Extending Layerwise Relevance Propagation using Semiring Annotations

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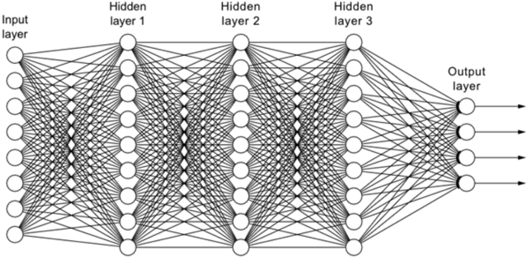
Tuesday, July 9th

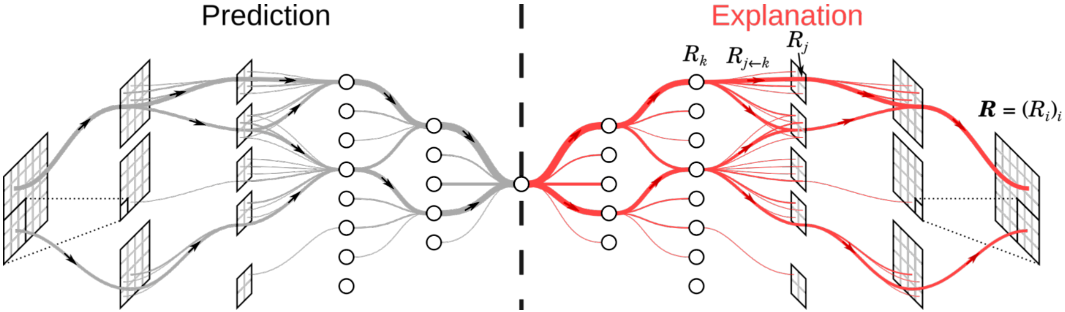
Plan

Problem statement



"castle"

Layerwise Relevance Propagation [5]



Layerwise Relevance Propagation

Initialization

Initialization:

$$R_i^{(L)} = \begin{cases} a_i^{(L)} & \text{if } i = y \text{ (the class we want)} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

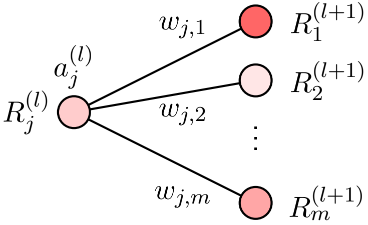
$$\begin{bmatrix} 0 \\ 0 \\ \vdots \\ \mathbf{4.2} \\ \vdots \\ 0 \end{bmatrix} \begin{array}{l} \longrightarrow \texttt{"goldfish"} \\ \longrightarrow \texttt{"street sign"} \\ \\ \longrightarrow \texttt{"castle"} \\ \\ \longrightarrow \texttt{"printer"} \end{array}$$

Layerwise Relevance Propagation

Propagation

LRP-0 rule:

$$R_j^{(l)} = \sum_k \frac{a_j^{(l)} w_{j,k}}{\sum_{j'} a_{j'}^{(l)} w_{j',k}} R_k^{(l+1)} \tag{2}$$



Other rules exist (LRP-ε, LRP-γ, $z^{\mathcal{B}}$)

LRP Results visualization

Multilayer Perceptron on MNIST dataset

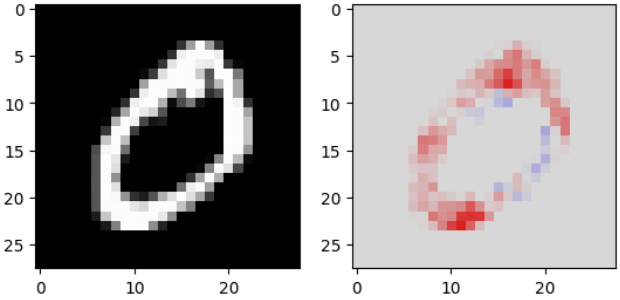


Figure: Reference image and relevance for the class 0

LRP Results visualization

VGG-16 on ImageNet dataset

Figure: Reference image

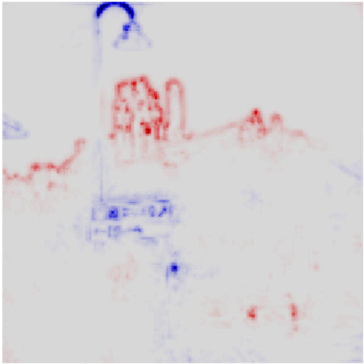
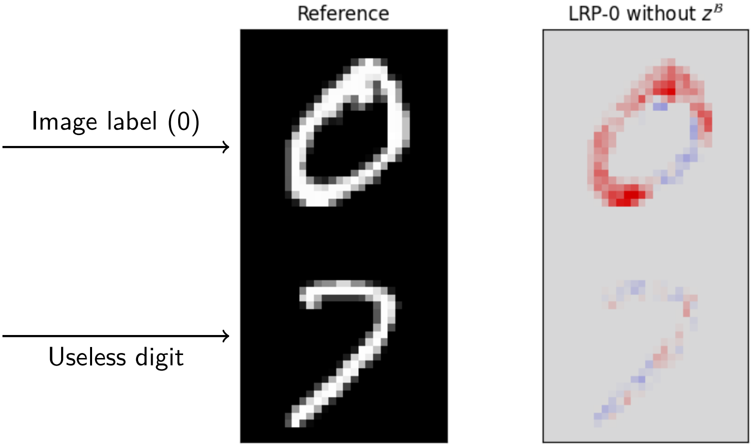


Figure: Relevance for the class "castle"

Pertinence of LRP results



Semiring-based provenance annotations [4, 6]

Definition (Semiring)

A semiring $(\mathbb{K}, \oplus, \otimes, \mathbf{0}, \mathbf{1})$ is such that:

- $\otimes$ distributes over $\oplus$,
- $(\mathbb{K}, \oplus, \mathbf{0})$ is a commutative monoid,
- $(\mathbb{K}, \otimes, \mathbf{1})$ is a monoid such that $\mathbf{0}$ is absorbing

Example

The following structures are semirings:

- Real semiring: $(\mathbb{R}, +, \times, 0, 1)$
- Boolean semiring: $(\{\bot, \top\}, \vee, \wedge, \bot, \top)$
- Counting semiring: $(\mathbb{N}, +, \times, 0, 1)$
- Viterbi semiring: $([0, 1], \max, \times, 0, 1)$

Plan

Simplifying LRP rule

Remove the denominator:

$$R_j^{(l)} = \sum_k \frac{a_j^{(l)} w_{j,k}^{(l)}}{\sum_{j'} a_{j'}^{(l)} w_{j',k}^{(l)}} R_k^{(l+1)} \quad \longrightarrow \quad R_j^{(l)} = \sum_k a_j^{(l)} w_{j,k}^{(l)} \cdot R_k^{(l+1)}$$

Use only LRP-0 rule (no ε, γ, $z^{\mathcal{B}}$, ...)

Semiring generalization of the LRP rule

Consider a semiring $(\mathbb{K}, \oplus, \otimes, \mathbf{0}, \mathbf{1})$
Conversion functions for activations and weights:

$$\Theta_a : \mathbb{R} \longrightarrow \mathbb{K}$$
$$\Theta_w : \mathbb{R} \longrightarrow \mathbb{K}$$

Initialization:

$$R_i^{(L)} = \begin{cases} \Theta_a\left(a_i^{(L)}\right) & \text{if } i = y \\ \mathbf{0} & \text{otherwise} \end{cases} \tag{3}$$

Propagation rule:

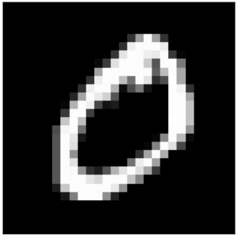
$$R_j^{(l)} = \bigoplus_k \Theta_a\left(a_j^{(l)}\right) \otimes \Theta_w\left(w_{j,k}^{(l)}\right) \otimes R_k^{(l+1)} \tag{4}$$

Boolean Semiring

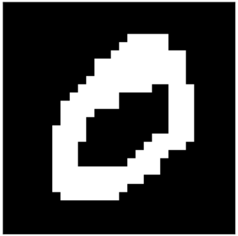
$$(\{\perp, \top\}, \vee, \wedge, \perp, \top)$$

$$\Theta_a = a \longmapsto \begin{cases} \top & \text{if } a \geq \theta_a \\ \perp & \text{otherwise} \end{cases} \qquad \Theta_w = w \longmapsto \begin{cases} \top & \text{if } w \geq \theta_w \\ \perp & \text{otherwise} \end{cases}$$

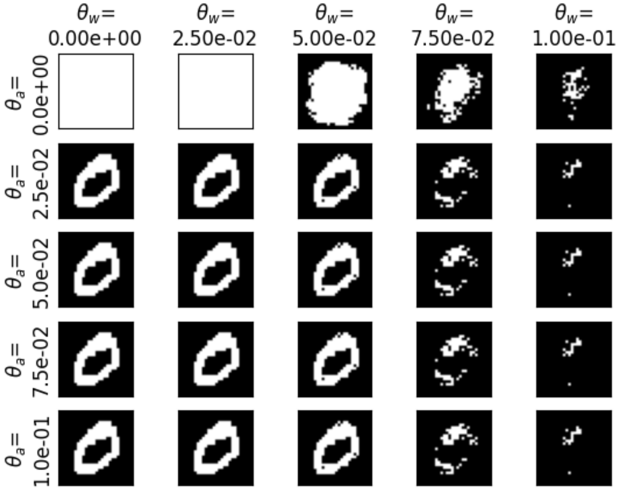
Reference



LRP-0 without $z^{\mathcal{B}}$
Boolean Semiring



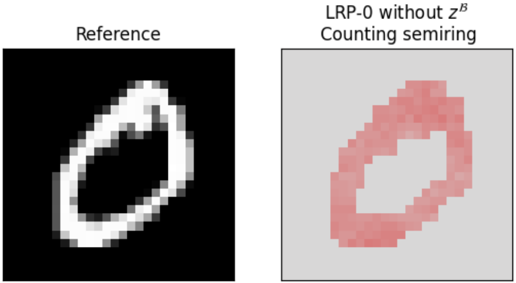
Influence of the thresholds



Counting Semiring

$$(\mathbb{N}, +, \times, 0, 1)$$

$$\Theta_a = a \longmapsto \begin{cases} 1 & \text{if } a \geq \theta_a \\ 0 & \text{otherwise} \end{cases} \qquad \Theta_w = w \longmapsto \begin{cases} 1 & \text{if } w \geq \theta_w \\ 0 & \text{otherwise} \end{cases}$$

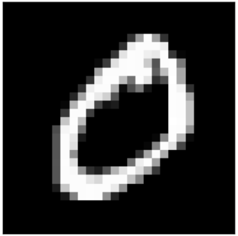


Viterbi Semiring

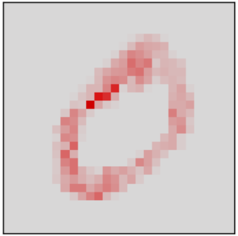
$([0,1], \max, \times, 0, 1)$

$$R_j^{(l)} = \max_k \underbrace{\left(\frac{\left| a_j^{(l)} w_{j,k}^{(l)} \right|}{\max_{j'} \left| a_{j'}^{(l)} w_{j',k}^{(l)} \right|} \right)}_{\in[0,1]} \cdot R_k^{(l+1)}$$

Reference

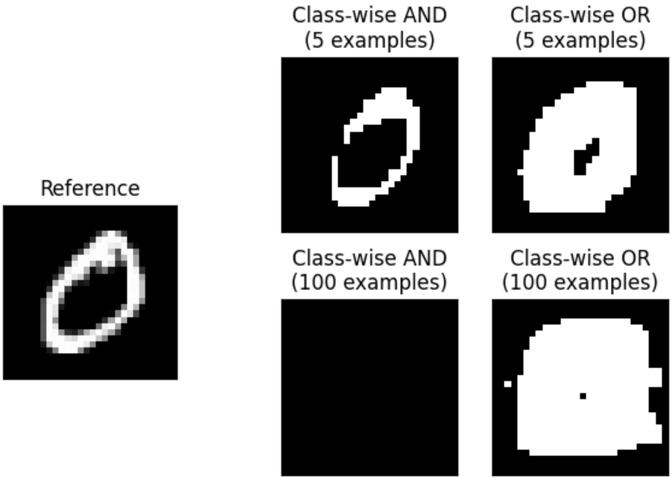


Viterbi semiring

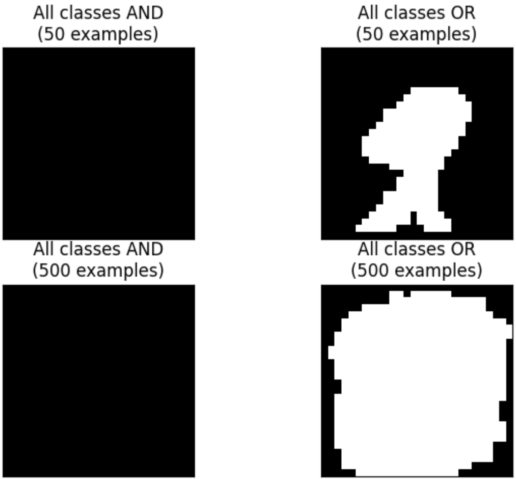


Plan

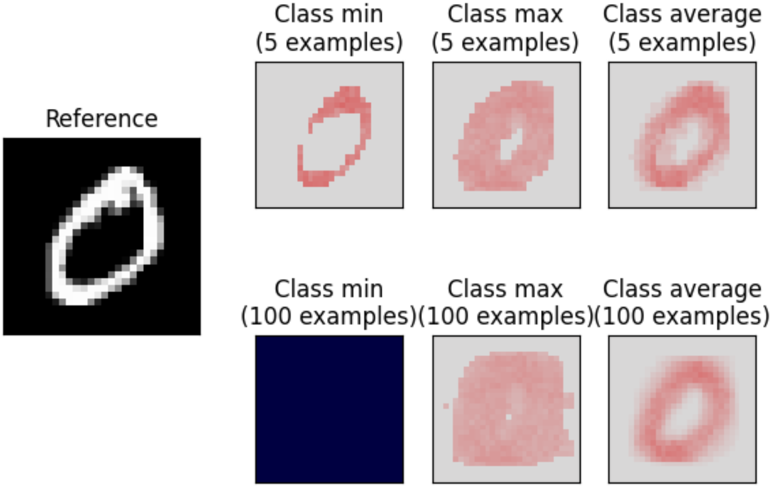
Class-wise mask – Boolean semiring



All classes mask – Boolean semiring

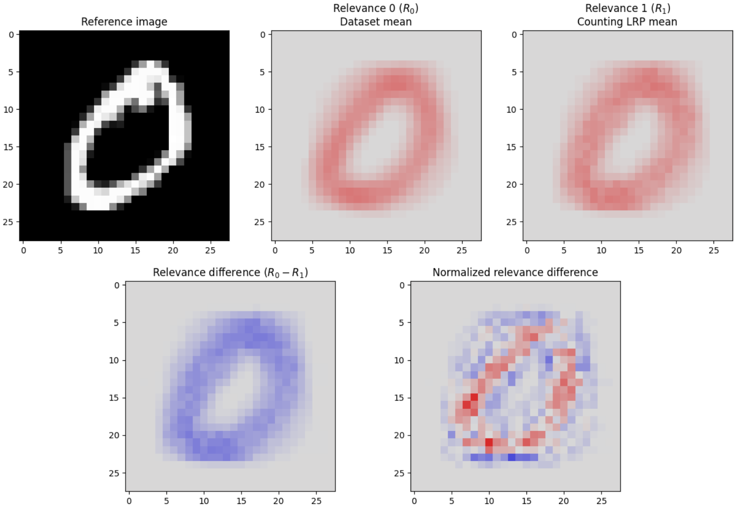


Class-wise mask – Counting semiring



All classes mask – Counting semiring

Comparison to dataset mean



Network pruning using LRP ranking

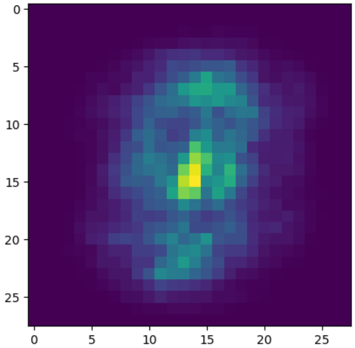
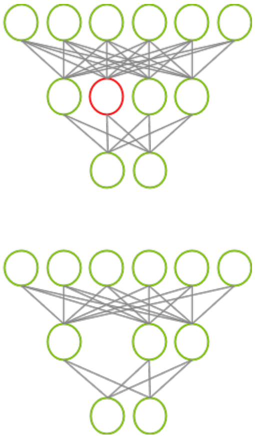
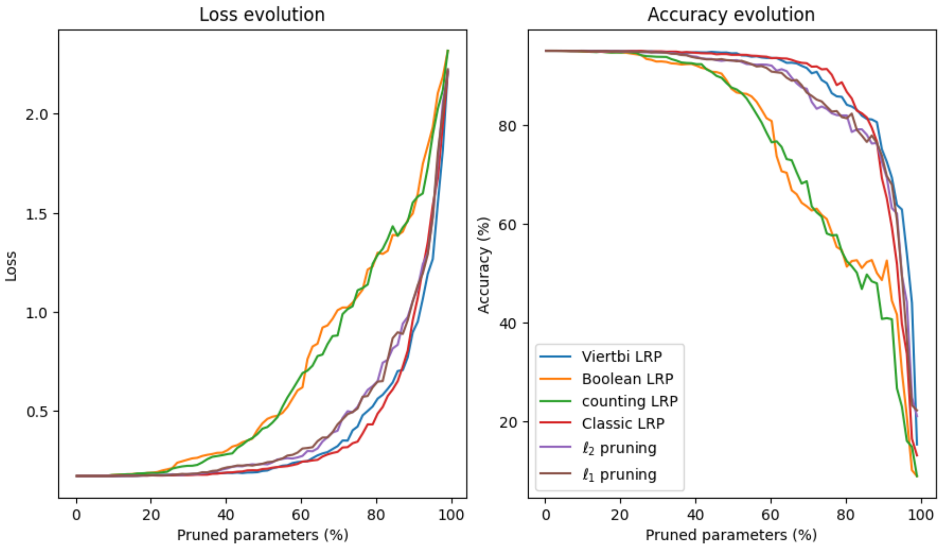


Figure: Relevance mean over the training dataset (Input layer)




Loss evolution
Loss
2.0
1.5
1.0
0.5
0
20
40
60
80
100
Pruned parameters (%)
Accuracy evolution
Accuracy (%)
80
60
40
20
0
20
40
60
80
100
Pruned parameters (%)
Viertbi LRP
Boolean LRP
counting LRP
Classic LRP
ℓ_2 pruning
ℓ_1 pruning

Comparison to image perturbation [2]

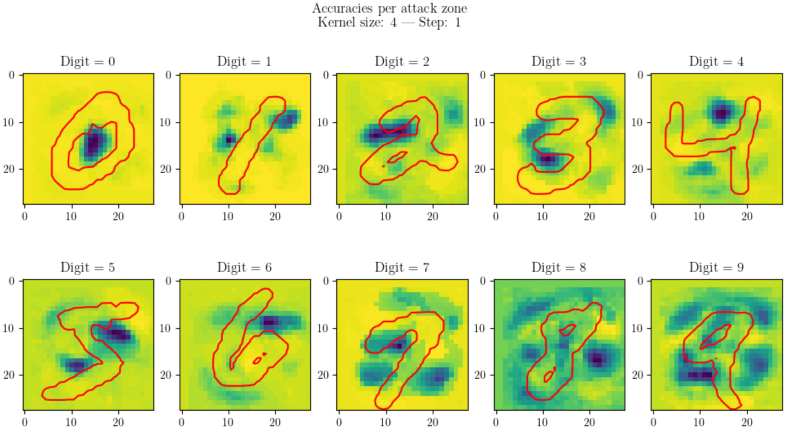


Figure: Accuracies per attack zone

[1] Sebastian Bach et al. "On Pixel-Wise Explanations for Non-Linear Classifier Decisions by Layer-Wise Relevance Propagation". In: *PLOS ONE* (2015), pp. 1–46. DOI: 10.1371/journal.pone.0130140. URL: https://doi.org/10.1371/journal.pone.0130140.

[2] Ruth C Fong and Andrea Vedaldi. "Interpretable explanations of black boxes by meaningful perturbation". In: *Proceedings of the IEEE international conference on computer vision*. 2017, pp. 3429–3437. URL: https://arxiv.org/abs/1704.03296.

[3] Robert Geirhos et al. "Shortcut learning in deep neural networks". In: *Nature Machine Intelligence* 2 (2020), pp. 665–673.

[4] Todd J Green, Grigoris Karvounarakis, and Val Tannen. "Provenance semirings". In: *Proceedings of the twenty-sixth ACM SIGMOD-SIGACT-SIGART symposium on Principles of database systems*. 2007, pp. 31–40.

[5] Grégoire Montavon et al. "Layer-Wise Relevance Propagation: An Overview". In: *Explainable AI: Interpreting, Explaining and Visualizing Deep Learning*. Springer International Publishing, 2019, pp. 193–209. URL: https://doi.org/10.1007/978-3-030-28954-6_10.

[6] Yann Ramusat, Silviu Maniu, and Pierre Senellart. "Provenance-Based Algorithms for Rich Queries over Graph Databases". In: *EDBT 2021 - 24th International Conference on Extending Database Technology*. 2021. URL: https://inria.hal.science/hal-03140067.