Movement planning in Robotics and Graphical Animation

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2.1 Introduction

Kinematics studies the movement of an object – in our case of a robot – without taking into acount the forces generating it. Instead, it only handles aspects such as position, orientation, speed and momentum of bodies in movement.

Consider for instance a robotic arm. We can design a simplified scheme of the robot and its environment, to create a kinematic pipeline and reference frames associated to each of these objects.

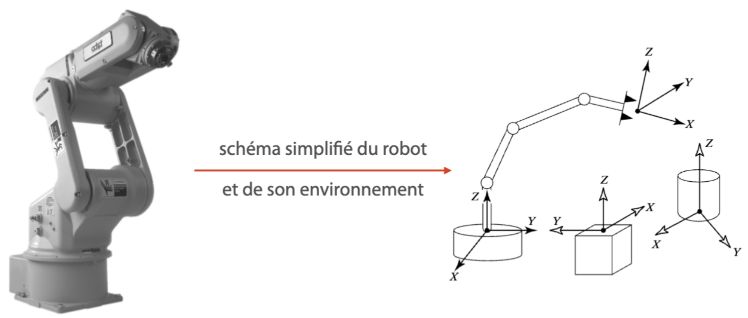
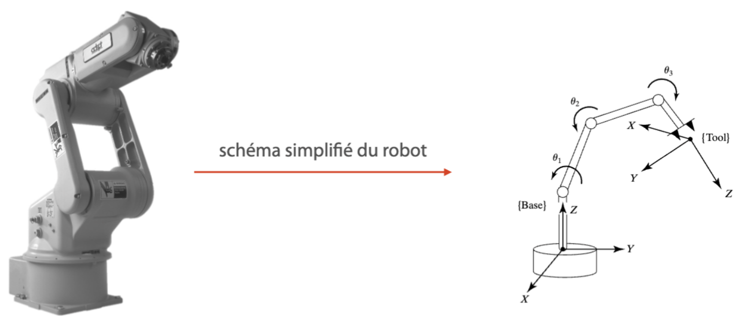


Figure 2.1: Simplified scheme of the robot feature the kinematic pipeline and reference frames.

Direct kinematics allows to compute the position and orientation of the terminal organ given, for instance, the angles of the articulations.



Invert kinematics answers the question the other way around: given the position and orientation of a boyd, how can we compute the values of the articulations angles. Invert kinematics is used for instance for trajectory tracking: given a reference trajectory, how can we compute the speed of the articulations?

2.2 Points, frames and transformations

2.2.1 Position of a point in space

Once that a frame of reference $\{A\}$ is defined, we can localize any point of the universe given a *position vector*:

$$^{A}P = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}$$

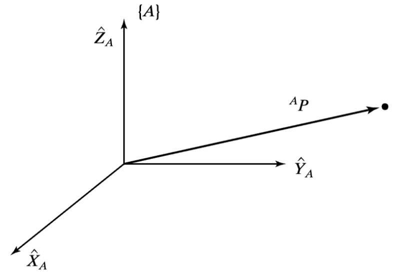


Figure 2.2: Vector and position of the point established in the frame $\{A\}$.

2.2.2 Position and orientation of a body in space

To define the orientation of a body in space, we need to define a frame of reference $\{B\}$ attached to this body. The orientation is therefore defined as the expression of this coordinate system in the reference frame $\{A\}$.

$$^{A}_{B}R = \begin{bmatrix} ^{A}\hat{X}_B & ^{A}\hat{Y}_B & ^{A}\hat{Z}_B \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

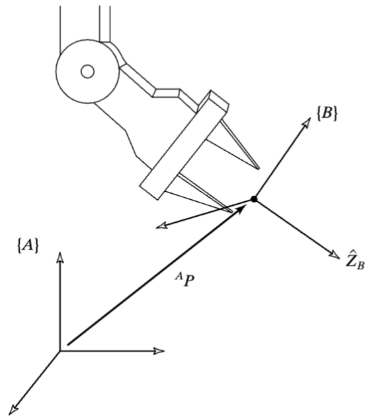


Figure 2.3: Expression of the coordinate system $\{B\}$ in the reference frame $\{A\}$, and the associated position and orientation of the body.

Each element of the matrix $^{A}_{B}R$ is the scalar product between the vectors of the two coordinate systems $\{B\}$ and $\{A\}$:

$$^{A}_{B}R = \begin{bmatrix} ^{A}\hat{X}_B & ^{A}\hat{Y}_B & ^{A}\hat{Z}_B \end{bmatrix} = \begin{bmatrix} \hat{X}_B \cdot \hat{X}_A & \hat{Y}_B \cdot \hat{X}_A & \hat{Z}_B \cdot \hat{X}_A \\ \hat{X}_B \cdot \hat{Y}_A & \hat{Y}_B \cdot \hat{Y}_A & \hat{Z}_B \cdot \hat{Y}_A \\ \hat{X}_B \cdot \hat{Z}_A & \hat{Y}_B \cdot \hat{Z}_A & \hat{Z}_B \cdot \hat{Z}_A \end{bmatrix}$$

The lines correspond to the axes of the frame $\{A\}$ expressed in the frame $\{B\}$. Note that the invert of a rotation matrix is its transpose:

$$^{A}_{B}R^{T} = {}^{A}_{B}R^{-1} = {}^{B}_{A}R$$

2.2.3 Rotation matrices

We showed that the orientation of the body in space could be expressed as a 3-dimensional rotation matrix. The group of 3-dimensional rotation matrices is denoted SO(3), and is composed of all the matrices that are orthonormal, that is orthogonal and with a determinant equal to 1:

$$\mathrm{SO}(3) = \left\{ R \in \mathcal{M}_3(\mathbb{R}) \,\middle|\, RR^T = I_3 \text{ and } \det(R) = +1 \right\}$$

If we write:

$$R = \begin{bmatrix} \hat{X} & \hat{Y} & \hat{Z} \end{bmatrix}$$

Then $R \in \mathrm{SO}(3)$ if and only if:

$$\begin{cases} \hat{X} \cdot \hat{Y} = 0 \\ \hat{Y} \cdot \hat{Z} = 0 \\ \hat{Z} \cdot \hat{X} = 0 \end{cases} \quad \text{and} \quad \begin{cases} \hat{X} \cdot \hat{X} = 1 \\ \hat{Y} \cdot \hat{Y} = 1 \\ \hat{Z} \cdot \hat{Z} = 1 \end{cases}$$

Note that we have 9 degrees of freedom and 6 independent constraints, so the group SO(3) is 3-dimensional.

2.3 Rotation representations

There are several ways to represent a rotation matrix, each with its own advantages and drawbacks. The most common representations are:

- Orthonormal 3 by 3 matrices — 9 components
- Euler angles — 3 components
- Axis-angle representation — 3 components
- Quaternions — 4 components

2.3.1 Euler angles

3 Direct Cinematic

4 Invert Cinematic

5 Reinforcement Learning

6 Locomotion