RedPRL

designing the people's refinement logic

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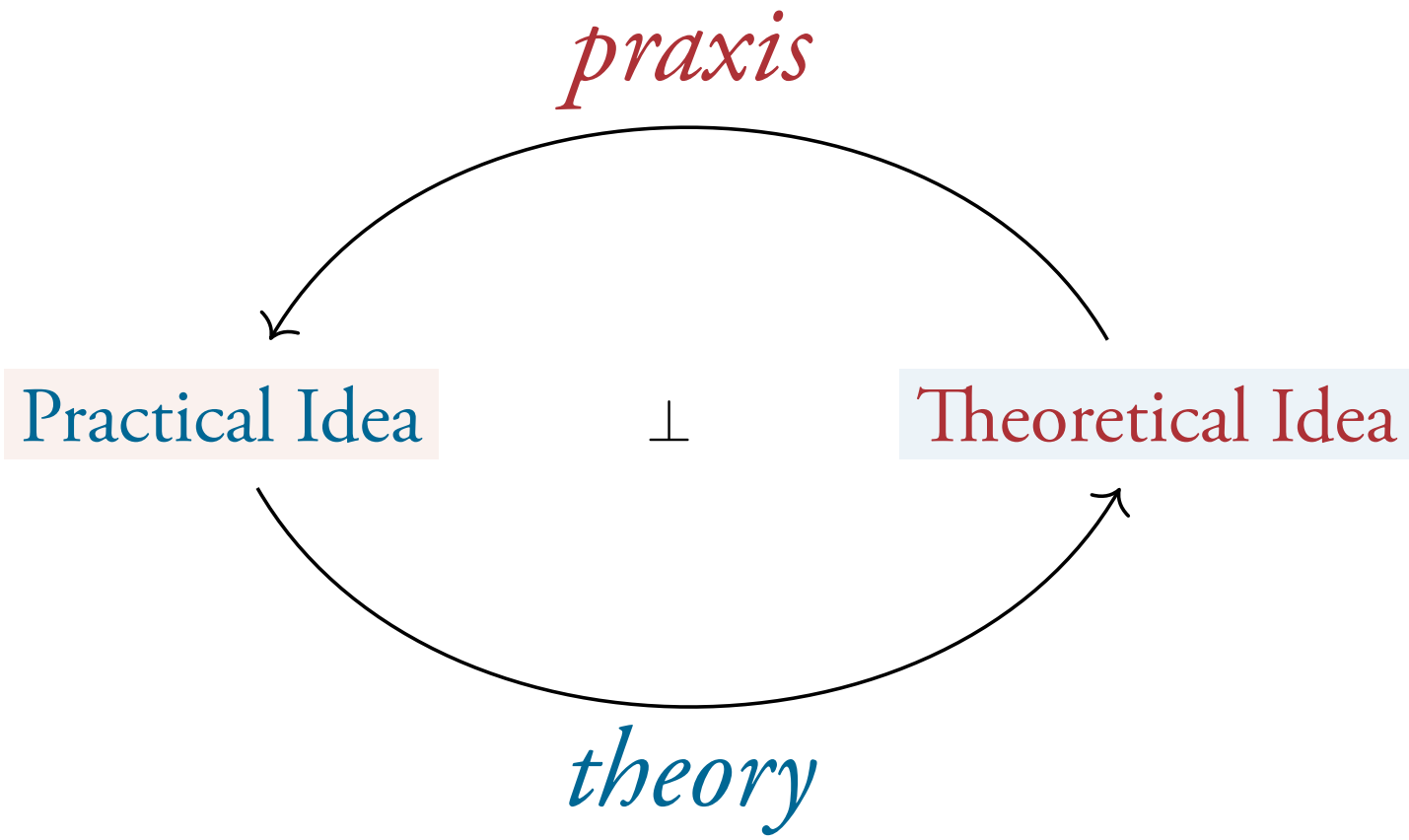
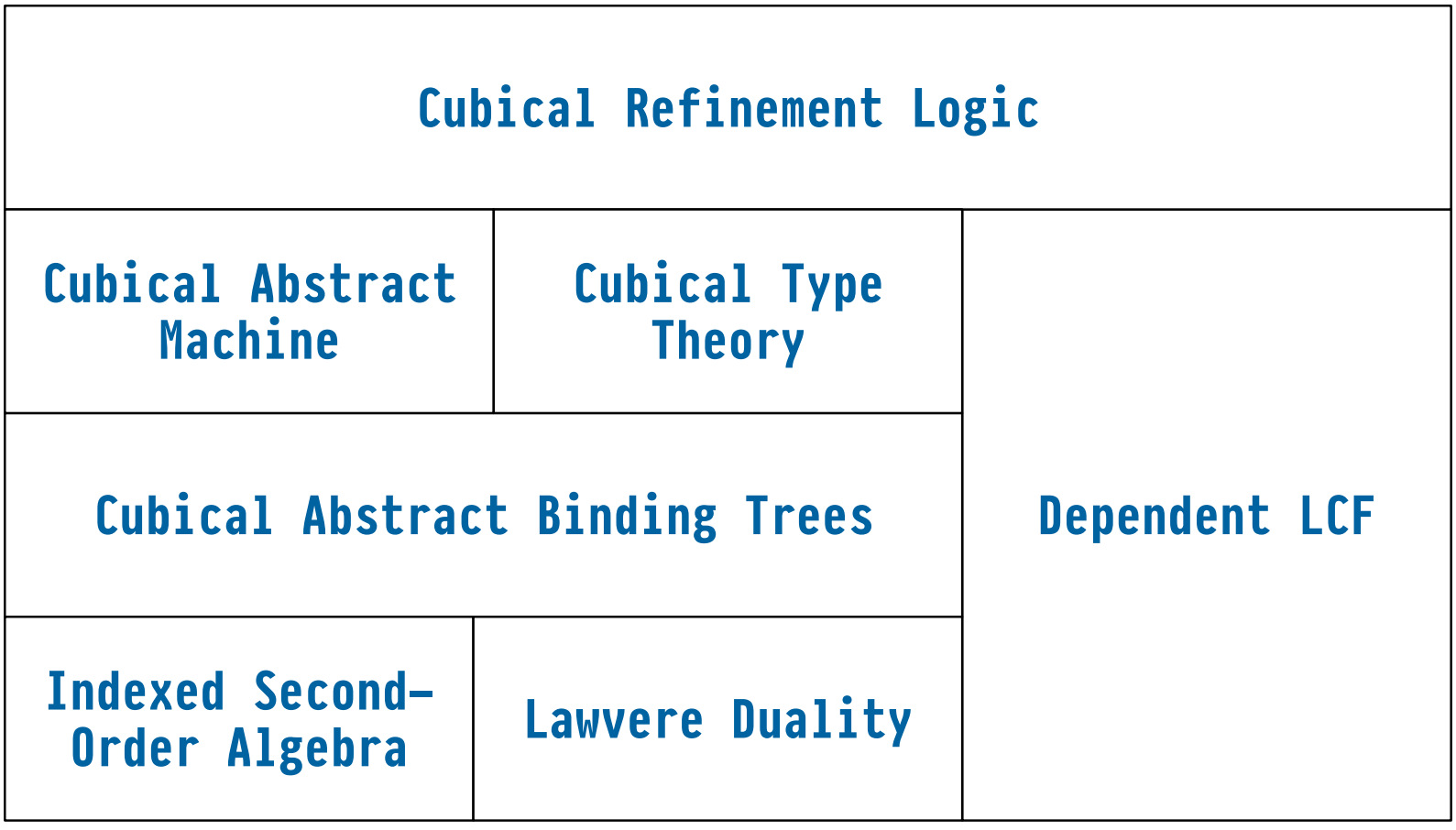
So, why bother?

the absolute idea



(With thanks to Hegel, Marx, Mao and Lawvere.)

overview of RedPRL



outline

1. The LCF Tradition
2. The Revisionists And Their Running Dogs
3. RedPRL: New Synthesis Of Proof Refinement

lcf architecture

```
type evd                                              (* evidence *)
type α state = α list ⊗ (evd list → evd)              (* proof state *)
type (α, β) tactic = α ⊸ β state
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(* proof state "monad" *)
val id : (α, α) tactic
val map : (α → β) → (α state, β) tactic
val mul : (α state state, α) tactic
val orelse : (α, β) tactic ⊗ (α, β) tactic → (α, β) tactic
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(* standard tacticals *)
val then : (α, β) tactic ⊗ (β, γ) tactic → (α, γ) tactic
fun then (t1, t2) = mul ∘ map t2 ∘ t1
val thenl : (α, β) tactic ⊗ (β, γ) tactic list → (α, γ) tactic
```

lcf architecture

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include LCF
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datatype prop =
    ∧ of prop ⊗ prop
  | ∨ of prop ⊗ prop
  | ⊃ of prop ⊗ prop
  | ⊤, ⊥
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datatype jdg = ⊢ of prop dict ⊗ prop
type rule = (jdg, jdg) tactic
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datatype jdg = ⊢ of prop dict ⊗ prop
type rule = (jdg, jdg) tactic

val ∧R, ∨R, ⊃R, ⊤R, ⊥R : rule
val hyp : string → rule
val ∧L : string ⊗ string ⊗ string → rule
...
```

programs as evidence

Abstract (!!) type of evidence implemented as functional programming language:

```
datatype evd =
  | var of string
  | λ of string ⊗ evd
  | ap of evd ⊗ evd
  | pair of evd ⊗ evd
  | π1, π2 of evd
  | inl, inr of evd
  | split of evd ⊗ evd ⊗ evd
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Other options possible: machine code, JavaScript, Perl, PHP, Julia ;-), etc.

inference rules

`inference rule` $\Leftrightarrow$ `ML function`

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inference rule $\Leftrightarrow$ ML function

$$\frac{}{\Delta, x : P, \Xi \vdash P}\ hyp[x] \quad \Leftrightarrow$$

```
fun hyp (x) (Γ ⊢ P) =
 let  (Δ, Q, Ξ) = split(Γ, x)
 and  true = (P = Q)
  in  ([], fn [] ⇒ var(x))
```

inference rules

$$\texttt{inference rule} \Leftrightarrow \texttt{ML function}$$

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$$\begin{array}{l}\textbf{fun}\ \mathrm{hyp}\,(x)\ \left(\ \Gamma \vdash P\ \right) = \\ \quad \textbf{let}\ \ (\Delta, Q, \Xi) = \mathrm{split}(\Gamma, x) \\ \quad \textbf{and}\ \ \mathrm{true} = (P = Q) \\ \quad\ \textbf{in}\ \ ([], \textbf{fn}\ [] \Rightarrow \mathrm{var}(x))\end{array}$$

$$\frac{\Gamma \vdash P \quad \Gamma \vdash Q}{\Gamma \vdash P \wedge Q}\ \wedge_R \quad \Leftrightarrow$$

$$\begin{array}{l}\textbf{fun}\ \wedge_R \left(\ \Gamma \vdash P \wedge Q\ \right) = \\ \left(\begin{array}{l}\left[\ \Gamma \vdash P\ ,\ \Gamma \vdash Q\ \right], \\ \ \textbf{fn}\ [M, N] \Rightarrow \mathrm{pair}(M, N)\end{array}\right)\end{array}$$

inference rules

$$\frac{\Delta, x : P \wedge Q, y : P, z : Q, \Xi \vdash R}{\Delta, x : P \wedge Q, \Xi \vdash R}\ \wedge_L[x, y, z]$$

$$\Updownarrow$$

$$\begin{array}{l}\textbf{fun}\ \wedge_L(x, y, z)\ \left(\Gamma \vdash R\right) = \\ \quad \textbf{let}\ \ (\Delta, P \wedge Q, \Xi) = \mathsf{split}(\Gamma, x) \\ \quad\ \textbf{in}\ \ \left(\begin{array}{l}\left[\,\Delta, x : P \wedge Q, y : P, z : Q, \Xi \vdash R\,\right], \\ \textbf{fn}\ [M] \Rightarrow [\pi_1(\mathsf{var}(x)), \pi_2(\mathsf{var}(x))/y, z]M\end{array}\right)\end{array}$$

proofs and scripts

$$\frac{}{x : P \wedge Q \vdash P \wedge Q}$$

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$$\dfrac{\dfrac{}{x : P \wedge Q, y : P, z : Q \vdash P \wedge Q}}{x : P \wedge Q \vdash P \wedge Q}\ \wedge_L [x, y, z]$$

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$$\wedge_L (x, y, z)$$

proofs and scripts

$$\dfrac{\dfrac{\dfrac{}{x : P \wedge Q, y : P, z : Q \vdash P} \quad \dfrac{}{x : P \wedge Q, y : P, z : Q \vdash Q}\ \wedge_R}{x : P \wedge Q, y : P, z : Q \vdash P \wedge Q}\ \wedge_L\,[x, y, z]}{x : P \wedge Q \vdash P \wedge Q}$$

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$\wedge_L\,(x, y, z)$
then $\wedge_R$

proofs and scripts

$$\dfrac{\dfrac{}{x : P \wedge Q, y : P, z : Q \vdash P}\ hyp[y] \qquad \dfrac{}{x : P \wedge Q, y : P, z : Q \vdash Q}\ hyp[z]}{\dfrac{x : P \wedge Q, y : P, z : Q \vdash P \wedge Q}{x : P \wedge Q \vdash P \wedge Q}\ \wedge_L [x, y, z]}\ \wedge_R$$

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$$\begin{array}{l} \wedge_L (x, y, z) \\ \texttt{then}\ \ \wedge_R \\ \texttt{then1}\ \left[\begin{array}{l} \texttt{hyp}(y), \\ \texttt{hyp}(z) \end{array}\right] \end{array}$$

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$\Updownarrow$

$$\wedge_L\,(x, y, z)\ \texttt{then}\ \wedge_R\ \texttt{thenl}\ \begin{bmatrix} \texttt{hyp}(y), \\ \texttt{hyp}(z) \end{bmatrix} \rightsquigarrow \texttt{pair}(\pi_1(\texttt{var}(x)), \pi_2(\texttt{var}(x)))$$

free program invariants

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All this is possible, whilst generating efficient codes in an arbitrary language. Proof structure does not need to appear in programs.

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- sequent calculus rules trivially translated into ML
- easy to check that a collection of rules is correct
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DECISIVELY SMASH THE FORMALIST CLIQUE!

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- ⋆ sadly, no dependent refinement (cf. “constructible subgoals property”)
- ⋆ ⇒ many sensible rules cannot be encoded (e.g. intro rule for Σ-types, bidirectional typing)
- ⋆ ⇒ complicated and brittle tactics are necessary for basic use

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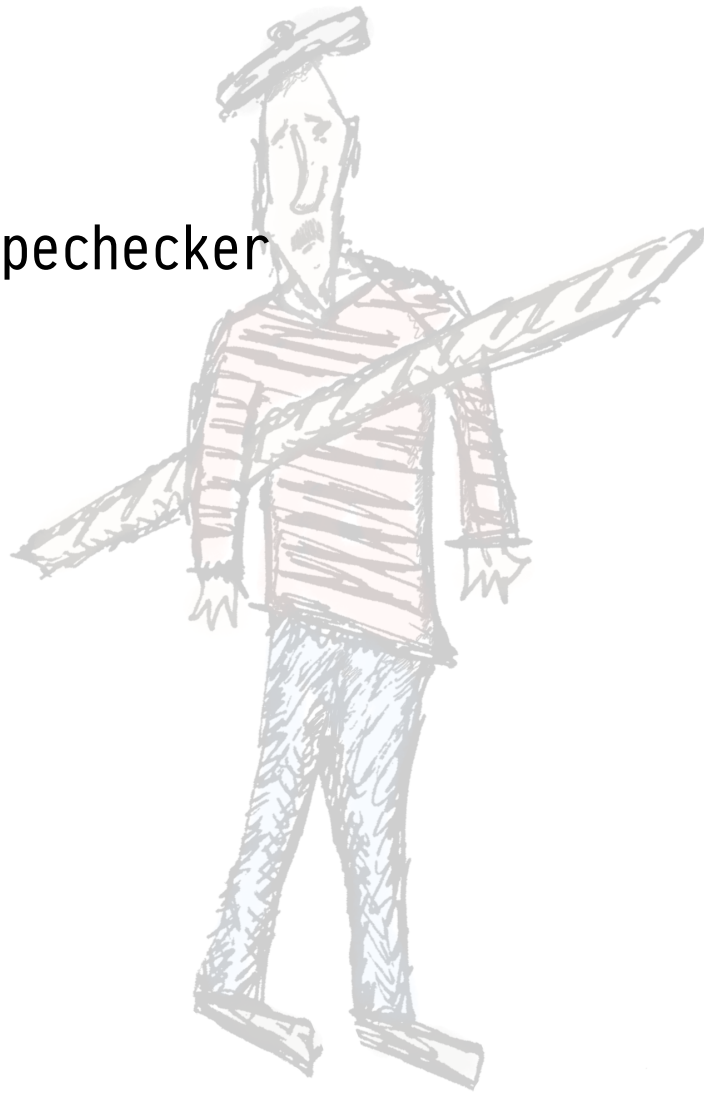

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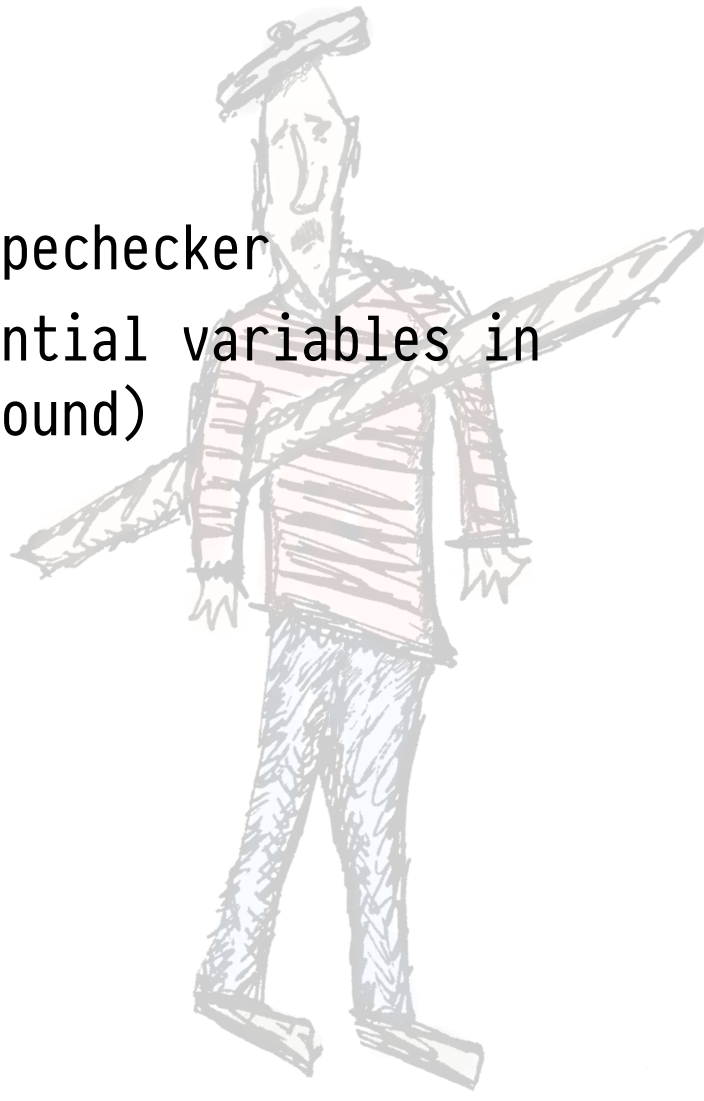

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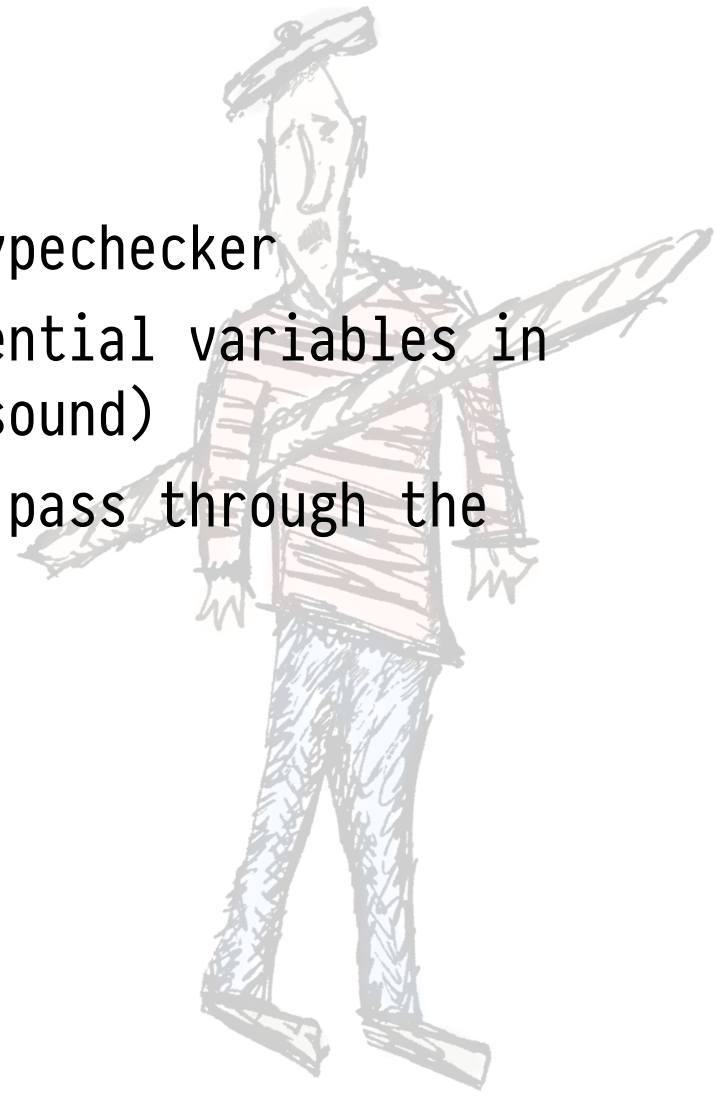

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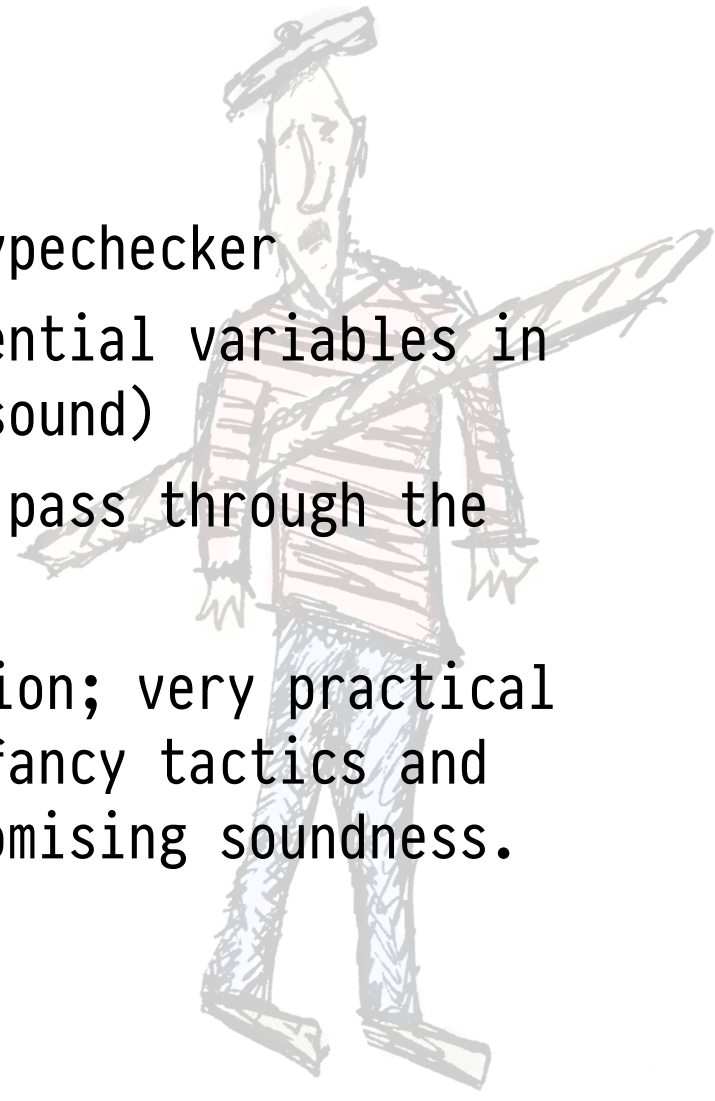

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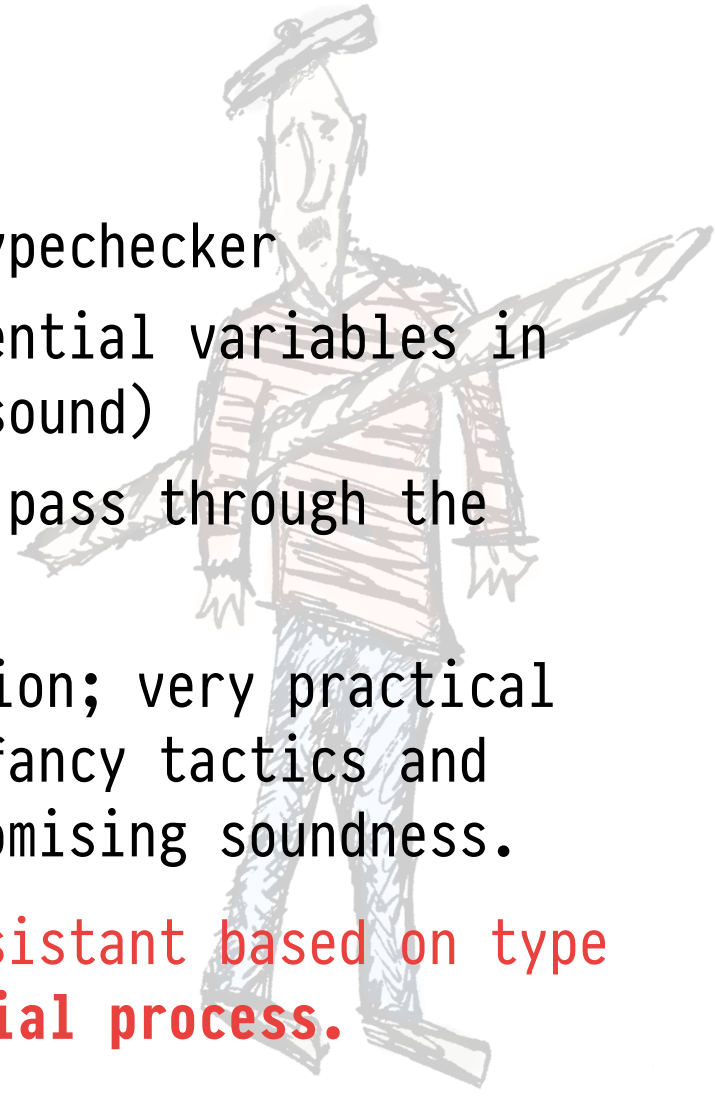

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Coq is the most successful proof assistant based on type theory in history. **Science is a social process.**

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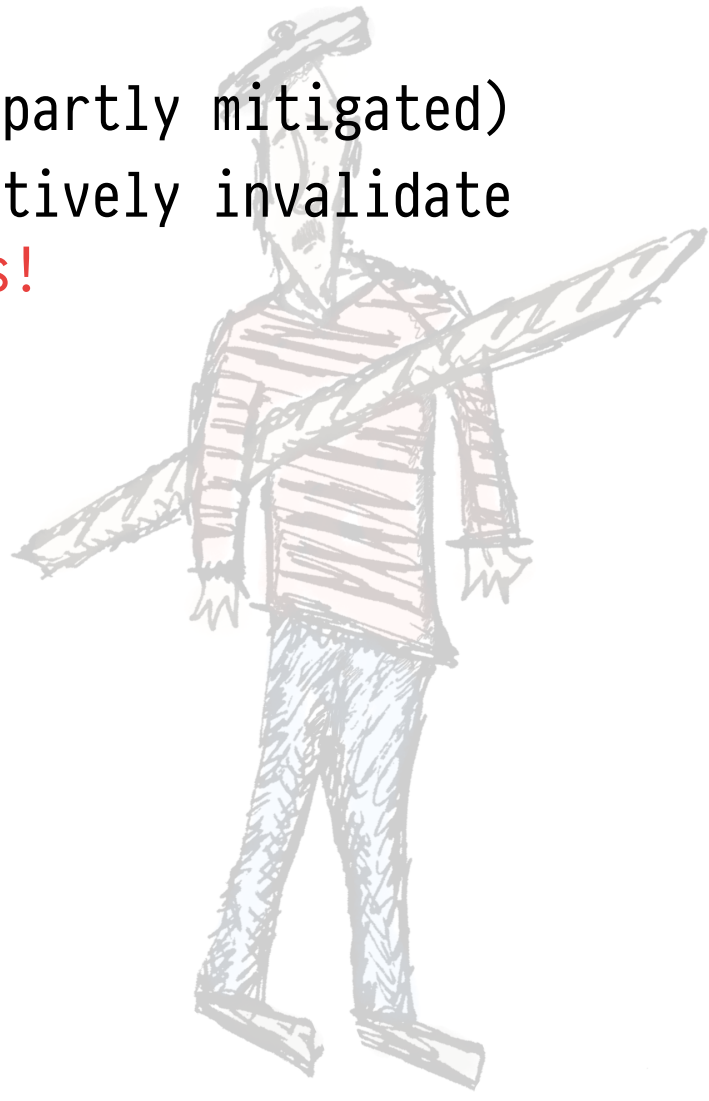

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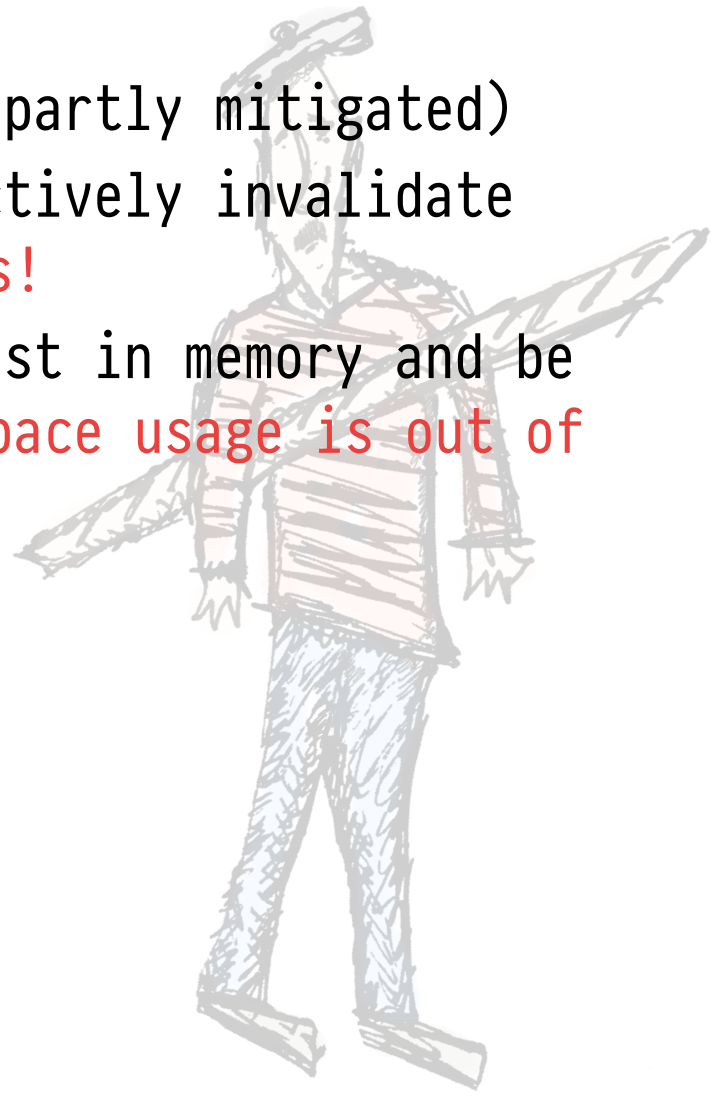

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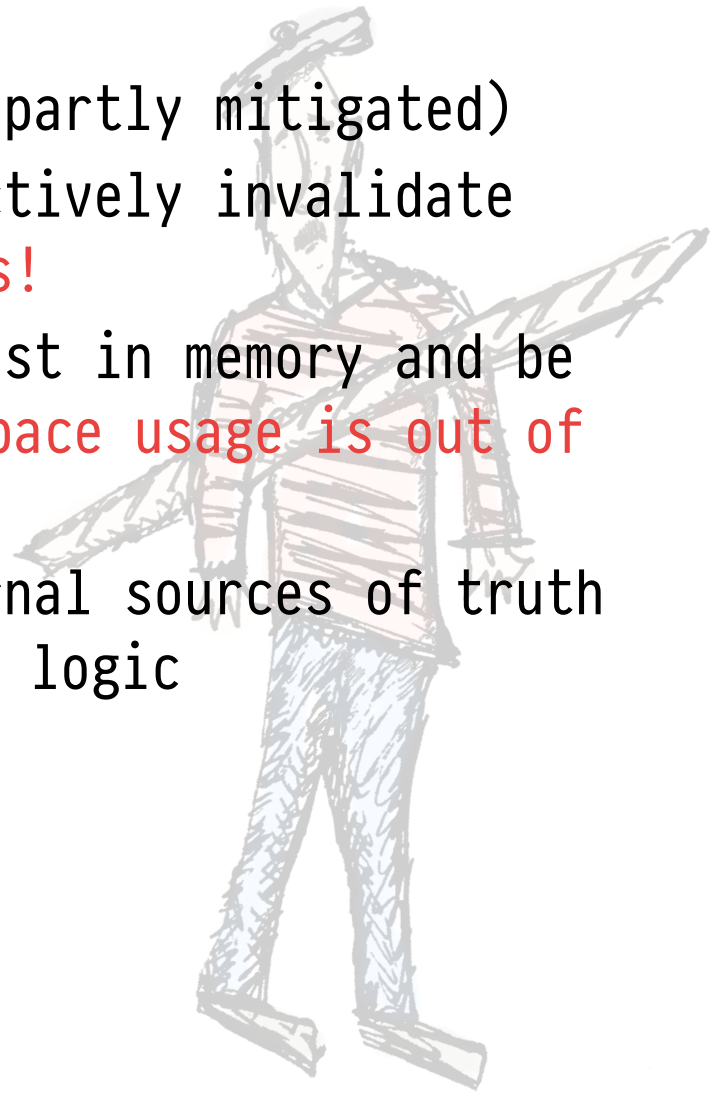

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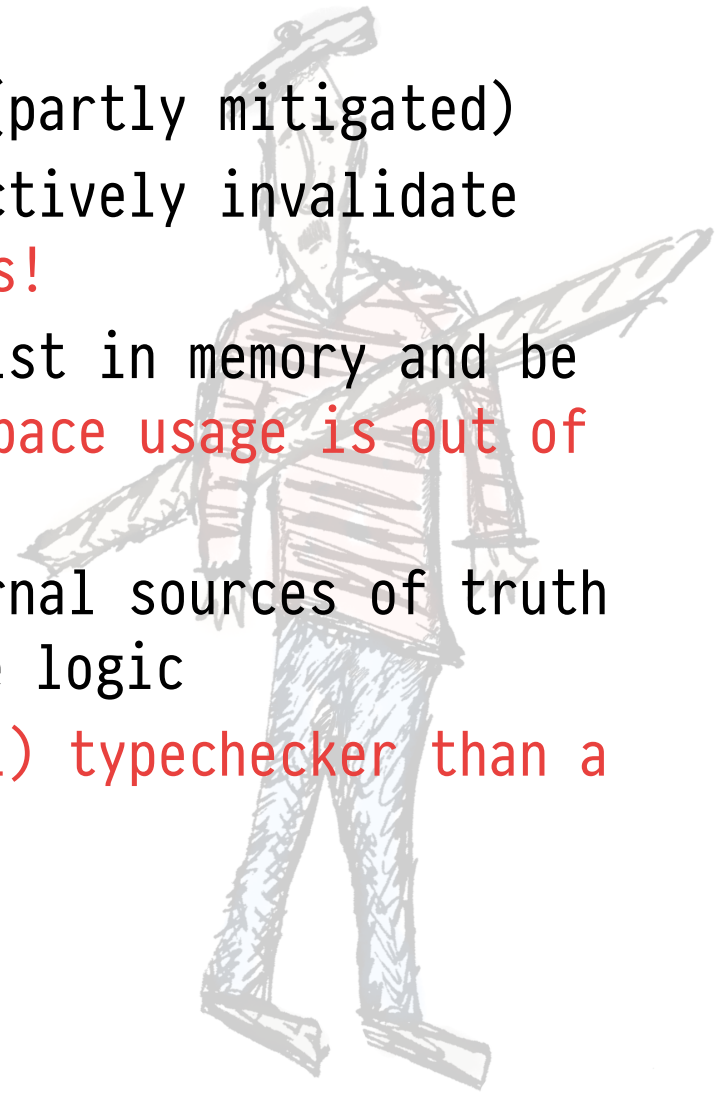

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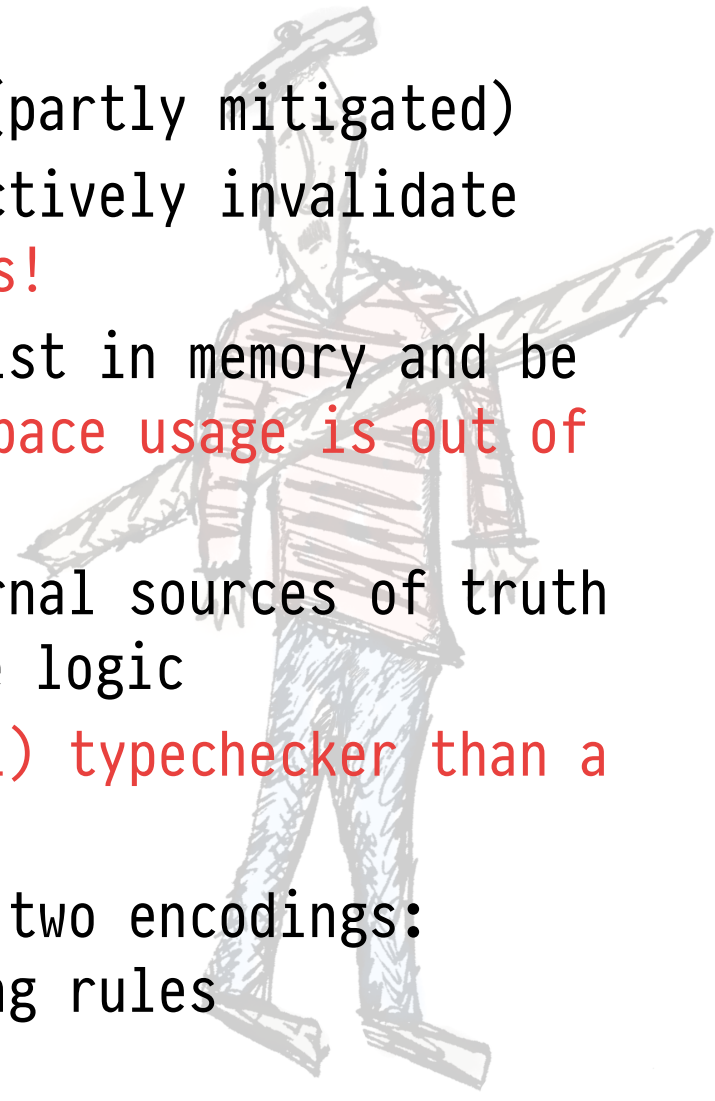

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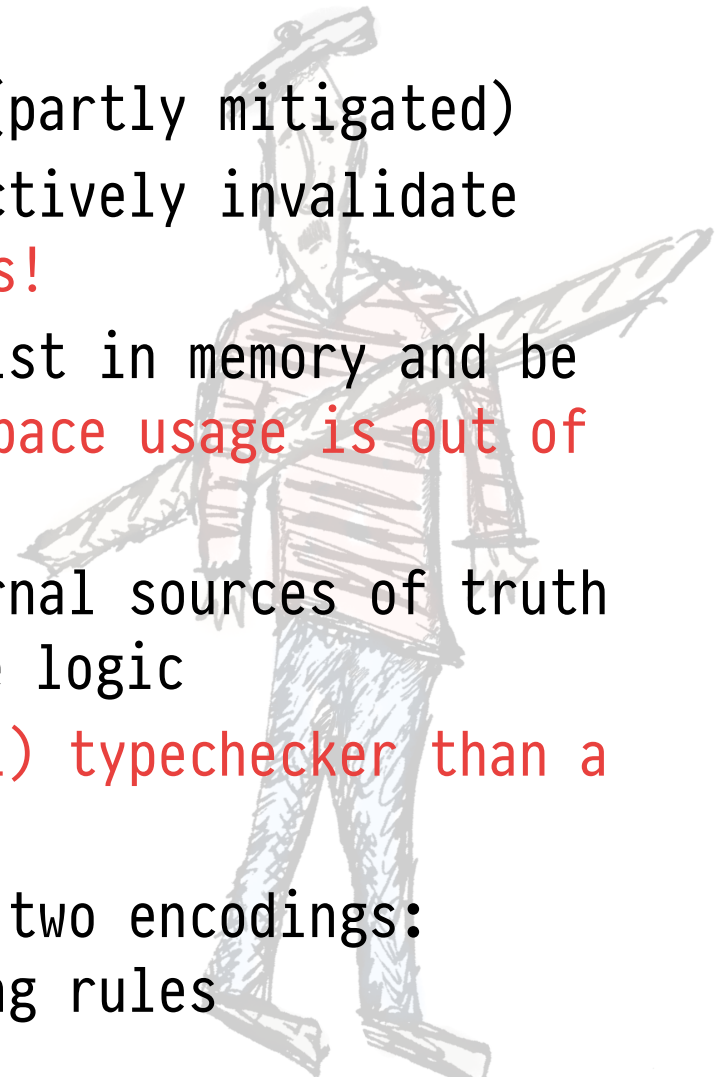

RESIST FRENCH IMPERIALISM AND UPHOLD ROBIN MILNER THOUGHT!

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COO

RedPRL: dependent refinement

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- **Metavariables can only be resolved *locally***, by refinement rules (NOT UNIFICATION).
- Adds nothing to the object logic: just a means of incremental construction / refinement.
- Precisely what is needed to encode existential instantiation, bidirectional typing rules.

remark on unification

Resolving existential variables via unification is so much fun! **But it induces non-local soundness conditions for a refiner** (very sad!).

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- ⋆ Works out fine in Coq because the refinement rules do not need to be sound.
- ⋆ Unification must be integrated as a judgment in your theory, not as part of a refinement framework. See Cockx/Devriese/Piessens ICFP 2016.

from classic lcf to *dependent lcf*

$$\frac{\begin{array}{l}\mathcal{J}_0 \rightsquigarrow \mathfrak{X}_0 \\ \mathcal{J}_1 \rightsquigarrow \mathfrak{X}_1 \\ \vdots \\ \mathcal{J}_n \rightsquigarrow \mathfrak{X}_n\end{array}}{\mathcal{J} \rightsquigarrow [\mathfrak{X}_0, \ldots, \mathfrak{X}_n].M}\ \texttt{my-rule}$$

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dependent Lcf: examples

Enables a straightforward encoding of sophisticated dependent refinement rules which are not expressible in Classic LCF or Nuprl.

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wow!!

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use *dependent lcf* today!

Implemented as a modular **Standard ML** library, which you can use in your own project today!

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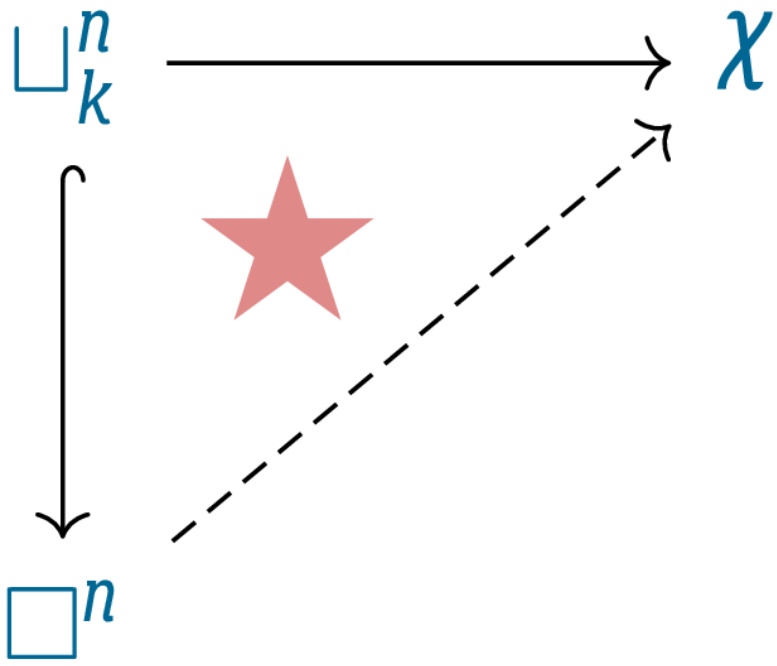
Restricts automatically to **Classic LCF** when instantiated without dependency/substitution structure.

RedPRL's cubical type theory



CUBICAL THOUGHT IS THE NEVER-SETTING SUN!

RedPRL's cubical type theory

Computational Higher-Dimensional Type Theory
[Angiuli/Harper/Wilson POPL 2017]

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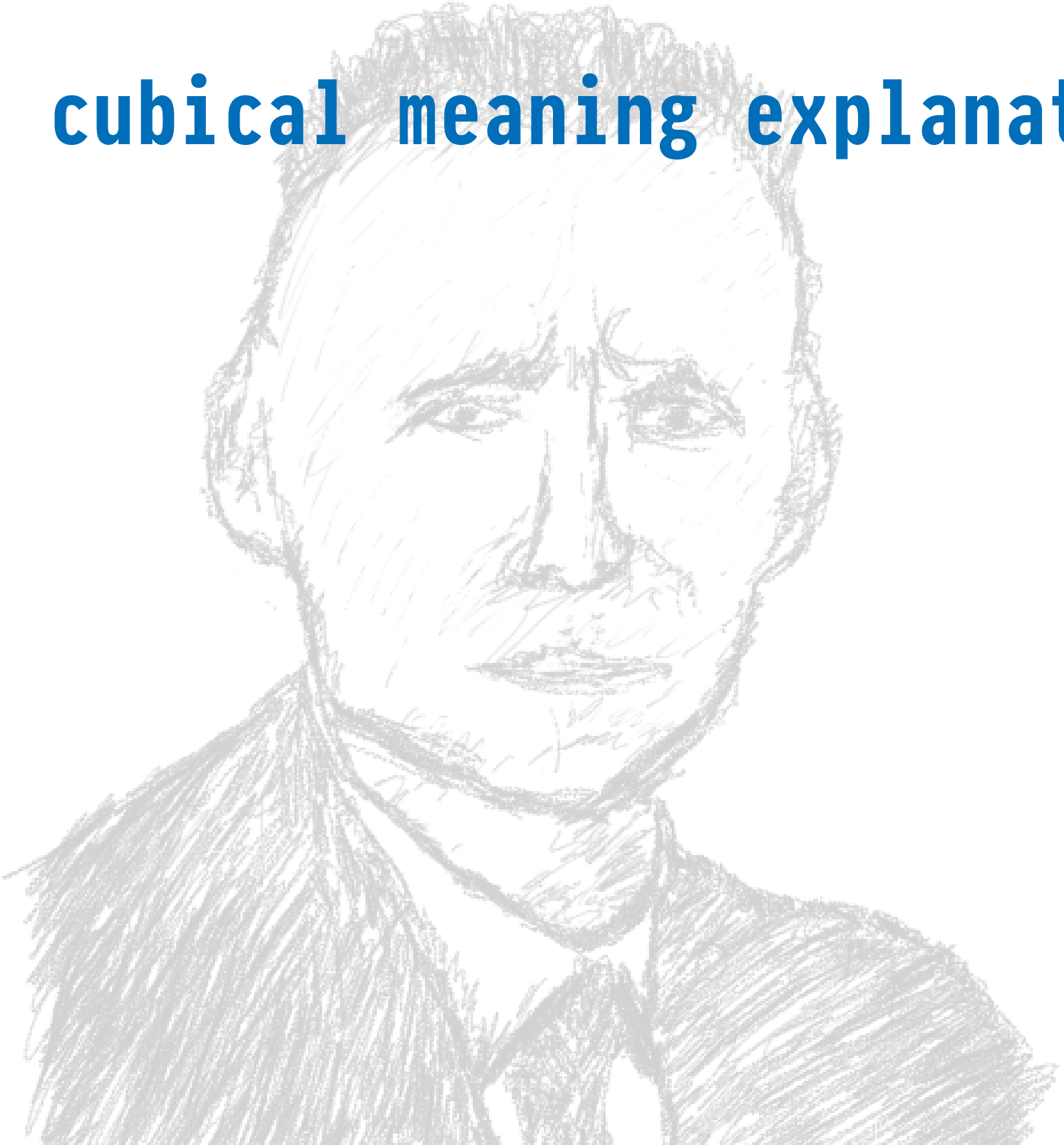
- ⋆ an instance of univalence, not_χ.
- ⋆ a deterministic and type-free operational semantics, amenable to cost analysis.

cubical meaning explanation



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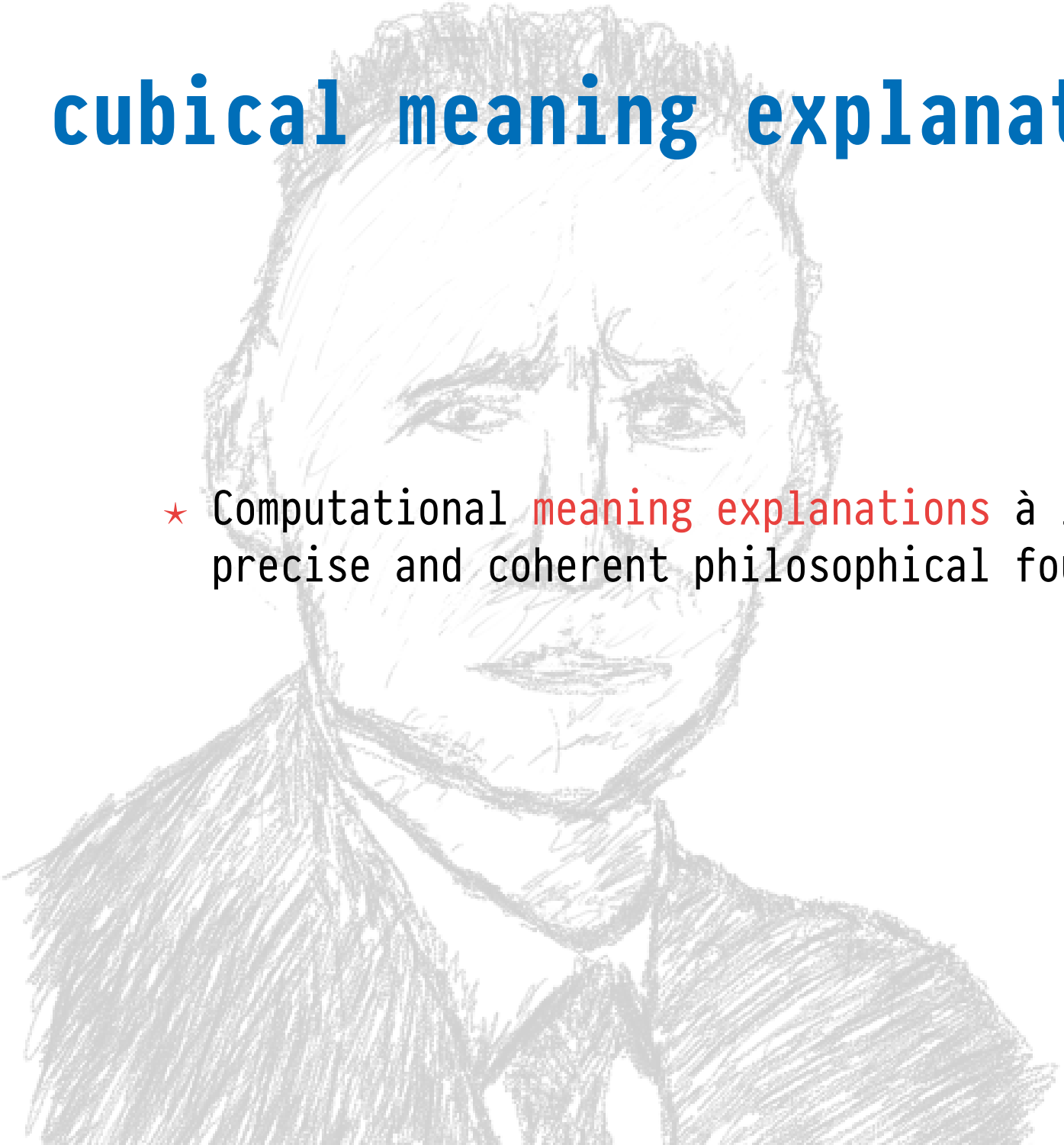

- ⋆ Computational meaning explanations à la Martin-Löf: precise and coherent philosophical foundation.

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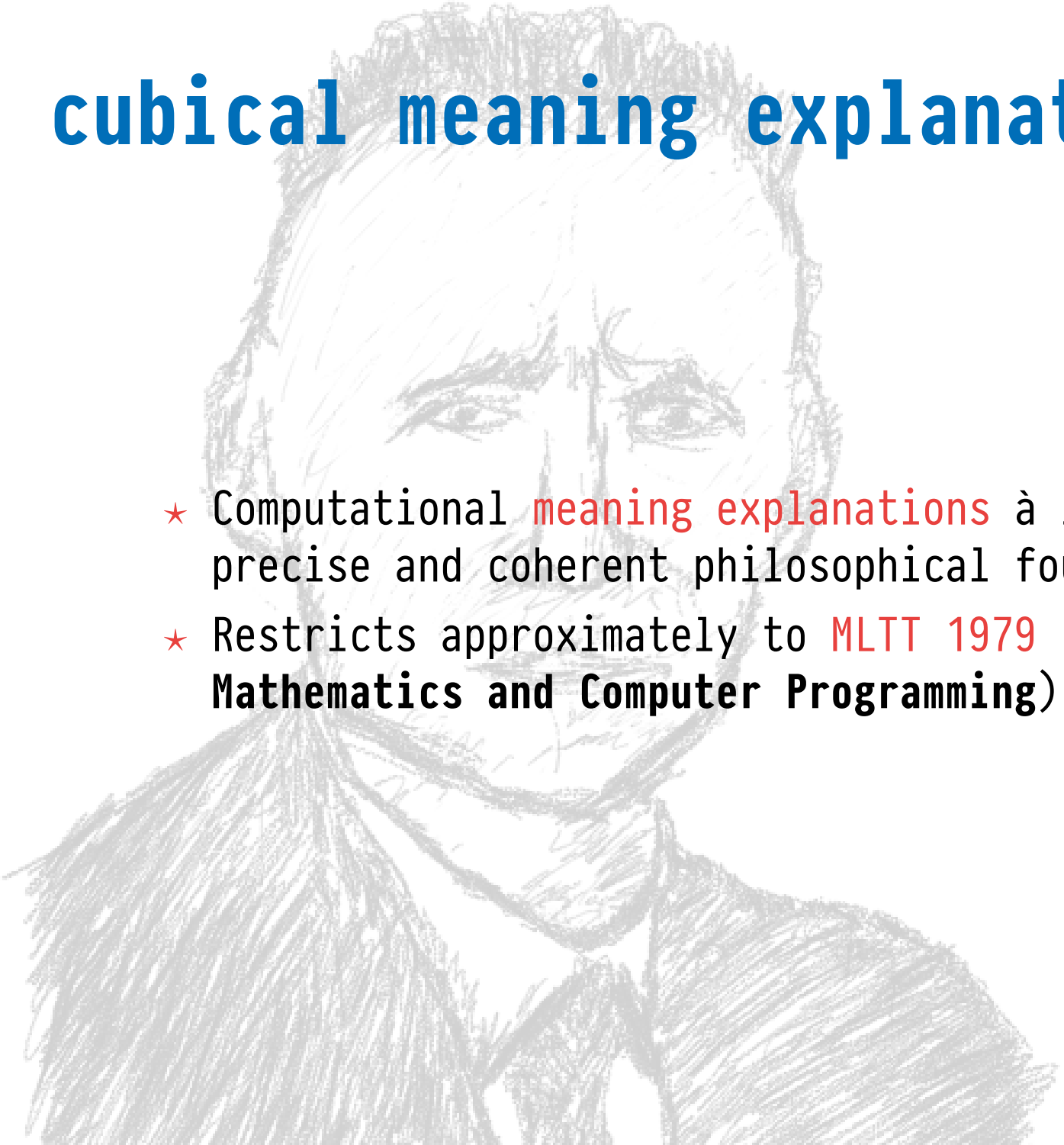

- Computational meaning explanations à la Martin-Löf: precise and coherent philosophical foundation.
- Restricts approximately to MLTT 1979 (**Constructive Mathematics and Computer Programming**) at dimension 0.

future work

- rule calculus
- $\Rightarrow$ MetaPRL-style derived rules
- univalence principle
- computational effects: gensym, exceptions, store, IO
- continuity principles, bar induction


COO