Extending Layerwise Relevance Propagation using Semiring Annotations

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Plan

Introduction
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- Layerwise Relevance Propagation
- Semiring-based provenance annotations

Extending LRP

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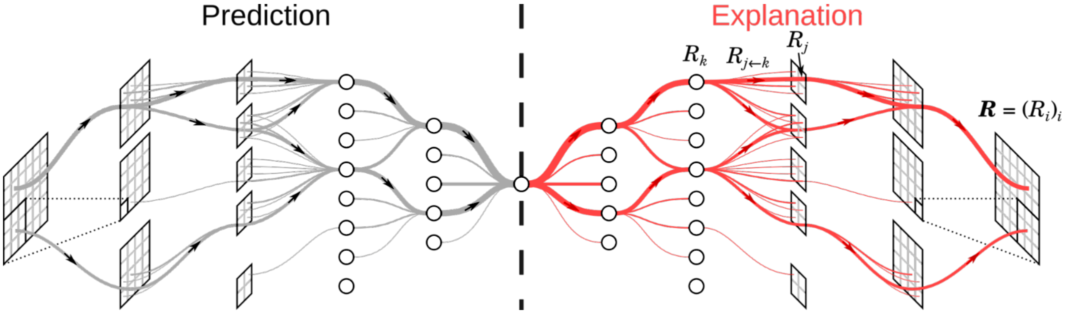
- Image mask computation
- Network pruning using LRP ranking
- Comparison to image perturbation

Problem statement

Layerwise Relevance Propagation



Layerwise Relevance Propagation

Propagation rules

Initialization:

$$R_i^{(L)} = \begin{cases} a_i^{(L)} & \text{if } i = y \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

LRP-0 rule:

$$R_j^{(l)} = \sum_k \frac{a_j^{(l)} w_{j,k}}{\sum_{j'} a_{j'}^{(l)} w_{j',k}} R_k^{(l+1)} \tag{2}$$

Other rules exist (LRP-ϵ, LRP-γ, $z^{\mathcal{B}}$)

LRP Results visualization

Multilayer Perceptron on MNIST dataset

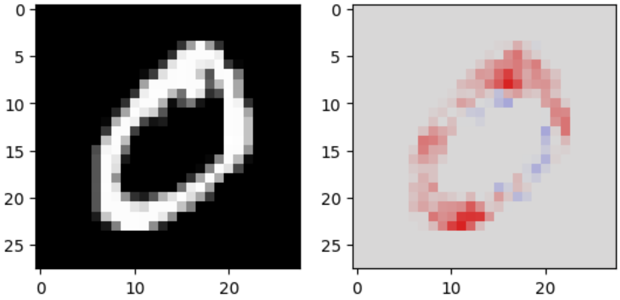


Figure: Reference image and relevance for the class 0

LRP Results visualization

VVG-16 on ImageNet dataset

Figure: Reference image

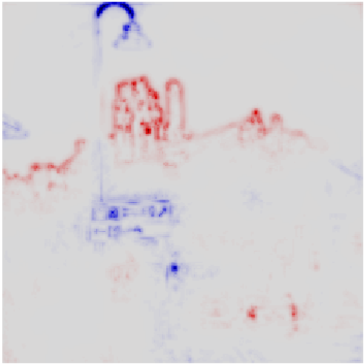
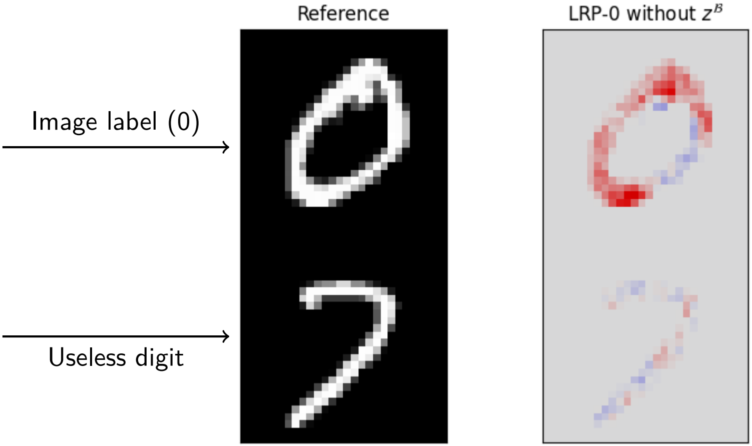


Figure: Relevance for the class `castle`

Pertinence of LRP results



Semiring-based provenance annotations

Definition (Semiring)

A semiring $(\mathbb{K}, \oplus, \otimes, \mathbf{0}, \mathbf{1})$ is composed of a set $\mathbb{K}$, binary operators $\oplus$ and $\otimes$ such that $\otimes$ distributes over $\oplus$, verifying the following properties:

- $(\mathbb{K}, \oplus, \mathbf{0})$ is a commutative monoid
- $(\mathbb{K}, \otimes, \mathbf{1})$ is a monoid such that $\mathbf{0}$ is absorbing

Example

The following structures are semirings:

- Real semiring: $(\mathbb{R}, +, \times, 0, 1)$
- Boolean semiring: $(\{\bot, \top\}, \vee, \wedge, \bot, \top)$
- Counting semiring: $(\mathbb{N}, +, \times, 0, 1)$
- Viterbi semiring: $([0, 1], \max, \times, 0, 1)$

Plan

Simplifying LRP rule

Remove the denominator:

$$R_j^{(l)} = \sum_k \frac{a_j^{(l)} w_{j,k}^{(l)}}{\sum_{j'} a_{j'}^{(l)} w_{j',k}^{(l)}} R_k^{(l+1)} \quad \longrightarrow \quad R_j^{(l)} = \sum_k a_j^{(l)} w_{j,k}^{(l)} \cdot R_k^{(l+1)}$$

Semiring generalization of the LRP rule

Consider a semiring $(\mathbb{K}, \oplus, \otimes, \mathbf{0}, \mathbf{1})$
Conversion functions for activations, weights:

$$\Theta_a : \mathbb{R} \longrightarrow \mathbb{K}$$
$$\Theta_w : \mathbb{R} \longrightarrow \mathbb{K}$$

Initialization:

$$R_i^{(L)} = \begin{cases} \Theta_a\left(a_i^{(L)}\right) & \text{if } i = y \\ \mathbf{0} & \text{otherwise} \end{cases} \tag{3}$$

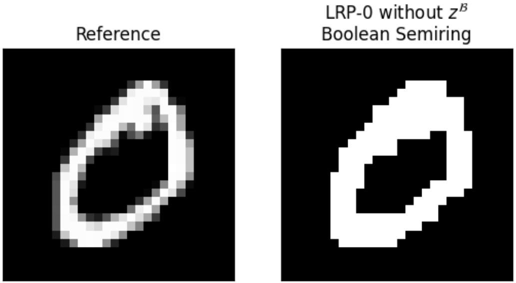
Propagation rule:

$$R_j^{(l)} = \bigoplus_k \Theta_a\left(a_j^{(l)}\right) \otimes \Theta_w\left(w_{j,k}^{(l)}\right) \otimes R_k^{(l+1)} \tag{4}$$

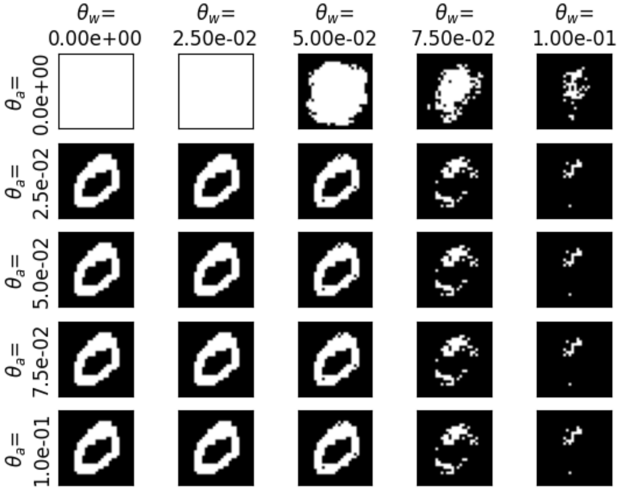
Boolean Semiring

$(\{\bot, \top\}, \vee, \wedge, \bot, \top)$

$$\Theta_a = a \longmapsto \begin{cases} \top & \text{if } a \geq \theta_a \\ \bot & \text{otherwise} \end{cases} \qquad \Theta_w = w \longmapsto \begin{cases} \top & \text{if } w \geq \theta_w \\ \bot & \text{otherwise} \end{cases}$$



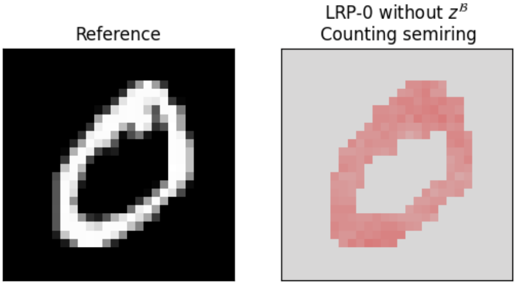
Influence of the thresholds



Counting Semiring

$$(\mathbb{N}, +, \times, 0, 1)$$

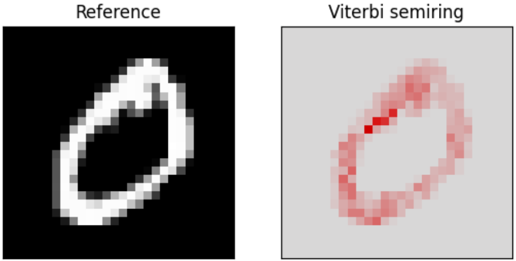
$$\Theta_a = a \longmapsto \begin{cases} 1 & \text{if } a \geq \theta_a \\ 0 & \text{otherwise} \end{cases} \qquad \Theta_w = w \longmapsto \begin{cases} 1 & \text{if } w \geq \theta_w \\ 0 & \text{otherwise} \end{cases}$$



Viterbi Semiring

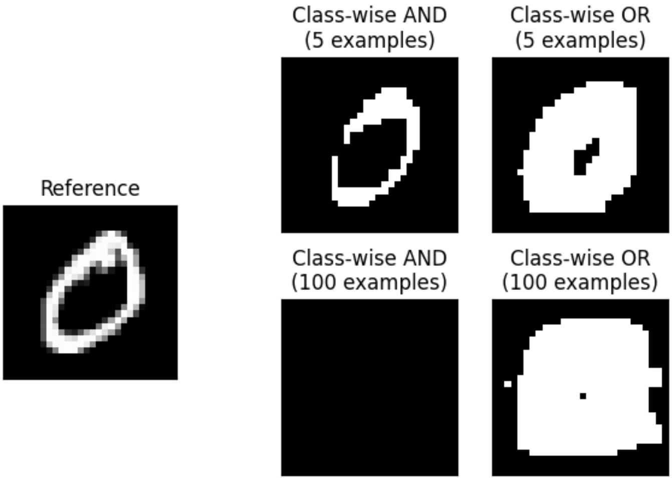
$$([0,1], \max, \times, 0, 1)$$

$$R_j^{(l)} = \max_k \underbrace{\left(\frac{\left| a_j^{(l)} w_{j,k}^{(l)} \right|}{\max_{j'} \left| a_{j'}^{(l)} w_{j',k}^{(l)} \right|} \right)}_{\in [0,1]} \cdot R_k^{(l+1)}$$

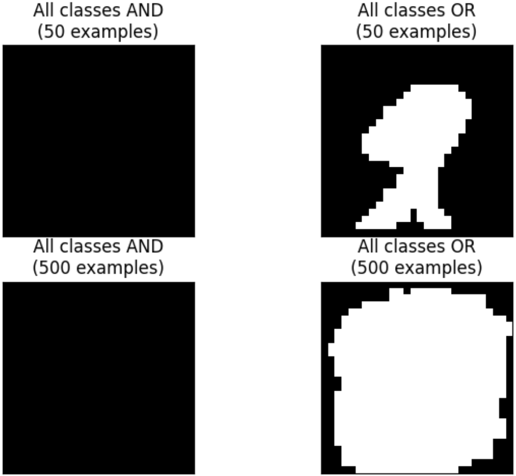


Plan

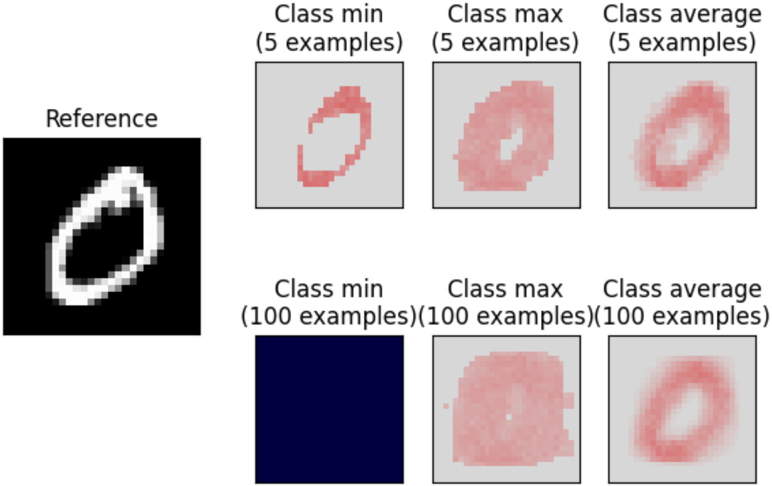
Class-wise mask - Boolean semiring



All classes mask - Boolean semiring

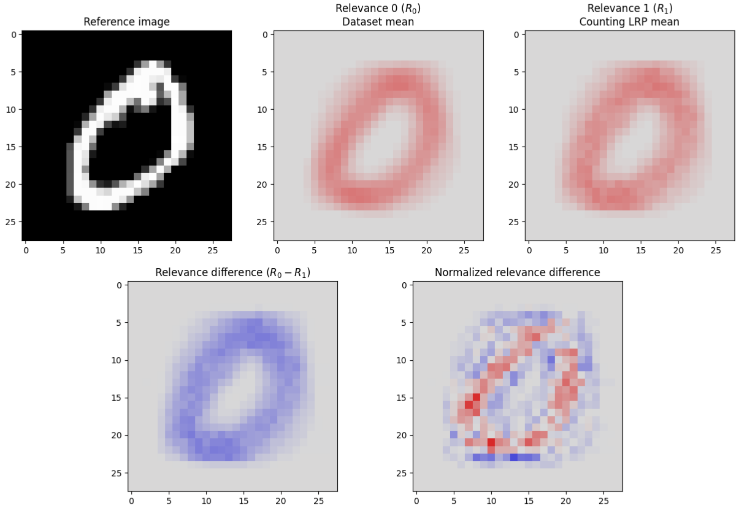


Class-wise mask - Counting semiring



All classes mask - Counting semiring

Comparison to dataset mean



Network pruning using LRP ranking

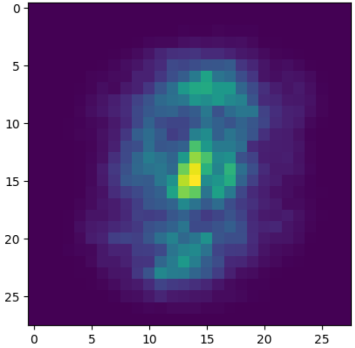
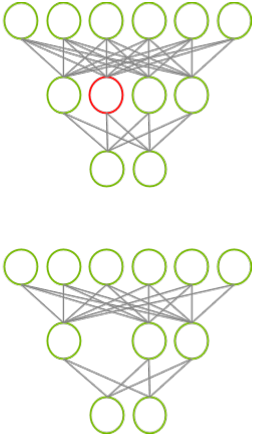
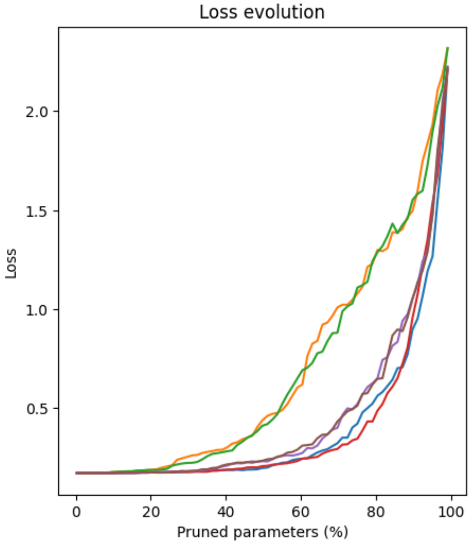
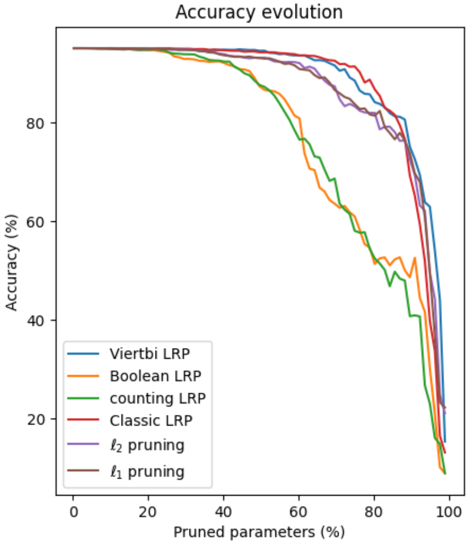


Figure: Relevance mean over the training dataset




Loss evolution
2.0
1.5
1.0
0.5
Loss
0
20
40
60
80
100
Pruned parameters (%)


Accuracy evolution
80
60
40
20
Accuracy (%)
0
20
40
60
80
100
Pruned parameters (%)
Viertbi LRP
Boolean LRP
counting LRP
Classic LRP
ℓ_2 pruning
ℓ_1 pruning

Comparison to image perturbation

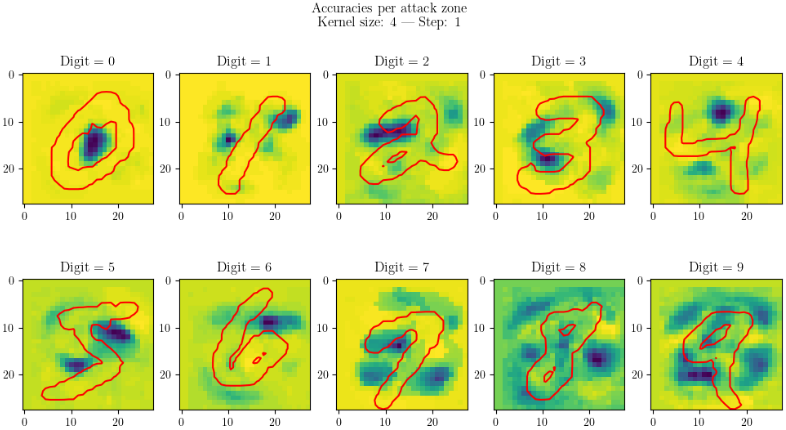


Figure: Accuracies per attack zone

[1] Sebastian Bach et al. "On Pixel-Wise Explanations for Non-Linear Classifier Decisions by Layer-Wise Relevance Propagation". In: *PLOS ONE* (2015), pp. 1–46. DOI: 10.1371/journal.pone.0130140. URL: https://doi.org/10.1371/journal.pone.0130140.

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[3] Robert Geirhos et al. "Shortcut learning in deep neural networks". In: *Nature Machine Intelligence* 2 (2020), pp. 665–673.

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[5] Grégoire Montavon et al. "Layer-Wise Relevance Propagation: An Overview". In: *Explainable AI: Interpreting, Explaining and Visualizing Deep Learning*. Springer International Publishing, 2019, pp. 193–209. URL: https://doi.org/10.1007/978-3-030-28954-6_10.

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