Circuit Theory and Electronics Fundamentals

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T1 Laboratory Report

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Contents

1 Introduction

The objective of this laboratory assignment is to study a circuit, Figure (1), containing 7 resistors, an indepent current source, an independent voltage source, a voltage-dependent current source and a current-dependent voltage source.
In Section (2) we performed a theoretical analysis of the circuit using the mesh current method and the nodal voltage method. The results were then calculated using an *Octave* script which allowed us to establish values for specific physical quantities in the circuit like V_b and I_c.
In Section (3) we present the results of a simulation of the circuit obtained from an *Ngspice* script and comment the difference between the values determined trough theoretical methods and by using a simulation.
Finally, in section (4) we recapitulate the results of this assignment, namely the comparison between the theoretical analysis and the simulation.

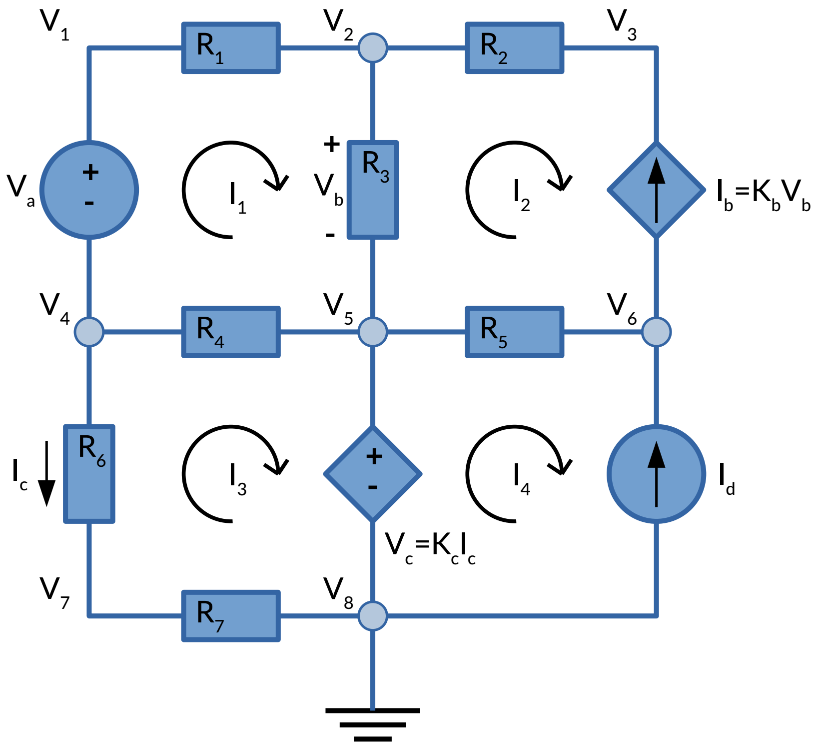

Figure 1: Circuit to be analysed with Node Voltages and Mesh Currents conventions

2 Theoretical Analysis

In this section, the circuit shown in Figure (1) is analysed theoretically using the node voltage method and the mesh current method.

2.1 Nodal Analysis Method

To apply the voltage node method we start by numbering each node as in Figure (1) and by choosing a ground node - in this case we choose node 8. Now we can define 8 node voltages (V_1, V_2, ..., V_8) and proceed with the method by applying KCL (Kirchoff's Current Law) in each node that is not connected to a voltage source. From there the ensuing equations follow:

$$\frac{V_2 - V_1}{R_1} + \frac{V_2 - V_3}{R_2} + \frac{V_2 - V_5}{R_3} = 0 \qquad \text{(Node 2)} \tag{1}$$

$$\frac{V_3 - V_2}{R_2} - K_b V_b = 0 \qquad \text{(Node 3)} \tag{2}$$

$$\frac{V_6 - V_5}{R_5} + K_b V_b - I_d = 0 \qquad \text{(Node 6)} \tag{3}$$

$$\frac{V_7 - V_4}{R_6} + \frac{V_7 - V_8}{R_7} = 0 \qquad \text{(Node 7)} \tag{4}$$

Currently we have 10 variables to solve for (node voltages, I_c and V_b). We can consider the following 5 additional equations:

$$V_8 = 0 \tag{5}$$

$$V_5 - V_8 = K_c I_c \tag{6}$$

$$V_1 - V_4 = V_a \tag{7}$$

$$I_c = \frac{(V_7 - V_8)}{R_7} \tag{8}$$

$$V_b = V_2 - V_5 \tag{9}$$

And by applying KCL in node 4 we can also obtain:

$$\frac{V_4 - V_7}{R_6} + \frac{V_4 - V_5}{R_4} + \frac{V_1 - V_2}{R_1} = 0 \tag{10}$$

Finally by combining the previous expressions we can write 8 linearly independent equations in matricial form:

$$\begin{bmatrix} -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_2} & 0 & -\frac{1}{R_3} & 0 & 0 & 0 \\ 0 & -\frac{1}{R_2} - K_b & \frac{1}{R_2} & 0 & K_b & 0 & 0 & 0 \\ 0 & K_b & 0 & 0 & -\frac{1}{R_5} - K_b & \frac{1}{R_5} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{R_6} & 0 & 0 & \frac{1}{R_6} + \frac{1}{R_7} & -\frac{1}{R_7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & \frac{1}{K_c} & 0 & -\frac{1}{R_7} & \frac{1}{R_7} - \frac{1}{K_c} \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ \frac{1}{R_1} & -\frac{1}{R_1} & 0 & \frac{1}{R_4} + \frac{1}{R_6} & -\frac{1}{R_4} & 0 & -\frac{1}{R_6} & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ I_d \\ 0 \\ 0 \\ 0 \\ V_a \\ 0 \end{bmatrix} \tag{11}$$

Using Octave to solve the matricial equations and considering that $V_b = V_2 - V_5$ and $I_c = \frac{1}{K_c} V_5 - \frac{1}{K_c} V_8$ we obtain the data in table (1).

Quantity	Value
V_1 (V)	8.223567
V_2 (V)	7.961484
V_3 (V)	7.415585
V_4 (V)	2.979997
V_5 (V)	7.999870
V_6 (V)	11.970808
V_7 (V)	0.990779
V_8 (V)	0.000000
V_b (V)	-0.038386
I_c (A)	0.000980

Table 1: Nodal Method, V_b and I_c obtained from the previous matricial system using Octave.

2.2 Mesh Analysis Method

We first start by arbitrarily assigning a current flow to each of the circuit's essential meshes - I_1, I_2, I_3 and I_4. For simplicity's sake, we decided to make the current flow in a counter-clockwise direction in every one of these meshes.
We can apply KVL (Kirchoff's Voltage Law) to mesh 1 and mesh 3 to obtain the following expressions:

$$R_1 I_1 + R_3(I_1 - I_2) + R_4(I_1 - I_3) - V_a = 0 \tag{12}$$

$$R_7 I_3 + R_6 I_3 + R_4(I_3 - I_1) + K_c I_c = 0 \tag{13}$$

By inspection (and using Ohm's Law) we can obtain the following additional equations:

$$I_4 = -I_d \tag{14}$$

$$I_3 = -I_c \tag{15}$$

$$I_2 = -I_b = -K_b V_b = -K_b R_3(I_1 - I_2) \tag{16}$$

Combining the previous equations and writing them in matricial form we obtain:

$$\begin{bmatrix} R_1 + R_3 + R_4 & -R_3 & -R_4 & 0 \\ -R_4 & 0 & R_7 + R_6 + R_4 - K_c & 0 \\ 0 & 0 & 0 & 1 \\ K_b R_3 & 1 - K_b R_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ -I_d \\ 0 \end{bmatrix} \tag{17}$$

Using Octave to solve the matricial equations and considering that $V_b = R_3(I_1 - I_2)$ and $I_c = -I_3$ we obtain the data in table (2).

Quantity	Value
I_1 (A)	0.000259
I_2 (A)	0.000271
I_3 (A)	-0.000980
I_4 (A)	-0.001035
V_b (V)	-0.038386
I_c (A)	0.000980

Table 2: Mesh Current, V_b and I_c obtained from the previous matricial system using Octave.

3 Simulation Analysis

In table (3) we can find the values obtained by the simulation.

Quantity	Value [A or V]
@gb[i]	-2.71033e-04
@id[current]	1.034681e-03
@r1[i]	2.588267e-04
@r2[i]	2.710331e-04
@r3[i]	-1.22064e-05
@r4[i]	-1.23910e-03
@r5[i]	-1.30571e-03
@r6[i]	9.802719e-04
@r7[i]	-9.80272e-04
v(1)	8.223567e+00
v(2)	7.961484e+00
v(3)	7.415585e+00
v(4)	2.979997e+00
v(5)	7.999870e+00
v(6)	1.197081e+01
v(7a)	9.907794e-01
v(7b)	9.907794e-01
v(2, 5)	-3.83855e-02
i(vaux)	9.802719e-04

Table 3: Simulation Results - Variables preceeded by @ are currents in Ampere, while all the other variables are voltages in Volt

We can see that the values obtained from both theoretical methods (which gave identical results) are the same as the ones that result from the simulation. It is worth noting that *v(7a)* and *v(7b)* are just auxilary voltages used to define I_c in *Ngspice* so that we can simulate the current dependent voltage source $V_c = K_c I_c$ and that *v(2,5)* corresponds to V_b and *i(vaux)* to I_c.

4 Conclusion

In this laboratory assignment we have analysed a simple linear circuit through both theoretical and simulation methods. We have obtained values for node voltages and mesh currents as well as specific physical quantities of the circuit (V_b and I_c).
In Section (3) we can see that the simulation values are coherent with the theoretical predictions, this is not surprising considering we are dealing with a very simple linear circuit and thefore there are no motives for significant differences.
We can conclude that the main objectives of this lab assignment have been achieved and that for simple linear circuits there are no significant differences between the theoretical models and the simulations.