Extending Layerwise Relevance Propagation using Semiring Annotations

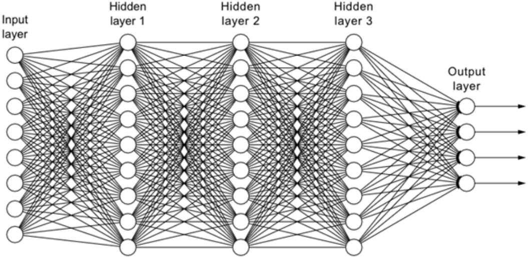
Antoine Groudiev
L3, ENS Ulm

Silviu Maniu – Supervisor
SLIDE Team, LIG

August 30, 2024

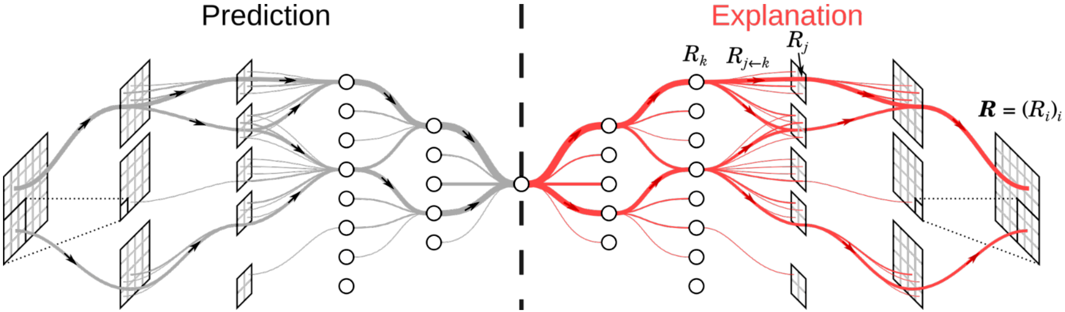
Plan

Problem statement



"castle"

Layerwise Relevance Propagation [11]



Layerwise Relevance Propagation

Initialization

Initialization:

$$R_i^{(L)} = \begin{cases} a_i^{(L)} & \text{if } i = y \text{ (the class we want)} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

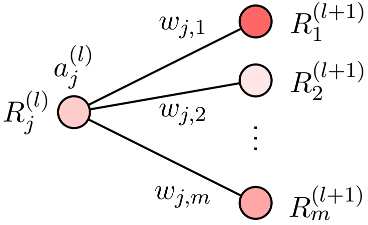
$$\begin{bmatrix} 0 \\ 0 \\ \vdots \\ \mathbf{4.2} \\ \vdots \\ 0 \end{bmatrix} \begin{array}{l} \longrightarrow \texttt{"goldfish"} \\ \longrightarrow \texttt{"street sign"} \\ \\ \longrightarrow \texttt{"castle"} \\ \\ \longrightarrow \texttt{"printer"} \end{array}$$

Layerwise Relevance Propagation

Propagation

LRP-0 rule:

$$R_j^{(l)} = \sum_k \frac{a_j^{(l)} w_{j,k}}{\sum_{j'} a_{j'}^{(l)} w_{j',k}} \cdot R_k^{(l+1)} \tag{2}$$



Other rules exist (LRP-ε, LRP-γ, $z^{\mathcal{B}}$)

LRP Results visualization

Multilayer Perceptron on MNIST dataset

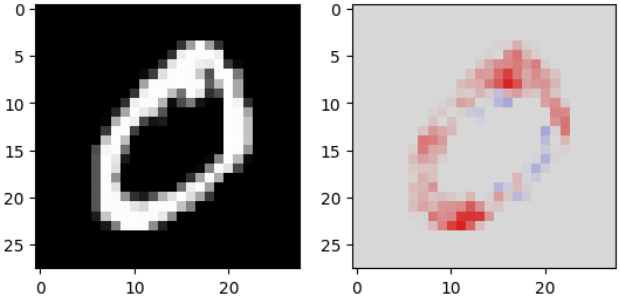


Figure: Reference image and relevance for the class 0

LRP Results visualization

VGG-16 on ImageNet dataset

Figure: Reference image

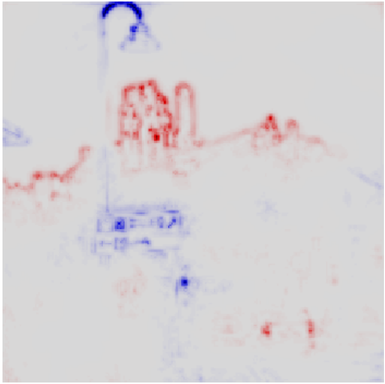


Figure: Relevance for the class "`castle`"

Semiring-based provenance annotations [7, 12]

Definition (Semiring)

A semiring $(\mathbb{K}, \oplus, \otimes, \mathbb{0}, \mathbb{1})$ is such that:

- $\otimes$ distributes over $\oplus$,
- $(\mathbb{K}, \oplus, \mathbb{0})$ is a commutative monoid,
- $(\mathbb{K}, \otimes, \mathbb{1})$ is a monoid such that $\mathbb{0}$ is absorbing

Example

The following structures are semirings:

- Real semiring: $(\mathbb{R}, +, \times, 0, 1)$
- Boolean semiring: $(\{\bot, \top\}, \vee, \wedge, \bot, \top)$
- Counting semiring: $(\mathbb{N}, +, \times, 0, 1)$
- Viterbi semiring: $([0, 1], \max, \times, 0, 1)$

Plan

Introduction
- Problem statement
- Layerwise Relevance Propagation
- Semiring-based provenance annotations

Extending LRP
- Rules modification
- MNIST dataset
- VGG-16 network

Applications
- Image mask computation
- Network pruning using LRP ranking
- Comparison to image perturbation

Semiring generalization of the LRP rule

Consider a semiring $(\mathbb{K}, \oplus, \otimes, \mathbb{0}, \mathbb{1})$
Conversion function:

$$\Theta : \mathbb{R} \longrightarrow \mathbb{K}$$

Initialization:

$$R_i^{(L)} = \begin{cases} \mathbb{1} & \text{if } i = y \\ \mathbb{0} & \text{otherwise} \end{cases} \tag{3}$$

Propagation rule:

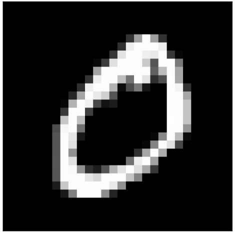
$$R_j^{(l)} = \bigoplus_k \Theta\left(\frac{a_j^{(l)} w_{j,k}}{\sum_{j'} a_{j'}^{(l)} w_{j',k}}\right) \otimes R_k^{(l+1)} \tag{4}$$

Boolean Semiring

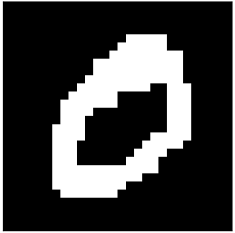
$(\{\perp, \top\}, \vee, \wedge, \perp, \top)$

$$\Theta = x \longmapsto \begin{cases} \top & \text{if } x \geq \theta \\ \perp & \text{otherwise} \end{cases}$$

Reference



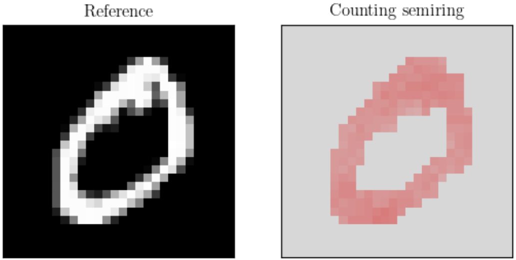
Boolean Semiring



Counting Semiring

$$(\mathbb{N}, +, \times, 0, 1)$$

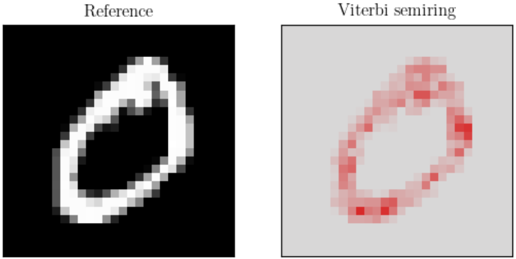
$$\Theta = x \longmapsto \begin{cases} 1 & \text{if } x \geq \theta \\ 0 & \text{otherwise} \end{cases}$$



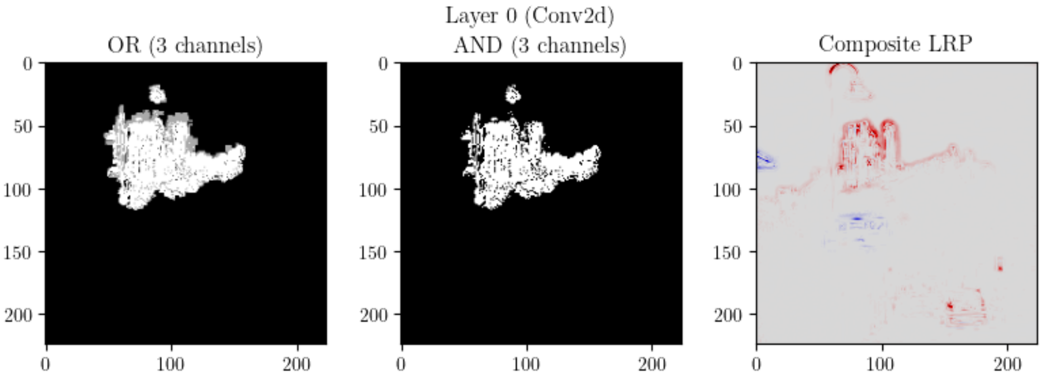
Viterbi Semiring

$$([0,1], \max, \times, 0, 1)$$

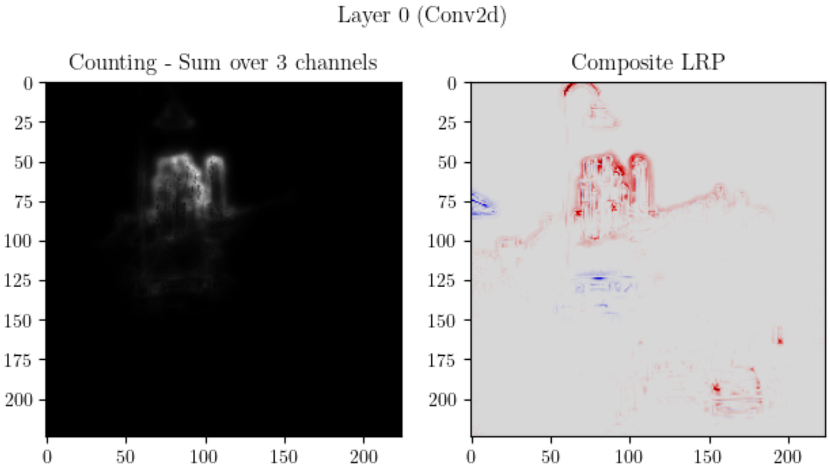
$$R_j^{(l)} = \max_k \underbrace{\left(\frac{\left| a_j^{(l)} w_{j,k}^{(l)} \right|}{\max_{j'} \left| a_{j'}^{(l)} w_{j',k}^{(l)} \right|} \right)}_{\in[0,1]} \cdot R_k^{(l+1)}$$



Boolean semiring

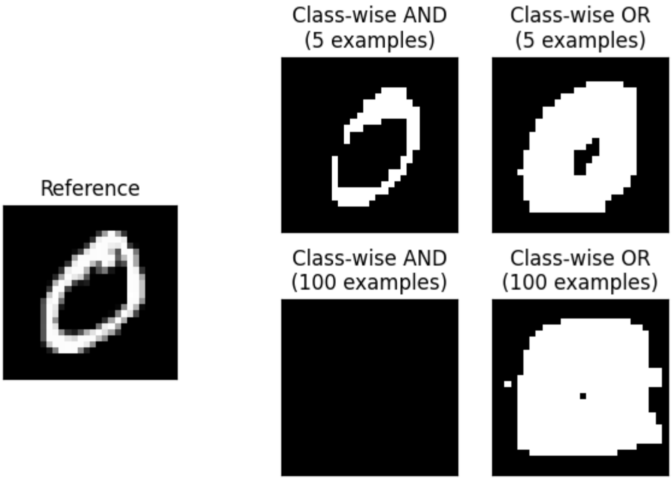


Counting semiring

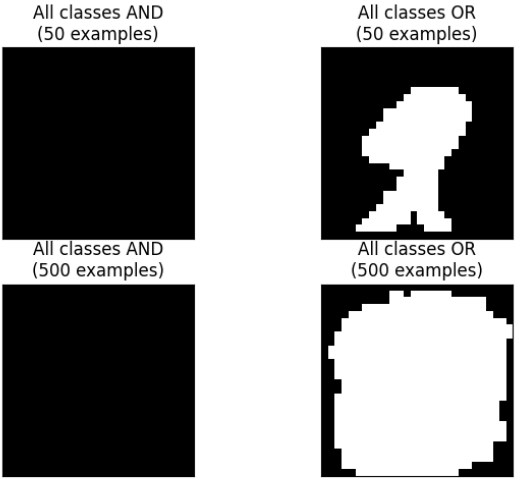


Plan

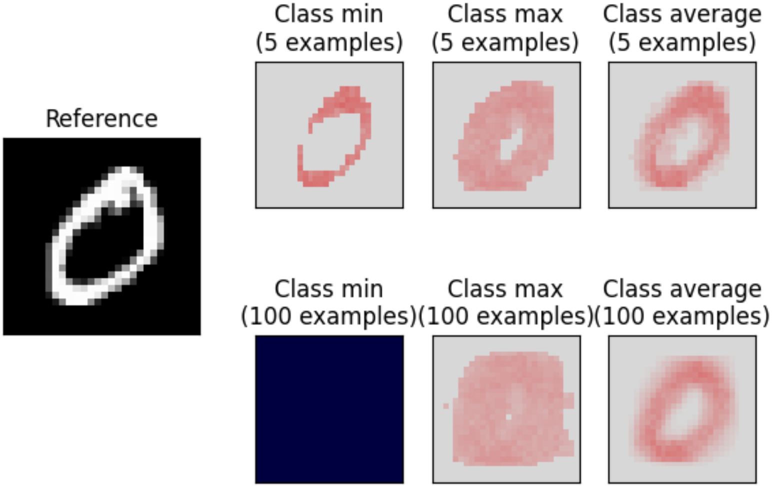
Class-wise mask – Boolean semiring



All classes mask – Boolean semiring

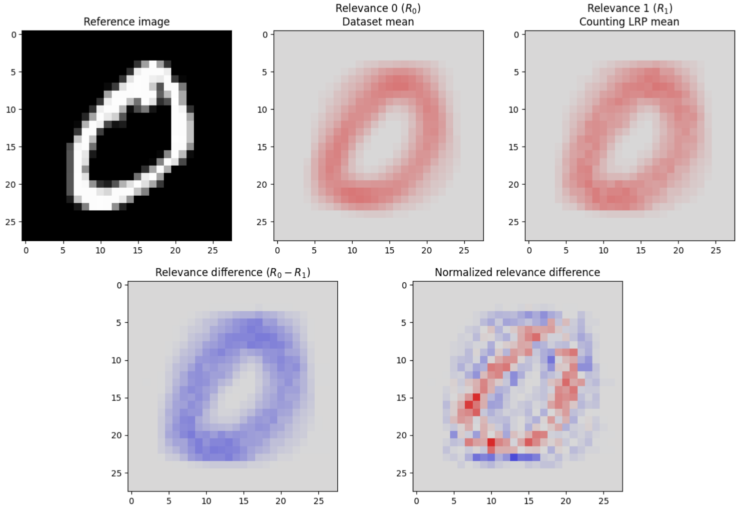


Class-wise mask – Counting semiring



All classes mask – Counting semiring

Comparison to dataset mean



Network pruning using LRP ranking

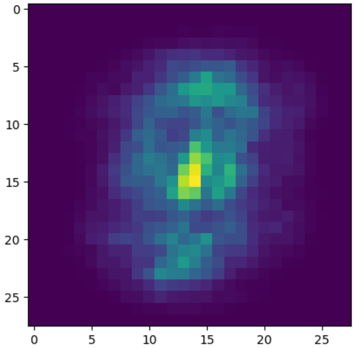
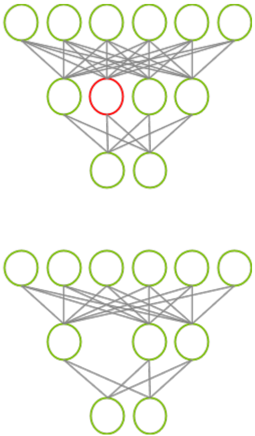
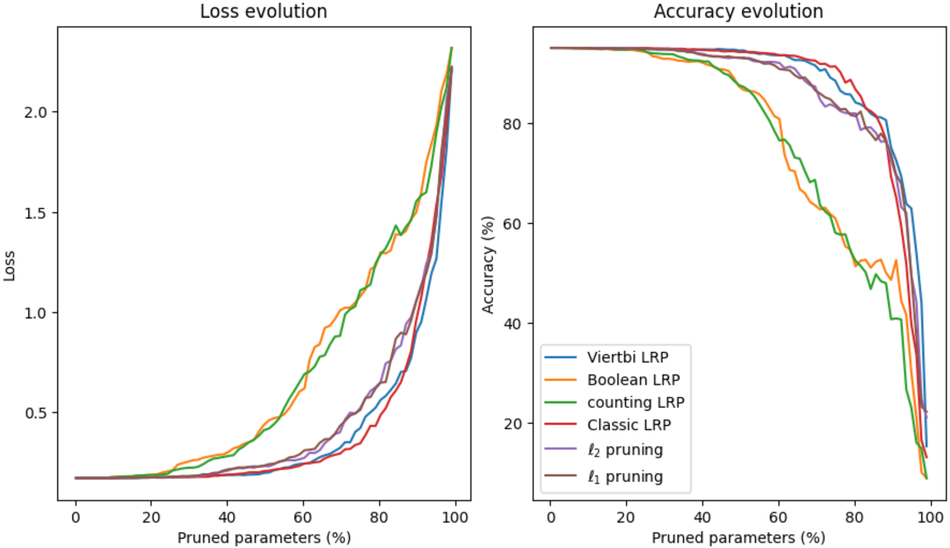


Figure: Relevance mean over the training dataset (Input layer)




Loss evolution
Loss
Pruned parameters (%)
Accuracy evolution
Accuracy (%)
Pruned parameters (%)
Viertbi LRP
Boolean LRP
counting LRP
Classic LRP
ℓ_2 pruning
ℓ_1 pruning

Comparison to image perturbation [5]

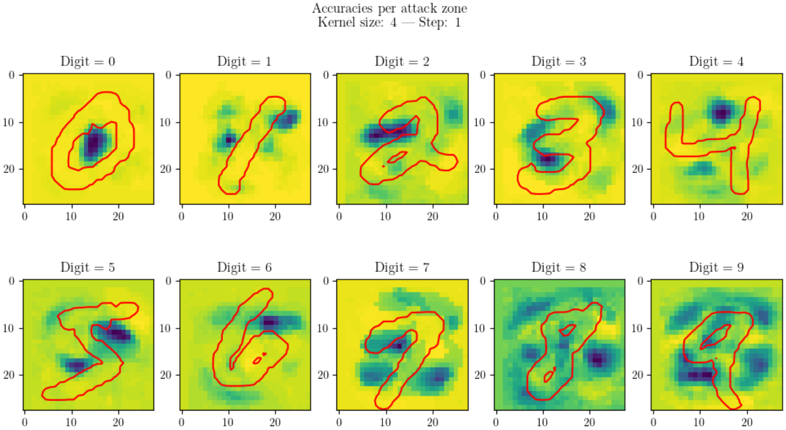


Figure: Accuracies per attack zone

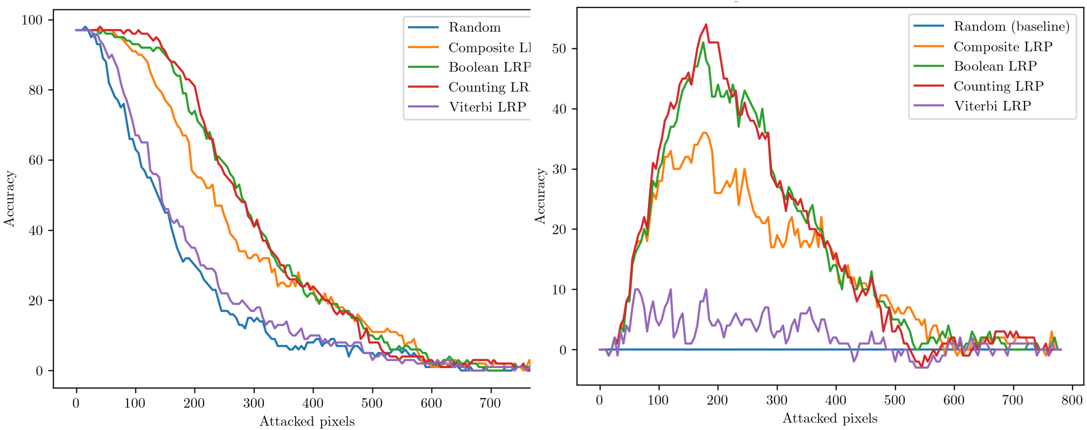


Figure: Accuracy drop for multiple pixels attacks strategies.

[1] Sebastian Bach et al. "On Pixel-Wise Explanations for Non-Linear Classifier Decisions by Layer-Wise Relevance Propagation". In: *PLOS ONE* (2015), pp. 1–46. URL: https://doi.org/10.1371/journal.pone.0130140.

[2] Nick Cammarata et al. "Thread: Circuits". In: *Distill* (2020). https://distill.pub/2020/circuits. DOI: 10.23915/distill.00024.

[3] Hongrong Cheng, Miao Zhang, and Javen Qinfeng Shi. "A Survey on Deep Neural Network Pruning-Taxonomy, Comparison, Analysis, and Recommendations". In: (2023). URL: https://arxiv.org/abs/2308.06767.

[4] Marina Danilevsky et al. "A survey of the state of explainable AI for natural language processing". In: *arXiv preprint* (2020). URL: https://arxiv.org/abs/2010.00711.

[5] Ruth C Fong and Andrea Vedaldi. "Interpretable explanations of black boxes by meaningful perturbation". In: *Proceedings of the IEEE international conference on computer vision*. 2017, pp. 3429–3437. URL: https://arxiv.org/abs/1704.03296.

[6] Robert Geirhos et al. "Shortcut learning in deep neural networks". In: *Nature Machine Intelligence* 2 (2020), pp. 665–673. URL: https://arxiv.org/abs/2004.07780.

[7] Todd J Green, Grigoris Karvounarakis, and Val Tannen. "Provenance semirings". In: *Proceedings of the twenty-sixth ACM SIGMOD-SIGACT-SIGART symposium on Principles of database systems*. 2007, pp. 31–40.

[8] Yann LeCun. *The MNIST database of handwritten digits*. 1998. URL: http://yann.lecun.com/exdb/mnist/.

[9] Aravindh Mahendran and Andrea Vedaldi. "Understanding deep image representations by inverting them". In: *Proceedings of the IEEE conference on computer vision and pattern recognition*. 2015, pp. 5188–5196. URL: https://arxiv.org/abs/1412.0035.

[10] Pavlo Molchanov et al. "Pruning Convolutional Neural Networks for Resource Efficient Inference". In: (2017). URL: https://arxiv.org/abs/1611.06440.

[11] Grégoire Montavon et al. "Layer-Wise Relevance Propagation: An Overview". In: *Explainable AI: Interpreting, Explaining and Visualizing Deep Learning*. Springer International Publishing, 2019, pp. 193–209. URL: https://doi.org/10.1007/978-3-030-28954-6_10.

[12] Yann Ramusat, Silviu Maniu, and Pierre Senellart. "Provenance-Based Algorithms for Rich Queries over Graph Databases". In: *EDBT 2021 - 24th International Conference on Extending Database Technology*. 2021. URL: https://inria.hal.science/hal-03140067.

[13] Wojciech Samek et al. “Evaluating the visualization of what a deep neural network has learned”. In: *IEEE transactions on neural networks and learning systems* 28.11 (2016), pp. 2660–2673. URL: https://arxiv.org/abs/1509.06321.

[14] Ramprasaath R Selvaraju et al. “Grad-CAM: Visual Explanations from Deep Networks via Gradient-based Localization”. In: *Proceedings of the IEEE international conference on computer vision*. 2017, pp. 618–626. URL: https://arxiv.org/pdf/1610.02391.

[15] Pierre Senellart. “Provenance and probabilities in relational databases”. In: *ACM SIGMOD Record* 46.4 (2018), pp. 5–15. URL: https://inria.hal.science/hal-01672566.

[16] Pierre Senellart et al. “ProvSQL: Provenance and probability management in postgresql”. In: *Proceedings of the VLDB Endowment (PVLDB)* (2018). URL: https://inria.hal.science/hal-01851538.

[17] Karen Simonyan and Andrew Zisserman. *Very Deep Convolutional Networks for Large-Scale Image Recognition*. 2015. URL: https://arxiv.org/abs/1409.1556.

[18] Matthew D Zeiler and Rob Fergus. “Visualizing and understanding convolutional networks”. In: *Computer Vision–ECCV 2014: 13th European Conference, Zurich, Switzerland, September 6-12, 2014, Proceedings, Part I 13*. Springer. 2014, pp. 818–833. URL: https://arxiv.org/abs/1311.2901.