BM IV 190321

Friday, March 19, 2021 2:45 PM

Properties of Modulus

(i) $|-x| = |x|$

(ii) $|x \cdot y| = |x| \cdot |y|$

(iii) $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$

(4) $|x+y| \leq |x| + |y|$

and $|x+y| = |x| + |y|$, { x, y are of same sign or zero

$xy \geq 0$

$|2+3| = |2| + |3|$ ✓

$|2+(-3)| < |2| + |-3|$ ✓

$|(-5)+(-7)| = |-5| + |-7|$ ✓

(5) i) $\big||x| - |y|\big| \leq |x-y|$ (difference)

$|x-y| = +ve.$ (small, big)

$= (y-x)$ (big, small)

$|x-y| = x-y$

ii) $\big||x| - |y|\big| = |x-y|$, when x & y are of same sign or zero

$\boxed{xy \geq 0}$

2, 3

$\big||2| - |3|\big| = |2-3|$

$\big||-2| - |3|\big| < |(-2) - 3|$

$\big||-5| - |-7|\big| = |(-5) - (-7)|$

$$|x+y| = |x| + |y|$$

$$|x| - |y| = |x-y| \quad , \quad |x| \geq |y|$$

x, y are of same sign or zero

(6) (i) $|x+y| = \big||x| - |y|\big|$,

when x, y are of opposite sign or zero.

(ii) $|x-y| = |x| + |y|$

x, y are of opposite sign or zero

(6) $|x| = \sqrt{x^2}$

or $|x|^2 = x^2$

Examples:

(1) Solve : $|x-1| + |x+3| = 7$

$|x-1|$: $\boxed{x-1}$, $x-1 \geq 0$, $x \geq 1$; $-(x-1)$, $x-1 < 0$, $\boxed{x < 1}$

$|x+3|$: $(x+3)$, $x+3 \geq 0$, $x \geq -3$; $-(x+3)$, $x+3 < 0$, $x < -3$

case I: $x < -3$; case II: $-3 \leq x < 1$; case III: $x \geq 1$

$$|x-1| + |x+3| = 7$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x & x < 0 \end{cases}$$

Case I $\quad x < -3$

Case I: $x < -3$

$$-(x-1) + \left(-(x+3)\right) = 7$$

$$-2x - 2 = 7 \Rightarrow 2x = -9$$

$$\Rightarrow \boxed{x = -9/2}$$ Check ✓ acceptable

Case II: $\boxed{-3 \le x < 1}$

$$\boxed{-(x-1) + (x+3) = 7}$$

$$\boxed{4 = 7}$$ → not true for any value of x

no solution within this case

$$|\overset{+}{x-1}| + |\overset{+}{x+3}| = 7$$

Case III: $x \geq 1$

$$(x-1) + (x+3) = 7$$

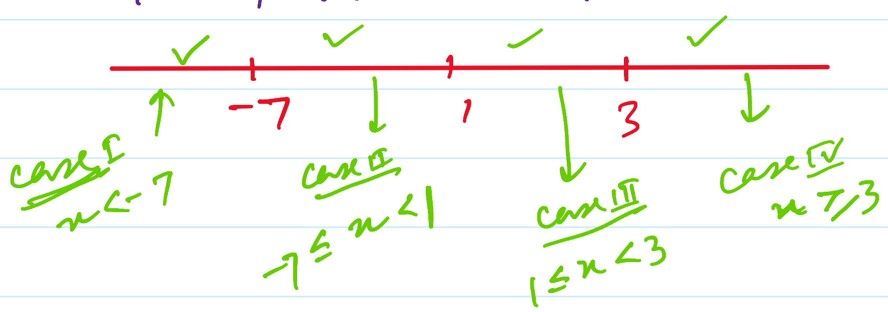
$$2x + 2 = 7$$

$$2x = 5$$

$$\boxed{x = 5/2}$$ check acceptable

∴ Final answer: $x = -\frac{9}{2}, 5/2$

Eg ② $|x-1| + |x+7| + |x-3| = 5$



Complete it yourself.

③ $|x-1| + |x+3| \geq 7$

$x \leq -9/2 \cup x \geq 5/2$

$\left(-\infty, -9/2\right] \cup \left[\frac{5}{2}, \infty\right)$

$\underbrace{\quad}_{C.I}$ -3 $\underbrace{\quad}_{C.II}$ 1 $\underbrace{\quad}_{C.III}$

Case I $x < -3$

$-(x-1) + \left(-(x+3)\right) \geq 7$

$-2x - 2 \geq 7 \Rightarrow 2x \leq -9$

$x \leq -9/2$ — Check

acceptable ✓

Case II : $\boxed{-3 \leq x < 1}$

$-(x-1) + (x+3) \geq 7$

$\boxed{4 \geq 7}$ → not true for any value of x

no solution within this case

$|x-1| + |x+3| = 7$

Case III : $x \geq 1$

$(x-1) + (x+3) \geq 7$

$2x + 2 \geq 7$

$2x \geq 5$

$\boxed{x \geq 5/2}$ — check

acceptable ✓

Final ans : $x \leq -9/2$ or $x \geq 5/2$

$\left(-\infty, -9/2\right] \cup \left[\frac{5}{2}, \infty\right)$ Ans

Ex: Solve $\big|\underbrace{|x-1|-1}\big| \le 1$ Ans: $[-1,3]$

$$\boxed{\underbrace{|x| \le a} \;\downarrow\; -a \le x \le a}$$

$$-1 \le |x-1| - 1 \le 1$$

$$0 \le |x-1| \le 2$$

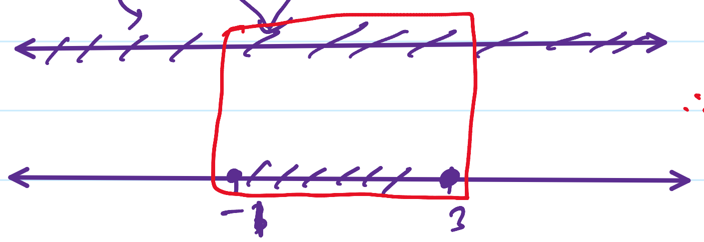
$\underbrace{|x-1| \ge 0}$ and $\underbrace{|x-1| \le 2}$

true for all x

Ans: $\boxed{R}$

$$-2 \le x-1 \le 2$$

$$\boxed{-1 \le x \le 3}$$



$\therefore$ Ans: $[-1,3]$

Ex. Solve $\underbrace{\big||x|-1\big|} < \underbrace{|1-x|}$ Ans: $x<0$

$(-\infty, 0)$ ✓

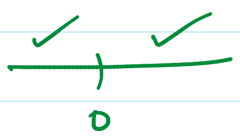
Square it

$$(|x|-1)^2 < (1-x)^2$$

$$\cancel{x^2} - 2|x| + \cancel{1} < \cancel{1} + \cancel{x^2} - 2x$$

$$2x - 2|x| < 0$$

$$\boxed{x - |x| < 0}$$



$x \ge 0$

$x - x < 0$

$0 < 0$

not true.

no solution in this case.

$x<0$

$x - (-x) < 0$

$2x < 0$

$\boxed{x<0}$ Ans

Final ans: $x<0$ or $(-\infty, 0)$ Ans

$\underbrace{x - |x| > 0}$

C.I $x \ge 0$ ✓ $\quad x - x > 0$

$0 > 0 \to$ false ✓ no sol^n

C.II $x<0$ $\quad x - (-x) > 0$

... no ans. final ans: no sol^n.

C. II $x<0$ $\quad x-(-x)>0$

$2x>0$

$x>0$ discard no ans. final ans: no ans.

2nd method: Take various cases.

$||x|-1| - |1-x| < 0$

$\longleftarrow$ 0 $\quad$ 1 $\longrightarrow$

Doubts (CPP)

1. $$\frac{4}{9} < \frac{(x-1)(x+3)}{(x-2)(x+4)} < 1$$

3. $$(\underbrace{x^2-x}_{y}-1)(\underbrace{x^2-x}_{y}-7) < -5.$$

$(y-1)(y-7) < -5$

$y^2 - 7y - y + 7 + 5 < 0$

$y^2 - 8y + 12 < 0$

$(y-6)(y-2) < 0$

$(x^2-x-6)(x^2-x-2) < 0$

$(x-3)(x+2)(x-2)(x+1) < 0$

$+ \quad -2 \quad - \quad -1 \quad + \quad 2 \quad - \quad 3 \quad +$

Ans: $(-2,-1) \cup (2,3)$

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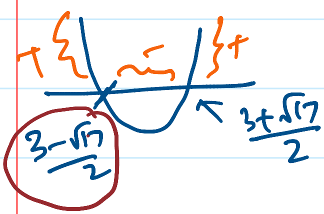
5. $$\frac{1}{x-2}-\frac{1}{x} \le \frac{2}{x+2}$$

$$\frac{1}{x-2}-\frac{1}{x}-\frac{2}{x+2} \le 0$$

$$\frac{x(x+2)-(x^2-4)-2x(x-2)}{x(x^2-4)} \le 0$$

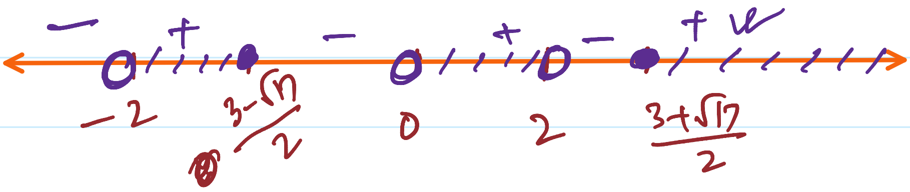
$$\frac{-2x^2+6x+4}{x(x+2)(x-2)} \le 0$$

$$\frac{-2(x^2-3x-2)}{x(x+2)(x-2)} \le 0$$



$$\frac{(x^2-3x-2)}{x(x+2)(x-2)} \ge 0$$

$\frac{3 \pm \sqrt{17}}{2}$



Ans: $\left[-2, \frac{3-\sqrt{17}}{2}\right] \cup (0,2) \cup \left[\frac{3+\sqrt{17}}{2}, \infty\right)$