

MATH1141: Higher Mathematics 1A

L. Cheung

February 17, 2026

Contents

1	Content Overview	2
2	Sets, inequalities and functions	2
2.1	Sets of numbers	2
2.2	Intervals	2
2.3	Solving Inequalities	2
2.4	Absolute Values	3
2.5	Functions	4
2.5.1	Domain, codomain and range	4
2.5.2	Forming new functions	4
2.6	The Elementary Functions	5
2.7	Implicitly Defined Functions	5
2.8	Continuous Functions	5

1 Content Overview

- Functions: continuity, differentiability, invertibility (inverse functions)
- Curve sketching
- Integration: area, Riemann sum (approximation), Fundamental theorems of calculus
- Logarithm: exponential, hyperbolic

2 Sets, inequalities and functions

2.1 Sets of numbers

A set is a collection of distinct objects. The objects in a set are elements or members of the set. Commonly used sets:

- The set of natural numbers: $\mathbb{N} = \{0, 1, 2, 3, 4, \dots\}$
- The set of integers: $\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
- The set of rational numbers: $\mathbb{Q} = \{\frac{p}{q} : p, q \text{ are integers and } q \neq 0\}$
- The set of \mathbb{R} real numbers may be represented as the collection of points lying on the number line

If A is a set of numbers and the number x is a member of the set A , then we write:

$$x \in A$$

If x is not a member of A , then we write

$$x \notin A$$

2.2 Intervals

- Parenthesis: excludes endpoints
- Bracket: includes endpoints
- Combination: neither open nor closed interval

2.3 Solving Inequalities

For $x, y, z \in \mathbb{R}$:

- if $x > y$, then $x + z > y + z$
- if $x > y$ and $z > 0$, then $xz > yz$
- if $x > y$ and $z < 0$, then $xz < yz$

Example

Solve the quadratic inequality $x^2 + 4x > 21$

$$\begin{aligned}x^2 + 4x > 21 &\iff x^2 + 4x - 21 > 0 \\ &\iff (x + 7)(x - 3) > 0\end{aligned}$$

Solution set: $x < -7$ or $x > 3$

$$x \in (-\infty, -7) \cup (3, \infty)$$

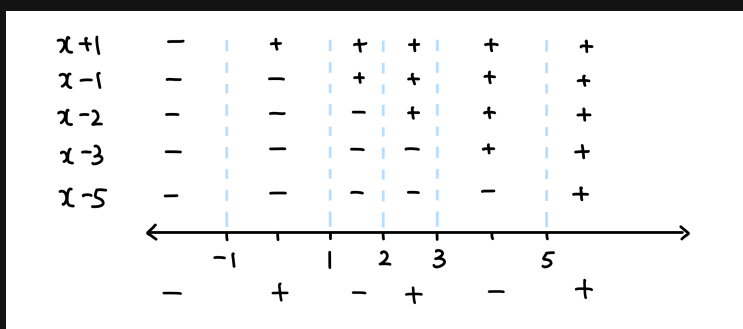
Example

Solve the rational inequality $\frac{1}{x+1} < \frac{1}{(x-2)(x-3)}$

Multiply both sides by $(x+1)^2(x-2)^2(x-3)^2$ (≥ 0),

$$\begin{aligned}\frac{1}{x+1} < \frac{1}{(x-2)(x-3)} &\iff (x+1)(x-2)^2(x-3)^2 < (x+1)^2(x-2)(x-3) \\ &\iff (x+1)(x-2)^2(x-3)^2 - (x+1)^2(x-2)(x-3) < 0 \\ &\iff (x+1)(x-2)(x-3)(x-4)(x-5) < 0\end{aligned}$$

Solution set: $(-\infty, -1) \cup (1, 2) \cup (3, 5)$



2.4 Absolute Values

The absolute value (or magnitude) of a real number x is

$$|x| = \begin{cases} x & \text{if } x \geq 0, \\ -x & \text{if } x < 0 \end{cases}$$

Example

Solve the inequality $|3x+1| \leq 4$

$$\begin{aligned}|3x+1| \leq 4 &\iff \left|x + \frac{1}{3}\right| \leq \frac{4}{3} \\ &\iff x + \frac{1}{3} \leq \frac{4}{3} \quad \text{or} \quad x + \frac{1}{3} \geq -\frac{4}{3} \\ &\iff x \leq \frac{3}{3} \quad \text{or} \quad x \geq -\frac{5}{3}\end{aligned}$$

Solution set: $[-\frac{5}{3}, 1]$

Example

Solve the inequality $\frac{|x+5|}{|x-11|} < 1$

$$\frac{|x+5|}{|x-11|} < 1 \iff |x+5| < |x-11| \quad (\text{multiply both sides by } |x-11|)$$

$$\iff |x+5|^2 < |x-11|^2$$

$$\iff x^2 + 10x + 25 < x^2 - 22x + 121$$

$$\iff 32x < 96$$

$$\iff x < 3$$

Solution set: $(-\infty, 3)$

Note: 11 is not in the solution set

2.5 Functions

2.5.1 Domain, codomain and range

Let A and B be subsets of \mathbb{R} : A function with domain A and codomain B is a rule which assigns to every $x \in A$ exactly one number $f(x) \in B$

$$f : A \rightarrow B$$

$$f : A \ni x \mapsto f(x) \in B$$

- $A = \text{Dom}(f)$: set of all inputs
- $B = \text{Codom}(f)$: set that contains all the outputs
- Range of f : set of actual outputs, defined by:

$$\text{Range}(f) = \{f(x) \in B : x \in A\}$$

- Hence, $\text{Range}(f) \subseteq \text{Codom}(f)$
- The **codomain does not have to be unique**
- The codomain is useful when the range is unknown

Example

Given: $f : [0, \infty) \rightarrow \mathbb{R}, f(x) = 2 + \sqrt{x}, \forall x \in [0, \infty)$

Then, $\text{Dom}(f) = [0, \infty)$, $\text{Codom}(f) = \mathbb{R}$, and $\text{Range}(f) = [2, \infty)$

Natural domain (or maximal domain) of f is the largest possible domain for which the rule makes sense (for real numbers)

2.5.2 Forming new functions

Let A, B, C and D be subsets of \mathbb{R} . Given two functions $f : A \rightarrow B$ and $g : A \rightarrow B$, then the functions

$$f + g, \quad f - g, \quad fg, \quad \frac{f}{g}$$

are defined by the rules

$$(f + g)(x) = f(x) + g(x), \quad \forall x \in A$$

$$(f - g)(x) = f(x) - g(x), \quad \forall x \in A$$

$$(fg)(x) = f(x)g(x), \quad \forall x \in A$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad \forall x \in A \text{ provided that } g(x) \neq 0.$$

To form these new functions, the domains of both f and g must be the same. Suppose that $f : C \rightarrow D$ and $g : A \rightarrow B$ are functions such that

$$\text{Range}(g) \subseteq \text{Dom}(f) = C$$

Then the composition function $(f \circ g)(x) = f(g(x))$, $\forall x \in A$

2.6 The Elementary Functions

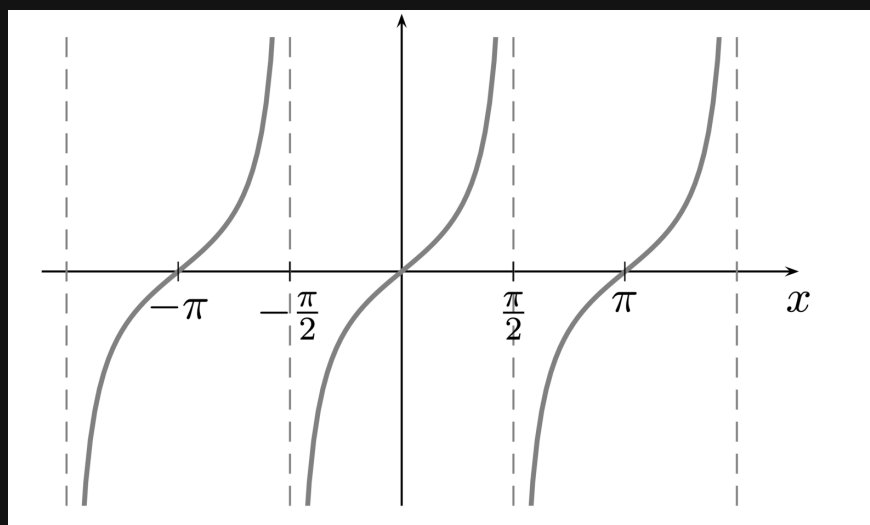
Elementary functions are those that can be constructed by combining a finite number of polynomials, exponentials, logarithms, roots, trigonometric and inverse trigonometric functions via function composition (\circ), $+$, $-$, \times , and \div .

2.7 Implicitly Defined Functions

Many curves on the plane can be described by the points (x, y) that satisfy some equation involving x and y . However, not all these curves are necessarily functions. Some cannot be expressed with a single y term ($y = \dots$). In this case, the curve may be decomposed into two or more functions that are implicitly defined by the curve.

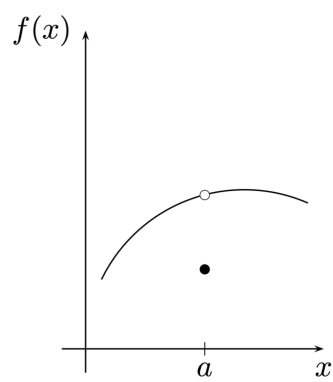
2.8 Continuous Functions

In the below example, the $\tan x$ graph breaks at $x = -\frac{\pi}{2}, \frac{\pi}{2}, \dots$

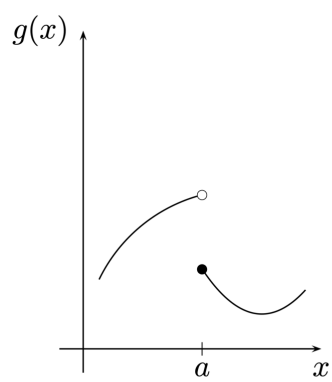


However, this break in the domain does not make it a discontinuity in the graph. Discontinuities can be categorised as following:

removable



jump



essential

