CCCN221 – Computer Architecture Lab 8 – Floating points in MIPS

In this lab, we will go through Floating-Point Number Representation (IEEE 754 Standard), have the basic understanding of MIPS Floating-Point Unit. Will write down programs using the MIPS Floating-Point Instructions that will have the input and output as the floating point numbers.

Floating-Point Number Representation

Floating-point numbers have been defined as follows

S	E = Exponent	F = Fraction
---	--------------	--------------

The Sign bit **S** is zero (positive) or one (negative).

For single-precision the Exponent field **E** has 8 bits and for double-precision, 11 bits. The exponent field is biased. The Bias is 127 for single-precision and 1023 for double-precision.

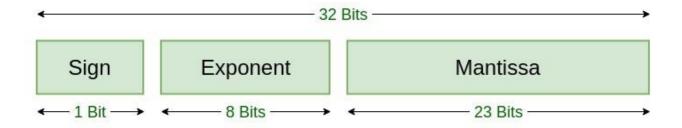
The Fraction field **F** is 23 bits for single-precision and 52 bits for double-precision. Floating-point numbers are normalized (except when **E** is zero). There is an implicit **1.** (not stored) before the fraction **F**. Therefore, the value of a normalized floating-point number is:

Value =
$$\square$$
 (1.F)₂ × 2 ^{E - Bias}

The QTSPIM simulator has a floating-point representation tool that illustrates single-precision floating-point numbers. The figure 1 shows the floating point representation.

Now use the tool to check the binary format and the decimal value of floating-point numbers.

Similarly, the 32-bit representation of: -2.7531 is 1 10000000 01100000011001011001010.



Single Precision IEEE 754 Floating-Point Standard

Figure 9.1: Floating-Point Representation

MIPS Floating-Point Registers

The floating-point unit (called coprocessor 1) has 32 floating-point registers. These registers are numbered as \$f0, \$f1, ..., \$f31. Each register is 32 bits wide. Thus, each register can hold one single-precision floating-point number. How can we use these registers to store 64-bit double-precision floating-point numbers? The answer is that the 32 single-precision registers are grouped into 16 double-precision registers. The double-precision number is stored in an even-odd pair of registers, but we only refer to the even-numbered register. For example, when we store a double-precision number in \$f0, it is actually stored in registers \$f0 and \$f1.

In addition, there are 8 condition flags, numbered from 0 to 7. These condition flags are used by floating-point compare and branch instructions. These are shown in Figure 9.2.

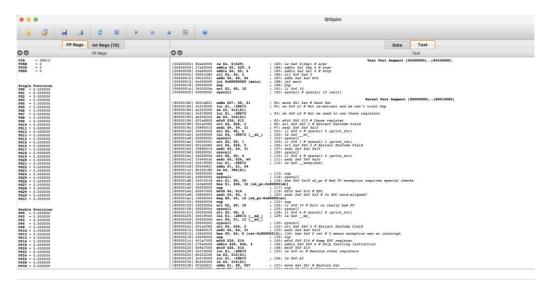


Figure 9.2: QTSPIM Floating-Point Registers and Condition Flags

MIPS Floating-Point Instructions

The FPU supports several instructions including floating-point load and store, floating-point arithmetic operations, floating-point data movement instructions, convert, and branch instructions. We start this section with the floating-point load and store instructions. These instructions load into or store a floating-point register. However, they use the same base-displacement addressing mode used with integer instructions. Notice that the base address register is an integer (not a floating-point) register.

Instruction	Example	Meaning
lwc1 or l.s	lwc1 \$f1,0(\$sp)	Load a word from memory to a single-precision floating-point register: \$f1 = MEM[\$sp]
ldc1 or l.d	ldc1 \$f2,8(\$t1)	Load a double word from memory to a double- precision register: \$f2 = MEM[\$t1+8]

Instruction	Example	Meaning
swc1 or s.s	swc1 \$f5,4(\$t2)	Store a single-precision floating-point register in memory: MEM[\$t2+4] = \$f5
sdc1 or s.d	sdc1 \$f6,16(\$t3)	Store a double-precision floating-point register in memory: MEM[\$t3+16] = \$f6

The floating-point arithmetic instructions are listed next. The $\bf \cdot s$ extension is used for single-precision arithmetic instructions, while the $\bf \cdot d$ is used for double-precision instructions.

Instruction	Example	Meaning	
add.s	add.s \$f0,\$f2,\$f4	\$f0 = \$f2 + \$f4 (single-precision)	
add.d	add.d \$f0,\$f2,\$f4	\$f0 = \$f2 + \$f4 (double-precision)	
sub.s	sub.s \$f0,\$f2,\$f4	\$f0 = \$f2 - \$f4 (single-precision)	
sub.d	sub.d \$f0,\$f2,\$f4	f0 = f2 - f4 (double-precision)	
mul.s	mul.s \$f0,\$f2,\$f4	$$f0 = $f2 \times $f4 \text{ (single-precision)}$	
mul.d	mul.d \$f0,\$f2,\$f4	$\$f0 = \$f2 \times \$f4 $ (double-precision)	
div.s	div.s \$f0,\$f2,\$f4	\$f0 = \$f2 / \$f4 (single-precision)	
div.d	div.d \$f0,\$f2,\$f4	\$f0 = \$f2 / \$f4 (double-precision)	
sqrt.s	sqrt.s \$f0, \$f2	Square root (single-precision)	
sqrt.d	sqrt.d \$f0, \$f2	Square root (double-precision)	
abs.s	abs.s \$f0, \$f2	Absolute value (single-precision)	
abs.d	abs.d \$f0, \$f2	Absolute value (double-precision)	
neg.s	neg.s \$f0, \$f2	Negative value (single-precision)	
neg.d	neg.d \$f0, \$f2	Negative value (double-precision)	

The data movement instructions move data between general-purpose and floating-point registers, or between floating-point registers.

Instruction	Example	Meaning
mfc1	mfc1 \$t0, \$f2	Move data from a floating-point register to a general-purpose register.
mtc1	mfc1 \$t0, \$f2	Move data from a general-purpose register to a floating-point register.
mov.s	mov.s \$f0, \$f1	Move single-precision data between two floating-point registers.
mov.d	mov.d\$f0, \$f2	Move double-precision data between two floating- point registers (move even-odd pair of registers).

The convert instructions convert the format of data in floating-point registers. Three data formats are supported: $\bullet s = \text{single-precision float}$, $\bullet d = \text{double-precision}$, and $\bullet w = \text{integer word}$.

Instruction	Example	Meaning
cvt.s.w	cvt.s.w \$f0,\$f2	\$f0 = convert \$f2 from word to single-precision
cvt.s.d	cvt.s.d \$f0,\$f2	\$f0 = convert \$f2 from double to single-precision
cvt.d.w	cvt.d.w \$f0,\$f2	\$f0 = convert \$f2 from word to double-precision
cvt.d.s	cvt.d.s \$f0,\$f2	\$f0 = convert \$f2 from single to double-precision
cvt.w.s	cvt.w.s \$f0,\$f2	\$f0 = convert \$f2 from single-precision to word
cvt.w.d	cvt.w.d \$f0,\$f2	\$f0 = convert \$f2 from double-precision to word
ceil.w.s	ceil.w.s \$f0,\$f2	\$f0 = Integer ceiling of single-precision float in \$f2
ceil.w.d	ceil.w.d \$f0,\$f2	\$f0 = Integer ceiling of double-precision float in \$f2
floor.w.s	floor.w.s \$f0,\$f2	\$f0 = Integer floor of single-precision float in \$f2
floor.w.d	floor.w.d \$f0,\$f2	\$f0 = Integer floor of double-precision float in \$f2
trunc.w.s	trunc.w.s \$f0,\$f2	\$f0 = Truncate single-precision float in \$f2
trunc.w.d	trunc.w.d \$f0,\$f2	\$f0 = Truncate double-precision float in \$f2

The floating-point compare instructions compare floating-point registers for equality, less than, and less than or equal. The FP compare instructions set the condition flags **0** to **7** to true (1) or false(0).

Instruction	Example	Meaning
c.eq.s	c.eq.s \$f2,\$f3	if $(\$f2 == \$f3)$ set flag 0 to true else false
c.eq.d	c.eq.s 3,\$f4,\$f6	Compare equal double-precision. Result in flag 3
c.lt.s	c.eq.s 4,\$f5,\$f8	if (\$f5 < \$f8) set flag 4 to true else false
c.lt.d	c.lt.d 7,\$f4,\$f6	Compare less-than double. Result in flag 7
c.le.s	c.le.s \$f10,\$f11	if (\$f10 <= \$f11) set flag 0 to true else false
c.le.d	c.le.d \$f14,\$f16	Compare less or equal double. Result in flag 0

The floating-point branch instructions (**bc1t** and **bc1f**) branch to the target address based on the value of the specified condition flag (true or false).

Instruction	Example	Meaning
bc1t	bc1t label	Branch to label if condition flag 0 is true
bc1t	bc1t 1, label	Branch to label if condition flag 1 is true
bc1f	bc1f label	Branch to label if condition flag 0 is false
bc1f	bc1f 4, label	Branch to label if condition flag 4 is false

System Call Services for Floating-Point Numbers

The MARS tool provides the following **Syscall** service numbers (passed in \$v0) to print and read single-precision and double-precision floating-point numbers:

Service	\$v0	Arguments	Result
Print Float	2	\$f12 = float to print	
Print Double	3	\$f12 = double to print	
Read Float	6		Float is returned in \$ f 0
Read Double	7		Double is returned in \$f0

MIPS Floating-Point Register Usage Convention

Compilers follow the MIPS register usage convention when translating functions and procedures into MIPS assembly-language code. The following table shows the MIPS software convention for floating-point registers. Not following the MIPS software usage convention can result in serious bugs when passing parameters, getting results, or using registers across function calls.

Registers	Usage	
\$f0 - \$f3	Floating-point procedure results	
\$f4 - \$f11	Temporary floating-point registers, NOT preserved across procedure calls	
\$f12 - \$f15	Floating-point parameters, NOT preserved across procedure calls. Additional floating-point parameters should be pushed on the stack.	
\$f16 - \$f19	More temporary registers, NOT preserved across procedure calls.	
\$f20 - \$f31	Saved floating-point registers. Should be preserved across procedure calls.	

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Lab Tasks

Task 1

1. Convert by hand the number **-123456789** into its 32-bit single-precision binary representation, and Show your work for a full mark.

```
First. We start with the positive version of the number |-123456789| = 123456789
Second. We divide the number repeatedly by 2
division = quotient + remainder;
123\ 456\ 789 \div 2 = 61\ 728\ 394 + 1;
61728394 \div 2 = 30864197 + 0;
30\ 864\ 197 \div 2 = 15\ 432\ 098 + 1;
15\ 432\ 098 \div 2 = 7\ 716\ 049 + 0;
7716049 \div 2 = 3858024 + 1;
3858024 \div 2 = 1929012 + 0;
1929012 \div 2 = 964506 + 0;
964\ 506 \div 2 = 482\ 253 + 0;
482\ 253 \div 2 = 241\ 126 + 1;
241\ 126 \div 2 = 120\ 563 + 0;
120\ 563 \div 2 = 60\ 281 + 1;
60\ 281 \div 2 = 30\ 140 + 1;
30\ 140 \div 2 = 15\ 070 + 0;
```

```
15\ 070 \div 2 = 7\ 535 + 0;
7535 \div 2 = 3767 + 1:
3767 \div 2 = 1883 + 1;
1883 \div 2 = 941 + 1;
941 \div 2 = 470 + 1;
470 \div 2 = 235 + 0;
235 \div 2 = 117 + 1;
117 \div 2 = 58 + 1;
58 \div 2 = 29 + 0;
29 \div 2 = 14 + 1;
14 \div 2 = 7 + 0;
7 \div 2 = 3 + 1;
3 \div 2 = 1 + 1;
1 \div 2 = 0 + 1;
Then we Construct the base 2 representation of the positive number.
123 456 789 (decimal) = 111 0101 1011 1100 1101 0001 0101 (binary)
After that. We Normalize the binary representation of the number.
111 0101 1011 1100 1101 0001 0101 (binary) = 1.1101 0110 1111 0011 0100 0101 01 (binary) \times 2^{26}
Now. We adjust the exponent. Exponent (unadjusted) = 26
Exponent (adjusted) = Exponent (unadjusted) + 2(8-1) - 1 = 26 + 2(8-1) - 1 = (26 + 127)(10)
= 153(10)
Then we use the same technique by dividing the number repeatedly by 2
division = quotient + remainder;
153 \div 2 = 76 + 1;
76 \div 2 = 38 + 0;
38 \div 2 = 19 + 0:
19 \div 2 = 9 + 1;
9 \div 2 = 4 + 1:
4 \div 2 = 2 + 0;
2 \div 2 = 1 + 0;
1 \div 2 = 0 + 1;
Exponent (adjusted) = 153 (decimal) = 1001 1001 (binary)
After that. We normalized the mantissa by:
                Remove the leading bit
      A-
      B-
                Adjust its length to 23 by removing the excess bits
```

Finally. The three elements that make up 32 bit single precision IEE 754 binary floating point are Sign (1 bit) = 1

1.1101 0110 1111 0011 0100 0101 01 (binary) = 110 1011 0111 1001 1010 0010

```
Exponent (8 bits) = 1001 1001 (binary)

Mantissa (23 bits) = 110 1011 0111 1001 1010 0010

= 1 10011001 11010110111100110100010
```

2. Convert by hand the floating-point number **1 10010100 10011000001100000000000** (shown in binary) into its corresponding decimal value. Show your work for a full mark.

Sign bit: 1 (negative)

Exponent: 10010100 (binary) = 148 (decimal)

Mantissa: 10011000001100000000000 (binary)

First. we subtract 127 from the exponent to get the true exponent: 148 - 127 = 21

Second. we convert the mantissa to decimal, by multiplying by 2^{N of number}:

```
1\times 2^{-1} + 0\times 2^{-2} + 0\times 2^{-3} + 1\times 2^{-4} + 1\times 2^{-5} + 0\times 2^{-6} + 0\times 2^{-7} + 0\times 2^{-8} + 0\times 2^{-9} + 0\times 2^{-10} + 1\times 2^{-11} + 1\times 2^{-12} + 0\times 2^{-13} + 0\times 2^{-14} + 0\times 2^{-15} + 0\times 2^{-16} + 0\times 2^{-17} + 0\times 2^{-18} + 0\times 2^{-19} + 0\times 2^{-20} + 0\times 2^{-21} + 0\times 2^{-22} + 0\times 2^{-23} = 0.594482421875 \text{ (decimal)}
```

Then we will add 1 to the mantissa: 0.594482421875 (decimal) + 1 = 1.594482421875 (decimal)

Finally, the floating-point value is then equal to $-1 \times 2^{21} \times 1.594482421875 = -3343872$

Task 3

3. Trace the following program by hand to determine the values of registers \$f0 thru \$f9. Notice that array1 and array2 have the same elements, but in a different order. Comment on the sums of array1 and array2 elements computed in registers \$f4 and \$f9, respectively. Now use the QTSPIM tool to trace the execution of the program and verify your results. What conclusion can be made from this exercise?

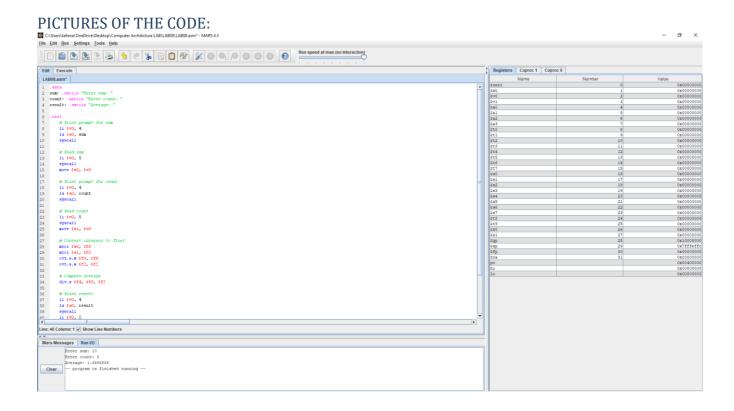
```
.data
array1: .float 5.6e+20, -5.6e+20, 1.2
array2: .float 1.2, 5.6e+20, -5.6e+20
.text
.globl main
main:
```

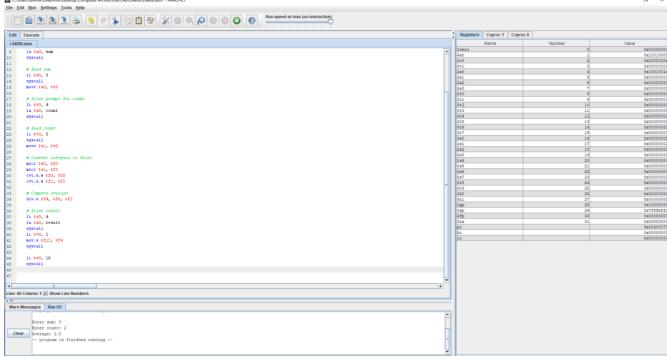
```
la
       $t0, array1
lwc1
       $f0, 0($t0)
lwc1
       $f1, 4($t0)
lwc1
       $f2, 8($t0)
add.s
      $f3, $f0, $f1
add.s
       $f4, $f2, $f3
la
       $t1, array2
       $f5, 0($t1)
lwc1
lwc1
       $f6, 4($t1)
lwc1
       $f7, 8($t1)
add.s $f8, $f5, $f6
add.s $f9, $f7, $f8
li $v0, 10
               # To terminate the program
syscall
```

.end main

The sum of elements of "array1" is 1.2 and the sum of elements of "array2" is 0. This shows that the order of elements in a floating-point addition operation can affect the result due to the limited precision of floating-point numbers.

4. Write an interactive program that inputs an integer **sum** and an integer **count**, computes, and displays the **average** = **(float) sum / (float) count** as a single-precision floating-point number. Hint: use the proper convert instruction to convert **sum** and **count** from integer word into single-precision float.





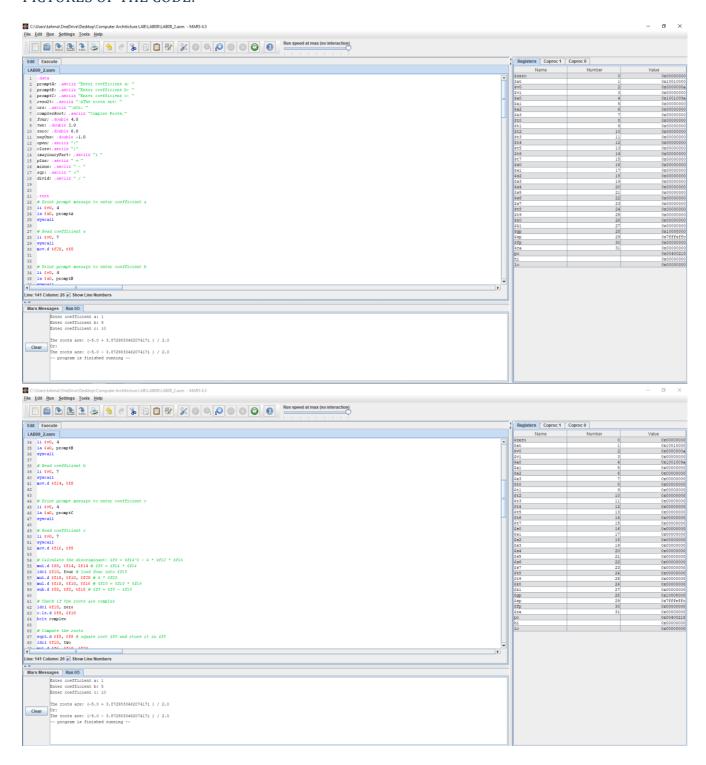
CODE: .data sum: .asciiz "Enter sum: " count: .asciiz "Enter count: " result: .asciiz "Average: " .text # Print prompt for sum li \$v0, 4 la \$a0, sum syscall # Read sum li \$v0, 5 syscall move \$s0, \$v0 # Print prompt for count li \$v0, 4 la \$a0, count syscall # Read count li \$v0, 5 syscall move \$s1, \$v0

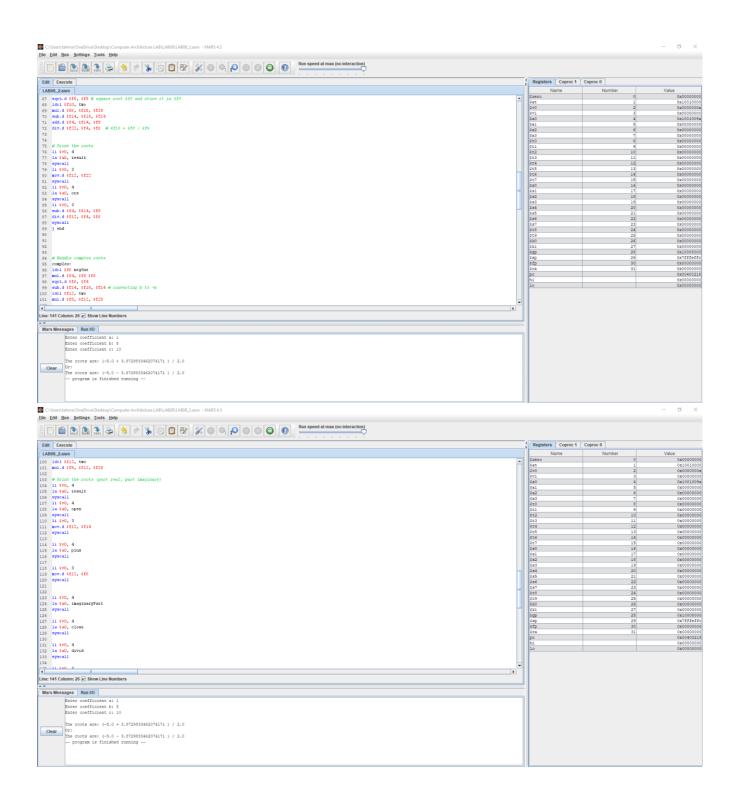
```
# Convert integers to float
mtc1 $s0, $f0
mtc1 $s1, $f2
cvt.s.w $f0, $f0
cvt.s.w $f2, $f2
# Compute average
div.s $f4, $f0, $f2
# Print result
li $v0, 4
la $a0, result
syscall
li $v0, 2
mov.s $f12, $f4
syscall
li $v0, 10
syscall
```

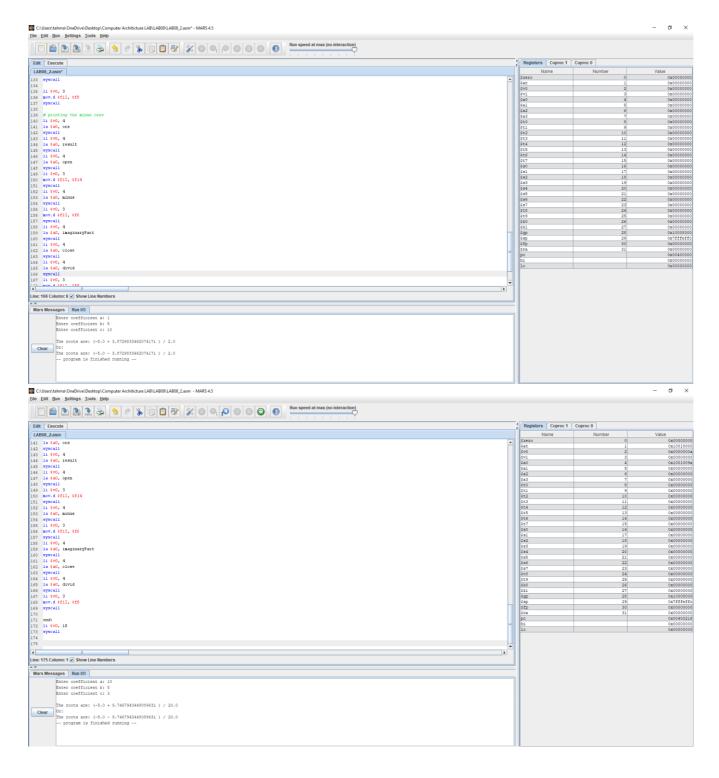
5. Write an interactive program that inputs the coefficient of a quadratic equation, computes, and displays the roots of the quadratic equation. All input, computation, and output should be done using double-precision floating-point instructions and registers. The program should handle the case of complex roots and displays the results properly.

$$x = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

PICTURES OF THE CODE:







OUTPUTS

```
Reset: reset completed.

Enter coefficient a: 25
Enter coefficient b: -30
Enter coefficient c: 9

The roots are: 0.6
Or: 0.6
-- program is finished running --

Reset: reset completed.

Enter coefficient a: 1
Enter coefficient b: 5
Enter coefficient c: 1

The roots are: -0.20871215252208009
Or: -4.7912878474779195
-- program is finished running --
```

```
Reset: reset completed.
Enter coefficient a: 10
Enter coefficient b: 200
Enter coefficient c: 50
The roots are: -0.2532056551910358
Or: -19.746794344808965
-- program is finished running --
presett ieset compieted.
Enter coefficient a: 10
Enter coefficient b: 20
Enter coefficient c: 30
The roots are: (-20.0 + 28.284271247461902i ) / 20.0
Or:
The roots are: (-20.0 - 28.284271247461902i ) / 20.0
-- program is finished running --
Enter coefficient a: 321.13
Enter coefficient b: 1.23
Enter coefficient c: 54.2
The roots are: (-1.23 + 263.8550190919248i ) / 642.26
Or:
The roots are: (-1.23 - 263.8550190919248i ) / 642.26
CODE:
.data
promptA: .asciiz "Enter coefficient a: "
promptB: .asciiz "Enter coefficient b: "
promptC: .asciiz "Enter coefficient c: "
result: .asciiz "\nThe roots are: "
ors: .asciiz "\nOr: "
complexRoot: .asciiz "Complex Roots."
four: .double 4.0
two: .double 2.0
zero: .double 0.0
negOne: .double -1.0
open: .asciiz "("
close:.asciiz ")"
```

minus: .asciiz " - "
sqr: .asciiz " √"
divid: .asciiz " / "

.text
Print prompt message to enter coefficient a li \$v0, 4
la \$a0, promptA
syscall
Read coefficient a li \$v0, 7
syscall
mov.d \$f20, \$f0

imaginaryPart: .asciiz "i "

plus: .asciiz " + "

Print prompt message to enter coefficient b li \$v0, 4 la \$a0, promptB syscall

```
# Read coefficient b
li $v0, 7
syscall
mov.d $f14, $f0
# Print prompt message to enter coefficient c
li $v0, 4
la $a0, promptC
syscall
# Read coefficient c
li $v0, 7
syscall
mov.d $f16, $f0
# Calculate the discriminant: $f8 = $f14^2 - 4 * $f12 * $f16
mul.d $f8, $f14, $f14 # $f8 = $f14 * $f14
ldc1 $f10, four # load four into $f10
mul.d $f10, $f10, $f20 # 4 * $f20
mul.d $f10, $f10, $f16 # $f10 = $f10 * $f16
sub.d $f8, $f8, $f10 # $f8 = $f8 - $f10
# Check if the roots are complex
ldc1 $f18, zero
c.lt.d $f8, $f18
bc1t complex
# Compute the roots
sqrt.d $f8, $f8 # square root $f8 and store it in $f8
ldc1 $f10, two
mul.d $f6, $f10, $f20
sub.d $f14, $f18, $f14
add.d $f4, $f14, $f8
div.d $f22, $f4, $f6 # $f10 = $f8 / $f6
# Print the roots
li $v0, 4
la $a0, result
syscall
li $v0, 3
mov.d $f12, $f22
syscall
li $v0, 4
1a $a0, ors
syscall
li $v0, 3
sub.d $f4, $f14, $f8
div.d $f12, $f4, $f6
syscall
j end
```

```
complex:
ldc1 $f6 negOne
mul.d $f4, $f8 $f6
sqrt.d $f6, $f4
sub.d $f14, $f18, $f14 # converting b to -b
1dc1 $f12, two
mul.d $f8, $f12, $f20
# Print the roots (part real, part imaginary)
li $v0, 4
la $a0, result
syscall
li $v0, 4
la $a0, open
syscall
li $v0, 3
mov.d $f12, $f14
syscall
li $v0, 4
la $a0, plus
syscall
li $v0, 3
mov.d $f12, $f6
syscall
li $v0, 4
la $a0, imaginaryPart
syscall
li $v0, 4
la $a0, close
syscall
li $v0, 4
la $a0, divid
syscall
li $v0, 3
mov.d $f12, $f8
syscall
# printing the minus case
li $v0, 4
1a $a0, ors
syscall
li $v0, 4
la $a0, result
syscall
li $v0, 4
la $a0, open
syscall
li $v0, 3
mov.d $f12, $f14
syscall
li $v0, 4
```

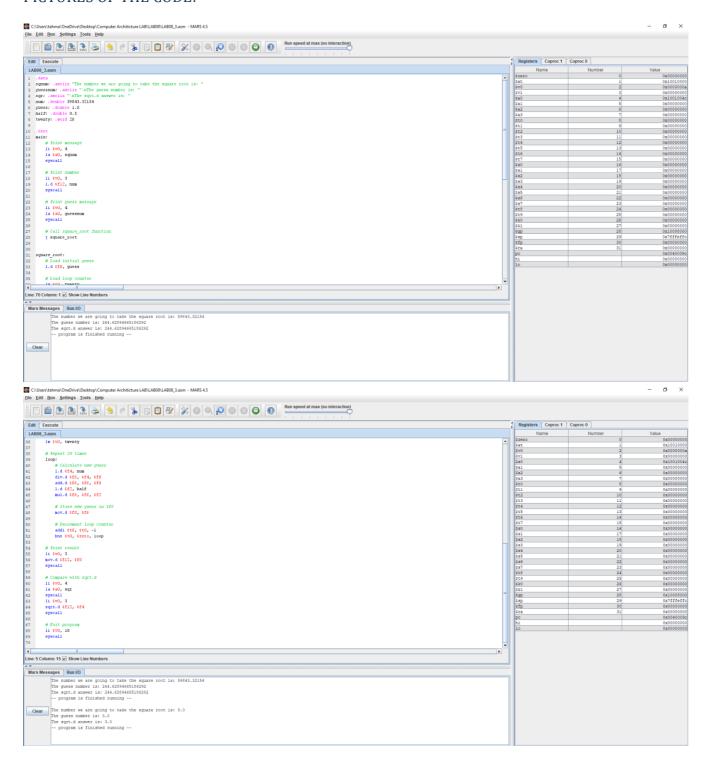
```
la $a0, minus
syscall
li $v0, 3
mov.d $f12, $f6
syscall
li $v0, 4
la $a0, imaginaryPart
syscall
li $v0, 4
la $a0, close
syscall
li $v0, 4
la $a0, divid
syscall
li $v0, 3
mov.d $f12, $f8
syscall
end:
li $v0, 10
syscall
```

6. Square Root Calculation: Newton's iterative method can be used to approximate the square root of a number **x**. Let the initial **guess** be **1**. Then each new **guess** can be computed as follows:

```
guess = ((x/guess) + guess) / 2;
```

Write a function called **square_root** that receives a double-precision parameter **x**, computes, and returns the approximated value of the square root of **x**. Write a loop that repeats 20 times and computes 20 **guess** values, then returns the final **guess** after 20 iterations. Use the MIPS floating-point register convention to pass the parameter **x** and to return the function result. All computation should be done using double-precision floating-point instructions and registers. Compare the result of the **sqrt.d** instruction against the result of your **square_root** function. What is the error in absolute value?

PICTURES OF THE CODE:



COMPARING RESULTS:

```
The number we are going to take the square root is: 9.984988675761652E15
The guess number is: 9.5227767724486E9
The sqrt.d answer is: 9.992491519016492E7
-- program is finished running --
```

Error = 9.5227767724486E9 - 9.992491519016492E7 = 852929148061692

```
The number we are going to take the square root is: 9.0
The guess number is: 3.0
The sqrt.d answer is: 3.0
-- program is finished running --

Error = 3.0 - 3.0 = 0.0

The number we are going to take the square root is: 5.43987564387543E14
The guess number is: 5.1913644684564495E8
The sqrt.d answer is: 2.3323540991614953E7
-- program is finished running --
```

Error = 2.3323540991614953E7 - 5.1913644684564495E8 = 4.85801037029501E8

The difference gets bigger when the number is big. We can fix that by increasing the loops from 20 to something bigger.

```
CODE:
.data
sqnum: .asciiz "The number we are going to take the square root is: "
guessnum: .asciiz "\nThe guess number is: "
sqr: .asciiz "\nThe sqrt.d answer is: "
num: .double 5.43987564387543E14
guess: .double 1.0
half: .double 0.5
twenty: .word 20
.text
main:
        # Print message
        li $v0, 4
        la $a0, sqnum
        syscall
        # Print number
        li $v0, 3
        1.d $f12, num
        syscall
        # Print guess message
        li $v0, 4
        la $a0, guessnum
        syscall
        # Call square_root function
        j square_root
```

```
square_root:
        # Load initial guess
        1.d $f0, guess
        # Load loop counter
        lw $t0, twenty
        # Repeat 20 times
        loop:
                # Calculate new guess
                1.d $f4, num
                div.d $f6, $f4, $f0
                add.d $f6, $f6, $f0
                1.d $f2, half
                mul.d $f6, $f6, $f2
                # Store new guess in $f0
                mov.d $f0, $f6
                # Decrement loop counter
                addi $t0, $t0, -1
                bne $t0, $zero, loop
        # Print result
        li $v0, 3
        mov.d $f12, $f0
        syscall
        # Compare with sqrt.d
        li $v0, 4
        1a $a0, sqr
        syscall
        li $v0, 3
        sqrt.d $f12, $f4
        syscall
        # Exit program
        li $v0, 10
        syscall
```