# Sequence modelling using neural networks

RNN's, GRU's and LSTM's

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Indian Institute of Techonology Madras

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#### Outline

- 1 Introduction
- 2 Recurrent Neural Networks
- 3 Long-Short Term Memory Networks
- 4 References

Introduction

# Introduction

- Fully connected networks and convolution networks cannot handle dynamic inputs
- All inputs have the exact same size.
- How do we handle audio or text?
- Use RNN's, LSTM's and GRU's<sup>1</sup>

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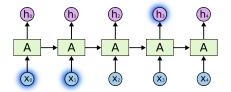
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### Natural language

- Natural language processing(NLP) is one domain with vast applications for sequence modelling.
- DL has completely changed the landscape for computer vision
- Images are comparatively easy to handle since
  - Images have symmetries to exploit
  - Pixels can take only 256 values, the English vocabulary has nearly 1 million words with varied meanings.
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  - Language has idioms, expressions, collocations and has evolved over time into a complex system.

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- Classification of text: Utility for sentiment/emotion analysis.
- Sequence-2-Sequence Models: Transliteration of text or extractive summarization.
- Text generation: Generating a corpus of text.
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#### Google's Speech recognition



# MS-Coco dataset for visual question answering



Figure: How many leftover donuts is the Red bicycle holding?

Recurrent Neural Networks

## **Recurrent Neural Networks**

- RNN consists of a state S<sub>t</sub> at any point t
- The input at time t is denoted by X<sub>t</sub>
- The output is denoted by  $H_t$

Note that the recurrent core(A) is the same for all t

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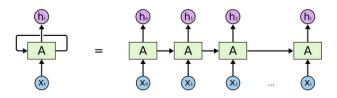


Figure: The RNN structure<sup>2</sup>

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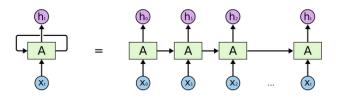


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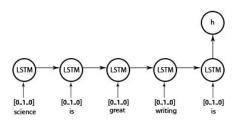
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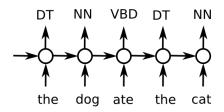
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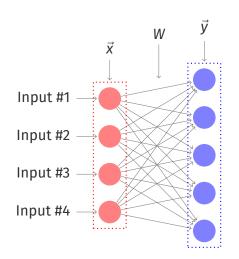
- The output  $H_t$  need not be utilized at all steps.
  - Only the last H<sub>T</sub> is utilized and trained on(Grammatical correctness of a sentence)



- The output  $H_t$  need not be utilized at all steps.
  - $h_0 \cdots h_T$  are utilized to generate an output(Finding POS tags for every word).



## The Notation - Fully Connected Layer

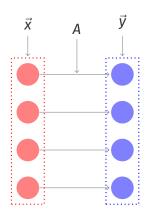


Let  $\vec{x}$  be a vector of size  $n_1$  and  $\vec{y}$  be a vector of size  $n_2$ . Let W be a matrix of dimensions  $n_2 \times n_2$ . Consider the following relation between  $\vec{x}$  and  $\vec{y}$ :

$$\vec{y} = W\vec{x}$$

This is represented by the feed-forward layer presented on the left.

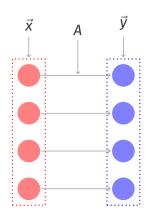
## The Notation - Activation Layer



Let  $\vec{x}$  be a vector of size n and  $\vec{y}$  be a vector of size n. Let A be an activation function from  $n \to n$ . Consider the following relation between  $\vec{x}$  and  $\vec{y}$ :

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The activation function is typically ReLu sigmoid or TanH. For example, if A was ReLu we have:

$$\vec{y} = [y_1, y_2, \cdots, y_n]$$

$$\vec{x} = [x_1, x_2, \cdots, x_n]$$

$$y_i = max(x_i, 0) \quad \forall i \in [1, \cdots, n]$$

## The Equations of an RNN<sup>3</sup>

The the primary idea behind RNN's is encapsulated by the following equation

$$\mathbf{S}^{(t)} = \mathbf{f}(\mathbf{S}^{(t-1)}, \mathbf{X}^{(t)}; \boldsymbol{\theta})$$

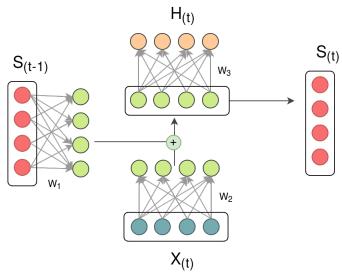
#### The Neural Network equations

Consistent with the notation earlier,  $S_t$  is the state,  $X_t$  is the input and  $H_t$  is the output.

$$\begin{split} \vec{s}_{(t)} &= \textit{tanH} \Big( \mathbf{W_1} \vec{s}_{(t-1)} + \mathbf{W_2} \vec{x}_{(t)} + \mathbf{b_1} \Big) \\ \vec{o}_{(t)} &= \mathbf{W_3} \vec{s}_{(t)} + \mathbf{b_2} \end{split}$$

The parameters to train/tune are  $W_1, W_2, W_3, b_1, b_2$ 

## A Pictorial Representation



# Points to ponder

- Note that the parameters are shared(like a CNN) which contributes to the sucess of this model.
- Some unanswered questions :
  - 1 How do we use the outputs  $H_{(t)}$ ?
  - 2 What are the complications with gradient descent?
  - 3 What loss functions do we train with?
- RNN, like CNNs can be used as a modular components.
   One can generate another sequence or feed the output to another neural network.

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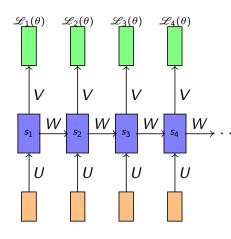
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#### The Loss Function



The loss function  $\mathcal{L}$  is given by

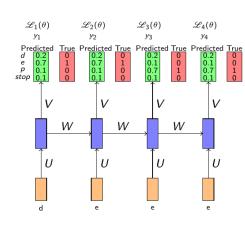
$$\mathcal{L} = \mathcal{L}_1(\theta) + \mathcal{L}_2(\theta) + \dots + \mathcal{L}_n(\theta)$$

Consider a task where we have to predict a sequence of characters, given an input of characters.(ex: Spell correction)

How do you utilize the output layer  $H_{(t)}$  ?

Note that unlike this example,  $\mathcal{L}_i$  can also be zero and we have only one non-zero  $\mathcal{L}_T$ .

# The Loss Function - Sequence prediction



Transform the output  $H_{(t)}$  into a probability distribution using a softmax layer i.e :

$$Q_{(t)} = \mathsf{softmax}(H_{(t)})$$

The loss function is the cross-entropy  $(\sum_n -p_n log(q_n))$ . Hence the loss is given by.

$$\begin{split} \mathcal{L} &= \mathcal{L}_1(\theta) + \mathcal{L}_2(\theta) + \mathcal{L}_3(\theta) + \mathcal{L}_4(\theta) \\ &= \sum_{t=1}^4 - \text{ln}(Q_{(t)}(\text{True class})_t) \\ &= -\text{ln}(0.7) - \text{ln}(0.7) \end{split}$$

 $-\ln(0.7) - \ln(0.7)$ 

# Backkpropagation through time

The expression for the gradients is complicated. Why?

$$\textbf{s}_{(t)} \propto \textbf{W} \textbf{s}_{(t-1)} + \textbf{W}^2 \textbf{s}_{(t-2)} \cdots + \textbf{W}^t \textbf{s}_{(0)}$$

- Hence, the expression for  $\frac{\partial \mathcal{L}}{\partial W}$  has terms involving derivative of power of matrix.
- A good reference for more of the math: https://www.deeplearningbook.org/contents/rnn.html
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Long-Short Term Memory Networks

# **Long-Short Term Memory Networks**

### Drawback's of RNN

- The expressions for the derivative diminish with time. Hence the effect of derivative of  $\mathcal{L}_{\mathcal{T}}$  has a smaller effect on  $S_{(k)}$  if k is farther away.
- The state information from earlier time steps are progressively diluted. There is no mechanism for retaining state information from earlier time steps.

$$S_{(t)} \propto W S_{(t-1)} + W^2 S_{(t-2)} \cdots + W^t S_{(0)}$$

 There is no ability to reject the information from the newest time-step, even if it is irrelevant.

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## Motivating LSTM's<sup>4</sup>

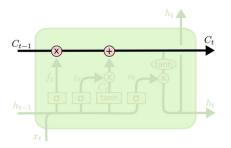
- $i_t$ : Input gate that filters the input  $x_t$ .
- o<sub>t</sub>: Output gate that determines the information to be passed on to the next time step.
- f<sub>t</sub>: Forget gate that retains only relevant information from the information passed on.
- · All the "gates" are vectors with same size as state vector.

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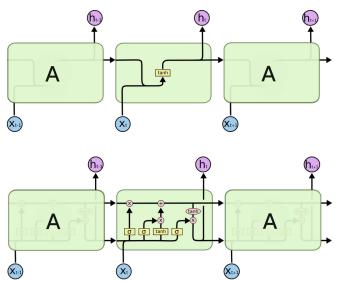
### The Notation

• Instead of a single state  $s_t$ , we utilize two states. One maintaining the long-term state( $c_t$ ), while the other maintains the short-term context( $s_t$ ).



• The long-term context  $C_t$  allows information to be easily passed on. This also allows the gradients to backpropagate for a longer period of time.

## **Comparing RNNs and LSTMs**



## The Equations

#### The LSTM structure

The equation for the gates are:

$$\begin{aligned} o_t &= \sigma(W_o h_{t-1} + U_o x_t + b_o) \\ i_t &= \sigma(W_i h_{t-1} + U_i x_t + b_i) \\ f_t &= \sigma(W_f h_{t-1} + U_f x_t + b_f) \end{aligned}$$

The equation for the states are:

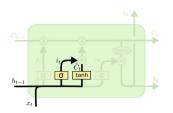
$$\bar{c}_t = \sigma(W_1 h_{t-1} + W_2 x_t + b_1)$$

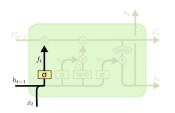
$$c_t = f_t \odot c_{t-1} + i_t \odot \bar{c}_t$$

$$s_t = o_t \odot \sigma(c_t)$$

$$h_t = \begin{cases} s_t \\ W_3 s_t + b_2 \end{cases}$$

## **Visualizing the Equations**



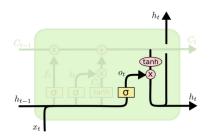


The input gate is used to filter the input content  $X_t$  and  $S_t$  and write into the long term context  $C_t$ .

The forget gate is used to filter the long term context  $C_t$  based on the short term context and current input  $X_t$ .

The two are used to modify the long term context.

## Visualizing the Equations

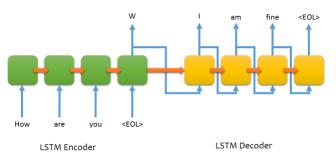


The output gate is used to determine the new local context, output vectors( $h_t$  and  $s_t$ ).

### **Encoder decoder Models**

Two LSTM's. The first LSTM generates an output  $H_T$  which is passed to the second LSTM.

The  $S_0$  of the second LSTM is set to be  $H_T$ . Then the first output is generated.



This first output is fed as input to the next step. This process is repeated till a <STOP> token is an output of the 2<sup>nd</sup> LSTM.

### How do you feed text to the model?

Let the vocabulary be of size  $|\mathcal{V}|$ . Assign every word a unique ID. For example, let the entire vocabulary be : ['Hello', 'World', 'Deep', 'Learning'].

We cannot use this unique ID as the reprentation, this assumes ordering among words. Instead use a one-hot vector representation.

Let the ID('hello') = 0 and ID('Learning') = 3. Then the representations are :

$$\vec{W}(\mathsf{Hello}) = [1, 0, 0, 0]$$
  
 $\vec{W}(\mathsf{Learning}) = [0, 0, 0, 1]$ 

References

## References

### Some Resources

- 1 A gentle introduction to RNN<sup>5</sup>
- 2 A nice intro to LSTM's<sup>6</sup>
- 3 Some overview on RNN's and encoder models<sup>7</sup>
- 4 Attention Mechanisms<sup>8</sup>

deep-learning-and-nlp/.

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<sup>5</sup>Jason Brownlee's Blog on RNN's.
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  <sup>6</sup>Colah's Blog on Understanding LSTM's. https:
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  <sup>7</sup>Jason Brownlee's Blog on Encoder-decoders. https:
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neural-network-models-neural-machine-translation/.
  <sup>8</sup>WildML's Blog on Attention Mechanisms.
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- WildML's Blog on Attention Mechanisms. http://www.wildml.com/2016/01/attention-and-a