HW2

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Problem 1

Calculate

$$\binom{8}{4}, \binom{10}{8}, \binom{15}{3}$$

$$\begin{aligned}\binom{8}{4} &= \frac{8!}{(8-4)!4!} = \mathbf{70}\\ \binom{10}{8} &= \frac{10!}{(10-8)!8!} = \mathbf{45}\\ \binom{15}{3} &= \frac{15!}{(15-3)!3!} = \mathbf{455}\end{aligned}$$

Problem 2

There are 8 apartments for 6 people. Each person chooses one apartment, and each apartment can host no more than one person. How many choices?

$$\begin{aligned}\text{Number of Choices} &= 8 * 7 * 6 * 5 * 4 * 3\\ &= \mathbf{20160}\end{aligned}$$

Problem 3

We need to choose a committee of six people: three French and three Germans, out of six French and seven Germans. How many ways?

$$\begin{aligned}\text{3 French} &= \binom{6}{3} \rightarrow (A)\\ \text{3 German} &= \binom{7}{3} \rightarrow (B)\\ Total &= (A) * (B)\\ &= \mathbf{700}\end{aligned}$$

Problem 4

Using the binomial theorem, expand the brackets and compute all coefficients in $(1-2x)^5$

$$(1-2x)^5 = \sum_{n=0}^{5} \binom{5}{n} (-2x)^{5-n}(1)^n = \mathbf{-32x^5 + 80x^4 - 80x^3 + 40x^2 - 10x + 1}$$

Problem 5

A collateralized debt obligation (CDO) is backed by 10 subprime mortgages. Five of them are from California, each of which defaults with probability 50%. Three mortgages are from Florida, each of which defaults with probability 60%. Two mortgages are from Nevada, each defaults with probability 40%. A senior tranch in this CDO defaults only if all of these mortgages default. Find the probability that the senior tranch does not default in the following cases:

(a) all independent;

$$\begin{aligned}
P(\text{Senior from California doesn't default}) &= 1 - 0.50 \\
&= 0.50 \\
P(\text{Senior from Florida doesn't default}) &= 1 - 0.60 \\
&= 0.40 \\
P(\text{Senior from Nevada doesn't default}) &= 1 - 0.40 \\
&= 0.60 \\
P(\text{Senior doesn't default}) &= .5^5 * .4^3 * .6^2 \\
&= \mathbf{0.00072}
\end{aligned}$$

(b) all mortgages from the same state default (or not default) simultaneously, but mortgages in different states are independent.

$$\begin{aligned}
P(\text{Senior doesn't default}) &= .5 * .4 * .6 \\
&= \mathbf{0.1200}
\end{aligned}$$

Problem 6

(SOA) An auto owner can purchase a collision coverage and a disability coverage. These purchases are independent of each other. He is twice as likely to purchase a collision coverage than a disability coverage. The probability that he purchases both is 15%. What is the probability that he purchases neither?

$$\begin{aligned}
D &= \text{Owner purchases disability coverage} \\
C &= \text{Owner purchases collision coverage} \\
&\text{C and D are independant} \\
P(C) &= 2P(D) \\
P(C \cap D) &= 0.15 \\
P(C \cap D) &= P(C)P(D) \\
0.15 = 2P(D)P(D) \rightarrow P(D) &= 0.2738 \\
P(C) = 2P(D) &= 0.5478 \\
P(D') = 1 - P(D) = 1 - .2738 &= 0.72614 \\
P(C') = 1 - P(C) = 1 - .5478 &= 0.45228
\end{aligned}$$

$$P(C' \cap D') = P(C')P(D') = 0.72614 * 0.45228 = \mathbf{.328}$$

Problem 7

Roll a die twice. Let A = first roll is even, B = sum of two rolls is 4. Find the conditional probability of A given B, and of B given A.

$$\text{Number of elements in sample} \rightarrow N(sample) = 36$$
$$A = \text{roll one is even}$$
$$B = \text{Sum of two rolls is 4}$$
$$N(A) = 18$$
$$N(B) = 2$$
$$P(A) = \frac{N(A)}{N(Sample)} = \frac{1}{2} \quad P(B) = \frac{N(B)}{N(Sample)} = \frac{1}{18} \quad P(A \cap B) = \frac{N(A \cap B)}{N(Sample)} = \frac{1}{36}$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \mathbf{\frac{1}{2}}$$
$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \mathbf{\frac{1}{18}}$$