BM V 240321

Wednesday, 24 March 2021 5:44 PM

⑥ $|x+1| + \sqrt{x-1} = 0$

$|x+1| = 0$

$x = -1$

$\sqrt{x-1} = 0$

$\boxed{x = 1}$

No solution

⑦ $|x^2-1| + \sqrt{x^2-3x+2} = 0$

$x = +1, -1$

$(x-1)(x-2) = 0$

$x = 1, 2.$

$\therefore$ Ans: $x = 1$

Logarithms

↳ reverse of exponentiation.

✓ Make large numbers look smaller.

✓ makes calculation simpler.

$\times \quad \div$

$\downarrow \quad \downarrow$

$+ \quad -$

$|x| \rightarrow (\ )$

$\log(x) = \sim$

$8 = 2^3$

$\log 8 = 3 \qquad \log 16 = 4$

$\log_2 8 = 3$ $\qquad \log_2 16 = 4$

$\log_{16} 16 = 1$

$\log_{20} 1 = 0$

$\log_{10} 10000 = 4 \qquad \log_{100} 10000 = 2$

base $\qquad$ base.

$$\boxed{\begin{array}{l} x = a^y \\ \log_a x = y, \quad x > 0 \\ \qquad\qquad a > 0, a \neq 1 \end{array}}$$

Two imp. bases

Common log (base 10)

$\log_{10} x$

Natural log (Base 'e')

$\log_e x$ or $\ln x$.

irrational no.

≈ 2.718

✓ Euler's Number

✓ Napier's constant

Standard Results

$\log_\alpha (a \cdot b \cdot c) = \log_\alpha a + \log_\alpha b + \log_\alpha c$

① $\log_\alpha (a \cdot b) = \underbrace{\log_\alpha a}_{m} + \underbrace{\log_\alpha b}_{n}$

Let

$\log_\alpha a = m \Rightarrow a = \alpha^m$

$\log_\alpha b = n \Rightarrow b = \alpha^n$

$a \cdot b = \alpha^m \cdot \alpha^n$

$(a.b) = \alpha^{m+n}$

$x = a^y$

$\log_a x = y$

$(a.b) = \alpha^{m+n}$

$$\underbrace{\log_\alpha ab} = m+n = \underbrace{\log_\alpha a + \log_\alpha b}$$

$$\boxed{\log_\alpha ab = \log_\alpha a + \log_\alpha b}$$

$\log_\alpha(a+b) \neq \log_\alpha a + \log_\alpha b$

$\log_\alpha(ab) \neq \log_\alpha a \cdot \log_\alpha b$

(2) $\log_\alpha a^n = n \cdot \log_\alpha a$

LHS $= \log_\alpha(a \cdot a \cdot a \cdot \ldots \cdot a)$

$= \underbrace{\log_\alpha a + \log_\alpha a + \ldots \log_\alpha a}_{n \text{ times}}$

$= n \cdot \log_\alpha a$ ✓

$(\log_\alpha a)^n \neq n \log_\alpha a$

(3) $\log_{\alpha}(a/b) = \log_{\alpha} a - \log_{\alpha} b$

$\log_{\alpha} a \cdot b^{-1} = \log_{\alpha} a + \log_{\alpha} b^{-1}$

$= \log_{\alpha} a - \log_{\alpha} b$

$\log_{b} a^{x} =$

$\log_{10} 10 = (\ )x$

$\log_{1} 1 = ?$

(4) Base change formule

$$\log_{b} a = \frac{\log_{c} a}{\log_{c} b}$$

c → any permissible base of our choice

Proof:

$\log_{b} a = \left(\frac{m}{n}\right) \longrightarrow (*)$

$\Rightarrow a = (b)^{\frac{m}{n}}$

$\Rightarrow a^{n} = b^{m}$

Taking log on both sides

$\log_{c} a^{n} = \log_{c} b^{m}$

$n \log_{c} a = m \log_{c} b$

$\Rightarrow \frac{m}{n} = \frac{\log_{c} a}{\log_{c} b}$

Using (*)

$$\boxed{\Rightarrow \log_{b} a = \frac{\log_{c} a}{\log_{c} b}}$$

(5) $\log_{b} a = \frac{1}{\log_{a} b}$

$= \frac{\log_{a} a}{\log_{a} b} = \frac{1}{\log_{a} b}$

Gre (i) $\log_{b} a \times \log_{a} b = 1$

Gre (ii) $\log_{b} a \times \log_{c} b \times \log_{a} c = 1$

$$\cancel{\frac{\log_\alpha a}{\log_\alpha b}} \times \cancel{\frac{\log_\alpha b}{\log_\alpha c}} \times \cancel{\frac{\log_\alpha c}{\log_\alpha a}} = 1$$

Ex (iii) $\log_b a \times \log_c b \times \log_d c = \log_d a$

$$\frac{\log_\alpha a}{\cancel{\log_\alpha b}} \times \cancel{\frac{\log_\alpha b}{\log_\alpha c}} \times \frac{\cancel{\log_\alpha c}}{\log_\alpha d} = \frac{\log_\alpha a}{\log_\alpha d} \rightarrow \log_d a$$

⑥ $\log_{b^n} a^m = \frac{m}{n} \log_b a$

$$\frac{\log_\alpha a^m}{\log_\alpha b^n} = \frac{m}{n}\frac{\log_\alpha a}{\log_\alpha b} = \frac{m}{n} \log_b a$$

Ex. $\log_{\sqrt{3}} 9 = \log_{3^{1/2}} 9 = \frac{1}{(1/2)} \log_3 9$

$= 2 \log_3 9$

$= 2 \times 2 = 4$

⑦ $\log_{b^n} a = \frac{1}{n} \log_b a$

⑧ $\underbrace{a^m = (b)^{m \cdot \log_b a}}$

$a^n \rightarrow$ exponential function

$e^n \rightarrow$

$$7^x = e^{x(\log_e 7)}$$

Take log

$$\log_\alpha a^m = \log_\alpha b^{m \cdot \log_b a}$$

$$m\log_\alpha a = m\cdot\underbrace{\log_b a\cdot \cancel{\log_\alpha b}}$$

$$= m\cdot \log_\alpha a$$

✓

$$A = B$$
$$\log_\alpha A = \log_\alpha B$$

\#

$$10^{x} = e^{x\log_e 10}$$

$$5^{x} = 7^{x\log_7 5}$$

Ex: $25^{8\log_5 3} = 3^{16}$ Ans

Examples:

① Find the value of $\begin{cases}\log_{3\sqrt2} 324 \longrightarrow 4 \\ \log_{5+2\sqrt6} 5-2\sqrt6 \longrightarrow -1\end{cases}$

② Find value of $2^{2\log_4 5} \longrightarrow 5$

③ Solve the equation

$$3\cdot\left(x^{\log_5 2}\right) + 2^{\log_5 x} = 64$$

$\boxed{x = 625}$

1) (i) $\log_{3\sqrt2} 324 = 4$

$$\begin{array}{c|c} 3 & 324 \\ \hline 3 & 108 \\ \hline 4 & 36 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$324 = 3^4\cdot 2^2$

$= (3\sqrt2)^4$

(ii) $\log_{5+2\sqrt6} 5-2\sqrt6 = ?$

$(5-2\sqrt6)(5+2\sqrt6) = 25-24 = 1$

$\Rightarrow 5-2\sqrt6 = (5+2\sqrt6)^{-1}$

$= \log_{5+2\sqrt6}(5+2\sqrt6)^{(-1)} = -1\left(\log_{5+2\sqrt6} 5+2\sqrt6\right) = -1\cdot 1 = -1$ Ans

③ $3\cdot x^{\log_5 2} + 2^{\log_5 x} = 64$

(3) $3 \cdot x^{\log_5 2} + 2^{\log_5 x} = 64$

$\Rightarrow 3 \cdot 2^{\log_5 x} + 2^{\log_5 x} = 64$

$\Rightarrow \cancel{4} \cdot 2^{\log_5 x} = \cancel{64}\,16$

$\Rightarrow 2^{\log_5 x} = 2^{4}$

$\Rightarrow \log_5 x = 4$

$x = 5^4 = 625$ Ans

---

(4) Value of the $\log_4[\log_2\{\log_2(\log_3 81)\}]$ is equal to 0

$= \log_4 \log_2 (\log_2 4)$

$= \log_4 (\log_2 2)$

$= \log_4 1 = 0$

---

(6) If $4^{\log_{16} 4} + 9^{\log_3 9} = 10^{\log_x 83}$, find x : $\to$ 10

$\Rightarrow \underbrace{4^{1/2} + 9^2} = 83^{\log_x 10}$

$\Rightarrow (2 + 81) = 83^{\log_x 10}$

$\Rightarrow 83^1 = 83^{\log_x 10}$

$\log_x 10 = 1 \Rightarrow 10 = x^1$

$\Rightarrow x = 10$

---

(7) If $\lambda^{\log_3 5} = 81$, find value of $\lambda^{(\log_3 5)^2}$

$\left(\lambda^{(\log_3 5)}\right)^{\log_3 5}$

$= \left(\lambda^{\log_3 5}\right)^{\log_3 5}$

$$= \left(\lambda^{\log_3 5}\right)^{\log_3 5}$$

$$= (81)^{\log_3 5} = 5^{\log_3 81} = 5^4 = \underline{625}$$

Ex:

$$\log_\alpha\left(\frac{a^3 \cdot b^2 \cdot c^4}{\sqrt{d}}\right) = \underbrace{3\log_\alpha a + 2\log_\alpha b + 4\log_\alpha c - \frac{1}{2}\log_\alpha d}$$

Ex:

$$a^{\sqrt{\log_a b}} - b^{\sqrt{\log_b a}} = 0$$

$$a^{\sqrt{\log_a b \cdot \log_b a \cdot \log_a b}} - b^{\sqrt{\log_b a}}$$

$$= \left(a^{\log_a b}\right)^{\sqrt{\log_b a}} - b^{\sqrt{\log_b a}}$$

$$= b^{\log_a a \sqrt{\log_b a}} - b^{\sqrt{\log_b a}}$$

$$= b^{\sqrt{\log_b a}} - b^{\sqrt{\log_b a}} = \underline{\underline{0}}$$