# Extending Layerwise Relevance Propagation using Semiring Annotations

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# Plan

## Introduction

- Problem statement
- Layerwise Relevance Propagation
- Semiring-based provenance annotations

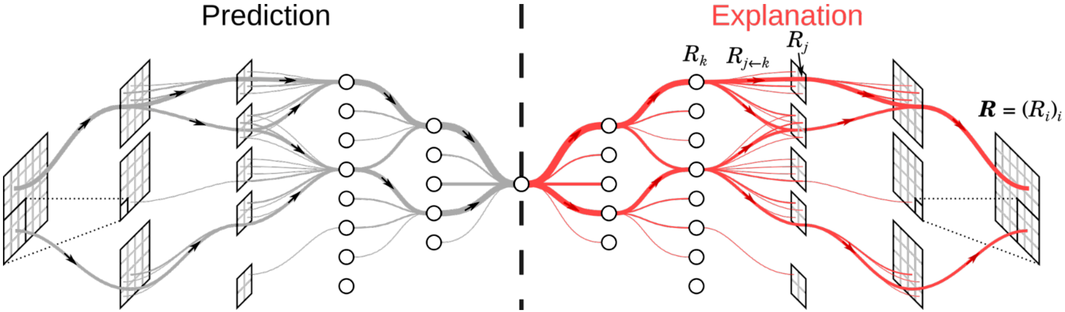
## Extending LRP

## Applications

- Image mask computation
- Network pruning using LRP ranking
- Comparison to image perturbation

# Problem statement

# Layerwise Relevance Propagation



# Layerwise Relevance Propagation

## Propagation rules

Initialization:

$$R_i^{(L)} = \begin{cases} a_i^{(L)} & \text{if } i = y \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

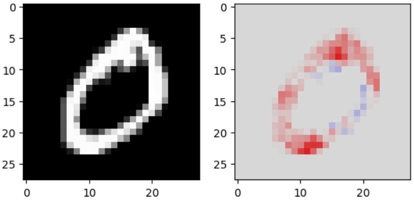
LRP-0 rule:

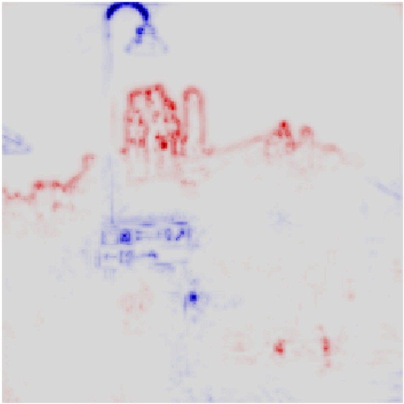
$$R_j^{(l)} = \sum_k \frac{a_j^{(l)} w_{j,k}}{\sum_{j'} a_{j'}^{(l)} w_{j',k}} R_k^{(l+1)} \tag{2}$$

Other rules exist (LRP-$\epsilon$, LRP-$\gamma$, $z^{\mathcal{B}}$)

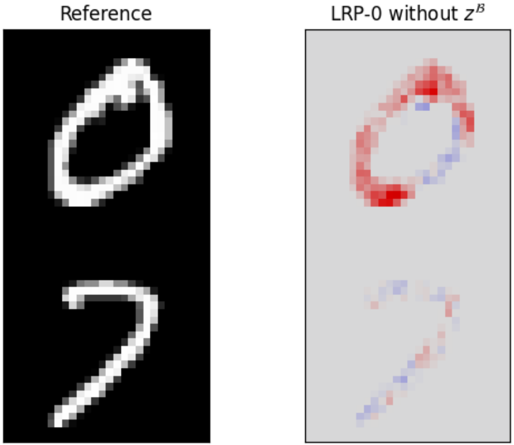
# Layerwise Relevance Propagation

## Results visualization



# Pertinence of LRP results



# Semiring-based provenance annotations

## Definition (Semiring)

A semiring $(\mathbb{K}, \oplus, \otimes, \mathbf{0}, \mathbf{1})$ is composed of a set $\mathbb{K}$, binary operators $\oplus$ and $\otimes$ such that $\otimes$ distributes over $\oplus$, verifying the following properties:

- $(\mathbb{K}, \oplus, \mathbf{0})$ is a commutative monoid
- $(\mathbb{K}, \otimes, \mathbf{1})$ is a monoid such that $\mathbf{0}$ is absorbing

## Example

The following structures are semirings:

- Real semiring: $(\mathbb{R}, +, \times, 0, 1)$
- Boolean semiring: $(\{\bot, \top\}, \vee, \wedge, \bot, \top)$
- Counting semiring: $(\mathbb{N}, +, \times, 0, 1)$
- Viterbi semiring: $([0, 1], \max, \times, 0, 1)$

# Plan

# Simplifying LRP rule

Remove the denominator:

$$R_j^{(l)} = \sum_k \frac{a_j^{(l)} w_{j,k}^{(l)}}{\sum_{j'} a_{j'}^{(l)} w_{j',k}^{(l)}} R_k^{(l+1)} \quad \longrightarrow \quad R_j^{(l)} = \sum_k a_j^{(l)} w_{j,k}^{(l)} \cdot R_k^{(l+1)}$$

## Semiring generalization of the LRP rule

Consider a semiring $(\mathbb{K}, \oplus, \otimes, \mathbf{0}, \mathbf{1})$

Conversion functions for activations, weights:

$$\Theta_a : \mathbb{R} \longrightarrow \mathbb{K}$$
$$\Theta_w : \mathbb{R} \longrightarrow \mathbb{K}$$

Initialization:

$$R_i^{(L)} = \begin{cases} \Theta_a\left(a_i^{(L)}\right) & \text{if } i = y \\ \mathbf{0} & \text{otherwise} \end{cases} \tag{3}$$

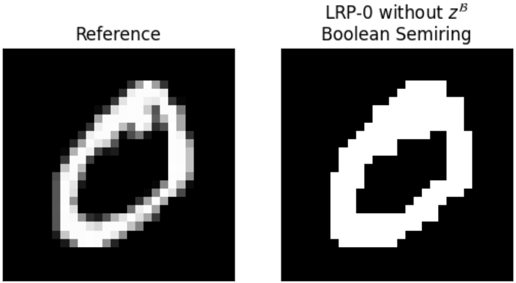
Propagation rule:

$$R_j^{(l)} = \bigoplus_k \Theta_a\left(a_j^{(l)}\right) \otimes \Theta_w\left(w_{j,k}^{(l)}\right) \otimes R_k^{(l+1)} \tag{4}$$

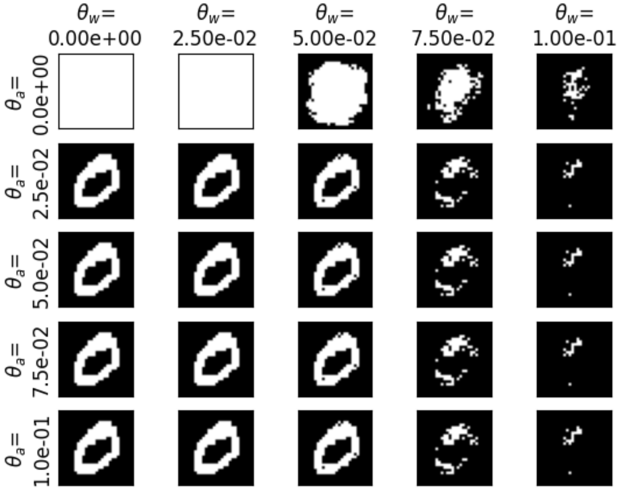
# Boolean Semiring

## $(\{\perp, \top\}, \vee, \wedge, \perp, \top)$

$$\Theta_a = a \longmapsto \begin{cases} \top & \text{if } a \geq \theta_a \\ \perp & \text{otherwise} \end{cases} \qquad \Theta_w = w \longmapsto \begin{cases} \top & \text{if } w \geq \theta_w \\ \perp & \text{otherwise} \end{cases}$$



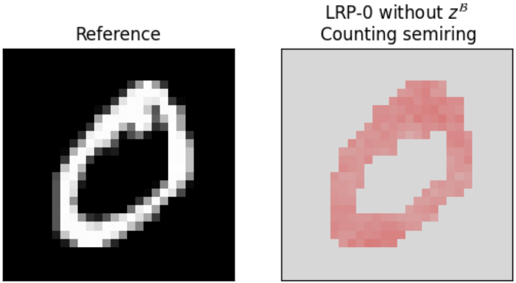
# Influence of the thresholds



# Counting Semiring

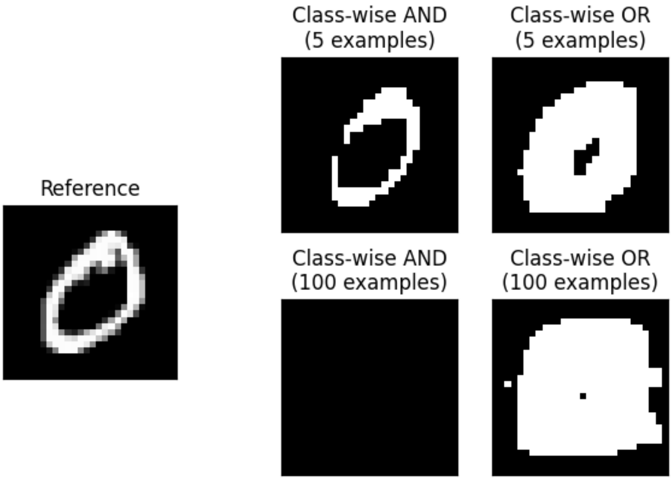
$$(\mathbb{N}, +, \times, 0, 1)$$

$$\Theta_a = a \longmapsto \begin{cases} 1 & \text{if } a \geq \theta_a \\ 0 & \text{otherwise} \end{cases} \qquad \Theta_w = w \longmapsto \begin{cases} 1 & \text{if } w \geq \theta_w \\ 0 & \text{otherwise} \end{cases}$$

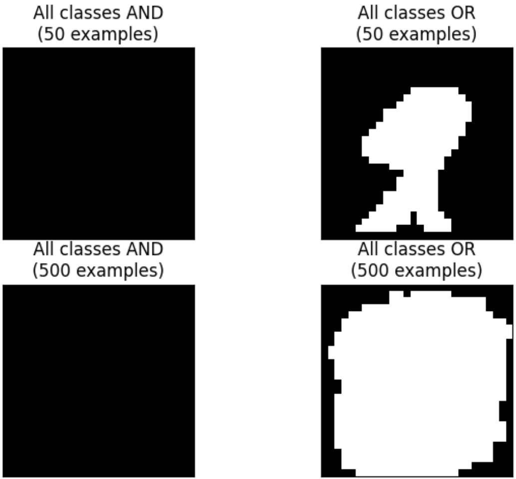


# Plan

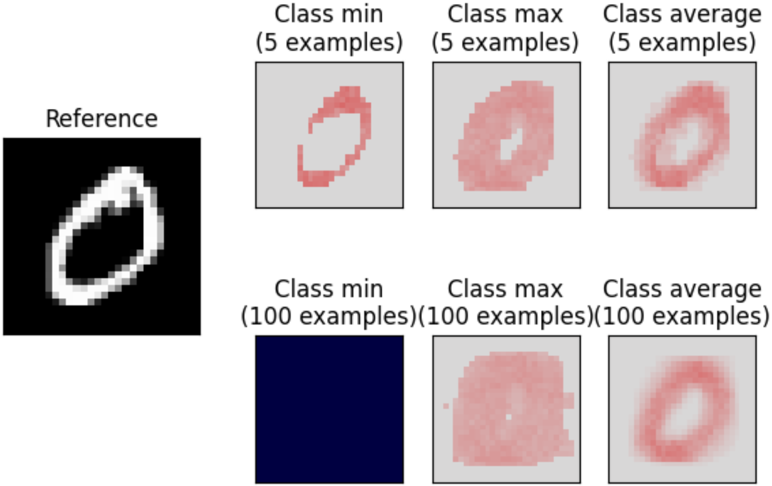
# Class-wise mask - Boolean semiring



# All classes mask - Boolean semiring

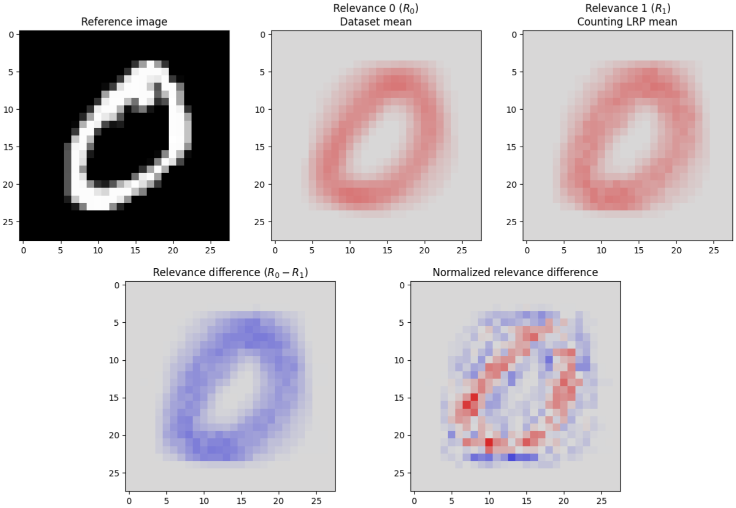


# Class-wise mask - Counting semiring



# All classes mask - Counting semiring

# Comparison to dataset mean



# Network pruning using LRP ranking

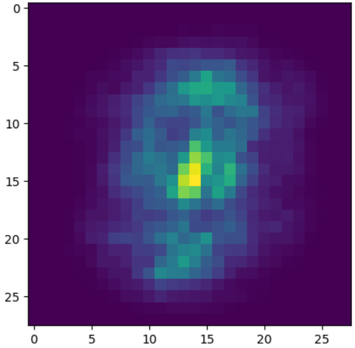
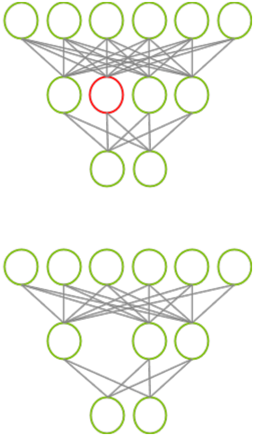
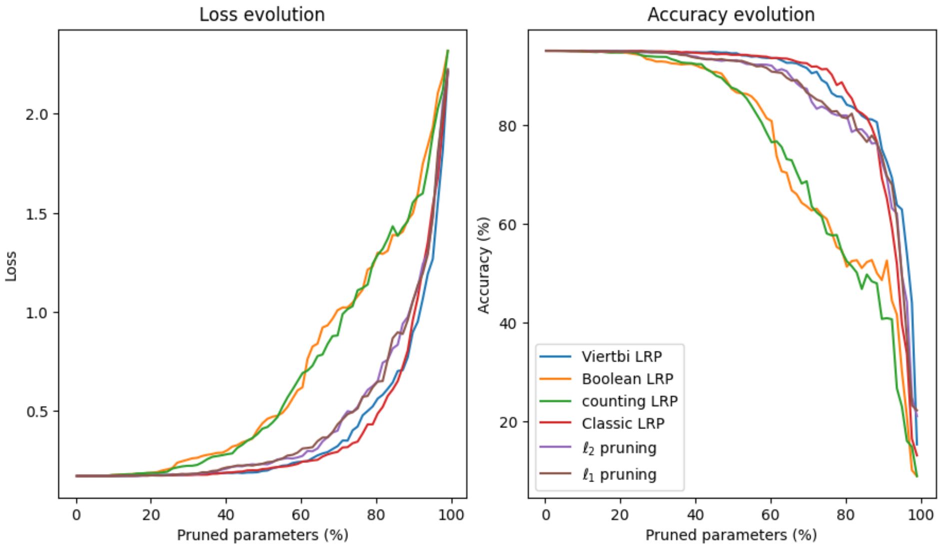



Figure: Relevance mean over the training dataset




Loss evolution
Loss
2.0
1.5
1.0
0.5
0
20
40
60
80
100
Pruned parameters (%)
Accuracy evolution
Accuracy (%)
80
60
40
20
0
20
40
60
80
100
Pruned parameters (%)
Viertbi LRP
Boolean LRP
counting LRP
Classic LRP
$\ell_2$ pruning
$\ell_1$ pruning

# Comparison to image perturbation

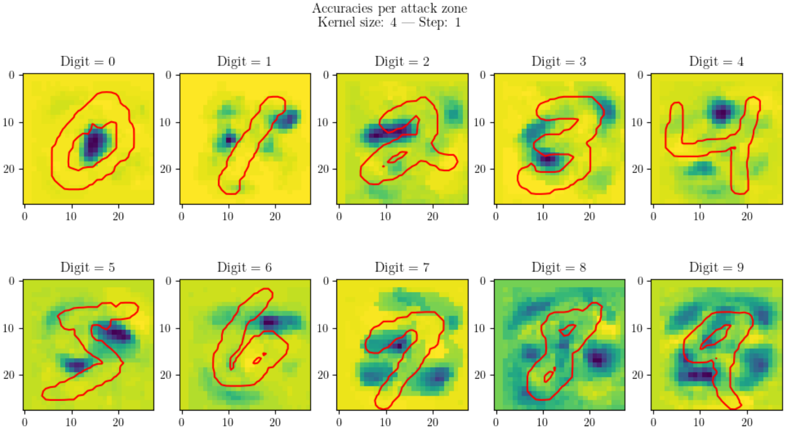



Figure: Accuracies per attack zone

[1] Sebastian Bach et al. "On Pixel-Wise Explanations for Non-Linear Classifier Decisions by Layer-Wise Relevance Propagation". In: *PLOS ONE* (2015), pp. 1–46. DOI: 10.1371/journal.pone.0130140. URL: https://doi.org/10.1371/journal.pone.0130140.

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[3] Robert Geirhos et al. "Shortcut learning in deep neural networks". In: *Nature Machine Intelligence* 2 (2020), pp. 665–673.

[4] Todd J Green, Grigoris Karvounarakis, and Val Tannen. "Provenance semirings". In: *Proceedings of the twenty-sixth ACM SIGMOD-SIGACT-SIGART symposium on Principles of database systems*. 2007, pp. 31–40.

[5] Grégoire Montavon et al. "Layer-Wise Relevance Propagation: An Overview". In: *Explainable AI: Interpreting, Explaining and Visualizing Deep Learning*. Springer International Publishing, 2019, pp. 193–209. URL: https://doi.org/10.1007/978-3-030-28954-6_10.

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