Ohm's Law and Power Law Lab

Lab Partner Names
Beth Momot
Angela Zhang

Introduction

In this lab, Ohm's law was tested by taking measurements of current and voltage and using the formula to determine the resistance of a resistor in a circuit. Resistance of a resistor and resistance of an unknown potentiometer was calculated by dividing voltage by current, as seen from the following relationship:

$$V = IR$$

Ohm's Law

$$\rightarrow R=V/I$$

This observed and calculated data was then compared to the stated resistance by the manufacturer (in the case of the resistor) and the measured resistance by the multimeter (in the case of the potentiometer), to verify that the value calculated using Ohm's law was reasonable.

The power law was tested on another day for a lightbulb in a circuit. The nonlinear relationship between current and voltage in a lightbulb is given by:

$$P = VI \text{ and } V = IR$$

$$\rightarrow I \propto V^{(3/5)}$$

For ideal blackbodies

$$\text{Or } \rightarrow I \propto V^{(0.5882)}$$

For a tungsten bulb

This relationship is analyzed by plotting current with voltage, and also by plotting the log of current with respect to the log of voltage, to confirm the value of the exponential in voltage.

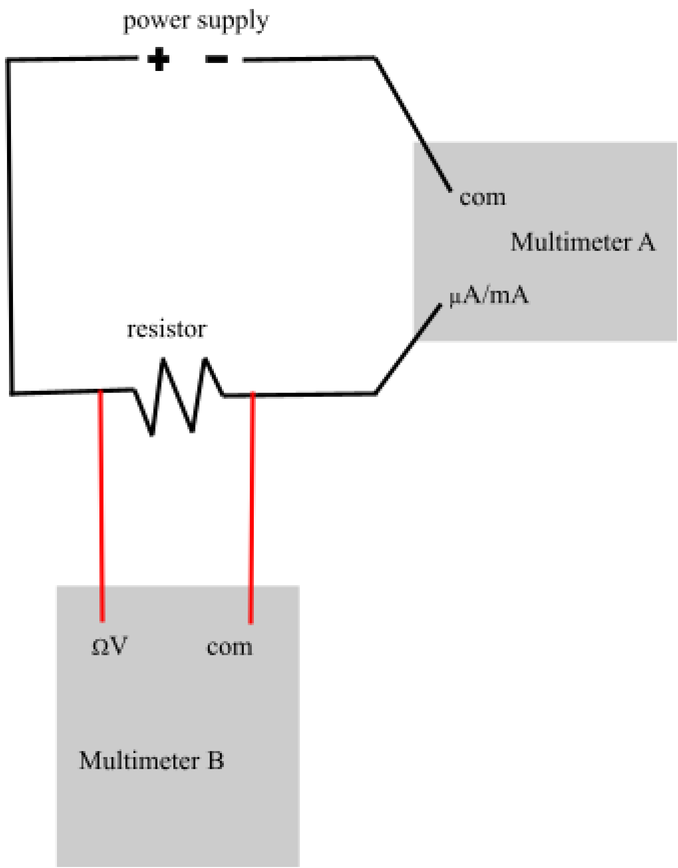
Equipment

The equipment used in this procedure is as follows:

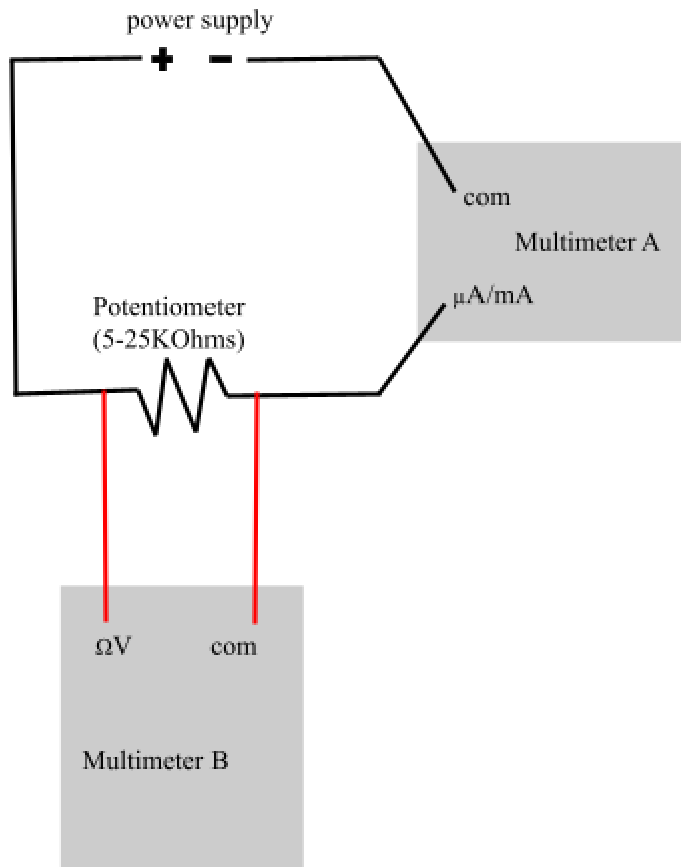
- 2 multimeters, one which measured currents for both experiments, and the other which measured voltage
- A 100-Ohm resistor, which was on the electrical components board
- A potentiometer, which had a range of resistances between 5 Ohms and 25 KOhms, which was also on the electrical components board
- A lightbulb, also on the electrical components board
- 5 lengths of wire
- Power supply

Methods

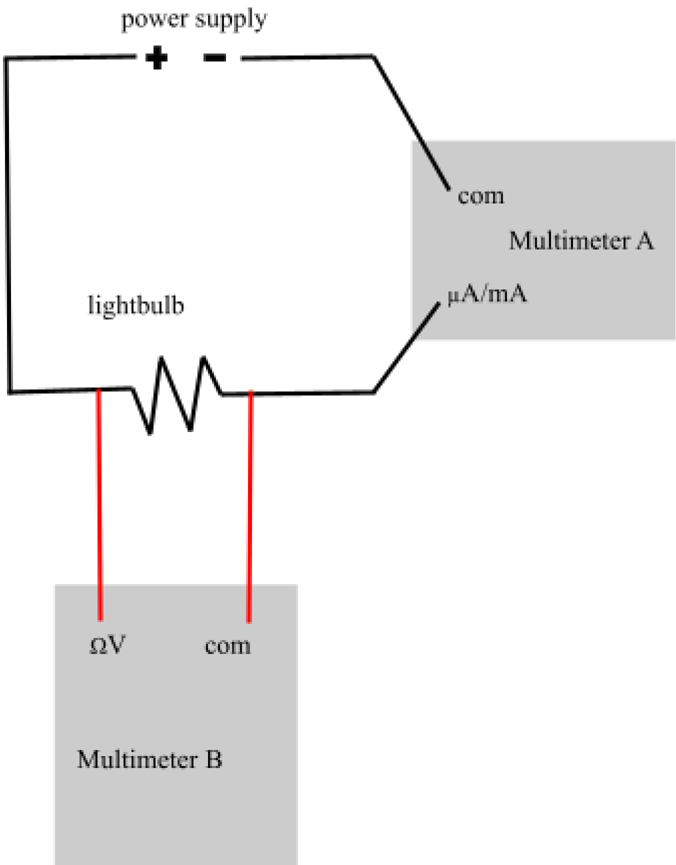
The power supply was off at the beginning. First, the wires were hooked up as follows: from the positive terminal on the power supply, to the left side of the resistor, then from the right side of the resistor to the µA/mA terminal on multimeter A, then from the com terminal on multimeter A to the negative terminal on the power supply. The second multimeter (B) was connected from its ΩV terminal to the wire on the left side of the resistor and from its com terminal to the wire on the right side of the resistor. The diagram below demonstrates this setup a bit more clearly:



Once this was all set up, the power supply was turned on, with an initial voltage of 15.001V (with an uncertainty of 0.001). Measurement of current was observed from multimeter A in mA, and measurement of voltage was observed and recorded in Excel from multimeter B in V. The voltage on the power supply was then turned down to 12.001V (the same uncertainty of 0.001 applies for all the power supply voltage statements), and the measurements and recordings of current and voltage were again observed. This process of turning down the voltage and observing and recording the current and voltage measurements by multimeters A and B respectively was repeated for 9.000V, 6.000V, and 3.000V. The power supply was then shut off, and a reading of resistance was taken by the multimeter, which was attached directly to the resistor, with the wires connecting the resistor to the power supply having been removed. The wires from multimeter B were connected on the same sides of the resistor as before. Finally, the color bars on the resistor were determined to be (from left to right) brown, black, brown, gold (100 Ohms). Next the circuit was rewired to have the potentiometer in place of the resistor, as follows:



The power supply was then turned on and the same process was repeated for the measurements and recordings of the voltage and current, for the same increments of 15.001V, 12.001V, 9.000V, 6.000V, and 3.001V as indicated on the power supply. The power supply was then again shut off and the resistance of the potentiometer was measured by multimeter B in the same method (directly attached to the resistor, with the wires to the power supply removed) as with the measurement of the resistor's resistance. It was determined to be 4.812 KOhms. Next, the process is repeated for a lightbulb in the place of a potentiometer.



The power supply is turned on and voltage and current over the lightbulb are recorded in the same way as previously described, but for voltages of 3.001V, 4.001V, 5.001V, 6.001, 7.001V, 8.001V, 9.001V, 10.001V, 11.001V, 12.001V, 13.002V, 14.002V, 15.002V, 16.002V, 17.002V, 18.003V and 19.003V as indicated on the power supply. Resistance across the lightbulb was not directly measured as it is not a point of interest in this instance. Instead, the data will be plotted to find the exponential relationship between voltage and current in this experiment, which arises from the power law.

The precision of the multimeter in measuring voltage was said to be 0.25% by the manufacturer, and the precision of its measurement of current was said to be 0.75%. It should also be noted that while taking the measurement of the current, the multimeter tended to increase its reading by 0.001mA every few 3seconds, so the measurement included in the charts was taken to be the reading directly after the fluctuation from the change in voltage from the power supply died down. In the case of the lightbulb measurements, the lightbulb also lights up visibly at a reading of 7.001V on the power supply and continues to get brighter with every voltage increment afterward, which may introduce uncertainties from energy loss to the environment. The room was most likely somewhere around room temperature, or 20°C, although this measurement was not taken. The room was neither notably hot or cold, so this is a reasonable estimate, and as no obvious fluctuations were noticed, it is also reasonable to assume the room temperature was constant. According to the link in the lab instructions, the manufacturer of the multimeter is Tegam and the model is 130A.

Results

Table 1: Measured Current and Voltage with Resistor Circuit

Voltage from power supply (V)	Voltage measured by multimeter (V)	Current measured by multimeter (mA)
15.001 ± 0.001	14.675 ± 0.037	147.040 ± 1.103
12.001 ± 0.001	11.740 ± 0.029	117.810 ± 0.884
9.000 ± 0.001	8.805 ± 0.022	88.450 ± 0.663
6.000 ± 0.001	5.870 ± 0.015	59.070 ± 0.443
3.001 ± 0.001	2.934 ± 0.007	29.540 ± 0.222

Table 2: Measured Current and Voltage with Potentiometer Circuit

Voltage from power supply (V)	Voltage measured by multimeter (V)	Current measured by multimeter (mA)
15.001 ± 0.001	14.994 ± 0.037	3.117 ± 0.023
12.001 ± 0.001	11.996 ± 0.030	2.494 ± 0.019
9.000 ± 0.001	8.996 ± 0.022	1.870 ± 0.014
6.000 ± 0.001	5.997 ± 0.015	1.246 ± 0.009
3.001 ± 0.001	2.999 ± 0.007	0.630 ± 0.004

Table 3: Measured Current and Voltage with Lightbulb Circuit

Voltage from power supply (V)	Voltage measured by multimeter (V)	Current measured by multimeter (mA)
3.001 ± 0.001	2.978 ± 0.007	10.754 ± 0.081
4.001 ± 0.001	3.974 ± 0.010	12.348 ± 0.093
5.001 ± 0.001	4.97 ± 0.012	13.866 ± 0.104
6.001 ± 0.001	5.967 ± 0.015	15.304 ± 0.115
7.001 ± 0.001	6.965 ± 0.017	16.672 ± 0.125
8.001 ± 0.001	7.962 ± 0.020	17.979 ± 0.135
9.001 ± 0.001	8.959 ± 0.022	19.237 ± 0.144
10.001 ± 0.001	9.957 ± 0.025	20.463 ± 0.153
11.001 ± 0.001	10.954 ± 0.027	21.652 ± 0.162
12.001 ± 0.001	11.952 ± 0.030	22.802 ± 0.171
13.002 ± 0.001	12.951 ± 0.032	23.911 ± 0.179
14.002 ± 0.001	13.949 ± 0.035	24.973 ± 0.187

15.002 ± 0.001	14.946 ± 0.037	26.014 ± 0.195
16.002 ± 0.001	15.944 ± 0.040	27.019 ± 0.203
17.002 ± 0.001	16.942 ± 0.042	27.996 ± 0.210
18.003 ± 0.001	17.941 ± 0.045	28.954 ± 0.217
19.003 ± 0.001	18.939 ± 0.047	29.893 ± 0.224

Analysis

Resistance was calculated using Ohm's law, rearranged to produce the formula R=I/V, with the mA units of the current converted to A, and the voltage left in units V. The uncertainty of this value was calculated using the uncertainty formula for a multiplication or division of values:

$$\frac{u(z)}{|z|} = \sqrt{\left(\frac{u(x)}{x}\right)^2 + \left(\frac{u(y)}{y}\right)^2}$$

Table 4: Calculated Resistances of Resistor

Voltage on power supply (V)	Resistance (Ohms)
15.001 ± 0.001	99.803 ± 0.789
12.001 ± 0.001	99.652 ± 0.788
9.000 ± 0.001	99.548 ± 0.787
6.000 ± 0.001	99.374 ± 0.786
3.001 ± 0.001	99.323 ± 0.785

Table 4: Calculated Resistances of Potentiometer

Voltage on power supply (V)	Resistance (Ohms)
15.001 ± 0.001	4810.395 ± 38.030
12.001 ± 0.001	4809.944 ± 38.026
9.000 ± 0.001	4810.695 ± 38.032
6.000 ± 0.001	4813.002 ± 38.050
3.001 ± 0.001	4760.317 ± 37.634

Averages of the resistances were calculated using the formula:

$$\overline{X} = \frac{1}{N}\sum_{i}^{N} X_i$$

and were determined to be 99.540 for the resistor and 4800.871 for the potentiometer. The uncertainties of these calculations will be discussed after the standard deviations are presented. The standard deviations were calculated using the formula:

$$s = \sqrt{\frac{\sum_i^N \left(X_i - \overline{X}\right)^2}{N-1}}$$

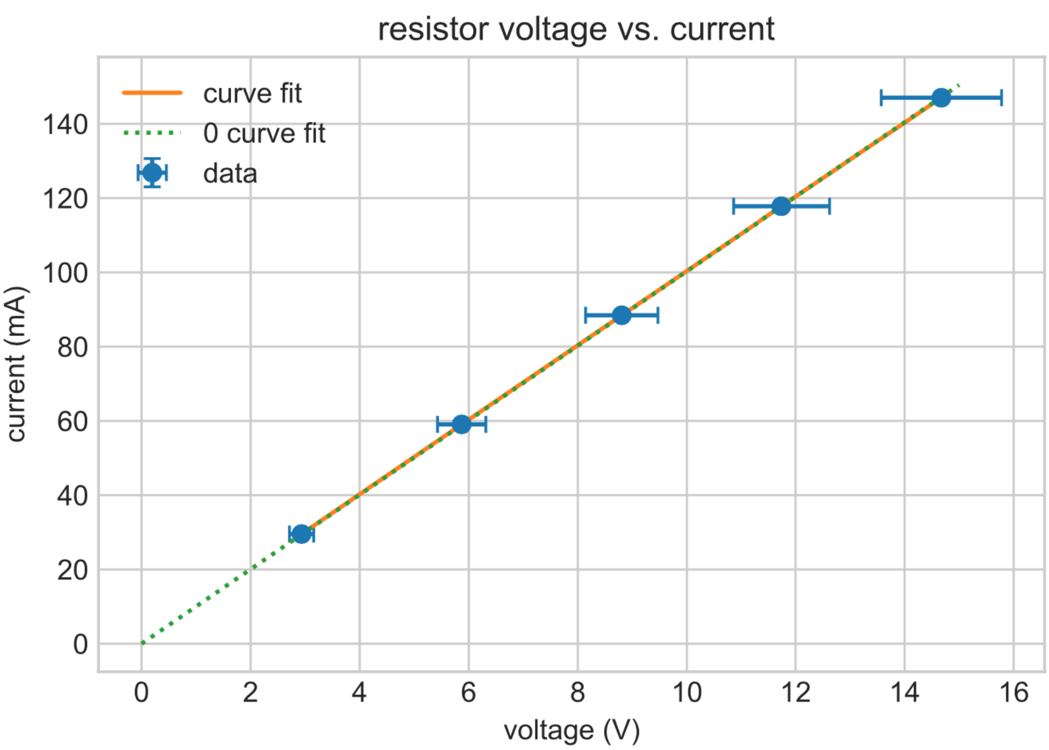
and were 0.198 for the resistor and 22.701 for the potentiometer. The standard error, or rather the uncertainty in the calculation of the average values of the resistances (as mentioned before) was found using the formula:

$$\sigma_{\overline{X}} = \frac{s}{\sqrt{N}}$$

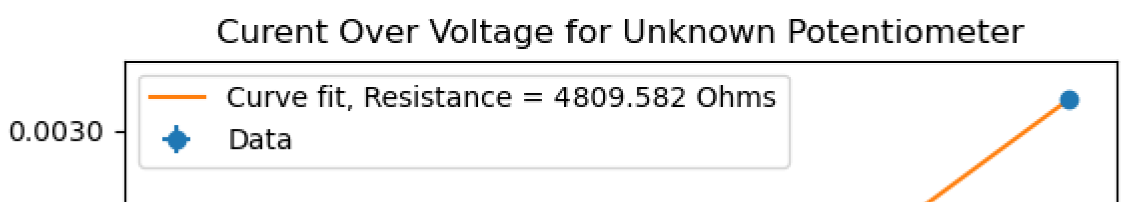
and was determined to be ± 0.088 for the resistor and ± 10.152 for the potentiometer.

Curves of best fit are generated for the data using the curvefit() function from the scipy.optimize module in Python 3.8.

The plot of voltage vs. current for the resistor came out to look like this:

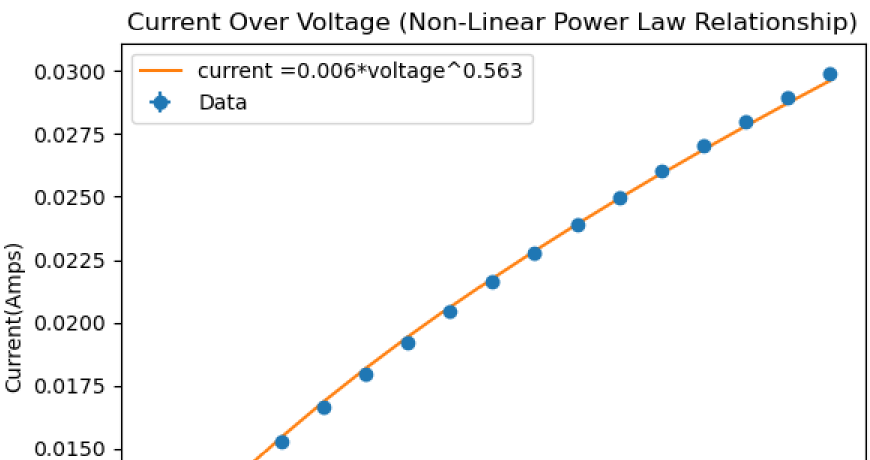


The plot of current over voltage for the potentiometer is as follows:

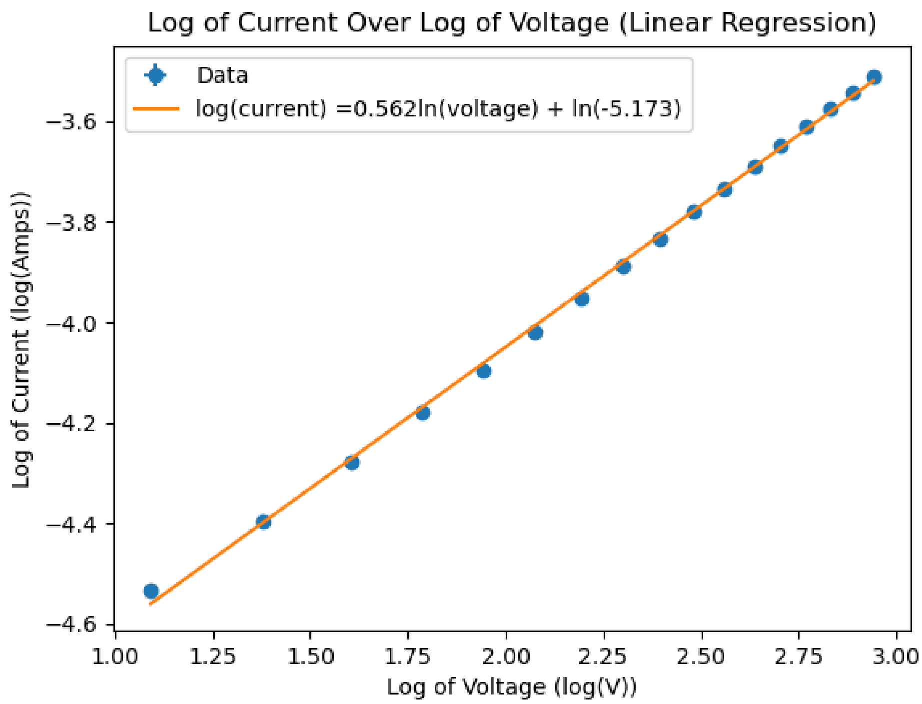


Since the curvefit() function takes into account the uncertainty values and weighs the data accordingly rather than taking a simple average, the optimized curve shows that the potentiometer has a resistance value of 4809.582 Ohms, slightly higher than the previously calculated average value of 4800.871 Ohms. Similarly, the optimized curve for the resistor shows the resistance to be 99.677 Ohms, very close to the stated value of 100 Ohms on the electrical components board, and slightly higher than 99.540 as determined by the average of all measurements.

Raw data of the lightbulb is plotted as follows in a non-linear relationship according to the power law.



A linearized version of the plot for the lightbulb data, where voltage and current are recorded in log scale is as follows:



The reduced chi squared value, which is an indicator for how well the curve of best fit represents the collected data, was found to be 8.575e-29 for the resistor and 0.523 for the potentiometer.

This value was calculated by taking this formula:

$$\chi^2_{\mathrm{red}} = \frac{1}{\nu} \sum_{i=1}^{N} \left(\frac{y_i - y(x_i)}{\sigma_i} \right)^2$$

where a chi-squared value of 1 would be an ideal fit, and values greater than 1 may indicate the curve does not fit the data enough, while values less than 1 may indicate the curve overfits the data.

Discussion

For the resistor, all calculations for uncertainty were within the range of uncertainty of the manufacturer's stated resistance value of 100 Ohms. For the potentiometer, the same can be said, with the exception of one outlier: the 3.001V case. This calculated value of resistance was, with the highest uncertainty level, still 14.049 Ohms away from the measured resistance with the multimeter. It is also the clear outlier in the calculated resistance table for the potentiometer. There is no discernible reason for why this occurred. Care was taken to make sure the dial was not turned mid-experiment. There was no rewiring between the measurements. The power supply remained on throughout this portion of the experiment, as well. There was no detected change in the environmental conditions of the classroom itself.

Given the above context, the value of 4809.582 Ohms provided by curvefit() may be closer to the true value of resistance for the potentiometer, as this is an optimized value for resistance that weighs the value with the most uncertainty less than other values with less uncertainty. While the chi-square value of the corresponding plot is less than 1, which could be indicative of the curve overfitting the data, given how consistent the data points are, however, it instead suggests an overall overestimate in the uncertainty of measurements for the potentiometer.

The average values for both the resistance of the resistor and of the potentiometer were a bit low (just outside the range of their uncertainties). The average for the resistor, with the highest range of uncertainty, is still 0.372 Ohms lower than the manufacturer specified resistance. The average for the potentiometer, with the highest range of uncertainty, is also low, by a value of 0.977 Ohms. This is notably closer by the percentage of total resistance to the measured value of resistance by the multimeter B. What this seems to indicate is that the multimeter may be a bit more inaccurate than it says in the manufacturer's specifications. The comparison of the multimeter's determination of the resistance is more similar than when the value of average resistance is compared to the value of the resistor not determined by the multimeter. This could also potentially be an indicator that the resistor's value is a bit lower than it is stated to be on the electrical board.

The chi-squared for the potentiometer seems to be reasonably close to the desired value of 1 in comparison with the comically small value of chi-squared for the resistor. The reason for the resistor's chi-squared data being so minute is that the measured currents and estimated currents are essentially the same, rendering chi squared almost zero. This could have been improved by having more data points, so that the fit was harder for curve_fit( ) to create accurately. Another way to reduce chi-squared would be to reduce the uncertainties in the measurements. This could be accomplished by using more precise measurement equipment.

Exercise Questions

Exercise 1:

Q1
As is shown in the plot of the resistor's voltage vs. current plot, and is clear from following the slope of the line on the potentiometer' voltage vs. current plot, our linear fits for these relationships did pass through zero.

Q2
The resistance is the same for both as was stated in the discussion, those being 99.677 for the resistor and 4009.582 for the potentiometer, since they already passed through zero with our initial fit.

Q3
The resistance from using curve_fit( ) was closer to the actual stated value of resistance of the resistor than the average value of the calculated resistances from the measured voltages and currents. The stated value was 100 Ohms, while the resistance from curve_fit was 99.677 Ohms and the average resistance from the calculations was 99.540.

For the potentiometer, the same was true. The measured value we took of the potentiometer's resistance using the multimeter was ~4812 Ohms, while the resistance from curve_fit was 4009.542 Ohms and the average value was 4800.871 Ohms.

Q4
The reduced-chi-squared values computed were both low, at 8.575e-29 for the resistor and 0.523 for the potentiometer. Obviously, the resistor chi-squared was much lower than the desired value of 1, being essentially zero, due to the fit being too good, with the line of fit passing through each data point. The potentiometer's reduced-chi-squared is also low, but more reasonably close to 1. These lower values for reduced-chi-squared could be improved by taking more data points, as well as using more precise equipment to create lower uncertainties.

Exercise 3:

Q1
The non-linear regression method produced the exponential value closer to the expected value of 0.588 for a tungsten bulb. The linear regression method produced 0.562 for the exponential, while the non-linear regression method produced 0.563. The difference is too small to be visible on the plot.

Q2
The standard deviation on the exponential in the linear regression method is 3.4*10^-4 to 2s.f., and the standard deviation on the exponential in the non-linear regression method is 4.5*10^-5 to 2s.f.. Standard deviation is smaller on the non-linear regression method. However, both are very small and neither cover the expected value in their range.

Q3
The chi-squared value in the non-linear regression method is slightly closer to the ideal chi squared value of 1. Linear regression produced chi-squared of 1.934 to 3s.f. while non-linear regression produced a chi-squared of 1.914 to 3.s.f., which seems to suggest the non-linear best fit curve was marginally better fit to the data than the linear best fit curve.

Both chi-squared values exceed one, meaning neither curve fits all measured data within the uncertainty, indicating an underestimated uncertainty, or a curve that is not precise enough to the data.

Conclusions

Overall, it seems that Ohm's law is a good theory for predicting the value of resistance given the voltage and current. None of the calculated values were so glaringly far off as to indicate inaccuracies in the theory, and fluctuations in the values outside of the range of uncertainty of the resistance are more likely due to errors with the equipment.

For the power laws, they also seem to be correct, since the obtained values for the exponentials of the voltage being very close to the expected values. There also does not seem to be any clear advantage to using one regression method over the other, both seem to provide accurate and correct results.