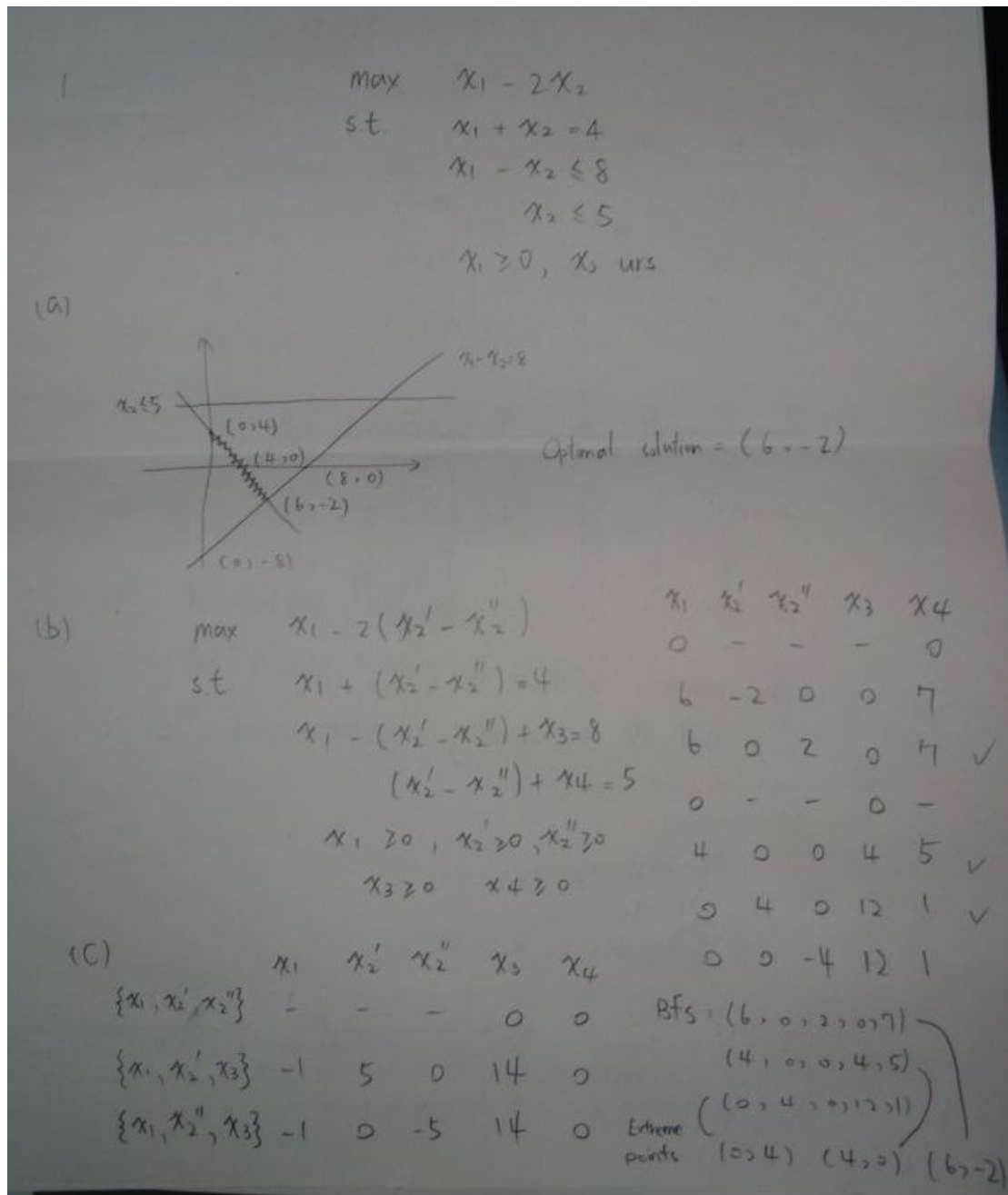


1. (a)(b)(c)



(d) 2.(a)

-1	2	-2	0	0	0	0
1	1	-1	0	0	1	$x_5=4$
1	-1	1	1	0	0	$x_3=8$
0	1	-1	0	1	0	$x_4=5$

0	3	-3	0	0	0	4
1	1	-1	0	0	1	$x_1=4$
0	-2	2	1	0	0	$x_3=4$
0	1	-1	0	1	0	$x_4=5$

0	0	0	$\frac{3}{2}$	0	0	10
1	0	0	$\frac{1}{2}$	0	0	$x_1=6$
0	-1	1	$\frac{1}{2}$	0	0	$x_2=2$
0	0	0	$\frac{1}{2}$	1	0	$x_4=7$

The optimal solution is $(6, -2)$
and the corresponding objective value is 10.

(a) $\max x_1 + 3x_2 + 2x_3$
s.t. $x_1 + x_2 + x_4 = 3$
 $x_1 + 2x_2 + x_3 - x_5 = 10$
 $x_3 - x_6 = 2$
 $x_i \geq 0 \forall i=1, \dots, 6$

-1	-3	-2	0	0	0	0
1	1	1	1	0	0	$x_4=3$
1	2	1	0	-1	0	$x_5=10$
0	0	1	0	0	-1	$x_6=2$

0	-2	-1	1	0	0	3
1	1	1	1	0	0	$x_1=3$
0	1	0	-1	-1	0	$x_5=10$
0	0	1	0	0	-1	$x_6=2$

2	0	1	3	0	0	9
1	1	1	1	0	0	$x_2=3$
-1	0	-1	-2	-1	0	$x_5=7$
0	0	1	0	0	-1	$x_6=2$

(b) It's unbounded.

Gurobi 8.1.0: unbounded; variable.unbdd returned.

2 simplex iterations

suffix unbdd OUT;

3.(a)(b)

x_{it} = number of products i produced in t , $t=1, \dots, T$, $i=1, \dots, N$
 y_{it} = number of need i doesn't fulfilled in t , $t=1, \dots, T$, $i=1, \dots, N$

$$\max \sum_{i=1}^N \sum_{t=1}^T D_{it} P_{it} - \sum_{i=1}^N \sum_{t=1}^T x_{it} C_{it} - \sum_{i=1}^N S_i \sum_{t=1}^T \max\{y_{it}, 0\}$$

s.t.

$$y_{i0} = 0 \quad \forall i=1, \dots, N$$

$$y_{i(t-1)} + D_{it} - x_{it} = y_{it} \quad \forall t=1, \dots, T \quad \forall i=1, \dots, N$$

$$x_{it} \leq K_{it} \quad \forall i=1, \dots, N \quad \forall t=1, \dots, T$$

$$x_{it} \geq 0, \quad y_{it} \text{ urs.} \quad \forall i=1, \dots, N \quad \forall t=1, \dots, T$$

\Downarrow Linearized

$$\max \sum_{i=1}^N \sum_{t=1}^T D_{it} P_{it} - \sum_{i=1}^N \sum_{t=1}^T x_{it} C_{it} - \sum_{i=1}^N S_i \sum_{t=1}^T y_{it}^+$$

s.t.

$$y_{i(t-1)} + D_{it} - x_{it} = y_{it} \quad \forall i=1, \dots, N \quad \forall t=1, \dots, T$$

$$y_{it}^+ \geq y_{it} \quad \forall i=1, \dots, N \quad \forall t=1, \dots, T$$

$$y_{it}^+ \geq 0$$

$$x_{it} \leq K_{it} \quad \forall i=1, \dots, N \quad \forall t=1, \dots, T$$

$$x_{it} \geq 0, \quad y_{it} \text{ urs} \quad \forall i=1, \dots, N \quad \forall t=1, \dots, T$$

(b)

Replace $x_{it} \leq K_{it} \quad \forall i=1, \dots, N \quad \forall t=1, \dots, T$

for $x_{it} + x_{i(t-1)} \leq K_{it} \quad \forall i=1, \dots, N \quad \forall t=1, \dots, T-1$

4.(a)(b)

x_{ij} = million barrels of oil shipped from city i to j

$$\begin{aligned} \max \quad & \sum_{i=1}^N \sum_{j=1}^M x_{ij} P_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^M x_{ij} \leq K_i \quad \forall i=1, \dots, N \\ & \sum_{i=1}^N x_{ij} \leq D_j \quad \forall j=1, \dots, M \\ & x_{ij} \geq 0 \quad \forall i=1, \dots, N \quad \forall j=1, \dots, M \end{aligned}$$

y_i = million barrels of oil expanded of city i

$$\begin{aligned} \max \quad & \sum_{i=1}^N \sum_{j=1}^M x_{ij} P_{ij} - \sum_{i=1}^N y_i \frac{F_i}{T} \\ \text{s.t.} \quad & \sum_{j=1}^M x_{ij} \leq K_i + y_i \quad \forall i=1, \dots, N \\ & \sum_{i=1}^N x_{ij} \leq D_j \quad \forall j=1, \dots, M \\ & x_{ij} \geq 0 \quad \forall i=1, \dots, N \quad \forall j=1, \dots, M \\ & y_i \geq 0 \quad \forall i=1, \dots, N \end{aligned}$$

(c)

Gurobi 8.1.0: optimal solution; objective 9950

9 simplex iterations

(d)

x_{ij} = Millions of barrels of oil shipped from city i to point j

y_i = Million barrels of annual refining capacity that is added to city i

$T=1,2,3,4$

optimal solution; objective 8850

$x :=$

1 1 1

1 2 5

1 3 0

1 4 0

2	1	6
2	2	0
2	3	0
2	4	0
3	1	0
3	2	0
3	3	7
3	4	0

y [*] :=

1	0
2	0
3	0

T=5
optimal solution; objective 9950

x :=

1	1	1
1	2	7
1	3	1
1	4	8
2	1	6
2	2	0
2	3	0
2	4	0
3	1	0
3	2	0
3	3	7
3	4	0

y [*] :=

1	11
2	0
3	0

T=6,7,8,9,10

optimal solution; objective 10683.33333

x :=

1 1 1

1 2 7

1 3 1

1 4 8

2 1 6

2 2 0

2 3 0

2 4 0

3 1 0

3 2 0

3 3 7

3 4 0

y [*] :=

1 11

2 0

3 0

Conclusion:

When T becomes larger, the average annual cost of expanding will be less, so it's better to expand when T is large enough.