EVALUATION SEMANTICS OF COERCION

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An idea for a *new* evaluation dynamics of coercion inspired by the denotational semantics of cubical type theory. The main idea is that coercions should *not* be evaluated in an ordinary environment, but instead an environment where each cell is a line.

$$\llbracket \Gamma \vdash A \rrbracket : \llbracket \Gamma \rrbracket \to \mathbf{Type}$$

$$\llbracket \Gamma \vdash a : A \rrbracket : \prod_{\gamma : \llbracket \Gamma \rrbracket} \llbracket \Gamma \vdash A \rrbracket_{\gamma}$$

$$\mathbf{coe}\llbracket \Gamma \vdash A \rrbracket : \prod_{\boldsymbol{\gamma} : \llbracket \Gamma \rrbracket^{\mathbb{I}}} \prod_{r,s:\mathbb{I}} \prod_{a : \llbracket \Gamma \vdash A \rrbracket_{\boldsymbol{\gamma}(r)}} \llbracket \Gamma \vdash A \rrbracket_{\boldsymbol{\gamma}(s)}$$

$$\llbracket \Gamma \vdash \mathbf{coe}_F^{r \to s} M : F(r) \rrbracket \gamma = \mathbf{coe}\llbracket \Gamma.\mathbb{I} \vdash F(i) \rrbracket^{\llbracket \Gamma \vdash r : \mathbb{I} \rrbracket_{\gamma} \to \llbracket \Gamma \vdash s : \mathbb{I} \rrbracket_{\gamma}}_{\langle \mathbf{refl}(\gamma), \mathbf{id}_{\mathbb{I}} \rangle} \llbracket \Gamma \vdash M : F(r) \rrbracket_{\gamma}$$

$$\mathbf{coe}\llbracket \Gamma \vdash \Pi(A, B) \rrbracket_{\boldsymbol{\gamma}}^{r \to s} f = \lambda a . \mathbf{coe}\llbracket \Gamma.A \vdash B \rrbracket_{\langle \boldsymbol{\gamma}, \lambda i . \mathbf{coe}\llbracket \Gamma \vdash A \rrbracket_{\boldsymbol{\gamma}}^{s \to i} a \rangle} \mathbf{coe}\llbracket \Gamma \vdash A \rrbracket_{\boldsymbol{\gamma}}^{s \to r} a$$

$$\mathbf{coe}\llbracket \Gamma \vdash \Sigma(A, B) \rrbracket_{\boldsymbol{\gamma}}^{r \to s} p = (\mathbf{coe}\llbracket \Gamma \vdash A \rrbracket_{\boldsymbol{\gamma}}^{r \to s} \pi_1(p), \mathbf{coe}\llbracket \Gamma.A \vdash B \rrbracket_{\langle \boldsymbol{\gamma}, \lambda i . \mathbf{coe}\llbracket \Gamma \vdash A \rrbracket_{\boldsymbol{\gamma}}^{r \to i} \pi_1(p) \rangle}^{r \to s} \pi_2(p))$$

$$\mathbf{coe}\llbracket \Gamma \vdash \mathsf{V}_i(A, B, E) \rrbracket_{\boldsymbol{\gamma}}^{r \to s}(v) = \begin{cases} ?0 & \text{if } \forall j : \mathbb{I}, \llbracket \Gamma \vdash i : \mathbb{I} \rrbracket_{\boldsymbol{\gamma}(j)} = j \\ ?1 & \text{otherwise} \end{cases}$$