Implementation of an Iterative Linear Quadratic Regulator (iLQR)

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Plan

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General formulation

Dynamics function:

$$x_{t+1} = f(x_t, u_t)$$

Goal: minimize a quadratic cost function

Cost function:

$$J(u) = \sum_{t=0}^{T-1} \left(x_t^\top Q x_t + u_t^\top R u_t \right) + \frac{1}{2}(x_T - x^*)^\top Q_f (x_T - x^*)$$

Q: state cost matrix

Q_f: final state cost matrix

R: control cost matrix

Example: Simple Pendulum

State: $x = [\theta \quad \dot{\theta}]$

Control: u, torque applied to the pendulum

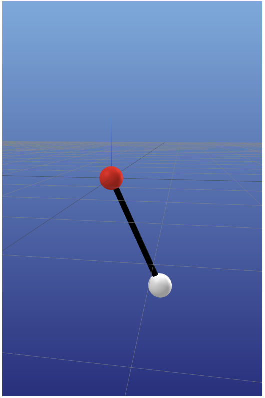
Dynamics: physical laws (simulator)

Target: $x = [0 \quad 0]$

Cost function:

$$J(u) = \frac{1}{2}\left(\theta_f^2 + \dot{\theta}_f^2\right) + \frac{1}{2}\int_0^T r u^2(t)\mathrm{d}t$$

corresponding to $Q_f = I_2$, $Q = 0_2$, $R = rI_1$



Example: Cartpole

State: $x = [y \quad \theta \quad \dot{y} \quad \dot{\theta}]$

Control: u, force applied to the cart

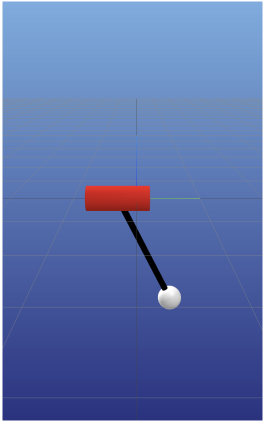
Dynamics: physical laws (simulator)

Target: $x = [0 \quad 0 \quad 0 \quad 0]$

Cost function:

$$J(u) = \frac{1}{2}\left(\theta_f^2 + \dot{\theta}_f^2 + y_f^2 + \dot{y}_f^2\right) + \frac{1}{2}\int_0^T r u^2(t)\mathrm{d}t$$

corresponding to $Q_f = I_4$, $Q = 0_4$, $R = rI_1$



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Problem statement

The iLQR algorithm