EVALUATION SEMANTICS OF COERCION

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An idea for a *new* evaluation dynamics of coercion inspired by the denotational semantics of cubical type theory. The main idea is that coercions should *not* be evaluated in an ordinary environment, but instead an environment where each cell is a line.

$$
\begin{aligned}
[\![\Gamma \vdash A]\!] &: [\![\Gamma]\!] \longrightarrow \mathbf{Type} \\
[\![\Gamma \vdash a : A]\!] &: \textstyle\prod_{\gamma : [\![\Gamma]\!]} [\![\Gamma \vdash A]\!]_{\gamma} \\
\mathbf{coe}[\![\Gamma \vdash A]\!] &: \textstyle\prod_{\boldsymbol{\gamma} : [\![\Gamma]\!]^{\mathbb{I}}} \prod_{r,s : \mathbb{I}} \prod_{a : [\![\Gamma \vdash A]\!]_{\boldsymbol{\gamma}(r)}} [\![\Gamma \vdash A]\!]_{\boldsymbol{\gamma}(s)}
\end{aligned}
$$

$$
[\![\Gamma \vdash \mathrm{coe}_{F}^{r \longrightarrow s} M : F(r)]\!]\gamma = \mathbf{coe}[\![\Gamma.\mathbb{I} \vdash F(i)]\!]_{\langle \mathbf{refl}(\gamma), \mathbf{id}_{\mathbb{I}} \rangle}^{[\![\Gamma \vdash r : \mathbb{I}]\!]_{\gamma} \longrightarrow [\![\Gamma \vdash s : \mathbb{I}]\!]_{\gamma}} [\![\Gamma \vdash M : F(r)]\!]_{\gamma}
$$

$$
\begin{aligned}
\mathbf{coe}[\![\Gamma \vdash \Pi(A,B)]\!]_{\boldsymbol{\gamma}}^{r \longrightarrow s} f &= \lambda a. \mathbf{coe}[\![\Gamma.A \vdash B]\!]_{\langle \boldsymbol{\gamma}, \lambda i. \mathbf{coe}[\![\Gamma \vdash A]\!]_{\boldsymbol{\gamma}}^{s \to i} a \rangle} \mathbf{coe}[\![\Gamma \vdash A]\!]_{\boldsymbol{\gamma}}^{s \longrightarrow r} a \\
\mathbf{coe}[\![\Gamma \vdash \Sigma(A,B)]\!]_{\boldsymbol{\gamma}}^{r \longrightarrow s} p &= (\mathbf{coe}[\![\Gamma \vdash A]\!]_{\boldsymbol{\gamma}}^{r \longrightarrow s} \pi_1(p), \mathbf{coe}[\![\Gamma.A \vdash B]\!]_{\langle \boldsymbol{\gamma}, \lambda i. \mathbf{coe}[\![\Gamma \vdash A]\!]_{\boldsymbol{\gamma}}^{r \to i} \pi_1(p) \rangle}^{r \longrightarrow s} \pi_2(p))
\end{aligned}
$$

$$
\mathbf{coe}[\![\Gamma \vdash \mathsf{V}_i(A,B,E)]\!]_{\boldsymbol{\gamma}}^{r \longrightarrow s}(v) = \begin{cases}
\mathbf{coe}[\![\Gamma, i = 0 \vdash A]\!]_{\langle \boldsymbol{\gamma}, \star \rangle}^{r \longrightarrow s}(v) & \text{if } \forall j : \mathbb{I},\ [\![i]\!]_{\boldsymbol{\gamma}(j)} = 0 \\
\mathbf{coe}[\![\Gamma \vdash B]\!]_{\boldsymbol{\gamma}}^{r \longrightarrow s}(v) & \text{if } \forall j : \mathbb{I},\ [\![i]\!]_{\boldsymbol{\gamma}(j)} = 1 \\
? & \text{if } \forall j : \mathbb{I},\ [\![i]\!]_{\boldsymbol{\gamma}(j)} = j \\
\mathbf{Vin}_{[\![i]\!]_{\boldsymbol{\gamma}(s)}}(\mathbf{coe}[\![\Gamma.i = 0 \vdash A]\!]_{\langle \boldsymbol{\gamma}, \star \rangle}^{r \longrightarrow s} v, \mathbf{com}[\![\Gamma \vdash B]\!]_{\boldsymbol{\gamma}}^{r \longrightarrow s}[\partial([\![i]\!]_{\boldsymbol{\gamma}(s)})](\ldots)) & \text{otherwise}
\end{cases}
$$

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