INTEGRATE THE FOLLOWING WITH RESPECT TO "X"

EXERCISE-I

Q.1 $x^{\frac{1}{3}} + x^{\frac{1}{2}} + x^{\frac{3}{5}}$.

Q.2 $(x^2 + 2x + 3) / x^4$.

Q.3 $\frac{(1+2x)^3}{x^4}$

Q.4 $\frac{2x^4+3}{x^2+1}$

Q.5 $\frac{x^2-1}{x^2+1}$

Q.6 $\frac{\left(\sqrt{x}+\sqrt[3]{x^2}\right)^2}{x}$

Q.7 $\frac{x^4+x^2+1}{2(x^2+1)}$

Q.8 $\frac{x^6-1}{x^2+1}$

Q.9 $5\cos x - 3\sin x - \frac{2}{\cos^2 x}$

Q.10 $\frac{5\cos^3 x + 7\sin^3 x}{2\sin^2 x \cos^2 x}$

Q.11 $\frac{\cos 2x}{\cos^2 x \sin^2 x}$

Q.12 $\sec^2 x \operatorname{cosec}^2 x$

Q.13 $\frac{3\cos x - 4}{\sin^2 x}$

Q.14 $\frac{1+2\sin x}{\cos^2 x}$

Q.15 $\tan^2 x$

Q.16 $\cot^2 x$

Q.17 $(\tan x + \cot x)^2$

Q.18 $\frac{1-\cos 2x}{1+\cos 2x}$

Q.19 $\sqrt{(1+\sin 2x)}$

Q.20 $\sqrt{(1+\cos 2x)}$

EXERCISE-II

Q.1 **(i)** $e^x \cos e^x$ **(ii)** $2xe^{x^2}$ **(iii)** $x^3 e^{x^4}$

(iv) $e^{\tan t} \sec^2 t$ **(v)** $\frac{e^{\ell n x}}{x}$ **(vi)** $\frac{e^{\tan^{-1} x}}{1+x^2}$

(vii) $\frac{e^{\sin^{-1} x}}{\sqrt{(1-x^2)}}$ **(viii)** $\frac{e^{\sqrt{x}}}{3\sqrt{x}}$ **(ix)** $\frac{e^x(1+x)}{\cos^2(xe^x)}$

Q.2 **(i)** $\sin^2 x \cos x$ **(ii)** $\sqrt[3]{\sin x}\cos x$ **(iii)** $\sin x \cos x$

(iv) $3 \sin x \sec^4 x$

Q.3 **(i)** $\frac{\cos x}{1+\sin^2 x}$ **(ii)** $\frac{\tan^2 x \sec^2 x}{1+\tan^6 x}$ **(iii)** $\frac{2}{x[1+(\ell n x)^2]}$

(iv) $\frac{3e^{2x}}{1+e^{4x}}$ **(v)** $\frac{2x}{1+x^4}$ **(vi)** $\frac{x^5}{1+x^{12}}$

(vii) $\frac{2x^3}{1+x^8}$ **(viii)** $\frac{1}{e^x+e^{-x}}$

Q.4 **(i)** $\frac{\cos x}{(1+\sin x)^2}$ **(ii)** $\frac{\sec^2 x}{(1+\tan x)^3}$ **(iii)** $\frac{(1-\ell n\, x)^2}{x}$

Q.5 **(i)** $4x^3 \operatorname{cosec}^2(x^4)$ **(ii)** $x^4 \sec^2(x^5)$ **(iii)** $x^3 \sin x^4$

(iv) $\frac{\cos\sqrt{x}}{\sqrt{x}}$ **(v)** $e^x \sec^2(e^x)$

Q.6 **(i)** $\frac{\cos(\ell n\, x)}{x}$ **(ii)** $\frac{\sec^2(\ell n\, x)}{x}$ **(iii)** $\frac{\sin(2+3\ell n\, x)}{x}$

(iv) $e^x \tan(e^x)\sec(e^x)$

Q.7 **(i)** $\frac{2x}{\sqrt{(1-x^4)}}$ **(ii)** $\frac{x^2}{\sqrt{(1-x^6)}}$ **(iii)** $\frac{2}{\sqrt{[2-(2x+3)^2]}}$

(iv) $\frac{\sec^2 x}{\sqrt{(1-\tan^2 x)}}$

Q.8 **(i)** $\frac{x^2 \tan^{-1} x^3}{1+x^6}$ **(ii)** $\frac{2x \sin^{-1} x^2}{\sqrt{(1-x^4)}}$ **(iii)** $\frac{\tan\sqrt{x}\sec^2\sqrt{x}}{\sqrt{x}}$

EXERCISE-III

Q.1 **(i)** $\sqrt{\sin x}\cos x$ **(ii)** $\tan^4 x \sec^2 x$ **(iii)** $\operatorname{cosec}^2 x\sqrt{\cot x}$

(iv) $\frac{(\tan^{-1} x)^3}{1+x^2}$ **(v)** $\frac{1}{(\tan^{-1} x)^2(1+x^2)}$ **(vi)** $\frac{\sin^{-1} x}{\sqrt{(1-x^2)}}$

(vii) $\frac{1}{\sqrt{(\sin^{-1} x)}\sqrt{(1-x^2)}}$ **(viii)** $\frac{x}{\sqrt{(1-x^2)}}$ **(ix)** $\frac{x}{\sqrt[3]{(x^2+1)}}$

(x) $x\sqrt{(x^2+1)}$ **(xi)** $e^x\sqrt{(1+e^x)}$ **(xii)** $\sin^3 x \cos x$

(xiii) $\frac{2x+3}{\sqrt{(x^2+3x-4)}}$ **(xiv)** $\frac{(x+1)(x+\ell n\, x)^2}{2x}$ **(xv)** $\frac{1}{x(1+\ell n\, x)^3}$

(xvi) $\frac{\ell n\, x}{x}$ **(xvii)** $\sqrt{(2+\sec^2 x)}\sec^2 x \tan x$ **(xviii)** $(e^x + e^{-x})(e^x - e^{-x})$

(xix) $\sec x\ \ell n(\sec x + \tan x)$

EXERCISE-IV

Q.1 **(i)** $\cos^2 x$ **(ii)** $\cos^3 x$ **(iii)** $\sin 4x \cos 2x$

(iv) $\sin 5x \sin 3x$ **(v)** $\cos x \cos 2x \cos 3x$

Q.2 **(i)** $\sin^4 x$ **(ii)** $\cos^4 x$

Q.3 **(i)** $\frac{1}{ax+b}$ **(ii)** $\sec^2(2x+3)$ **(iii)** $\cot(4x+5)$

(iv) $\cos\frac{x}{2}$ **(v)** $\sec^2\frac{x}{2}\cos ec^2\frac{x}{2}$ **(vi)** $\frac{1}{1+\cos x}$

(vii) $\sqrt{(1-\cos x)}$ **(viii)** $\sqrt{(1+\sin x)}$ **(ix)** $\sqrt{(1+\cos x)}$

(x) $\sqrt{(1-\sin x)}$ **(xi)** $\sec(ax+b)$ **(xii)** $\text{cosec}(ax+b)$

EXERCISE-V

Q.1 **(i)** $x^2 e^x$ **(ii)** $x^3 e^x$ **(iii)** $x\left(e^x - e^{-x}\right)$

Q.2 **(i)** $\ell n\ x$ **(ii)** $x\ \ell n\ x$ **(iii)** $x^2\ \ell n\ x$ **(iv)** $x^n\ \ell n\ x$

Q.3 **(i)** $x(\ell n\ x)^2$ **(ii)** $(x\ \ell n\ x)^3$ **(iii)** $\sqrt{x}(\ell n\ x)^2$

Q.4 **(i)** $\tan^{-1} x$ **(ii)** $x\tan^{-1} x$ **(iii)** $\cot^{-1} x$ **(iv)** $x\cot^{-1} x$

(v) $x^3\tan^{-1} x$

Q.5 **(i)** $\sin^{-1} x$ **(ii)** $x\sin^{-1} x$ **(iii)** $\sec^{-1} x$ **(iv)** $x\sec^{-1} x$

Q.6 **(i)** $x\cos x$ **(ii)** $x^2\sin x$ **(iii)** $x^3\cos x$ **(iv)** $x^2\sin^2 x$

Q.7 **(i)** $\frac{x}{\sin^2 x}$ **(ii)** $\frac{x}{\cos^2 x}$ **(iii)** $x\ \ell n\ (1+x)$

Q.8 **(i)** $\int e^{4x}\cos 5x\,dx$ **(ii)** $\int e^x\cos^2 x\,dx$

(iii) $\int e^{2x}\cos^2 x\,dx$

Answers

EXERCISE-I

1. $\frac{3}{4}x^{\frac{4}{3}}+\frac{2}{3}x^{\frac{3}{2}}+\frac{5}{8}x^{\frac{8}{5}}+C$ **2.** $-(x^2+x+1)/x^3+C$ **3.** $-(1+9x+36x^2)/3x^3+8\ \ell n|x|+C$

4. $\frac{2}{3}(x^3-3x)+5\tan^{-1}x+C$ **5.** $x-2\tan^{-1}x+C$ **6.** $x+\frac{3}{4}x^{4/3}+\frac{12}{7}x^{7/6}+C$

7. $\frac{1}{6}(x^3+3\tan^{-1}x)+C$ **8.** $\frac{1}{5}x^5-\frac{1}{3}x^3+x-2\tan^{-1}x+C$ **9.** $5\sin x+3\cos x-2\tan x+C$

10. $\frac{7}{2}\sec x-\frac{5}{2}\operatorname{cosec} x+C$ **11.** $-\sec x\operatorname{cosec} x+C$ **12.** $-2\cot 2x+C$

13. $4\cot x-3\operatorname{cosec} x+C$ **14.** $\tan x+2\sec x+C$ **15.** $\tan x-x+C$

16. $-\cot x-x+C$ **17.** $-2\cot 2x+C$ **18.** $\tan x-x+C$

19. $-\cos x+\sin x+C$ **20.** $\sqrt{2}\sin x+C$

EXERCISE-II

1. (i) $\sin e^x+C$ **(ii)** $e^{x^2}+C$ **(iii)** $\frac{1}{4}e^{x^4}+C$

(iv) $e^{\tan t}+C$ **(v)** $x+C$ **(vi)** $e^{\tan^{-1}x}+C$

(vii) $e^{\sin^{-1}x}+C$ **(viii)** $\frac{2}{3}e^{\sqrt{x}}+C$ **(ix)** $\tan(xe^x)+C$

2.(i) $\frac{1}{3}\sin^3 x+C$ **(ii)** $\frac{3}{4}\sin^{4/3}x+C$ **(iii)** $-\frac{1}{4}\cos 2x+C$

(iv) $\sec^3 x+C$ **3.(i)** $\tan^{-1}(\sin x)+C$ **(ii)** $\frac{1}{3}\tan^{-1}(\tan^3 x)+C$

(iii) $2\tan^{-1}(\ln x)+C$ **(iv)** $\frac{3}{2}\tan^{-1}(e^{2x})+C$ **(v)** $\tan^{-1}x^2+C$

(vi) $\frac{1}{6}\tan^{-1}x^6+C$ **(vii)** $\frac{1}{2}\tan^{-1}x^4+C$ **(viiii)** $\tan^{-1}(e^x)+C$

4.(i) $-1/(1+\sin x)+C$ **(ii)** $-\frac{1}{2}(1+\tan x)^{-2}+C$ **(iii)** $-\frac{1}{3}(1-\ell n\,x)^3+C$

5.(i) $-\cot x^4+C$ **(ii)** $\frac{1}{5}\tan x^5+C$ **(iii)** $-\frac{1}{4}\cos x^4+C$

(iv) $2\sin\sqrt{x}+C$ **(v)** $\tan e^x + C$ **6.(i)** $\sin(\ell n\, x)+C$

(ii) $\tan(\ell n\, x)+C$ **(iii)** $-\frac{1}{3}\cos(2+3\ell n\, x)+C$ **(iv)** $\sec e^x + C$

7.(i) $\sin^{-1} x^2 + C$ **(ii)** $\frac{1}{3}\sin^{-1} x^3 + C$ **(iii)** $\sin^{-1}\left[(2x+3)/\sqrt{2}\right]+C$

(iv) $\sin^{-1}(\tan x)+C$ **8.(i)** $\frac{1}{6}(\tan^{-1} x^3)^2+C$ **(ii)** $\frac{1}{2}(\sin^{-1} x^2)^2+C$

(iii) $\left(\tan\sqrt{x}\right)^2+C$

EXERCISE-III

1. (i) $\frac{2}{3}(\sin x)^{3/2}+C$ **(ii)** $\frac{1}{5}\tan^5 x+C$ **(iii)** $-\frac{2}{3}\cot^{\frac{3}{2}} x+C$ **(iv)** $\frac{1}{4}(\tan^{-1} x)^4+C$

(v) $-1/\tan^{-1} x + C$ **(vi)** $\frac{1}{2}(\sin^{-1} x)^2+C$ **(vii)** $2\sqrt{(\sin^{-1} x)}+C$ **(viii)** $-\sqrt{(1-x^2)}+C$

(ix) $\frac{3}{4}(1+x^2)^{2/3}+C$ **(x)** $\frac{1}{3}(1+x^2)^{3/2}+C$ **(xi)** $\frac{2}{3}(1+e^x)^{3/2}+C$ **(xii)** $\frac{1}{4}\sin^4 x+C$

(xiii) $2\sqrt{(x^2+3x-4)}+C$ **(xiv)** $\frac{1}{6}(x+\ell n\, x)^3+C$ **(xv)** $-\frac{1}{2}(1+\ell n\, x)^{-2}+C$ **(xvi)** $\frac{1}{2}(\ell n\, x)^2+C$

(xvii) $\frac{1}{3}(2+\sec^2 x)^{3/2}+C$ **(xviii)** $\frac{1}{2}(e^x-e^{-x})^2+C$ **(xix)** $\frac{1}{2}[\ell n(\sec x+\tan x)]^2+C$

EXERCISE-IV

1.(i) $\frac{1}{2}(x+\sin x\cos x)+C$ **(ii)** $\frac{1}{12}(9\sin x+\sin 3x)+C$ **(iii)** $-\frac{1}{12}(\cos 6x+3\cos 2x)+C$

(iv) $\frac{1}{16}(4\sin 2x-\sin 8x)+C$ **(v)** $\frac{1}{48}(12x+6\sin 2x+3\sin 4x+2\sin 6x)+C$

2.(i) $\frac{1}{32}(12x-8\sin 2x+\sin 4x)+C$ **(ii)** $\frac{1}{32}(12x+8\sin 2x+\sin 4x)+C$ **3.(i)** $\frac{1}{a}\ell n(ax+b)+C$

(ii) $\frac{1}{2}\tan(2x+3)+C$ **(iii)** $\frac{1}{4}\ell n\sin(4x+5)+C$ **(iv)** $2\sin\frac{1}{2}x+C$ **(v)** $-4\cot x+C$

(vi) $\tan\frac{1}{2}x+C$ **(vii)** $-2\sqrt{2}\cos\frac{x}{2}+C$ **(viii)** $2\left(\sin\frac{1}{2}x-\cos\frac{1}{2}x\right)+C$ **(ix)** $2\sqrt{2}\sin\frac{x}{2}+C$

(x) $2\left(\sin\frac{1}{2}x+\cos\frac{1}{2}x\right)+C$ **(xi)** $\frac{1}{a}\ell n\tan\left[\frac{1}{2}(ax+b)\frac{1}{4}\pi\right]+C$ **(xii)** $\frac{1}{a}\ell n\tan\frac{ax+b}{2}+C$

EXERCISE-V

1. (i) $(x^2 - 2x + 2)e^x + C$ **(ii)** $(x^3 - 3x^2 + 6x - 6)e^x + C$ **(iii)** $(x-1)e^x - (x+1)e^{-x} + C$

2.(i) $x\ \ell n\,(x/e) + C$ **(ii)** $\frac{1}{4}x^2 \ell n(x^2/e) + C$ **(iii)** $\frac{1}{4}x^3 \ell n(x^3/e) + C$

(iv) $[x^{n+1}/(n+1)^2]\ \ell n\,(x^{n+1}/e) + C$ **3.(i)** $\frac{1}{4}x^2[2(\ell n\,x)^2 - 2\ell n\,x + 1] + C$

(ii) $\frac{1}{128}x^4[32(\ell n\,x)^3 - 24(\ell n\,x)^2 + 12\ell n\,x - 3] + C$ **(iii)** $\frac{1}{27}[18(\ell n\,x)^2 - 24\,\ell n\,x + 16]x^{3/2} + C$

4.(i) $x\tan^{-1}x - \ell n\sqrt{(x^2+1)} + C$ **(ii)** $\frac{1}{2}(x^2+1)\tan^{-1}x - \frac{1}{2}x + C$

(iii) $x\cot^{-1}x + \ell n\sqrt{(x^2+1)} + C$ **(iv)** $\frac{1}{2}(x^2+1)\cot^{-1}x + \frac{1}{2}x + C$

(v) $\frac{1}{4}(x^4-1)\tan^{-1}x - 1/12(x^3 - 3x) + C$ **5.(i)** $x\sin^{-1}x + \sqrt{(1-x^2)} + C$

(ii) $\frac{1}{4}(2x^2-1)\sin^{-1}x + \frac{1}{4}x\sqrt{(1-x^2)} + C$ **(iii)** $\ell n\left|x + \sqrt{x^2-1}\right| + C$

(iv) $\frac{1}{2}[x^2\sec^{-1}x - \sqrt{(x^2-1)}\,] + C$ **6.(i)** $x\sin x + \cos x + C$

(ii) $-x^2\cos x + 2(x\sin x + \cos x) + C$ **(iii)** $(x^3 - 6x)\sin x + 3(x^2 - 2)\cos x + C$

(iv) $\frac{1}{6}x^3 + \frac{1}{8}(1-2x^2)\sin 2x - \frac{1}{4}x\cos 2x + C$

Q.7(i) $-x\cot x + \ell n\ \sin x + C$ **(ii)** $x\tan x - \ell n\sec x + C$

(iii) $\frac{1}{2}(x^2-1)\ell n(1+x) - \frac{1}{4}(x^2 - 2x) + C$

8. (i) $\sqrt{\frac{1}{41}}e^{4x}\cos(5x - \tan^{-1}\frac{15}{4}) + C$ **(ii)** $\frac{1}{2}e^x + \frac{1}{2}\sqrt{\frac{1}{5}}e^x\cos(2x - \tan^{-1}2) + C$

(iii) $\frac{1}{4}e^{2x}\left[\frac{3}{\sqrt{5}}\cos\left(x - \tan^{-1}\frac{1}{2}\right) + \frac{1}{\sqrt{13}}\cos\left(3x - \tan^{-1}\frac{3}{2}\right)\right) + C$