Summary of Xiaoyan Zhang's Master's Thesis

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22-2-2021

Introduction

The structure of the Rough Fractional Stochastic Volatility (RFSV) model is:

$$\frac{dS(t)}{S(t)} = \mu(t)dt + \sigma(t)dZ(t)$$

Where $\mu(t)$ is the drift term, $Z(t)$ is a standard Brownian motion, $X(t)$ is a Ornstein-Uhlenbeck process driven by fractional Brownian motion $W^H(t)$ given by:

$$dX(t) = -\alpha(X(t) - m)dt + \nu dW^H(t)$$

where $m \in \mathbb{R}$ and ν, α are positive constants, with Z and W^H correlated in general.

Summary of chapter 1

In chapter 1 there were 3 main parts

The first talked about some generic facts about volatility and modeling assets, both theoretical and practical aspects. One practical aspect which is sometimes neglected in theory is leverage - the more debt a firm has w.r.t. its earning, its volatility is going to be higher. In practice volatility also clusters and has a long memory.

The next part talked about how to actually find the volatility of a stock price. One method was using the derived from the Black & Scholes framework. This is inconsistent with practice because in practice the volatility is not constant.Another idea was time-dependent volatility model, which is still built upon B&S. It considers the dynamic of a stock price with time-dependent(yet deterministic) volatility:

$$\frac{dS(t)}{S(t)} = \mu dt + \sigma(t)dW(t)$$

Merton(1973) proposed the pricing formula using dependent volatility. If we use the annualized variance:

$$\sigma_a = \sqrt{\frac{1}{T-t}\int_t^T \sigma^2(\tau)d\tau}$$

And view σ_a as a constant dependent upon t and T, you can plug it into B&S and receive a formula which when solved gives you time-dependent volatility. Local volatility and mixture models were also mentioned in this part.

The last part talks about the following model: $\frac{dS(t)}{S(t)} = \mu_t dt + \sigma(t)dW(t)_1$

The stock price is driven by the wiener process W_1 and μ_t is the drift term. However the volatility $\sigma(t)$ now is also stochastic. Typically the volatility will be governed by a SDE which is driven by another Wiener process W_2 that is correlated with W_1. The author then described popular SDEs that are used with this model. For example:

- Scott(1987)

$dS(t) = \mu_1 S(t)dt + \sigma(t)S(t)^{\kappa} dW_1(t),$
$d\sigma(t) = \mu_2 \sigma(t)dt + \delta\sigma^{\beta}(t)dW_2(t)$
and that is the end of chapter 1.

Summary of chapter 2