

HW2

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Problem 1

Calculate

$$\binom{8}{4}, \binom{10}{8}, \binom{15}{3}$$

Problem 2

There are 8 apartments for 6 people. Each person chooses one apartment, and each apartment can host no more than one person. How many choices?

$$\begin{aligned}\text{Number of Choices} &= 8 * 7 * 6 * 5 * 4 * 3 \\ &= \mathbf{20160}\end{aligned}$$

Problem 3

We need to choose a committee of six people: three French and three Germans, out of six French and seven Germans. How many ways?

$$\begin{aligned}3 \text{ French} &= \binom{6}{3} \rightarrow (A) \\ 3 \text{ German} &= \binom{7}{3} \rightarrow (B) \\ \text{Total} &= (A) * (B) \\ &= 700\end{aligned}$$

Problem 4

Using the binomial theorem, expand the brackets and compute all coefficients in $(1 - 2x)^5$

Problem 5

A collateralized debt obligation (CDO) is backed by 10 subprime mortgages. Five of them are from California, each of which defaults with probability 50%. Three mortgages are from Florida, each of which defaults with probability 60%. Two mortgages are from Nevada, each defaults with probability 40%. A senior tranche in this CDO defaults only if all of these mortgages default. Find the probability that the senior tranche does not default in the following cases:

(a) all independent;

$$\begin{aligned}
P(\text{Senior from California doesn't default}) &= 1 - 0.50 \\
&= 0.50 \\
P(\text{Senior from Florida doesn't default}) &= 1 - 0.60 \\
&= 0.40 \\
P(\text{Senior from Nevada doesn't default}) &= 1 - 0.40 \\
&= 0.60 \\
P(\text{Senior doesn't default}) &= .5^5 * .4^3 * .6^2 \\
&= \mathbf{0.00072}
\end{aligned}$$

(b) all mortgages from the same state default (or not default) simultaneously, but mortgages in different states are independent.

$$\begin{aligned}
P(\text{Senior doesn't default}) &= .5 * .4 * .6 \\
&= \mathbf{0.1200}
\end{aligned}$$

Problem 6

(SOA) An auto owner can purchase a collision coverage and a disability coverage. These purchases are independent of each other. He is twice as likely to purchase a collision coverage than a disability coverage. The probability that he purchases both is 15%. What is the probability that he purchases neither?

Problem 7

Roll a die twice. Let A = first roll is even, B = sum of two rolls is 4. Find the conditional probability of A given B, and of B given A.