Hw3

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1.

(a) False

(b) False

(c) True

(d) True

(e) False

2.

2.

(a) $$\max_h \quad U(h) = -4\left(a^2 + (2h)^2 + \frac{\sqrt{6}}{2}ah\right) - h^2$$

$$\text{s.t.} \quad h \le \frac{a}{3}, \quad h \ge 0$$

$$U(h) = -4a^2 - 17h^2 + 2\sqrt{6}\,ah$$

$$\frac{d}{dh}U(h) = -34h + 2\sqrt{6}a$$

$$-34h + 2\sqrt{6}a = 0$$

$$h = \frac{\sqrt{6}}{17}a \quad \#$$

Utility:

$$-4\left(a^2 + \left(\frac{2\sqrt{6}}{17}a\right)^2 - \frac{6}{17}a^2\right) - \frac{6}{17^2}a^2$$

$$= -4a^2 + \frac{24}{289}a^2 - \frac{6}{17}a^2 - \frac{6}{289}a^2$$

$$= \frac{-1240}{289}a^2 \quad \#$$

(b) $$\max U(h,v) = \frac{-4\left(a^2 + (2h)^2 - \frac{\sqrt{6}}{2}ah\right)}{v^2} - h^2 - v$$

$$\text{s.t.} \quad h \le \frac{a}{3}, \quad h \ge 0, \quad v \ge 0 \quad \#$$

(c) $$U(h,v) = -\frac{4}{v^2}a^2 - \frac{16}{v^2}h^2 + \frac{2\sqrt{6}}{v^2}ah - h^2 - v$$

$$\nabla U(h,v) = \begin{bmatrix} -\frac{32}{v^2}h - 2h + \frac{2\sqrt{6}a}{v^2} \\ \frac{32h^2}{v^3} + \frac{4\sqrt{6}ah}{v^3} - \frac{8a^2}{v^3} - 1 \end{bmatrix} \quad \#$$

$$\nabla^2 U(h,v) \begin{bmatrix} -\frac{2(v^2+16)}{v^2} & \frac{64h - 4\sqrt{6}a}{v^3} \\ \frac{64h - 4\sqrt{6}a}{v^3} & -\frac{24\left(4h^2 - \frac{\sqrt{6}}{2}ah + a^2\right)}{v^4} \end{bmatrix} \quad \#$$

Content:

(d) $H = \left(\frac{3(v^3 - 32h^2 + 4\sqrt{6}ah - 8a^2)}{v^4} - \frac{3}{v} \right)\left(\frac{-2(v^2+16)}{v^2} \right) - \left(\frac{64h - 4\sqrt{6}a}{v^3} \right)^2 > 0$

$$\frac{\partial^2}{\partial x^2} f = -\frac{2(v^2+16)}{v^2} > 0$$

(e) Primal Feasibility:

$$h \le \frac{a}{3}$$
$$h \ge 0$$
$$v \ge 0$$

Dual Feasibility:

$$\frac{32h^2}{v^3} - \frac{4\sqrt{6}ah}{v^3} + \frac{8a^2}{v^3} - 1 - \lambda_3 = 0 \qquad \lambda_1 \ge 0$$

$$\lambda_2 \ge 0$$

$$-\frac{32}{v^2}h - 2h + \frac{2\sqrt{6}a}{v^2} - \lambda_1 + \lambda_2 = 0 \qquad \lambda_3 \ge 0$$

Complementary slackness:

$$\lambda_1 \left(\frac{a}{3} - h \right) = 0$$
$$\lambda_2 \times h = 0$$
$$\lambda_3 \times v = 0$$

(f)

According to the solution given by MINOS, v = 3.08701, h=0.191894 and utility = -4.667335231. That is, while moving at v= 3.08701 cross the pyramid at the route whose highest altitude is h = 0.191894, he will have the highest utility.

3. $\min_{x \in \mathbb{R}^2} \quad f(x_1, x_2) = x_1 e^{3x_2} - 3x_1^2$

(a)

$$\nabla f = \begin{bmatrix} e^{3x_2} - 6x_1 \\ 3x_1 e^{3x_2} \end{bmatrix}$$

$$\nabla^2 f = \begin{bmatrix} -6 & 3e^{3x_2} \\ 3e^{3x_2} & 9x_1 e^{3x_2} \end{bmatrix}$$

#

(b)

$$x^1 = x^0 - [\nabla^2 f(x^0)]^{-1} \nabla f(x^0)$$

$$= \begin{bmatrix} -1 \\ -1 \end{bmatrix} - \frac{1}{54e^{-3} - 9e^{-6}} \begin{bmatrix} -9e^{-3} & -3e^{-3} \\ -3e^{-3} & -6 \end{bmatrix} \begin{bmatrix} e^{-3} + 6 \\ -3e^{-3} \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ -1 \end{bmatrix} + \begin{bmatrix} \frac{9e^{-3}}{54e^{-3} - 9e^{-6}} & \frac{3e^{-3}}{54e^{-3} - 9e^{-6}} \\ \frac{3e^{-3}}{54e^{-3} - 9e^{-6}} & \frac{6}{54e^{-3} - 9e^{-6}} \end{bmatrix} \begin{bmatrix} e^{-3} + 6 \\ -3e^{-3} \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ -1 \end{bmatrix} + \begin{bmatrix} \frac{\cancel{9e^{-6}} + 54e^{-3}}{54e^{-3} - 9e^{-6}} - \frac{\cancel{9e^{-6}}}{54e^{-3} - 9e^{-6}} \\ \frac{3e^{-6} + \cancel{18e^{-3}}}{54e^{-3} - 9e^{-6}} - \frac{\cancel{18e^{-3}}}{54e^{-3} - 9e^{-6}} \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ -1 \end{bmatrix} + \begin{bmatrix} \frac{54e^{-3}}{54e^{-3} - 9e^{-6}} \\ \frac{3e^{-6}}{54e^{-3} - 9e^{-6}} \end{bmatrix} = \begin{bmatrix} \frac{54e^{-3}}{54e^{-3} - 9e^{-6}} - 1 \\ \frac{3e^{-6}}{54e^{-3} - 9e^{-6}} - 1 \end{bmatrix}$$

#

(c) $x^0 = (1, 0)$ $\quad f(x^0) = -2$

$\nabla f(x^0) = (-5, 3)$

$f(x^0 - a\nabla f(x^0)) = f(1+5a, -3a)$

$= f(11, -6)$

$= 11e^{-18} - 3 \times 121$

$= 11e^{-18} - 363$

$2 + 11e^{-18} - 363 = -361 + 11e^{-18}$ #

(d)

$\underset{a \geq 0}{\operatorname{argmin}} f(1+5a, -3a) = (1+5a)e^{-9a} - 3(1+5a)^2$

$\Rightarrow -9(5a+1)e^{-9a} + 5e^{-9a} - 30(5a+1) = 0$

By graph, $a \doteq 0.14231$ #

(e) $x^0 = (0, 4)$ $\quad f(x^0) = 0$

$\nabla f(x^0) = (e^{12}, 0)$

$f(x^0 - a\nabla f(x^0)) = f(-e^{12}a, 4)$

$\operatorname{argmin}_{a \geq 0} f(-e^{12}a, 4) \Rightarrow -e^{12}a \cdot e^{12} - 3e^{24}a^2$

$= -e^{24}a - 3e^{24}a^2$

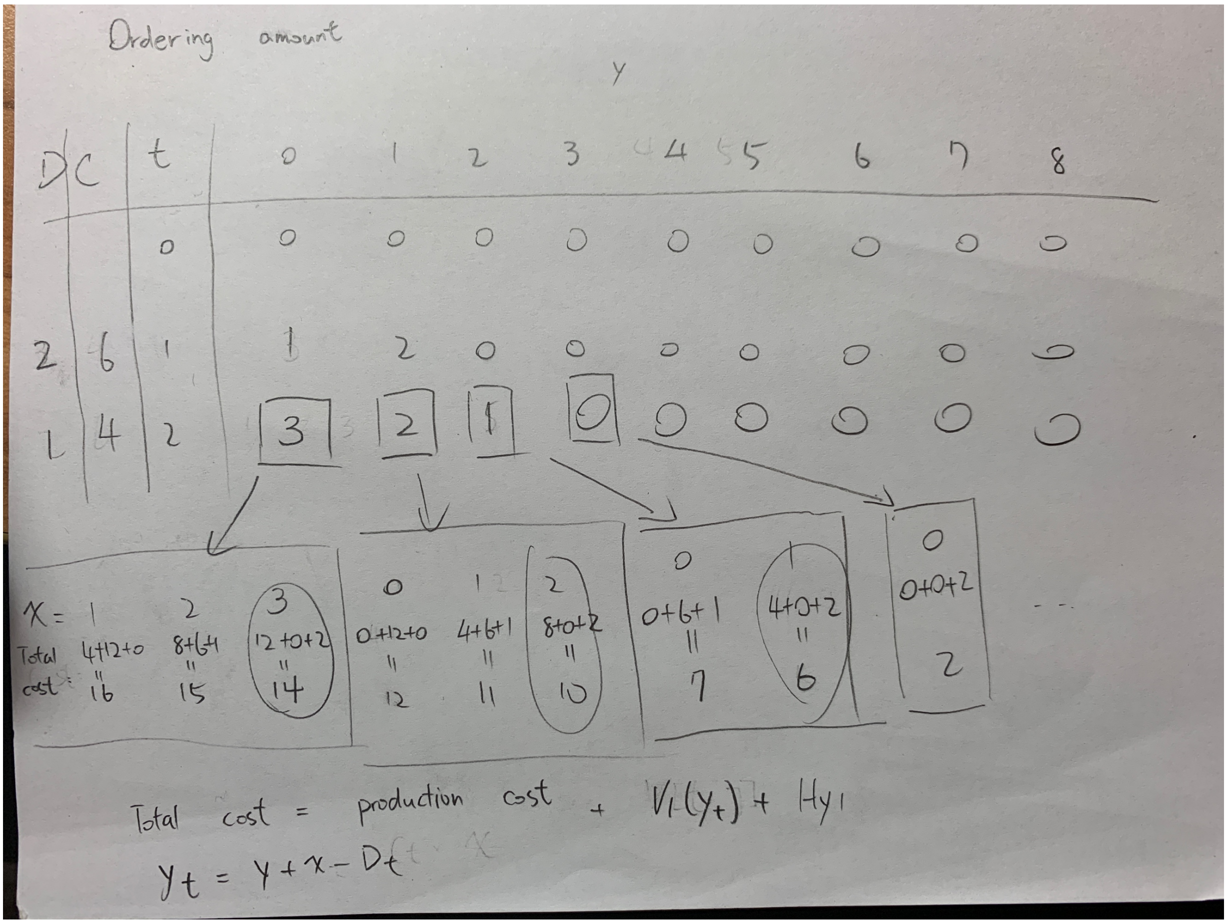
$f' = -6e^{24}a - e^{24} = -e^{24}(6a+1) = 0$

$a = 0, \cancel{-\frac{1}{6}}$

Because the optimal step size, it cannot move to a better solution. #

4.

(a)



(b)

```r
v=seq(0,10)

v4=vector(mode = "numeric",length = 10)

v3=vector(mode = "numeric",length = 10)

for (y in 1:10) {

for (q in 1:10) {

  u=0

  a=0

  z=0

  temp=0

  for (x in 0:q) {

    a=a+x*dbinom(x,10,0.3)
```

```
  }
  for (x in q+1:10) {
    z=z+q*dbinom(x,10,0.3)
  }
  u = (a+z)*5 -max((q-y),0)*2
  for (x in 0:q) {
    temp=temp+v[q-x+1]*dbinom(x,10,0.3)
  }
  v4[q]=u+temp
}
  v3[y]=max(v4)
}
print(v3)
```

```
 [1]  9.160202 11.160202 13.160202 15.160202 16.761276 17.950672
 [7] 18.993040 19.999402 20.999976 22.000000
```

By using R, we find that the optimal number of $V_1(4)$ is 15.160202.