\ln 2 +$

$+ 32$

(8)

$(xy^2+2ixy)dy+iy^2dx=0 \quad |:y$

$2ixdy+y(xdy+idx)=0 \qquad y\to y=0$

$(xy+2ix)dy=-iydx$

$x(y+2i)dy=-iydx \quad |\frac{x}{:y}$

$\frac{(y-2i)dy}{y}=-\frac{idx}{x} \quad |\int \qquad x\to x=0$

$\int\frac{y+2i}{y}dy=\int-\frac{i}{x}dx$

$2i\ln(y)+y=C-i\ln(x)$

$2i\ln(y)+y=C-i\ln x, \quad y=0, x=0$

$64\ln(y)+y=C-32\ln x, \quad y=0, x\neq 0$