THE PEOPLE'S REFINEMENT LOGIC

RedPRL

nuprl family & friends

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THE PRL FAMILY IS THE VANGUARD OF THE PEOPLE!

nominal abstract binding trees

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- ⋆ introduced by Harper in Practical Foundations for Programming Languages

computational type theory

Set-theoretic semantics:

$$\mathcal{J} \in \mathbf{SET} ::= \left|\begin{array}{l} A = B\ \mathit{type} \\ M = N \in A \\ M \preccurlyeq N \\ \ldots \end{array}\right. \qquad \text{(categorical judgment)}$$

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(categorical judgment)

$$\mathcal{J}_2 \ (\mathcal{J}_1) \triangleq {\mathcal{J}_2}^{\mathcal{J}_1}$$

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$$|_{\mathfrak{x}:\tau} \ \mathcal{J}[\mathfrak{x}] \triangleq \prod_{m \in \tau} \mathcal{J}[m]$$

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(sequent judgment)

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Sheaf-theoretic semantics, let $\mathbb{I}[\mathcal{S}] \triangleq \left((\mathbb{I} \downarrow \mathcal{S})^{\mathrm{o}}, J_{\mathtt{atomic}}\right)$:

$$\mathcal{J} \in \mathbf{Sh}(\mathbb{I}[\mathcal{S}]) ::= \left| \begin{array}{l} A = B \ \mathit{type} \\ M = N \in A \\ M \preccurlyeq N \\ \cdots \end{array} \right.$$ (parametric judgment)

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(*parametric* judgment)

$$\Upsilon \Vdash \mathcal{J}_2 \ \ (\mathcal{J}_1) \triangleq [\mathbf{y}^{\mathsf{o}}(\Upsilon) \times \mathcal{J}_1, \mathcal{J}_2]$$

(hypothetical judgment)

$$\Upsilon \Vdash |_{\text{ъ}:\tau} \ \mathcal{J}[\text{ъ}] \triangleq \left\{ s \in \left[\mathbf{y}^{\mathsf{o}}(\Upsilon) \times \tau, \oint \mathcal{J}\right] \mid \pi_{\mathcal{J}} \circ s = \mathsf{id} \right\}$$

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DECISIVELY SMASH THE FORMALIST CLIQUE!


PRL
WE CAN PROVE IT

dependent refinement

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Nupri, MetaPRL, JonPRL:

$$\frac{H, x : A \gg (B[x] \in \mathbb{U}_i)\ \mathit{true} \rightsquigarrow \lfloor \star \rfloor \qquad \begin{array}{l} H \gg (M \in A)\ \mathit{true} \rightsquigarrow \lfloor \star \rfloor \\ H \gg B[M]\ \mathit{true} \rightsquigarrow \lfloor N \rfloor \end{array}}{H \gg \sum_{(x \in A)} B[x]\ \mathit{true} \rightsquigarrow \lfloor \langle M, N \rangle \rfloor}\ \textstyle\sum\text{-intro}\{M, i\}$$

No general refinement rules expressible for $\sum$-introduction, $\prod$-elimination ("constructible subgoals property")

dependent refinement

RedPRL:

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Dependent LCF allows subgoals to depend on the syntheses of previous subgoals, enabling <u>bidirectional refinement</u>.

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CRITICIZE THE OLD WORLD AND BUILD A NEW WORLD WITH JON STERLING THOUGHT AS A WEAPON!

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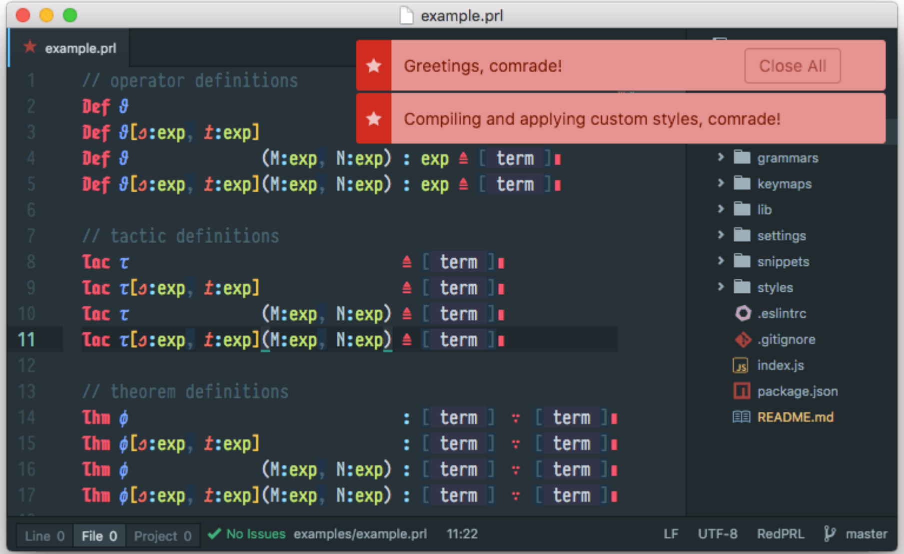
OPEN-SOURCE CONTRIBUTORS ARE THE SHOCK TROOPS OF THE REVOLUTION!


example.prl
example.prl
Greetings, comrade!
Close All
Compiling and applying custom styles, comrade!
// operator definitions
// tactic definitions
// theorem definitions
grammars
keymaps
lib
settings
snippets
styles
.eslintrc
.gitignore
index.js
package.json
README.md
Line 0
File 0
Project 0
No Issues
examples/example.prl
11:22
LF
UTF-8
RedPRL
master


COQ