Analysis of Mechanics in Jenga

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Abstract

An analysis of the mechanics of extracting blocks from a tower of stacked blocks in the game of Jenga is presented. Translational and rotational moves are analyzed for the likelihood of disturbing other blocks in the tower, with an aim to determine moves which are safe from disturbing other blocks and are robust against change in outcome with different block configurations. Optimal moves are found using translational moves which are robust against tower configurations and keep the tower undisturbed.

1 Introduction

Playing Jenga well requires players to operate at several different strategic levels, ranging from strategies to extract individual blocks to choosing "easy" blocks as well as choosing blocks to leave other players with poor options. Research has been done looking into tower stability [1] and counting scenario strategies involved in Jenga [5], but this paper focuses on the local strategies of removing individual blocks. In order to focus on local strategies, a simplified version of the Jenga tower is considered, as shown in Figure 1. Using this tower, only moves extracting blocks from the middle tower are considered. This choice derives from two factors: the top level of blocks cannot be chosen during actual play in the normal rules, and the bottom level provides similar outcomes to the middle level. In order to eliminate confusion when referring to individual blocks, blocks will be referred to by level as "top", "bottom", and "middle", while position in a level is designated either by "forward", "back", and "center" or "left", "right", and "center" depending on the orientation of the level.

This paper focuses on analyzing possible manipulations of blocks, and the resulting mechanics. The first analysis focuses on moves involving translation of blocks only, with discussion of the optimal movements resulting from these strategies. The possibility of rotational maneuvers is also analyzed, using both kinematic analysis and implications from translational moves.

1.1 Gameplay

The game of Jenga is a challenging game with significant physical intricacy. The game uses fifty-four hardwood blocks, stacked in groups of three oriented perpendicular to the previous layer, similar to Figure 1. A full game has eighteen levels of blocks. The rules are as follows - players take turns removing blocks from

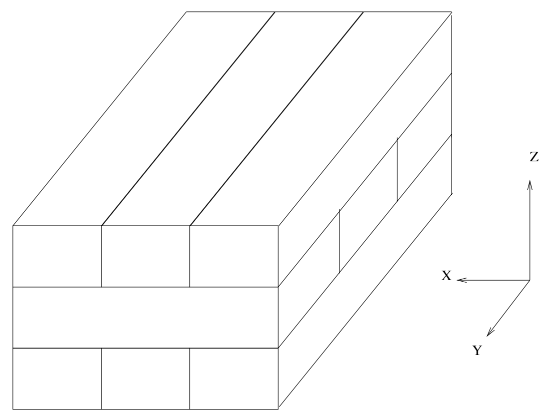

Figure 1: Simplified Jenga Tower.

any level other than the top, without causing the tower to topple. The player who safely removed the last block before the tower topples wins. Since keeping the tower stable is the primary goal of the game, a first look at the physical aspects of the game analyzes the movement of blocks, minimizing the motion of blocks within the tower.

1.1.1 Jenga Blocks

Jenga blocks are hardwood blocks made into rectangular prisms. For the actual game, the exact physical dimension of each individual block varies, but using a set of random samples taken in [4] the average block weighs 19.6 g, and is 8.1 cm long, 2.6 cm wide, and 1.8 cm tall. The static coefficient of friction was taken from [2], which gives a typical value $\mu_s = 0.4$ for hardwood, independent of wood species. Kinetic friction is typically less than static friction, but a value is not given in [2]. In this analysis an actual value of kinetic friction is not used, except insofar that $\mu_s > \mu_k$. While the actual game includes physical variation in blocks for additional complexity, this paper assumes Jenga blocks are ideal. Blocks are of uniformly dense, perfect rectangular prisms, and have no variation in size or mass.

2 Translation Moves

Extraction of blocks using translation only provides the most straightforward execution for extracting a block. A zeroth order analysis of translations utilizing physical constraints yields simplification - blocks cannot move along the z-axis shown in Figure 1. For any block being extracted, there is complete contact on faces normal to the z-axis. These movements are not considered as part of optimal strategies since they must cause movement in other blocks. This leaves only movements along the x- and y-axes of the tower to be considered.

2.1 Side Blocks

Extraction of front and back middle blocks from the tower provides the most freedom in strategy for extraction, allowing movement along both x- and y-axes. First, the analysis with extraction occurring by moving the block along the y-axis.

2.1.1 Y-Axis Translation

Extraction of a front or back middle block along the y-axis results in the free body-diagram shown in Figure 2. Extraction is performed by applying equal force to both sides of the block, resulting in a single equivalent force F_{app} to be exerted through the center of mass of the block. There are three groups of blocks to consider for this: the top blocks, the target block, and bottom blocks. The equations for the top layer are the most straightforward, with only frictional forces to consider. While Figure 2 only shows frictional forces on the top middle block, the forces are identical for the left and right top blocks, but are not shown to keep the figure clean. The forces acting on the top center block are shown as F_{slide}, which represents the frictional interaction between the target block and the top center block, and f_1 and f_2, which represent the frictional forces exerted by the middle center and middle back block.

Assuming that F_{app} is strong enough to produce an acceleration a_{app} within the target block, the frictional force between the top center block and the target block will attempt to keep the two blocks in sticking together. However, the other two middle blocks exert frictional forces resisting the motion of the top center block. The maximum frictional force that any of these blocks exert is given by Coulomb's Law:

$$f_s = \mu_s * m_{block} * g \tag{1}$$

The top center block is thus given a scenario wherein $fslide_{max} < f_{1_{max}} + f_{2_{max}}$. This means the top center block is forced into a sliding condition with the target block and sticking to the other middle blocks. This result is key - no

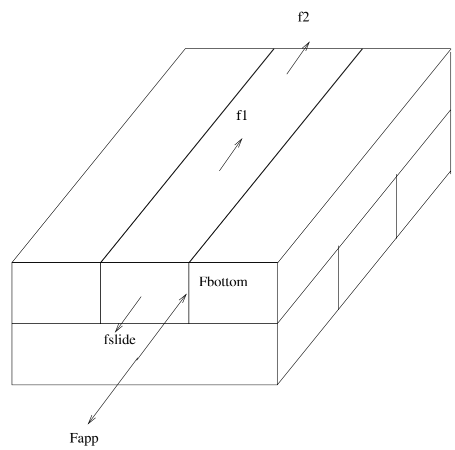

Figure 2: Free body diagram of sliding a side block along the width.

F_{app} can produce a force capable of moving the top center block. The free body diagrams for the top left and right blocks have identical results, meaning that the top layer is secure in its position.

For the target block, the free body diagram contains two frictional forces, f_{bottom} and $-f_{slide}$, and one applied force, F_{app}. For this block, the equation governing it's motion is shown in Eq. 3. This equation shows that for $F_{app} > 0.23N$, the block will slide, otherwise the target block will remain at rest.

$$\frac{1}{2} * f_{bottom} = -f_{slide} = \mu_s * m_{block} * g \tag{2}$$

$$F_{app} - f_{bottom} - f_{slide} = m_{block} * a_{app} = F_{app} - 3 * \mu_s * m_{block} * g \tag{3}$$

$$F_{app} = 3 * \mu_s * m_{block} * g = 0.23N \tag{4}$$

For the bottom row, the results are approximately similar to the top layer. The most notable difference comes from increased normal forces due to mass in higher levels pushing down on the bottom level. This serves to increase the friction keeping the blocks steady, meaning that this layer is safe from motion while extracting the target block.

The move shown in Figure 2 is robust against the absence of other blocks. In removing a side block, the middle center block is required to maintain a stable tower. A single block can generate enough friction to resist acceleration in the top layer, since $f_{slide_{max}} = f_{1_{max}} = \mu_s * g * m_{block}$.

2.1.2 X-axis Translation

The second extraction method for a side block slides the block along the x-axis. This method is shown in Figure 3. Once again, the analysis is broken down into top layer, target block, and bottom layer.

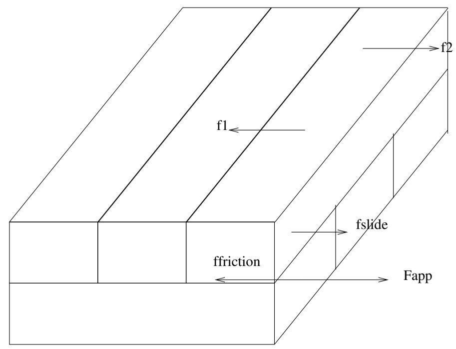

Figure 3: Free body diagram of sliding a side block along the length.

For the top layer, the frictional forces are f_{slide}, the force attempting to maintain sticking to the target block, and f_1 and f_2 which attempt to resist sliding on the other center blocks. As the diagram shows, the frictional forces can potentially produce torques on the blocks. The only force capable of counteracting the torque generated by f_{slide} is f_2, since f_1 acts through the center of gravity of the top blocks. Since $f_{2_{max}} = f_{slide_{max}} = \mu_s * g * m_{block}$, the target block must have a force applied that exceeds the maximum static friction and causes the top blocks to slide relative to the target block.

$$F_{app} - 2 * \mu_s * g * m_{block} - \mu_s * g * m_{block} = m_{block} * a_{app} \tag{5}$$

$$a_{app} = \frac{F_{app}}{m_{block}} - 3 * \mu_s * g \tag{6}$$

$$a_{top_{max}} = \mu_s * g \tag{7}$$

$$F_{app} < 4 * \mu_s * g * m_{block} = 0.307N \tag{8}$$

Working through the equations, we finally get the result seen in Eq. 8. Any $F_{app} > 0.307N$ will result in a torque applied to the top layer. Comparing this to Eq. 4, there is now a range of forces, $0.23N < F_{app} < 0.307N$ in which the block will slide but result in a torque being applied to the top layer. For any block in the tower in its normal orientation, a torque τ will induce an angular acceleration as given by Eq. 9, where The value of I is calculated in 10, which indicates a small moment of inertia, and thus a high angular acceleration for a given τ.

$$\alpha = \frac{\tau}{I} \tag{9}$$

$$I = \frac{m_{block} * (l^2 + w^2)}{12} = 1.18E - 5kg * m^2 \tag{10}$$

For the target block, the frictional forces are $f_{friction}$, which is the friction resisting from the bottom layer, and $-f_{slide}$, which is the force resisting the force from the top layer, while the applied force F_{app} is the force attempting the block move. The result here in the same as Eq. 4, with the block beginning to slide with an applied force of 0.23 N.

For the bottom level, the difference with the top level is the increased normal force due to the extra mass pressing on these blocks. The f_2 force for the bottom has a maximum value of $f_{2_{max}} = 3 * \mu_s * g$. The additional normal force benefits the bottom level, since $f_{2_{max}} > f_{friction}$, meaning the bottom level can counteract the torque produced by the target block.

This movement is not robust to the removal of additional blocks. If the other side block has been removed, f_2 is not present to counteract the torque produced by f_{slide}. In this situation, if $F_{app} > 0.23N$ torque will be generated by F_{app}, resulting in rotation of the top layer.

2.2 Middle Center Block

The movement of the middle center block is restricted to a single translation along the x-axis. While translation along the x-axis results in the possibility of generating torques in the case of middle side blocks, it cannot do so with the middle center block since forces operate through the center of gravity of the blocks. By appropriately translating the origin shown in Figure 1, the middle center block's translation results in the same results as Eq. 4 and shown in Figure 2. As a result, the situation remains identical - the target block can be extracted without disturbing the surrounding blocks.

The removal of the center block is also robust to the absence of other blocks in the tower. Since the two side blocks in the center level must be present in order to remove the middle center block, there will always be frictional forces capable of resisting disturbance of the tower.

3 Rotation Moves

Rotating blocks poses a significant challenge for analysis, due to the complications of applying forces to produce a rotation. In particular, the limit surface technique shown in [3], while capable of dealing with pure rotations, has two limitations which prevent its application: applied forces must act in the support plane, and accelerated motion requires the center of gravity to be in the support plane. Neither of these conditions are met with Jenga blocks, meaning that the pressure distribution is unknown. Furthermore, the pressure distribution changes as a block is partially removed, changing the support area over which the pressure distribution acts.

It is important to note that rotation moves can only be considered for side blocks. The center block is physically constrained along x- and z-axes, resulting in no allowable rotation for the center middle block. However, the pitch and roll cases of the side blocks can be solved with simple kinematic considerations, and yaw can be considered through translational moves.

3.1 Pitch and Roll

Pitch and roll moves are moves which are easily ruled out by some fairly simple kinematic considerations. Looking at the tower shown in Figure 1, we can see that the blocks are constrained in the z-axis by contact with other blocks. With no space between one level of the tower and the next, any move which shifts part of a block along the z-axis will produce movement in the surrounding blocks. As a result, these moves are eliminated from consideration, since both pitch and roll will cause some portion of a block to push another block.

3.2 Yaw

The yaw moves are the only rotation movements which potentially can produce a reasonable move, since yaw will not violate the physical constraints of the tower. Since rotation movements cannot be analyzed due to constraints in generating the limit surface, lessons from the translational maneuvers can be used to consider yaw movements. As we saw in Section 2.1, applying a force which does not act through the center of gravity of blocks on the top level can induce torques. Applying a torque to blocks results in high angular acceleration due to the small moment of inertia. Considering that optimal moves minimize the movement of blocks other than the target block, any yaw move producing forces which can induce torques is sub-optimal. Any yaw on the side blocks will eventually result in forces potentially capable of generating torques, which is given as sufficient motivation to eliminate yaw from consideration.

4 Conclusion

Analysis of the possible moves for extracting blocks from the Jenga tower results in a single move reaching a perfect outcome. Figure 2 shows a movement which assures that only the target block is disturbed. This move is robust to having fewer blocks present in the tower, and does not have any potential for resulting in disturbance of other blocks. For the center middle block, the only move available maintains the same benefits as the optimal moves for side blocks. Fully analyzing the rotational moves cannot be performed with current techniques, but with some reasoning about the translational cases, these moves are found to be uninteresting as part of an optimal local strategy.

The results presented here are interesting when compared against observations of the actual game in play. The optimal moves developed typically work well in the actual game, but there are cases where these results do not appear to always hold. There are several factors which complicate the actual game of Jenga when compared to this simplified model. The biggest factor is variability in game pieces. As shown in [4], blocks vary in physical dimension and weight, resulting in some blocks becoming load bearing and others free floating within the structure. Errors in grip and manipulation also increase difficulty, since pressure distributions and contact areas change as blocks are moved incorrectly. Also, with more levels present in the tower, normal forces are increased, but also not acting solely through the center of gravity for any given level. With blocks offset from ideal positions, normal forces begin to apply torques on the tower, resulting in additional forces.

References

[1] Peter Barnum. Algorithmic analysis of the stability of stacked objects, 2006.

[2] Forest Products Laboratory. *Wood Handbook - Wood as an Engineering Material*, 2002.

[3] Matthew Mason. *Mechanics of robotic manipulation*. The MIT Press, 2001.

[4] Matthew South. A real-time physics simulator for jenga. Master's thesis, University of Sheffield, Sheffield, UK, 2003.

[5] Uri Zwick. Jenga. In *Proceeding of 13th SODA*, 2002.